Temporal Variability in Corrosion Modeling and Reliability Updating

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Many parameters influencing corrosion degradation are time variant and the corrosion process is thus subject to temporal variability, the real characteristics of which are commonly neglected in reliability assessment. After a short overview on the quantitative modeling of corrosion loss, a comparison is made between different temporal models of corrosion degradation and the consequences of applying an inappropriate model are investigated. The effect of temporal variability is then investigated in detail and illustrated in an example considering CO_2 *corrosion in pipelines. It is demonstrated how the time-variant corrosion process can be consistently represented by time-invariant random variables, using equivalent values of the corrosion rate. Finally, the influence of temporal variability on reliability updating following inspections is investigated and it is shown how this effect can be accounted for in inspection planning.* [DOI: 10.1115/1.2355517]

Introduction

Degradation due to corrosion is an issue of high importance not least for pipelines, pressure vessels, and components of process systems in general. The reliable operation of such systems is a key factor for the safety of personnel operating on offshore facilities and has a major influence on the economical benefit. Moreover, failures of pipelines have been the cause of significant environmental damages. This fact has been recognized by the engineering profession and during the last decade significant efforts have been directed toward the formulation of engineering models for the prediction of corrosion degradation, both in deterministic and probabilistic terms.

Most of the models suggested so far take basis in the assumption that the rate of corrosion is a constant which may be assessed by a time average of degradation measurements over a sufficiently long time interval. Furthermore, most published work assumes that the condition assessment of pipelines and pressure vessels can be based on models which represent the conditions at only one given location. It is obvious that these approaches disregard the aspects of both temporal and spatial variations of the corrosion processes. Whereas such a basis for the probabilistic modeling of corrosion degradation might be useful in certain cases (e.g., for the purpose of design of pipelines and pressure vessels), it is of limited value when dealing with results of in-service inspections and measurements and, consequently, when applying the models to inspection and maintenance planning. In this context it is crucial to capture both the spatial as well as the temporal variability of the corrosion degradation process in order to be able to quantify the benefit of inspection coverage as well as inspection frequency.

In the present paper emphasis is placed on the probabilistic modeling of the temporal variability of corrosion processes and its effects on the reliability before and after inspections. After a short overview on the quantitative modeling of corrosion loss, a comparison is made between different temporal models of corrosion degradation and the consequences of applying an inappropriate model are investigated. The effect of temporal variability of the parameters influencing the corrosion progress is investigated in detail and illustrated in an example considering CO_2 corrosion in pipelines. It is then proposed to model the time-variant corrosion process by means of equivalent values of the corrosion rate. On this basis it is demonstrated how the influence of the temporal variability on reliability updating following inspections can be accounted for in a highly practical but fully consistent manner.

Quantitative Corrosion Modeling for Engineering Purposes. Quantitative corrosion models are models that predict the extent of corrosion loss (at a specific location and time) as a function of the influencing parameters of the environment and the material. Although great effort is spent on understanding, predicting, and controlling corrosion on steel structures, only relatively few research projects dealing with the development of quantitative corrosion models are reported. However, in practice such models are constantly applied for both the design of structures and the planning of inspection and maintenance activities. Unfortunately, the applied models are often highly simplistic and do not (explicitly) address the related uncertainties, due to the limited understanding of the underlying processes, the limited availability of relevant statistical data, and failure to consider the temporal and spatial variability in the corrosion behavior. The current paper focuses on the last aspect; it will be demonstrated how the temporal variability can be modeled and its effect on the optimal design and inspection maintenance will be investigated.

The complex corrosion process is first addressed by considering the geometrical characteristics of the corrosion defects. If, simplifying, the geometrical characteristics of the corrosion loss at a given point in time t are described by either uniform (general) corrosion or by localized corrosion, then most corrosion degradation problems encountered in the real world are a combination of these two forms. Consequently the total corrosion depth at any location x and time t can be described by the sum of the two types, Eq. (1).

$$d_C(x,t) = d_{\mathrm{UC}}(t) + d_{\mathrm{LC}}(x,t) \tag{1}$$

$d_C(x,t)$ is the total depth of the corrosion at the location x at time t, $d_{\mathrm{LC}}(x,t)$ is the depth of the localized corrosion defect (depending on x), and $d_{\mathrm{UC}}(t)$ is the depth of the uniform corrosion.

As illustrated by Eq.(1), a complete quantitative corrosion model includes the modeling of both the temporal (t) and the spatial (x) characteristics of the corrosion loss. Hereafter the spatial variability is not explicitly addressed and focus is put on the temporal characteristics of the corrosion process.

Contributed by the Ocean Offshore and Arctic Engineering Division of ASME for publication in the Journal of Offshore Mechanics and Arctic Engineering. Manuscript received July 21, 2005; final manuscript received May 26, 2006. Assoc. Editor: Bernt Leira. Paper presented at The 24rd International Conference on Offshore Mechanics and Arctic Engineering (OMAE2005), June 12–17, 2005, Halkidiki, Greece.

