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Parameter Estimation for Incompressible Cake Filtration: Advantages of a Modified Fitting Method

There is a widely used linear strategy to determine the parameters specific cake resistance and filter medium resistance in incompressible cake filtration. In this article, it is intended to demonstrate that this strategy has some disadvantages and should be replaced by an alternative nonlinear approach which yields more exact results. Even though the gains in precision are small for most cases, the nonlinear strategy is favored because it involves no extra effort and is grounded in the same physical theory as the original approach. This claim is based on a broad simulation study using noisy data with known parameter values to compare both fitting strategies and judge their accuracies.

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1 Introduction

In cake filtration, important parameters are filter cake resistance and filter medium resistance. Resistance of cake and medium cause pressure drop in the fluid phase and, therefore, influence the filtration setup, such as choice of filter apparatus and suitable pumps as well as integration of filtration into the larger process. As long as filter cakes can be considered incompressible, these two parameters are indeed decisive and are the only descriptors to be determined by parameter fitting as all others are known from the experimental conditions [1,2]. Filter cakes can be considered incompressible depending on their material properties and the operation conditions. Material properties are, e.g., compressibility of the primary particles and friction between the particles [3].

Important operation conditions are flow rate or overall differential pressure, respectively, and maximal cake height. Compressibility effects become more pronounced when these variables increase. In case of compressible filter cakes, further parameters need to be determined that characterize the compression behavior [4,5]. Even though many substances can exhibit compressible behavior during filtration, also the assumption of incompressibility is often valid, especially for incompressible primary particles and moderate operation conditions. For this reason, we focus only on incompressible cake filtration in this article and, therefore, on the parameters filter cake resistance and filter medium resistance.

Additionally, it must be assured that the substance system considered exhibits pure cake filtration behavior, i.e., that all newly separated particles are captured on the cake surface only. If, on the contrary, also depth filtration occurs, i.e., small particles are separated within the already existing filter cake, the phenomenological behavior changes and other analysis tools

have to be used [6,7]. Depth filtration effects can be expected especially when very broad and possibly multimodal particle size distributions of the dispersed phase are encountered because in that case small particles can pass through the pores created by the larger particles and internally block the previously built-up cake. However, just as compression, this effect is neglected and only ideal cake filtration is considered here.

There is a relatively simple and widely used procedure to determine the parameters filter cake resistance and filter medium resistance, which is reported classically in the guideline VDI 2762, Part 2 [1]. The approach is also described in overview articles and established textbooks [1,8], used as a reference for validating alternative strategies [9], and is still employed in very recent publications [10–12]. It is also standard in industrial research. For this reason, the classical procedure must be seen as state-of-the-art.

Before more details are given on the general approach, a comment on the two characteristic parameters is in order. In most cases, the sought-for filter cake resistance is used as specific resistance. In some cases, it refers to filter cake mass, i.e., resistance per filter cake mass; sometimes it is expressed in relation to filter cake height, i.e., resistance per cake height. In the latter case, specific cake resistance is the inverse of the filter cake permeability. In the remaining article, we will rely on height-specific cake resistances; this does, however, not restrict the generality of the found results because both height- and

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mass-specific resistances can easily be converted into each other, as will be shown later on.

Additionally, it is important to note that the so-called medium resistance does not characterize the used filter medium or septum alone, but is characteristic for the used medium together with the first layers of separated substances. Additional resistance effects resulting from these first layers are, therefore, also called “interference resistance”, referring to the interference between medium and the separated dispersed phase [13]. The interaction of the filter medium with the particles to be separated is, e.g., discussed by Hund et al. [14]. In case of precoat filtration, the interaction was investigated by Rainer et al. [15,16]. Due to the fact that in classical experiments specific cake resistance is the decisive parameter because it is usually much larger than the medium resistance, including the interference effect [1], we will focus in the remaining article mainly on this variable.

Considering the experimental strategy to determine the sought-for parameters, a suspension is created with known mass concentration of the substance to be separated. This suspension is filtered in a laboratory filter cell, either in the mode of constant flow or constant pressure, where the latter is the dominant mode of operation because no process control system is needed to assure a constant flow rate while the overall flow resistance increases due to cake growth. For this reason, also constant pressure filtration is focused upon in this article. During the whole separation process, the accumulated liquid mass at the filter outlet is measured by an automatic scale. Using either the filter cake height at the end of filtration or the mass of separated matter together with the collected liquid volume, converted from the mass using the density, as a function of time, the parameters cake resistance and medium resistance can be calculated [1, 2].

In the next section, the mathematics behind this calculation procedure is described. So far, it is only noted that the already mentioned standard method is based on a linear representation of the measured data and a corresponding linear model formulation that is fitted to these data points [1, 2, 17]. In our opinion, such strategies were very helpful to evaluate experimental data manually on paper sheets; however, current computer tools make them obsolete. The aim of this article is to show that the said linear representation leads to a decreased accuracy in the parameter fitting procedure. Therefore, an alternative procedure is proposed. It is based on the same basic model equations, i.e., the same physical theory, only the data are not represented linearly and instead a root function is fitted to the raw data. It is shown that this latter fitting procedure, even though it does not cause any extra effort and is, as mentioned, based on the same established theory, leads to more accurate results.

