

# Covariant errors in ocean retrackerers evaluated using along-track cross-spectra

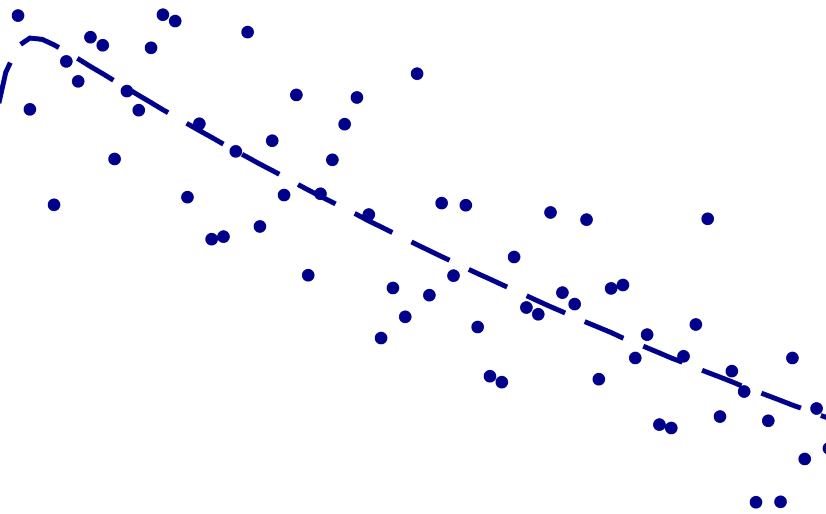
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# Random inputs yield random outputs



Waveforms graph the power in a random process.

Retrackers estimate model parameters: range, SWH,  $\sigma^0$ ,  $(\text{ONA})^2$ .

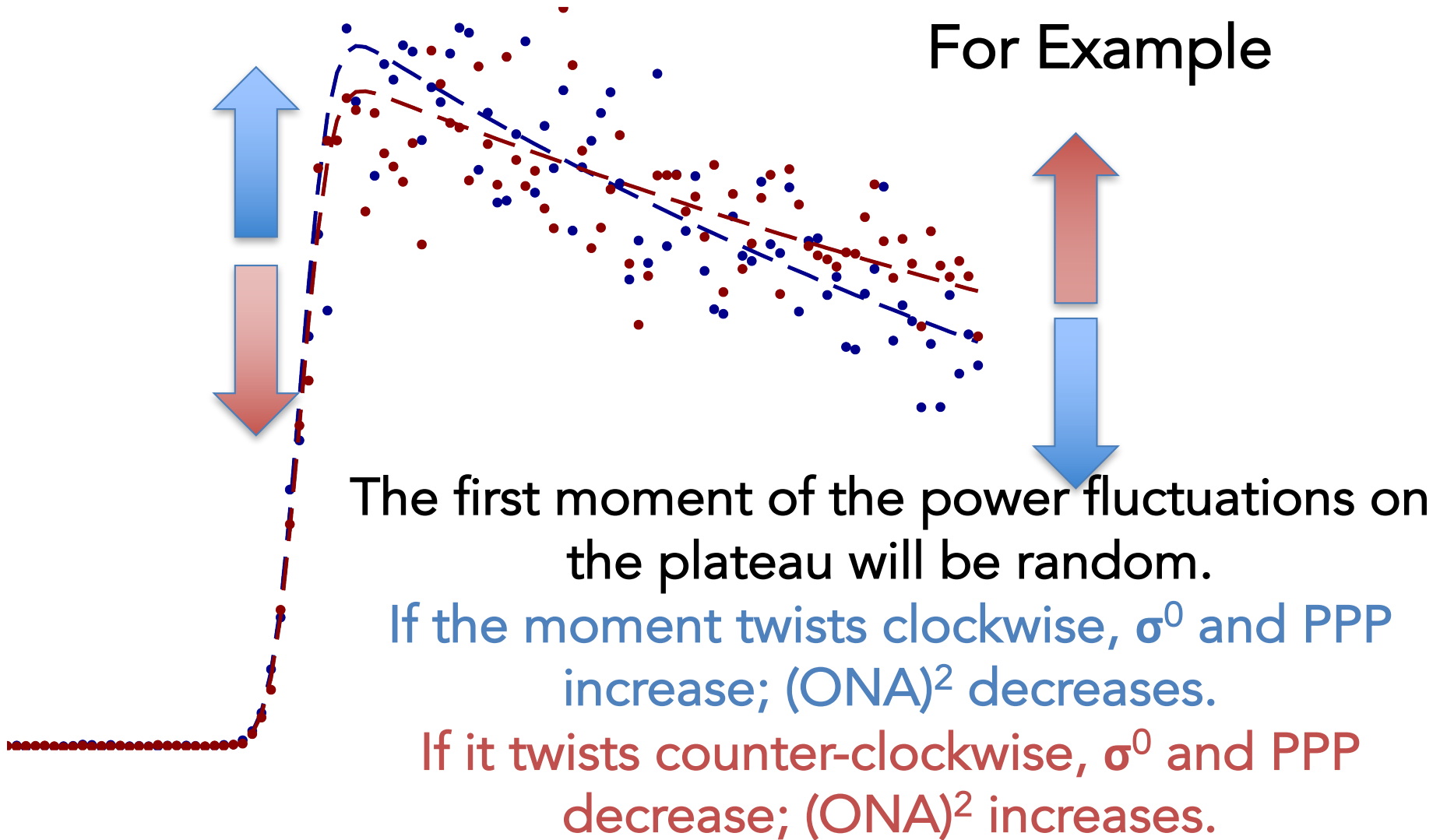
The estimates have errors.

The errors are inevitably correlated.



