QUANTIFYING THE EFFECT OF INSPECTIONS IN SHIPS CONSIDERING THE SPATIAL VARIABILITY OF CORROSION

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ABSTRACT

Ship structures are inspected regularly to reduce uncertainty associated with deterioration and help identifying optimal maintenance actions. The effect of inspections can be quantified by updating probabilistic models of the ship structure and deterioration processes with the information obtained during inspections. Most deterioration processes in ship structures, such as corrosion, are spatially distributed. In this contribution, we investigate how the spatial distribution and dependence of corrosion can be adequately addressed in Bayesian updating and how it affects the ship reliability. To model the spatial variability of corrosion in a ship structure, we apply a hierarchical spatial model. Inspections of the structure by means of thickness measurements are considered. Bayesian updating of the spatial corrosion model and the ship reliability is performed by means of the BUS approach. Finally, comparison is made with reliability estimates obtained with a model that neglect spatial variability and with the classical approach that defines separate random variables for the corrosion groups.

1. INTRODUCTION

Ship structures are exposed to multiple deterioration processes during their lifetime, among which corrosion loss is the most common and often the most critical one [1]. Corrosion types relevant to ship structures are uniform (general) corrosion, pitting corrosion, crevice corrosion and galvanic corrosion [2]. Uniform corrosion, which is the focus of this contribution, can reduce the structural capacity by a widespread reduction of plate thickness, leading to a loss in cross section. Corrosion progress is influenced by a variety of factors, such as age, type, cargo type, area of operation. These factors vary by ship and by position and type of elements within a ship structure. Empirical models are available to describe corrosion progress [3-6]. Considering the significant uncertainty associated with the many influencing factors and their effect, such models should be formulated probabilistically.

To manage corrosion and the associated uncertainties, inspections are carried out regularly on ship structures. Inspection results enable the determination of repair and replacement of corroded elements. Because these measurements are necessarily incomplete and subject to uncertainty, inspection results should themselves be described in a probabilistic format. These can be included in a Bayesian framework, in which the prior corrosion model is combined with the information from the inspection.

Corrosion processes in ship structures are spatially variable depending on the location and type of elements. For example, the bottom shell exposed to sea water or a tank with corrosive cargo are more vulnerable to corrosion than other elements in the ship. The mutual distance between different elements affects their dependence. Accordingly, plate elements of similar type and located closely to each other are likely to be in similar condition.

In this study, we investigate how the spatial variability and dependence of corrosion can be adequately addressed in a Bayesian analysis and how it effects the reliability. The spatial dependence of corrosion process among a mid-ship section is described by hierarchical spatial model following [7]. Effect of the spatial dependence on the time-variant reliability updated with thickness measurement data is quantified by means of the BUS approach [8, 9]. In the numerical example, the framework is exemplarily applied to a mid-ship section with thickness measurements. It is found that the effect of inspections on spatial dependent corrosion and the associated reliability can be quantified effectively through the Bayesian updating scheme.

2. PROBABILISTIC SPATIAL MODELING OF CORROSION IN SHIP STRUCTURES

2.1 TIME-DEPENDENT PLATE THICKNESS DUE TO CORROSION

The thickness w of a plate at time *t* due to uniform corrosion can be expressed as follows:

$$w(t) = w_0 + M - D(t) \quad (1)$$

where w_0 is the design plate thickness, *M* is the thickness margin, and *D* is the corrosion loss. The thickness margin accounts for the difference between the as-built and the planned thickness that is caused by plate fabrication and the ship building process. In this study, the thickness margin *M* is modeled as a lognormally distributed variable assuming non-negative margins [7]. The corrosion loss is calculated with a bi-linear corrosion model, which assumes that the corrosion loss is zero before the coating breaks and afterwards increases at a constant rate [10].

2.2 HIERARCHICAL REPRESENTATION OF CORROSION IN SHIP STRUCTURE

Classically, spatial variability of corrosion in ship structures has been addressed by grouping structural elements depending on the location and type (e.g. bottom shell, side shell, deck plating), and modeling them with separate random variables, assuming independence among different groups [2, 11, 12]. Spatial dependence among corrosion at different locations can be described through random fields [13]. However, similarities in the corrosion processes at elements that are distant but belong to the same type of structural elements are not adequately represented by the random field. For this reason, Luque et al. [7] proposed to represent the spatial distribution of corrosion rate *R,* coating life *C* and thickness margin *M* through a hierarchical model. They define multiple hierarchical levels, e.g. fleet, frame, compartment, structural element according to which the elements are grouped (Figure 1). The basic element of the model is the plate.

