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Parametric Nonlinear Model Reduction

for Structural Dynamics

calm smooth smart SPP 1897

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Abstract

Finite element analyses of advanced mechanical structures that undergo large deformations usually lead to high-dimensional systems of nonlinear equations. For applications such as parameter studies and optimization, where the equations of motion have to be solved several times, it is highly demanded to reduce the computation time for solving the equations. Model order reduction can satisfy this demand by projecting the equation of motion onto a subspace and approximating the nonlinear term by using Hyperreduction techniques.

However, the best subspace for projection as well as the hyperreduction strongly depend on the system properties which is a main issue for applications like parameter studies, optimization and control, where the system properties, such as shape, material, and boundary conditions, are parametric.

This contribution gives an overview over the reduction process for parametric structures with large deflections that are simulated with the finite element method. It also shows the main features and modular structure of the open source research code "AMfe", which is used by the authors to develop new reduction techniques.

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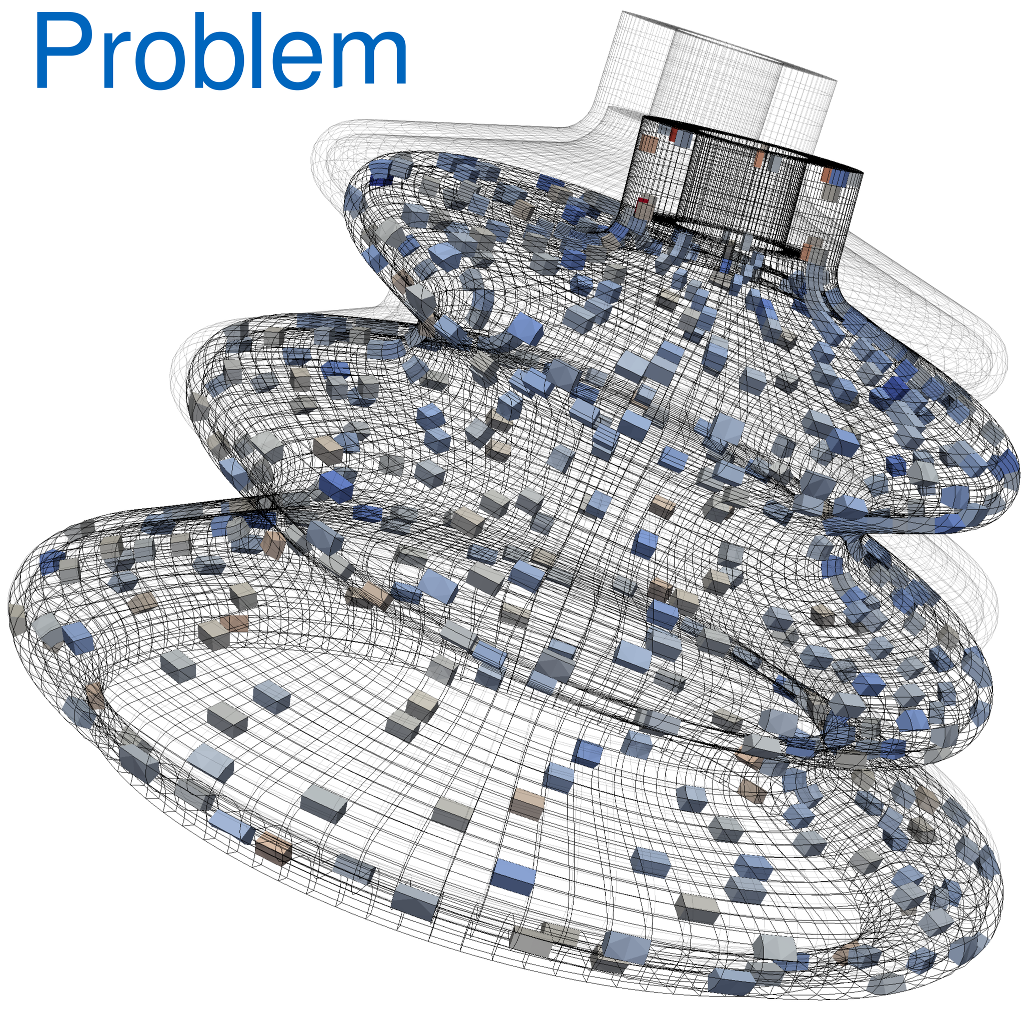
Problem

