Reliability updating with inspection and monitoring data in deteriorating reinforced concrete slabs

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ABSTRACT: Corrosion is a common phenomenon in engineering structures; examples include corrosion of the reinforcement in RC structures, corrosion of steel plates in ship hulls or localized corrosion in pipelines. These corrosion mechanisms are generally subject to random spatial variability, due to random changes in influencing factors over space. When assessing the effect of inspections and measurements on the reliability of such structures, it is essential to account for this spatial variability: Due to correlation among influencing factors at separate points, the measurement results obtained at one location contain information on the corrosion process at other locations. In this paper, a novel algorithm for reliability updating [Straub D. Probabilistic Engineering Mechanics, 26(2): 254–258, 2011] is adopted to the spatial reliability analysis of RC corrosion conditional on measurements. The algorithm is computationally efficient and robust, thus facilitating the applications to reliability updating of large-scale structural systems subject to corrosion. The method is applied to an example concrete slab subject to chloride induced corrosion of the reinforcement, for which spatial measurements of concrete cover depth and chloride profiles are available.

1 INTRODUCTION

Corrosion of the reinforcement in RC structures is one of the major causes for aging of civil engineering structures. Due to random spatial variability of influencing factors, notably cover depth, chloride surface concentration and concrete properties, corrosion should ideally be modeled through spatial random fields. Such a random field modeling becomes particularly relevant when considering inspection and monitoring data gathered during a reassessment of the structure. The effect of such information on the reliability strongly depends on the spatial correlation of corrosion and its influencing factors: The higher the correlation between these factors at two locations Y_1 and Y_2, the more information on the corrosion at site Y_1 is obtained by measurements at site Y_2.

Random field modeling of corrosion of the reinforcement in RC structures has been considered in a number of studies (e.g. Hergenröder 1992; Sterritt et al 2001; Li et al. 2004; Stewart and Mullard 2007). In addition, it has been demonstrated how inspection results can be incorporated in the analysis by means of Bayesian updating when using hierarchical models to represent the spatial variability of corrosion in RC (Maes 2002; Faber et al 2006; Straub et al. 2009). However, to the author's knowledge, Bayesian updating has not been previously applied together with a full random field model of corrosion in RC surfaces. Main reasons for this omission are computational difficulties associated with such spatial reliability updating.

In this paper, an efficient algorithm for spatial reliability updating is presented, based on a novel approach to reliability updating developed in Straub (2011). The algorithm can consider both spatially distributed data, e.g. measurements of concrete cover depth or half-cell potential measurements, and local (discrete) observations, e.g. chloride profiles. The use of the algorithm is illustrated through an application to a concrete slab subject to chloride-induced corrosion of the reinforcement. This case study demonstrates the benefit of combining all data within a single probabilistic (quantitative) model.

2 BAYESIAN UPDATING OF DETERIORATION RELIABILITY

When assessing the deterioration reliability of structures, the probability of the structure (or components thereof) reaching an adverse state F is of interest. In RC structures subject to corrosion of the reinforcement, F might represent initiation of corrosion, spalling of the concrete, a critical loss of cross-section or failure of the structure due to corrosion (Stewart and Val 2003).

In many instances, information on the deterioration process becomes available during the service life of the structure, e.g. through measurements, inspection or monitoring of structures, which can be used to update the reliability estimate. This information is commonly uncertain and often indirect. In probability theory, information can be represented by an event Z. The updated reliability is then represented by the conditional probability of failure given the information event Z, defined as

$$\Pr(F \mid Z) = \frac{\Pr(F \cap Z)}{\Pr(Z)} \tag{1}$$

The process of computing this conditional probability is commonly referred to as Bayesian updating or information updating. It has been applied in the context of structural reliability since the 1970s (e.g. Tang 1973, Madsen 1987).

In structural reliability, failure events F and information events Z are described by domains Ω in the outcome space of the basic random variables $\mathbf{X} = (X_1, X_2, \ldots, X_n)$.

The failure domain Ω_F is defined in terms of continuous limit state functions $g(\mathbf{x})$. In the simplest case, it is

$$\Omega_F = \{g(\mathbf{x}) \le 0\} \tag{2}$$

In the general case, Ω_F is defined in terms of a number of limit state functions (e.g., Der Kiureghian 2005), corresponding to systems of components that are defined by limit state functions. For the purpose of this paper, the formulation in Eq. (2) is sufficiently general; extension to the system application is straightforward (Straub 2011).