It is worth mentioning that an analogous development took place in a completely different field of research, namely enzyme kinetics based on the Michaelis-Menten equation [18–20]. In this area, also first linear representations were used to determine the characteristic parameters. Afterwards, nonlinear fitting of algebraic equations was applied and still later the development shifted to the direct use of differential equations [21]. Whereas working with differential equations directly is not required in case of incompressible cake filtration as all equations

can be solved analytically, the analogy between enzyme kinetics and filtration holds for the decision between linear and nonlinear fits. As will be seen in Sect. 3, the quality of fits crucially depends on the distribution of errors. Accordingly, it was found in enzyme kinetics research that linear plots result in “error bars which are asymmetrical” [18] and “suffer from a highly biased weighting of points and should never be used.” [19].

2 Model Equations

Modeling of incompressible cake filtration is briefly recapitulated now. For deriving the general model equation, from which specific cake and medium resistance are derived, one starts by decomposing the overall pressure drop across the filter Δp ¹⁾ into the pressure drop across the cake Δp_C and across the medium Δp_M , i.e.:

$$\Delta p = \Delta p_C + \Delta p_M \quad (1)$$

For the cake, Darcy's law is used in the following form:

$$\Delta p_C = \frac{Q \mu H}{A k} = \frac{Q \mu H r}{A} \quad (2)$$

where Q is the volumetric flow rate, i.e., dV/dt , μ is the dynamic viscosity, A is the filter's cross-sectional area, k denotes the permeability, and r the height-specific resistance. Cake height H can be substituted and expressed by:

$$H = \frac{V K}{A} \quad (3)$$

with the factor K , often referred to as concentration constant [1], being defined as:

$$K = \frac{c}{(1 - \varepsilon)} = \frac{H A}{V} \quad (4)$$

and c being the volumetric concentration of impurities in the suspension; ε is the cake's porosity. Also, for the filter medium Darcy's law is applied. However, as the medium height does not play a role here, specific resistance and height are combined into the total resistance of the medium R_M , yielding:

$$\Delta p_M = \frac{Q \mu R_M}{A} \quad (5)$$

Putting these components together and writing dV/dt for Q yields:

$$\Delta p = \frac{r \mu K}{A^2} V \frac{dV}{dt} + \frac{R_M \mu}{A} \frac{dV}{dt} \quad (6)$$

Solving this differential equation by separation of variables and using the initial condition $V(t=0) = 0$ leads to:

$$t = \frac{r \mu K}{2 \Delta p A^2} V^2 + \frac{R_M \mu}{\Delta p A} V = P_2 V^2 + P_1 V \quad (7)$$

1) List of symbols at the end of the paper.

which can be rearranged to

$$\frac{t}{V} = \frac{r \mu K}{2 \Delta p A^2} V + \frac{R_M \mu}{\Delta p A} = P_2 V + P_1 \quad (8)$$

as traditionally used for determining the parameters r and R_M . As can be easily seen, in both equations, the parameters P_1 and P_2 have the same meaning, namely $P_1 = R_M \mu (\Delta p A)^{-1}$ and $P_2 = r \mu K (2 \Delta p A^2)^{-1}$. As μ , Δp , and A are known from the experimental conditions as well as the used setup, and the final cake height H , contained in K , can be determined after the experiment, R_M can be directly computed from P_1 and r from P_2 . However, the quadratic Eq. (7) can also be resolved for V , yielding the root function:

$$V = \frac{-P_1 + \sqrt{P_1^2 + 4 P_2 t}}{2 P_2} \quad (9)$$

The claim of the present article is that it makes a difference whether Eq. (8) or (9) is used for fitting it to the experimental data and determining R_M and r . Before moving on to the parameter fitting strategy, a remark is made on the specific resistance r . As mentioned, here r is considered as height-specific; therefore, also the final cake height H is included in the factor K . However, substituting $K_m = m/V$ for K in Eq. (4), where m is the filter cake mass at the end of filtration, the equation stays the same, only r takes the meaning of r_m , i.e., the mass-specific resistance [1, 2]. This was meant when we mentioned earlier that the height- and mass-specific form are equivalent and that we do not limit the generality of our approach due to the primary focus on height-specific filter cake resistances.

