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A conventional value for the geoid reference potential W_0

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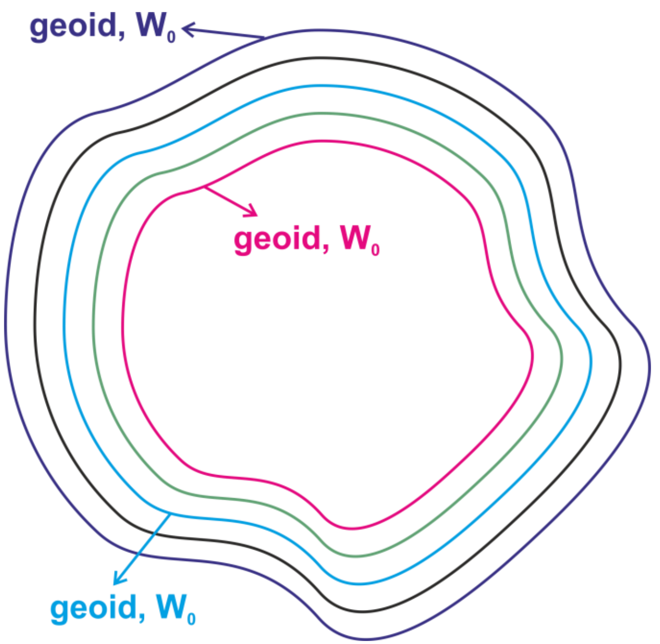
Unified Analysis Workshop 2017 (UAW2017)
Paris, July 10 – 12, 2017

Basics

- W_0 is understood as the gravity potential value of the geoid; i.e. the geoid potential;
- The geoid is an equipotential surface of the Earth's gravity field;
- Any equipotential surface of the Earth's gravity field may be selected as the geoid;
- Since there is an infinite number of equipotential surfaces, the geoid (and consequently W_0) is to be defined arbitrarily by convention;
- Everyone can select "his own geoid (or W_0)" according to his convenience. In local approximations, any selection works. In global applications, everyone should select
  - — one and the same equipotential surface as the conventional geoid and
  - — one and the same potential value as the conventional W_0.



Gauss-Listing definition of the geoid

- Usual convention: the geoid is the equipotential surface of the Earth's gravity field that best fits (in a least-squares sense) the undisturbed mean sea level;
- As to satisfy the "undisturbed" condition is not possible and as the sea level changes, a convention about mean sea level (time span, area, removal of disturbing effects, etc.) is also needed:
- Local approaches:
  - mean value at a local tide gauge: $W_0 = W_0^{(i)}$

    Example: Tide gauge in Amsterdam for Europe.

  - mean value at several tide gauges: $W_0 = \frac{1}{n}\sum_{i=1}^{n} W_0^{(i)}$

    Example: in the new vertical datum for North America, W_0 is the averaged value of the potentials determined at 35 tide gauges along the Atlantic coastline and 22 tide gauges along the Pacific coastline.

Gauss-Listing definition of the geoid (cont.)

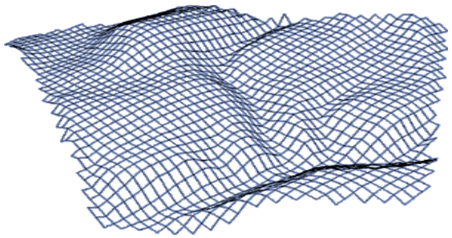
- Global approaches:
  - Determination of a reference ellipsoid, implies
    a) selection of the definition parameters (a, f or J_2, ω, GM)
    b) computation of U_0 as function of those parameters and then $W_0 = U_0$

    Examples:

    Mean Earth ellipsoid GRS80 (Moritz 1980): $W_0 = U_0$ = 62 636 860.850 m^2s^{-2}
    Best fitting ellipsoid for the Topex/Poseidon (T/P) mean sea surface (Rapp 1995): $W_0 = U_0$ = 62 636 856.88 m^2s^{-2}

  - Mean value over ocean areas sampled globally $\int_S (W - W_0)^2 \, dS = \min$

    Implies:
    a) a discrete representation of the mean sea surface MSS (from satellite altimetry data)
    b) estimation of the potential values at the sea surface using global gravity models (GGM).



Examples:

MSS: GEOSAT, GGM: GEM-T2 (Burša et al., 1992) W_0 = 62 636 856.5 m^2s^{-2}
MSS: T/P (1993-1996), GGM: EGM96 (Burša et al., 1998) W_0 = 62 636 855.611 m^2s^{-2}

The value Burša et al. (1998) was rounded to 62 636 856 m^2s^{-2} and it is included in the IERS conventions as “best estimate”.

