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RISK BASED ACCEPTANCE CRITERIA FOR JOINTS SUBJECT TO FATIGUE DETERIORATION

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ABSTRACT

Different approaches to determine the acceptance criteria for fatigue induced failure of structural systems and components are discussed and compared. The considered approaches take basis in either optimization (societal cost-benefit analysis) or are derived from past and actual practice or codes (revealed preferences). The system acceptance criteria are expressed in terms of the maximal acceptable annual probability of collapse due to fatigue failure. Acceptance criteria for the individual fatigue failure modes are then derived using a simplified system reliability model. The consequence of fatigue failure of the individual joints is related to the overall system by evaluating the change in system reliability given fatigue failure. This is facilitated by the use of a simple indicator, the Residual Influence Factor. The acceptance criteria is thus formulated as a function of the system redundancy and complexity. In addition, the effect of dependencies in the structure on the acceptance criteria are investigated. Finally an example is presented where the optimal allocation of the risk to different welded joints in a jacket structure is performed by consideration of the necessary maintenance efforts.

INTRODUCTION

For an offshore operator responsible for the safe and efficient operation of an entire facility it is important that the overall facility specific requirements to the acceptable risk to personnel, environment and economy can be verified and documented to the relevant authorities. A prerequisite for this is that the risk arising from the different sub-systems of the facility can be quantified. Design and inspection requirements are, however, generally given on a component or element level and consider different failure modes separately. They normally focus on the probability of failure and neglect the consequences of the individual failure on the system. To prove that the design and maintenance specifications for the individual components are in compliance with the overall facility risk acceptance criteria, it is necessary to relate the individual component risk to the entire facility or the sub-system. In the present paper fatigue failure modes in an offshore steel structure are considered and a simplified but consistent approach to derive risk acceptance criteria for the individual joints is presented. Previous efforts in this research area include work performed by Kirkemo [1], Moan and Vardal [2], Faber et al. [3] and Stahl et al. [4], however, so far no practically applicable approaches have been identified addressing how to derive component specific risk acceptance criteria from an overall facility perspective.

The different approaches considered within the present paper are divided into three main directions: 1) derivation of acceptance criteria from expressed preferences, such as acceptable *FAR* (Fatal Accident Rate); 2) derivation of acceptance criteria based on revealed preferences (best practice); 3) derivation of acceptance criteria based on an optimization approach. The main focus is on the second part in view of a pragmatic and applicable methodology for deriving acceptance criteria. The consistent consideration of system complexity and redundancy in the assessment of the acceptance criteria is presented.

Definitions

The term collapse is in the following reserved for the event of failure of the overall structure. The term failure is reserved for the event of fatigue failure of welded joints. A crack in a joint can occur at different hot spots. Only the term hot spot will be used in the following. A failed hot spot signifies a failed joint, where it is assumed that a failed joint has no residual load carrying capacity.

Acceptance criteria are given in terms of maximum allowable annual probabilities of failure, in accordance with Rackwitz [5].

Nomenclature

b	factor relating load to wave height, Eq. (A1);
B	expected benefit from the operation of the facility;
C_D	cost of design;
C_F	expected cost of failure;
C_M	cost of maintenance;
COL	event of collapse caused by structural failure;
$CORR$	cost of risk reduction;
F	event of having one or more fatigue failures;
$\overline{F}$	event of having no fatigue failures in the structure;
F_i	event of fatigue failure of hot spot i;
FA	event of a fatality;

FAR	fatal accident rate;
FAR_{acc}	acceptable FAR;
FAR_{OS}	FAR related to all accident scenarios no related to fatigue failures;
FDF	fatigue design factor;
H	maximum annual wave height;
I	income from the operation of the facility;
$ICAF$	implied cost of averting a fatality;
N	number of critical hot spots in the structure;
N_F	number of failed hot spots;
p_{acc}	accepted annual probability of fatigue induced structural collapse;
p_{acc,F_i}	accepted annual probability of fatigue failure for hot spot i;
p_{COL}	annual probability of collapse due to structural failures;
$p_{F\,i}$	annual probability of fatigue failure of hot spot i;
$p_{COL\|F_i}$	conditional annual probability of collapse given fatigue failure of hot spots i;
$p_{COL\|\overline{F}}$	annual probability of collapse given no fatigue failure;
$p_{COL\|F}$	annual probability of collapse given one or more fatigue failures;
R	resistance against wave load, Eq. (A1);
RIF	residual influence factor (defined in Annex B);
RSR	reserve strength ratio (defined in Annex A);
X_C	characteristic value of X;
α	risk mitigation action;
δ	factor relating load to wave height, Eq. (A1);
ψ	chosen ratio of the risk related to fatigue introduced collapse to the collapse risk related to extreme environmental loads.

Part A – Direct Acceptance criteria

Some authorities and codes specify a maximum risk to different groups of people exposed (personnel, public), see e.g. Paté-Cornell [6]. One current format for such criteria is the allowable Fatal Accident Rate (FAR), Aven [7] and Vinnem [8]. The

FAR is defined as the number of fatalities per 100 million hours of exposure; it is a measure for the acceptable risk to the individual and provides no direct information on the acceptable expected loss of lives. The FAR is very often used in quantitative risk analysis (QRA).

Stahl et al. [4] show how the acceptable probability of platform collapse can be derived from the FAR criteria. Taking basis in an overall facility acceptable FAR (denoted FAR_{acc}) the derivation assumes that the FAR share that can be allocated to structural collapse is the difference between FAR_{acc} and the FAR contributions due to all other failure and accident scenarios. The apparent shortcoming of this approach is that the derived acceptable risk due to structural collapse is much higher than observed in practice (approx. $2/3$ of the total risk in the example given by Stahl et al. (1998)), and therefore is in contradiction to present practice (discussed in part B). Furthermore FAR values from non-structural sources generally change (and can be changed) much more frequently than risk due to structural failures. Allocating large portions of the allowable risk to the inflexible structural system seems not to be an economical solution.

