Beyond Positivity Bounds and the Fate of Massive Gravity

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We constrain effective field theories by going beyond the familiar positivity bounds that follow from unitarity, analyticity, and crossing symmetry of the scattering amplitudes. As interesting examples, we discuss the implications of the bounds for the Galileon and ghost-free massive gravity. The combination of our theoretical bounds with the experimental constraints on the graviton mass implies that the latter is either ruled out or unable to describe gravitational phenomena, let alone to consistently implement the Vainshtein mechanism, down to the relevant scales of fifth-force experiments, where general relativity has been successfully tested. We also show that the Galileon theory must contain symmetry-breaking terms that are at most one-loop suppressed compared to the symmetry-preserving ones. We comment as well on other interesting applications of our bounds.

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The idea that physics at low energy can be described in terms of light degrees of freedom alone, which goes under the name of effective field theory (EFT), is one of the most satisfactory organizing principles in physics. The effect of ultraviolet (UV) dynamics is systematically accounted for in the resulting infrared (IR) EFT by integrating out heavy degrees of freedom, which generate an effective Lagrangian made of infinitely many local operators. Since the symmetries of the underlying UV theory are retained in the IR, EFTs are predictive even when the UV dynamics is unknown: only a finite number of symmetric operators contribute, at a given accuracy, to observable quantities.

Remarkably, extra information about the UV can always be extracted if the underlying Lorentz invariant microscopic theory is unitary, causal, and local. These principles are encoded in the fundamental properties of the S matrix such as unitarity, analyticity, crossing symmetry, and polynomial boundedness [1,2]. These imply a UV-IR connection in the form of dispersion relations that link the (forward) amplitudes in the deep IR with the discontinuity across the branch cuts integrated all the way to infinite energy [3,4]. Unitarity ensures the positivity of such discontinuities, and in turn the positivity of (certain) Wilson coefficients associated with the operators in the IR effective Lagrangian. This UV-IR connection can be used to show that coefficients with the "wrong" sign cannot be generated by a Lorentz invariant, unitary, causal, and local UV completion [5]: the corresponding EFT, even if compatible with the symmetries of the system, is thrown to the "swampland." Positivity bounds have found several applications, including the proof of the a theorem [6,7]; the study of chiral perturbation theory [8] and WW scattering; and theories of composite Higgs [9–14], quantum gravity [15], massive gravity [16–18], Galileons [18–21], inflation [22,23], the weak gravity conjecture [24,25], and conformal field theory [26–28]. The approach has been recently extended to particles of arbitrary spin [18], leading to a general no-go theorem on the leading energy-scaling behavior of the IR amplitudes, with applications to massive gravity [16] and Goldstini [29–31]. References [21,32,33] extended this technique beyond the forward limit.

In this Letter, we show that qualitatively new bounds, stronger than standard positivity constraints, can be derived by taking into account the irreducible IR cross sections under the dispersive integral, which are calculable within the EFT. We discuss for what models our bounds can be important and focus explicitly on two relevant applications: the EFT for a weakly broken Galileon [34,35], and the ghost-free massive gravity theory [36,37]. Figure 1 gives a preview of our results for the latter.

Let us consider the center-of-mass 2-to-2 scattering amplitude $\mathcal{M}^{z_1 z_2 z_3 z_4}(s,t)$, where the polarization functions are labeled z_i. The Mandelstam variables are defined by $s=-(k_1+k_2)^2,\ t=-(k_1+k_3)^2,\ u=-(k_1+k_4)^2$ and

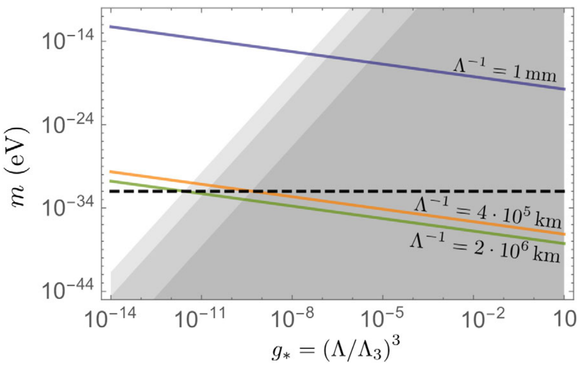

FIG. 1. Exclusion region for massive gravity in the plane of graviton mass m and coupling $g_* \equiv (\Lambda/\Lambda_3)^3$, with Λ being the physical cutoff and $\Lambda_3 = (m^2 m_{\mathrm{Pl}})^{1/3}$ the strong coupling scale. Our theoretical bound Eq. (14) excludes with accuracy $\delta = 1\%$ the darkest gray region, and with 10%, 30% the gradually lighter gray regions. Solid lines mark the fixed cutoff Λ, whereas the dashed black line shows the upper experimental bound $m = 10^{-32}$ eV. Our constraint gives rise to a tension between high Λ and a small m: the graviton mass can only be below the experimental bound at the expense of a premature breakdown of the EFT at macroscopically large distances.

