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# Statistical methods in sterile neutrino experiments

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**Abstract.** We compare the current statistical methods applied to short-baseline neutrino oscillation experiments looking for light sterile neutrinos. The interpretation of the reconstructed confidence regions differs because of the different definition of the alternative hypothesis in the hypothesis test. When the alternative hypothesis extends over the whole physical allowed parameter space of the oscillation parameters, proper confidence regions are obtained both in case of a limit and a signal. Since the different definition of the alternative hypothesis prevents from a direct comparison of the results, a standardized analysis is proposed.

## 1. Introduction

After several experimental hints [1, 2, 3], a large number of experiments are currently looking for sterile neutrinos with a mass in the eV-range. Besides the gravitational force, sterile neutrinos can not take part in any standard model interaction and are hence not directly observable. Since the hypothetical sterile neutrino carries the mass  $m_4$ , it can mix with the mass eigenstates of the standard neutrinos with the masses  $m_{1,2,3}$  and contribute to the phenomenology of neutrino oscillations.

On the one hand, a sterile neutrino can introduce a transition from a neutrino with flavor  $\alpha$  to a neutrino with different flavor  $\beta$  ( $\alpha, \beta = e, \mu, \tau, \alpha \neq \beta$ ). This so-called appearance probability can be approximated by:

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sin^2(2\theta_{\alpha\beta}) \cdot \sin^2(1.27 \cdot \Delta m^2[\text{eV}^2] \cdot L[\text{m}]/E[\text{MeV}]). \quad (1)$$

On the other hand, the standard neutrino of flavor  $\alpha$  can oscillate into the sterile neutrino state and reduce the visible neutrino flux by the disappearance probability:

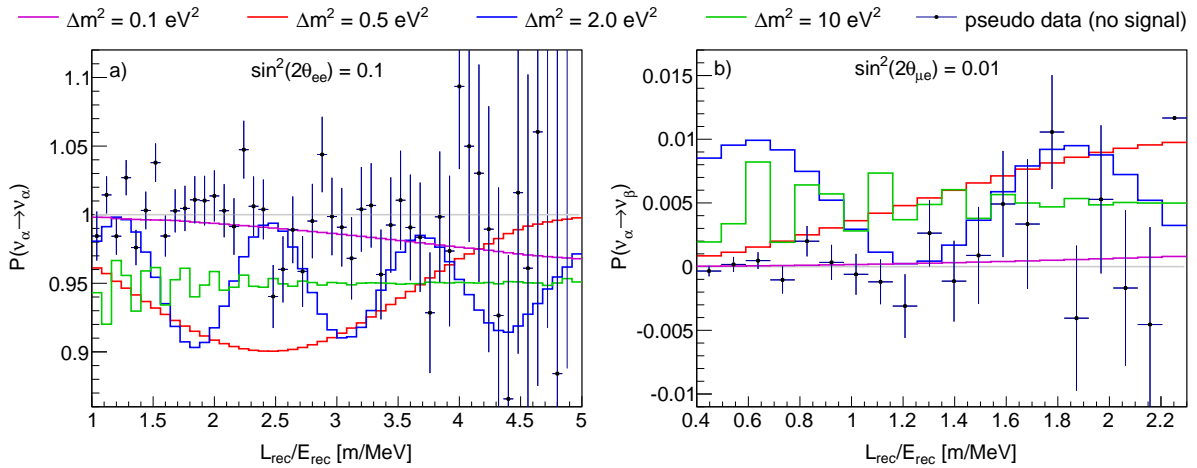
$$P_{\nu_\alpha \rightarrow \nu_\alpha}(L, E) = 1 - \sin^2(2\theta_{\alpha\alpha}) \cdot \sin^2(1.27 \cdot \Delta m^2[\text{eV}^2] \cdot L[\text{m}]/E[\text{MeV}]). \quad (2)$$

In both cases, the oscillation probability is a function of the travelled distance ( $L$ ) and energy ( $E$ ) of the neutrino. Measuring  $L$  and  $E$  allows to deduce the physical parameters  $\theta_{\alpha\beta}/\theta_{\alpha\alpha}$  (mixing angle) and  $\Delta m^2$  (difference between the squared mass values  $\Delta m^2 = m_4^2 - m_1^2$ ). Fig. 1 shows the reconstructed oscillation probability of a toy-disappearance/appearance experiment for different mixing angles and  $\Delta m^2$ -values. The mixing angle affects the amplitude of the oscillation and the mass value determines the oscillation frequency. For the experimentally motivated  $\Delta m^2$ -value of  $\sim 1 \text{ eV}^2$  the oscillation period  $(L/E)_{\text{osc}}$  is  $\sim 1 \text{ m/MeV}$  which could be directly observed through a detector with a reconstructed  $L/E$ -range of several m/MeV (e.g. a detector located only a few meters away from a reactor core emitting MeV-neutrinos). This oscillatory pattern



would be a smoking gun signature for sterile neutrinos and is hence the signal that appearance and disappearance experiments are looking for. The signature and the sensitivity of such an short-baseline experiment can be divided into three  $\Delta m^2$ -regions (see Fig. 1 and 2):