In the hierarchical model, the correlation between different elements is defined according to the level of hierarchies that are shared by two plates. In the lowest level of hierarchy, i.e. two plates in the same structural element, a random field defines their correlation. This model is capable to represent spatial dependence in function of location, type and mutual distance of elements. In [7], the proposed model was learnt with thickness measurement data from several inspection campaigns.

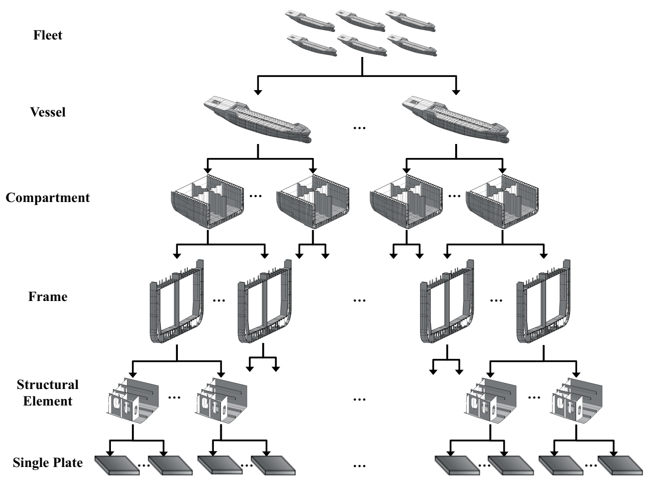
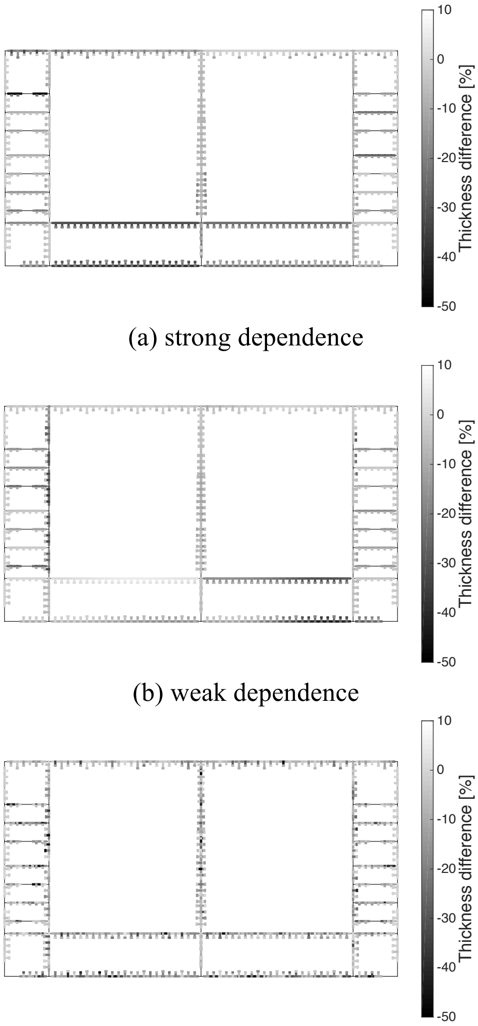


Figure 1. Hierarchical structure of the spatial corrosion model taken from [7]

In this study, the four hierarchical levels frame, cell, structural element, and single plate are considered to represent the spatial dependence of corrosion parameters in a mid-ship section. Hereafter the four hierarchical levels are denoted by level 1, 2, 3 and 4. The correlation between the logarithmic corrosion rate at two different plate elements is calculated as

$$\gamma_q = \rho\left(\log R_{x_1}, \log R_{x_2}\right) = \frac{\sum_{k=1}^{q} \sigma_k^2}{\sum_{k=1}^{4} \sigma_k^2} \quad (2)$$

where x_1 and x_2 indicate the spatial coordinates of two plate elements; σ_k is the standard deviation of the logarithm of corrosion rate variability at level i_k ; q is the lowest level of hierarchy that is common to two plates. Correlation at level i_4, i.e. between two plate elements in the same frame, cell and structural element type, additionally includes the correlation matrix $\mathbf{\Pi}_{1:3}$ that defines a random field [7]. For simplicity, the same hierarchical model is here used for representing spatial dependence of corrosion rate, coating life and thickness margin.



