Probabilistic Modeling of System Deterioration with Inspection and Monitoring Data using Bayesian Networks

Jesus Luque
PhD Student, Engineering Risk Analysis Group, Technische Universität München, Munich, Germany

Daniel Straub
Professor, Engineering Risk Analysis Group, Technische Universität München, Munich, Germany

ABSTRACT: To facilitate the estimation of the reliability of deteriorating structural systems conditional on inspection and monitoring results, we develop a modeling and computational framework based on Bayesian Networks (BNs). The framework enables accounting for dependence among deterioration at different system components, for dependence due to the structural system behavior, but also dependence introduced by information obtained on selected parts of the system, which effects the reliability estimates of other system parts. The proposed model and algorithm is applicable to aging structures, including offshore platforms, bridges, ships, aircraft structures, considering deterioration process such as corrosion and fatigue. To efficiently model dependence among component deterioration states, a hierarchical structure is defined. This structure facilitates the solution of the Bayesian model updating of the components in parallel. For illustration, a Daniels system subjected to fatigue is used as a case study. The computational efficiency of the proposed algorithm is compared with that of Markov Chain Monte Carlo and found to be orders of magnitude higher.

1. INTRODUCTION

Deterioration processes are present in all type of engineering structures, reducing their service life and affecting the safety of the environment, people and the structure itself. For this reason, significant resources have been invested to identify, quantify, mitigate, model and prevent the deterioration processes in structures.

Predictive deterioration models are typically associated with significant uncertainty, and ideally they are used probabilistically. Such probabilistic deterioration models are available mainly at the structural component level. However, deterioration at different locations in a structural system is typically correlated, and system considerations should be made (Straub and Faber 2005). Models have been proposed to deal with the complexity of probabilistically modeling the deterioration of large systems, and they were applied to different types of applications (e.g. Guedes Soares and Garbatov 1997, Kang and Song 2010, Straub 2011, Luque et.al 2014). Most of these models still present challenges related to their computational complexity, especially when the number of system components and available observations increase.

During recent years, Bayesian models have been used to represent deterioration processes in structures (e.g. vessels, offshore platforms, bridges, tunnels) in a probabilistic manner. One of the main advantages of the Bayesian approach is the possibility of updating the estimations and model parameters as new observations become available. In the case of deterioration processes, observations of the current condition of the structure are easily obtained from inspections and monitoring systems. As a consequence, Bayesian models are used to quantify the impact of these observations on the reliability of the structure, to facilitate maintenance decisions and the planning of future inspections (e.g. Straub and Faber 2005, Luque and Straub 2013).

Recently, Bayesian Networks (BNs) have become popular for Bayesian modeling in engineering risk analysis due to their intuitive nature and their ability to handle many dependent random variables (Jensen 2001). BNs are a powerful framework to graphically describe complex probabilistic problems and efficiently perform Bayesian inference. The graphical structure of the BN is formed by nodes and directed links. The former represent random or deterministic variables, and the latter the conditional dependencies among nodes. For example, if the amount of deterioration D is modeled as a function of an external random factor S (e.g. load) and an internal parameter M (e.g. material property), then the corresponding BN has three nodes representing each random variable and two directed links pointing from S and M towards D. Additionally, if an observation Z of the deterioration state D is potentially available (e.g. from an inspection), this would be modeled as a child of D (Figure 1).

Many engineering applications, including deterioration modeling, involve random processes BN, and nodes are a function of time or space. In this case, the BN is conveniently composed of a chain of sub-BNs. This type of BN is termed dynamic Bayesian network (DBN). Extending the previous BN example, we can model the deterioration D_t at each time step t as function of a time-dependent external factor S_t and a time-independent internal factor M with observations Z_t, where $t = 1, \dots, T$ (Figure 2). Each vertical "slice" of the DBN is a sub-BN that corresponds to a particular time step in the analysis.

In this paper, a DBN at the component level is extended to the system level and an algorithm is provided to efficiently assess the reliability of a deteriorating system when partial observations of its condition are available. In the following section, the concept of dynamic Bayesian networks and its application to efficiently model component deterioration are presented. Thereafter, in Section 3, the model is extended to represent the complete structural system. Finally, section 4 presents a case study.

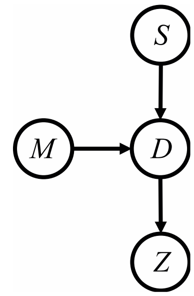

Figure 1. BN deterioration model example.