- $\Delta m^2 < 1 \text{ eV}^2 \rightarrow (L/E)_{\text{osc}} > 1 \text{ m/MeV}$ : the oscillation period is larger than the reconstructed  $L/E$ -range and can only partially be observed. The sensitivity decreases with smaller  $\Delta m^2$ -values.
- $\Delta m^2 \sim 1 \text{ eV}^2 \rightarrow (L/E)_{\text{osc}} \sim 1 \text{ m/MeV}$ : the oscillation period is within the reconstructed  $L/E$ -range and the oscillations can be resolved within the detector. This is the most sensitive  $\Delta m^2$ -region to sterile neutrinos.
- $\Delta m^2 > 1 \text{ eV}^2 \rightarrow (L/E)_{\text{osc}} < 1 \text{ m/MeV}$ : the oscillation period is smaller than the detector resolution. An overall increased/reduced rate can be measured. This results in a constant sensitivity for large  $\Delta m^2$ -values.



**Figure 1.** Reconstructed oscillation probability for a toy-disappearance (a) and appearance (b) experiment for different  $\Delta m^2$ -values. The data points correspond to pseudo-data from the no-signal hypothesis.

Although the experiments look for the same signal, the methods used to analyze the data vary from each other. Table 1 lists the short-baseline experiments that have already published results and groups it according to the same test statistic. In the following we will discuss the differences of the reconstructed confidence regions and their interpretations in terms of hypothesis testing.

## 2. Definition of the test statistic and the alternative hypothesis

The construction of a confidence region (CR) can be interpreted as the inversion of a family of hypothesis tests. Each hypothesis in the physical parameter space is tested and the set of accepted hypotheses constitutes to the CR. In a hypothesis test the tested hypothesis (null hypothesis  $H_0$ ) and the alternative hypothesis ( $H_1$ ) have to be defined. The outcome of a hypothesis test is either to accept  $H_0$  or to reject it in favor of  $H_1$ . This results in accepting at least one hypothesis out of  $H_0$  or  $H_1$ . The dimensionality of  $H_1$  defines the dimensionality of the CR. For example a two-dimensional alternative hypothesis leads to a two-dimensional CR [9].

To decide whether to accept or reject a hypothesis, a test statistic, a real-valued one-dimensional number, is introduced. This test statistic indicates how likely it is that the data

**Table 1.** Test statistics used in current short-baseline experiments. The definition of the alternative hypothesis with its free parameters is given in the second and third column. The associated techniques and the experiments using it are shown in the fourth and fifth column.

test statistic	alternative hypothesis $H_1: \{\sin^2(2\theta), \Delta m^2\}$	number of free parameters	associated techniques	experiments
$T_2$	$0 \leq \sin^2(2\theta) \leq 1$ $\Delta m^2 \geq 0 \text{ eV}^2$	2	2D/global scan global p-value	LSND[2], MiniBooNE[4] PROSPECT[5]
$T_1$	$0 \leq \sin^2(2\theta) \leq 1$ $\Delta m^2 = Y$	1	raster scan local p-value	NEOS[6] STEREO[7]
$T_0$	$\sin^2(2\theta) = 0$ $\Delta m^2 = 0$	0	Gaussian $CL_s$	DANSS[8]

comes from  $H_0$  and not from  $H_1$ . This can be achieved using a likelihood ratio test statistic:

$$T = -2 \log \left( \frac{\mathcal{L}(H_0)}{\max(\mathcal{L}(H_1))} \right) \quad (3)$$