For most structural elements risk is mainly related to collapse of the entire facility, but for fatigue failures leading to local (in a limited area) endangering of personnel, the direct criteria can be applied to specify a maximum failure probability. The acceptable probability of fatigue failure per year p_{acc,F_i} for a hot spot i is then

$$p_{acc,F_i} = \frac{\left(FAR_{acc} - FAR_{OS}\right)}{P\left(FA \middle| F_i\right)} \cdot \frac{24 \cdot 365}{10^8} \qquad (1)$$

where FAR_{acc} is the acceptable FAR value, FAR_{OS} is the risk due to all other accident scenarios and $P(FA|F_i)$ is the probability of a fatal accident given fatigue failure for any person in the area.

Part B - Acceptance Criteria based on Revealed Preferences

Deriving acceptance criteria based on revealed preferences, it is assumed that the socio-economic risk associated with current practice and codes is generally accepted by the society. Such inherent acceptance criteria is evaluated

1) by calculating the probability of failure for different limit state functions that comply with code requirements, using Structural Reliability Analysis (SRA).

2) taking basis in the overall collapse capacity (as given by the RSR), assuming that the prevailing structural collapse mechanism is extreme environmental loading.

3) by Quantitative Risk Assessments (QRA), determining the failure rate observed in practice. This approach is difficult to apply for fatigue failures in practice due to the lack of available reliable data.

The general shortcoming of the approach is that progress in society (e.g. the increase in life expectancy) is not accounted for. A general discussion of acceptance criteria based on revealed preferences and its implications can be found in Slovic [9].

Acceptance criteria derived directly from codes

As noted in Faber [10], reliability indices from different sources and models should not be compared directly. Therefore, the acceptable probability of failure and the probability of failure as evaluated for a specific structure should be calculated based on the same assumptions, applying the same methodology. A simple but (in this way) consistent approach is presented in the following, based on Faber et al. [10] and Moan and Vardal [2].

The design criteria given in the NORSOK standard [11] for welded joints that cannot be inspected is a Fatigue Design Factor (FDF)[a] larger than 10 when the consequences of fatigue failure are large and $FDF \geq 3$ when the consequences are minor. From these criteria it can be concluded that the probability of failure in the last year of service for a hot spot with FDF = 10 corresponds to the acceptable annual probability of collapse due to fatigue failure. Collapse is implied here in consistency with the assumed high consequences of fatigue failure. It should, however, be noted that a large majority of the joints in service

1) have FDF 's larger than the minimum requirement of the codes;

2) are not in their last year of service, therefore having a smaller annual probability of failure;

3) are part of a redundant system, where collapse is not equivalent to first member failure.

[a] The FDF is defined as the ratio of the calculated design fatigue life to the service life of the structure [3].

These factors lead to a “hidden” safety inherent in present structures. Focusing on the minimum required reliability without accounting for this additional safety may thus lead to an increase in the failure rate compared to present practice. The non-consideration of system redundancy on the other hand leads to inconsistency in the criteria, which, however, is common to all actual structural codes. The advantage of the method is that the same modelling can be applied to derive the acceptance criteria and to prove compliance of existing structures and maintenance policies with these criteria. Due to its simplicity and consistency (regarding the reliability model) this approach is by now the most commonly applied. An additional advantage is that the approach allows to derive a serviceability criterion (the minimum fatigue reliability required by the code, eg. $FDF \geq 3$ in NORSOK). For serviceability criteria, the non-consideration of the system effects has no implication. Fig. 1 illustrates the application of the NORSOK criteria:

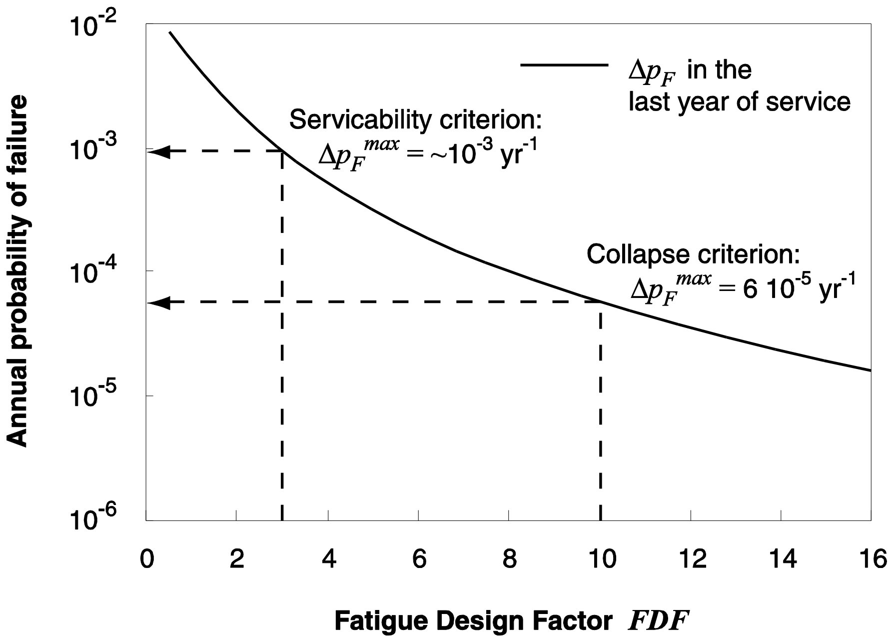

Figure 1. Annual probability of fatigue failure in the last year of service given no prior fatigue failure as a function of the FDF (applying the probabilistic model from Straub [12]).

Acceptance criteria derived from the Reserve Strength Ratio (*RSR*)

If the fatigue performance of a structure is considered separately from the other failure modes, and if collapse due to fatigue failure is not considered as the main collapse mode, then acceptance criteria for fatigue failure can be derived from the acceptable risk of collapse due to the predominant structural collapse mechanism. For fixed jacket structures this will generally be an extreme weather event. The annual probability of collapse due to wave loads can be estimated as a function of the RSR, as described in Annex A. In accordance with Fig. A1 it can e.g. be said that a structure with RSR = 2 corresponds to a $p_{acc} = 1.2 \cdot 10^{-4}\ \mathrm{yr}^{-1}$.

