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Uncertainty modeling with the open source framework urbs

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ABSTRACT

The transition of the energy system to a renewable energy source based system requires methods on how to incorporate uncertainty in modeling the energy system. There are different approaches starting from mainly variation based approaches up to including stochastic programming.

For this work, a modified version of stochastic dual dynamic programming (SDDP) has been implemented into the open source framework urbs. The framework consists of a linear optimization for energy dispatch and expansion planning and has been extended to include uncertain inputs for volatile energy sources like wind or solar. Different paths on how much these sources are providing for the feed-in can be modeled by packing one or more time steps to so-called realizations with different probabilities. The solution algorithm itself is based on a modified Benders decomposition approach, which is adapted to the constraints specifically relevant for power system analysis. The relation of SDDP and Benders decomposition is used to overcome the exponential growth of variables typically involved in classic stochastic programming.

The novel approach is tested on a case study of Germany and shows how a more realistic economic dispatch can be calculated with a stochastic approach compared to a deterministic one.

1. Introduction

Today's energy systems face new challenges due to the integration of renewable energy sources. Instead of few large power plants, many smaller units are distributed over the country. System operators have to take these changes into account, especially as renewable energy sources bring more volatility into the system. Hence, methods on how to incorporate these uncertainties into system models are needed to provide more robust and reliant insights into the system operation.

There are many different approaches in how to include uncertainties in optimization [1] or more specific in context of power systems [2,3]. Typical examples include robust optimization [4,5], fuzzy programming and stochastic optimization [6]. Robust optimization is often referred to as too conservative, as it optimizes the worst-case of the modeled uncertainty by using an uncertainty set. The optimization itself is therefore highly dependent on a reasonable modeling of this uncertainty set and is, hence, subject to research [7,8]. Another important aspect in the research efforts for robust optimization lies within solving the bilinear problem resulting from reformulating the minimax problem into a pure maximization by using duality theory. Different approaches like reformulating the problem into a linear [9] or using special algorithms are the main proposed solutions [10].

Stochastic programming represents the uncertain parameter by a random variable [6,11]. For each so-called stage, the random variable can take several possibilities according to its probability. This has the disadvantage of computational complexity with an increased number of realizations of the random variable as the tree of possibilities grows exponentially over the stages.

For the later, stochastic dual dynamic programming (SDDP) has been developed to overcome the so-called *curse of dimensionality* resulting from formulating the stochastic optimization problem as its deterministic equivalent, where each possible state gets its own variable. The idea of SDDP is to approximate the expected costs of the later stages, also called *future costs* by hyperplanes, which are generated out of the dual variables of the later stages.

As the structure of the stochastic multi-stage problem and its SDDP algorithm resembles the general workings of Benders decomposition, we use this structural benefits by applying a novel cut generation developed for Benders decomposition to the SDDP algorithm. The general idea behind the new cut generation is explained and the resulting algorithm is applied to a case study for Germany.

To increase transparency, the method is implemented into an open-source framework *urbs* [12]. It is written in python and originally used to solve linear optimization problems for energy dispatch and expansion

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planning (c.f. [13,14]). The user can define the number of modeled regions and which processes per region exist or are allowed to be expanded. Furthermore, transmission lines between the regions, energy storage and other energy-specific features can be defined. The implementation of all these features is kept flexible, hence it is easily adaptable to applications including technologies like P2X [15].

Over the past years, many such frameworks for the context of energy system optimization have been published. One of the reasons for choosing urbs as basis for the implementation of SDDP lies within the fact, that no additional licenses for a program is needed to work with it, in contrast e.g. to Balmorel [16], where GAMS is the underlying language. As it has been pointed out for other open-source frameworks, the reproducibility and transparency they provide by publishing their code is important for acceptance in policy decisions [17,18]. Additionally to the code, the input data and all steps to process the input as well as the output data for analysis are published [19], too.

The structure of the paper is as follows: First, basics of SDDP and the general idea of the cuts will be explained. Section 3 focuses on the improved cut generation, while section 4 shows a practical application of the introduced method on a model of Germany.

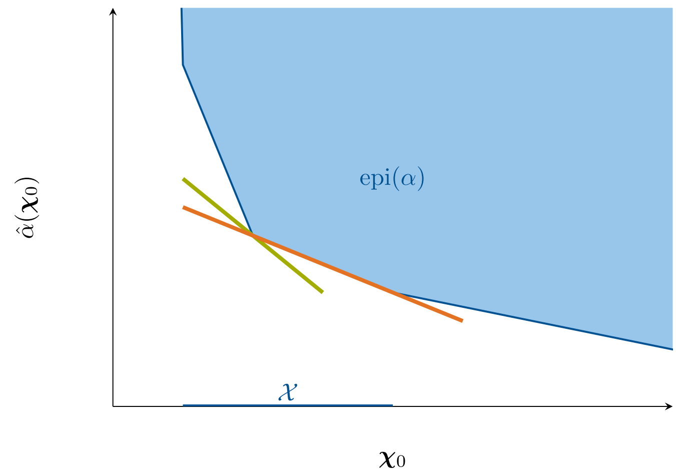

Fig. 1. Illustration of improved cut idea [25].