Information obtained on the system, e.g. in the form of measurements, monitoring, inspection or observed system performance, is also described through continuous limit state functions $h(\mathbf{x})$ and corresponding domains. Information is said to be of the inequality type if it can be written as

$$\Omega_Z = \{h(\mathbf{x}) \le 0\} \tag{3}$$

and it is said to be of the equality type if it can be written as

$$\Omega_Z = \{h(\mathbf{x}) = 0\} \tag{4}$$

Structural reliability methods (SRM) solve Eq. (1) by computing integrals in the space of the basic random variables $\mathbf{X}$:

$$\Pr(F \mid Z) = \frac{\int_{\mathbf{x} \in \{\Omega_F \cap \Omega_Z\}} f_{\mathbf{X}}(\mathbf{x}) \mathrm{d}\mathbf{x}}{\int_{\mathbf{x} \in \Omega_Z} f_{\mathbf{X}}(\mathbf{x}) \mathrm{d}\mathbf{x}} \tag{5}$$

where $f_{\mathbf{X}}$ is the joint probability density function of $\mathbf{X}$.

If information is exclusively of the inequality type, Eq. (3), evaluation of the above integrals is straightforward using any of the available SRM. However, if the information event Z is of the equality type, the integrals result in zero, since these events have zero probability a-priori. Direct application of SRM is thus not possible in this case.

Solutions to overcome this problem have been suggested by Madsen (1987) and the group of Rackwitz (e.g. Schall et al. 1988). Madsen's solution is based on inserting a dummy random variable and then computing probability sensitivities with respect to this variable. The solutions of the Rackwitz group are based on computing surface integrals, using first- or second order approximations of the surfaces $h_i(\mathbf{x}) = 0$. These solutions are implemented in the Strurel software (Gollwitzer et al. 2006). Both Madsen's and Rackwitz' methods are efficient and, in many cases, represent a sufficiently accurate approximation. However, in cases where FORM/SORM solutions are not sufficiently accurate or in which it is difficult to identify the joint design point, these methods should not or cannot be used. Furthermore, it is often difficult to appraise the error made by the first- or second-order approximation.

Recently, the author introduced a novel method for solving Eq. (5) using SRM when information is of the equality type (Straub 2011). The method is based on transforming equality information into inequality information, which enables the direct use of Eq. (5) using any SRM. The aim of the present paper is to study the application of the methodology to deterioration reliability problems in spatially distributed systems. For such systems, commonly a large number of reliability problems must be solved simultaneously, which requires that the applied algorithms are robust and efficient.

In some instances, direct measurements of basic random variables are made. As an example, in the application considered in this paper, measurements of concrete cover depth are available. In such cases, it can be more efficient and convenient to first update the model of the measured random variables and then perform the reliability analysis with the updated probability distribution of the random variable model as an input. This procedure is outlined in Section 2.3.

2.1 *Reliability updating with equality information*

This section presents a summary of the method described in Straub (2011), with slight modifications in view of the considered application to deterioration reliability updating in spatially distributed systems.

2.1.1 *Likelihood function*

In statistics, information is not commonly described in the form of domains Ω_Z. Instead, the usual way to describe (uncertain) information Z on $\mathbf{X}$ is by means of the likelihood function, which is defined as

$$L(\mathbf{x}) \propto \Pr(Z \mid \mathbf{X}=\mathbf{x}) \tag{6}$$

As noted in Straub (2011), any domain Ω_Z can be translated into a likelihood function. However, it is often more convenient to directly identify the likelihood function. As an example, consider a measurement s_m of a system characteristic $s(\mathbf{X})$. The measurement has an additive error ε that is a zero mean random variable uncorrelated with $\mathbf{X}$. The limit state function $h(\mathbf{x},\varepsilon)$ describing this equality information as well as the corresponding likelihood function are given in the following, with $f_\varepsilon()$ being the PDF of ε.

$$h(\mathbf{x},\varepsilon) = s(\mathbf{x}) - s_m + \varepsilon \tag{7}$$

$$L(\mathbf{x}) = f_\varepsilon(s_m - s(\mathbf{x})) \tag{8}$$

For the case of several observations $Z_1,...,Z_m$ with corresponding likelihood functions $L_i(\mathbf{x})$, it is always possible to combine the likelihood functions into a single likelihood function $L(\mathbf{x})$. E.g., if measurements are uncorrelated for given $\mathbf{X}=\mathbf{x}$, it is simply $L(\mathbf{x}) = \Pi_{i=1}^{m} L_i(\mathbf{x})$. It is thus sufficient to consider only the case of a single likelihood function describing combined observations $Z = Z_1 \cap ... \cap Z_m$ in the following.