The acceptable risk due to structural collapse generally includes the entire set of possible collapse mechanisms. Collapse due to fatigue failure at one or several hot spots represents only a fraction of all possible mechanisms. Risk acceptance criteria for collapse due to fatigue failure should therefore be stricter than the overall criteria. In accordance with Faber et al. [13] the problem may be approached by taking basis in the risk analysis as part of the concept studies and design verification (FMECA, RAM, QRA), as well as experiences from similar facilities. There is, however, little data on severe structural fatigue failures available. Establishing a representative statistic is thus not possible. Using risk analysis to determine the share ψ of the acceptable risk due to structural collapse that can be attributed to fatigue failures has the advantage that the acceptance criteria are in accordance with the general design philosophy. A factor of $\psi = 0.1$ is assumed in the following; in Straub [12] it is demonstrated how this factor can be obtained from calibration to present standards.

Indicators for the system characteristics

Given the acceptable annual probability of collapse, p_{acc}, the acceptance criteria for the individual hot spots are derived in the following. The allocation of the risk to the different hot spots must thereby be based on the following factors:

1) Redundancy (of the structural system)

2) Complexity (the number of fatigue critical hot spots)

3) Dependency (between the different failure and collapse modes)

Effectively, the acceptance criteria for each hot spot are based on indicators for these three system characteristics as presented in the following.

Modeling the structural system

System strength is represented by the annual probability of collapse, p_{COL}, which is a function of the state of the individual hot spots. For the purpose of simplification the hot spots are modeled as either intact or failed, i.e. no continuous decrease of the hot spot's performance is considered. F_i denotes the event of fatigue failure of the i th hot spot and $\overline{F_i}$ the event of survival of the i th hot spot. The principal form of the degradation of the system strength towards collapse is illustrated in Fig. 2, where one possible realization of p_{COL} is shown.

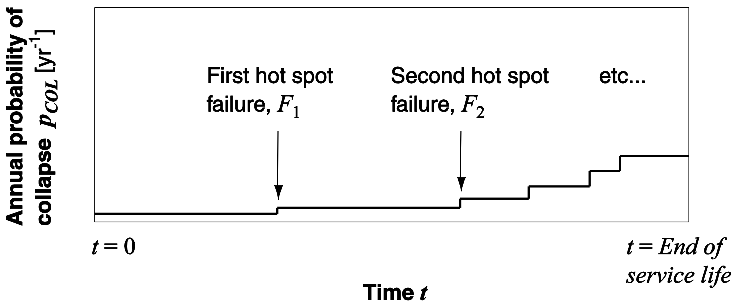

Figure 2. Illustration of system deterioration model: One possible realization.

The simplification allows for modeling the system as a series system, illustrated in Fig. 3 for a structure with two fatigue critical hot spots. For a structure with N critical hot spots the system is accordingly, consisting of 2^N elements.

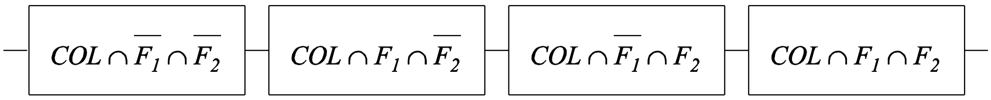

Figure 3. System reliability model (with 2 fatigue critical hot spots).

With the introduction of conditional events of collapse the model presented in Fig. 3 is modified to Fig. 4. Therein the event of any (i.e. one or more) fatigue failure is denoted by F.

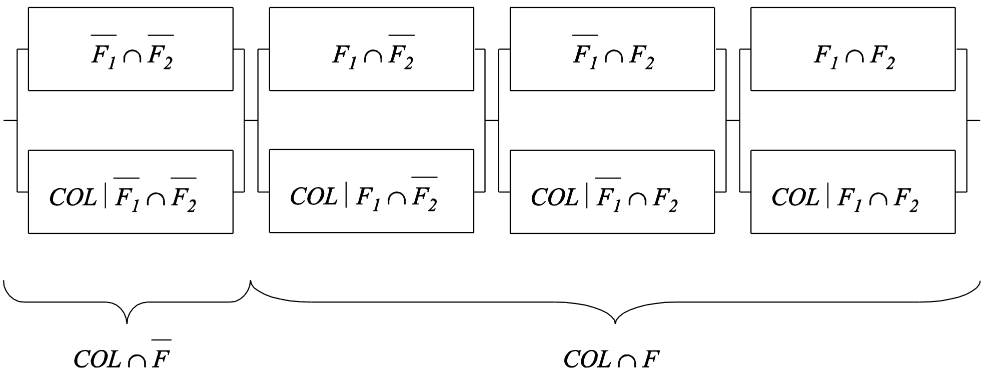

Figure 4. System reliability model (with 2 fatigue critical hot spots) using conditional collapse events.

The total probability of collapse is evaluated from the probabilities of the individual events as shown in Fig. 4. For this system with 2 fatigue critical hot spots the probability is calculated as in Eq. (2). It is the probability of failure of a series system with mutually exclusive events.

$$
\begin{aligned}
p_{COL} = \; & P\left(COL \middle| \overline{F_1} \cap \overline{F_2}\right) \cdot P\left(\overline{F_1} \cap \overline{F_2}\right) + \\
& P\left(COL \middle| F_1 \cap \overline{F_2}\right) \cdot P\left(F_1 \cap \overline{F_2}\right) + \\
& P\left(COL \middle| \overline{F_1} \cap F_2\right) \cdot P\left(\overline{F_1} \cap F_2\right) + \\
& P\left(COL \middle| F_1 \cap F_2\right) \cdot P\left(F_1 \cap F_2\right)
\end{aligned} \tag{2}
$$

Eq. (2) is simplified further by the following approximations, which are justified by the generally large fatigue reliability of welded joints in offshore structures:

$$
\begin{aligned}
& P\left(F_1 \cap \overline{F_2}\right) \approx p_{F_1} \qquad P\left(COL \middle| F_1 \cap \overline{F_2}\right) \approx p_{COL|F_1} \\
& P\left(\overline{F_1} \cap F_2\right) \approx p_{F_2} \qquad P\left(COL \middle| \overline{F_1} \cap F_2\right) \approx p_{COL|F_2}
\end{aligned} \tag{3}
$$

$p_{COL|F_i}$, the annual probability of collapse given fatigue failure of hot spot i, may be estimated as a direct function of the Residual Influence Factor (RIF). This important step is shown in Annex B. Thereby the RIF is a main indicator for the redundancy the structure.