satisfy $s + t + u = 4m^2$, where m is the mass of the scattered particles. Our arguments will require finite $m \neq 0$, yet they hold even for some massless theories [scalars, spin-1/2 fermions, and softly broken $U(1)$ gauge theories], which have a smooth limit $m \to 0$. We call

$$\mathcal{M}^{z_1 z_2}(s) \equiv \mathcal{M}^{z_1 z_2 z_1 z_2}(s, t = 0) \tag{1}$$

the forward elastic amplitude at $t = 0$ and study the analyticity properties of $\mathcal{M}^{z_1 z_1}(s)/(s - \mu^2)^3$, integrating along a closed contour Γ in the complex s plane, enclosing all the physical IR poles s_i associated with stable light degrees of freedom entering the scattering (or its crossed symmetric process), together with the point $s = \mu^2$ lying on the real axis between $s = 0$ and $s = 4m^2$. We define

$$\Sigma_{\mathrm{IR}}^{z_1 z_2} \equiv \frac{1}{2\pi i} \oint_\Gamma ds \frac{\mathcal{M}^{z_1 z_2}(s)}{(s - \mu^2)^3} = \sum_{s = s_i, \mu^2} \mathrm{Res}\left(\frac{\mathcal{M}^{z_1 z_2}(s)}{(s - \mu^2)^3}\right), \tag{2}$$

which is calculable within the EFT. Using Cauchy's integral theorem, the contour can be deformed into a new contour that runs around the s-channel and u-channel branch cuts, and goes along a big circle eventually sent to infinity. The boundary contribution at infinity vanishes, due to the Froissart-Martin asymptotic bound $|\mathcal{M}(s \to \infty)| < \mathrm{const} \times s \log^2 s$, which is always satisfied in any local massive QFT [38,39]. This leads to a dispersion relation that connects the IR, Eq. (2), to an integral (UV) of the total cross section

$$\Sigma_{\mathrm{IR}}^{z_1 z_2} = \sum_X \int_{4m^2}^\infty \frac{ds}{\pi} \sqrt{1 - 4\frac{m^2}{s}} \times \left[\frac{s\sigma^{z_1 z_2 \to X}(s)}{(s - \mu^2)^3} + \frac{s\sigma^{-\bar{z}_1 z_2 \to X}(s)}{(s - 4m^2 + \mu^2)^3}\right]_{\mathrm{IR}}, \tag{3}$$

where we use crossing symmetry relating the s and u channels in the forward limit even for particles with spin [18]: in the helicity basis, $\mathcal{M}^{z_1 z_2}(s) = \mathcal{M}^{-\bar{z}_1 z_2}(u = -s + 4m^2)$, and for particles that are their own antiparticles, $\bar{z} = z$. We also invoke the optical theorem to relate the imaginary parts of the amplitudes (across the branch cuts) to the cross section.

For any theory where particles 1 and 2 are interacting, as long as $0 < \mu^2 < 4m^2$, the right-hand side (rhs) of Eq. (3) is always positive, and one obtains the rigorous positivity bound, $\Sigma_{\mathrm{IR}}^{z_1 z_2} > 0$. Since $\Sigma_{\mathrm{IR}}^{z_1 z_2}$ is calculable in the IR in terms of the Wilson coefficients, this provides a nontrivial constraint on the EFT.

We can in fact extract more than positivity bounds by noticing that the total cross section on the rhs of the dispersion relation Eq. (3) contains an irreducible contribution from IR physics, which is also calculable within the EFT, by construction. The other contributions, e.g., those from the UV, are incalculable with the EFT but are nevertheless always strictly positive, by unitarity. Moreover, each final-state X in the total cross section contributes positively too. Therefore, an exact inequality follows from truncating the rhs of Eq. (3) at some energy $s_{\max} \equiv E^2 \ll \Lambda^2$ below the cutoff Λ of the EFT. To leading order (LO) in powers of $(E/\Lambda)^2$ and $(m/E)^2$ [hence also $(\mu/E)^2$], the bound in Eq. (3) becomes