2. Problem formulation

For explaining our methodology, we use a general stochastic two-stage linear optimization problem:

$$\min_{\chi_0,\chi_{1r}} \boldsymbol{c}_0^{\mathrm{T}}\boldsymbol{\chi}_0 + \sum_{r=1}^{R} p_{1r}\boldsymbol{c}_1^{\mathrm{T}}\boldsymbol{\chi}_{1r} \tag{1}$$

$$\text{s.t. } \boldsymbol{A}_0\boldsymbol{\chi}_0 \geq \boldsymbol{b}_0 \tag{2}$$

$$\boldsymbol{E}_0\boldsymbol{\chi}_0 + \boldsymbol{A}_{1r}\boldsymbol{\chi}_{1r} \geq \boldsymbol{b}_{1r}\ \forall r \in \{1,\dots,R\} \tag{3}$$

$$\boldsymbol{\chi}_0, \boldsymbol{\chi}_{1r} \geq 0\ \forall r \in \{1,\dots,R\} \tag{4}$$

The variables of the first and second stage are denoted by χ_0 and χ_1, respectively. The second stage variable is dependent of the outcome of the first stage variable, hence, the stages cannot be easily decoupled. Uncertainty is included by realization $\boldsymbol{r}$ of the uncertain parameters $\boldsymbol{A}_{1r}$ and b_{1r}. In literature, this structure of uncertainty (only included in the matrix and right hand side parameter of the constraints), is said to have technology and objective constraints [20]. There are in total R realization, which the second stage can take.

This formulation for a stochastic problem results from expressing the – mostly continuous – probability of the uncertainty with help of a sampling as discrete probabilities. We will not further go into the details about the sample average approximation, as many papers and books focus on that method [6,11] and other methods on how to create respective scenarios [21,22].

We will keep the basic SDDP approach, introduced by Pereira and Pinto [23], which is basically a Benders decomposition or L-shaped method [6]. As the classic stochastic approach includes the exponential growth of variables due to realizations per stage, the idea of SDDP is to bundle same realizations of stages into one and hence, reduce the scenario tree. For this, mostly independence or at least an underlying Markov process is assumed, which is, of course, not always the case. However, we also require these assumptions for our algorithm as it is closely based on the original idea of the SDDP algorithm.

The idea of the algorithm introduced by Pereira and Pinto is to split the problem into its stages and approximate the later stages by a so-called *future cost function* α. The first stage reads as:

$$\min_{\chi_0}\ \boldsymbol{c}_0^{\mathrm{T}}\boldsymbol{\chi}_0 + \alpha(\boldsymbol{\chi}_0) \tag{5}$$

$$\text{s.t. } \boldsymbol{A}_0\boldsymbol{\chi}_0 \geq \boldsymbol{b}_0 \tag{6}$$

$$\boldsymbol{\chi}_0 \geq 0, \tag{7}$$

while the secoond stage represents the future costs function:

$$\alpha(\boldsymbol{\chi}_0) = \min_{\chi_{1r}} \sum_{r=1}^{R} p_{1r}\boldsymbol{c}_1^{\mathrm{T}}\boldsymbol{\chi}_{1r} \tag{8}$$

$$\text{s.t. } \boldsymbol{A}_{1r}\boldsymbol{\chi}_{1r} \geq \boldsymbol{b}_{1r} - \boldsymbol{E}_0\boldsymbol{\chi}_0 \tag{9}$$

$$\boldsymbol{\chi}_{1r} \geq 0\ \forall r \in \{1,\dots,R\} \tag{10}$$

Instead of calculating every possible outcome to get a representation for the future cost function α, an approximation is created by generating cuts from the solution of the dual problem of the second stage:

$$\alpha \geq \sum_{r=1}^{R} p_{1r}\boldsymbol{\gamma}_{1r}^{*,T}(\boldsymbol{b}_{1r} - \boldsymbol{E}_0\boldsymbol{\chi}_0) \tag{11}$$

The dual variables γ_{1r} are the dual variables corresponding to constraints (9). One problem of this particular representation lies within the extension to the multistage case as noted by Velásquez et al. [24], as for every new cut, another dual variable will be added and hence, the cut will become longer and longer. With the improved cut we use derived for Benders decomposition, this problem can easily be avoided. We will show not only this benefit, but the main idea behind the cut generation and apply it to an optimal dispatch problem for an energy system.