3 Parameter Estimation

In this section, the fitting strategy is described and some theoretical background on parameter estimation is provided. Regarding notation, the independent variable is denoted by x , the dependent variable by y . Measured data are marked using a hat sign, i.e., \hat{x} and \hat{y} , and data from the model f depend on the parameter vector P . In case of the usual least-squares strategy for N experimentally determined points, the cost function is formulated as:

$$J = \sum_{i=1}^N [\hat{y}_i - f(\hat{x}_i, P)]^2 \quad (10)$$

and the corresponding optimization problem is:

$$\min_P J(\hat{x}_i, \hat{y}_i, P) \quad (11)$$

Now, different strategies for parameter estimation are formulated; the first is based on the linear model of Eq. (8), the second on the nonlinear form of Eq. (9). In the remaining article, the two cases are referred to as Strategy 1 and 2, respectively. Strategy 1 is the classical approach known from the literature [1, 2]. Strategy 2 is the alternative method proposed in this paper. For Strategy 1, the cost function becomes:

$$J_1 = \sum_{i=1}^N \left[\left(\frac{\hat{t}_i}{\hat{V}_i} \right) - (P_2 \hat{V}_i + P_1) \right]^2 \quad (12)$$

i.e., a linear parameter fitting problem is encountered. Correspondingly, Strategy 2 yields the following nonlinear cost function

$$J_2 = \sum_{i=1}^N \left[\hat{V}_i - \left(\frac{-P_1 + \sqrt{P_1^2 + 4 P_2 \hat{t}_i}}{2 P_2} \right) \right]^2 \quad (13)$$

To assess the adequacy of the two fitting strategies proposed so far, some theoretical background is required. As mentioned, both Strategy 1 and 2 rely on a least-squares approach. Due to the fact that both fitting equations, Eqs. (8) and (9), can be analytically recast into each other, fitting results are expected to be identical for perfect experimental data, which is, however, not realistic in an actual experimental setting. So, the main question is how both approaches deal with real data that are inevitably noisy.

Least-squares methods can be shown to be reliable for linear as well as nonlinear cost functions as long as all errors are normally or at least approximately normally distributed [22]. At this point, the crucial question is how the errors occurring in filtration experiments behave. t/V , i.e., the decisive quantity in case of Strategy 1, is intuitively not expected to scatter in a normally distributed way. However, if this were the case, Strategy 1 would be reliable to determine the parameters. On the contrary, Strategy 2 is reliable as long as the errors of V are normally distributed. The actual distribution of errors for both strategies is discussed in the next sections when we turn to our simulation study.

Please note that an increasing error magnitude with larger values of the independent variable, i.e., liquid volume in case of Strategy 1 and time for Strategy 2, is not a problem with respect to the quality of the found parameter values. As the mentioned heteroscedasticity, i.e., the increasing spread or dispersion of the observed variables, does not affect the accuracy but only the efficiency with which the parameters are determined, it is not further accounted for in this work. Should computational efficiency be an issue, different strategies to handle heteroscedasticity are known in the literature, e.g., by scaling of variables [22, 23]. To substantiate our claim further, i.e., that the applied fitting strategy makes a difference, a numerical study is presented in the following article.

4 Computational Methods and Simulation Strategy

Strategy 1 and 2 are tested for determining the decisive parameters P_1 and P_2 , respectively, R_M and r . In this respect, as mentioned, the focus is laid on the more important parameter of specific filter cake resistance r . To truly test a fitting strategy, the actual parameter values must be known. For that reason, forward simulations are conducted in which Eq. (9) is solved with known values of P_1 and P_2 for 100 s with one data point per second. These ideal solutions are perturbed with different levels of noise, and the obtained noisy data are in turn used as

fictional experiments to determine the parameter values. In each case, it is checked how well the true parameter values are found again as is a common strategy when testing parameter fitting procedures [24].

Forward simulations as well as parameter fitting are conducted in MATLAB (version 2018a; Mathworks, Natick, MA). Noise is generated using the function *randn* which yields normally distributed random numbers; the optimization problem is solved with *fminsearch* in which the maximal number of iterations and of function evaluations are both set to 1 000 000 while the other options remained at their default settings.

Next, the exact generation of noisy data is explained in more detail. In all cases, noise is assumed to be normally distributed with previously defined standard deviations. Flow rate $dV/dt = Q$ and specific filter cake resistance r are supplied with noise. In both cases, the level of noise, i.e., its standard deviation, is given in % of the nominal variable value. Please note that this also implies that the standard deviation changes over time for the flow rate as dV/dt decreases with time in the mode of constant pressure filtration. Noise is added to dV/dt because the flow rate was observed to show some scattering in published studies [9] as well as in our own experiments [11].

It is important to mention that normally distributed noise on dV/dt causes also scattered values of the cumulated volume V as schematically shown in some publications [1, 2]. As the noise on dV/dt simply adds up, scattering on V is also normally distributed because any linear combination of independent randomly distributed variables is also randomly distributed [9]. Due to this summing up, the error bands on V can become larger over time which is also in agreement with published experimental findings [11]. For the reasons discussed, we believe that our noise model simulates the true flow behavior in constant-pressure filtration quite well, a point we will also elaborate more in the discussion of the results.

However, there is also another possible source of experimental errors. It can be imagined that r itself is prone to uncertainties, e.g., due to biases when taking samples of the powder or particle system used for the experiments. To account for this effect, also normally distributed noise is added to specific cake resistance r . Noise on dV/dt and r , therefore, accounts for two different effects: the first covers non-idealities when conducting the experiment, the second includes uncertainties when preparing the experiment.