Because collapse not related to fatigue failures is treated elsewhere, only the probability of collapse combined with fatigue failure, $p_{COL \cap F}$, is considered here. It is

$$
p_{COL \cap F} = p_{COL} - p_{COL \cap \overline{F}} \approx p_{COL} - p_{COL|\overline{F}} \tag{4}
$$

Approximating $p_{COL\cap\overline{F}}$ by $p_{COL|\overline{F}}$ is again justified by the generally large fatigue reliability.

If the fatigue failures are fully dependent, then the middle terms in Eq. (2) become zero. If the fatigue failures are independent[b], then the last term in Eq. (2) (the one of higher order) can be neglected as a consequence of $P(F_1 \cap F_2) << \max\left[P(\overline{F_1} \cap F_2), P(F_1 \cap \overline{F_2})\right]$. In that case the probability of collapse due to a fatigue failure is rewritten as

$$\begin{aligned} p_{COL\cap F} &= \left(p_{COL|F_1} - p_{COL|\overline{F}} \right) \cdot p_{F_1} \\ &\quad + \left(p_{COL|F_2} - p_{COL|\overline{F}} \right) \cdot p_{F_2} \end{aligned} \tag{5}$$

In order to relate the overall fatigue acceptance criteria p_{acc} with criteria for the individual hot spots, $p_{COL\cap F}$ is replaced by p_{acc} and p_{F_i} is replaced by p_{acc,F_i}, the fatigue acceptance criteria for the individual hot spots. When p_{acc} is given, a second condition is needed to the derivation of the p_{acc,F_i}. Lacking specific information a practical approach is to require that the contribution to the risk is equal for all hot spots[c], therefore

$$\left(p_{COL|F_1} - p_{COL|\overline{F}} \right) \cdot p_{acc,F_1} = \left(p_{COL|F_2} - p_{COL|\overline{F}} \right) \cdot p_{acc,F_2} = q \tag{6}$$

then

$$p_{acc} = 2 \cdot q \tag{7}$$

---

[b] Dependency between fatigue performance at different hot spots is investigated in the next section.

[c] In the example in part C this simple requirement will be replaced by the optimal allocation of the risk reducing measures.

For hot spot 1 the acceptance criteria is therefore

$$p_{acc,F_1} = \frac{1}{2} \frac{p_{acc}}{\left(p_{COL|F_1} - p_{COL|\overline{F}} \right)} \tag{8}$$

For N fatigue critical hot spots the acceptance criteria are derived accordingly, for hot spot i being

$$p_{acc,F_i} = \frac{1}{N} \frac{p_{acc}}{\left(p_{COL|F_i} - p_{COL|\overline{F}} \right)} \tag{9}$$

Fig. 5 shows the acceptable probability of failure as a function of the RIF value, according to Eq. (9) and the relationship illustrated in Fig. B1.

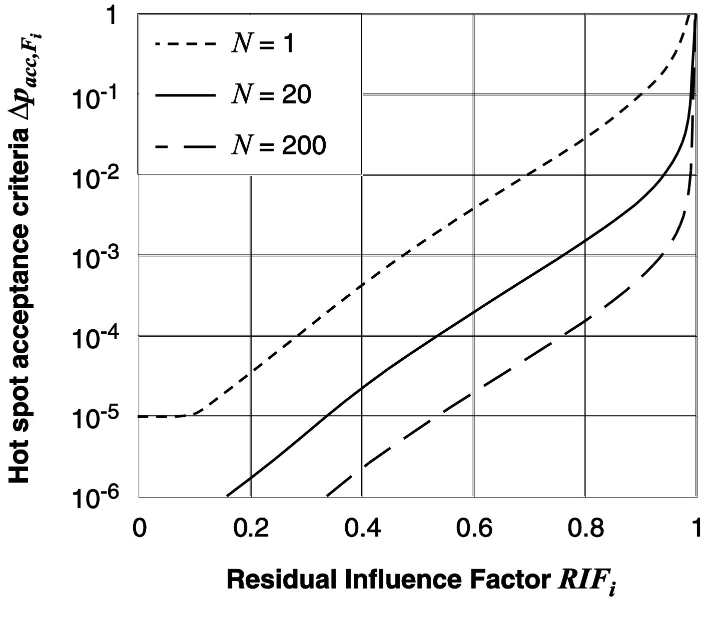

Figure 5. Acceptance criteria for independent hot spots in a structure containing N fatigue critical hot spots (RSR_{intact} = 2.05, $\psi = 0.1$).

Accounting for the effect of dependencies

The above presented derivation of the acceptance criteria for individual fatigue subjected hot spots does not account for all dependencies. The elements shown in Fig. 4 are in general not independent. In the previous modeling it has been taken into account that the conditional collapse events ($COL| \cdot$) are mutually exclusive. Moreover it is assumed that the collapse events are independent from the fatigue failure events, and that the fatigue failure events themselves are independent of each other. These dependencies will be treated in the following.

Fatigue failures at different locations are generally dependent. Straub and Faber [14] present a methodology for the consideration of this dependency in inspection planning. Whereas the dependency has positive effects on the information obtained from inspections, it has adverse effects on the system probability of collapse and must thus be accounted for. In Eq. (2) the term of higher order was omitted, reasoning that (due to the assumption of independence) the probability of coincidence of two or more fatigue failures is low. The number of fatigue failures given independence is in fact binomial distributed. This binomial distribution is in the following compared to the distribution of the number of fatigue failures given a dependency between the individual fatigue failures. The (illustrative) model from Straub and Faber [14] is used. The model assumes a 100% correlation between the stress ranges in the fatigue sensitive hot spots. A system with $N = 20$ hot spots is considered, all having a fatigue design factor $FDF = 4$.