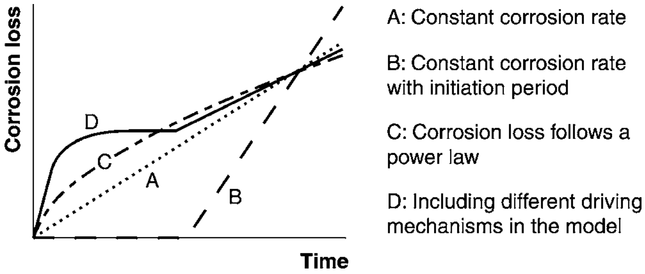

Fig. 1 Common quantitative temporal corrosion models

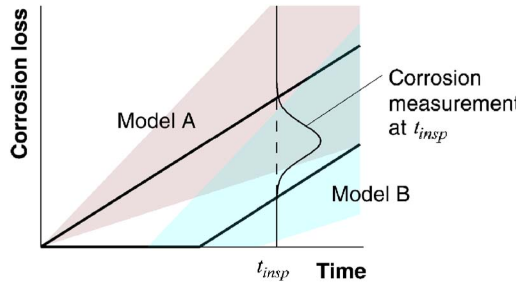

Fig. 2 Comparing two corrosion models. The shaded areas represent the 95% confidence interval.

Temporal Modeling. In practical applications, corrosion is, with few exceptions, quantitatively modeled by a constant corrosion rate. In the scientific literature, often a power law is proposed for the description of corrosion loss as a function of time, Eq. (2). As shown in Turnbull [1], at least for pitting corrosion this law has some theoretical foundations when the corrosion process is kinetically controlled. The same holds for diffusion-controlled corrosion, Melchers [2].

$$d_C(T) = A \times T^B \tag{2}$$

In Eq. (2), T is the time period in service, A and B are parameters of the model. When a coating is present, often an additional initiation time is included in the model, see, e.g., Paik et al. [3] or Guedes Soares and Garbatov [4].

For many types of corrosion problems the degradation processes are governed by several driving mechanisms. Such a behavior is explicitly modeled in Melchers [5], for marine immersed steel specimens. For other environments, such a phenomenological modeling may be more difficult to achieve, because the influencing parameters are less homogenous.

In Fig. 1 these different types of temporal corrosion models are illustrated in such a way that all predict the same corrosion loss at a particular point in time.

When phenomenological modeling (also referred to as process, physical, or mechanistic modeling) of the corrosion process is considered, the underlying mechanisms must be understood in order to determine the limiting factors of the process and these must then be modeled. This is discussed in several publications, including Melchers [2,6], Cole [7], and Roberge et al. [8]. The engineering corrosion models termed as phenomenological in the literature are actually semiempirical models, because their parameters do not represent any physical quantities. Instead they must be fitted to measured corrosion degradation. Fully physical models would need, e.g., knowledge on the concentration of O_2 in the environment and the diffusion coefficient in the corrosion product. Clearly, these input parameters are not generally available and fully physical models do not provide a satisfactory solution at present, see also Cole [7]. On the other hand, it is stated in Melchers [6] that "purely empirical models have little value," because extrapolation of the models outside the range of data to which they were calibrated is not possible. It is therefore argued that quantitative corrosion models for prediction and inspection/maintenance planning purposes must be based on consideration of the governing mechanisms and their driving or limiting factors. This is especially true when considering inspection updating, as demonstrated later.

Uncertainties in Corrosion Modeling. The quantitative corrosion models applied by the profession are generally developed for design purposes and represent a sort of "worst-case model" without clear definition of the underlying uncertainties or the assumed reference size in the case of localized phenomena. Apart from the models in Melchers [5] and Sydberger et al. [9], the authors are not aware of any published quantification of the uncertainties involved in the corrosion predictions for real operational situations. Because such models are crucial for the development of a risk based approach to corrosion control, the reluctance of corrosion engineers to quantify the uncertainties related to their models constitutes a significant drawback in corrosion modeling.

In addition to the uncertainties related to the corrosion models, in many instances there are additional large uncertainties regarding the environmental and operational conditions determining the input parameters to the models. The total uncertainty in quantitative corrosion predictions is thus generally very large, which motivates the application of in-service inspections for corrosion control and for model updating. However, the latter requires that the initial uncertainties are quantified and that the applied quantitative corrosion model is able to accurately represent the phenomena. This issue is considered in more detail in the following.

Application of Temporal Corrosion Models in Inspection Updating. Inspections in many instances represent a highly efficient means of corrosion control and risk reduction measure. Quantitative corrosion models may form the basis for determining the optimal inspection efforts (i.e., what to inspect, when to inspect, and how to inspect). Such inspection planning procedures are based on the application of Bayes' rule to update the uncertain corrosion model using inspection results, see, e.g., Hellevik et al. [10] or Straub [11].

Whereas a conservative "worst-case model" may be appropriate for design purposes, such models can lead to nonconservative predictions when combined together with inspection results. Consider the following simple case: For a component, corrosion is modeled by a constant but uncertain corrosion rate r (corresponding to model A in Fig. 1). The actual corrosion mechanism includes an initiation period t_I (model B in Fig. 1), yet the engineer considers neglecting this effect in his model as conservative. The two models are illustrated in Fig. 2 together with an assumed inspection result. In the following, it will be demonstrated that the omission of the initiation time in model A in combination with the inspection result may lead to an underestimation of the actual corrosion loss in the future and thus to an overestimation of the reliability