# Retracker errors correlated by the waveform plateau

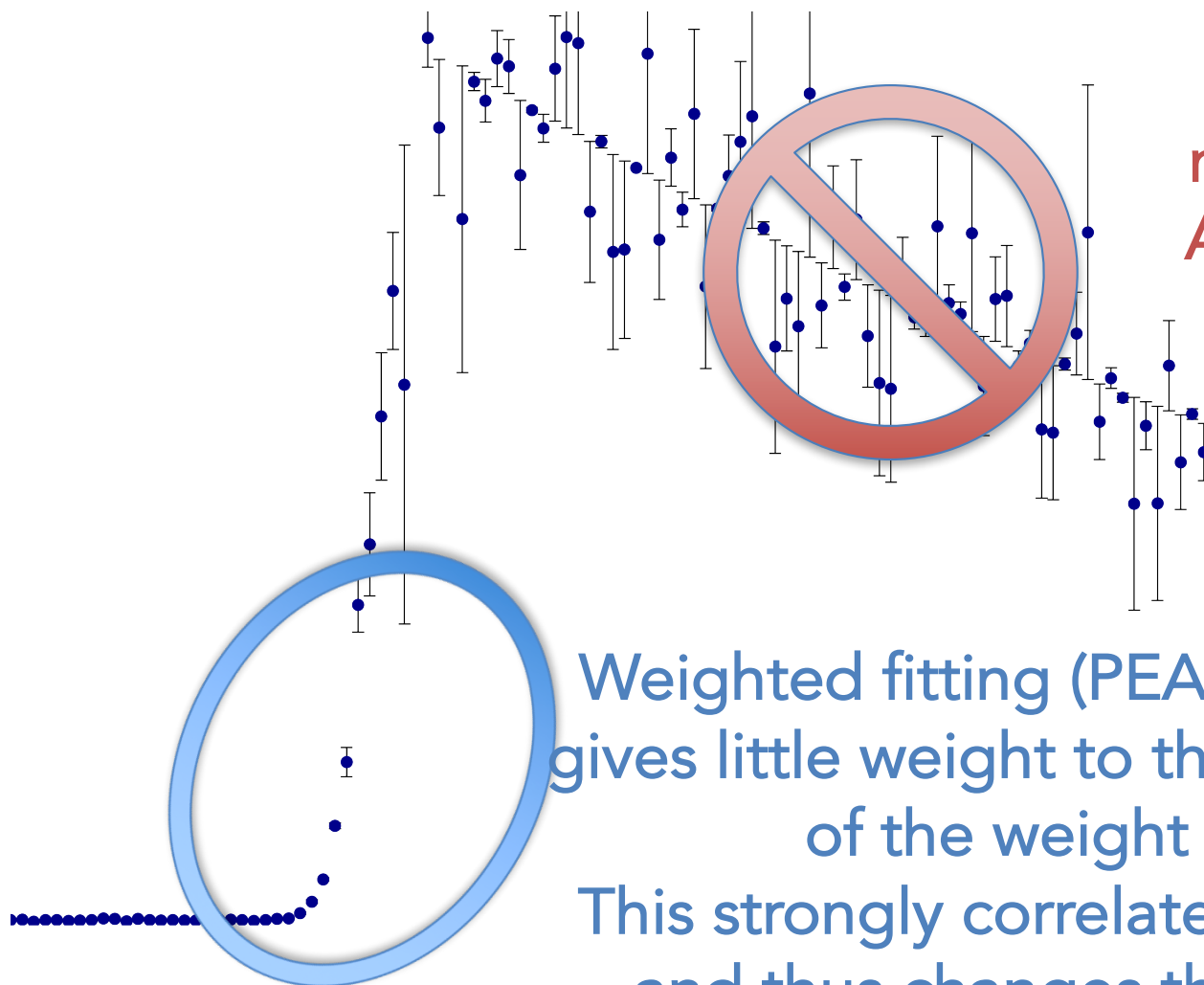
For Example





## Different trackers yield different error correlations

MLE3 and ALES do not estimate  $(ONA)^2$ . ALES does not fit the plateau much at all.



Weighted fitting (PEACHI Nelder-Mead) gives little weight to the plateau and most of the weight to the toe.

This strongly correlates SWH and range, and thus changes the sea state bias.



## Retrackers Analyzed

### Unweighted Retracker:

- ALES: Fits leading edge only; no  $(ONA)^2$
- MLE3 Fits all waveform without  $(ONA)^2$
- MLE4 Fits all waveform with  $(ONA)^2$
- PEACHI Newton-Raphson, all with  $(ONA)^2$

### Weighted Retracker:

- PEACHI Nelder-Mead, all with  $(ONA)^2$



## Parameters analyzed

- RAWSSHA (orbit height minus retracked range minus mean sea surface)
- SWH
- $\sigma^0$
- PPP (pulse peakiness parameter)
- $(ONA)^2$ , if the retracker estimates this

Data analysis is at 20 Hz sampling for Jason-2 and Jason-3, and at 40 Hz sampling for SARAL.