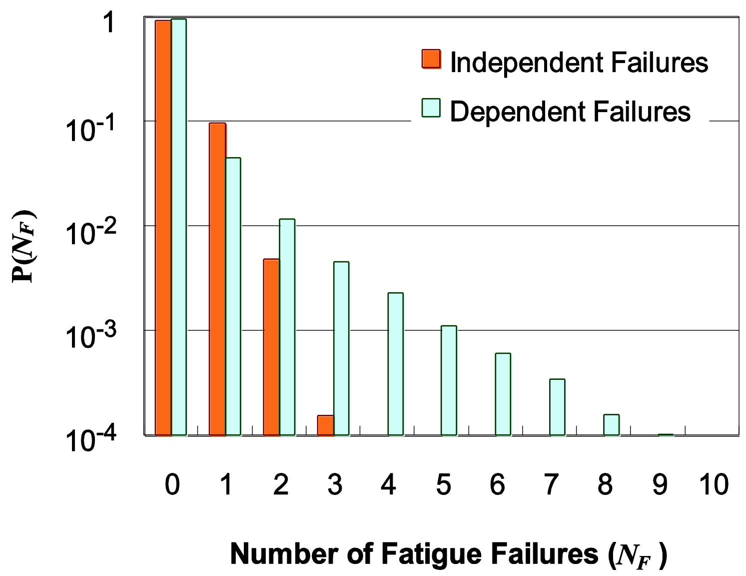

Figure 6. Distribution of the number of fatigue failures during service life.

Fig. 6 shows that the omission of the terms of higher order (2 and more fatigue failures) is generally not appropriate when fatigue failure events are dependent. The effect, however, is only quantifiable if the probability of collapse given N_F fatigue failures would be known. This will generally not be the case, except for $N_F = 0$ and $N_F = 1$; therefore two illustrative systems are introduced, which represent different levels of redundancy. They are shown in Fig. 7.

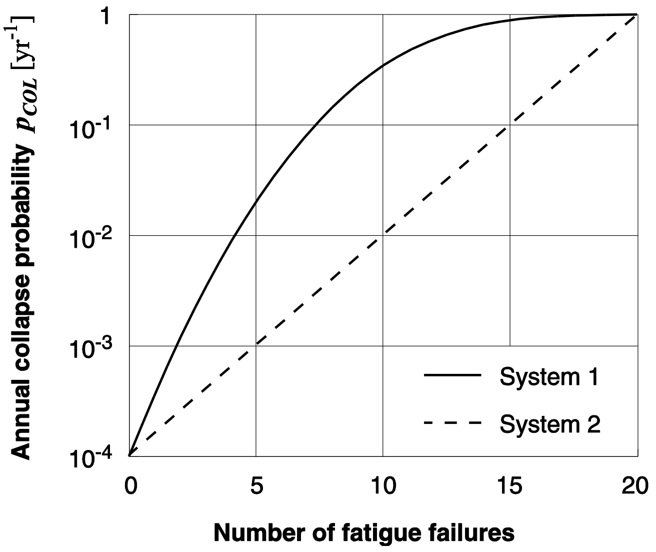

Figure 7. Two illustrative systems.

The omission of the higher order terms in Eq. (2) can now be investigated: The probability of collapse including one fatigue failure is compared to the probability of collapse including more than one fatigue failure (which is the omitted part), Table 1:

Table 1. Probabilities of collapse (in the last year of service life) given N_F fatigue failures.

	Independent hot spots			Dependent hot spots	
	$N_F = 1$	$N_F > 1$		$N_F = 1$	$N_F > 1$
System 1	$3.4 \cdot 10^{-5}$	$6.1 \cdot 10^{-6}$		$1.6 \cdot 10^{-5}$	$2.6 \cdot 10^{-4}$
System 2	$1.5 \cdot 10^{-5}$	$1.3 \cdot 10^{-6}$		$7.0 \cdot 10^{-6}$	$1.5 \cdot 10^{-5}$

Table 1 shows that for independent hot spots the probability of collapse originating from one fatigue failure is dominant, justifying the simplifications. For dependent hot spots this is not the case. It is seen that the probability of collapse due to more than one fatigue failure is dominant and therefore must not be omitted. Treatment of dependency would necessitate not only a model of the dependency but also knowledge of the collapse probability as a function of several fatigue failures (as shown in Fig. 7). This is not applicable to date and an approach to circumvent the problem is therefore suggested in the following.

The distribution of failures as shown in Fig. 6 is valid for the occurrence of fatigue failures during the total service life (20 years in the example). If inspections are performed, most existing failures will be detected (remembering that failure was defined as through crack). If inspections are performed each year, and assuming that all failed hot spots are found, then the relevant question is what is the distribution in Fig. 6 considering a time frame of only one year. The results are shown in Fig. 8 for the last year of service life.

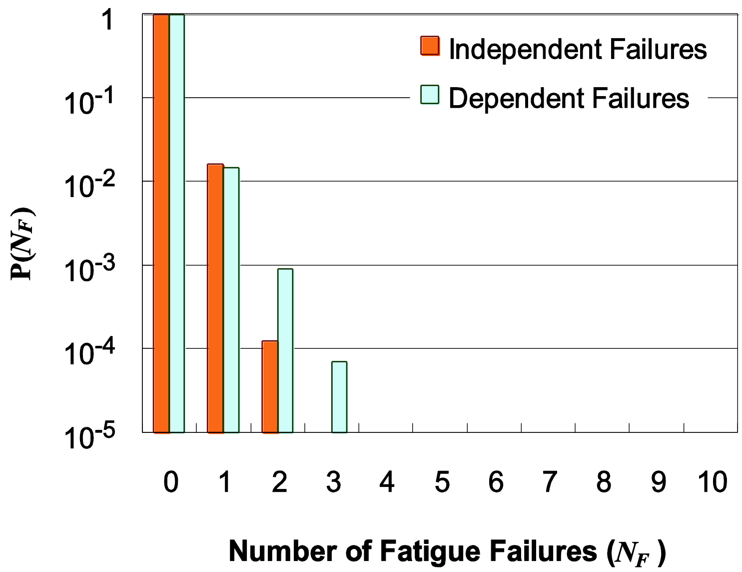

Figure 8. Distribution of the number of fatigue failures in the last year of service.