Our two noise modes are illustrated in Fig. 1 where it is shown how a noisy flow rate (a) influences cumulated volume V (b); the effect of variations in specific filter cake resistance on the resulting trajectory of V is also displayed (c). The figure reveals that a noisy flow rate results in jagged curves whereas variations in r still give smooth, but diverging curves. This behavior is due to the fact that noise on r mimics variations when preparing the experiment as explained, and, therefore, only affects the model parameter r in a time-invariant way. A noisy flow rate, on the other hand, is intended to model transient effects. As in reality both phenomena often occur together, their combined effects will be studied later on.

The filter medium resistance R_M has not been subjected to noise because, as justified above, it is not studied in detail here. Please note that data on experimentally occurring noise are scarce in the literature; raw data and error bands are often not

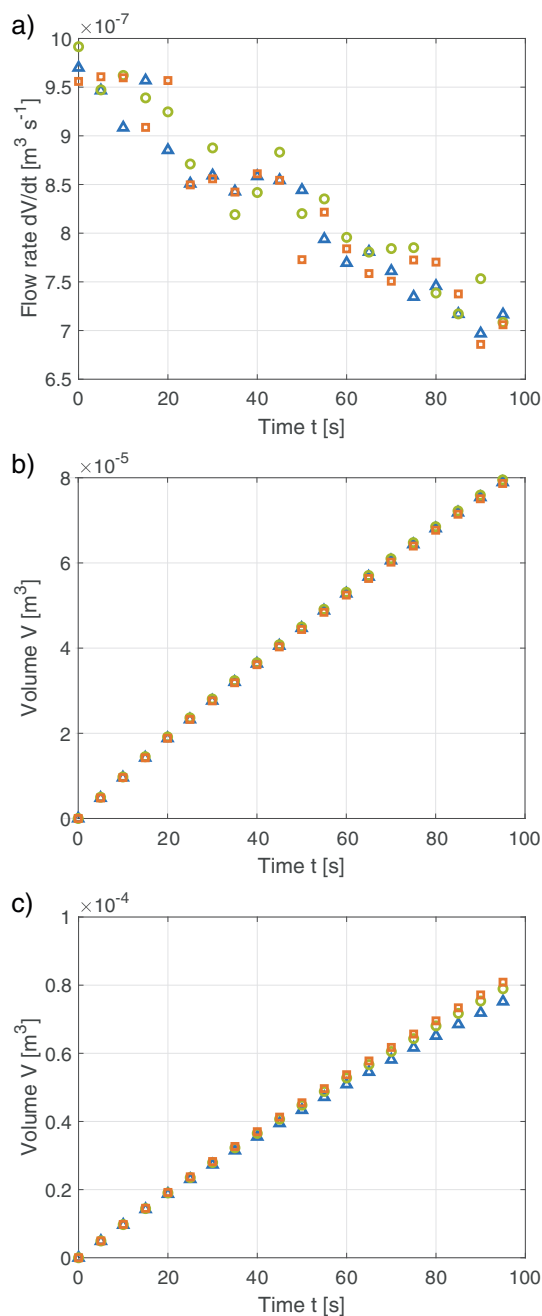


Figure 1. Illustration of the noise generation procedure. A noisy flow rate (a) leads to noise on the cumulative volume (b); variations in specific filter cake resistance also affect the cumulative volume (c); three randomly generated data sets are shown (marked by different symbols) as used in each inner iteration of the Monte-Carlo method; the noise level on the flow rate is 2.5 % and 20 % on specific filter cake resistance.

shown or only single experimental runs are discussed. Therefore, we also had to rely on our own lab experiences to choose realistic noise intensities. However, it is claimed that all noise levels analyzed in the remaining article are within the range commonly encountered when conducting filtration experiments.

To evaluate the model fits, a two-stage strategy is employed which is divided into an inner and an outer iteration. In the inner iteration, three-fold experiments are mimicked by generating three different sets of noisy data, i.e., noise is generated with three different random number seeds, as displayed in Fig. 1. These three-fold datasets are used together for one model fit. In an outer iteration, 1000 repetitions of such triple experiments are performed, again each run with different random number seeds. By the outer iteration, statistical information is obtained on the determined parameters, i.e., mainly error bars on r . Therefore, conclusions can be drawn about which fitting strategy performs better “in the long run”. As computational methods where repeated use is made of random numbers are often referred to as Monte-Carlo methods, also the described procedure can be classified as a Monte-Carlo approach.

5 Results

Using the simulation strategy described in the last section, various data sets were generated to test and compare parameter fitting Strategy 1 and 2. For didactic reasons, first some single selected cases are shown and subsequently turned to overview representations. The first are more tangible but contain less information, the second are denser information-wise but, therefore, also more difficult to interpret.