W_0 and the IERS Conventions

- In 1991, the International Astronomical Union (IAU) introduced timescales for the relativistic definition of the celestial space-time reference frame;
- The relationship between Geocentric Coordinate Time (TCG), and Terrestrial Time (TT) depends on the constant $L_G = W_0 / c^2$
- For this reason, the IERS Conventions included a W_0 value and updated this value regularly according to new best-estimates:

Year	W_0	L_G
1991	62 636 860 ± 30 m²s⁻² (Chovitz 1988)	$6.969\,291 \times 10^{-10} \pm 3 \times 10^{-16}$ (IAU 1991, Recommendation IV, note 6)
1992	62 636 856.5 ± 3 m²s⁻² (Burša et al. 1992)	$6.969\,290\,19 \times 10^{-10} \pm 3 \times 10^{-17}$ (Fukushima 1995)
1995	62 636 856.85 ± 1 m²s⁻² (Burša 1995)	$6.969\,2903 \times 10^{-10} \pm 1 \times 10^{-17}$ (McCarthy 1996, Tab. 4.1)
1999	62 636 856.0 ± 0.5 m²s⁻² (Burša et al. 1998, Groten 1999)	$6.969\,290\,134 \times 10^{-10}$ (as defining constant) (IAU2000, Resolution B1.9)

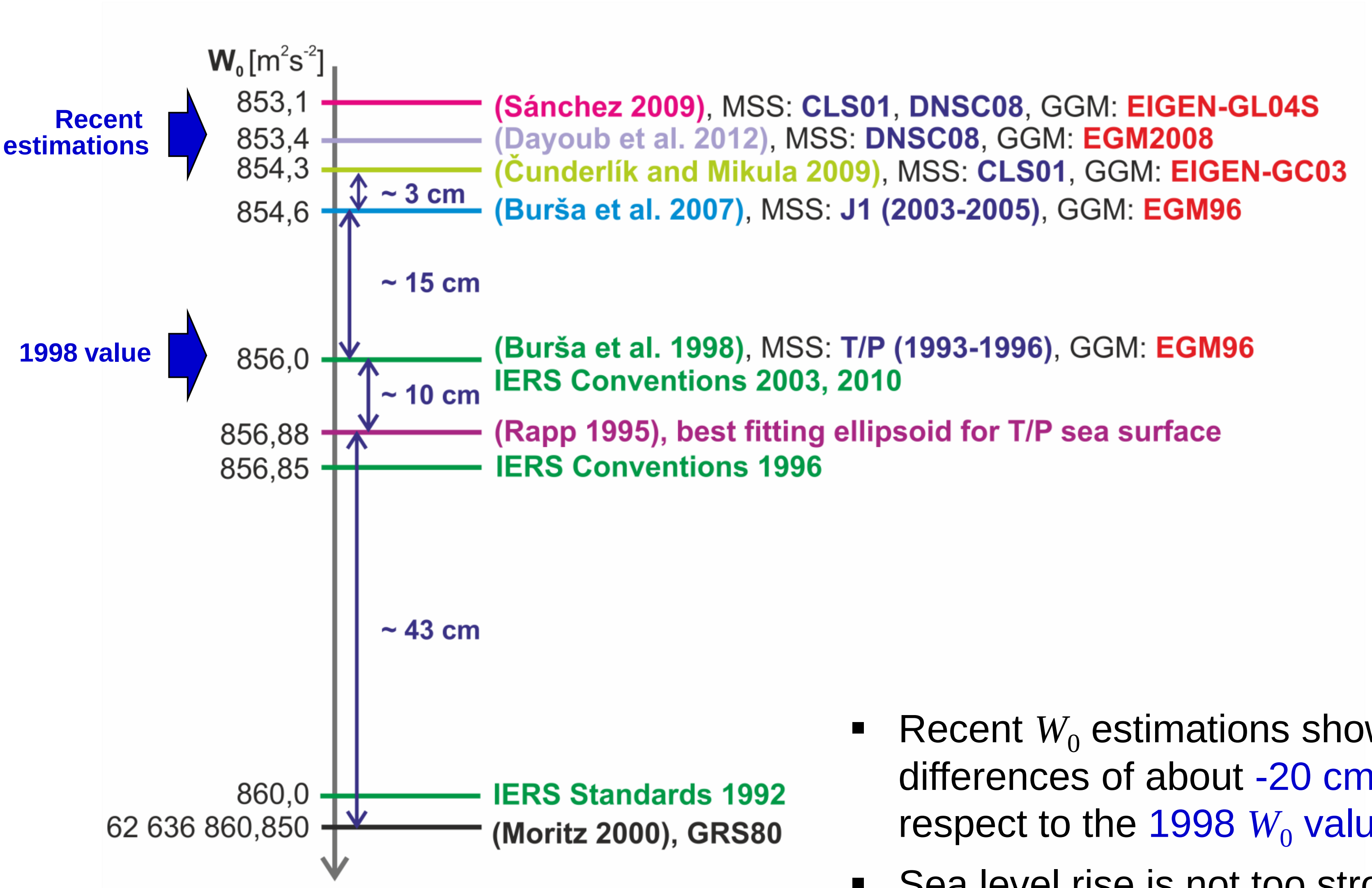
- In 2000, L_G is declared as "defining constant", i.e. it should not be changed with new estimations of W_0. The corresponding W_0 value is the best-estimate available in 1998.

