Enactive Self: a study of engineering perspectives to obtain the sensorimotor self through enaction*

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Abstract— **In this paper we discuss the *enactive self* from a computational point of view and study the suitability of current methods to instantiate it onto robots. As an assumption, we consider any cognitive agent as an autonomous system that constructs its identity by continuous interaction with the environment. We start examining algorithms to learn the body-schema and to enable tool-extension, and we finalise by studying their viability for generalizing the enactive self computational model. This paper points out promising techniques for bodily self-modelling and exploration, as well as formally link sensorimotor models with differential kinematics. Although the study is restricted to basic sensorimotor construction of the self, some of the analysed works also traverse into more complex self constructions with a social component. Furthermore, we discuss the main gaps of current engineering approaches for modelling enactive robots and describe the main characteristics that a synthetic sensorimotor self-model should present.**

I. Introduction

> *We may define a system as "cognitive" if and only if it generates its actions, and the feedback sensations serve to guide actions, in a very specific way so as to maintain its autopoiesis[1] and hence its very existence [2, p. 3].*

Enactive robots are artificial agents that construct their identity and their knowledge about the world by means of continuous interaction in the environment. According to [3], the system must be capable of: (1) generating its own systemic identity at some level of description and (2) actively regulate its ongoing sensorimotor interaction in relation to a viability constraint.

The vagueness of the enaction concept makes difficult to formalize an algorithm or mathematical model, apart from settling some guidelines or requirements [4]. However, the utility of such model goes beyond the improvement of current robotic systems [5]. From a simplified biological perspective, according to [6], humans start learning the sensorimotor mapping while being inside the uterus. Modality and inter-modality patterns are acquired from timed and contingent events [7], becoming the basis for further learning that includes goal-directed and prosocial interactions during the entire life of the agent [8]. Any form of identity extracted from these patterns will provide the agent a way to distinguish the self from the others. In the enaction terminology, by constructing a sensorimotor self from multisensory contingent patterns, maintaining the autopoiesis, we will provide a unique identity to the cognitive agent. From this definition, the enactive self is compatible with the pattern approach [9], the existential[2] and the ecological self [11]. In [12], five types of self were identified: the ecological, the interpersonal, the temporally extended, the conceptual and the private self. Conversely, Gallagher proposed that all notions of self can be reduced into two categories [13]: (1) the minimal self, where the sense of agency and ownership are encoded and (2) the narrative self, where the stories of the past that we and others tell about ourselves construct the self that we experience in our daily lives [14]. Furthermore, self differentiation is a very prominent candidate for agency interpretation, as some studies in neurophysiology point out [15]. In conclusion, a model of enactive self will provide some clues on the construction of the identity and the agency process, which is an important pillar for self-awareness.

This paper is focused on models that implicitly construct the minimal self from a bottom-up approach perspective. Despite of the numerous works on active learning, the literature only provides partial and sparse solutions for different aspects of enaction. Thus, with this work we target the final goal by some of its parts: we investigate current engineering techniques, e.g., their advantages and drawbacks, which could be valuable for modelling an enactive robot guaranteeing the implicit requirement of identity construction. On the one hand, a vast majority of the computational works related to the enactive paradigm are focus on body-schema learning, tool-extension and reaching (Sec. II). Within those topics, active state exploration techniques are an important aspect of the learning (Sec. III).

On the other hand, much less effort has been done in the social contribution to the learning stage outside pure imitation techniques (Sec. IV). Section V summarizes and compares some of the most relevant methods studied from the enaction point of view. Section VI provides a final remark about designing enactive robots with the capacity of building its own sensorimotor identity. Finally, in the Appendix we give an overview of the connections between sensorimotor models and differential kinematics.

II. Body-schema, tool extension and reaching

First we have to clarify, as posed in [16], what learning the body means. On the one hand, we have the body

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*This manuscript includes some corrections and missing references from the original IEEE ICDL-Epirob 2017 proceedings version.

[1]An autopoietic system presents an invariant processes network and has the capacity of creating and destroying in its own system in response to the environmental perturbations [1]. Even if the system structurally changes, the network maintains its own identity.