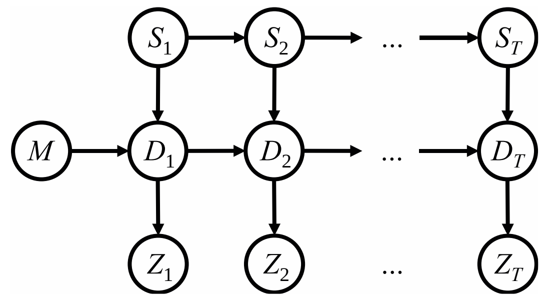

Figure 2. DBN deterioration model example.

2. DYNAMIC BAYESIAN MODEL OF COMPONENT DETERIORATION

The DBN model developed in Straub (2009) is used to describe the deterioration of components. This model includes the following elements:

- Time-invariant model parameters $\boldsymbol{\theta} = \boldsymbol{\theta}_0 = \dots = \boldsymbol{\theta}_t$.
- Time-variant parameters model $\boldsymbol{\omega}_0$, ..., $\boldsymbol{\omega}_t$.
- Deterioration model: A parametric function h for describing the deterioration amount D as a function of t, $\boldsymbol{\theta}$, $\boldsymbol{\omega}_0$, ..., $\boldsymbol{\omega}_t$ and the deterioration at the previous time D_{t-1}, i.e.

$$D_t = D(t) = h(t, D_{t-1}, \boldsymbol{\theta}, \boldsymbol{\omega}_1, \dots, \boldsymbol{\omega}_t) \quad (1)$$

- Observations: Information on the condition of a model parameter or deterioration D_t may be available (e.g. from inspections or monitoring systems). These observations are denoted by $Z_{\boldsymbol{\theta},t}$, $Z_{\boldsymbol{\omega},t}$, and $Z_{D,t}$, depending on the random variables to which they relate.

Figure 3 shows the generic DBN deterioration model for a single component. Each set $\{\boldsymbol{\theta}_t, \boldsymbol{\omega}_t, D_t, Z_{\boldsymbol{\theta},t}, Z_{\boldsymbol{\omega},t}, Z_{D,t}\}$ represents a time step t of the DBN.

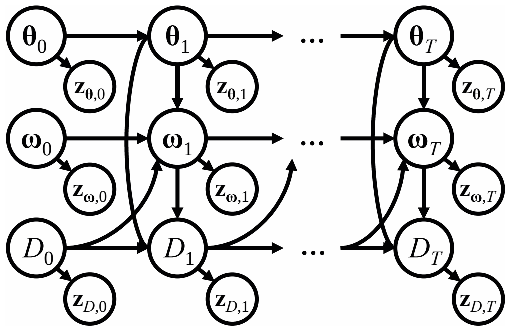

Figure 3. Generic DBN of the deterioration model at the component level (following Straub 2009).

To estimate the posterior distribution of the random variables in the model given a set of observations, several methods have been developed. For models with continuous or mixed (i.e. discrete and continuous) random variables, sampling-based methods are available. The most popular among these belong to the family of Markov Chain Monte Carlo (MCMC) methods. For BNs, MCMC using Gibb's sampler is particularly effective (Gamerman and Lopes 2006). Nevertheless, the computational cost of MCMC increases considerably as the number of observations included in the model increases and/or the probability of failure of interest decreases. This motivates the use of exact inference algorithms with discretized random variables, whose performance does not deteriorate with increasing amount of observation and which is independent of the magnitude of the probabilities of interest. However, the approach requires that all continuous random variables entering the deterioration model are discretized. For a description of the applied discretization approach see (Straub 2009).

3. BAYESIAN NETWORK MODEL OF SYSTEM DETERIORATION

One challenging aspect of modeling systems is the modeling and representation of the interrelationship among its components and the common factors that affect their condition. Only a limited number of investigations of the dependence among component deterioration states can be found in the literature (e.g. Maes et al. 2008, Vrouwenvelder 2004). The two most common mathematical representations of such dependence are hierarchical models and random field models. The latter are suitable for systems where dependence among component deterioration is a function of the geometrical location. Hierarchical models are suitable where the dependence among component deterioration depends on common features and common influencing factors. They have computational advantages over random fields, in particular in the context of DBN modeling.