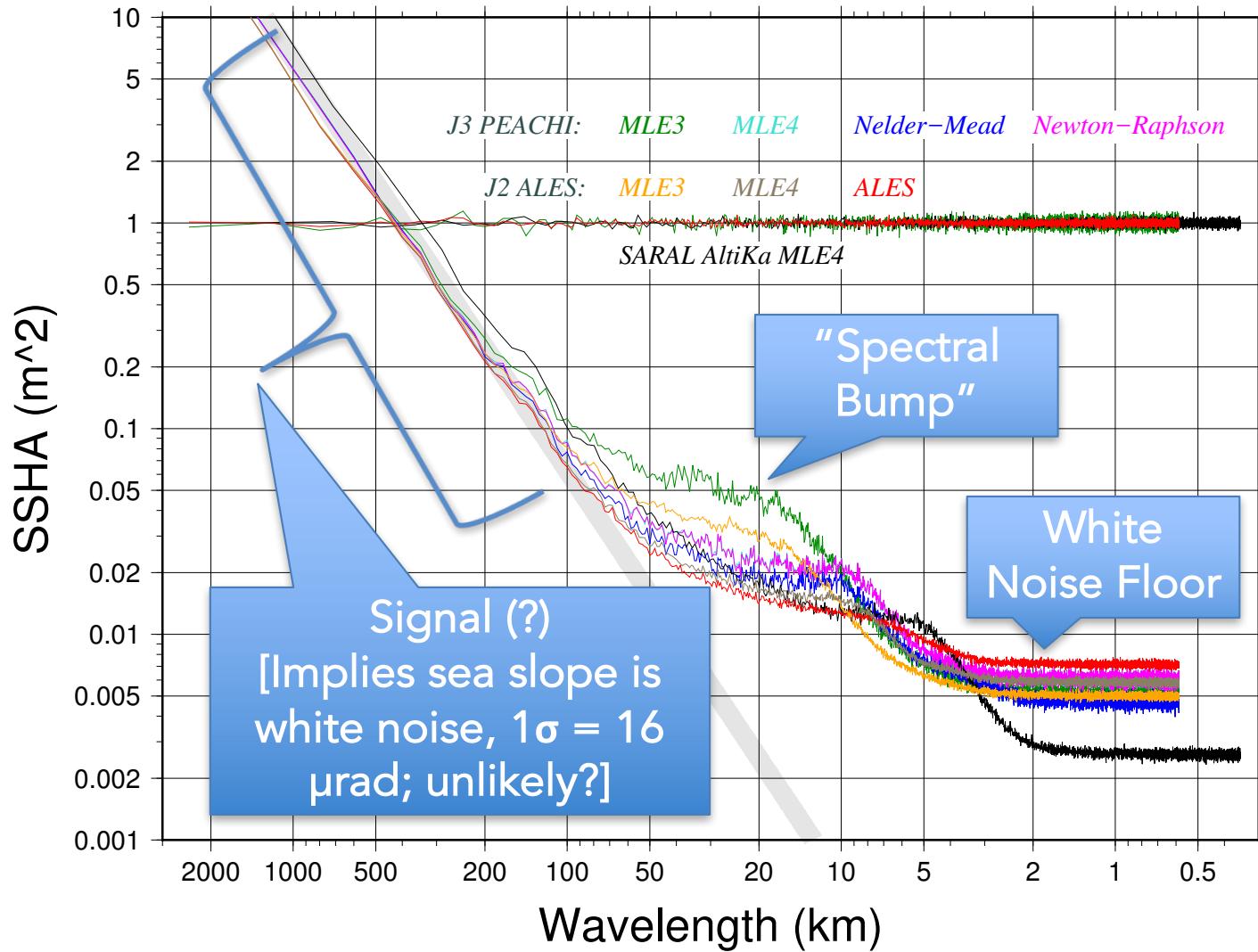
# Data Analyzed

Passes through the “South Pacific SAR Box”  
(longitude  $200^{\circ}$  to  $275.2^{\circ}$ , latitude  $-25.5^{\circ}$  to  $-2.5^{\circ}$ ) where F.  
Boy identified a “spectral bump”

- Jason-2 Cycles 1-40: ALES, MLE3, MLE4
  - 397 days beginning 12 July 2008 (1633 passes)
- Jason-3 Cycles 1-13: PEACHI, MLE3, MLE4
  - 129 days beginning 17 February 2016 (442 passes)
- SARAL Cycles 9-12: MLE4
  - 140 days beginning 19 December 2013 (1115 passes)