As expected, no detectable differences between Strategy 1 and 2 are found in case of non-noisy data, i.e., the true parameter values can be reliably identified with both approaches. Thus, this case is not further discussed. Before both strategies are compared in detail, a problem is considered that can occur with Strategy 1, i.e., the linear fit. Linear representation of the data can cause highly nonlinear segments at the beginning of the experiments. Therefore, no meaningful fits can be conducted without cropping the data for small times. A comparison of a full, non-cropped data set and an adapted data set is shown in Fig. 2 together with the corresponding linear fit; a 10 % noise level on flow rate is used.

In case of the non-cropped data, the fitting error of specific cake resistance r is 9.66 %; after cropping the data, it can be reduced to 5.65 %. Error, here and in the remaining article, is defined as the difference between the parameter value determined by fitting r_f and the true value r_t relative to r_t , i.e.:

$$\text{Error} = \frac{r_f - r_t}{r_t} \times 100\% \quad (14)$$

For the results of Fig. 2, the first 15 data points, i.e., 15 s, were cropped. It is important to note that this behavior is no artefact of the simulation strategy or noise generation method, as the same behavior was also observed with our own experimental data. The necessity to crop data sets is also mentioned in the literature [1]. As cropping of data is not required in case of Strategy 2, here already is the first advantage of the proposed method because it avoids the decision of how many data points to drop, which is always to some degree arbitrary. In the remaining, article only cropped data are used in case of Strategy 1 to allow a meaningful comparison of the two approaches; the same cut-off of 15 s is used throughout this work as this proved a good threshold.

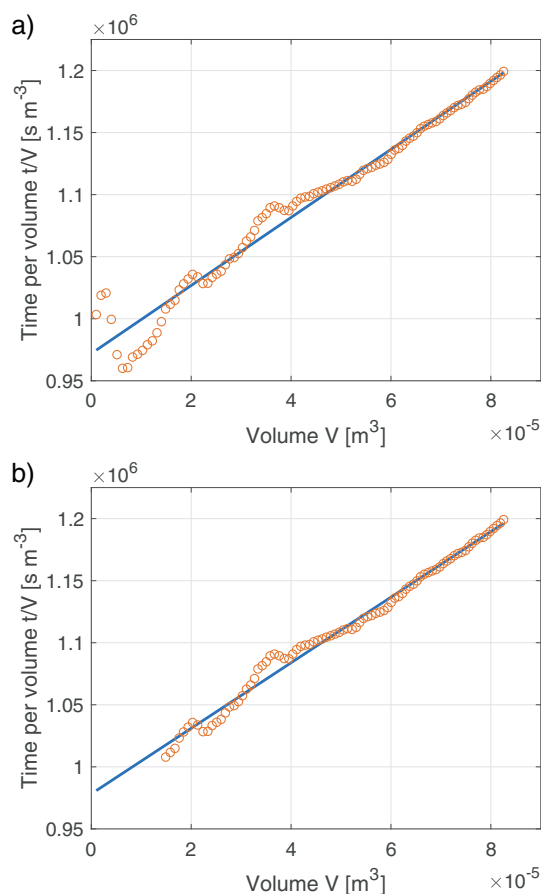


Figure 2. Exemplary comparison of Strategy 1 for the non-cropped data set (a) and cropped data (b); raw data are displayed as circles; continuous lines are the linear fits.

For uncropped data, Strategy 1 would yield a significantly worse performance than the results shown in the following. Also, whereas in Fig. 2 only a single experimental run is shown to illustrate the problem, all subsequent results will be given for a triple determination, which was described in Sect. 4 as the internal iteration of our Monte-Carlo method. Please note that in the following figures the raw data (displayed by circles in Fig. 2) are omitted and only the fitted lines are presented together with the confidence intervals of the raw data (given for every fifth data point) in order to keep the plots clear and understandable.

Fig. 3 presents curve fits with both strategies for a case with a low level of noise, i.e., a standard deviation of 1 % on the nominal value of the flow rate. Already in this case it can be observed that the fitting with Strategy 2 leads to more exact results: The fitting error of the specific filter cake resistance is 1.944 % as determined by Strategy 1, compared to a value of 1.75 % when using Strategy 2. Insignificant as this difference may be for all practical purposes, it proves already that the two methods deviate. Also, the shown fictitious experiments are quite close to ideal data that are hardly found in real experiments.

