Covariance Based Signal Parameter Estimation of Coarse Quantized Signals

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***Abstract*—The use of low resolution analog-to-digital converters is an effective solution to reduce the complexity and the power consumption of the analog front-end, especially in the context of massive multiple-antenna and large system bandwidth. However, most well-known signal parameter estimation algorithms assume that the receiver has access to the observations data with infinite precision. In this paper, we investigate a method to improve the performance of covariance based algorithms that operate on quantized signals. To this end, we use a particular transformation between the input and output second order statistics when affected by non-linear processing. By applying this transformation, we could then utilize standard algorithms with almost no additional complexity. The introduced method is also of particular interest in the context of adaptive signal processing, since many adaptive processing algorithms are based on estimating the received signal covariance matrices. Through simulations, we show that the method is capable of improving the estimation performance especially in the extreme case of one-bit quantization.**

I. Introduction

The mobile data traffic grew from $0.820 \cdot 10^{18}$ bytes to $1.5 \cdot 10^{18}$ bytes per month from the end of 2012 to the end of 2013 and is expected to reach $15.9 \cdot 10^{18}$ bytes per month by 2018 [13]. The deployment of multiple antennas at the transmitter and receiver promises large channel capacity gain over single antenna systems and is one of the major techniques to improve the spectral efficiency. Since each antenna has its own analog front-end, the power consumption of a large number of antennas is quite high. A major contributing factor to the overall hardware complexity is the analog to digital converter (ADC). As shown in [14] the power consumption can be significantly reduced by decreasing the resolution. Information theoretical evaluation showed that a 1-bit quantized system has only a small performance loss compared to infinite resolution systems, but has the potential to greatly reduced the hardware complexity and the power consumption [1], [2], [3], [4] and [5]. In [11], the authors analyze the performance of joint Angle of Arrival (AoA) and Timing of Arrival (ToA) estimation of Ultra-wideband signals (UWB) quantized with 1 bit. Similarly, a method is presented in [9] to improve the performance of 1-bit quantized signals. An iterative decoder based on the message passing algorithm for quantized MIMO channel output is given in [10]. It is shown that the iterative decision detector performance is significantly better than the linear MMSE detector while having lower complexity for large system size. The method represented in [12] gives an approximation of the correlation matrices for evaluating the mean square error using quantized channel outputs. The main contribution of this paper is the application of a method that is capable of improving the performance of covariance based estimation algorithms under coarse quantization. Our method is generic and can be applied to any ADC resolution. We will show that our method significantly outperforms existing methods particularly for a 1-bit quantized system.

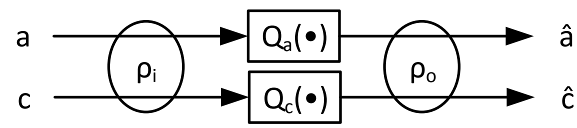

Fig. 1. System model for input output covariance.

Our paper is organized as follows: first the relationship between the input and output covariance of a quantizer operating on Gaussian random variables is represented. Then this relationship is generalized to circular symmetric complex Gaussian random vectors. Afterwards, two examples are shown, where the consideration of this relationship helps to improve the system performance. Throughout the paper we use boldface lower and upper case letters to represent vectors and matrices. $\boldsymbol{A}^*$, $\boldsymbol{A}^T$, $\boldsymbol{A}^H$ and $\boldsymbol{A}^{-1}$ represent the complex conjugate, the transpose, the hermitian and the inverse of the operand. $\mathrm{E}[\cdot]$ is the expectation operator. The abbreviations $\rho_{ab} = \mathrm{E}[(a-\mathrm{E}[a])(b-\mathrm{E}[b])^*]$ and $\boldsymbol{R_{ab}} = \mathrm{E}[(\boldsymbol{a}-\mathrm{E}[\boldsymbol{a}])(\boldsymbol{b}-\mathrm{E}[\boldsymbol{b}])^H]$ are used.

II. Covariance Matrix perturbation of Gaussian random variables due to quantization

In [6], the author described a method to relate the covariance before and after a non-linear function if the random variable is Gaussian. Figure 1 shows the system model of the relationship, where ρ_i is ρ_{ac}, and ρ_o is $\rho_{\hat{a}\hat{c}}$. The equation proved for Gaussian random variables in [6] is:

$$\frac{\partial^k \rho_o}{\partial^k \rho_i} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q_a^{(k)}(a) Q_c^{(k)}(c) f_{ac}(a,c)\; da\; dc, \qquad (1)$$

which describes the relation of the covariance coefficients ρ_i and ρ_o. $Q_a^{(k)}$ and $Q_c^{(k)}$ are the kth derivative of the non-linear processing functions. f_{ac} is the joint probability density function of the variables a and c. This differential equation is only valid if the variables a and c are jointly Gaussian.