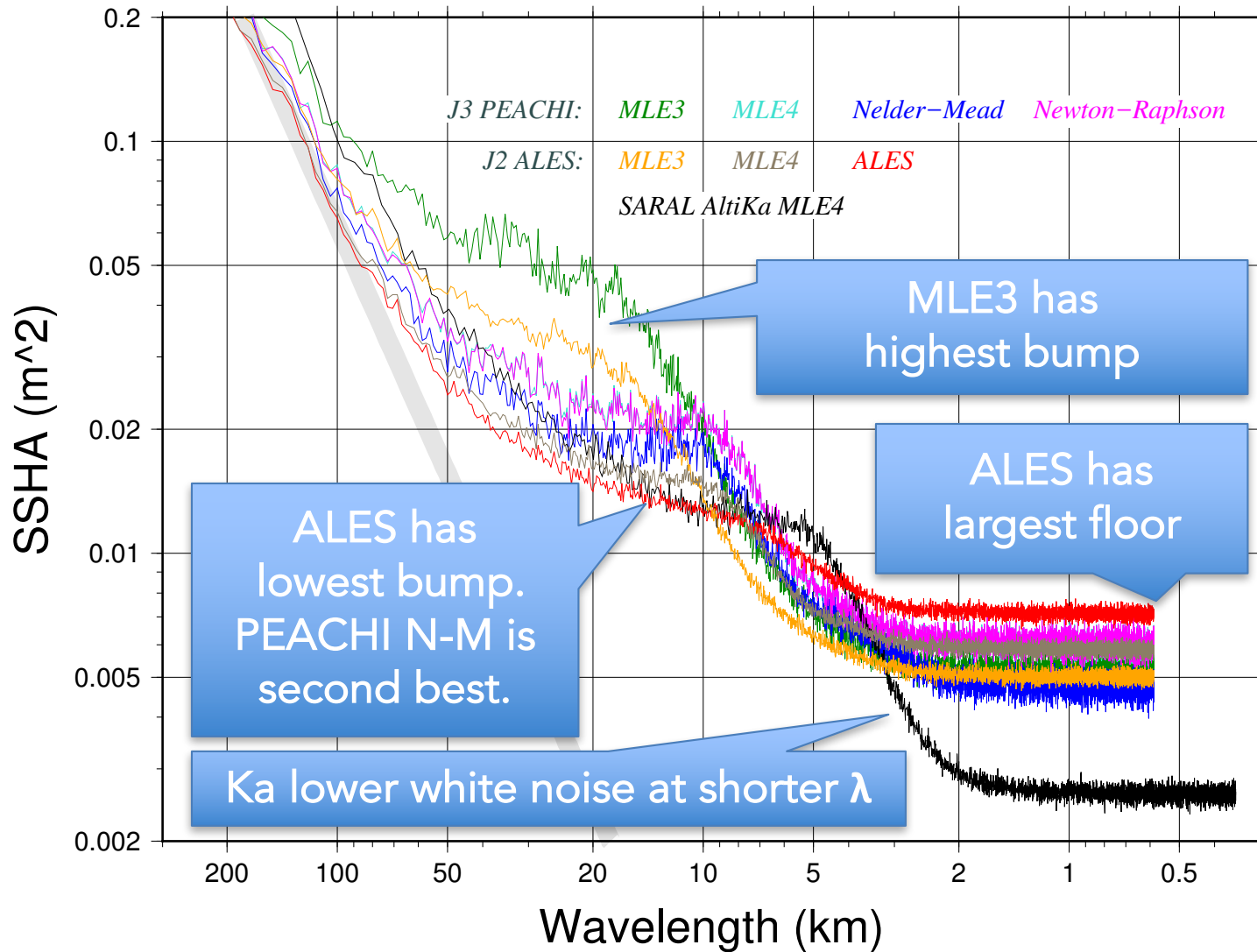
# SSHA (Auto-) Spectra







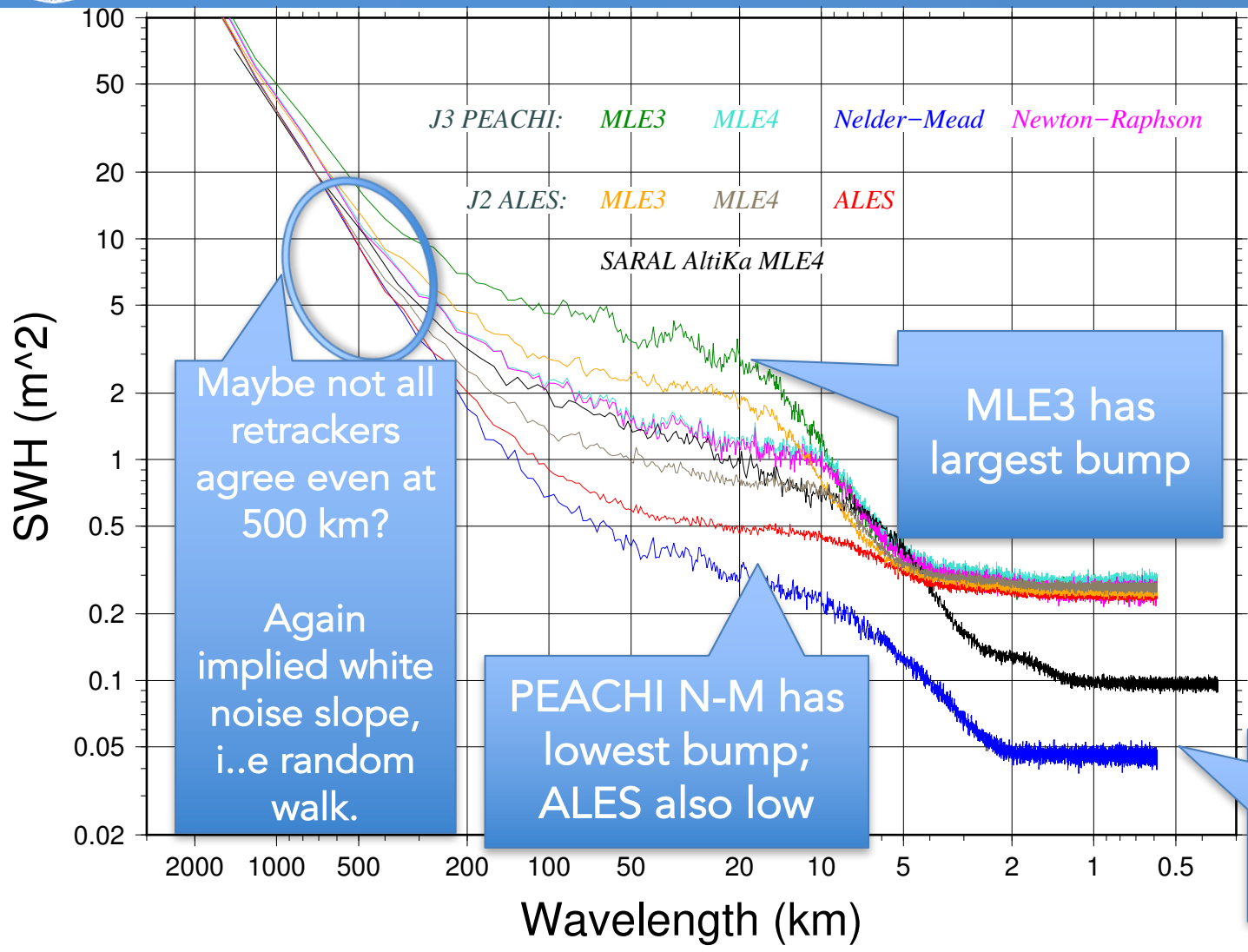
# SSHA (Auto-) Spectra: bump and noise



ALES has higher terminal noise because it is constrained by fewer data (it ignores the tail of the plateau). But this is an advantage in the spectral bump region, where it has lowest noise.



# SWH (Auto-) Spectra



Peachi Nelder-Mead has the lowest noise in SWH. ALES is also very good.

Maybe not all retracker agree even at 500 km?

Again implied white noise slope, i.e random walk.