3.1. Hierarchical models

In hierarchical models, multiple levels are used to group element properties (Raudenbush and Bryk 2008). The random variables within a level are interrelated through common influencing parameters, which are modeled at a higher level in the hierarchy. The random variables at the highest level are known as hyperparameters. As a simple example, Figure 4 shows a BN representing a set of random variables $\{V_1, V_2, \dots, V_n\}$ with common mean value α. As long as the value of α is uncertain, the random variables $\{V_1, V_2, \dots, V_n\}$ are statistically dependent. The correlation between V_i and V_j will depend on the distribution parameters. If the random variables V_i conditional on α all have standard deviation σ_V, and α has mean value α_0 and standard deviation σ_0 , then the linear correlation between any pair V_i and V_j, $i \neq j$, is $\rho(V_i, V_j) = \sigma_0^2/(\sigma_0^2 + \sigma_V^2)$.

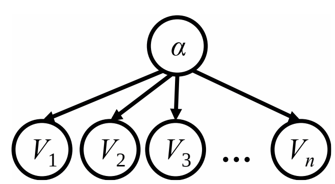

Figure 4. Hierarchical BN with a hyperparameter α.

3.2. Correlation among components

In many instances, influencing parameters are not modeled explicitly, as in the example above, but instead models of the correlation among components are available. As long as the correlation among any pair of components is the same (equi-correlation) or if the correlation structure follows the Dunnett-Sobel class, such correlation can be represented by a hierarchical model (see e.g. Thoft-Christensen and Murotsu 1986, Kang and Song 2009). Further details are presented in Luque and Straub (2015).

3.3. DBN model of the system

The hierarchical modeling approach is applied to model dependence among component deterioration in structures. Extending the DBN model of section 2 for single components, a set of hyperparameters $\boldsymbol{\alpha} = \left[\alpha_{\boldsymbol{\theta}}, \alpha_{\boldsymbol{\omega}}, \alpha_{D_0}\right]^T$ is defined. In the system model, all components are connected through these hyperparameters $\boldsymbol{\alpha}$ (Figure 5). All random variables in the DBN are now indexed by the component number i and the time step t, i.e. $D_{3,10}$ is the damage of component 3 at time step 10.

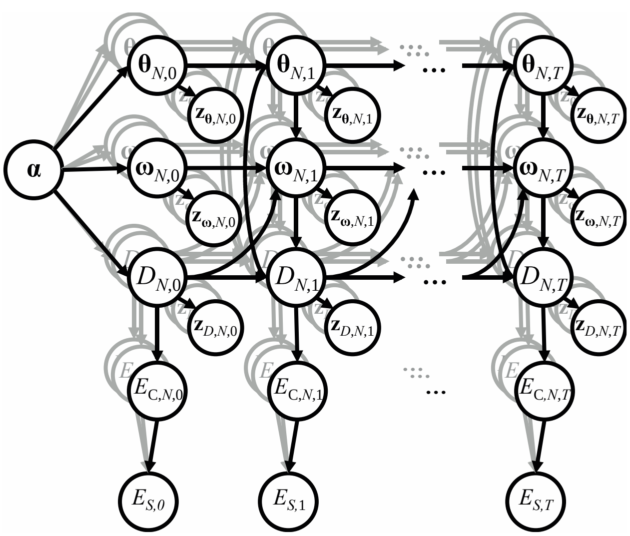

Figure 5. DBN model of the structural system deterioration.

In the full system model DBN of Figure 5, the binary random variable $E_{C,i,t}$ represents the condition (i.e. $E_{C,i,t} = 0$: safe, $E_{C,i,t} = 1$: fail) of component i at time step t. $E_{C,i,t}$ is a (possibly probabilistic) function of the deterioration state $D_{i,t}$. The binary random variable $E_{S,t}$ represents the system condition (i.e. $E_{S,t} = 0$: safe, $E_{S,t} = 1$: fail) as a function of all component conditions. The relation between the system condition $E_{S,t}$ and the condition of its components $E_{C,i,t}$, $i = 1, \dots, N$, is quantified by the probability of system failure given the conditions of its components. To obtain these conditional probabilities, a probabilistic model of the structural system is necessary and structural reliability computations must be performed in a pre-processing step.