(a) strong dependence

(b) weak dependence

(c) independence

Figure 2. Realizations of thickness reduction due to corrosion loss with different spatial dependence

Figure 2 illustrates the effect of spatial dependence on random realizations of corrosion loss in a midship section. Variation of line color shows how much the simulated corrosion loss deviates from the average corrosion loss for the structural type. Clearly, neglecting spatial dependence would lead to an unrealistic representation of the corrosion processes in ship structures.

3. LOAD AND RESISTANCE MODELS

3.1 ULTIMATE BENDING MOMENT CAPACITY

The thickness loss caused by corrosion leads to a reduction of capacity. Because of the time-dependent property of corrosion loss, the strength of a ship section M_u becomes a function of time t.

The resistance of the hull girder section against bending is the ultimate moment capacity. It can be calculated by the incremental curvature method described in IACS guideline [13]. In this contribution, we alternatively utilize an approach based on optimization.

3.2 VERTICAL BENDING MOMENT

The total bending moment is calculated as the sum of stillwater moment and wave-induced moment. These are a function of operational condition, cargo history, sea states, and additional parameters.

The stillwater bending moment M_{sw} is here represented by a normal distribution [12, 10-17]. Based on [19], mean value and standard deviation are defined as $\mu_{sw} = 0.70 M_{sw,d}$ and $\sigma_{sw} = 0.20 M_{sw,d}$, where $M_{sw,d}$ is the design stillwater moment calculated with IACS guidelines [14].

The probability distribution of the extreme wave-induced moment M_{wv} is the Gumbel distribution with scale parameter a_{wv} and location parameter b_{wv} [17]. These parameters are associated with the design load and wave period by $b_{wv} = w(\log n_{w,T})^{1/k}$ and $a_{wv} = \frac{w}{k}(\log n_{w,T})^{\frac{1-k}{k}}$, where $n_{w,T}$ is the mean number of wave loads in the return period T. w and k are the scale and shape parameters of the associated long-term distribution of the wave-induced moment. k is dependent on the environmental conditions but can be taken as 1.0 if no further information is available [20]. w is calculated by requiring that the design load $M_{wv,d}$ is exceeded by the long-term wave-induced moments with a probability of 10^{-8}, i.e. $\Pr[M_{wv} > M_{wv,d}] = 10^{-8}$. The design load $M_{wv,d}$ is also calculated based on IACS guidelines [14]. To calculate $n_{w,T}$ for a return period $T = 1$ year, the wave period is defined as 8s, which is representative of the North Atlantic [17], and it is assumed that 45% of operation time was is spent in ballast condition. Eventually, summing up the normal distribution of stillwater moment M_{sw} and the extreme value distribution of wave-induced moment M_{wv} determines the total bending moment acting on the ship cross section.

3.3 RELIABILITY

In structural reliability, the failure event F, e.g. collapse of a ship due to an exceedance of the vertical bending moment capacity, is expressed by the failure domain Ω_F. The associated random variables $\mathbf{X}$ and the limit state $g(\mathbf{X})$ define the failure domain as $\Omega_F = \{g(\mathbf{X}) \leq 0\}$.

The reliability is here expressed by the annual probability of failure without strict consideration of survivor or failure of the ship in preceding years. The limit state function defining failure is:

$$g(\mathbf{X}, t) = X_u M_u(\mathbf{X}_s, t) - X_{sw} M_{sw} - X_{wv} M_{wv} \quad (4)$$

where $\mathbf{X}_s$ are the random variables affecting the ultimate moment strength, X_u represents the model uncertainties associated with ultimate moment strength calculation, X_{sw} and X_{wv} are the model uncertainties related to the stillwater and wave-induced moment load calculations. They are defined in Table 1 following [18]. All random variables are combined in the vector $\mathbf{X} = [\mathbf{X}_s, X_u, X_{sw}, X_{wv}, M_{sw}, M_{wv}]^T$.