Since we are interested in a simple relationship, we use the first order partial derivative, which means k is equal to 1. In addition we choose Q_a and Q_c to be the step function of a uniform quantizer with N_q quantization levels. Since the quantization functions Q_a and Q_c are step functions, their first order derivatives are $N_q - 1$ Dirac impulses δ at the position of the quantization steps a_i^s and c_i^s with the area Δ_a and Δ_c equal to the step-size, i.e.,

$$Q_a^{(1)}(x) = \Delta_a \sum_{l=1}^{N_q-1} \delta(x - a_l^s) \ , \ Q_c^{(1)}(x) = \Delta_c \sum_{j=1}^{N_q-1} \delta(x - c_j^s). \tag{2}$$

Tthe joint probability density function $f_{ac}(a, c)$ of a and c with variance σ_a^2 and σ_c^2 is defined as:

$$\frac{1}{2\pi\sigma_a\sigma_c\sqrt{1-\rho_i^2}} \exp\left(-\frac{1}{2(1-\rho_i^2)}\left[\frac{a^2}{\sigma_a^2} + \frac{c^2}{\sigma_c^2} - \frac{2\rho_i ac}{\sigma_a\sigma_c}\right]\right). \tag{3}$$

Because the functions $Q_a^{(1)}(x)$ and $Q_c^{(1)}(x)$ are only non zero at the position of the Dirac impulses, the double integral in Equation (1) is reduced to the double sum:

$$\frac{\partial\rho_o}{\partial\rho_i} = \Delta_a\Delta_c \sum_{l=1}^{N_q-1}\sum_{j=1}^{N_q-1} f_{ac}(a_l^s, c_j^s). \tag{4}$$

With the initial condition $\rho_i = 0$ at $\rho_o = 0$, the output covariance ρ_o for a specific input covariance ρ_i' is:

$$\rho_o = \int_0^{\rho_i'} \Delta_a\Delta_c \sum_{l=1}^{N_q-1}\sum_{j=1}^{N_q-1} f_{ac}(a_l^s, c_j^s)\ d\rho_i. \tag{5}$$

This equation has in general no closed form solution and has to be evaluated numerically. But if the signals a and c are quantized with a 1 bit uniform quantizer with its only step at 0, the previous formula is reduced to

$$\rho_o = \frac{\Delta_a\Delta_c}{2\pi\sigma_a\sigma_c}\int_0^{\rho_i'} \frac{1}{\sqrt{1-\rho_i^2}}\ d\rho_i. \tag{6}$$

For this equation the closed form solution is:

$$\rho_o = \frac{\Delta_a\Delta_c}{2\pi\sigma_a\sigma_c}\sin^{-1}(\rho_i). \tag{7}$$

In the following we are going to verify the derived relationship by a simulation. To normalize the graph the correlation is used instead of the covariance. In this simulation the input-output correlation relationship is calculated by solving equation (6) and then compared to the results of numerically evaluation of the correlation. Figure 2 shows that the theoretical and simulated curves are almost identical. The slight difference comes from the fact that only finite number of realizations are simulated. For the next curves the minimum distortion uniform quantizers for Gaussian input symbols according to [8] are used. Figure 3 shows that the more quantization bits we use the more the function converges to a straight line with slope 1, which means that the correlation at the input is the same as the correlation at the output. If we use a minimum

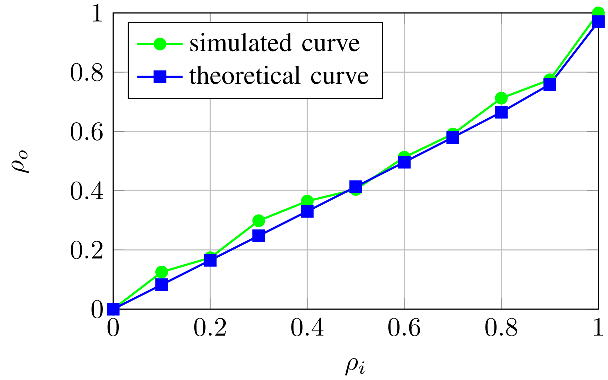

Fig. 2. Correlation relationship simulation uniform 2-bit quantizer.