Turning from this singular example with only the threefold inner iteration to the full data set obtained by the additional

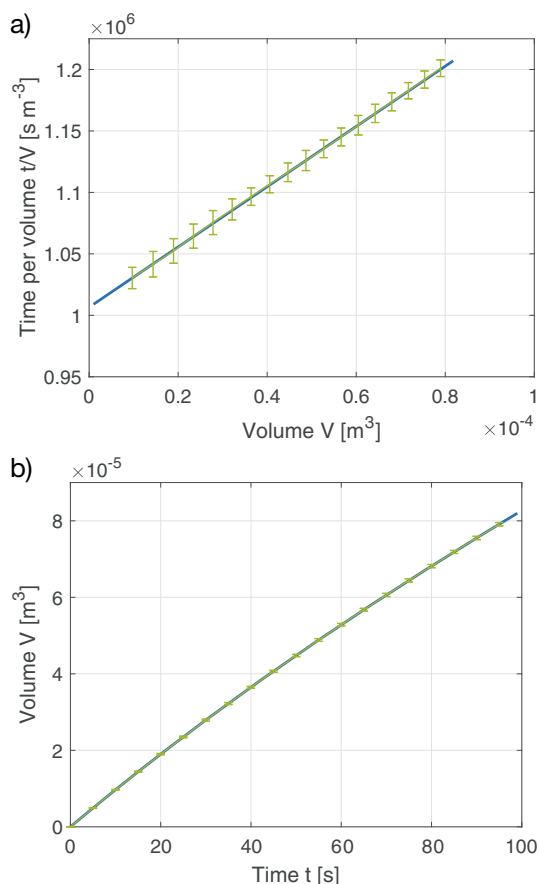


Figure 3. Fits of data with low noise level on the flow rate and a true specific filter cake resistance of 10^{12} m^{-2} using the classical Strategy 1 (a) and the new, nonlinear Strategy 2 (b); confidence intervals of the raw data are displayed for every fifth value (green).

1000 outer iterations, Strategy 1 yields an average error of -0.0128% ($\pm 0.060\%$) compared to the value of -0.001% ($\pm 0.053\%$) in case of Strategy 2. Here and in the following, the percentage values within brackets are the confidence intervals of the fitting errors based on a confidence level of 99%. The same confidence level is also used in all plots where confidence intervals are shown. As the reported confidence intervals refer to the errors, smaller values are not per se an advantage. If, e.g., the average error is large but the obtained confidence interval for that error is small, this only means that the wrong parameter value is identified with a low variation.

All results shown so far also underline the trivial truth that repetition of experiments is important for a reliable determination of parameters. Single experimental runs, as, e.g., displayed in Fig. 2, can exhibit large variations and, therefore, lead to inexact parameter values. Three-fold repetitions, as used in Fig. 3 and all remaining fits of this article, already lead to an increased accuracy. If many such three-fold experimental runs are considered, as modeled by our outer iteration (see Sect. 4), the accuracy can still be improved remarkably.

Next, data with a higher level of noise but the same true specific filter cake resistance are considered. In Fig. 4, an analogous

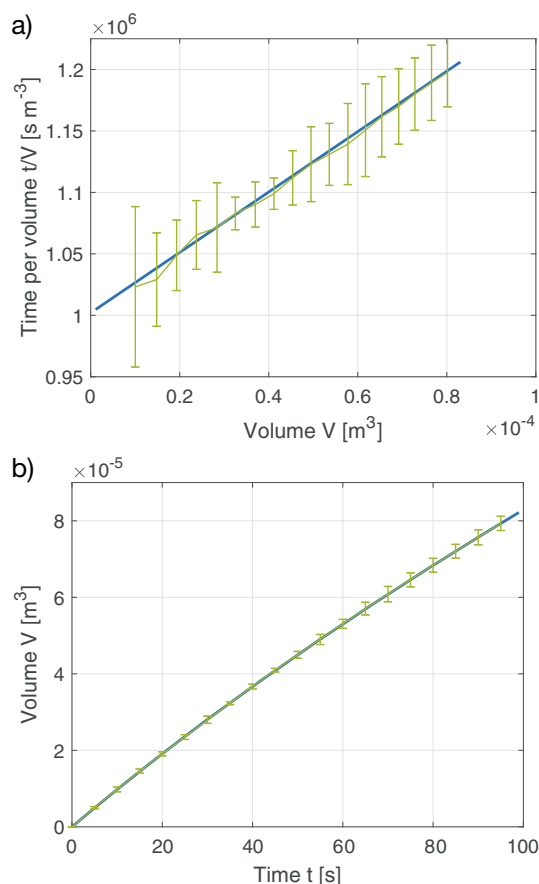


Figure 4. Fits of data with high noise level on the flow rate and a true specific filter cake resistance of 10^{12} m^{-2} using the classical Strategy 1 (a) and the new, nonlinear Strategy 2 (b); confidence intervals of the raw data are displayed for every fifth value (green).

comparison is shown for a flow rate-specific noise of 10%. Here, the fitting errors become 1.69% and -0.31% for Strategy 1 and 2, respectively. When turning to the full data set including the 1000 outer iterations, Strategy 1 yields an average error of -1.024% ($\pm 0.6\%$) compared to the value of 0.0499% ($\pm 0.525\%$) from Strategy 2. Thus, it can be observed that the new Strategy 2 becomes more effective, the higher the noise level on the experimental data is. It also becomes apparent what was already conjectured in Sect. 3, namely that the distribution of errors matters for the fitting procedure. Whereas Strategy 2 relies on data with normally distributed errors, a distorted scaling of errors is encountered in Strategy 1 which is the reason for the worse quality of the determined parameters.