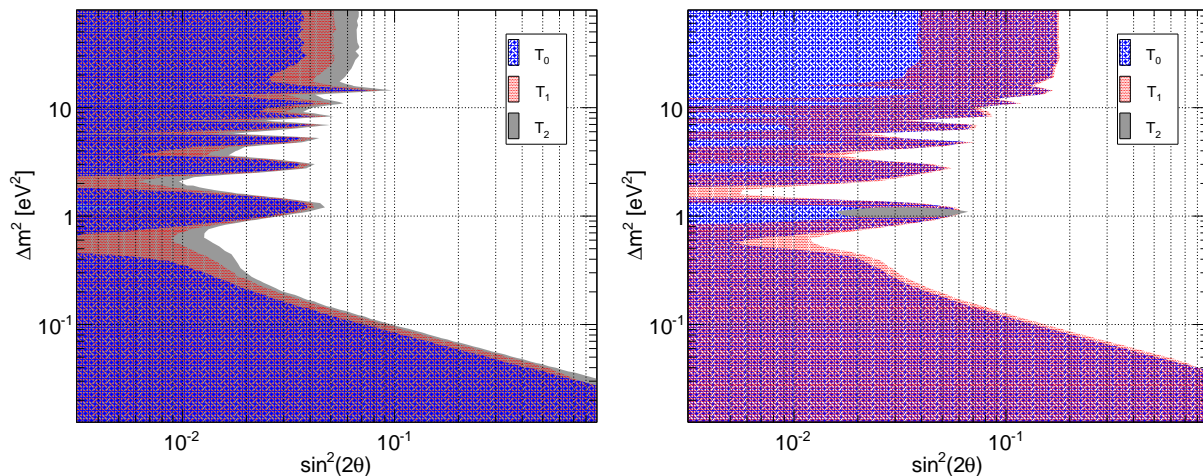
where  $\mathcal{L}$  is the likelihood function.<sup>1</sup> The denominator is the maximum of the likelihood function in the defined alternative parameter space. Table 1 groups the used test statistics according to the definition of the alternative hypothesis. We found three different definitions:

- $T_2$ :  $H_1$  is defined by the physically allowed space of the mixing parameters and will naturally result into a two-dimensional CR. This test is agnostic towards the mixing parameters and is the natural choice in a sterile neutrino experiment at the moment (given the lack of predictive theories or measurements).
- $T_1$ :  $H_1$  fixes the  $\Delta m^2$ -value to the  $\Delta m^2$ -value of  $H_0$ . This test can be used when  $\Delta m^2$  is already known, e.g. by previous measurements or theoretical calculations. The one-dimensionality of  $H_1$  leads to a one-dimensional CR. Nevertheless, the test is used to obtain a two-dimensional region. This can be done by repeating the construction of the one-dimensional CR for each  $\Delta m^2$ -value. The two-dimensional region created through this test will contain all hypotheses that are describing the data best for each  $\Delta m^2$ -value.
- $T_0$ :  $H_1$  is the no-sterile neutrino hypothesis. This test can be used when there are predicted values for the mixing parameters. The obtained CR is only point-like and its construction needs to be repeated for each  $\sin^2(2\theta) - \Delta m^2$  - couple to obtain a two-dimensional region. Due to the definition of  $H_1$ , the test accepts all hypotheses more likely than the no-signal hypothesis. Since for the test of the no-signal hypothesis  $H_0$  and  $H_1$  are the same, the no-signal hypothesis cannot be tested and is always accepted. This is the reason why the test can only set a limit.

### 3. Comparison of reconstructed confidence regions

We study the differences between the test statistics with pseudo-data from a toy-disappearance experiment that is representative of the current reactor experiments. The toy-experiment has a flat energy spectrum between 2-7 MeV and can measure distances in the range between 7-10 m.

<sup>1</sup> Typically, experiments incorporate systematic uncertainties as nuisance parameters in the likelihood function and maximize the likelihood against them. To simplify the discussion, we focus only on the oscillation parameters. For the definition of the likelihood function used in this work we refer to Reference [10].



**Figure 2.** Confidence regions reconstructed by the different test statistics. The regions are computed from a pseudo-data sample of a toy-disappearance experiment assuming no signal (left) and a signal with  $\sin^2(2\theta) = 0.04$  and  $\Delta m^2 = 1\text{eV}^2$  (right).

The expected number of neutrino events for the no-signal hypothesis is  $10^5$  and the expected background is  $10^4$  events. The relative uncertainties are assumed to be 2%.

Fig. 2 shows the reconstructed confidence regions of the different test statistics for a pseudo-data sample from the no-signal and a signal hypothesis ( $\sin^2(2\theta) = 0.04$  and  $\Delta m^2 = 1\text{eV}^2$ ). The size of the confidence regions increases with the number of free parameters in  $H_1$ . Hence, the limits obtained with  $T_0$  and  $T_1$  are typically stronger than the limit obtained with  $T_2$ . In case of a signal,  $T_2$  is the only test statistic that can properly reconstruct the signal. The confidence regions of  $T_1$  and  $T_0$  will always contain some  $\sin^2(2\theta)$ -region at each  $\Delta m^2$ -value. This results from the non-standard construction of the confidence regions that are rather the union of multiple confidence regions. We obtain similar results for a pure rate and shape analysis as well as for a toy-appearance experiment (see Ref.[10] for a detailed study).

#### 4. Conclusions

We discussed the current methods applied to short-baseline experiments looking for light sterile neutrinos. Their difference can be traced back to the definition of the alternative hypothesis in the hypothesis test. This affects the interpretation of the obtained confidence regions. It would be beneficial for the field to converge to a standardized analysis using the test statistic  $T_2$ . Since statistical fluctuations can easily mimic a sterile neutrino signature, a MC construction of the test statistic distributions is mandatory. For more information we refer the reader to Ref. [10].

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