[2]The *existential self* is the perceptual sensitivity that discriminates self from non-self at least momentarily [10].

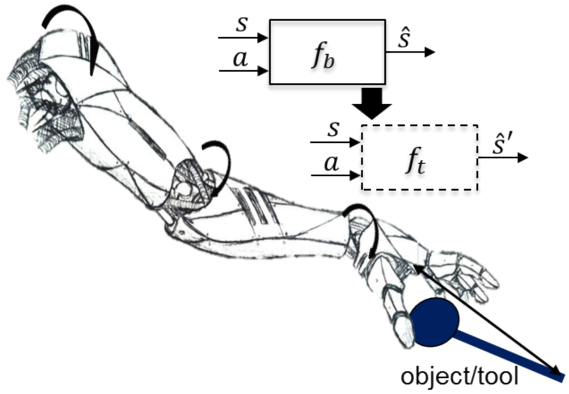


Fig. 1. Learning the body and tool-extension from sensorimotor experience.

configuration, which is composed of joints, limbs, sensors and actuators. On the other hand, we have models that govern the body, such as forward and inverse models, e.g., effectors kinematics and dynamics. We can find several efficient solutions to compute these models in the literature when the body configuration is known. For instance, in [17] a $O(N)$ algorithm for computing the forward dynamics is proposed. When the body configuration is not known, we can either learn the body representation or directly learn models that govern the body. From the sensorimotor point of view, the body-schema generalizes to the relation between the sensations (produced by the sensors) and the actions (exerted by the actuators). Appropriately, one of the ways to mathematically formalize the body-schema is also through the forward and inverse models paradigm. These functions are non linear and sparse; a black box that can be learnt using supervised learning techniques, such as artificial neural networks (e.g., Hopfield [18] or deep learning [19]). Anyhow, the most interesting algorithms are online and unsupervised as self-organizing maps [20] or space partitioning [21]. The number of approaches for modelling the body-schema and tool-extension [22] is wide, from connectionist methods to general purpose regressors. A comprised review can be found in [23]. This section explains some of the main methods for different body modelling purposes, such as body-schema, tool-extension and reaching.

From the enactive paradigm the robot should learn the models only by means of interaction until constructing an embodied identity. Ideally, the learning has to be incremental, on-line and life-long. Let define the body state as the actions ($a \in A$) and their sensory consequences ($s \in S$). Then, we define the sensorimotor self as the spatio-temporal patterns $A \times S \times T$. We formalize the forward body model f_b as unique for a specific agent. However, f_b should adapt according to changes on the agent and the environment. A possible description of f_b expressed as a forward model is the predicted sensory response $\hat{s}$ given an action a [24],

$$\hat{s}^{t+1} = f_b(s^t, a^t) \tag{1}$$

In the case that the robot has been able to learn the body forward model, it should be able to reuse that model for body changes, tool-extension and environmental interferences. For instance, the robot has to learn a new function f'_b to predict the perceptual consequences $\hat{s}'$ to adapt to a new situation (e.g., using a tool) exploiting its previous learnt model f_b. Thus, we have three scenarios to obtain new models: complete substitution $f_b \leftarrow f'_b$, recursion $f'_b = f_t(f_b)$ and aggregation $f'_b = f_t \odot f_b$. Furthermore, we expect that learning f_b could be helpful for other similar tasks or tools. Finally, the reaching capability has been faced as an emergent behaviour of learning the coordination between visual and proprioceptive cues or as a direct use of a forward model f_b, which is, computing its inverse f_b^{-1}.

A. Advanced non-linear regressors and learning algorithms

Apart from the stochastic gradient descent algorithm used by Rolf et al. [25] for learning the robot model, we can find in the literature several methods to learn forward and inverse models. In [26], they used a combination of Circle Point Analysis (for estimating the kinematics parameters) and the Levenberg-Marquardt algorithm (for refining the model) to learn the visual and joint kinematics at the same time. This method can be adapted for tool-extension and perspective transformation. The Locally Weighted Projection Regression (LWPR) [27] can be also used to learn the forward/inverse model in an on-line schema, where the learning parameter controls the level of adaptation to changes on the system.