3.4. Inference algorithm

To perform inference with the system DBN, i.e. to compute the probability of component and system failure given inspection and monitoring results, the forward-backward algorithm presented in Straub (2009) for exact inference is extended to the system level. It solves the filtering problem, i.e. it computes the posterior distribution of the random variables $\boldsymbol{\alpha}$, $\boldsymbol{\theta}_i$, $\boldsymbol{\omega}_{i,t}$, $D_{i,t}$, $E_{C,i,t}$ and $E_{S,t}$ for all $i = 1, \dots, N$ given the observations up to time t. The algorithm is formulated in a recursive manner for each time step and exploits the property of the hierarchical model that all components are statistically independent for given hyperparameters. The algorithm is described in detail in Luque and Straub (2015).

4. NUMERICAL INVESTIGATION

The numerical example serves to investigate and illustrate the workings of the proposed model and inference algorithm. For validation purposes, the results obtained with the exact inference algorithm are compared to those obtained with two alternative methods: 1) Monte Carlo simulation (MCS) for the case without observations, and 2) MCMC for the cases with and without observations. The MCMC computations are implemented with OpenBUGS (Lunn et.al 2009).

4.1. Structural system

For illustration purposes, we consider a Daniels system (Daniels 1945), which consists of a set of N components with independent and identically distributed capacities R_i for $i = 1, \dots, N$ and the system is affected by a random load L (Figure 6).

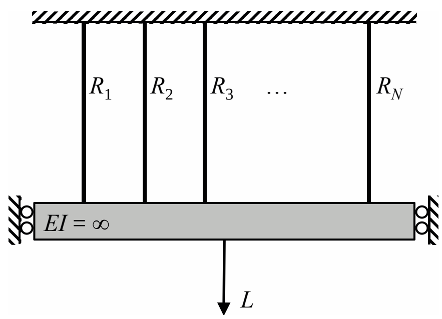

Figure 6. Daniels system.

Prior to the application of the load, each component in the system is in one of two possible states: a) full capacity, or b) zero capacity due to a fatigue failure. For a discussion of this model see Straub and Der Kiureghian (2011).

4.2. Deterioration model

The system components are subject to fatigue deterioration, which - for illustration purposes - is modeled by the simple fracture-mechanics-based crack growth model presented e.g. in (Ditlevsen and Madsen 1996). It uses Paris' law to describe the growth of the crack length D_i at component i:

$$\frac{\mathrm{d}D_i(n)}{\mathrm{d}n} = C_i\left[\Delta S_{e,i}\sqrt{\pi D_i(n)}\right]^{M_i} \tag{2}$$

where $n =$ number of stress cycles; $\Delta S_{e,i} = (\mathrm{E}[\Delta S_i^M])^{\frac{1}{M}} =$ equivalent stress range per cycle; $\mathrm{E}[\cdot]$ is the expectation operator; $\Delta S_i =$ stress range per cycle; C_i, $M_i =$ empirically determined material parameters. The long term distribution of the fatigue stress range ΔS_i follows a Weibull distribution with scale and shape parameters K_i and λ_i (Madsen 1997), and $\Delta S_{e,i}$ is given by:

$$\Delta S_{e,i} = K_i \Gamma\left(1 + \frac{M_i}{\lambda_i}\right)^{\frac{1}{M_i}} \tag{3}$$

where $\Gamma(\cdot)$ is the Gamma function. Using the initial condition $D_i(n = 0) = D_{i,0}$, the following analytical solution for the crack length after n stress cycles can be obtained from Eq. (2) as:

$$D_i(n) = \left[\left(1 - \frac{M_i}{2}\right) C_i \Delta S_{e,i}^{M_i} \pi^{\frac{M_i}{2}} n + D_{i,0}^{1-\frac{M_i}{2}}\right]^{\left(1-\frac{M_i}{2}\right)^{-1}} \tag{4}$$

4.3. Observations and probability of detection

In this example, we only consider observations of the deterioration state through inspections, e.g. visual inspections or non-destructive evaluation of the fatigue hot spots. The observation $Z_{D,i,t}$ is a binary random variable with possible states "no crack detection" (i.e. $Z_{D,i,t} = 0$), and "crack detection" (i.e. $Z_{D,i,t} = 1$). The inspection is defined by an exponential probability of detection (POD) model as a function of the crack length D and a deterministic parameter ξ:

$$\Pr(Z = 1|D) = \mathrm{POD}(D) = 1 - \exp\left(-\frac{D}{\xi}\right) \tag{5}$$

4.4. Relation between the system condition and its components

Failure of the i-th component after t time steps (equivalent to $n = n(t)$ stress cycles) occurs when the crack length exceeds the critical value D_c . It is expressed through the limit state function $g_{i,t}$:

$$g_{i,t} = D_c - D_{i,t} = D_c - D_i(n) \tag{6}$$

The performance of the i-th component at time step t , represented through $E_{C,i,t}$, is characterized by the failure event (i.e. $g_{i,t} \leq 0$) and the survival event (i.e. $g_{i,t} > 0$). If the component has not failed, it is assumed to have its full capacity.