2.1.2 *Transform equality information described by a likelihood function into equivalent inequality information*

Let P be a random variable with uniform distribution in the range [0,1] and let c be a constant that is selected so that $0 \le cL(\mathbf{x}) \le 1$ for any $\mathbf{x}$. In this case, the following identity holds for given values of $\mathbf{X}=\mathbf{x}$:

$$L(\mathbf{x}) = \frac{\Pr[P \le cL(\mathbf{x})]}{c} \tag{9}$$

Let α denote the proportionality constant in the likelihood definition given in Eq. (6). By combining with Eq. (9), we obtain

$$\Pr(Z \mid \mathbf{X}=\mathbf{x}) = \alpha L(\mathbf{x}) = \frac{\alpha}{c}\Pr[P \le cL(\mathbf{x})] \tag{10}$$

By application of the total probability theorem, it follows that the probability of the information event Z is

$$\begin{aligned}\Pr(Z) &= \int_{\mathbf{X}} \Pr(Z \mid \mathbf{X}=\mathbf{x}) f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x} \\ &= \frac{\alpha}{c}\int_{\mathbf{X}} \Pr[P \le cL(\mathbf{x})] f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}\end{aligned} \tag{11}$$

Next, we define the event $Z_e = \{P \le cL(\mathbf{x})\}$ through the limit state function

$$h_e(\mathbf{x}, p) = p - cL(\mathbf{x}) \tag{12}$$

and the corresponding domain $\Omega_{Ze} = \{h_e(\mathbf{x}, p) \le 0\}$. This has the same form as the domains describing inequality information, Eq. (3). Equation (11) can now be rewritten to

$$\begin{aligned}\Pr(Z) &= \frac{\alpha}{c}\int_{\mathbf{x},p\in\Omega_{Ze}} f_P(p) f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p \\ &= \frac{\alpha}{c}\int_{\mathbf{x},p\in\Omega_{Ze}} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p\end{aligned} \tag{13}$$

The second identity follows from $f_P(p)=1$. Accordingly, $\Pr(F \cap Z)$ is obtained as

$$\begin{aligned}\Pr(F \cap Z) &= \int_{\mathbf{X}} \Pr(Z \mid \mathbf{X}=\mathbf{x}) \Pr(F \mid \mathbf{X}=\mathbf{x}) f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x} \\ &= \frac{\alpha}{c}\int_{\mathbf{x},p\in\{\Omega_F \cap \Omega_{Ze}\}} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p\end{aligned} \tag{14}$$

The conditional probability of F given Z is therefore

$$\Pr(F \mid Z) = \frac{\int_{\mathbf{x},p\in\{\Omega_F \cap \Omega_{Ze}\}} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p}{\int_{\mathbf{x},p\in\Omega_{Ze}} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p} \tag{15}$$

Here, the proportionality constant α disappears. Both integrals in Eq. (15) can be computed using any SRM. The denominator corresponds to a component reliability problem, the nominator to a parallel system reliability problem.

2.2 *Simple importance sampling techniques for evaluating the conditional reliability*

When considering spatial deterioration reliability, the conditional probability $\Pr(F \mid Z)$ must be computed for different locations in space. Therefore, a large number of evaluations of the integrals in Eq. (15) are potentially required. It is therefore important to employ a SRM that provides an optimal trade-off between computational robustness and efficiency. Robustness means that a method can be applied without problem specific adjustments and efficiency means that only a limited number of evaluations of the limit state functions are required.

As shown in Straub (2011), FORM/SORM techniques are not generally suitable due to the fact that the limit state surfaces $h_e(\mathbf{x}, p) = 0$, which describe the information, can be highly non-linear. Alternatively, a number of advanced simulation methods can be used, e.g. importance sampling schemes around the design point or subset simulation, as employed in Papaioannou and Straub (2010). The subset simulation is more robust, since it does not require the identification of the design point, but the choice of the MCMC sampling algorithm is not without difficulties.

Monte Carlo simulation (MCS) is the most robust SRM. However, as shown in Straub (2010), MCS becomes highly inefficient when more than just a few observations are available, due the fact that the effective number of samples available to compute the conditional reliability diminishes with increased information content of Z. For this reason, direct application of MCS to solve the integrals in (15) is not recommended.

For the considered application, the use of a simple importance sampling (IS) scheme is suggested, providing a good trade-off between robustness and efficiency. The IS estimator for the conditional probability $\Pr(F \mid Z)$ in Eq. (15) is

$$\Pr(F \mid Z) \approx \frac{\sum_{i=1}^{n_s} I\left[h_e(\mathbf{x}_i, p_i) \leq 0\right] I\left[g(\mathbf{x}_i) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i, p_i)}}{\sum_{i=1}^{n_s} I\left[h_e(\mathbf{x}_i, p_i) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i, p_i)}} \quad (16)$$

wherein the samples $\mathbf{x}_i$ and p_i are simulated from a distribution with sampling density $\psi(\mathbf{x}, p)$. Following Straub (2010), $\psi(\mathbf{x}, p)$ is split into

$$\psi(\mathbf{x}, p) = \psi_1(\mathbf{x}) \psi_2(p \mid \mathbf{x}) \quad (17)$$

where $\psi_1(\mathbf{x})$ is the sampling PDF of $\mathbf{X}$ and $\psi_2(p \mid \mathbf{x})$ is the conditional sampling density of P given $\mathbf{X} = \mathbf{x}$. An optimal conditional sampling density $\psi_2(p \mid \mathbf{x})$ that is valid for any application of Eq. (16) is given as

$$\psi_2(p \mid \mathbf{x}) = \frac{1}{cL(\mathbf{x})}, \quad 0 \leq p \leq cL(\mathbf{x}) \quad (18)$$