Table 1. Random variables related to model uncertainties

Random variables	Mean	COV	Distribution
X_u	1.0	0.10	Normal
X_{sw}	1.0	0.05	Normal
X_{wv}	0.9	0.15	Normal

4. BAYESIAN UPDATING

In Bayesian analysis, the data (or observation event) Z is expressed by a likelihood function. Here, the observation event is a set of plate thickness measurements. Following the BUS approach (Bayesian Updating with Structural reliability methods) proposed in [8, 9, 21], the likelihood function L describing Z can be cast into a structural reliability framework, by defining the observation limit state function:

$$h(\mathbf{X}, U_0) = \{U_0 - \Phi^{-1}[cL(\mathbf{X})] \leq 0\} \quad (5)$$

U_0 is a standard normal random variable, Φ^{-1} is the inverse standard normal cumulative distribution function, and c is a positive constant that can be chosen following [8]. The observation limit state function defines a corresponding domain $\Omega_Z = \{h(\mathbf{X}, U_0)\}$. Then, the updated reliability conditional on the observation event Z, can be calculated as:

$$\Pr(F|Z) = \frac{\Pr(F \cap Z)}{\Pr(Z)} = \frac{\int_{\mathbf{X}, u_0 \in \{\Omega_F \cap \Omega_Z\}} f_{\mathbf{X}}(\mathbf{X}) \mathrm{d}\mathbf{X} \mathrm{d}u}{\int_{\mathbf{X}, u_0 \in \Omega_Z} f_{\mathbf{X}}(\mathbf{X}) \mathrm{d}\mathbf{X} \mathrm{d}u} \quad (6)$$

Any classical structural reliability method can be used to solve Eq. (6). Here, we employ subset simulation, which was originally proposed in [22]. This method expresses the failure event as the intersection of intermediate events and the probability of failure is estimated as a product of conditional probabilities of the intermediate events. In this way small failure probabilities can be estimated efficiently even in high-dimensional problems. The adaptive MCMC algorithm proposed in [23] is used to improve accuracy and efficiency of subset simulation.

5. NUMERICAL EXAMPLE

5.1 SHIP EXAMPLE

The mid-ship section of a tanker, taken from [12], is modified for this numerical example. The properties of the ship are as follows: length L=255m, breadth B=57m, height H=31.1m, block coefficient C_B=0.842, mean value of the Young's modulus E=207,000MPa, mean value of the yield stress σ_y=353MPa. The mid-ship cross section is shown in Figure 4. The section consists of 400 stiffened plate elements.

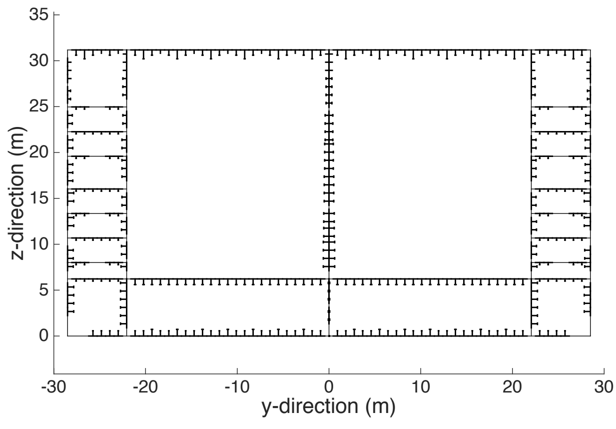


Figure 4. Geometry of the hull girder section at mid-ship

Corrosion parameters of the structural elements are defined for 8 groups depending on their position and type (Table 2). The corrosion rate R and the coating life C_t are modeled as lognormal random variables. The remaining random variables are summarized in Table 3.