In a Daniels system, due to the exchangeability of the components, the probability of having a system failure at time step t is a function only of the total number of component failures. To avoid a convergent definition of the total number of component failures as a child node of all component conditions $E_{C,i,t}$, $i = 1, \dots, N$, the cumulative number of component failures up to component i, $N_{f,1:i,t}$ is defined as follows:

$$N_{f,1:i,t} = \sum_{j=1}^{i} E_{C,j,t} = E_{C,i,t} + N_{f,1:i-1,t} \tag{7}$$

With this recursive definition, the total number of component failures, $N_{f,1:N,t}$, does not have a convergent relation anymore. The complete DBN of the Daniels system is presented in Figure 7.

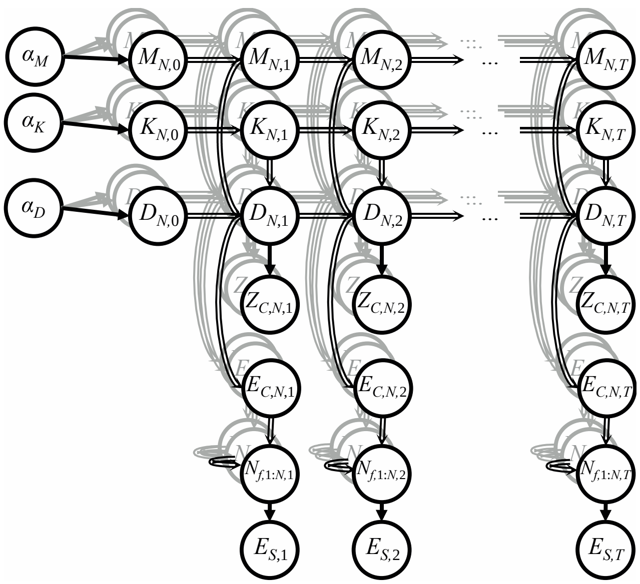

Figure 7. DBN of the Daniels system

The conditional probability of failure of the system given j failed components is given by:

$$\Pr\left(E_{S,t} = 1 \middle| N_{f,1:N,t} = j\right) = \Pr\left(\sum_{i=1}^{n-j} R_i - L \le 0\right) \tag{8}$$

4.5. Probabilistic model for fatigue deterioration

A Daniels system with $N = 10$ components and $T = 100$ time steps is investigated. The probabilistic model for the fatigue deterioration is summarized in Table 1 and the corresponding discretization scheme is presented in Table 2. Each time step corresponds to $\Delta n = 10^5$ fatigue stress cycles. The correlation of fatigue parameters among components are $\rho_{D_0} = 0.5$, $\rho_M = 0.6$, and $\rho_K = 0.8$.

The load L is lognormal distributed with coefficient of variation $\delta_L = 0.25$, the capacities R_i, $i = 1, \ldots, 10$, are independent and normal distributed with $\delta_R = 0.15$ and the mean safety factor is $n\mu_{R_i}/\mu_L = 2.9$. The resulting conditional probability of failure of the system given the j failed components is computed according to Eq. (8).

Table 1. Probabilistic model for fatigue deterioration in the Daniels system.

RV	Distribution	Mean	StDev
$\alpha_{D_0,K,M}$	*Normal*	0	1
$D_{0,i}$ (mm)	*Exponential*	1	1
$M_{0,i}$	*Normal*	3.5	0.3
$M_{t,i}$	$M_{t,i} = M_{t-1,i}$		
$ln\,C_{t,i}$	$ln\,C_{t,i} = -3.34 M_{t,i} - 15.84$		
$K_{0,i}$	*Lognormal*	1.6	0.22
$K_{t,i}$	$K_{t,i} = K_{t-1,i}$		
ξ (mm)	*Deterministic*	10	

RV: Random variable.