Neural networks as a general non-linear regressor can cope with the forward models learning. Perceptron like neural-networks with one hidden layer with reduced number of neurons (e.g., 6 neurons for 2DOF learning) are also able to learn the model using the standard back-propagation algorithm [18]. Deep learning, and in particular multiple time-scales recurrent neural networks (MTRNN) and deep neural networks (DNN), has shown generalization capabilities for the body model acquisition and its extension for tool-use [19]. The method relies on several stages of motor babbling for acquiring the body-schema and tool using. Finally, network architectures that combine Hopfield, RNN and associative networks, as proposed by Tani in [28] has shown great potential to construct a dynamical system able to learn from interaction.

We can also find in the literature probabilistic approaches with nice properties. Gaussian processes (GP) as a generalized regressor have been used to compute the forward model. In [29] the robot learns how to reach desired state (e.g., location) using GP for learning the model parameters. This works differentiates between updating and adapting the model depending on the distance between the new sample and the training data. This allows to refine the model when needed without breaking the on-line scheme. Infinite Mixture of Linear Experts (IMLE) algorithm [30] has been used to learn multi-valued forward and inverse kinematics, improving in that aspect the LWPR. Here, the parameters learning is reserved for an Expectation-Maximization (EM) methodology. This approach has shown generalization capabilities for multiple tool models [31]. Finally, Dynamical

Bayesian Networks (DBN) have been proposed for inferring the parts of the body that belong to the robot exploiting the multimodal correlations [32], i.e., self-detection [33]. In [34], an extra simplified DBN model was proposed to differentiate own body from the others and we recently presented a hierarchical Bayesian model that relates the spatio-temporal sensory signals using a more plausible visual attention system [32].

B. Inter-modality models

Another way to learn the body-schema is to capture the interrelation between several sensor modalities or senses. For instance, the robot learns the model that relates between the visual f_v and the proprioceptive f_p sensing. Inter-modality learning is really important from the point of view of the construction of the self, as they reflect unique timing patterns that can be used for self-detection [7]. In some literature this is also referred as cross-modal learning. We formalize mathematically the inter-modal model as:

$$\hat{s} = f_v \wedge f_p \tag{2}$$

where the state in one of the modality manifolds or in a new one is given by the interaction of different sensors.

Recurrent neural networks have been used as associative memory for this purpose. For instance, Nabeshima et al. were able to learn simple reaching capabilities by integrating visual and tactile information in a two joints planar robot [18]. Another popular method to address these inter-modality models are variants of Self-Organizing Maps (SOM), as it permits on-line learning and performs dimensionality reduction. The main drawbacks of this approach is the elevated quantity of samples needed for proper learning and potential drifting of the resultant weights [35]. Hikita et al. used one SOM for each modality and then one specific SOM for integrating both senses for simple body and tool-extension learning [22]. More complex visuo-motor coordination, i.e., maintaing the end-effector in the visual field, has been studied by Schillaci et al. by means of two Dynamic SOM layers (one for the arm and one for the head) connected trough Hebbian links [20]. MTRNN architectures have also being used to learn the inter-modality and a self-motion predictor at the same time [36], [19]. Furthermore, spike-timing-dependent plasticity learning has been employed to produce inter-modal binding in neural spiking networks with accurate timing patterns [37]. Finally, as proposed in [7], inter-modality models can be formalized as Bayesian networks.

III. EXPLORATION

In terms of exploration algorithms the most common method is random babbling (e.g., motor babbling), where the robot exerts random feasible movements and then it uses motor and sensing data as the sampling vector. This approach has been employed in [38], [20], [19], among others. An interesting improvement of random babbling is generating the movements by chaotic units (e.g., logistic functions) that connect sensors and actuators [39]. Chaos and resonance compete for increasing the entropy of the system, providing dense inter-modality correlation, and "stable" spatio-temporal patterns.

On the other hand, Rolf et al. proposed goal babbling [25], where the sparsity of the exploration is reduced as the task contextual information has been included into the exploration. Body inverse models can be more efficiently learn with this method. In order to reduce the elevated number of samples required for the learning, [36] has also proposed restricted DOF exploration to learn a rough model through and then refine it during later interaction (e.g., while reaching).