Table 2. Corrosion rate [mm/year] for different locations and structural element type (Adapted with modifications from [11, 12])

Corrosion group	Location	Mean	COV	Distribution
1	Bottom shell plating	0.34	0.5	Lognormal
2	Bottom stiffener/web	0.13	0.5	Lognormal
3	Deck plating	0.13	0.5	Lognormal
4	Deck stiffener/web	0.13	0.5	Lognormal
5	Side shell plating	0.06	0.1	Lognormal
6	Side stiffener/web	0.06	0.1	Lognormal
7	Inner plating	0.13	0.1	Lognormal
8	Inner stiffener/web	0.13	0.1	Lognormal

Table 3. Random variables related to ship structure

Random variables	Mean	COV	Distribution
Coating life [year]	5	0.40	Lognormal
Thickness margin [mm]	1.0	0.15	Lognormal
Yield stress [MPa]	353	0.10	Lognormal
Young's modulus [MPa]	207,000	0.03	Lognormal

To numerically represent the spatial variability of corrosion processes, the ship section is discretized by individual plating, web and flange. The 400 stiffened plates are divided into 1,200 discretization elements. The corrosion loss at each element is determined by the probabilistic models of corrosion rate, coating life and the thickness margin. The hierarchical spatial model defines the correlation between the corrosion process at the elements considering their location, type and mutual distance according to the four hierarchical levels. The ship cross section consists of one frame (level 1), 22 cells (level 2) and 5 structural element types (level 3). Correlation lengths in y- and z-direction are defined as $L_y = 5\,m$ and $L_z = 3\,m$ assuming stronger spatial dependence in transverse direction.

For illustration purposes, we define a weak and a strong spatial dependence case, as summarized in Table 4. These are compared to the classical approach, in which separate random variables are defined for the 8 corrosion groups of Table 2. In addition, we consider a model with all elements treated as independent.

Table 4. Correlation at each hierarchical level defined for two spatial dependence cases

Correlation of elements	Weak dependence	Strong dependence
γ_1 (Frame)	0.04	0.10
γ_2 (Cell)	0.19	0.32
γ_3 (Struct. elem.)	0.53	0.70

5.2 RELIABILITY ANALYSIS

Figure 5 shows the time-variant reliability for the different model assumptions. In the early phase (until year 5), only minor differences are observed, since the effect of corrosion is limited because of coating. With increasing time t, the effect of spatial dependence becomes more apparent. Larger spatial dependence is associated with smaller reliability. The classical approach and the hierarchical model with strong dependence lead to similar reliability estimates.

The effect of the dependence model is further illustrated in Figure 6, which shows the distribution of ultimate moment capacity. The mean value is constant, but the standard deviation of the capacity is strongly influenced by the spatial dependence. This is to be expected, since the capacity corresponds to a linear function of the element properties.

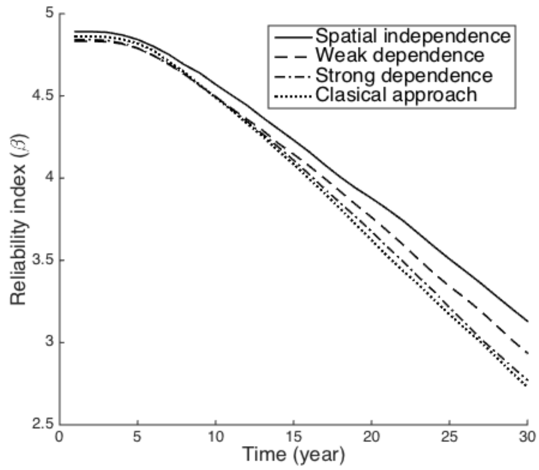


Figure 5. Time-variant reliability under varying dependence assumptions

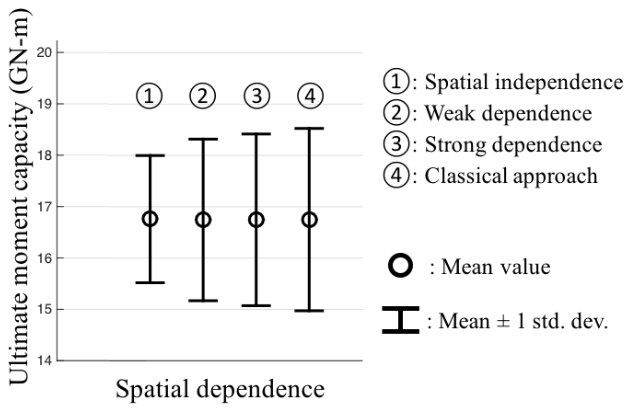


Figure 6. Statistical properties of the ultimate moment capacity under varying dependence assumptions (at year 25)

These results indicate that the classical approach assumes a strong spatial dependence overall by representing spatial variability of the elements with a few random variables. The statistical properties of the ultimate moment strength are also summarized in Table 5.