Table 2. Discretization scheme

RV	Interval boundaries
$\alpha_{D_0,K,M}$	$\Phi^{-1}(0{:}\,0.2{:}\,1)$
D	$0, exp\left[ln(0.01) : \frac{ln(50) - ln(0.01)}{78} : ln(50)\right], \infty$
M	$0, ln\left[exp(2.2) : \frac{exp(4.8) - exp(2.2)}{18} : exp(4.8)\right], \infty$
K	$0, \left\{0.86 : \frac{2.83 - 0.86}{18} : 2.83\right\}, \infty$

4.6. Results

A good agreement among the three methods is observed at the component level. For the unconditional case (i.e. without observations), the reliability index β calculated with the proposed inference algorithm is compared to the results obtained using MCS and MCMC for a single component (Figure 8) and the system (Figure 9). The reliability index is defined as $\beta = -\Phi^{-1}[\Pr(E = fail)]$, with Φ^{-1} being the inverse standard normal CDF.

At the system level, the difference between the probability estimates from the proposed DBN model and the Monte Carlo methods is due to the discretization of the hyperparameters $\boldsymbol{\alpha}$ in the DBN. The relatively coarse discretization of α_{D_0},

α_M, and α_K leads to an underestimation of the correlation in the fatigue performance among components. This in turn leads to an overestimation of the system reliability in a redundant system, such as the Daniels system.

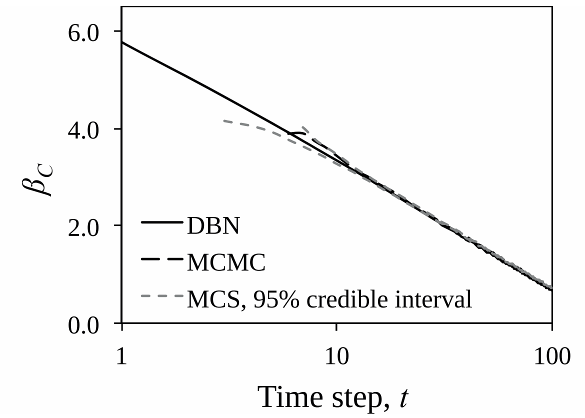

Figure 8. Reliability index β_C of a single component.

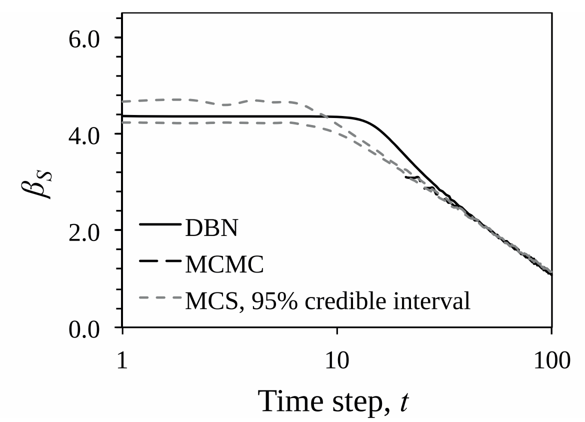

Figure 9. Reliability index β_S of the Daniels system.

The relevant case for the DBN model is the conditional case, i.e. including inspections results. It is assumed that one component is inspected every 10^6 cycles, i.e. after every 10 time steps, without detecting any crack. The updated reliability index of the inspected component is considerably larger with respect to the unconditional case (Figure 10). The observation also affects the non-inspected components, due to the correlation defined by the hyperparameters (Figure 11). The reliability of the system is affected by the reliability of both the inspected and the non-inspected components (Figure 12). By inspection only one component every 10 time steps, and assuming that the inspections always result in a no-detection, the system reliability at the end of the service life increases from 1.1 to 2.1.

In Figure 10 to Figure 12, the results of the DBN model are compared with results obtained by MCMC for verification. The results from two algorithms match well, and the slight differences observed in the unconditional case (Figure 9) are not seen here.