If it holds that $L(\mathbf{x}) > 0$ for any $\mathbf{x}$, then $I[h_e(\mathbf{x}_i, p_i) \leq 0] = 1$ for any value of p_i that is sampled from the above conditional density $\psi_2(p \mid \mathbf{x})$. In this case, Eq. (16) reduces to

$$\Pr(F \mid Z) \approx \frac{\sum_{i=1}^{n_s} I\left[g(\mathbf{x}_i) \leq 0\right] \frac{f_{\mathbf{X}}(\mathbf{x}_i) L(\mathbf{x}_i)}{\psi_1(\mathbf{x}_i)}}{\sum_{i=1}^{n_s} \frac{f_{\mathbf{X}}(\mathbf{x}_i) L(\mathbf{x}_i)}{\psi_1(\mathbf{x}_i)}} \quad (19)$$

Note that the constant c disappears when using this sampling density. For the case of selecting the prior PDF of $\mathbf{X}$ as its sampling density, i.e., $\psi_1(\mathbf{x}_i) = f_{\mathbf{X}}(\mathbf{x}_i)$, the above reduces to the MCS solution of

$$\Pr(F \mid Z) = \int_{\mathbf{X}} I\left[g(\mathbf{x}) \leq 0\right] f''(\mathbf{x} \mid Z) \mathrm{d}\mathbf{x} \quad (20)$$

where $f''(\mathbf{x} \mid Z)$ is the posterior distribution of $\mathbf{X}$ given the information Z.

For spatially distributed systems described by homogenous probabilistic deterioration models, it is suggested in Straub (2010) to use as sampling density $\psi_1(\mathbf{x})$ a distribution centered around the design point $\mathbf{x} = \mathbf{u}_i$ corresponding to failure at location i (a-priori, i.e. before the observation). This implies the use of a different sampling density for computing the reliability at every location i. However, the identification of the design point $\mathbf{u}_i$ is straightforward, since it suffices to find the values of the design point for the variables at the location i. These values are identical for any i, due to the assumption of homogeneity. The design point values of the random variables at the other locations are then found as the mode of the conditional distributions, which are readily identified if the random fields are modeled by a Gaussian copula (the Nataf model).

In Straub (2010) it was found that a sampling density $\psi_1(\mathbf{x})$ with a Multinormal distribution with mean equal to the design point $\mathbf{u}_i$ and covariance function equal to that of the prior distribution $f_{\mathbf{X}}(\mathbf{x}_i)$ performs well. This sampling density is utilized in the application presented later.

2.3 *Bayesian updating when direct measurements of model parameters are available*

When basic random variables are measured or observed, it can be convenient to first update the probability distribution of the measured random variables and then use the resulting posterior probability distribution as an input to the reliability analysis. This is true in particular for the case when the Bayesian updating of the random variables has an analytical solution.

Let $\mathbf{x}_m = [x_{m1}, x_{m2}, ..., x_{mk}]$ be measurements outcomes of a model parameter X at k discrete locations in space. In the general case, these can be described by the likelihood function

$$L_m(\mathbf{x}) = f_{\mathbf{X}_m|\mathbf{X}}(\mathbf{x}_m \mid \mathbf{x}) \quad (21)$$

where $f_{\mathbf{X}_m|\mathbf{X}}(\mathbf{x}_m \mid \mathbf{x})$ is the joint PDF of $\mathbf{X}_m$ for given $\mathbf{X} = \mathbf{x}$.

In the special case that the measurements have additive jointly Normal distributed measurement errors with zero mean and covariance matrix $\mathbf{C}_m$, the likelihood function becomes

$$L_m(\mathbf{x}) = \frac{1}{(2\pi)^{k/2} \left|\mathbf{C}_m\right|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}_m - \mathbf{x})^{\mathrm{T}} \mathbf{C}_m^{-1} (\mathbf{x}_m - \mathbf{x})} \quad (22)$$