5.3 RELIABILITY UPDATING WITH THICKNESS MEASUREMENTS

To demonstrate the reliability updating, a set of hypothetical thickness measurement data is generated based on a Monte Carlo simulation of the plate thicknesses. Measurements have an error that follows a normal distribution with σ_{ϵ}=1mm, i.e. $\epsilon \sim N(0, \sigma_{\epsilon})$. It is assumed that 13 elements in the mid-ship section are inspected at 5, 15, and 25 years (Figure 7). The utilized thickness measurement data are summarized in Figure 8.

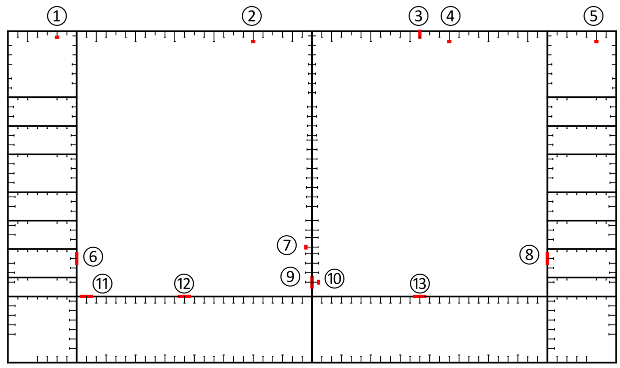


Figure 7. Locations of 13 elements assumed to be inspected regularly

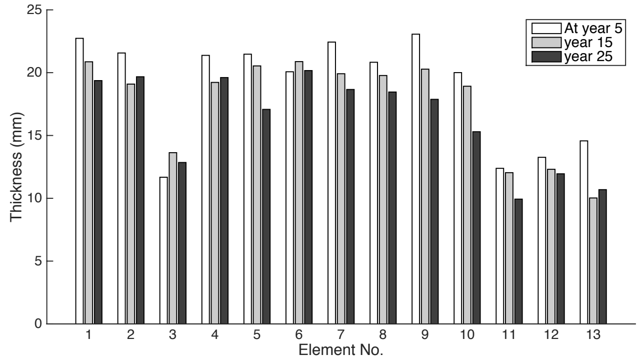
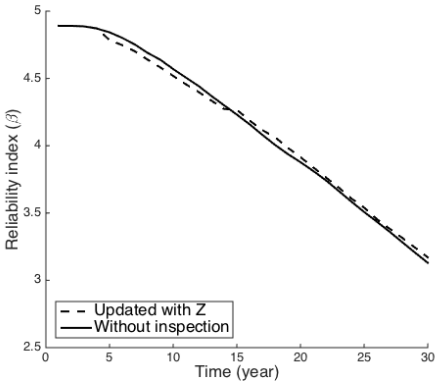


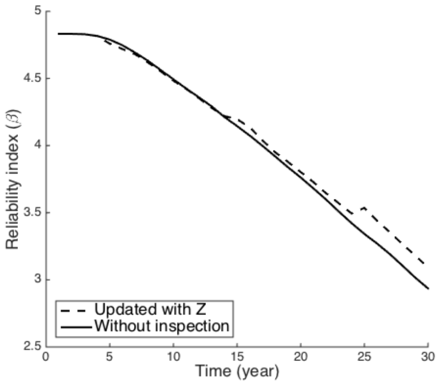
Figure 8. Thickness measurements of the inspected elements at 5, 15 and 25 years

The time-variant reliability updated with the thickness measurements is shown in Figure 9. Note that the assumption of spatial independence results in negligible changes of reliability estimates.

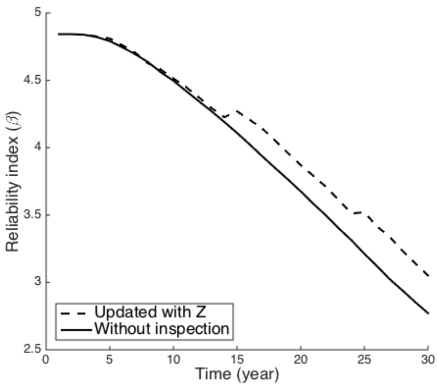
To better understand the effect of spatial dependence assumptions on the updated reliability estimate, statistical properties of the updated ultimate moment capacity at year 25 are shown in Figure 10. The reference moment capacity calculated with the remaining plate thicknesses underlying the generated measurement data is also plotted. Statistical properties of the updated ultimate moment capacity and the reference value are summarized in Table 5.