$$\Sigma_{\mathrm{IR,LO}}^{z_1 z_2} > \sum_X \int^{E^2} \frac{ds}{\pi s^2} [\sigma^{z_1 z_2 \to X}(s) + \sigma^{z_1 - \bar{z}_2 \to X}(s)]_{\mathrm{IR,LO}} \times \left[1 + O\left(\frac{m}{E}\right)^2 + O\left(\frac{E}{\Lambda}\right)^2\right], \tag{4}$$

where the subscript IR highlights the fact that both sides of the equation are computable within the EFT; the main source of error for small masses is the truncation of the tower of higher-dimensional operators. Choosing E at or slightly below the cutoff Λ gives just an order-of-magnitude estimate for the bound [18,21], as originally suggested in Ref. [19]. A rigorous bound can instead be obtained by choosing a sufficiently small $(E/\Lambda)^2$: percent accuracy can be achieved already with $E/\Lambda \approx 1/10$.

The $\Sigma_{\mathrm{IR}}^{z_1 z_2}$ must therefore be not only positive but *strictly larger* than something which is itself positive and calculable within the EFT. Moreover, we can include any final states X, elastic or inelastic: the more channels and information are retained, the more refined the bound will be. Notice that the 2-to-2 cross section retained on the rhs is obtained by integrating over t; thus, effectively, our bounds capture as well the behavior of the amplitude away from the forward limit.

The implications of our bound Eq. (4) are particularly interesting in theories where the elastic forward amplitude $\mathcal{M}^{z_1 z_2}$, appearing on the left-hand side (lhs), is parametrically suppressed compared to the nonforward or inelastic ones appearing on the rhs. The Galileon, massive gravity, the dilaton, and WZW-like theories, as well as other models where $2 \to 2$ is suppressed while $2 \to 3$ is not, are other simple examples of theories that get nontrivial constraints, on the couplings and/or masses of the corresponding EFTs, that include and go beyond the positivity of $\Sigma_{\rm IR}$. Even in situations without parametric suppression, our bound carries important information: it links elastic and inelastic cross sections that might depend on coefficients of the EFT.

Galileon.—The Lagrangian for the weakly broken Galileon [34,35],

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \pi)^2 \left[1 + \frac{c_3}{\Lambda^3}\Box\pi + \frac{c_4}{\Lambda^6}((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2) + c_5(\ldots)\right] + \frac{\lambda}{4\Lambda^4}[(\partial\pi)^2]^2 - \frac{m^2}{2}\pi^2, \tag{5}$$

with physical cutoff Λ, has suppressed symmetry-breaking terms $\lambda \ll c_3^2$, c_4 and $m^2 \ll \Lambda^2$. Forward $2 \to 2$ scattering is controlled at $O(s^2)$ by the symmetry-breaking interactions: the lhs of Eq. (4) is $\Sigma_{\rm IR} = (\lambda/\Lambda^4) + (c_3^2 m^2/2\Lambda^6)$. On the other hand, the hard scattering is controlled by the symmetry-preserving interactions, $\sigma^{\pi\pi\to\pi\pi} \approx 3(c_3^2 - 2c_4)^2 s^5/(5120\pi\Lambda^{12})$.

In the massless limit, or more generally for $c_3^2 m^2/\Lambda^2 \ll \lambda$ (a natural hierarchy, given that λ preserves a shift symmetry while m^2 does not), the bound Eq. (4) shows not only that λ must be positive, but (parametrically) at most one loop factor away from $(c_3^2 - 2c_4)/4$:

$$\lambda > \frac{3}{640}\frac{(c_3^2 - 2c_4)^2}{16\pi^2}\left(\frac{E}{\Lambda}\right)^8 \quad \text{for} \quad \frac{c_3^2 m^2}{\Lambda^2} \ll \lambda. \tag{6}$$

For a massive Galileon with negligible λ and $c_3 \neq 0$, one gets a lower bound on the mass,

$$m^2 > \Lambda^2 \frac{3(c_3 - 2c_4/c_3)^2}{320 \times 16\pi^2}\left(\frac{E}{\Lambda}\right)^8 \quad \text{for} \quad \frac{c_3^2 m^2}{\Lambda^2} \gg \lambda, \tag{7}$$

where $(E/\Lambda)^8 \approx 10^{-2}$ for a 30% accuracy. Therefore, the Galileon symmetry-breaking terms cannot be arbitrarily suppressed, the general lesson being that $O(s^2)$ terms in the amplitude cannot be too suppressed compared to the $O(s^3)$ terms. The results of Eqs. (6) and (7) hold when loop effects are included, as they simply generate terms that are subleading, e.g., $\sim(m/\Lambda)^6 c_3^4 m^2/16\pi^2\Lambda^6$ in $\Sigma_{\rm IR}$ (recall that $m \ll E \ll \Lambda$).