Assuming that the component fails when the corrosion loss exceeds the critical depth d_{cr}, the limit state function for component failure is written as

$$g = d_{cr} - d_C(t) \tag{3}$$

$d_C(t)$ is the corrosion loss at time t and corresponds to the general formulation in Eq. (1) when the spatial variability is neglected. Here, $d_C(t)$ is

$$d_C(t) = \begin{cases} 0, & t < t_I \\ r(t - t_1) & t \geq t_I \end{cases} \tag{4}$$

The reliability index β, respectively, the probability of failure p_F, is evaluated from the limit state function using structural reliability analysis (SRA). The discontinuity at t_I in the first order partial derivative of $d_C(t)$ with respect to time may lead to numerical problems; in that case a differentiable function may be fitted to

Table 1 Parameters of the example

Parameter		Dim.	μ	σ	Distribution
Corrosion rate r		mm/yr	1	0.3	W
Initiation time t_I	Model A	yr	0	—	D
	Model B	yr	5	2	LN
Critical depth d_{cr}		mm	20	—	D
Insp. Time t_{insp}		yr	8	—	D
Corrosion measurement d_m		mm	6	1	N

Note: μ: Mean value; σ: standard deviation; D: deterministic; W: Weibull distr.; LN: lognormal distr.; N: normal distr.

Eq. (4), see Friis-Hansen [12].

The inspection result, i.e., the corrosion measurement d_m, is taken into account by updating the probabilistic model using Bayes' rule, see, e.g., Madsen et al. [13]. The limit state function for the measurement event represents an equality event and is defined as

$$g_m = d_m - d_C(t) \tag{5}$$

The reliability is now evaluated for the two models with and without considering the inspection/measurement outcome by means of FORM (first order reliability methods) using the parameter values in Table 1. The results are shown in Fig. 3.

The results in Fig. 3 demonstrate that although model A is a "worst-case model" as long as no inspection is performed, this is not the case when accounting for the inspection result. After 20 years, the reliability index β is close to zero with model B (p_F=0.5), whereas β is equal to 2 with model A (p_F=0.01).

In many applications it may be unclear which is the appropriate model; this additional uncertainty must be explicitly addressed in the probabilistic modeling. Thereby the probability density function (PDF) of the corrosion loss is predicted as the weighted average of the pdf's of corrosion loss predicted by the two (or more) models. The applied weights correspond to the degree of belief in the models. A similar approach is proposed and described in some details for crack growth modeling in Zhang and Mahadevan [14]. In Faber and Maes [15] it is demonstrated how such system uncertainties can be taken into account not at the damage level but at the level of decision making.

Temporal Variability

In the modeling of uncertainty within the context of engineering decision making, it is of utmost importance to formulate probabilistic models which are consistent with the available information and the characteristics of the decision making problem. It is, e.g., possible to develop a probabilistic model for the corrosion loss which would be appropriate for design optimization purposes but which would be inadequate for inspection and maintenance planning purposes. In the latter context, the issue is to establish a model that is consistent with regard to the time dependency of the knowledge about future degradation. As explained in Faber [16], the time dependency structure of the probabilistic models can be understood as being governed by the epistemic uncertainty which by inspections may be updated and affect the probabilistic model of degradation at times after the inspection.

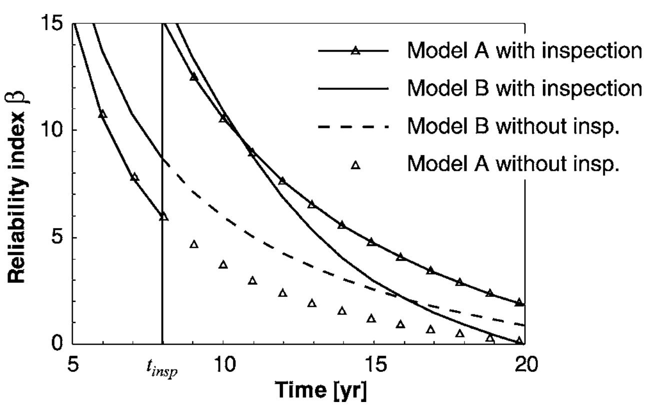

Fig. 3 Reliability index obtained with the two corrosion models, with and without inspection

Consider the simple deterioration model in Eq. (4): So far it has been assumed that all parameters are constant with time, i.e., they have one realization over the lifetime. In reality, many of the influencing parameters vary with time; parameters like the corrosion rate r in Eq. (4) are thus stochastic processes. However, such temporal variability is not explicitly modeled in corrosion predictions for engineering applications. This is not necessarily simplifying or wrong, as it is often sufficient to account for the temporal variability through the use of equivalent values. Such is the case for the stress ranges ΔS in fatigue modeling: Because the fatigue damage is cumulative and proportional to ΔS^m, with m being a parameter of the fatigue model, the stochastic process describing ΔS can be replaced by the expected value of ΔS^m when the number of stress cycles is large, see, e.g., Madsen et al. [13]; this is the standard procedure for fatigue design and inspection planning. A similar approach will be proposed in the following for corrosion modeling. This requires that the conditions for such a simplification are investigated, yet a rigorous mathematical treatment is not intended here; instead focus will be put on the practical aspects of the approach. The procedure will be illustrated in an example considering CO_2 corrosion in pipelines; the applied model is presented in the next section, followed by a discussion of the temporal variability for this example and a proposal for the definition of equivalent values.