3. Improved SDDP algorithm

The main improvement of our SDDP algorithm lies within the used cut generation based on Benders decomposition cuts, which ensure a faster convergence of the algorithm as shown by Stursberg [26]. The motivation behind finding a "good" cut is visualized in Fig. 1. Both, the green and orange cut touch the epigraph of the future cost function $\hat{\alpha}$. As the orange cut not only touches the set in one point, but a facet, the cut orange is a better cut than the green cut.

The cut generation is nothing else, than finding a hyperplane which touches the surface of the epigraph of the future cost function, hence, the underlying problem results in a geometric problem: Finding a hyperplane which separates the current solution from the solution space, while touching most of surface. The generated cut is dependent on the problem formulation. Brandenberg and Stursberg [25,26] explain the mathematical background. The underlying idea is as follows: A cut which not only touches the epigraph in one point but a facet is better than the other cut. Therefore, these so-called *facet cuts* would be beneficial for a cut generating algorithm to get a better approximation of the

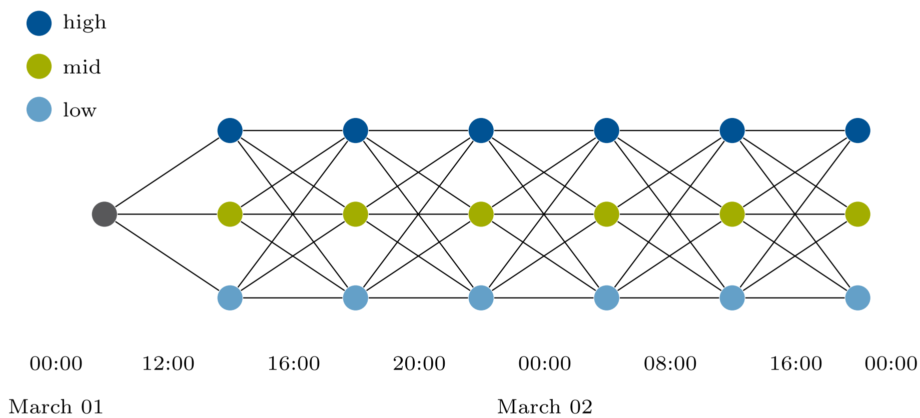

Fig. 2. Representation of implemented realizations.

future cost function. They also explain the relation to *Pareto-optimal* cuts [27], which dominate a cut with respect to the domain. In Fig. 1 the domain is indicated by $\mathscr{X}$.

Brandenberg and Stursberg [25] draw the relation between the alternative polyhedron and the reverse polar set based on findings by Cornuéjols and Lemaréchal [28] and Fischetti et al. [29]. The later set is of interest as it contains all possible normal vectors of separating hyperplanes, while the former set can be easily derived from the optimization problem. The separation problem can be reduced to finding the optimal vertex of the reverse polar set, as every vertex corresponds to a facet of the original set. As it cannot be derived easily, the relation between the reverse polar set and the alternative polyhedron is important: Stursberg [26] derives that for a certain problem formulation, the reverse polar set is a linear projection of the alternative polyhedron. Meaning that finding a vertex of the easily computable alternative polyhedron leads to a vertex of the reverse polar set under special conditions.

For this paper, using the method derived for Benders decomposition on a stochastic optimization problem the second stage problem formulation for the *r*-th realization results in:

$$\min_{\lambda_r,\chi_0,\chi_{1r}} \lambda_r \tag{12}$$

$$\text{s.t. } \boldsymbol{A}_{1r}\chi_{1r}+\boldsymbol{E}_0\chi_0 \geq \boldsymbol{b}_{1r} \tag{13}$$

$$\chi_0=\chi_0^*+\lambda\boldsymbol{\omega} \tag{14}$$

$$\boldsymbol{c}_{1r}^{\mathrm{T}}\chi_{1r} \leq \alpha^*+\omega_0\lambda \tag{15}$$

The difference to the above problem (8)-(10) lies in the relaxation with λ. Additionally, the first stage variable χ_0 is also optimized in the second stage, set with the relaxed constraint (14) to the optimal value of the first stage optimization. This constraint, while increasing the number of variables, also ensures that it is highly probable that a facet cut is calculated. Using equation (14), χ_0 could be substituted in equation (13) by the corresponding expression, eliminating all additional variables except λ. A corresponding substitution is typically also performed by the optimization algorithm used to solve the problem, making the computational cost of the additional variables negligible. Instead of directly optimizing the cost function, the relaxation parameter λ is minimized. The tuning parameters $\boldsymbol{\omega}$ and ω_0 are set to 1 for this paper like done by Fischetti et al. [29] with the aim of reducing the cardinality of the support of the optimal vertex. As proposed by Stursberg [26], additional information about the problem can be used to improve the choice of the parameters, or even to adapt them in each iteration. Evaluating the effect of different choices of the parameter in the context of SDDP would be an interesting topic for future research. For further implementations, these tuning parameters could also be optimized for each iteration according to Brandenberg and Stursberg [25]. Stursberg [26] shows in his thesis, that his relaxed approach speeds up convergence of Benders decomposition by a factor between 2 and 3. As a classic SDDP approach closely resembles a Benders decomposition, it is reasonable to assume an improvement of the convergence speed for SDDP as well.