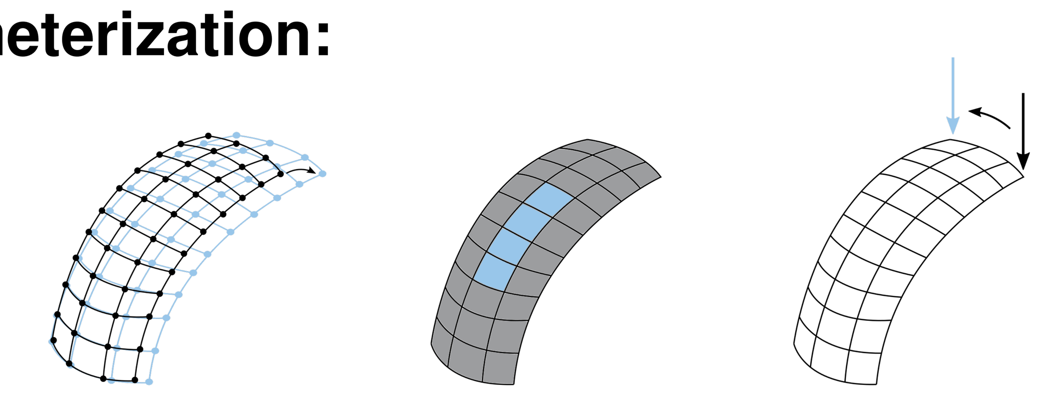


PDE: $\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} - \rho \ddot{\boldsymbol{u}} = \boldsymbol{0}$ + BCs + ICs

FE: $\boldsymbol{M}(\boldsymbol{p})\ddot{\boldsymbol{u}}(t) + \boldsymbol{f}(\boldsymbol{p}, \boldsymbol{u}(t)) = \boldsymbol{B}(\boldsymbol{p})\boldsymbol{F}(\boldsymbol{p}, t)$

Nonlinear term $\boldsymbol{f}(\boldsymbol{p}, \boldsymbol{u})$ allows for large deformations

Parameterization:

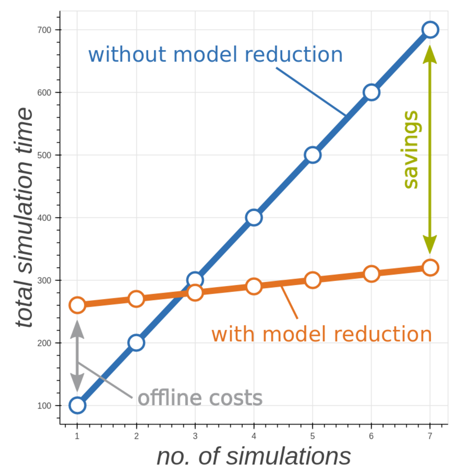


Parameter-Set: $\boldsymbol{p} = \{p_1,\ p_2,\ \dots\ p_p\} \in \mathbb{P}$

Model Reduction

Problem: Parameter studies and optimization of large models are *very time consuming*

Idea: Reduce computational effort for solving equations of motion by applying model reduction



Galerkin Projection

$$\boldsymbol{u} = \boldsymbol{V}\boldsymbol{q} + \boldsymbol{\varepsilon} \approx \boldsymbol{V}\boldsymbol{q} \quad \text{with} \quad \boldsymbol{V} = \left[\boldsymbol{V}_{\text{lin}} \middle| \boldsymbol{V}_{\text{nl}}\right]$$

$$\rightsquigarrow \quad \boldsymbol{V}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{V}\ddot{\boldsymbol{q}} + \boldsymbol{V}^{\mathrm{T}}\boldsymbol{f}(\boldsymbol{V}\boldsymbol{q}) = \boldsymbol{V}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{F}$$

Methods:

	Linear part V_{lin}	*Nonlinear part V_{nl}*
Modal truncation	Vibration modes	Modal derivatives Static derivatives
Moment matching	Krylov directions	Krylov derivatives Static derivatives
Other linear methods	Linear basis vectors	Exact derivatives Static derivatives