The posterior distribution of $\mathbf{X}$ given the measurements is obtained through Bayes' rule as

$$f''_{\mathbf{X}}(\mathbf{x}) \propto L_m(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) \quad (23)$$

where the proportionality constant is determined by the condition $\int_{\mathbf{X}} f''_{\mathbf{X}}(\mathbf{x}) \mathrm{d}\mathbf{x} = 1$. Unfortunately, this constant can only be determined analytically for special cases. However, numerical methods for determining the constant for high dimensions k are available, in particular Markov Chain Monte Carlo (MCMC), e.g. Gilks et al. (1996). For the special case that the $\mathbf{X}$ are jointly Normal distributed and the likelihood function is described by Eq. (22), the posterior distribution of $\mathbf{X}$ is the joint Normal distribution with mean vector $\mathbf{M}''_{\mathbf{X}}$ and covariance matrix $\mathbf{C}''_{\mathbf{XX}}$ given as

$$\mathbf{M}''_{\mathbf{X}} = \mathbf{M}_{\mathbf{X}} + \mathbf{C}_{\mathbf{XX}} \left(\mathbf{C}_{\mathbf{XX}} + \mathbf{C}_m\right)^{-1} \left(\mathbf{x}_m - \mathbf{M}_{\mathbf{X}}\right) \quad (24)$$

$$\mathbf{C}''_{XX} = \mathbf{C}_{XX} - \mathbf{C}_{XX}\left(\mathbf{C}_{XX} + \mathbf{C}_m\right)^{-1}\mathbf{C}_{XX} \quad (25)$$

Once the posterior PDF $f''_{\mathbf{X}}(\mathbf{x})$ is available, it can be used instead of the prior $f_{\mathbf{X}}(\mathbf{x})$ as an input to further reliability updating following the algorithm described earlier.

3 APPLICATION TO SPATIAL RELIABILITY UPDATING OF CORROSION IN RC STRUCTURES

We consider a reinforced concrete (RC) surface that is subject to corrosion of the reinforcement caused by chloride ingress. The method presented in the previous section is applied to compute the spatial probability of corrosion conditional on measurements of concrete cover depth and chloride penetration. The cover depth measurements are made on the entire surface using a continuous device (see Gehlen and Greve-Dierfeld 2010). Information on chloride penetration is obtained from cores taken at discrete locations of the surface.

The considered surface area has size $10\text{m} \times 20\text{m}$; For the purpose of the analysis, the surface is discretized in 800 elements of size $0.5\text{m} \times 0.5\text{m}$. This choice is made based on the correlation lengths of the random variables that vary with space (see Malioka (2009) for a review of discretization approaches). Thus, the continuous random fields describing corrosion and its influencing factors are replaced by discrete random fields, which are defined by means of the midpoint method (Der Kiureghian and Ke 1998). Each element is represented by a random variable corresponding to the value of the random field in its midpoint (centre of gravity).

3.1 *Spatial model of chloride-induced reinforcement corrosion*

A diffusion model is utilized to describe chloride ingress and initiation of corrosion at the reinforcement, corresponding to a simplified version of the probabilistic models developed in the Duracrete project (fib 2006). The chloride concentration C_z at a depth z at time t is described by the following solution of the one-dimensional linear diffusion equation:

$$\frac{C_s - C_z}{C_s - C_0} = \text{erf}\left(\frac{z}{\sqrt{4Dt}}\right) \quad (26)$$

where C_s is the concentration of chloride at the concrete surface, C_0 is the concentration of chlorides in the concrete at time zero, D is the diffusion coefficient and $\text{erf}()$ is the error function. In the Duracrete model, C_s is given for different environmental conditions, C_0 is set equal to zero and D is expressed as a function of several variables that represent various material characteristics. For simplicity, D is here modeled by a single random variable.

The random variables, including their probabilistic model, are explained in Table 1. The values approximately correspond to a concrete surface in a parking deck with water-to-cement ratio equal to 0.4, which is exposed to splash water containing deicing salts.

Table 1. Probabilistic model for one location.

Parameter	Dimension	Distrib.	Parameters
W: Cover depth	mm	LN	μ = 40.0; σ = 8.0.
D : Diffusion coefficient	mm²/yr	LN	μ = 20.0; σ = 10.0.
C_S : Cl surface concentration	Mass-% of cement	Normal	μ = 3.10; σ = 1.23.
C_{cr} : Critical Cl concentration	Mass-% of cement	Normal	μ = 0.8; σ = 0.1.

The random variables that are subject to spatial variability are summarized in Table 2, together with the corresponding correlation length r_X.

Table 2. Modeling of spatial variability.

Parameter	Correlation length r_X [m]
W: Cover depth	1m
D : Diffusion coefficient	2m
C_S : Cl surface concentration	2m
C_{cr} : Critical Cl concentration	1m

All spatially varying random variables X are described by homogenous isotropic random fields with exponential covariance function:

$$\text{Cov}[X_i, X_j] = \sigma_X^2 \exp\left(-d_{ij}/r_X\right) \quad (27)$$

wherein d_{ij} is the distance between two points i and j on the concrete surface. The joint distribution of the random variables in the random field is described by a Gaussian copula (i.e. the Multinormal distribution in the case of C_s and C_{cr}, and the Multilognormal distribution in the case of W and D).