The DeWaards-Milliams Model for CO_2 Corrosion in Pipelines. It is noted that this corrosion model was originally developed for design purposes. Its application in reliability analysis, as presented in the following, has illustrative character, and the model must be reviewed before a real application can be advocated.

The DeWaards-Milliams model is the most common model for carbonic acid corrosion (CO_2 corrosion) in pipelines. It was originally published in DeWaard and Milliams [17] with modifications in DeWaard et al. [18,19]. A simplified version of this model is applied here, in accordance with CRIS [20]. It predicts a constant corrosion rate based on the main influencing parameters: operating temperature T_o and pressure P_o, as well as the partial pressure of CO_2, P_{CO_2}. Other influencing parameters, such as the flow rate or the pH are not explicitly accounted for. The corrosion rate r_{CO_2} is

$$r_{CO_2} = 10^{(5.8 - 1710/T_o + 0.67 \times \log_{10} f_{CO_2})} \tag{6}$$

where the temperature T_o is expressed in Kelvin and the CO_2 fugacity f_{CO_2} is calculated from

$$f_{CO_2} = P_{CO_2} \times 10^{P_o(0.0031 - 1.4/T_o)} \tag{7}$$

The partial pressure of CO_2 is a function of the operating pressure and the fraction of CO_2 in the gas phase n_{CO_2}

Table 2 Parameters of the CO_2 corrosion model

Parameter	Dim.	μ	σ	Distribution
Critical depth d_{cr}	mm	30	1.5	Normal
Temperature T_o	K	Defined in Table 3		
Pressure P_o	bar	Defined in Table 3		
Fraction of CO_2 in gas phase n_{CO_2}	—	0.01	—	Deterministic
Model unc. X_M	—	0.4	0.32	Weibull

$$P_{CO_2} = n_{CO_2} P_o \tag{8}$$

The calculation of the corrosion depth $d_C(t)$ at time t includes a model uncertainty, which is, following Sydberger et al. [9] described by a multiplicative factor X_M

$$d_C(t) = X_M r_{CO_2} t \tag{9}$$

The parameter values used in the example are provided in Table 2. CO_2 corrosion is typically of localized nature. The spatial characteristics of the deterioration are thus of importance, however, these are not accounted for in the original references and the model only predicts the maximum defect in a pipe element (whose size is not stated in the references). For the sake of the example it is here assumed that the reference size corresponds to the size assumed for inspection, i.e., pipe elements with length 2–3 m and diameters 110–220 mm. This reference size is of importance when the spatial variability of the corrosion process is addressed.

Simplifying it is assumed that failure can be represented by the critical corrosion depth d_{cr}, which is a function of other parameters, including the wall thickness. Note that if the burst failure mode is decisive, then d_{cr} becomes a function of the time-variant parameter P_o, see Stewart et al. [21] or Ahammed and Melchers [22].

Temporal Variability in the CO_2 Corrosion Model. Most quantitative corrosion models are fitted to observed data. Temporal variability which is inherent to the corrosion process, e.g., due to changes in the chemical and physical characteristics of the material with depth, is part of the observation. Because this variability is not explicitly modeled, it will increase the scatter in the observations and is accounted for by the statistical model uncertainty of the corrosion process. Temporal variability due to the inherent characteristic of the corrosion process is therefore modeled by means of time-invariant random variables. On the other hand, the influencing environmental parameters, which in experiments are generally held constant, often vary significantly with time in operational conditions. This temporal variability should therefore be addressed in the models. This is considered in the following exemplarily for the CO_2 corrosion model.

The operating temperature T_o and pressure P_o are modeled by two Poisson square-wave processes, see Madsen et al. [13]. These processes consist of different intervals, whose starting points are generated by a Poisson process with intensity ν. In each interval i the value of the variable of interest, $X(t)$, is defined by Y_i, where the Y_i are independent, identically distributed random variables. The expected value of $X(t)$ for such a process is

$$\mathrm{E}[X(t)] = \mathrm{E}[Y] \tag{10}$$

and its covariance function is

$$\mathrm{Cov}[X(t_1), X(t_2)] = \mathrm{Var}[Y]\exp|-\nu(t_2 - t_1)| \tag{11}$$

An additional assumption is made, namely, that the underlying Poisson processes are identical and fully correlated for T_o and P_o. This is reasonable considering that the temperature and pressure will both change simultaneously at times when the operating conditions are modified. With this additional assumption, the corrosion rate $r_{CO_2}(t)$ also follows a Poisson square-wave process with the same intensity ν; its amplitude Y_r is a function of all other random variables. Furthermore, a correlation coefficient of 0.8 is assumed between Y_{T_o} and Y_{P_o} at any point in time. Figure 4 shows possible realizations of T_o and P_o, whose characteristics are summarized in Table 3.