Massive gravity.—The action for ghost-free massive gravity, also known as Λ_3 or dRGT massive gravity, is [36,37] (for reviews, see Refs. [40,41])

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm Pl}^2}{2}R - \frac{m_{\rm Pl}^2 m^2}{8}V(g,h)\right], \tag{8}$$

where $m_{\rm Pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ is an effective metric written in terms of the Minkowski metric $\eta_{\mu\nu}$ (with mostly + signature) and a spin-2 graviton field $h_{\mu\nu}$ in the unitary gauge, R is the Ricci scalar for $g_{\mu\nu}$, and

$$\begin{aligned} V(g,h) = {} & b_1\langle h^2\rangle + b_2\langle h\rangle^2 + c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3 \\ & + d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 \\ & + d_5\langle h\rangle^4 \end{aligned}$$

is the soft graviton potential, with $\langle h\rangle \equiv h_{\mu\nu}g^{\mu\nu}$, $\langle h^2\rangle \equiv g^{\mu\nu}h_{\nu\rho}g^{\rho\sigma}h_{\sigma\mu}$, etc. Absence of ghosts implies that the coefficients of this potential depend on just two parameters, c_3 and d_5; see Ref. [37,42].

Note that, though tempting, the results obtained above for the Galileon cannot be directly interpreted in the context of massive gravity (even if the Galileon is the longitudinal component of the massive graviton), since the IR dynamics is different: for example, in the scattering of the Galileon scalar mode, the helicity-2 mode exchanged in the t channel contributes as much as the scalar mode.

Since the graviton is its own antiparticle, it is convenient to express Eq. (4) in terms of linear polarizations [15,16,18]:

$$\Sigma_{\rm IR,LO}^{z_1 z_2} > \sum_X \frac{2}{\pi}\int^{E^2}\frac{ds}{s^2}[\sigma^{z_1 z_2\to X}(s)]_{\rm IR,LO}. \tag{9}$$

We adopt the basis of polarizations of Ref. [16] to calculate the amplitudes for different initial- and final-state configurations, finding that, generally, $\Sigma_{\rm IR}^{z_1 z_2} \sim m^2/\Lambda_3^6$ is suppressed by the small graviton mass, with $\Lambda_3 \equiv (m^2 m_{\rm Pl})^{1/3}$ the strong coupling scale [43]. For instance,

$$\begin{aligned} \Sigma_{\rm IR}^{SS} &= \frac{2m^2}{9\Lambda_3^6}(7 - 6c_3(1+3c_3) + 48d_5) > 0, \\ \Sigma_{\rm IR}^{VV} &= \frac{m^2}{16\Lambda_3^6}(5 + 72c_3 - 240c_3^2) > 0, \\ \Sigma_{\rm IR}^{VS} &= \frac{m^2}{48\Lambda_3^6}(91 - 312c_3 + 432c_3^2 + 384d_5) > 0. \end{aligned} \tag{10}$$

In contrast, the hard-scattering limits of the amplitudes that enter the rhs of Eq. (9) are unsuppressed. For $s, t \gg m^2$, e.g., elastic amplitudes read

$$\begin{aligned} \mathcal{M}^{SS} &= \frac{st(s+t)}{6\Lambda_3^6}(1 - 4c_3(1-9c_3) + 64d_5), \\ \mathcal{M}^{VV} &= \frac{9st(s+t)}{32\Lambda_3^6}(1-4c_3)^2, \\ \mathcal{M}^{VS} &= \frac{3t}{4\Lambda_3^6}\left(c_3(1-2c_3)(s^2+st-t^2) - \frac{5s^2+5st-9t^2}{72}\right). \end{aligned} \tag{11}$$