3.2 *Failure event*

Here, we define failure F as the event of corrosion initation, which occurs when the chloride concentration C_w exceeds the critical chloride concentration C_{cr}. With the diffusion model, the limit state function for corrosion in element i at time t is obtained as

$$g_{i,t}(\mathbf{X}) = C_{cr,i} - C_{s,i}\left[1 - \text{erf}\left(\frac{W_i}{\sqrt{4D_i t}}\right)\right] \quad (28)$$

3.3 *Measurements of concrete cover depth*

Concrete cover depth measurements are made over the entire surface area, following the discretization scheme described earlier. The measurement outcomes are summarized in Figure 1.

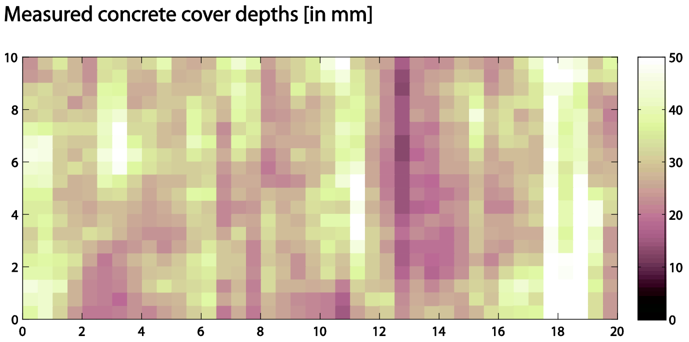

Figure 1. Concrete cover depth measurements $\mathbf{w}_m$ *[mm].*

The concrete cover depth measurements are subject to measurement error. It is assumed that the error can be modeled by a multiplicative factor ε_m, which has the Lognormal distribution with parameters $\mu_{ln\varepsilon} = 0$ and $\sigma_{ln\varepsilon} = 0.2$. Measurement errors at different locations are assumed to be statistically independent. With this model, the likelihood function for the vector of concrete cover depths $\mathbf{W} = [W_1,\ldots,W_{800}]$ is

$$L_m(\mathbf{w}) = \frac{e^{-\frac{1}{2}[\ln(\mathbf{w}_m-\mathbf{w})]^{\mathrm{T}}\mathbf{C}_m^{-1}[\ln(\mathbf{w}_m-\mathbf{w})]}}{(2\pi)^{k/2}\sigma_{ln\varepsilon}^{k}\prod_{i=1}^{800}(w_{mi}-w_i)} \qquad (29)$$

Since $\ln w = \ln w_m + \ln \varepsilon_m$, the posterior distribution of $\ln \mathbf{W}$ is the joint Normal distribution, whose parameters are given by Eqs. (24) and (25), wherein $\mathbf{M}_{\mathbf{X}} = \mathbf{M}_{\ln \mathbf{W}}$ and $\mathbf{C}_{\mathbf{XX}} = \mathbf{C}_{\ln \mathbf{W} \ln \mathbf{W}}$ are defined according to the prior probabilistic model of cover depth presented above, $\mathbf{x}_m = \ln \mathbf{w}_m$ and $\mathbf{C}_m = \sigma_{ln\varepsilon}\mathbf{I}_{800}$, where $\mathbf{I}_{800}$ is the identity matrix of size 800.

3.4 *Measurements of chloride concentration*

We consider measurements of chloride concentration made by taking cores from the concrete at selected locations $\mathbf{x}_{m,i}$. The chloride concentrations C_z at various depths z_m are obtained by chemical analysis of the ground-up concrete. The cores are taken at time $t_m = 10$ years and for each core, the chloride content is measured at two depths $z_{m1} = 20\text{mm}$ and $z_{m2} = 40\text{mm}$. The considered hypothetical measurement outcomes are summarized in the following table:

Table 3. Measurements of chloride concentration [in mass-% of cement].

Location (x and y coordinates)	$c_m(j,20\text{mm})$	$c_m(j,40\text{mm})$
a: x = 1m, y = 1m	0.3	0.1
b: x = 19m, y = 1m	0.5	0.3
c: x = 1m, y = 9m	0.6	0.1
d: x = 19m, y = 9m	0.9	0.3
e: x = 10m, y = 5m	1.4	0.5

The uncertainty in the concentration measurement is modeled by an additive Normal distributed error with zero mean and standard deviation $\sigma_\varepsilon = 0.2$ [mass-% of cement], which is assumed to be statistically independent from one measurement to the next.