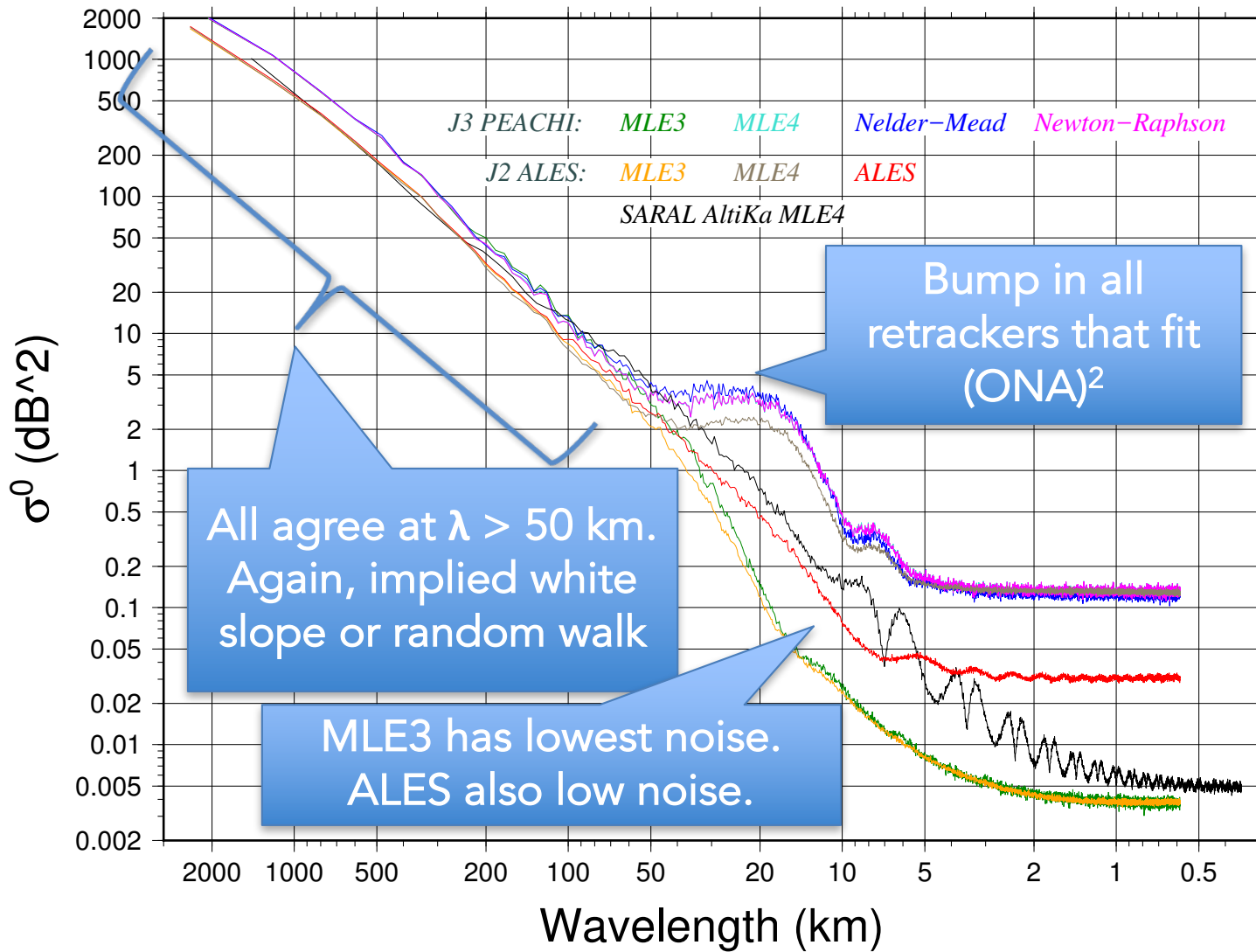
MLE3 has largest bump

PEACHI N-M has lowest bump; ALES also low

Peachi N-M has lowest white noise floor



# $\sigma^0$ (Auto-) Spectra



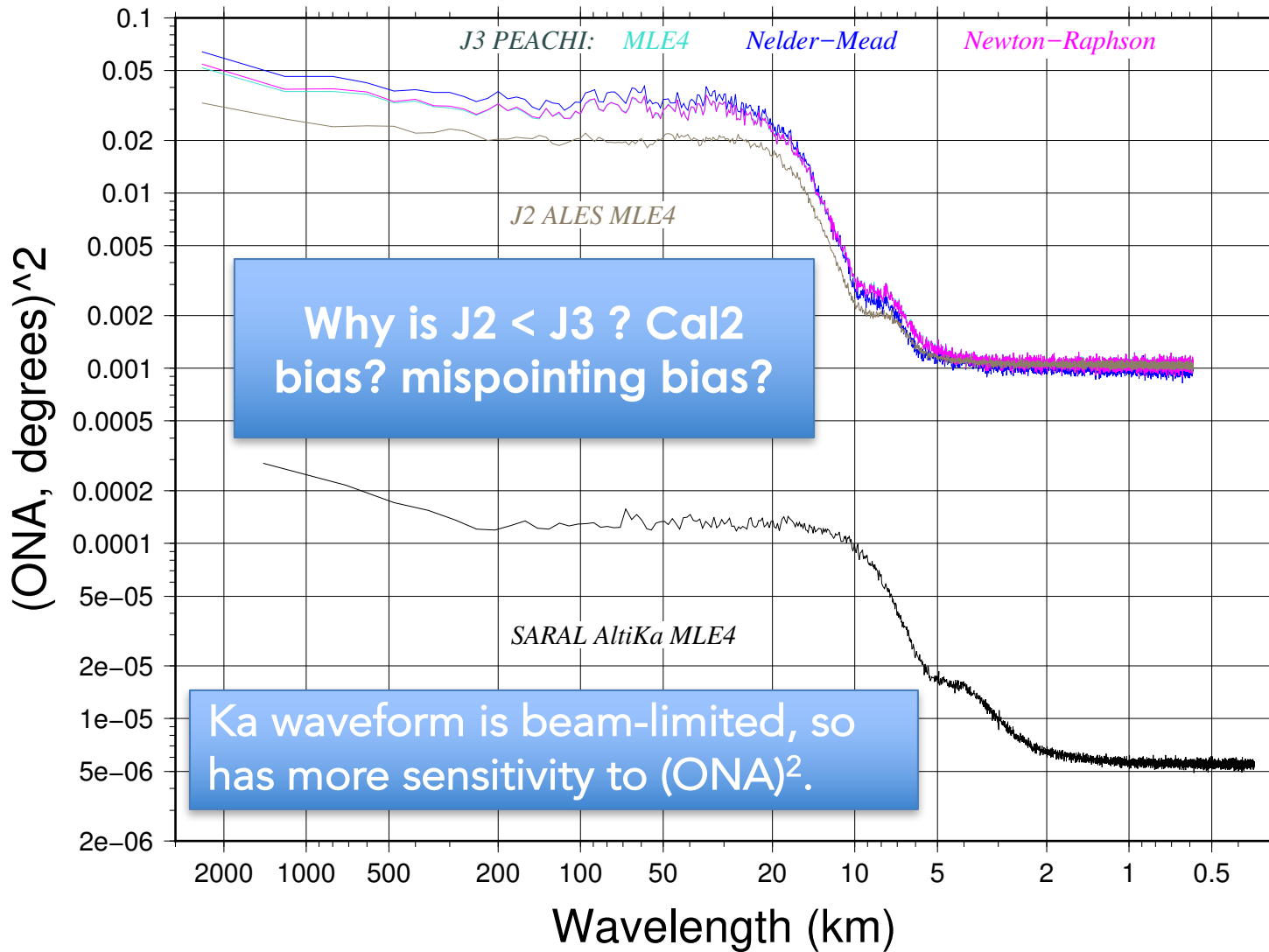
All retrackerers that fit the  $(ONA)^2$  have a bump in  $\sigma^0$ , including PEACHI N-M.

Those that do not (MLE3, ALES) do not have a bump.

The lowest noise is from MLE3.



# (Off-Nadir Angle)<sup>2</sup> (Auto-) Spectra



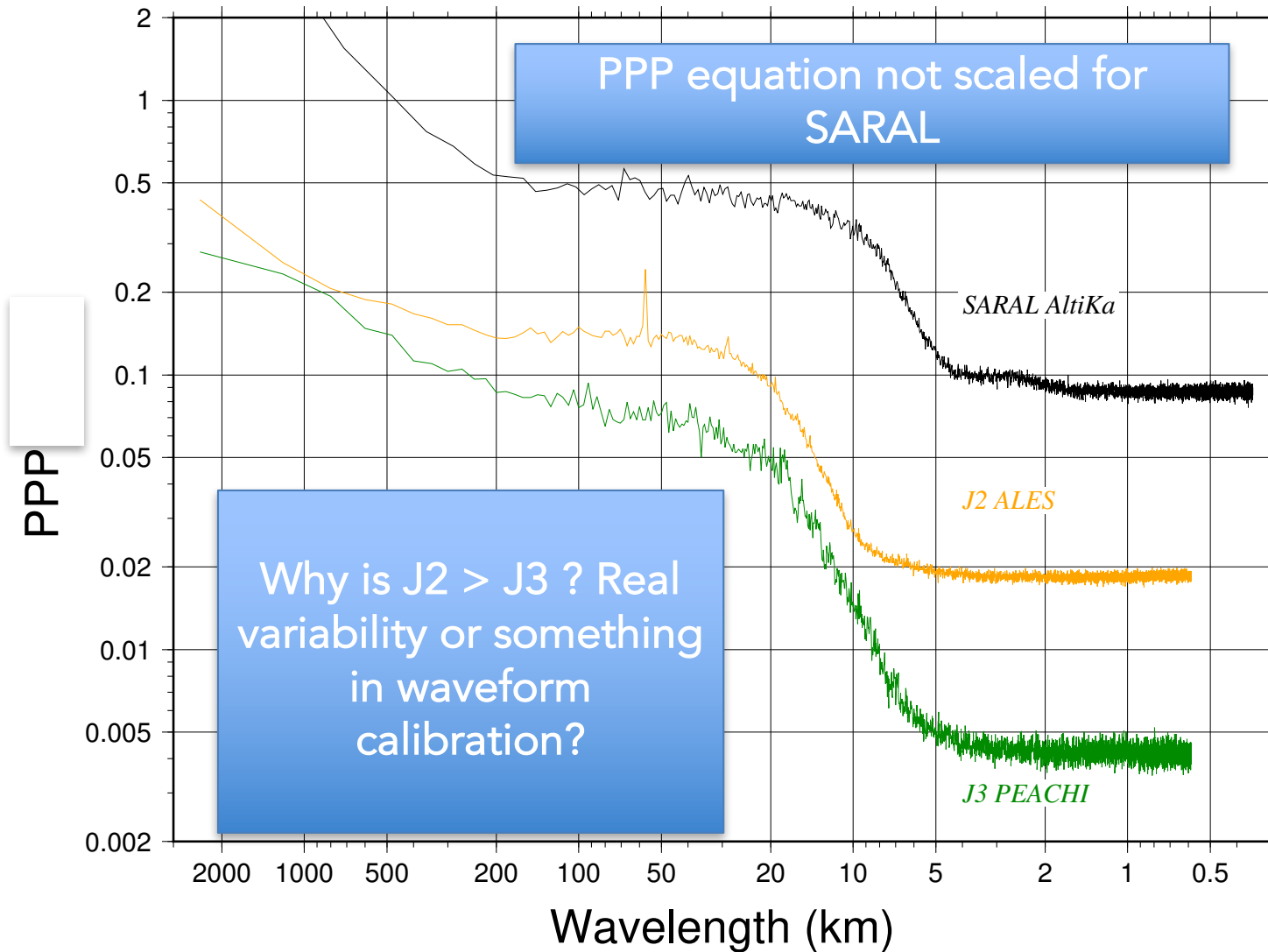
A step from one white noise at large scale to a lower white noise at small scale.