Recent computations of W_0

In the 2000s, new W_0 computations are performed:

- To extend in time the sea surface mapped by satellite altimetry (the 1998 W_0 value includes only the period from 1993 to 1996).
- To include new satellite altimetry missions (the 1998 W_0 value is based only in Topex/Poseidon).
- To apply newest processing standards and conventions in the satellite altimetry data analysis (for instance, the dynamic atmospheric correction to reduce the uncertainty associated to the inverse barometric effect was not available in 1998).
- To take into account observations from the satellite gravity missions CHAMP, GRACE, GOCE (the 1998 W_0 value is based on the EGM96 model).

Recent computations of W_0 (cont.)



- Recent W_0 estimations show level differences of about -20 cm with respect to the 1998 W_0 value.
- Sea level rise is not too strong to explain this discrepancy.

Recent computations of W_0 (cont.)

- In a workshop of the IAG inter-commission project "Vertical Reference Frames" held in Prague in April, 2006, Burša et al. presented a detailed comparison between their computations and the new ones:

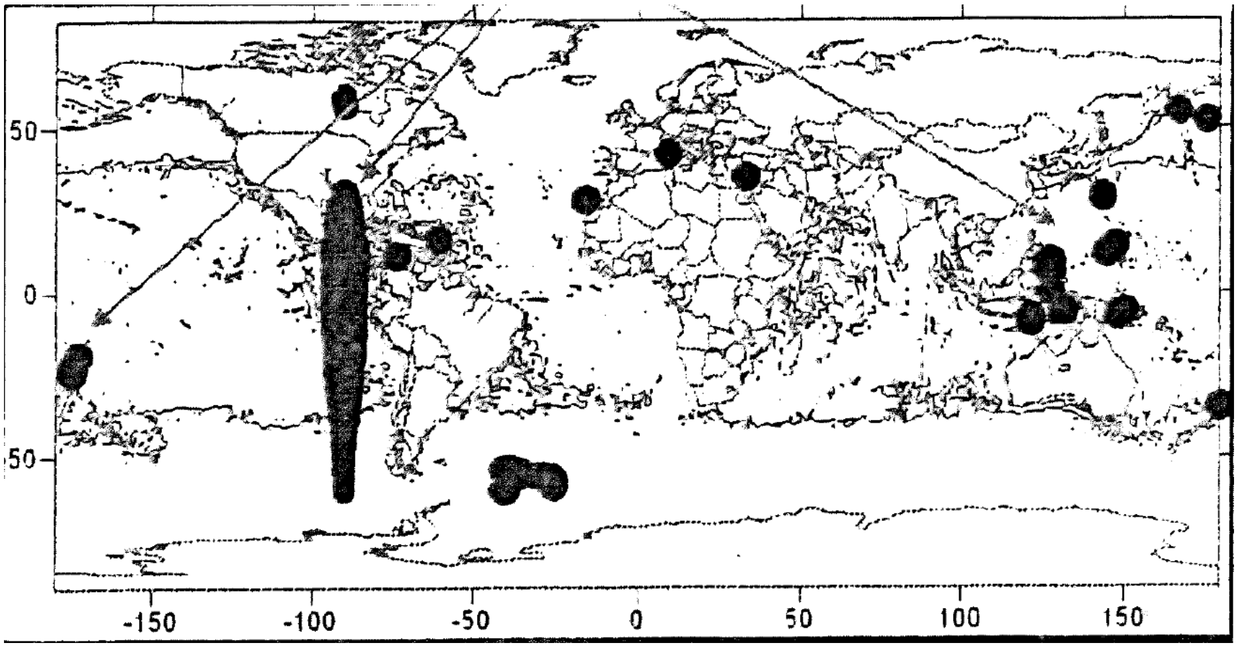


Figure taken from Burša et al. (2006): "Confirmation of the W_0 value derived by SSG GGDA – explanation of the difference between the two W_0 determinations", slide 14.

- The dark zones represent level differences between 20 m and 70 m. They mainly occur at the longitude -90° (evaluation of the co-tangent in GGM?) and in shallow waters, where satellite altimetry is problematic.
- Conclusion (2006): A new standardised computation should be performed towards a conventional W_0 value.

Assessment of W_0 (activities 2011 - 2015)

- GGOS established a working group with four different teams performing the W_0 computations parallelly, using different estimation methodologies but the same input data and models (to ensure reproducibility and to avoid programming/software mistakes).