Complementary techniques have been studied by Odeyer et al. in [40], where partitioning the state space allows to evaluate which action should be exerted by the robot to improve the robot skill. This exploration methods assume that intrinsic motivation drives the robot towards the accumulation of competence. In this sense, other information-based measures have been also investigated by Martius et al., such as predictive information [41].

IV. BODY/SOCIAL MODELS

Accounting for social interaction within the construction of the self has been less studied from the computational point of view. However, a complete enactive self model should incorporate social cues.

One of the most interesting works has been proposed by Nagai and her colleges in [42], [43] where the robot incrementally learns its own self-perceptual schema and then integrates the cues from other agents. This approach, which exploits a predictive model learning, emphasizes the importance of self/other distinction. Thomas and Brezal, in [44] started investigating about learning with human interaction and Cedeborg et al. analysed theoretically the influence of teachers at the agent learning [45]. With this philosophy, Nguyen and Oudeyer proposed a socially-motivated method by switching between imitation and intrinsic motivation [46].

The social component should be further studied despite of the complexity of the cues to provide a coherent enactive self.

V. APPROACHES ANALYSIS

First of all, we should highlight that just a few works in the robotics literature have proposed explicit mathematical models of self construction and the majority can only be used to implicitly build a model of the self. Secondly, it is important to mention that the theory of predictive coding [47] have had a strong influence in these studies and therefore, the idea of using the error between the current stimuli and the predicted one as the cue for self-distinction have gained popularity (Fig. 2). For instance, in [28] a dynamical system approach using neural networks has been presented as a generic construction of the self. Despite of the fact that the proposed architecture fits the paradigm of bottom-up learning vs top-down prediction, the self model was presented just as a philosophical discussion. An adaptation of this concept is the predictive learning approach [42], where the prediction of others actions also plays an important role in the development of the self. Another interesting point of view, presented in [7], is to picture self-construction just as an emergent characteristic of sensorimotor information integration. In this case, just stimuli contingency patterns can provide body-ownership and agency through error prediction or sensory attenuation, in coherence with the rubber hand illusion [48], as it has been presented in [49].

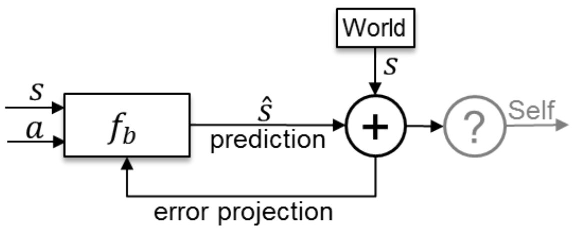


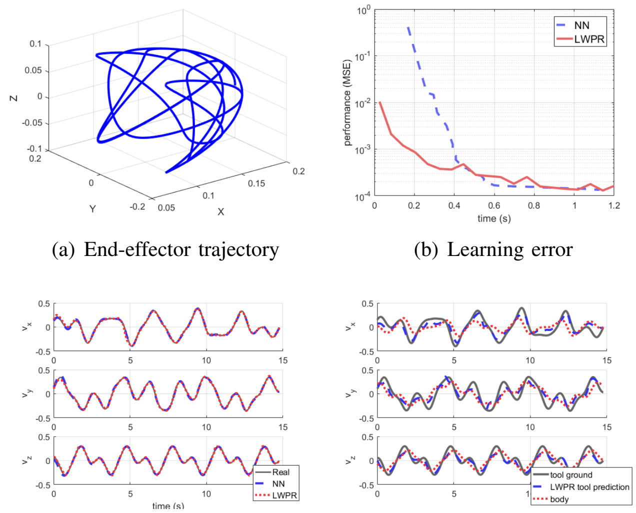
Fig. 2. Simplified conceptual self-construction using error prediction.

We have analysed some of the most relevant approaches to model the enactive self taking into account three main designing aspects that should be faced in an enactive model:

- *Learning*. Describes the algorithm that the agent uses to learn from the stimuli. The method should account for plasticity and drifting countermeasures
- *Exploration*. Defines the active method to sample the state space and the utility functions that drive the learning.
- *Memory retrieval*. Where to store the information depending on the followed representational approach and how to retrieve the already learned information.