The uncertainties in the mean values of T_o and P_o represent

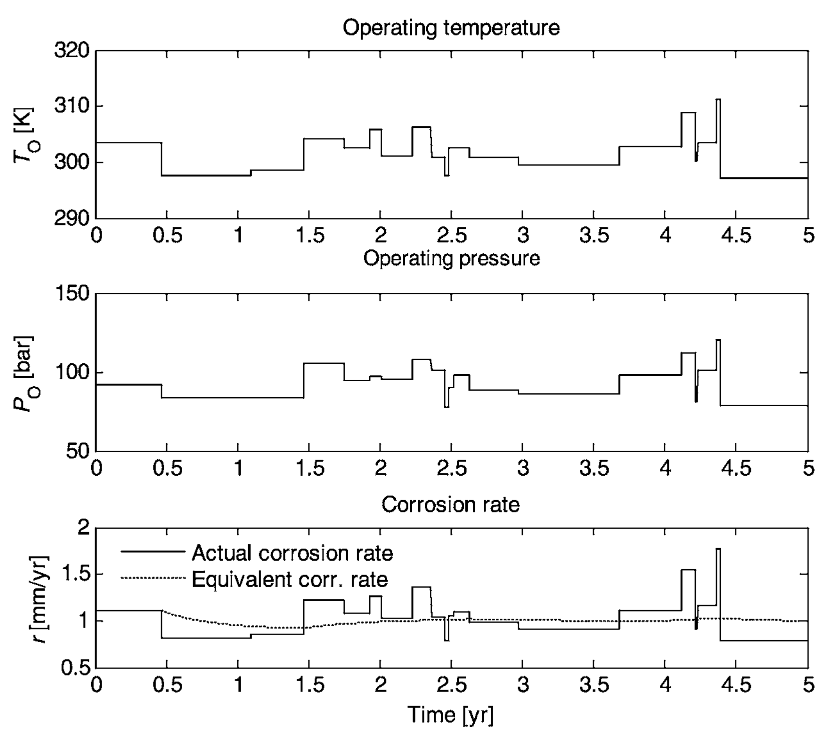

Fig. 4 One realization of the two random processes and the consequent corrosion rate

Table 3 Characteristics of T_o and P_o

Parameter	Dim.	μ	σ	Distribution
Intensity ν (T_o and P_o)	yr^{-1}	4	—	Deterministic
Amplitude Y_{T_o}	K	μ_{T_0}	15	Normal
Mean temperature μ_{T_0}	K	303	3	Normal
Amplitude Y_{P_o}	bar	μ_{P_0}	15	Normal
Mean pressure μ_{P_0}	bar	100	10	Normal
Correlation factor $\rho_{Y_{T_0},Y_{P_0}}$	—	0.8	—	Deterministic.

model uncertainty. These mean values have only one realization during the service life. The individual Y_i in the processes are thus only conditionally independent, given μ_{T_o} or μ_{P_o}; the same is valid for r_{CO_2}. Equation (11) does therefore not apply for the processes considered here; instead the covariance function is derived as

$$\begin{aligned}\mathrm{Cov}[X(t_1),X(t_2)] &= \mathrm{E}[(X(t_1)-E[\mu_Y])^2|X(t_1)=X(t_2)]\cdot \mathrm{e}^{-\nu|(t_2-t_1)|} \\ &\quad + \mathrm{E}[(X(t_1)-\mathrm{E}[\mu_Y])(X(t_2)-\mathrm{E}[\mu_Y])|X(t_1)\neq X(t_2)] \\ &\quad \times(1-\mathrm{e}^{-\nu|(t_2-t_1)|}) \\ &= \mathrm{Var}[Y]\mathrm{e}^{-\nu|(t_2-t_1)|}+\mathrm{Var}[\mu_Y](1-\mathrm{e}^{-\nu|(t_2-t_1)|})\end{aligned} \quad (12)$$

where $\mathrm{Var}[Y]$ corresponds to the variance of the point-in-time distribution of $X(t)$ and $\mathrm{Var}[\mu_Y]$ is the variance of the mean value of Y. For the corrosion rate process, these are in the following determined numerically.

The dashed line in the lower diagram of Fig. 4 is the realization of the equivalent corrosion rate. This equivalent corrosion rate is defined as the constant corrosion rate that would lead to the same corrosion loss as the actual, time-variant corrosion rate. It can be calculated as:

$$r_e(t)=\frac{d_O(t)}{t}=\frac{1}{t}\int_0^t r_{CO_2}(t)\mathrm{d}t \quad (13)$$

Equation (13) states that the equivalent corrosion rate $r_e(t)$ is obtained by integration of the corrosion rate process. Its moments are thus given by Eqs. (14) and (15), see, e.g., Parzen [23]

$$\mathrm{E}[r_e(t)]=\frac{1}{t}\int_0^t \mathrm{E}[r_{CO_2}(t)]\mathrm{d}t=\mathrm{E}[Y_r] \quad (14)$$

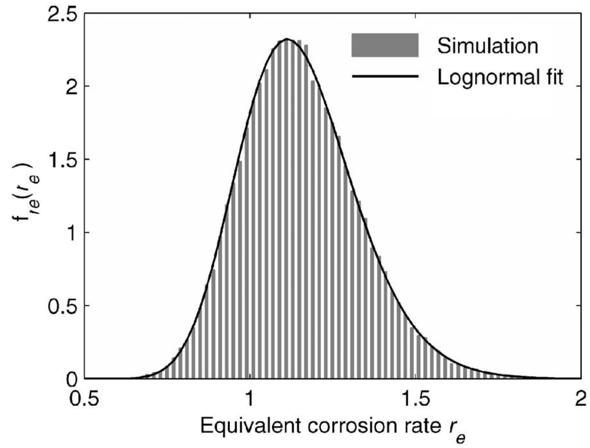

Fig. 5 The distribution of $r_e(t=5$ year)

$$\mathrm{Var}[r_e(t)]=\frac{1}{t^2}\int_0^t\int_0^t \mathrm{Cov}[r_{CO_2}(t_1),r_{CO_2}(t_2)]\mathrm{d}t_1\mathrm{d}t_2 \quad (15)$$