Following aspects were evaluated:

- Sensitivity of the W_0 estimation on the Earth's gravity field model.
- Dependence of W_0 on the omission and commission error of the global gravity model.
- Influence of the time-dependent Earth's gravity field changes on W_0.
- Sensitivity of the W_0 estimation on the mean sea surface model.
- Influence of time-dependent sea surface changes on W_0.
- Effects of the sea surface topography on the estimation of W_0.
- Dependence of the W_0 empirical estimation on the tide system .
- Rigorous error propagation analysis to estimate the influence of the input data uncertainties on the W_0 estimation.

- Details are provided in back-up slides.

Assessment of W_0 (activities 2011 - 2015), cont.

Following models were used:

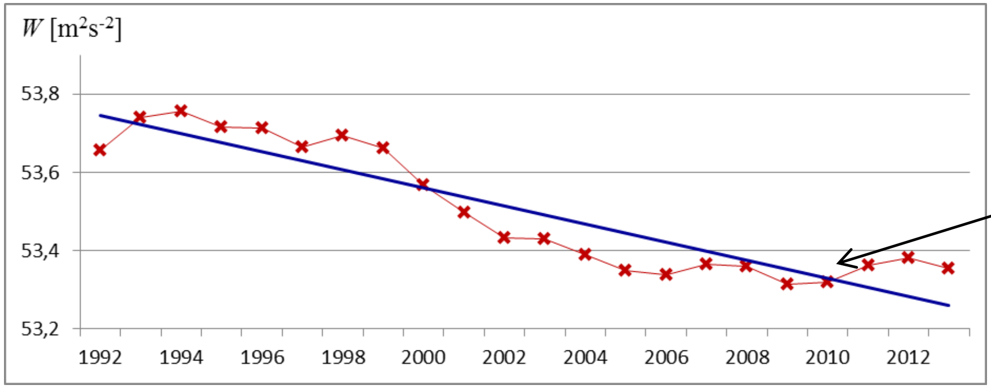
- Sea surface models:
  - CLS11 16 years, 9 missions (Schaeffer et al. 2012)
  - DTU10 17 years, 11 missions (Andersen 2010)
  - own computed yearly models (1992-2014)
  - cross calibrated data from the DGFI-OpenADB (Schwatke et al. 2010) with covariance matrixes (Bosch et al. 2014), 9 satellite altimetry missions.
- Global gravity models:
  - EGM96 (Lemoine et al. 1998), EGM2008 (Pavlis et al. 2012), EIGEN-6C and 6EIGEN-6C3stat (Förste et al. 2012), GOCO03S (Mayer-Gürr et al. 2012), DIR-R4 (Bruinsma et al. 2013), TIM-R4 (Pail et al. 2011), GGM05S (Tapley et al. 2013), monthly models from GRACE GFZ Release 05.
  - Gravity models including satellite laser ranging observations (LAGEOS) and GRACE and GOCE data provide practically the same results.

Main conclusions

- Computations carried out within the GGOS-WG demonstrate that the 1998 W_0 value (62 636 856.0 ± 0.5 m^2s^{-2}) is not in agreement (and consequently it is not reproducible) with the newest geodetic models describing geometry and physics of the Earth.
- Like any reference parameter, W_0 should be based on adopted conventions that guarantee its uniqueness, reliability, and reproducibility; otherwise, there would be as many W_0 reference values as computations.
- W_0 should be computed following the Gauss-Listing geoid definition.
- As a totally calm condition of the sea surface is not achievable, a quasi-stationary representation of the sea surface is needed; i.e., time-dependent effects affecting the instantaneous sea surface should be reduced previously.
- Due to the time-dependent variations of the sea surface, the realisation of the Gauss-Listing definition necessarily has to be associated to a certain epoch. The selection of a time span (e.g. 1992-2010, or 1995-2013, etc.) for the computation of a mean value is problematic because the inter-annual oceanic variability. Depending on the time-span, different results will be obtained.

Main conclusions (cont.)

- To avoid effects of the inter-annual oceanic variability; it is recommended to determine the linear trend of the potential value at the sea surface by yearly mean sea surface models and to adopt the value corresponding to a certain epoch.



In 2010.0:
W_0 = 62 636 853.353 m^2s^{-2}
rounded to
W_0 **= 62 636 853.4** m^2s^{-2}

- Based on the yearly W_0 estimations performed for the time span 1993.0 - 2014.0, it was recommended to adopt the W_0 value obtained for the epoch 2010.0 after fitting the time series by means of a lineal regression.
- As this value is adopted as a convention, an accuracy indicator can be omitted. However, it can be mentioned that its formal error is $\pm 0.02\ m^2\ s^{-2}$.
- This W_0 value was officially adopted by the International Association of Geodesy as the geoid reference potential value for the definition and realisation of the International Height Reference System- IHRS (see IAG resolution No. 1, 2015).