Evaluating the works presented in Table I, an enactive robot should count with: the multivalued learning provided by [31] or [19], the efficiency and adaptability of [27], the inter-modal learning as [20], [7], incremental refinement of the model using new knowledge [36], [50], multisensory self-detection and causality inference as [32], be able to progressively switch from chaotic generators to goal-babbling [25], and incorporate self-perception learning with social cues [46], [51]. The design of the algorithm should also enforce knowledge reusing [52], although in some cases this can be computationally expensive.

In order to show how the forward model can be learned we have compared two state-of-the-art methods in a 2DOF robot armfeedforward neural network (NN) using off-line back propagation and on-line Locally Weighted Projection Regression (LWPR) [27]. The distance between the joints $(L1, L2)$ are 10 cm length. We conduct periodic movements on the robot during 30 seconds following the pattern presented in Fig. 3(a). The first 15 seconds are used for training and the last 15 seconds are used for testing the methods. Figure 3(b) shows the learning curves for both algorithms. NN achieves a mean squared error (MSE) performance of 0.000090 for the training data in 1.989476 seconds and gets a MSE performance for the test data of 0.000098. On the other hand, LWPR performance is 0.000123 and 0.000191 for



(a) End-effector trajectory (b) Learning error

(c) Predicted velocity (d) Tool extension

Fig. 3. Comparison between NN and LWPR for learning a differential forward model for $\dot{x}$. (a) End-effector trajectory generated. (b) Training error. (c) Comparison between the predicted $\dot{x}$ and the real one (only the velocities). (d) Tool-extension adaptation using LWPR with 15 seconds of new data.

training and test respectively. Although there is a small error, the predicted velocities are quite accurate as Fig. 3(c) shows. This means that kinematic models are easy to learn with current methods. What it has not been sufficiently studied is the plasticity of those methods for incorporating changes on the robot and environment while maintaining the main processes that construct the agent identity. We have tested to incorporate a rigid tool (e.g., a stick) with 15 cm length and rotated $90°$ with respect to the end-effector z axis. The NN should be retrained as the parameters are overfitted (standard incremental NN methods present unreliable performance). LWPR has shown better behaviour and as long as sequences of the movement are performed, the model converges to the correct tool forward model. Fig. 3(d) shows the LWPR prediction when 15 seconds of the new tool effector movements have been observed. However, this does not provide any type of identity, as the forward model has been changed and the body forward model is implicitly encoded. Moreover, these methods need the observed velocities of the effector. In this sense, both [46] and [42] go one step further by explicitly treating the error of the predictor machine to new observations for generating different learning behaviours.

In conclusion, these state-of-the-art methods are able to compute forward and inverse models. However, they do not fully account for sensorimotor self learning.

VI. Concluding Remarks: Artificial Enactive Robots and Biological Plausibility

Timing, spatial correlations and contingency [18], [53] are three main known pillars for learning self patterns. One of the potential models for the enactive self is to learn the forward body model (some interesting methods have been presented here) and then use the prediction error to adapt

TABLE I

STUDY OF POTENTIAL APPROACHES FOR MODELLING THE ENACTIVE SELF.