With this relaxed formulation, the resulting and novel cut for SDDP reads as:

$$\sum_{r=1}^{R} p_{1r}\lambda_r^* \leq \sum_{r=1}^{R} p_{1r}\left(\gamma_{\omega r}^{\mathrm{T}}(\chi_0-\chi_0^*)+\gamma_{\alpha r}^{\mathrm{T}}(\alpha-\alpha^*)\right). \tag{16}$$

This formulation can easily be expanded for the multistage case as it has been done for the application of this paper.

3.1. Application

The presented novel approach for cut generation for SDDP is used for a short-term study of Germany represented by 17 regions (16 states + one offshore region). The goal of the study is to minimize system costs for the German power system while fulfilling a given demand and CO_2 targets. This minimization is conducted with a classic deterministic perfect foresight approach, meaning all inputs are known and mostly data from 2015, and the presented stochastic approach. The uncertain parameter in this study is the volatile wind production of the system. In urbs the available percentage of capacity is modeled by a time series. This so-called *capacity factor* is the uncertain parameter. With help of the study, differences in the results because of modeling choices will be explained.

The data is conducted openly [19] and processed as input for an extension of the open-source framework urbs [12]. The installed capacities of the power plants are given in the appendix (Table 2 its visualization in Fig. 7). As CO_2 emission bounds, official statistics of 2015 have been used [30]. This new implementation featuring the presented cut generation method and the SDDP algorithm in general is also published under an open license. The complete package (data + code) can be downloaded for further analysis and reproduction [31].

3.2. Uncertainty modeling

The first two days of March 2015 are taken as a time horizon in an hourly resolution for the optimization. These weeks are chosen because they both do not fall in neither extreme conditions like winter (nearly no PV) or summer (more than average PV production). Additionally, no major holidays fall in that time period which indicates a more regular power demand.

With the presented method also longer time periods can be modeled, however, more iterations might be needed and hence more time will pass until the method converges. This also highly depends on the chosen

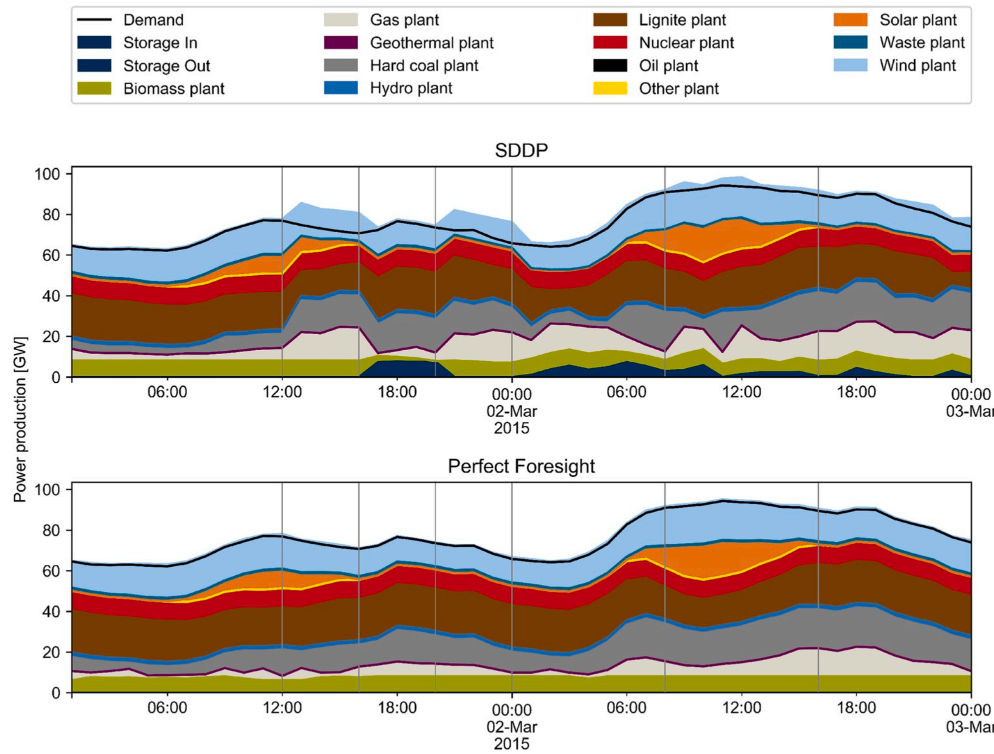

Fig. 3. Resulting economic dispatch in path *mid* with SDDP and perfect foresight approach. Vertical lines indicate modeling ranges.