Closing comments

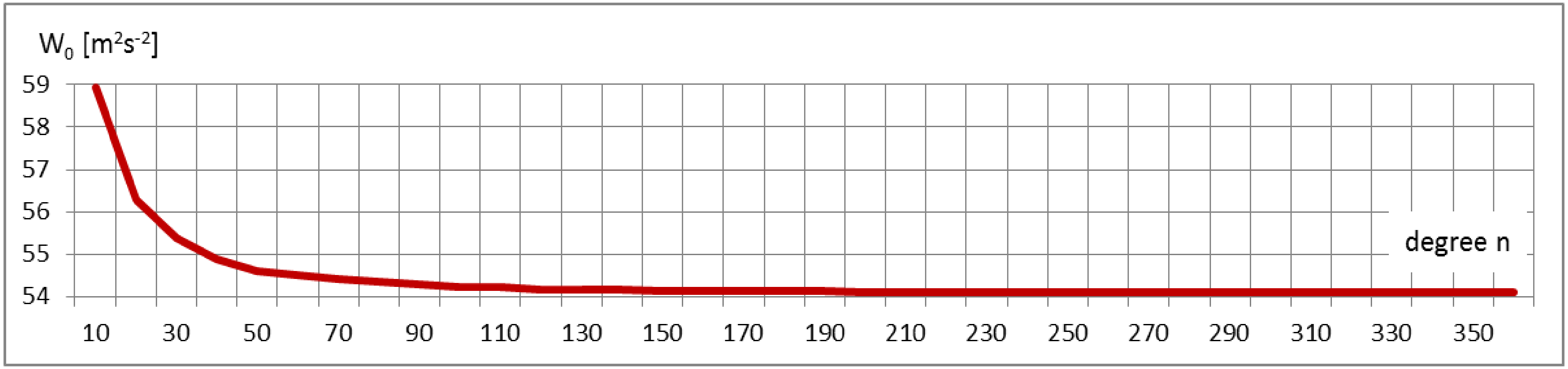
- As a reference parameter, W_0 should be time independent (i.e., quasi-stationary) and it should remain fixed for a long-term period (e.g., 30 years).
- It is recommended to monitor the changes of the potential value at the sea surface to compare it with the adopted W_0 value.
- When large differences appear (e.g., more than $\pm 2\ m^2s^{-2}$; i.e. ± 20 cm), the adopted W_0 may be replaced by an updated (best estimate) value.
- The difference between the IAG conventional W_0 value and the 1998 W_0 value (included in the IERS conventions) is $-2.6\ m^2s^{-2}$ (-25 cm) Consequently, we recommend to replace the 1998 W_0 value by the IAG conventional W_0 value in all applications requiring a geoid reference potential.

Further reading:

Sánchez L., Čunderlík R., Dayoub N., Mikula K., Minarechová Z., Šíma Z., Vatrt V., Vojtíšková M.: A conventional value for the geoid reference potential W_0 . Journal of Geodesy 90(9), 815-835, 10.1007/s00190-016-0913-x, 2016