Work	Learning	Exploration	Memory retrieval	Advantages	Limitations
General frameworks					
Tani1998 [28]	Hopfield, Associative and RNN	Fixed + attention	RRN prediction	top-down modulation	complex learning dynamics
Kuniyosi2004 [38]	two-layer Hopfield nets	Random signalling	Connections	Close loop learning	Drifting and unpredicted behaviours
Nabeshima2006 [18]	SOM	Motor babbling	Connections	Multisensory learning	only 2DOF
Forward/inverse models and exploration					
Baranes2009 [21]	Region partition + Regressor(not specified)	Intrinsic motivation (Goal-babbling through competence)	Nearest neighbours	unsupervised learning of the sensory consequences and explicit predictive error treatment	Depends on local regressors of the sparse sampled state space
Vijayakumar2005 [27]	Locally Weighted Projection Regression	Not defined	Projection weights	Tackles high-dimensionality efficiently	Learns only one model
Rolf2010 [25]	Gradient descent	Goal babbling	Parameters	Inverse kinematics learning of redundant robots	A goal is needed
Damas2012 [30], [31]	Mixture of Linear Experts + EM	Motor Babbling	Parameters	Learns multiple inverse and forward models	Elevated number of experts for high-dimensionality contexts
Hart2015 [26]	Levenberg-Marquardt	Circle point analysis	Parameters	Simultaneous visual and body calibration	Predefined visual model and kinematic chain
Ghadirzadeh2016 [29]	Gaussian Process + Resilient backpropagation	Min. distance between the current state and the goal state	Hyper-parameters	Dimensionality reduction by relevance	Model retrain for new samples
Wieser2016 [36]	MTRNN + customized gradient	restricted DOF	Forward net (Weights)	Small number of training samples and allows refinement	Assumes end-effector and object distinction
Takahasi2017 [19]	Deep Learning (MTRNN)	Different stages of motor babbling	Forward net (Weights)	Grasping and ungrasping learning	Elevated amount of training samples processed off-line
With social component					
Nagai2011 [51]	Visual X-means clustering + Hebbian learning	Vertical and horizontal patterns	Network activation	Incorporates self/other distinction through predictor error	Poor scalability
Odeyer2014 [46]	Region partition + Regressor(not specified)	Intrinsic motivation / Imitation	nearest neighbours+interpolation	Learning by demonstration integrated	Depends on local regressors of the sparse sampled state space and the social development is not gradual.
Inter-modal approaches					
Schillaaci2014 [20]	DSOM	Motor babbling	Connections	Visuo-motor coordination	Scalability and high num. of samples
Lanillos2016 [7]	Dynamical Bayesian nets	Fixed	Posterior distribution	Self-detection model	Does not learn forward or inverse kinematics
Pitty2017 [49]	Rank order coding algorithm	Not defined	Associative and Recurrent maps	Inter-modal timing relations	Depends on the network dynamics

or refine the model as well as differentiating the inbody from outbody/other sources. Here, the challenge is to encode multisensory forward and inverse models for the whole perceptual schema. Another approach is to learn the perceptual patterns that unequivocally represent the agent (some of the inter-modality approaches have been described). This will yield, oversimplifying, to basic self/other distinction. Many challenges arise from this approach as classical kinematics and dynamics cannot be employed as tool-extension, and reaching should appear as an emergent behaviour.

The studied methods have interesting properties for an enactive model of the self. However, we are still far from a full fledged computational model that harmonises both body and social interaction. Furthermore, although some of the proposed techniques overlap in different robotic fields, we enforce the enactive developmental approach [54], [5] in contrast to other paradigms, such as active perception or interactive perception [55] arguing that real adaptability emerge from the mastery of inter-modality contingency patterns during the self-perception exploration stage [32], and executing current machine learning algorithms during interaction is not enough for modelling enactive robots. Embodied intelligence [56] from the enactive paradigm should maintain agent's identity, hopefully building the first step of many for achieving artificial self-awareness. Furthermore, there might be a relation between active learning [57] and self-construction that enables voluntary interaction with the environment.

The biological plausibility of all the methods analysed is debatable. The well-known sensorimotor mapping is just one of the contributors to self construction and agency. It is unclear how the basal ganglia [58] and the dopamine segregation affects error prediction or agency. Reinforcement learning and surprising events detection should also play an important role. Furthermore, the engineering approaches for learning body models omit the relation between the cerebellum motor coordination and the sensorimotor body-schema mapping.

APPENDIX

A. Linking forward sensorimotor models and differential forward models in robotics

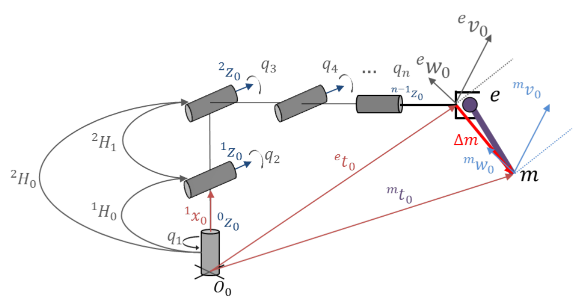


Fig. 4. Rigid body limb and tool extension description and notation. The end-effector e is connected through a set of n joints to the origin O_0. The new rigid body is attached to the end-effector. The angular and linear velocity of the new point m with respect to O_0 are ${}^m w_0$ and ${}^m v_0 \in \mathbb{R}^{3\times 1}$ respectively.