As a third example, noise is added to the specific filter cake resistance r along with a flow rate-specific noise of 5%; the standard deviation of r is 1% of its nominal value. For this case, scattered data and fitting results are presented in Fig. 5. Based on this noisy r , the errors are -4.35% and -2.95% for Strategy 1 and 2, respectively. Including again the outer iterations, average errors of -0.271% ($\pm 0.29\%$) and 0.076% ($\pm 0.26\%$) are obtained from Strategy 1 and 2, respectively. First of all, this example demonstrates how important the outer

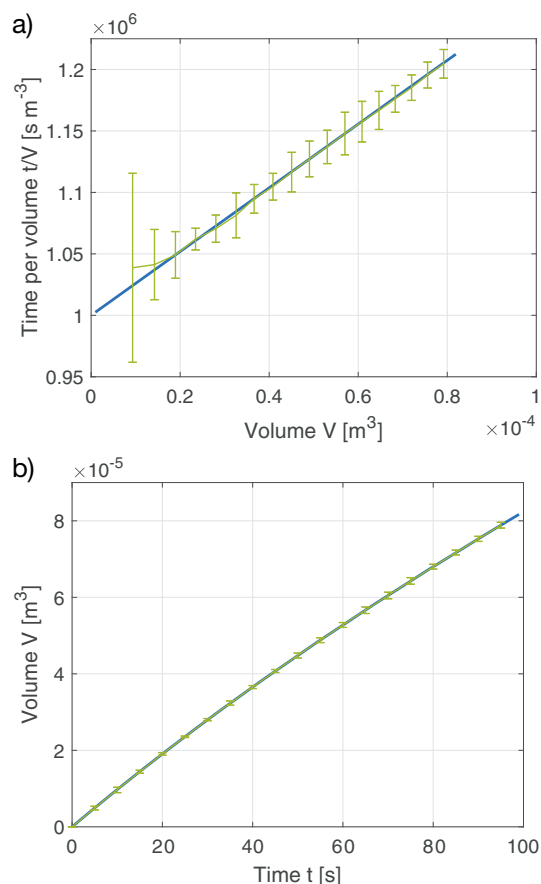


Figure 5. Fits of data with noise both on flow rate and specific filter cake resistance; the mean true specific filter cake resistance is $10^{12}\ m^{-2}$; comparison of Strategy 1 (a) and the new, nonlinear Strategy 2 (b); confidence intervals of the raw data are displayed for every fifth value (green).

iteration is. Whereas the single example shown in Fig. 5 led to relatively high fitting errors for both approaches, on the average Strategy 1 and 2 perform better than in this example. This shows again that repetition of experiments cannot be overestimated. However, also here Strategy 2 is superior to Strategy 1.

Now the more complex display of results is considered. The heat maps displayed in Fig. 6 comprise the results of many singular fits as they were discussed so far; all are shown again for 1000 outer iterations. Various different levels of noise on the flow rate dV/dt and on specific filter cake resistance r as well as combinations of both are displayed. The color scale denotes the resulting fitting errors; areas of green color indicate that the correct parameter values were identified, blue and red colors symbolize found parameter values that are too low or too high, respectively.

In general, Fig. 6 confirms what the single examples discussed so far already indicated: Strategy 2 consistently performs better than Strategy 1, i.e., it allows to determine the specific filter cake resistance with a higher accuracy. Additionally, it can be seen that, on the average, fitting error increases both with ascending noise levels on dV/dt and r . For small levels of noise, e.g., less than 10% and certainly less than 5%, both strategies

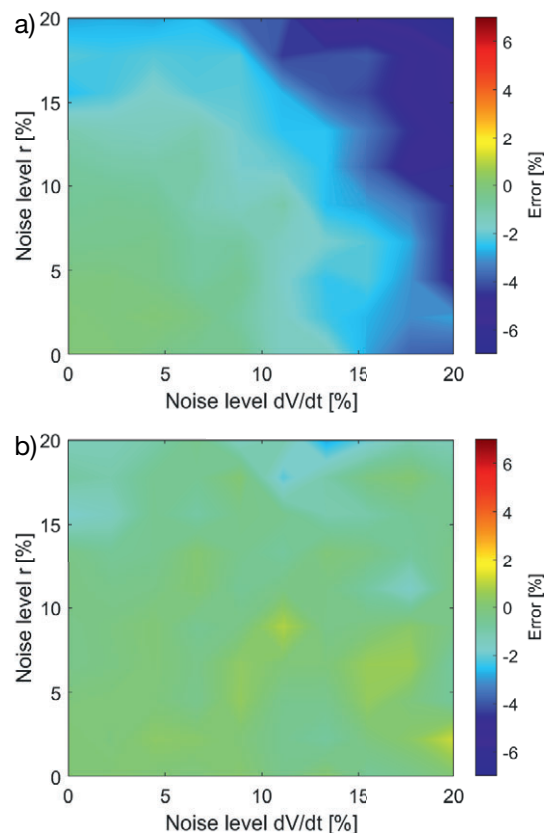


Figure 6. Heat maps showing the average fitting errors obtained by Strategy 1 (a) and the new, nonlinear Strategy 2 (b) for different levels of noise added to the flow rate dV/dt and specific filter cake resistance r ; the mean true specific resistance is $10^{12}\ m^{-2}$. Dark blue and dark red on the color scale denote the found parameter values that are too low or too high, respectively.