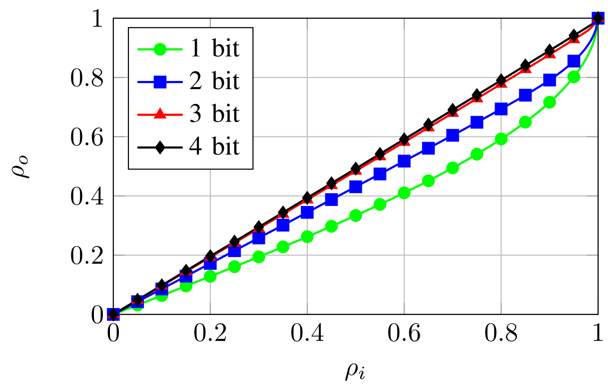

Fig. 3. Correlation relationship for different resolution for a uniform quantizer.

distortion uniform quantizer, the covariance perturbation due to quantization is almost negligible if a quantizer with more then 3 bit resolution is used. A quantization with 1 bit shows by far the greatest perturbation. The variance of the signal $\hat{a}$ or $\hat{c}$ can be calculated as

$$\sum_{l=1}^{N_q} \hat{a}_l^2 \left[\Phi\left(\frac{a_l^s}{\sigma_a}\right) - \Phi\left(\frac{a_{l-i}^s}{\sigma_a}\right)\right], \tag{8}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the normal Gaussian distribution, the a_l^s are the positions of the quantization steps and $\hat{a}_l$ is the output value of the quantizer for the input region l between a_{l-1}^s and a_l^s. The first and last step are set to be $a_0^s = -\infty$ and $a_{N_q}^s = \infty$.

III. Generalization for Circular Symmetric Gaussian Vectors

Now it is possible to completely calculate the correlation and variance of two correlated real Gaussian variables after quantization. In most communication applications the signals are represented by circular symmetric complex Gaussian signals. The random variables that are going to be treated from now on are complex circular symmetric Gaussian vectors $\boldsymbol{a}$ with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{R_{aa}}$. The relationship in (6) can be used to either calculate the covariance matrix after quantization from the version before or vice versa. The first possibility is shown in the following paragraph.

First we stack the real and imaginary part of $\boldsymbol{a}$ into one vector, i.e.:

$$\boldsymbol{a}_{RI} = \begin{bmatrix} \Re(\boldsymbol{a}) \\ \Im(\boldsymbol{a}) \end{bmatrix}. \tag{9}$$

The elements of $\boldsymbol{a}_{RI}$ are real Gaussian distributed. The elements of $\hat{\boldsymbol{a}}_{RI}$ are the quantized elements of $\boldsymbol{a}_{RI}$. So the jth element of $[\hat{\boldsymbol{a}}_{RI}]_j$ is equal to $Q_j([\boldsymbol{a}_{RI}]_j)$, where $Q_j(\cdot)$ is the quantization function of the jth element. The matrix

$$\boldsymbol{R}_{\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}} = \mathrm{E}\left[\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}^T\right] \tag{10}$$

is the covariance matrix of the quantized vector $\hat{\boldsymbol{a}}_{RI}$. Similarly,

$$\boldsymbol{R}_{\boldsymbol{a}_{RI}\boldsymbol{a}_{RI}} = \mathrm{E}\left[\boldsymbol{a}_{RI}\boldsymbol{a}_{RI}^T\right] \tag{11}$$

is the covariance matrix before the quantizer. To calculate the j,k's element of $\boldsymbol{R}_{\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}}$ where $j \neq k$, the j's and k's elements of $\boldsymbol{a}$ are taken as the inputs a and c of the previously described system. For the jth diagonal element of $\boldsymbol{R}_{\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}}$ we just need to calculate the variance of the signal $[\hat{\boldsymbol{a}}_{RI}]_j$. Exploiting the fact that $\boldsymbol{R}_{\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}}$ is a symmetric matrix the computational burden can be reduced. The covariance matrix $\boldsymbol{R}_{\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}}$ can be divided into sub-matrices in the following way:

$$\boldsymbol{R}_{\hat{\boldsymbol{a}}_{RI}\hat{\boldsymbol{a}}_{RI}} = \begin{bmatrix} \boldsymbol{R}_{RR} & \boldsymbol{R}_{RI} \\ \boldsymbol{R}_{RI}^T & \boldsymbol{R}_{II} \end{bmatrix}. \tag{12}$$

It turns out that the signal $\hat{\boldsymbol{a}}$ is also circular, therefore $\boldsymbol{R}_{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}}$ can be calculated from these sub-matrices as:

$$\boldsymbol{R}_{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}} = \boldsymbol{R}_{RR} + \boldsymbol{R}_{II} + j(\boldsymbol{R}_{RI}^T - \boldsymbol{R}_{RI}). \tag{13}$$

Now we derived a way to calculate the covariance matrix of Gaussian random variable after quantization.

IV. Examples

In the following two examples, where the described method is capable of improving the system performance are represented. The first one is an MMSE-receiver for a quantized MIMO system. In this example the covariance matrix after the quantizer is calculated from $\boldsymbol{R}_{\boldsymbol{aa}}$. The second example shows how this algorithm could improve the performance of covariance matrix based angle of arrival estimation algorithms. Here the algorithm is used to invert the perturbation of the covariance matrix, that is estimated from samples after the quantizer.