The realizations of $r_{CO_2}(t)$ are only conditionally independent (conditional on the realization of the time-invariant random variables). $r_{CO_2}(t)$ therefore corresponds to a modified Poisson square-wave process and its covariance function is given by Eq. (12). The evaluation of Eq. (15) thus requires that $\mathrm{Var}[Y_r]$ and $\mathrm{Var}[\mu_{Y_r}]$ are evaluated numerically. However, the distribution of $r_e(t)$ is readily obtained using Monte Carlo simulation (MCS). Figure 5 shows the stochastic model of $r_e(t=5$ yr). It is observed that the simulated r_e fits closely to a log normal distribution for any t. For $t=5$ yr the moments of the log normal distribution are obtained by a maximum likelihood estimator (LME) as

$$\mu_{r_e}=1.15 \text{ mm/yr}$$

$$\sigma_{r_e}(t=5\text{yr})=0.18 \text{ mm/yr}$$

The mean value of r_e is not dependent on the time, Eq. (14), and is equal to $\mathrm{E}[Y_r]$. The standard deviation of r_e, however, is dependent on the considered time t, as seen from Eq. (15) (actually on νt, the expected number of realizations of the time-variant random variables T_o and P_o.) The results as obtained by means of MCS are presented in Fig. 6. From Eq. (12) it is observed that $\mathrm{Var}[Y]$ corresponds to $\sigma^2_{r_e}(\nu t=0)$ and $\mathrm{Var}[\mu_{Y_r}]$ to $\sigma^2_{r_e}(\nu t=\infty)$. From the simulation results we can thus obtain

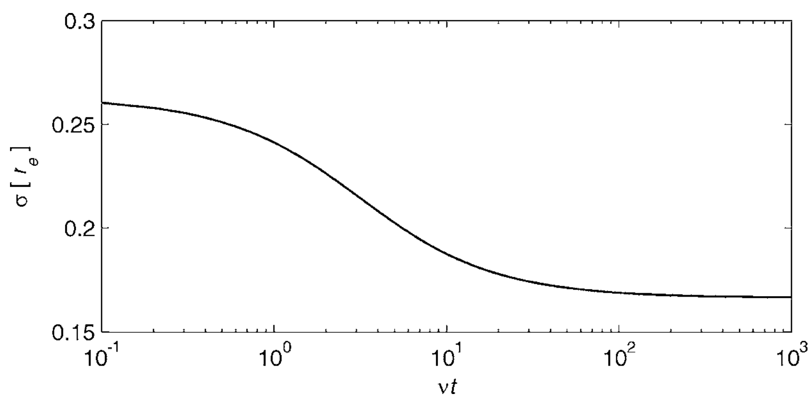

Fig. 6 The standard deviation of the equivalent corrosion rate σ_{r_e} as a function of νt

$$\mathrm{Var}[Y_r] \approx 0.263^2\ \mathrm{mm}^2\ \mathrm{yr}^{-2}$$

$$\mathrm{Var}[\mu_{Y_r}] \approx \sigma^2_{r_e}(\nu t = 1000) \approx 0.167^2\ \mathrm{mm}^2\ \mathrm{yr}^{-2}$$

The Variance of r_e can now be derived analytically from Eqs. (12) and (15) as

$$\begin{aligned}\mathrm{Var}[r_e(t)] &= \frac{1}{t^2}\int_0^t \int_0^t (\mathrm{Var}[Y_r]\mathrm{e}^{-\nu|t_2-t_1|} + \mathrm{Var}[\mu_{Y_r}] \\ &\quad \times (1-\mathrm{e}^{-\nu|t_2-t_1|}))\mathrm{d}t_1\mathrm{d}t_2 \\ &= \mathrm{Var}[\mu_{Y_r}] + 2(\mathrm{Var}[Y_r] - \mathrm{Var}[\mu_{Y_r}]) \\ &\quad \times \left(\frac{1}{\nu t} + \frac{(\mathrm{e}^{-\nu t}-1)}{(\nu t)^2}\right)\end{aligned} \tag{16}$$

The results obtained with Eq. (16) correspond to the results shown in Fig. 6. Note that:

$$\lim_{\nu t\to\infty} \mathrm{Var}[r_e(t)] = \mathrm{Var}[\mu_{Y_r}] \tag{17}$$

which is in accordance with the central limit theorem.

The parameter values of $\mathrm{Var}[Y_r]$ and $\mathrm{Var}[\mu_{Y_r}]$ can be precalculated and stored for different combinations of the stochastic variables. The equivalent corrosion rate $r_e(t)$ is then obtained for all specific cases directly from the database and Eq. (16), all reliability evaluations can then be performed using time-invariant reliability analysis. However, the largest benefit of using an equivalent value is in inspection planning, yet this requires that the effect of the temporal variability on the updated reliability is investigated.