seem to yield acceptable results. However, it must be taken into account, that Fig. 6 only shows the long-term behavior. Both types of noise can still result in considerable fitting errors when only few experimental runs are considered, as usually done in experimental practice and as indicated in Fig. 5. An additional disadvantage of Strategy 1 is that it has a clear tendency to underestimate the true specific filter cake resistance (mostly the blue color range is present in Fig. 6a); compare Eq. (14) for our definition of fitting error. This underestimation might be a problem for process design because, as a consequence, equipment such as pumps might be chosen undersized.

Before providing some overall conclusions, a few additional clarifications are required. It is important to note that the qualitative behavior displayed in Fig. 6 is the same also for different nominal values of specific filter cake resistance r . Even though the nominal value $r = 10^{12}\ m^{-2}$ was used throughout the article, the findings are unaffected if the decimal power is varied in the realistic range from 10^{11} to 10^{16} . Furthermore, the overall time span and discretization of time points could affect the fitting results, i.e., the whole experimental time considered and the intervals at which data points are saved. However, it was found that, excluding unrealistically short experimental times, this

effect is negligible. Also, it might be objected that only the new Strategy 2 was compared with the traditional t/V - V plot (Strategy 1) whereas also some sources suggest to use a dV/dt - V plot [1]. In answer to this, it must be said that t/V is much more common in the literature, and, even more importantly, the uneven scaling of errors does equally occur in the dV/dt - V approach as in Strategy 1. Also, some conducted simulation studies confirmed the hypothesis and demonstrated that the latter strategy performs even worse than Strategy 1. A last remark about the parameter fitting itself: Both investigated fitting strategies result in well-posed optimization problems with pronounced minima regarding the sought-for parameter r ; the obtained solutions are insensitive to the provided initial values. Fitting errors in case of Strategy 1, therefore, result from a systematic bias rather than from multiple minima.

6 Conclusions

The present article had a very simple aim, namely, to show that a nonlinear fit, based on a root function, is superior to the classical linear strategy when it comes to determining the parameters for incompressible cake filtration, mainly specific filter cake resistance. There are different advantages of the nonlinear approach. First of all, a cropping of data, as is often necessary for small times when using the linear strategy, is not required. This eliminates a subjective factor in the fitting process and prevents that useful data points are discarded. However, the main advantage is that the nonlinear strategy consistently leads to more exact results.

In order to warrant this claim, a broad Monte-Carlo simulation study with two different noise models was presented. Firstly, noise was added to the flow rate which mimics variations in conducting the filtration experiments; secondly, the nominal values of specific filter cake resistance were supplied with noise in order to model variations when preparing the experiment. For both modes of noise, it was checked how well the two different fitting approaches were able to identify the true parameter values. In this respect, it must be stressed that such an evaluation is only possible by a simulation study because only then the true parameter values are known and, therefore, the fitting quality can be assessed adequately.

In conclusion, all filtration experimentalists who deal with incompressible substances, or substances that can be considered approximately as such under the given process conditions, are advised to evaluate their data by the nonlinear method proposed in this article. Even though the gain in precision is in the order of some percent, there is no reason to refrain from using the nonlinear fit. It is more exact and involves no extra effort because practically all available software tools nowadays allow fitting nonlinear functions with a comparable computational efficiency as linear models. Also, the nonlinear fitting strategy removes arbitrariness in data point selection. Finally, it requires no shift to a new theoretical framework as this new approach is based on the familiar and well-tested theory of incompressible cake filtration.

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Symbols used

A	[m ²]	cross-sectional area of filter
c	[-]	volumetric concentration or volume fraction of impurities
H	[m]	filter cake height
J	[variable]	value of cost function
k	[m ²]	permeability
K	[-]	concentration constant, reference to filter cake height
K_m	[kg m ⁻³]	concentration constant, reference to solid mass
m	[kg]	filter cake mass
N	[-]	maximal index of measured points
Δp	[kg m ⁻¹ s ⁻²]	differential pressure
P	[variable]	fit parameters
Q	[m ³ s ⁻¹]	volumetric flow rate
r	[m ⁻²]	specific or relative resistance
R_M	[m ⁻¹]	resistance of filter medium
t	[s]	time
V	[m ³]	liquid volume
x	[-]	independent variable
y	[-]	dependent variable

Greek letters

ε	[-]	porosity or void fraction
μ	[kg m ⁻¹ s ⁻¹]	dynamic viscosity

Sub- and superscripts

C	cake
f	parameter value determined by fitting
i	index
m	mass
M	medium
t	true parameter value

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