Transition scale related to field of view (?) because SARAL is shorter.

SARAL is beam-limited so noise levels are lower.



# Pulse Peakiness (Auto-) Spectra



Pulse Peakiness Parameter (PPP) spectrum looks like  $(ONA)^2$  spectrum. This parameter depends on the waveform only, and not on the retracker, so only 3 cases are shown here, one from each of the three data sets analyzed.



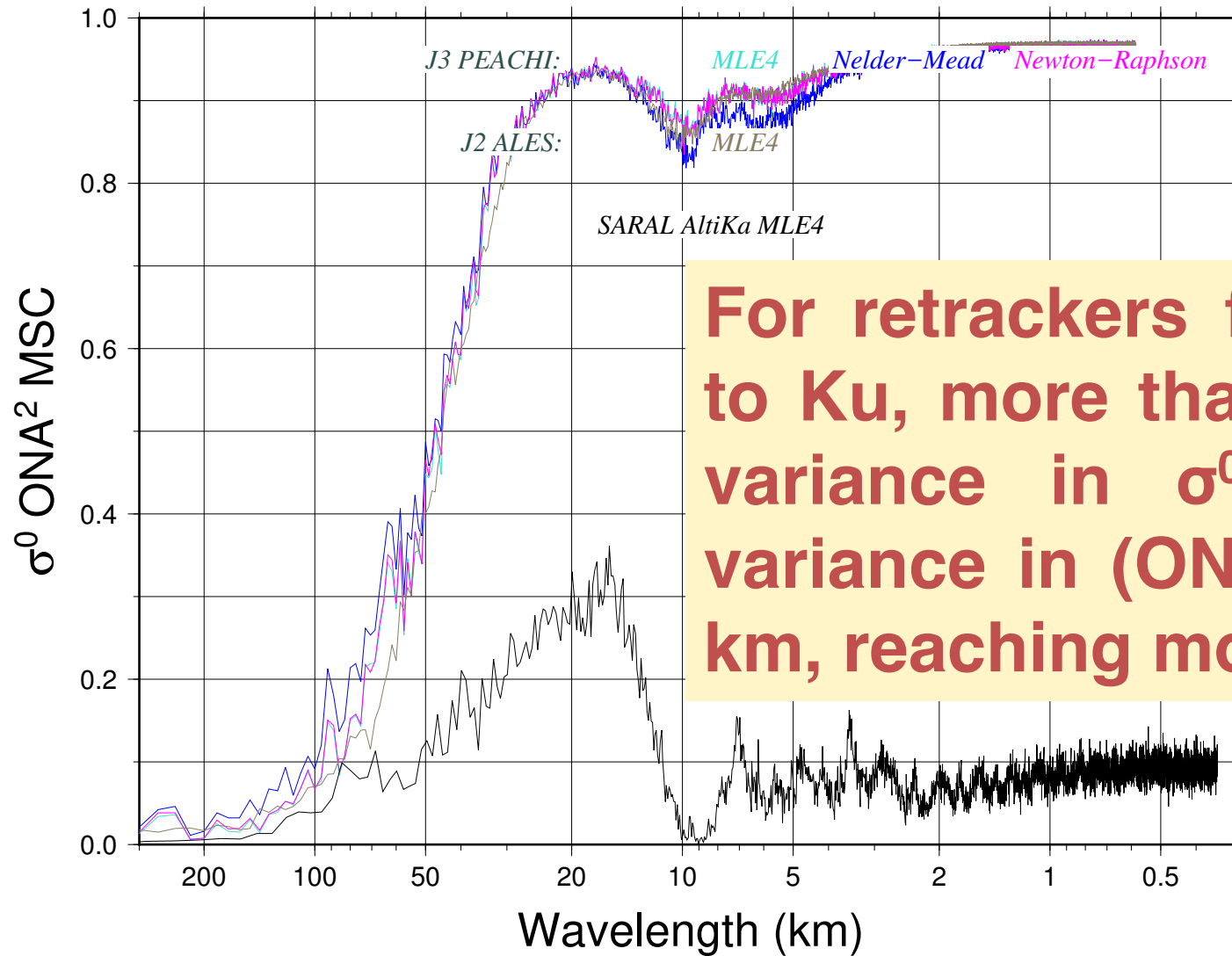
# Cross-Spectral Analyses

Two types of cross-spectral analysis:

1. MSC (magnitude-squared coherency), the square of the linear correlation coefficient between two variables. This shows us where one parameter is correlated with another.
2. Admittance, the ratio (variable 2):(variable 1). This shows us, e.g., meters of SSHA per meter of SWH in sea-state bias.