At this point, we choose the energy scale E in Eq. (9) below the cutoff, $E \ll \Lambda$, so that the EFT calculation of the cross sections is trustworthy, and above the mass, $E \gg m$, so that the amplitudes Eq. (11) dominate such cross sections. We define $\delta \equiv (E/\Lambda)^2$, that controls the accuracy of the EFT calculation, and obtain

$$F_{z_1 z_2}(c_3, d_5) > \left(\frac{4\pi m_{\rm Pl}}{m}\right)\left(\frac{g_*}{4\pi}\right)^4 \delta^6, \tag{12}$$

where $g_* \equiv (\Lambda/\Lambda_3)^3$ and, e.g.,

$$F_{SS} = \left[960 \frac{7 - 6c_3(1+3c_3) + 48 d_5}{(1 - 4c_3(1-9c_3) + 64 d_5)^2}\right]^{3/2},$$
$$F_{VV} = \left[\left(\frac{2560}{27}\right)\frac{5 + 72 c_3 - 240 c_3^2}{(1-4c_3)^4}\right]^{3/2},$$
$$F_{VS} = \left[\frac{80640(91 - 312 c_3 + 432 c_3^2 + 384 d_5)}{1975 - 29808 c_3 (1 - 2c_3)(1 - 4c_3 + 8c_3^2)}\right]^{3/2}. \tag{13}$$

A more complete set of $\Sigma_{\rm IR}^{z_1 z_2}$, $\mathcal{M}^{z_1 z_2}$, as well as the resulting inequalities involving the $F_{z_1 z_2}(c_3, d_5)$ functions, are reported in the Supplemental Material [44].

The inequalities following from Eq. (12) are the main result of this discussion and can be read in several ways: as constraints on the plane of the graviton potential parameters (c_3, d_5) for a given graviton mass m and ratio $(\Lambda/\Lambda_3)^3 \equiv g_*$, as a constraint on g_* for fixed m at a given point in the $(c_3,\ d_5)$ region allowed by positivity, or equivalently as a bound on the graviton mass for fixed coupling at that point.

An important aspect of Eq. (12) is that it is possible to find an absolute maximum value of g_* above which our bounds do not allow for a solution: we write Eq. (12) as $m > m_{\rm min} \propto 1/F_{z_1 z_2}(c_3, d_5)$ and note that at each point (c_3, d_5) the bound is determined by the smallest $F_{z_1 z_2}$. Now, the positivity constraints Eq. (10) provide a compact allowed region in the $(c_3,\ d_5)$ plane, within which the (continuous) function $\min\{F_{z_1 z_2}\}(c_3, d_5)$ has a maximum. This corresponds to $(\hat{c}_3, \hat{d}_5) \approx (0.18, -0.017)$ and $F_{VS} \approx 4.6 \times 10^6$; thus, the most conservative bound

$$m > 10^{-32}\ {\rm eV}\left(\frac{g_*}{4.5 \times 10^{-10}}\right)^4 \left(\frac{\delta}{1\%}\right)^6. \tag{14}$$

Taking $m = 10^{-32}$ eV as a benchmark experimental upper bound on the graviton mass (see Ref. [52] for a critical discussion), any value $g_* \gtrsim 4.5 \times 10^{-10}$ is excluded, irrespectively of the values of $(c_3,\ d_5)$, a situation that we summarize in Fig. 1. Slightly stronger bounds can be obtained by working with the nonelastic channels, while if we were to admit a slightly larger uncertainty, e.g., $\delta = 5\%$, the upper bound on g_* would increase by 1 order of magnitude. Smaller values of the graviton mass, e.g., $m \simeq H_0 \simeq 10^{-33}$ eV, as is customary in cosmology, require an even smaller coupling. For a given mass, only as g_* is lowered sufficiently according to Eq. (14), a region allowed by our bounds eventually materializes inside the positivity region.

At this point, the crucial question is what the physical meaning of g_* is, and if it can be arbitrarily small, $g_* \lesssim 10^{-10}$ [18]. To our knowledge, most literature of massive gravity has so far taken $\Lambda = \Lambda_3$, or $\Lambda \gg \Lambda_3$, corresponding to $g_* \gtrsim 1$. These values are now grossly excluded by our bounds. From a theoretical point of view, Λ and Λ_3 scale differently with $\hbar$, so that their ratio actually changes when units are changed, in such a way that g_* indeed scales like a coupling constant. This is analogous to the difference between a vacuum expectation value v and the mass of a particle $\sim$coupling $\times\ v$ (e.g., the W-boson mass $m_W \sim gv$). The point, then, is that the cutoff Λ is a physical scale, which differs from Λ_3 that instead does not have the right dimension to represent a cutoff. Since $\Lambda_3^{-1} \approx 320\ {\rm km}(m/10^{-32}\ {\rm eV})^{-2/3}$, a very small coupling g_* translates into a very low cutoff (large in units of distance),

$$\Lambda \simeq (4.1 \times 10^5\ {\rm km})^{-1}\left(\frac{g_*}{4.5 \times 10^{-10}}\right)^{1/3}\left(\frac{m}{10^{-32}\ {\rm eV}}\right)^{2/3}. \tag{15}$$