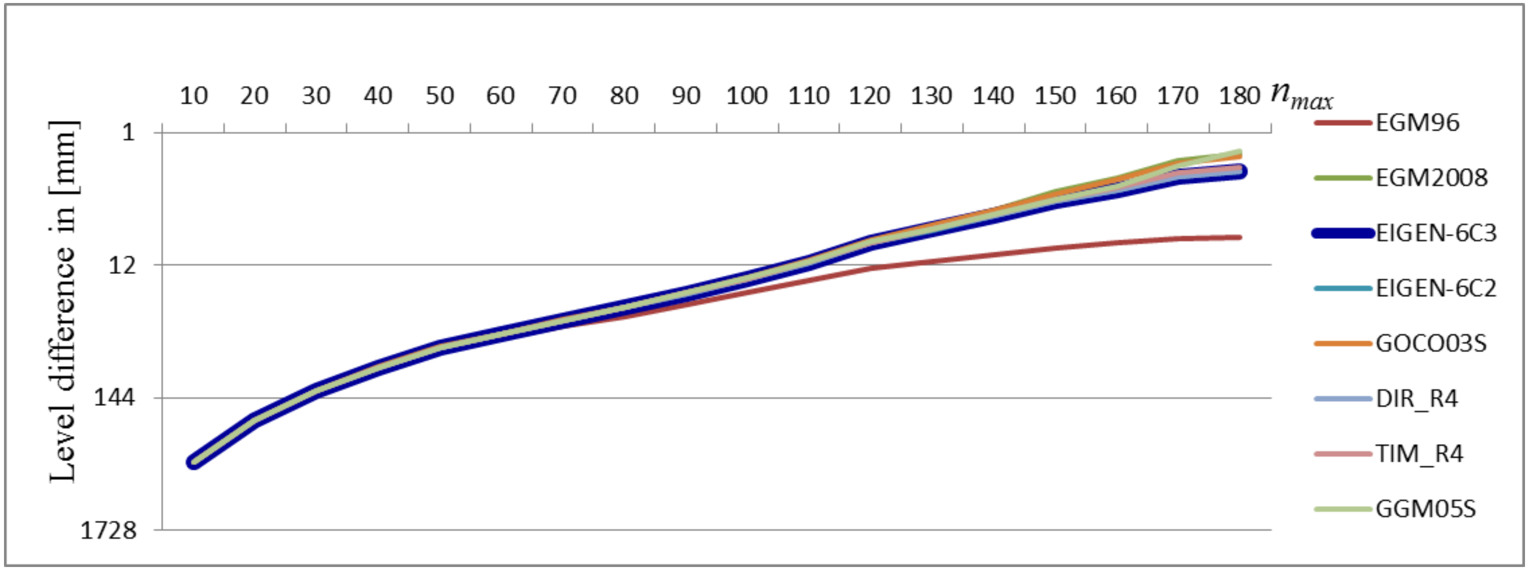
Back-up

Dependence of the W_0 estimate on the choice of the gravity model



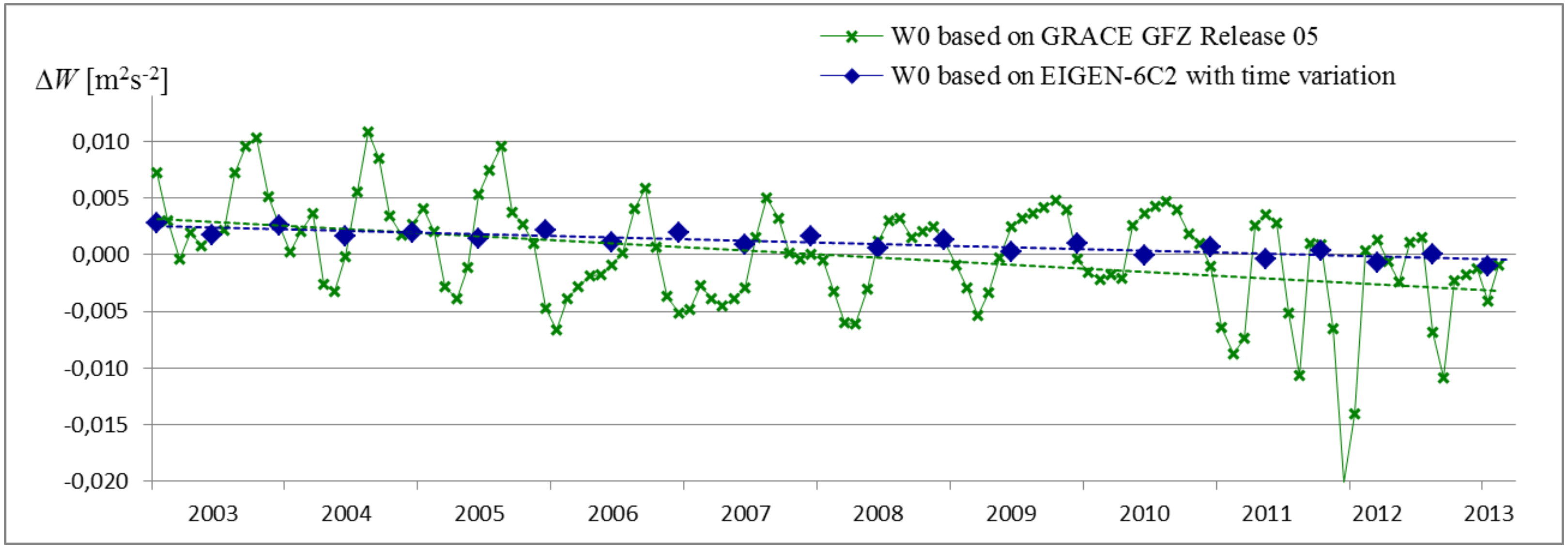
1) The use of a satellite-only gravity model is suitable. With n,m higher than 200 the largest differences are 0,001 m^2s^{-2}. These small differences are negligible.

Dependence of the W_0 estimate on the choice of the gravity model



2) W_0 estimations based on models including GRACE, GOCE and Satellite Laser Ranging (Lageos) data are practically identical. Max. differences 0.01 m^2s^{-2}.