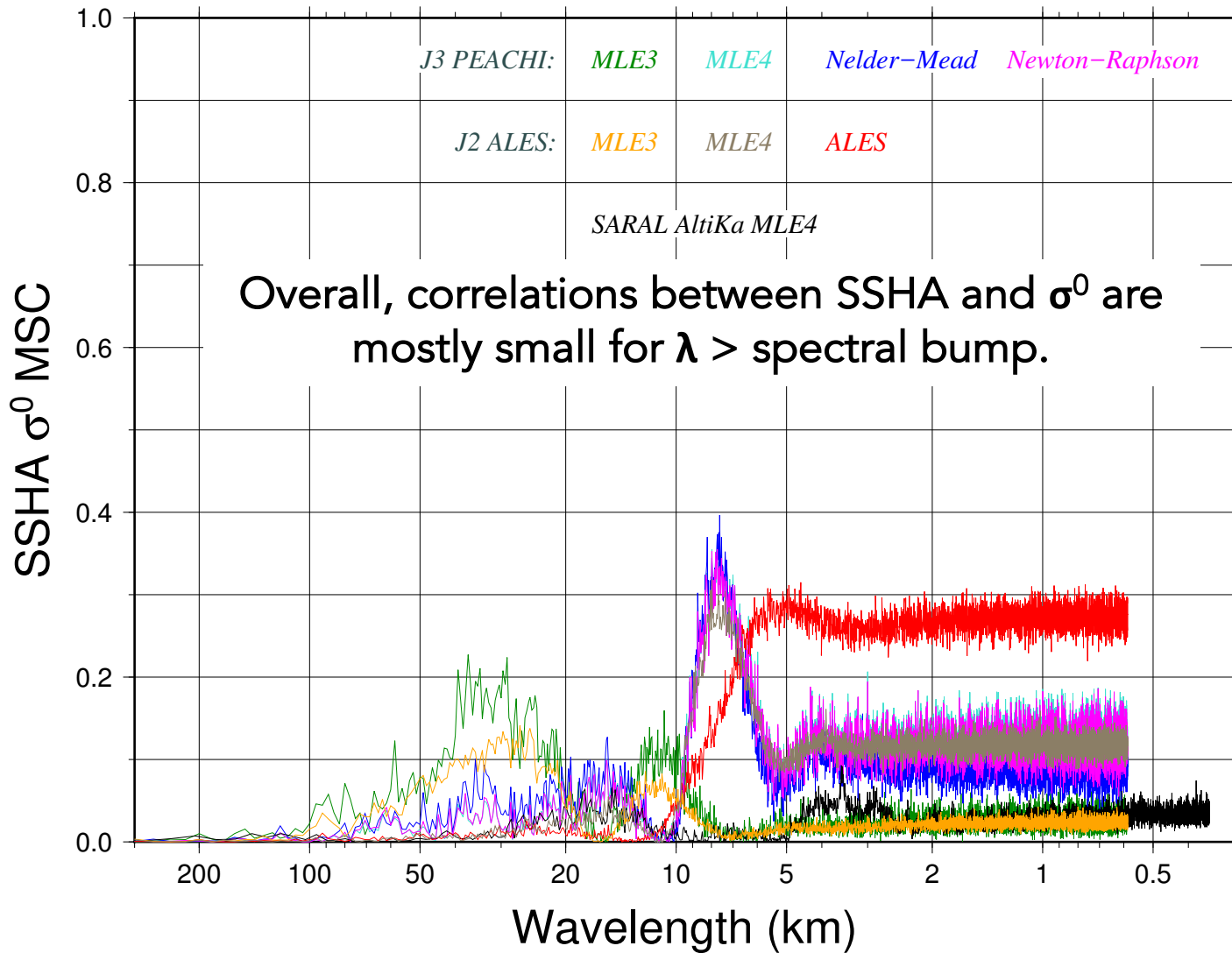
# $\sigma^0 - (\text{ONA})^2$ Cross-Spectrum: MSC



**For retrackerers fitting  $(\text{ONA})^2$  to Ku, more than 50% of the variance in  $\sigma^0$  is due to variance in  $(\text{ON})^2$  at  $\lambda < 50$  km, reaching more than 90%.**