Comparing Fig. 8 to Fig. 6 it is seen that the dependency in fatigue performance is less crucial if only failures occurring in one year are considered. If these are combined with the respective conditional probabilities of collapse from Fig. 7 then the equivalent to Table 1 is obtained, Table 2.

Table 2. Probabilities of collapse given N_F fatigue failures in the last year of service life.

	Independent hot spots			Dependent hot spots	
	$N_F = 1$	$N_F > 1$		$N_F = 1$	$N_F > 1$
System 1	$5.8 \cdot 10^{-6}$	$1.4 \cdot 10^{-7}$		$5.1 \cdot 10^{-6}$	$1.3 \cdot 10^{-6}$
System 2	$2.6 \cdot 10^{-6}$	$3.2 \cdot 10^{-8}$		$3.2 \cdot 10^{-6}$	$2.6 \cdot 10^{-7}$

Table 2 shows that when only failures in the last year are considered, the omission of more than one fatigue failures is not crucial. Given that there are no failed hot spots present before the last year, what can be ensured by yearly inspections, Eq. (9) is a sufficiently accurate description of the acceptable risk in a structure. The appropriate inspection interval depends on the correlation assumed between fatigue performance at the individual hot spots (the lower the correlation, the less inspections are needed). In addition the probability of detection (PoD) of a failed hot spot has to be considered.

Part C – Optimization

Acceptable probabilities of failure of engineering systems and individual components of systems can be derived by means of optimization as outlined already by Rosenblueth and Mendoza [15]. More recent publications include Rackwitz [5], Kübler and Faber [16] and Faber [17]. Following this approach the acceptable probabilities of failure for the system as well as for its components can be identified as those yielding a positive service life benefit B, given for illustrational purposes as

$$B = I - C_D - C_M - C_F \tag{10}$$

where I denotes the service life income of the facility, C_D are the design costs, C_M are the expected inspection and maintenance costs and C_F are the expected failure cost, which includes consequences to people, the environment and financial consequences. The Costs Of Risk Reduction ($CORR$) is, for each risk reducing measure α defined as the cost of decreasing C_F, e.g. for a possible maintenance action it is [d]

$$CORR(\alpha) = -\frac{\mathrm{d}\,C_M(\alpha)}{\mathrm{d}\,C_F(\alpha)} \tag{11}$$

The variable α is in the following omitted for simplicity. From Eq. (11) it follows that the optimum is reached when $CORR = 1$. In case C_F and C_M cannot be expressed in the same unit (e.g. when C_F is expressed as the expected loss of life) the optimal set of actions always fulfills the condition that the optimal $CORR$ is equal for all possible risk mitigating alternatives. It should be noted that the assessment of the service life benefit and the $CORR$ is a little more complicated than it appears from Eq. (10 - 11) as the individual contributions have to be converted to their net present values taking into account the point in time where they occur.

The general formulation of Eq. (10) is valid for both the owner of a facility as well as the public. Acceptance criteria derived on the basis of optimization is extensively described by Rackwitz [18] who also outlines the difference between the owner's and the public's objective function. Shortly, there is a social acceptable domain of the design parameters where the public's benefit function is positive, yet the optimum is generally not the same for the public and the owner. Acceptance criteria must thus ensure that Eq. (10) yields positive values for society, but not necessarily the optimal benefit. This criteria is formulated in terms of e.g. a p_{acc} or, in case potential fatalities are considered, an $ICAF$ (implied cost of averting a fatality) that may be evaluated

[d] It is here assumed that I and C_D are constant with respect to the action α. This is typical for in-service structures.

based on the Life Quality Index (LQI) concept from Nathwani et al. [19], see also Skjong and Ronold [20]. If the $ICAF$ is prescribed then the following condition must be fulfilled for all possible risk mitigation alternatives:

$$CORR \geq ICAF \tag{12}$$

For the design or for the planning of inspection and maintenance activities for fatigue sensitive joints of a structural system, the optimal allocation of resources, i.e. the efficient distribution of fatigue safety and condition control, can be assessed directly by the maximization of Eq. (10). However, in most practical cases the question is how to allocate the efforts for risk reduction among the individual components of an existing structural system in the most efficient manner. An example is presented in the following where the p_{acc} is assumed given (either identified by optimization or by the methods described in part B) and the optimal allocation of p_{acc} to the individual hot spots is searched. At the optimal solution the $CORR$ value must be equal for all considered risk reducing activities, here maintenance actions on the different hot spots. In other words – the last Euro invested in to risk reduction for any of the components of the system shall be equally efficient in risk reduction. This characteristic might have potential for designing effective algorithms for the optimization of risk based design and inspection and maintenance strategies for structures and other technical facilities. Even if the consequences of failure are not expressed in monetary terms, the $CORR$ values can still be calculated.

Example

The example considers an in-service structure where the income I and the design costs C_D are fixed and only the maintenance costs C_M and the expected failure costs C_F are variable. Applying the probabilistic model (fracture mechanics and inspections) and the cost model as in Straub and Faber [14], the expected failure costs are calculated as a function of the maintenance effort. This is performed by use of the generic approach to risk based inspection planning as introduced in Faber et al. [3] and elaborated in Straub [12].

Three different hot spots are considered, characterized by their FDF and RIF value. It is assumed that RIF = 0.8 corresponds to failure costs of 1. The cost of failure for RIF = 0.7 is then (applying the simplified system model and in accordance with Fig. B1)

$$C_F\left(RIF=0.7\right)\approx C_F\left(RIF=0.8\right)\cdot\frac{p_{COL|F_{(RIF=0.8)}}}{p_{COL|F_{(RIF=0.9)}}}=2.3 \qquad (13)$$

$p_{COL|F_{(RIF=0.8)}}$ is the conditional probability of collapse given fatigue failure of a hot spot with RIF = 0.8. The resulting functions are illustrated in Fig. 9. Each point along the graphs corresponds to a reliability level. The curves in Fig. 9 do not represent continuous functions because the effect of maintenance (inspection plans) is only calculated for specific maximal annual probabilities of failure, Straub [12].