To establish the likelihood function, the relation between the measured concentration and the model parameters must be established. According to the diffusion model, the chloride concentration c_z at location j and depth z_m is given as

$$c_{z_m,j}(\mathbf{X},t) = C_{s,j}\left[1 - \mathrm{erf}\left(\frac{z_m}{\sqrt{4D_j t}}\right)\right] \qquad (30)$$

The likelihood function for one measurement of chloride concentration $c_m(j,z_m,t)$ at location j, depth z_m and time t is accordingly:

$$L_j(\mathbf{x}) = \frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{c_{z_m,j}(\mathbf{x},t) - c_m(j,z_m,t)}{\sigma_\varepsilon}\right)^2\right]$$

(31)

3.5 *Numerical investigations*

The a-priori probability of corrosion as a function of time is shown in Figure 2. Since no location-specific information is available prior to the measurements, this probability is identical at all locations.

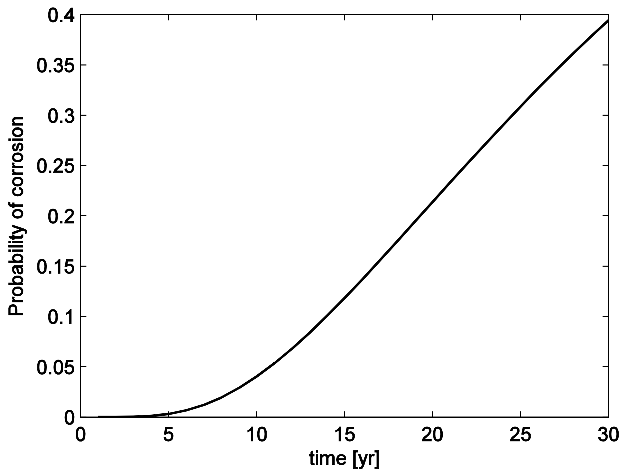

Figure 2. Probability of corrosion prior to measurements (SORM solution).

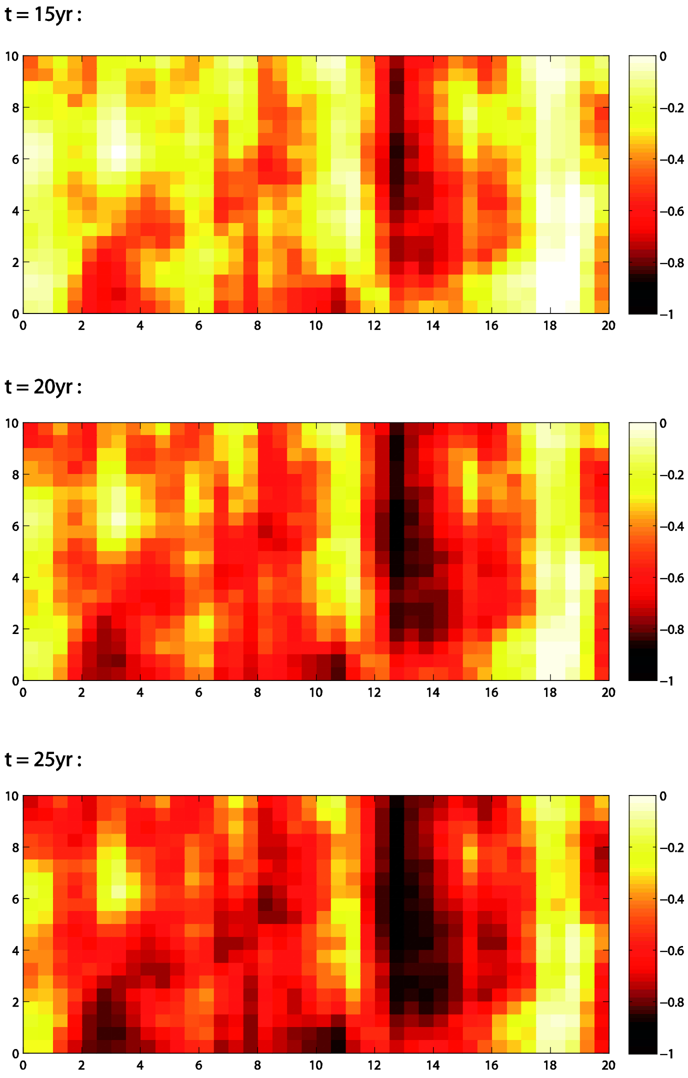

Figure 3. Probability of corrosion initiation at different times conditional on the concrete cover depth measurements of Figure 1.