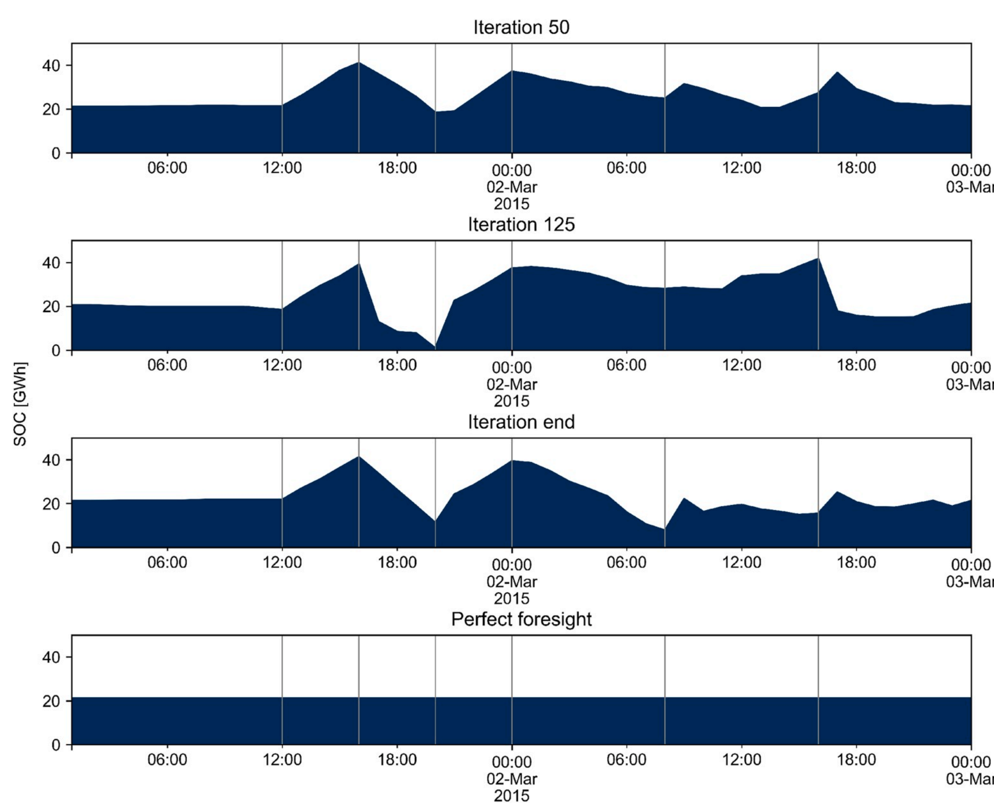

Fig. 4. State of charge over iterations for path *mid* with SDDP and perfect foresight. Vertical lines indicate modeling ranges.

Table 1
Costs for short term study scaled to one year in billion €

	Perfect foresight	SDDP	Worst-case
Cost	13.41	14.04	14.45

time ranges which are taken as one stage. As each stage can be calculated in parallel, more realizations for uncertain variables can be easily be incorporated. In this case, calculation times might vary due to more path options. This highly depends on the additional realizations which will be modeled, e.g. are they even more extreme realizations or just refining the probability space.

While for the first 12 hours the capacity factor time series for wind is taken as a certain prediction, the later hours can take three realizations *low*, *mid* and *high*. Realization *mid* is based on historical wind capacity factors for 2015, *low* and *high* are shifted time series based on the mean positive and negative deviations of the last 20 years from the 2015 time series.

The modeled time is not divided hourly but in certain ranges as illustrated by Fig. 2. The first 12 hours of the optimization horizon are assumed to be known (indicated by). The next 12 hours are represented by three 4 hour blocks which can take one of three possible realizations for the wind time series. The second day is then separated into three 8 hour blocks with the same three realization possibilities.

The presented formulation leads to a relaxation of several constraints: First, the state of charge at the overlapping time steps can be relaxed by λ. Second, the already emitted CO_2 is collected in a variable. The state of this variable is passed between subproblems and can hence be relaxed. At the end of the modeled time horizon the total emitted emissions has to be smaller than the user defined bound. In case of an expansion scenario, not only these two groups of variables can be relaxed, but also the installed capacities of power plants, storages and transmission lines.

3.3. Results

To analyze the results, it has to be clarified what conclusions can be drawn from the calculated case study. With the given setup, $3^6 = 729$ paths have been taken into account. As it is not sure, which path will happen, the main focus has to be on the first certain 12 hours. For comparison, the path which consists only of the realization *mid* is presented against a complete perfect foresight approach with the same inputs as the path *mid*. However, the dispatch of the shown SDDP result will hold for all possible paths.