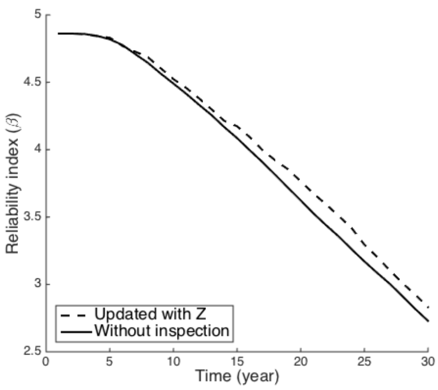
(a) spatial independence



(b) weak dependence



(c) strong dependence



(d) classical approach

Figure 9. Reliability updated with the observation events Z under varying dependence assumptions

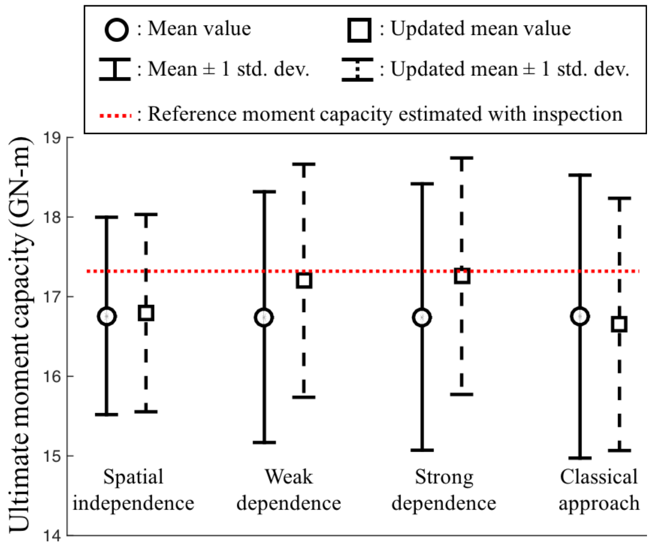


Figure 10. Comparative plots of statistical properties of the ultimate moment strength updated with different observation events and spatial dependence (at 25^{th} year)

Table 5. Comparison of prior/updated statistical properties of the ultimate moment strength and the exact ultimate moment strength [in GN-m] (at 25th year)

Spatial dependence	Without inspection		Updated		$M_{u,ref}$
	μ_{M_u}	σ_{M_u}	μ_{M_u}	σ_{M_u}	
Spatial independence	16.74	1.24	16.79	1.24	17.32
Weak dependence	16.74	1.56	17.20	1.46	
Strong dependence	16.75	1.68	17.26	1.48	
Classical approach	16.75	1.77	16.65	1.58	

For the investigated example, the classical approach is less effective than the hierarchical models when computing the reliability conditional on the inspection data, even though it assumes a strong dependence and leads to accurate results in the prior case. It remains to investigate to what degree this result can be generalized.

5. SUMMARY AND CONCLUSIONS

The effect of spatial dependence of the corrosion process on the time-variant reliability is investigated. In particular, the conditional reliability estimate given inspection data is compared for different assumptions on the dependence structure.

The spatial variability of corrosion in a mid-ship section was described through a hierarchical approach. Results from this model were compared to those obtained under the assumption of spatially independent elements and with the classical approach, in which a few random variables represent the parameters of the corrosion process in groups of elements that belong to the same structural element type and location.

The exemplarily results show that the simplified assumptions of the classical approach lead to good estimates of the reliability when no inspections are available. When including inspection results, it leads to a slight misestimation of the posterior mean, and an underestimation of the remaining reliability. However, it remains to be investigated if and to what extend this result can be generalized. Nevertheless, the hierarchical model better reflects the real situation and is preferable. As demonstrated in this paper, reliability analysis and updating with this model is computationally feasible with subset simulation, whose performance does not depend directly on the number of random variables.

It is expected that the effect of the implemented hierarchical spatial model will be more crucial when the inspected locations are few and not well distributed along the ship section. The effect of the correlation assumptions will also be more crucial when assessing the whole ship structure or a fleet of ships.

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