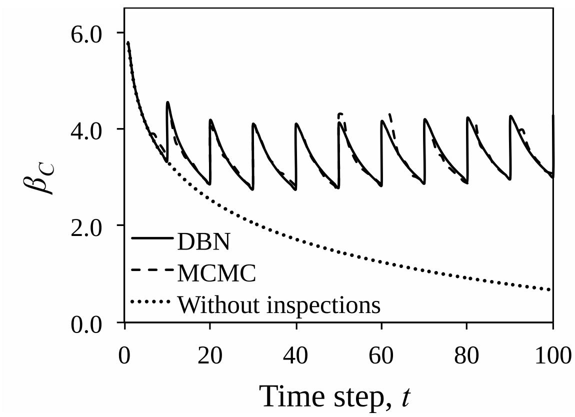

Figure 10. Reliability index β_C of the inspected component after no detection of a crack at inspections every 10 time steps.

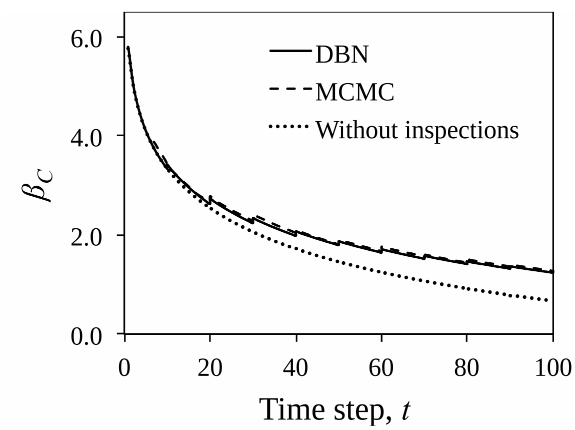

Figure 11. Reliability index β_C of a non-inspected component given no-detection of the inspected component.

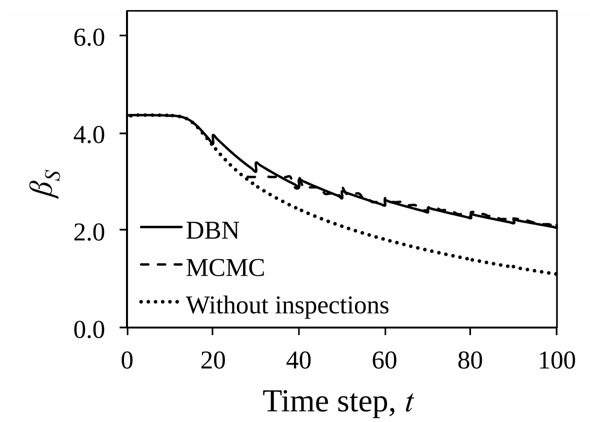

Figure 12. Reliability index β_S of the system after no detection of a crack in all inspection times.

5. CONCLUDING REMARKS

A hierarchical Bayesian network to model the deterioration process in structural systems is proposed. The model includes the spatial and temporal relation among the system components, in order to assess the effect of (partial) observations of system components on the probability of system failure. An efficient algorithm for performing inference and calculating reliability estimates at the system level is provided. The accuracy of the model and the algorithm is tested using a Daniels system as a case study. The results are compared with two alternative methods (MCS and MCMC) showing good agreement and considerably smaller computation time.

A main advantage of the proposed exact inference algorithm is its computational complexity. The system deterioration model presented in Figure 5 can be solved with almost linear complexity with respect to the time steps and the number of components. This property can be affected by the convergent links between the system and components, but in some cases this can be simplified as shown for the Daniels system. Additionally, the computation time is not affected by the number of observations used to update the model or the order of magnitude of the probability of failure. The main limitation of the approach is related to the discretization of the random variables. Depending on the size of the CPTs used for each node of the network, the algorithm can require considerable amount of memory resources. This requirement becomes more relevant if the number of hyperparameters or the refinement of their discretization increases.

In terms of absolute total computation time, for the presented example, the exact inference algorithm takes at least one order of magnitude less than the time needed for MCMC using OpenBUGS. Although the comparison of the total time is not necessarily fair (due to the usage of different programs to solve the two methods), the difference in the complexity is noticeable. MCMC quickly becomes intractable when the number of components or observations increases whereas the computation time for the exact inference algorithm is almost proportional to the number of components and it is independent of the number of observations included in the model. Additionally, the number of samples MCMC needs to reasonably estimate the probability of failure of the system is inversely proportional to the magnitude of that probability whereas in the exact inference algorithm the computation time is independent of that probability.

In conclusion, the efficiency and robustness of the proposed algorithm make it suitable to planning and optimizing monitoring, inspection and maintenance activities in an integral manner for structural systems. Such an approach was previously hindered by the computational limitations of sampling-based inference algorithms.