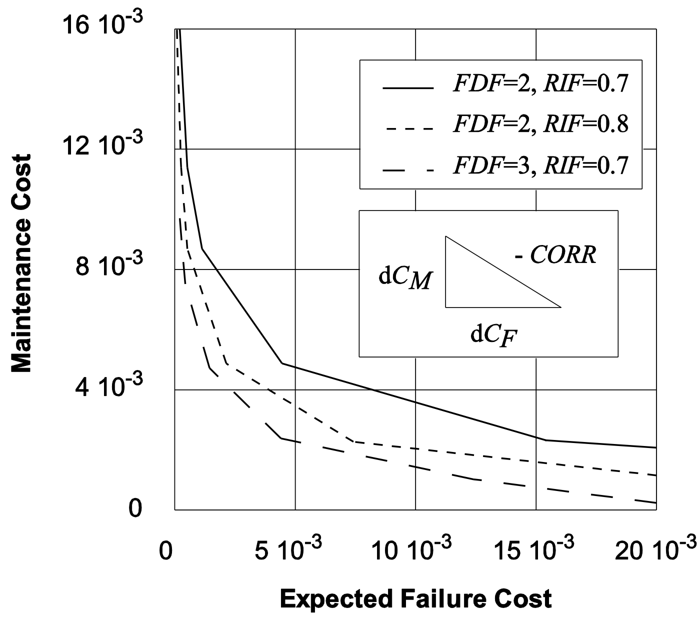

Figure 9. Expected failure cost as a function of the maintenance effort.

Because at the optimal solution the *CORR* value is equal for all hot spots, the acceptable risks should be allocated so that all *CORR* are the same and such that the total acceptable system failure probability is not exceeded. Fig. 10 shows the annual probability of collapse due to fatigue failure, $p_{COL\cap F_i}$, for the different C_M.

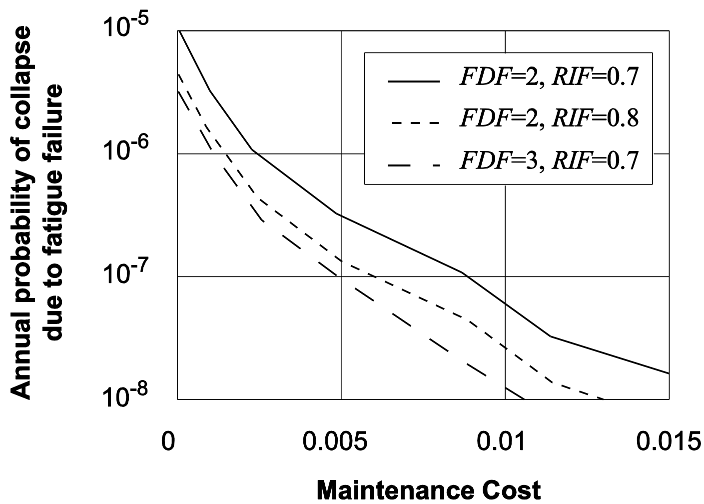

Figure 10. Maximum annual probability of failure for different maintenance expenditures.

The optimal allocation of the acceptable risk is now determined by the two conditions

1) $$p_{acc} = \sum_{i=1}^{N} p_{COL \cap F_i} \approx \sum_{i=1}^{N} \left(p_{COL|F_i} - p_{COL|\overline{F}} \right) \cdot p_{F_i}$$

2) $$CORR_1 = CORR_2 = \ldots = CORR_N \tag{14}$$

This is equivalent to the acceptance criteria derived in Eq. (2-9) but instead of demanding equal risk contribution now equal $CORR$ is demanded. This is the most optimal solution (with regard to the maintenance costs) that still fulfills the total acceptance criteria. Given a total accepted $p_{acc} = 1 \cdot 10^{-5}$, and given that there are 10 of each three example hot spots (i.e. in total the system consist of 30 fatigue sensitive joints), the solution is $p_{acc,F_i} = 10^{-3}$ for all hot spots with ($FDF = 2$, $RIF = 0.8$), $p_{acc,F_i} = 3 \cdot 10^{-4}$ for all others. Approximately the same is found when the approach from part B (Eq. (9)) is applied.

Discussion & Conclusions

Different approaches to the derivation of risk acceptance criteria for fatigue sensitive details are discussed. It is shown that based on two indicators, RSR and RIF, consistent acceptance criteria can be derived. The approach is based on the fact that the RSR implies an accepted risk of structural collapse due to extreme events, to which the acceptable fatigue induced collapse risk is related. The RIF is a simple but consistent indicator for the system redundancy, whose relation to the acceptance criteria is shown in the paper (Eq. (9) & Annex B). In this way all the system characteristics (redundancy, complexity, dependency) are included in the formulation. The importance of regular inspections for failed joints is outlined; they ensure that fatigue failures of the individual joints may be regarded as independent. This assurance of independency may also be a valid concept for highly redundant structures (such as ship hulls), where the RIF concept is not appropriate (because in these cases it is generally very close to one). In addition it is shown how the maintenance costs can be accounted for when allocating the risk to the individual joints. In this case the uniform distribution of the risk to the individual hot spots is replaced by the optimal allocation.

A highly practical way of deriving acceptance criteria is to combine the approach based on RSR and RIF, as illustrated in Fig. 5, with the serviceability criteria (Fig. 1) which is obtained directly from the code requirements. This can be directly implemented in practice.

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ANNEX A - Relation between *RSR* and overall collapse probability

In this section it is demonstrated how a simple indicator for the overall collapse probability can be derived. The indicator is then used to account for the redundancy in the structure.