This section shows the relation between the sensorimotor forward models expressed in Eq. 1 and the differential kinematics used in robotics. This enforces the idea that sensorimotor models are a generalization of rigid body models. We further show that the aggregation operation (Sec. II) for tool extension in differential forward kinematics holds.

From the geometrical point of view the robot rigid body is defined by a set of homogeneous transformations $H = [R\ t] \in \mathbb{R}^{3\times 4}$ that spatially relates every joint to a common coordinate frame. Figure 4, depicts the configuration of a general robotic limb with a tool at the end-effector. Here the forward differential kinematics model describes the velocities (linear v and angular w) of the joints or end-effector depending on the rest of the joint positions q. At Fig. 4, the notation assumes that the common coordinate frame is O_0. We remind that the linear and angular velocity (${}^e v_0$ and ${}^e w_0$ respectively) of the end-effector e with respect to O_0 is:

$$\begin{bmatrix} {}^e v_0 \\ {}^e w_0 \end{bmatrix} = \begin{bmatrix} {}^0 z_0 \times {}^e t_0, & \dots, & {}^{n-1}z_0 \times ({}^e t_0 - {}^{n-1}t_0) \\ {}^0 z_0, & \dots, & {}^{n-1}z_0 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \vdots \\ \dot q_n \end{bmatrix} \tag{3}$$

where $\dot q_i$ is the joint i velocity. The matrix that multiplies the set of $\dot q$ is referred as the kinematics Jacobian $J_b(q)$ and defines the gradients of task motion depending on the joint position (robot state ${}^e \dot x$) depending on the joint positions q. Note that this model is a particular case of sensorimotor model previously defined, where s is substituted by the joint angle measurement q and a by the derivative of the joint angle or joint velocity $\dot q$. Then, the predicted sensation is the relative change in the location of the body in the space $\dot x = f_b(q, \dot q)$[3]. Note that the derivative of the effector state can express, for instance, the change on the position of the effector that is perceived by the proprioceptive and visual senses.

1) Differential forward model with a Jacobian: Let us assume that the robot has learnt the kinematics model of the end-effector f_b as a function of q and $\dot q$, the tool extension is then defined by the combination of J_b and the tool model J_t. This combination in general form is (see Appendix B for proof):

$$\dot x = [J_b + J_t]\dot q \tag{4}$$

In other words, assuming that we already have $J_b(q)$ and we observe and estimate the new Jacobian $J_t(q)$ we can extract the Jacobian $J_m(q)$ that is contributing for point m as follows:

$$J_m(q) = J_t(q) - J_b(q) \tag{5}$$

Or in the case that the kinematic model has full rank, it can be alternatively described as the product [18]:

$$\dot x = J_b J_t \dot q \tag{6}$$

In order to learn the forward kinematic model with a tool (rigid body extension) a Jacobian of size number of joints $\times 3$ has to be learnt. Note that the angular velocities remain the same.

2) Differential forward model as a function of the kinematics parameters: The forward model Y_b can also be written as a function of the parameters θ_b:

$$\dot x = Y_b \theta_b \tag{7}$$

Thus, if the robot knows Y_b, the tool extension becomes:

$$\dot x = Y_b \theta_b + Y_t \theta_t \tag{8}$$

Figure 5(a) shows an example of a tool extension without rotation, i.e., it has the same direction as the end-effector. Here, the body kinematics parameters are defined by the length of the arms $\theta = [L_1, L_2]$ and Y_b is described as a 3×6 function.