ACKNOWLEDGEMENT

This work is supported by the Deutsche Forschungsgemeinschaft (DFG) through Grant STR 1140/3-1 and the Consejo Nacional de Ciencia y Tecnología (CONACYT) through Grant No. 311700.

REFERENCES

Daniels, H.E. (1945). The statistical theory of the strength of bundles of threads, Part I. Proc. Roy. Soc., A, 183(995), 405-435.

Ditlevsen, O., Madsen, H.O. (1996). Structural reliability methods, Wiley, New York.

Gamerman D, Lopes H F, 2006. Markov Chain Monte Carlo, stochastic simulation for Bayesian inference. 2nd Ed. Chapman & Hall/CRC, Florida, USA.

Guedes Soares C, Garbatov Y, (1997). Reliability assessment of maintained ship hulls with correlated corroded elements. Marine Structures 10, pp. 629-653.

Jensen, F.V. (2001). Bayesian networks and decision graphs. Springer, New York.

Kang, W.H., Song, J., (2010). Evaluation of multivariate normal integrals for general systems by sequential compounding. Journal of Structural Safety 32(1), pp 35–41.

Kang, W.H., Song, J. (2009). Efficient reliability analysis of general systems by sequential compounding using Dunnett-Sobel correlation model. Proc. 10th International Conference on Structural Safety and Reliability (ICOSSAR2009), Osaka, Japan.

Kim, D.S., Ok, S.Y., Song, J., Koh, H.M., (2013). System reliability analysis using dominant failure modes identified by selective searching technique. Reliability Engineering & System Safety, 119, 316-331.

Liu P.-L., Der Kiureghian A. (1986). Multivariate distribution models with prescribed marginals and covariances. Probabilistic Engineering Mechanics, 1(2), 105-112.

Lunn, D., Spiegelhalter, D., Thomas, A. and Best, N. (2009) The BUGS project: Evolution, critique and future directions (with discussion), Statistics in Medicine 28: 3049-3082

Luque J, Straub D. (2015). Reliability analysis of deteriorating structural systems with inspection and monitoring data using Bayesian networks. Working paper, Engineering Risk Analysis Group, TU München.

Luque J., Hamann R., Straub D. (2014). Spatial model for corrosion in ships and FPSOs. Proc. OMAE2014, San Francisco, CA.

Luque J., Straub D. (2013). Algorithms for optimal risk-based planning of inspections using influence diagrams. Proc. 11th International Probabilistic Workshop IPW11 2013, Brno University of Technology, Brno, Czech Republic.

Madsen, H.O. (1997). Stochastic modeling of fatigue crack growth and inspection. IN SOARES, C.G. (Ed.) Probabilistic Methods for Structural Design. Kluwer Academic Publishers. Printed in the Netherlands.

Maes M, Dann M, Breitung K, Brehm E., (2008). Hierarchical modeling of stochastic deterioration. Graubner, Schmidt & Proske: Proceedings of the 6th International Probabilistic Workshop, Darmstadt.

Raudenbush, S.W., Bryk, A.S., 2008. Hierarchical linear models, applications and data analysis methods. 2nd Ed. SAGE Publications, Inc. California, USA.

Russell, S.J., Norvig, P. (2003). Artificial intelligence: A modern approach. 2^{nd} Ed. Prentice-Hall, Englewood Cliffs, N.J.

Straub D. (2011) Reliability updating with inspection and monitoring data in deteriorating reinforced concrete slabs. Proc. ICASP11, Zürich, Switzerland.

Straub, D., Der Kiureghian, A., (2011). Reliability acceptance criteria for deteriorating elements of structural systems. Journal of Structural Engineering, Trans. ASCE, 137(12): 1573–1582.

Straub, D., (2009). Stochastic modeling of deterioration processes through dynamic Bayesian networks. Journal of Engineering Mechanics, Trans. ASCE, 135(10), pp. 1089-1099.

Straub D., Faber M.H. (2005). Risk Based Inspection Planning for Structural Systems. Structural Safety, 27(4): pp 335-355.

Thoft-Christensen P., Murotsu Y. (1986). Application of structural systems reliability theory, Springer.

Vrouwenvelder, T. (2004). Spatial correlation aspects in deterioration models. Proc., 2^{nd} Int. Conf. Lifetime-Oriented Design Concepts, F. Stangenberg, ed., Ruhr-Universität Bochum, Germany.