The Reserve Strength Ratio (RSR) is defined as the ratio of the mean system capacity to the nominal design load, Stahl et al. [4]. Generally the nominal capacity is used, but Stahl et al. [4] argue that the use of the mean value is an appropriate approximation. RSR values are generally obtained by push-over analysis. A general relation between RSR and the probability of collapse can be obtained by consideration of the following limit state function (References are provided in Stahl et al. [4]):

$$g(x) = R - bH^{\delta} \qquad \text{(A1)}$$

where R is the effective capacity of the platform, H is a stochastic variable modeling the maximum annual value of the wave height, and b and δ are factors relating the wave height to the structural load. b is modelled as a stochastic variable to account for the model uncertainty. The wave height exponent is modelled deterministically as $\delta = 2.2$, in accordance with Stahl et al. [4]; this value represents a good approximation for large jacket structures in the Gulf of Mexico and in general for drag-dominated jacket structures. For other structural configurations this value may be different, as investigated in Ronalds et al. [21].

The RSR value as evaluated by a push-over analysis can be related to characteristic values of R, b and H in the following way (where the index C denotes the characteristic values):

$$RSR = \frac{R_C}{b_C H_C^{\delta}} \tag{A2}$$

It is assumed that R and b can be modeled probabilistically as log-normal distributed random variables and H as a Gumbel distributed random variable. The characteristic values for R, b and H are defined as 5%, 50% and 98% quantile values of their probability distributions. The coefficients of variation are chosen as

$$COV_R = 0.15,\ COV_b = 0.10,\ COV_H = 0.16$$

This stochastic model is as in Stahl et al. [4] except for the choice of the Gumbel distribution for H.

The reliability corresponding to a RSR value is then determined by structural reliability analysis (SRA) using the above given limit state function, Eq. (A1). The mean values of R, b and H are determined by choosing two and evaluating the third according to Eq. (A2). The results are given in Fig. A1.

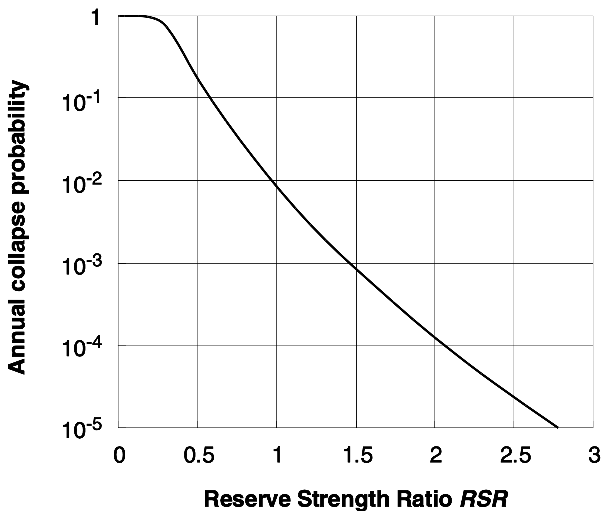

Figure A1. Relation between the RSR and the annual probability of collapse.

Annex B – indicator for the Redundancy in the structure

The redundancy determines the influence of a fatigue failure on the system capacity. In accordance with Faber et al. [3] it is represented by the conditional probability of collapse given fatigue failure of component i, $p_{COL|F_i}$. A deterministic measure for this probability is the Residual Influence Factor (RIF). The RIF is defined as the ratio of the load carrying capacity of the damaged structure to the capacity of the intact structure, Eq. (B1).

$$RIF_i = \frac{RSR_{F_i}}{RSR_{intact}} \tag{B1}$$

where RSR_{F_i} is the Reserve Strength Ratio given failure of the i th hot spot. The RSR_{F_i} values for the individual hot spots are determined by performing pushover analysis of the structure without the element.

If the RSR for the intact structure is not known, it can be evaluated from the overall acceptance criteria, p_{acc}, if fatigue failure is not the mayor source of collapse. For $p_{acc} = 10^{-4}$ and the above given values, $RSR_{intact} = 2.06$ is required. Stahl et al. [4] derived the RSR_{intact} as implicitly demanded when meeting a 10'000 year ultimate load return period. Their criteria is considerably lower, $RSR_{intact} = 1.63$, though consistent with the principles of revealed preferences. For existing structures, the RSR is given. Considering fatigue in new-built structures, the general design and the related RSR value may also be regarded as fixed. The RSR of existing structures may even be taken as a measure for the overall accepted collapse probability, as described in a preceding section.

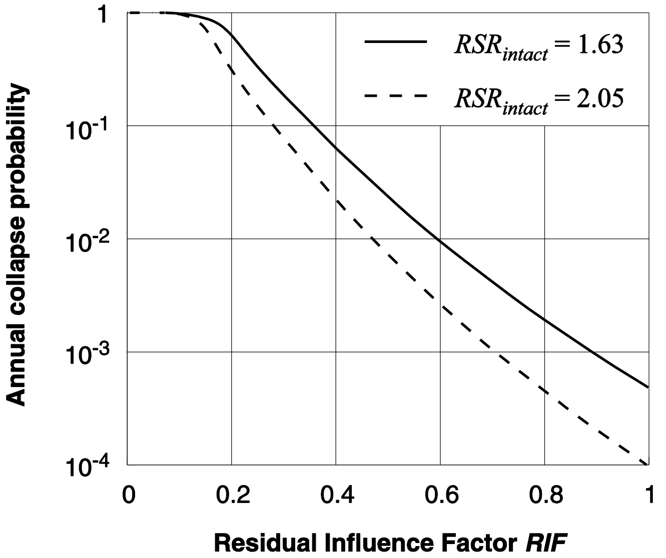

Figure B1. Conditional probability of collapse given fatigue failure as a function of the RIF for different RSR_{intact}.

The presented model implicitly assumes independence between the variables in the collapse limit state and the fatigue limit state. This is in accordance with common fatigue design procedures, as they model the fatigue loading by a long term stress range distribution, Almar-Naess [22]. Following this standard approach, large (extreme) load events are not explicitly accounted for in the fatigue modeling. The fatigue damage is thereby uncoupled from the extreme events. It seems reasonable to use this simplification also in the present context.