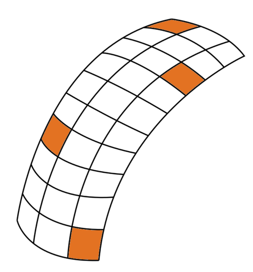
Evaluation & optimization via system norm

+

Hyperreduction

$$\boldsymbol{V}^T \boldsymbol{f}(\boldsymbol{V}\boldsymbol{q}) = \sum_{e \in E} \boldsymbol{V}^T \boldsymbol{B}^T \boldsymbol{f}_e(\boldsymbol{B}_e \boldsymbol{V}_e \boldsymbol{q})$$

$$\approx \sum_{e \in \tilde{E} \subset E} \boldsymbol{V}^T \boldsymbol{L}_e \boldsymbol{B}_e^T \boldsymbol{f}_e(\boldsymbol{B}_e \boldsymbol{V}_e \boldsymbol{q})$$



Hyperreduced Mesh.

Methods:

- DEIM
- ECSW
- Polynomial Expansion

Parametric Reduction

Same methods at each parameter sampling point $\boldsymbol{p}_i \in \mathbb{P}$ $(i = 1, 2, \dots, N)$

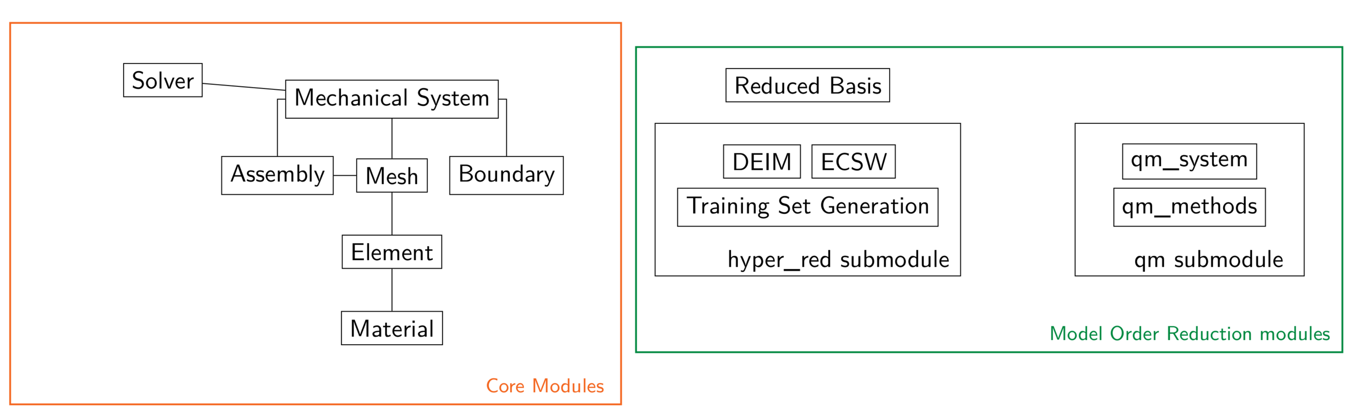
$$\rightsquigarrow \quad \boldsymbol{V}(\boldsymbol{p}_i)$$

Methods:

Local approaches	*Global approaches*
Basis updating $\boldsymbol{V}(\boldsymbol{p}_i)$	Concatenation to global basis
Basis interpolation $\boldsymbol{V}(\boldsymbol{p}_i)$	$[\boldsymbol{V}(\boldsymbol{p}_1)\ \boldsymbol{V}(\boldsymbol{p}_2)\ \cdots\ \boldsymbol{V}(\boldsymbol{p}_N)]$
Matrix (system) interpolation $\mathcal{S}_r(\boldsymbol{p}_i)$	Global parameter-dependent basis $\boldsymbol{V}(\boldsymbol{p})$

Research FE Code for Nonlinear Model Reduction

Module Structure:



AppliedMechanics/AMfe.git

Main features:

- Solve nonlinear structural dynamics problems
- Modular Structure
- Interpretable (no input files!)
- Easy access to internal computations
- Rapid Prototyping of new model reduction methods

Model Reduction Features:

- Calculation of reduced bases
- Hyperreduction Techniques