Assuming that Y_b is known, when there is an extension of the arm of length Δ, then $L2$ should be substituted by $L2 + \Delta$. Operating over Eq. 7 we have that the new linear velocities ${}^m v_0$, which correspond to the three first components of $\dot x$, are:

$$\dot x = Y_b \theta_b + Y_t \begin{bmatrix} 0 \\ \Delta \\ 0 \end{bmatrix}$$

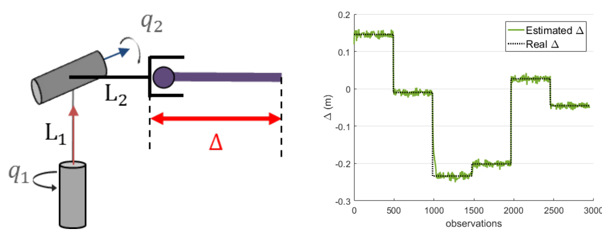
Here Y_t corresponds to all zeros with the exception of the elements $Y_t(1,2), Y_t(2,2), Y_t(3,2)$ that get the same values

[3]The explicit use of the same notation f_b has been adopted to emphasize the similarity between the functions.

as in the original Y_b. The problem here is reduced to compute the parameter Δ. This tool extension, described by Y_t and θ_t, can be computed off-line using simple least squares regression or in a stochastic gradient descent on-line scheme. For the Fig. 5(a) the iterative algorithm is:

$$F = {}^{t}v_0 + Y_b \cdot [0, \hat{\Delta}, 0]^T$$
$$G = [Y_b(1,2), Y_b(2,2), Y_b(3,2)]^T$$
$$\hat{\Delta} = \hat{\Delta} - \alpha \sum_i (F_i - {}^{e}v_0) \cdot G_i$$

Note that α parameter controls the adaptability of the system to changes on the input $\dot{x}$ (observed end-effector position). Figure 5(b) shows this on-line stochastic gradient descent algorithm for the parameter learning when changing the length of the tool. The data has been generated synthetically and the tool extreme location input has a Gaussian noise of mean 0 and standard deviation 0.02 meters.



(a) Robot configuration (b) noise $\sim N(0, 0.02^2)$

Fig. 5. Rigid body tool extension without rotation. (a) robot configuration with two rotating joints and an extension in the same axis of the last joint. L_1, L_2 and Δ express the length of each segment. (b) Learning $\theta_t = \{\Delta\}$ adaptively with different extensions of the arm. We can observe that with just a small amount of observations the system re-adapts to the new situation.

B. Rigid body tool extension Jacobian as a summation

We write the tool extension kinematics forward model as Eq. 3, but considering m as the extreme point of the tool instead of e. Note that the angular velocity does not change with a rigid body tool extension. Then, defining Δm as the translation vector from the end-effector e to the new point m: ${}^{m}t_0 = {}^{e}t_0 + \Delta m$, the tool velocity ${}^{m}v_0$ becomes:

$${}^{m}v_0 = \left[{}^{0}z_0 \times ({}^{e}t_0 + \Delta m), \ldots, {}^{n-1}z_0 \times ({}^{e}t_0 + \Delta m - {}^{n-1}t_0)\right] \dot{q} \quad (9)$$

Let the forward kinematics with tool extension be:

$$\begin{bmatrix} {}^{m}v_0 \\ {}^{m}w_0 \end{bmatrix} = f_t(q)\dot{q} \quad (10)$$

Using the distributive property of the cross product over the addition, we can rearrange the terms of Eq. 9 ${}^{1}z_0 \times ({}^{e}t_0 + \Delta m - {}^{1}t_0)$ as ${}^{1}z_0 \times ({}^{e}t_0 - {}^{1}t_0) + {}^{1}z_0 \times \Delta m$. This makes that we can separate the function $f_t(q)$ as:

$$f_t(q) = J_e(q) + f_m(q) \quad (11)$$

where $f_t(q)$ is:

$$f_t(q) = \begin{bmatrix} {}^{0}z_0 \times \Delta m & {}^{1}z_0 \times \Delta m & \ldots & {}^{n-1}z_0 \times \Delta m \\ 0 & 0 & \ldots & 0 \end{bmatrix} \quad (12)$$

Therefore, the generalized kinematic forward model with tool extension is given by:

$$\dot{x} = [f_b + f_t]\,\dot{q} \quad (13)$$

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