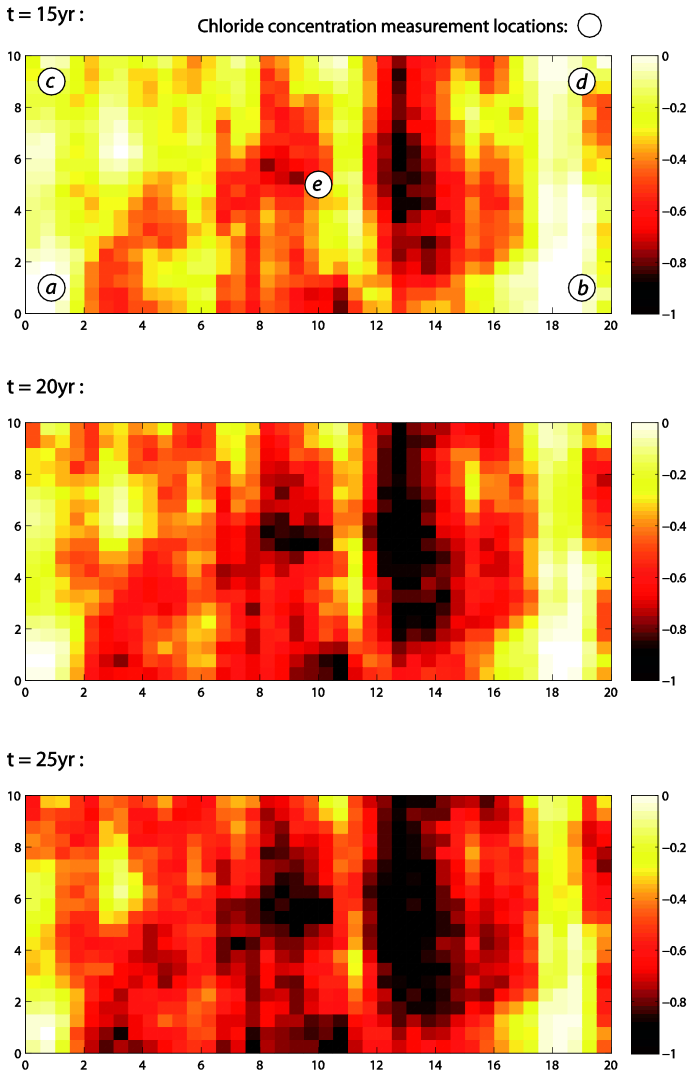

Figure 4. Probability of corrosion initation at different times conditional on the concrete cover depth measurements of Figure 1 and on the chloride measurements from Table 3.

In a first step, the measurements of concrete cover depth shown in Figure 1 are considered, by computing the posterior probability distribution of the cover depth at all locations, $f''_{\mathbf{W}}(\mathbf{w})$. The probability of corrosion initiation is then computed by MCS, with $f''_{\mathbf{W}}(\mathbf{w})$ as the probability distribution for cover depth, and shown in Figure 3. Due to the variability of the measurements, the probability of corrosion initiation is no longer uniform over the surface. In many locations, the a-posteriori probability of corrosion is significantly higher than the a-priori probability, which was 12% for year 15 and 30% for year 25, due to low measured values of cover depth.

In a second step, the results of the chloride measurements are included. To this end, the probability of failure conditional on the chloride measurements is evaluated with the importance sampling method described earlier with 10^5 samples. This computation utilizes the posterior distribution of concrete cover depth $f''_{\mathbf{W}}(\mathbf{w})$. The results of the analysis are shown in Figure 4.

By comparing Figure 3 and Figure 4, the influence of the chloride concentration measurements can be observed. Distinctly different values of the probability of corrosion are obtained in the vicinities of the locations where chloride profiles were taken; e.g. the probability is reduced around measurements at *c*, which show low concentrations, and it is increased around measurements at *e*, which show high concentration. However, overall, the influence of the chloride measurements is less than that of the cover depth measurements, which can be explained by the fact that the former are limited to five locations whereas the latter are available for all locations. Despite the spatial correlation, a larger number of measurements provides more information and thus has a larger impact on the probability estimate.

4 CONLUDING REMARKS

The application presented in this paper demonstrates the potential of the method proposed in Straub (2011) for Bayesian updating of spatial probabilistic models of deterioration with information that is obtained at discrete points in space. In the presented application, this information is the measured chloride content at discrete points in the concrete surface. Additionally, when direct measurements of individual model parameters are available, the method can be combined with classical methods for Bayesian updating of random variables. In the presented example, such measurements were available on concrete cover depths.

The new updating method proceeds by transforming equality information into equivalent inequality information. In this way, Bayesian updating of the probabilistic model and the reliability estimate with any information can be performed using simple simulation techniques, such as importance sampling. These techniques have the advantage of being robust, which is of particular relevance in the context of spatially distributed systems, where a large number of conditional probabilities must be computed.

As the amount of information increases, the efficiency of the presented simulation techniques decreases, due to a decrease of $\Pr(Z_e)$, the probability of the equivalent inequality observation. This limitation becomes relevant when measurements, which are not directly on an input variable, are made continuously in space; an example being large-scale half-cell potential measurements. Further investigations are ongoing on how the efficiency of methods for Bayesian updating can be improved for such applications.

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