This is clearly problematic. For example, let us consider the experimental tests of massive gravity in the form of bounds on fifth forces from the precise measurements of the Earth-Moon precession $\delta\phi$ [41,53,54]. Due to the Vainshtein screening [55,56], which is generically dominated by the Galileon cubic interactions in the $(c_3,\ d_5)$ region allowed by our bounds, the force mediated by the scalar mode compared to the standard gravitational one is $F_S/F_{\rm GR} \sim (r/r_V)^{3/2}$, where $r_V = (M/4\pi m_{\rm Pl})^{1/3}\Lambda_3^{-1} = (M/4\pi m^2 m_{\rm Pl}^2)^{1/3}$ is the Vainshtein radius associated with the (static and spherically symmetric) source under consideration, in this case the Earth, $M = M_\oplus$. Before our bound, one would find that at lunar distances, $r = r_{\oplus L} \approx 3.8 \times 10^5$ km, the ratio of forces, and thus also the precession $\delta\phi \sim \pi(F_S/F_{\rm GR})$, even if very small for $m = 10^{-32}$ eV, would be borderline compatible with the very high accuracy of present measurements $\sim 10^{-11}$. Now our result in Eq. (15) shows that the EFT is not valid already for $r \sim 1/\Lambda > r_{\oplus L}$. This implies that the Vainshtein screening should receive important corrections before reaching the (inverse) cutoff $1/\Lambda$, and moreover, it means that new degrees of freedom should become active at that scale: two effects that likely impair the fifth-force suppression and hinder the agreement with the precise measurement of the Earth-Moon precession. (Note that the Vainshtein redressing [57] deep inside the Vainshtein region, $\Lambda_3 \to z\Lambda_3$ with $z \gg 1$, does not generically extend to the physical cutoff Λ, since the rescaling of the kinetic term in a background does not affect the extra derivative terms that come without extra field insertions. In other words, a potentially large local kinetic term translates into a small local coupling, but it does not change the location of poles and thresholds associated with the UV degrees of freedom).

In summary, our theoretical bound Eq. (14) leads to a tension between direct limits on the graviton mass and fifth-force experiments. This either rules out Λ_3 massive gravity or implies that the theory is unable to make predictions at scales where GR instead agrees with experimental observations (this EFT of massive gravity cannot tell, e.g., whether an apple would fall to the ground or else go up). This calls for new ideas on extending the theory in the UV, corresponding here to macroscopic distances, a few 10^5–10^6 km as shown in Fig. 1, in such a way as to describe the relevant gravitational phenomena while remaining consistent with experimental tests (i.e., the new gravitational dynamics remaining undetected) not only in lunar experiments but also down to the millimeter.

Needless to say, our bounds apply neither to Lorentz-violating models of massive gravity (e.g., Ref. [58]), nor to theories with a massless graviton: one can avoid our bounds by dropping any of the assumptions on the S matrix that led to them. This is in practice not very different from finding explicit UV completions, since it also requires nontrivial dynamics in the UV.

There are several directions where our bounds can find fruitful applications. Immediate ideas involve theories with Goldstone particles, e.g., the EFT for the Goldstino from SUSY breaking or the R axion from R symmetry breaking, and the dilaton from scale symmetry breaking (all of which have interesting phenomenological applications [29–31,59,60]), as well as theories with suppressed 2-to-2 amplitudes but unsuppressed 2-to-3 amplitudes, e.g., Ref. [61]. It is also attractive to recast our bounds in diverse spacetime dimensions, such as massive gravity in $d = 3$ [62,63], or the conjectured a theorem in $d = 6$ (see, e.g., Ref. [64]); for the latter, we have obtained promising preliminary results. Another stimulating avenue is to use our bounds to extend the no-go theorems for massless higher-spin particles in flat space (see, e.g., Refs. [65–69]) to the case of small but finite masses. This could bring new insight on why light higher-spin particles cannot emerge, even in principle, in nongravitational theories without decoupling them or sending the cutoff to zero. One important open question, that for the time being remains elusive, is whether it is possible to extend our results to theories with massless particles with spin $J \geq 2$, thus providing new insights into the long-distance universal properties of the UV completion of quantum gravity, such as string theory, or into IR modifications of GR where the graviton remains massless, such as Horndeski-like theories.

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