Fig. 3 shows the dispatch for whole Germany for the complete time horizon of two days. Even in the predicted first 12 h, differences in the dispatch of the gas plants are clearly visible. As already mentioned, the shown result for SDDP only consists of the realization *mid*. As the first 12 hours of both approaches (SDDP and perfect foresight) are assumed as certain, it is interesting to see how the system behaves differently in both cases. For SDDP, the gas power plants are already dispatched more often. This is mainly due to the fact, that a lower wind realization would lead to higher CO_2 emissions and hence, a technology has to be chosen which does not emit as much CO_2 as for example a lignite power plant would. Additionally, regional differences have to be taken into account, as a lower wind realization has only small impact in a region with less wind installations than in a state which highly depends on wind (c.f. Table 2 and Fig. 7). From Fig. 3 it is also clearly visible, that the storage plays a vital role in ensuring a resilient system which can surpass low wind realizations easily.

This is even more visible in Fig. 4, which visualizes the state of charge over several iterations in contrast to the perfect foresight approach. Steep ramps indicate the need of relaxation between subproblems. While the iterations proceed, the relaxation of the state of charge between subproblems gets more and more tightened. Hence,

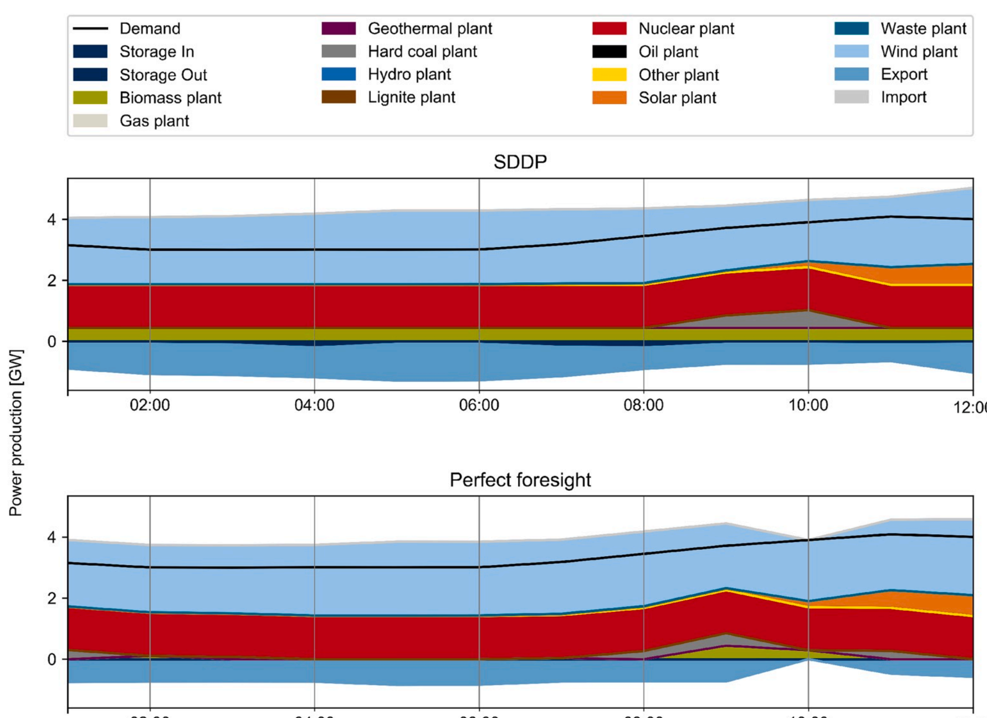

Fig. 5. Resulting economic dispatch in path *mid* with SDDP and perfect foresight approach for the first 12 h in Schleswig-Holstein.