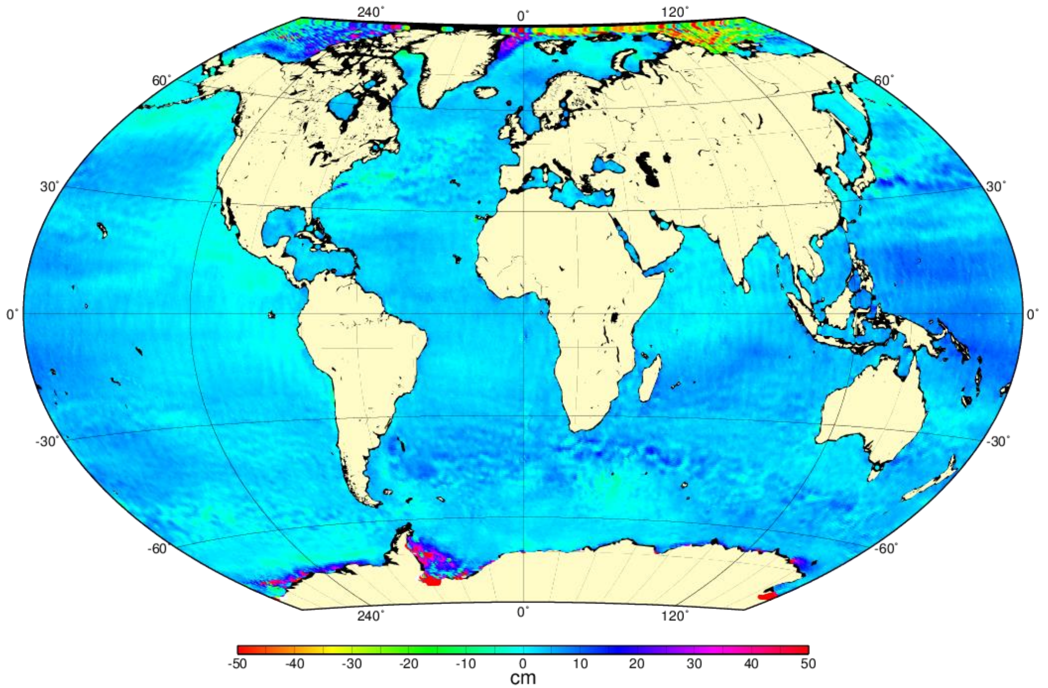
Dependence of the W_0 estimate on the choice of the gravity model



Changes in the W_0 estimates after applying the monthly GRACE-based models GFZ Release 05 and the time-dependent harmonics of the model EIGEN-6C2. The linear trend of W_0 using the GFZ Release 05 is -6.617×10^{-4} $m^2s^{-2}a^{-1}$*, while the linear trend using EIGEN-6C2 is* $-2.647 \times 10^{-4} m^2s^{-2}a^{-1}$.

3) Seasonal variations of the Earth's gravity model can be neglected (max. variation 0.03 m^2s^{-2}).

Dependence of the W_0 estimate on the mean sea surface model



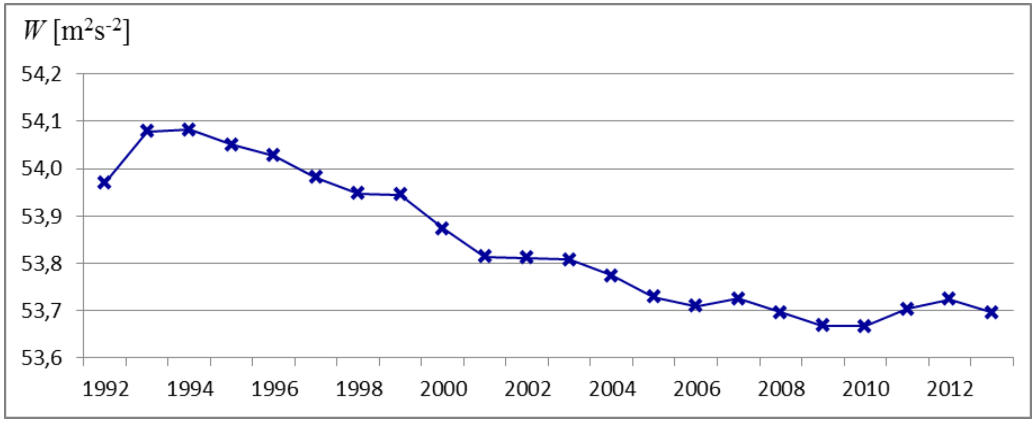
Potential differences (divided by the normal gravity) between the estimations derived from the models MSS-CNES-CLS11 and DTU10 (computations in zero tide system with the GGM EIGEN-6C3).

By using the models CLS11 and DTU10 there is a difference of 0.31 m^2s^{-2}, which reflects the mean discrepancy of ~ 3 cm between both models. Possible causes:

- Different strategies to process the altimetry data;
- Different reductions taken into account in each model;
- Different periods (inter-annual ocean variability).