# SSHA – $\sigma^0$ Cross-Spectrum: MSC

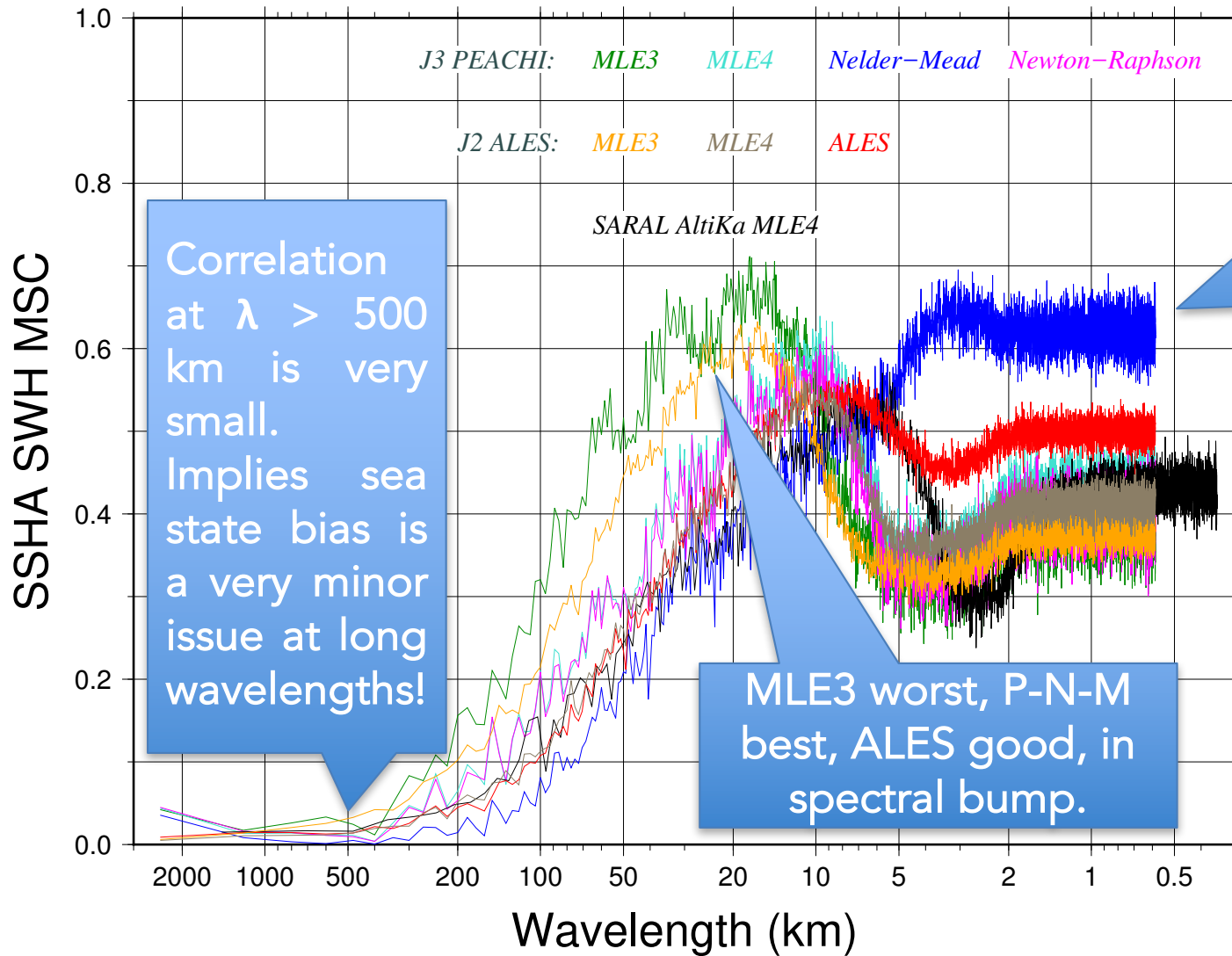


For MLE3, as much as 20% of SSHA variance is due to  $\sigma^0$  variance in the spectral bump. Ku (ONA)<sup>2</sup> fitting spikes to 35% in a narrow band. ALES does best overall at minimizing the covariance of SSHA and  $\sigma^0$ .





# SSHA – SWH Cross-Spectrum: MSC

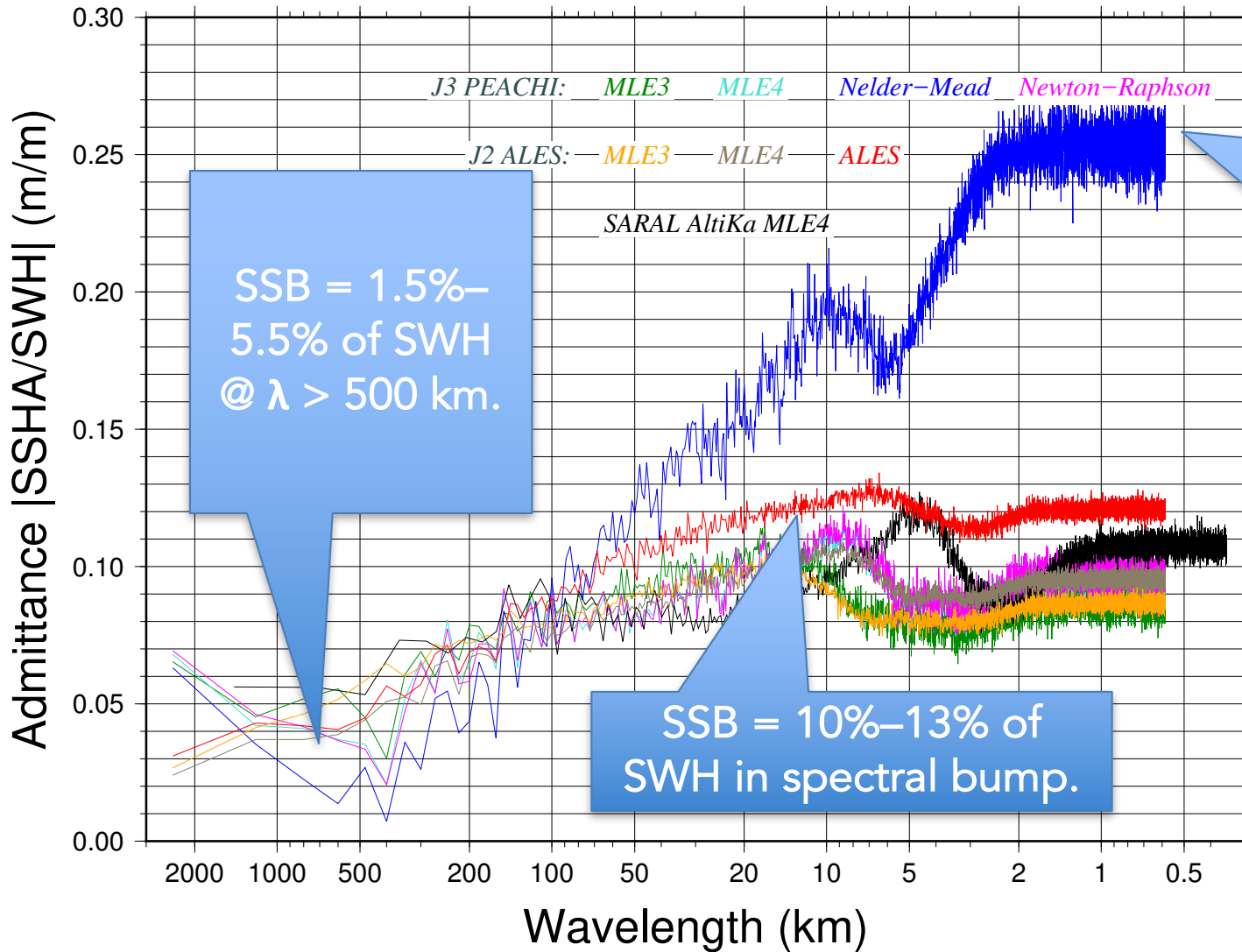


Weighted fitting most strongly correlates range and SWH noise in the white noise zone.

More than half the variance in SSHA is due to correlation with SWH in the spectral bump, if MLE3 is used. PEACHI Nelder-Mead minimizes this correlation.



# SSHA – SWH Admittance: Sea State Bias



SSB = 1.5%–5.5% of SWH @  $\lambda > 500$  km.

SSB = 10%–13% of SWH in spectral bump.

SSB rises above 25% of SWH if weighted fitting is used. Note steeper slope at all wavelengths.

Admittance is negative; SSHA goes down as SWH goes up; absolute value shown here.



# Summary / Conclusions, 1/2

- Sea state bias is not a constant percentage; it is wavelength- and retracker-dependent.
- The "spectral bump" is due to correlated errors.
- Fitting  $(\text{ONA})^2$  increases  $\sigma^0$  errors.
- Overall, ALES has the best noise spectrum (low covariant errors, low/moderate SSB)
- PEACHI Nelder-Mead minimizes SSHA variance but at the cost of strong correlation with SWH and new and larger SSB.



## Summary / Conclusions, 2/2

- Comparison with SARAL AltiKa is instructive, as it has a narrower field of view and is beam-limited as well as pulse-limited.
- This study was in a relatively quiet area of the sub-Equatorial Pacific, so very large SWH or extreme weather are uncommon. A more global study may be needed to explore the full range of SSB conditions.