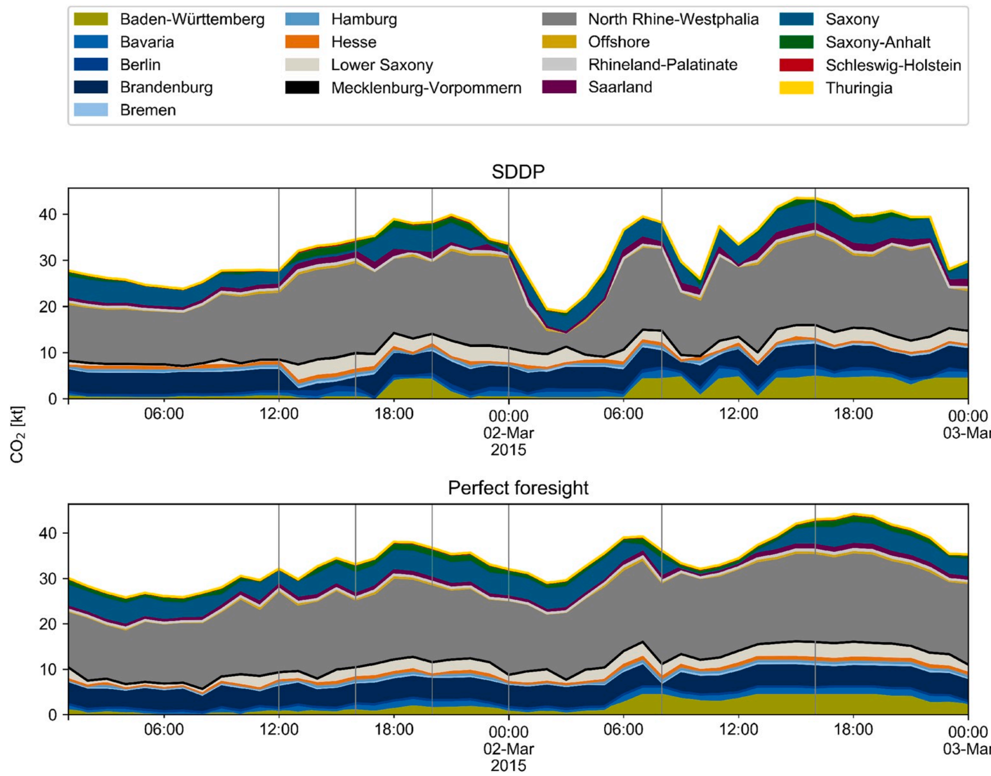

Fig. 6. CO_2 emissions per state for the SDDP and perfect foresight approach. Vertical lines indicate modeling ranges.

high ramps will vanish. In contrast to the perfect foresight approach, the storage is more often used in the SDDP case, as the system has to be able to deal with worse wind constellations.

Table 1 present the costs for three approaches scaled to one year. This is calculated by upscaling hourly costs like production and variable costs to one year. Investment costs are multiplied with an annuity factor calculated from the deprecation time of the installed capacities. More information on the cost structure in urbs can be found in the mathematical documentation [32]. Regarding the total costs, the SDDP approach has 4.7% higher expected costs compared to the perfect foresight approach, as it takes several paths into account which are worse. In contrast to a complete worst-case analysis, where the system would have been optimized for the worst-case path, the total system costs are 2.8% lower for SDDP. This worst-case analysis has been calculated by using the same approach as for the deterministic perfect foresight case, but changing the time series for the capacity factor to the *low* realization.

Comparing the values one might ask why an SDDP approach should be used compared to a deterministic one regarding the higher costs: the reason for this lies in the value of the insight into the system performance. The costs calculated by a perfect foresight approach do not meet realistic conditions as there is no perfect foresight and redispatch actions are taken every day to fulfill system requirements. Hence, using an SDDP approach gives more insight and a more realistic price than the deterministic perfect foresight approach.

A more regional analysis of the dispatch in the first 12 hours shows how SDDP changes the scheduling of some plants. Schleswig-Holstein, a state in the North of Germany with 4.7GW installed wind power, can be analyzed as an example: Fig. 5 visualizes how the biomass plant runs more evenly in the SDDP case, meaning that is runs all 12 hours and not only in the morning hours (8–11). More power is exported to neighboring states as there is a constant over-fulfillment of the demand. These observations, especially the usage of technologies with lower CO_2 emissions, can me made for several states and can be explained by regional CO_2 emissions: Not only the whole country but also each state wants to fulfill certain restrictions and hence, the whole system will act differently. The CO_2 emissions per region are presented in Fig. 6. It shows that coal relying states such as North Rhine-Westphalia have to reduce their amount of emitted CO_2 as they might violate their bound otherwise. States like Hesse on the other hand emit more CO_2 during the first 12 hours as the gas power plants in this region are used more often to minimize CO_2 emissions in neighboring states like North Rhine-Westphalia.

4. Conclusion

The paper presented two main contributions: A novel cut generation for the SDDP algorithm and a transparent case study for a short-term dispatch problem of Germany.

The cut generation, which was adapted from Benders decomposition, was explained and the implications on the problem formulation stated. Additionally, insight in how to implement the cuts have been presented. Furthermore, the implementation can be accessed as it is published under an open license [31].

To highlight how valuable a stochastic perspective on a typical dispatch problem can be, a case study on Germany was presented. The presented method was applied to a 48 hours short-term dispatch optimization problem of a 17-node representation of Germany. While not only the preparation of the input assumptions was published, also the processing of the output data is available online [19]. The case study shows how valuable it is to be able to react to low wind realizations and how this affects not only the storage usage but the dispatch of power plants as well. A stochastic modeling seems to be able to present the current situation more realistically than a perfectly planned optimizations, as the higher usage of the reserve market indicates.

Integrating renewables into the current energy system still pose a challenge to the community and policy makers. Studies like these can help operators to be able to face uncertain futures while staying cost optimal.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

The following table and figure show the installed capacities of the German electricity system.