Dependence of the W_0 estimate on the mean sea surface model

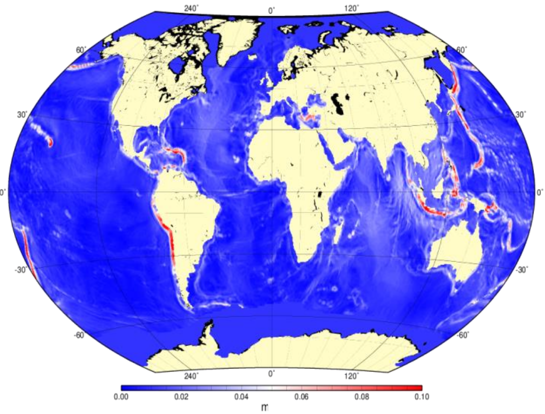
Alternative to long-term mean sea surface models: use of yearly mean sea surface models



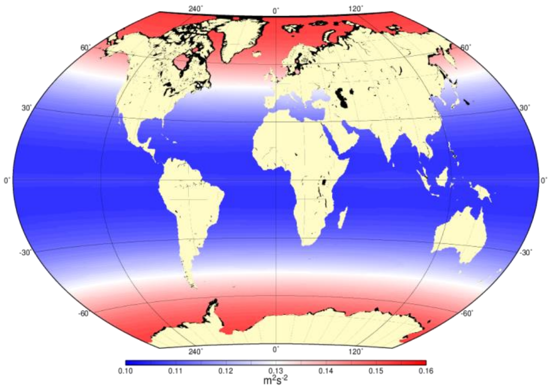
- The W_0 estimates reflect (with opposite sign) the sea level rise measured by satellite altimetry;
- Max. difference 0.46 m^2s^{-2};
- These variations shall not be understood as a change in W_0, but in the sea level; i.e. the conventional geoid is not growing/decreasing with the mean sea level!
- This only means that the mean sea level coincides with a different equipotential surface depending on the period utilized for the average of the sea surface heights.

Reliability of the W_0 estimate

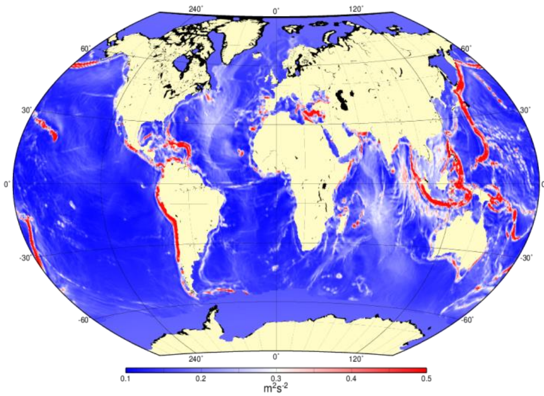
Until now, all the computations assumed error free input data (MSS and GGM). By rigorous error propagation analysis (as Sjöberg 2011 proposed), the W_0 value estimate decreases by about 0.3 m^2s^{-2}.



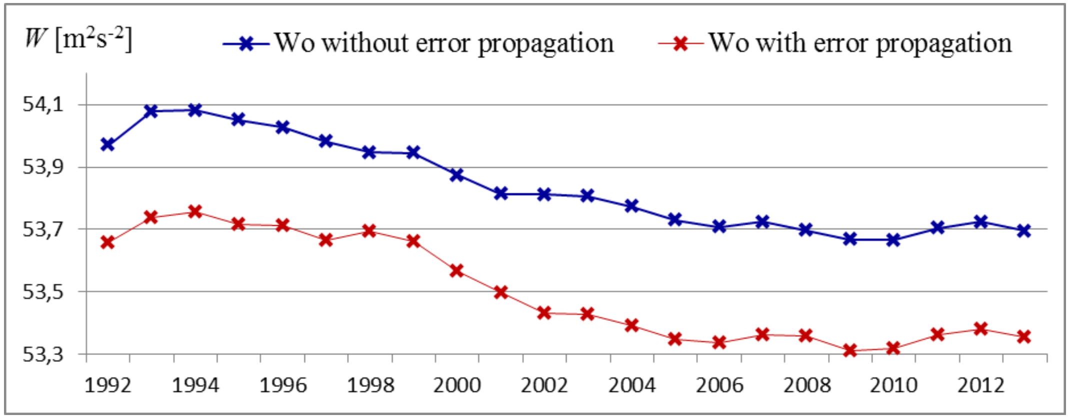
Standard deviation (σ_h) of the mean sea surface heights for the year 2005.



Standard deviation (σ_T) of the anomalous potential derived from the model EIGEN-6C3 (n = 200).



Standard deviation ($\sigma_W = \sqrt{\sigma_T^2 + \gamma^2 \sigma_h^2}$) of the gravity potential values computed at the sea heights (h) for the year 2005 with the model EIGEN-6C3 (n = 200).



W_0 estimates assuming error free input data (blue series) and applying a proper error propagation computation (red series).