The Effect of Temporal Variability on the Updated Corrosion Reliability. Whereas it is sufficient to apply equivalent values in the corrosion models for design purposes, this is not necessarily true when considering reliability updating following inspections, as in Straub and Faber [25], due to the fact that the future degradation may be different from the past degradation, as observed in Vrouwenvelder [24]. In the inspection planning procedures documented in the literature it is generally assumed that all random variables are time invariant. Information obtained at an inspection is used to update all random variables and those are then applied for the prediction of future deterioration. However, for the time-variant components of the model, the inspection cannot give any information on the future realizations and the usefulness of the inspection will thus depend on the ratio between the time-variant and time-invariant components and the expected number of realizations νt before and after the inspection. For inspection planning it is thus required to quantify this effect. If a component is put in service at year $t=0$ and an inspection is performed at year t_{insp}, an indicator for the value of the inspection in regard to the prediction of corrosion loss in the future is the correlation coefficient between the equivalent corrosion rate at time t_{insp} and time $t > t_{\mathrm{insp}}$: $\rho[r_e(t_{\mathrm{insp}}), r_e(t)]$. Following it is shown how this indicator can be derived and how it must be interpreted in the context of inspection planning.

The covariance of the summation of a process over two different time intervals is obtained from Eq. (18), see Parzen [23]

$$\begin{aligned}\mathrm{Cov}[r_e(t_{\mathrm{insp}}), r_e(t)] &= \frac{1}{t_{\mathrm{insp}}t}\mathrm{Cov}\left[\int_0^{t_{\mathrm{insp}}} r_{CO_2}(t_1)\mathrm{d}t_1, \int_0^t r_{CO_2}(t_2)\mathrm{d}t_2\right] \\ &= \frac{1}{t_{\mathrm{insp}}t}\int_0^{t_{\mathrm{insp}}}\mathrm{d}t_1\int_0^t \mathrm{d}t_2 \mathrm{Cov}[r_{CO_2}(t_1), r_{CO_2}(t_2)]\end{aligned} \tag{18}$$

By combining Eq. (12) with Eq. (18) we obtain the covariance function, Eq. (19)

$$\begin{aligned}\mathrm{Cov}[r_e(t_{\mathrm{insp}}), r_e(t)] &= \frac{1}{t_{\mathrm{insp}}t}\int_0^{t_{\mathrm{insp}}}\mathrm{d}t_2\int_0^t \mathrm{d}t_1(\mathrm{Var}[Y_r]\mathrm{e}^{-\nu|(t_2-t_1)|} + \mathrm{Var}[\mu_{Y_r}](1-\mathrm{e}^{-\nu|(t_2-t_1)|})) = \frac{1}{t_{\mathrm{insp}}t}\int_0^{t_{\mathrm{insp}}}\mathrm{d}t_2\left(\int_0^{t_2}\mathrm{d}t_1(\mathrm{Var}[Y_r]\mathrm{e}^{-\nu(t_2-t_1)}\right. \\ &\quad \left. + \mathrm{Var}[\mu_{Y_r}](1-\mathrm{e}^{-\nu(t_2-t_1)})) + \int_{t_2}^t \mathrm{d}t_1(\mathrm{Var}[Y_r]\mathrm{e}^{-\nu(t_1-t_2)} + \mathrm{Var}[\mu_{Y_r}](1-\mathrm{e}^{-\nu(t_1-t_2)}))\right) \\ &= \mathrm{Var}[\mu_{Y_r}] + (\mathrm{Var}[Y_r] - \mathrm{Var}[\mu_{Y_r}])\left(\frac{2}{\nu t} + \frac{1}{\nu t_{\mathrm{insp}}t}(-\mathrm{e}^{-\nu(t-t_{\mathrm{insp}})} + \mathrm{e}^{-\nu t} + \mathrm{e}^{-\nu t_{\mathrm{insp}}} - 1)\right)\end{aligned} \tag{19}$$

The covariance function is dependent on the expected number of realizations of the random process before the inspection νt_{insp} and in total νt. The correlation between the equivalent corrosion rate before and after the inspection can now be calculated as

$$\rho[r_e(t_{\mathrm{insp}}), r_e(t)] = \frac{\mathrm{Cov}[r_e(t_{\mathrm{insp}}), r_e(t)]}{\sqrt{\mathrm{Var}[r_e(t_{\mathrm{insp}})]\mathrm{Var}[r_e(t)]}} \tag{20}$$

Figure 7 shows the correlation for different combinations of νt and νt_{insp}. The correlation coefficient of $r_e(\nu t_{\mathrm{insp}})$ and $r_e(\nu t)$ becomes 1 for $t=t_{\mathrm{insp}}$ and decreases with increasing t. As seen from Equation (19), the covariance function becomes equal to $\mathrm{Var}[\mu_Y]$ as $\nu t \to \infty$; the correlation coefficient has thus a lower limit which is a function of νt_{insp} only. A low value of νt_{insp} signifies that the inspection is performed after only few realizations of the random processes and the inspection result thus reflects the additional variability due to the time-variant random variables, which is also observed in Fig. 6.

The calculated correlation coefficients between $r_e(\nu t_{\mathrm{insp}})$ and $r_e(\nu t)$, Fig. 7, are very close to one for most cases. E.g., for the case where $\nu=4\ \mathrm{yr}^{-1}$ and $t_{\mathrm{insp}}=5$ yr, the equivalent corrosion rate r_e at the time of inspection is correlated to r_e after 25 years with $\rho[r_e(t_{\mathrm{insp}}), r_e(t=25\ \mathrm{yr})]=0.98$. It is thus reasonable to assume a time-invariant equivalent corrosion rate for inspection updating in this case. For situations where the correlation coefficient is relatively low (<0.9), this can be accounted for by applying different values of r_e in the limit state functions for failure and for inspection (Eqs. (3) and (5)), taking into account the correlation between the two.