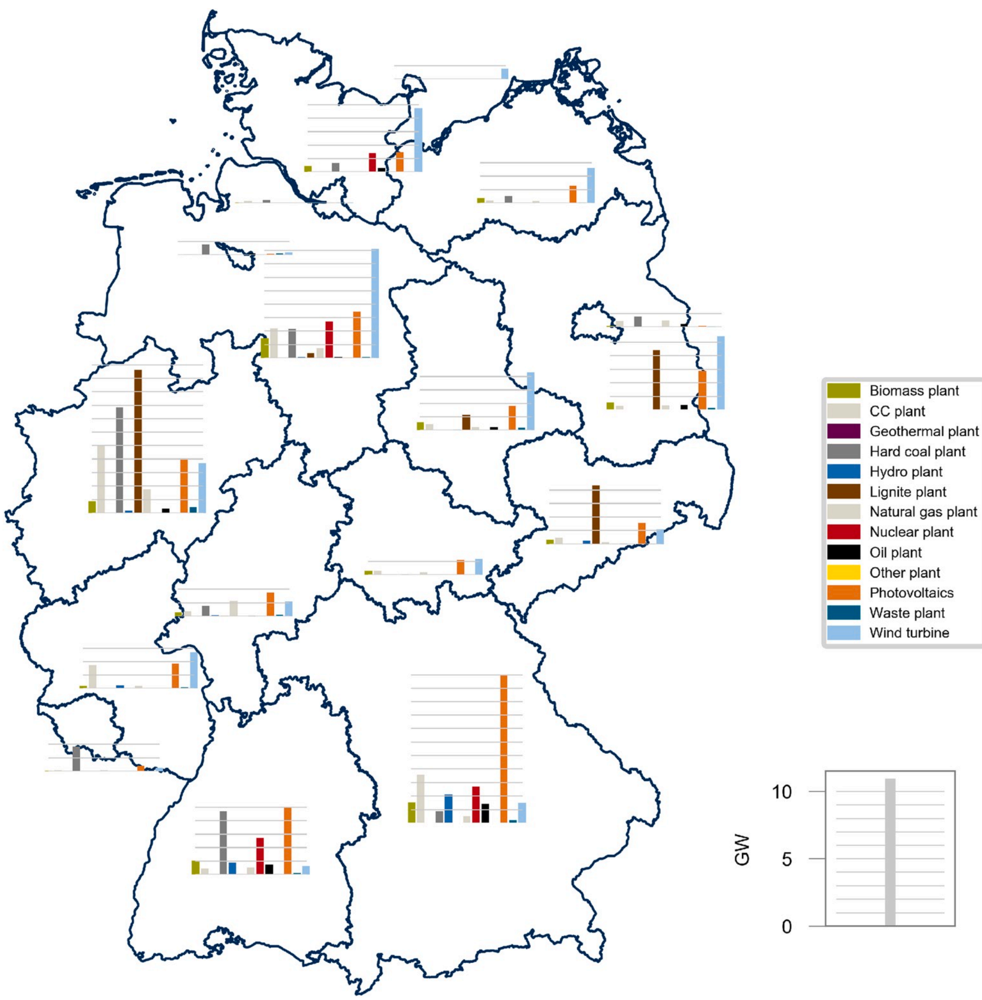

Fig. 7. Visualization of input data for process capacities in Germany per state (c.f. Table 2)

Table 2
Installed process capacities in Germany per state in MW[33,34].

Site Process	BW	BY	BE	BB	HB	HH	HE	NI	MV
Biomass	1,022	1,535	61	539	9	61	294	1,476	379
CC	434	3,568	444	282	15	127	345	2,174	183
Geothermal	1	25	0	0	0	0	0	0	0
Hard coal	4,667	847	777	0	772	194	753	2,162	514
Hydro	864	2,091	0	4	20	0	81	58	3
Lignite	0	0	0	4,409	0	0	34	352	0
Natural gas	518	487	467	301	0	22	1,114	701	136
Nuclear	2,712	2,698	0	0	0	0	0	2,696	0
Oil	702	1,384	218	334	86	0	25	56	0
Other	0	0	0	0	0	0	0	19	0
Solar	4,985	10,943	78	2,861	38	36	1,761	3,429	1,270
Waste	98	214	36	118	91	24	112	73	17
Wind	605	1,465	4	5,445	151	51	1,080	8,095	2,594
Site		NW	OF	RP	SL	SN	ST	SH	TH
Process									
Biomass		866	0	176	19	282	589	441	272
CC		4,968	0	1,728	75	440	449	0	291
Geothermal		0	0	8	0	0	0	0	0
Hard coal		7,827	0	13	1,822	0	0	680	0
Hydro		159	0	232	11	210	26	5	33
Lignite		10,618	0	0	0	4,325	1,136	0	0
Natural gas		1,714	0	185	58	113	238	22	177
Nuclear		0	0	0	0	0	0	1,410	0
Oil		332	0	0	0	17	212	320	0
Other		0	0	0	0	0	0	0	0
Solar		4,039	0	1,845	404	1,558	1,785	1,474	1,095
Waste		432	0	83	28	16	185	17	12
Wind		3,684	776	2,694	239	1,082	4,306	4,726	1,185

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