Discussion

It is seen from Fig. 3 that the use of an inappropriate temporal corrosion model may lead to critical errors in the prediction of the actual failure probability and thus to inappropriate inspection efforts. It is thus crucial that the temporal model, although not nec-

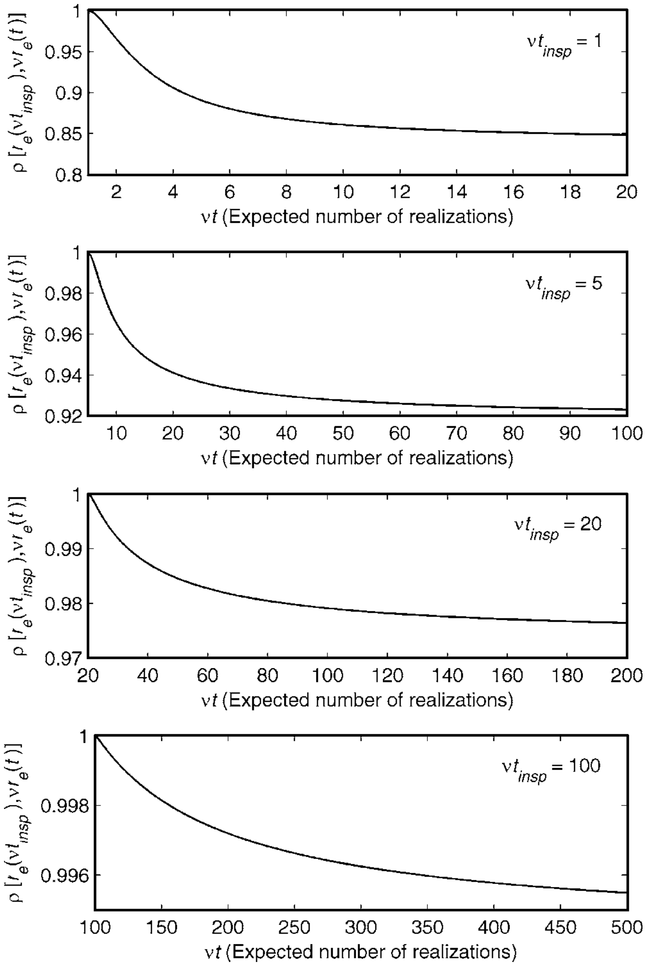

Fig. 7 Correlation of the equivalent corrosion rate in the period before the inspection and the total time period *t*

essarily fully accurate, is able to represent the actual corrosion phenomena. Whereas the inaccuracy in the parameters can be addressed by the stochastic modeling, this is not possible when the model itself is inaccurate. Doubts about the actual nature of the corrosion phenomenon must be accounted for by using a mixed model (i.e., a corrosion model consisting of two or more different corrosion models) with weights representing the degree of confidence in the individual models, see, e.g., Zhang and Mahadevan [14].

The proposed equivalent corrosion rate together with the presented algorithms allow for a highly efficient reliability calculation and updating procedure. A basic limitation of the approach at present is that the corrosion process must be independent from the failure mechanism, i.e., that the critical corrosion depth d_{cr} and the corrosion loss d_C in Eq. (3) are independent. For pressurized pipelines, the probability of failure given a specific corrosion damage depends on the operating pressure, which is also a parameter influencing the corrosion process and d_{cr} and d_C are thus dependent. In this case the presented approach must be modified to account for this dependency. However, it is noted that for most applications the uncertainty related to the failure mechanism is much smaller than the uncertainty related to the corrosion process and the assumption of independency between d_{cr} and d_C is thus not necessarily critical.

Conclusions

It is demonstrated that an appropriate representation of the phenomenological nature of the corrosion process is crucial when reliability updating and inspection planning is considered. If uncertainty on the choice of appropriate quantitative corrosion model exists, this must be accounted for in the formulation of the decision problem in the context of which the corrosion modeling is applied.

For corrosion processes whose influencing parameters are time variant, it is proposed to use an equivalent corrosion rate $r_e(t)$ for reliability calculations. This concept facilitates the consistent consideration of temporal variability in reliability calculations in a highly efficient manner. For the special but common case when the corrosion rate follows a Poisson square wave process, the derivation of $r_e(t)$ is presented.

The effect of the temporal variability in corrosion on the reliability updating following inspections is investigated. It is observed that this is of relevance only for cases with very low numbers of expected realizations of the corrosion rate process before the inspection. The calculation of the correlation coefficient of $r_e(t)$ before the inspection and for any time t is derived. This correlation coefficient, together with the concept of the equivalent corrosion rate $r_e(t)$, allows for an efficient inspection updating for situations where the temporal variability of the corrosion process is of importance.

The considerations outlined in the present paper can be seen as being valid also for other types of deterioration mechanisms. E.g., for low-cycle fatigue, where only a few load cycles may lead to failure, the dependency structure must be accounted for explicitly, as illustrated in the paper.

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