A. Quantized MMSE-receiver

The following section compares the results of calculating a quantized MMSE MIMO equalizer based on the presented approach to the approximation done in [7]. Figure 4 shows the equivalent baseband system model of a quantized MIMO system. The vector $\boldsymbol{x} \in \mathbb{C}^M$ is zero mean circular symmetric distributed with the covariance matrix $\boldsymbol{R}_{\boldsymbol{xx}} = \mathrm{E}[\boldsymbol{x}\boldsymbol{x}^H]$ and represents the input signal to the system. The matrix $\boldsymbol{H} \in \mathbb{C}^{N \times M}$ is the channel matrix. The noise $\boldsymbol{n} \in \mathbb{C}^N$ is zero mean circular symmetric Gaussian distributed with the covariance matrix $\boldsymbol{R}_{\boldsymbol{nn}} = \mathrm{E}[\boldsymbol{n}\boldsymbol{n}^H]$. The vector $\boldsymbol{y}$ is the channel output:

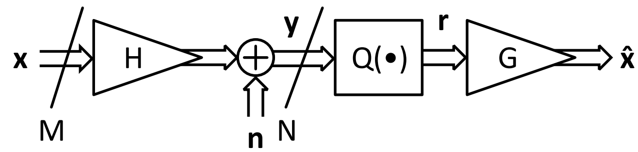

Fig. 4. System model quantized MIMO.

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}. \tag{14}$$

The operator $Q(\cdot)$ represents a quantization, which means that the real and imaginary parts of each element of $\boldsymbol{y}$ are quantized separately. The vector $\boldsymbol{r}$ is equal to:

$$\boldsymbol{r} = \boldsymbol{y} + \boldsymbol{q}, \tag{15}$$

where $\boldsymbol{q}$ represents the quantization error. The linear filter $\boldsymbol{G}$ and $\boldsymbol{r}$ form the estimate of the input signal $\boldsymbol{x}$:

$$\hat{\boldsymbol{x}} = \boldsymbol{G}\boldsymbol{r}. \tag{16}$$

The Mean Square Error (MSE) is defined as:

$$\mathrm{E}[||\boldsymbol{x} - \hat{\boldsymbol{x}}||_2^2] = \mathrm{E}[||\boldsymbol{x} - \boldsymbol{G}\boldsymbol{r}||_2^2]. \tag{17}$$

If we minimize the MSE, the optimal linear equalizer is:

$$\boldsymbol{G}_{opt} = \boldsymbol{R}_{\boldsymbol{xr}}^{-1}\boldsymbol{R}_{\boldsymbol{rr}}. \tag{18}$$

Therefore the minimal MSE (MMSE) is:

$$\mathrm{MMSE} = tr(\boldsymbol{R}_{\boldsymbol{xx}} - \boldsymbol{R}_{\boldsymbol{xr}}\boldsymbol{R}_{\boldsymbol{rr}}^{-1}\boldsymbol{R}_{\boldsymbol{xr}}^H). \tag{19}$$

For a general input distribution the covariance matrices $\boldsymbol{R}_{\boldsymbol{xr}}$ and $\boldsymbol{R}_{\boldsymbol{rr}}$ cannot be calculated in a closed form solution. But if we assume that $\boldsymbol{x}$ and $\boldsymbol{n}$ are Gaussian, it is possible to calculate $\boldsymbol{R}_{\boldsymbol{xr}}$ and $\boldsymbol{R}_{\boldsymbol{rr}}$ based on the results from Section II.

B. Simulation Results Quantized MMSE-receiver

To evaluate the proposed receiver, an uncoded BER simulation was carried out. The result was averaged over 1000 channel realizations. For each realization the channel matrix $\boldsymbol{H}$ was constructed out of i.i.d. zero mean circular symmetric Gaussian random entries with unit variance. The input signals were chosen to be independent QPSK symbols with unit variance. This means the $\boldsymbol{R}_{\boldsymbol{xx}}$ is equal to $\boldsymbol{I}$. The Gaussian noise was chosen to be white with variance $\sigma_{\boldsymbol{n}}^2$. Therefore the covariance matrix $\boldsymbol{R}_{\boldsymbol{nn}}$ is equal to $\sigma_{\boldsymbol{n}}^2\boldsymbol{I}$. The proposed quantized Gaussian Wiener Filter (QGWF), the approximated Wiener Filter (AWF) [7] and the Wiener Filter (WF) [15] were compared. The AWF is the modified Wiener Filter given in [7] and the WF is standard Wiener Filter for a system with infinite resolution used in the quantized system. Which means that here $\boldsymbol{G} = \boldsymbol{R}_{\boldsymbol{xy}}^{-1}\boldsymbol{R}_{\boldsymbol{xx}}$. In Figure 5 it is shown that the proposed method is significantly improving the performance of the system in the high SNR regime. The new receiver outperforms the other receivers in the quantization limited

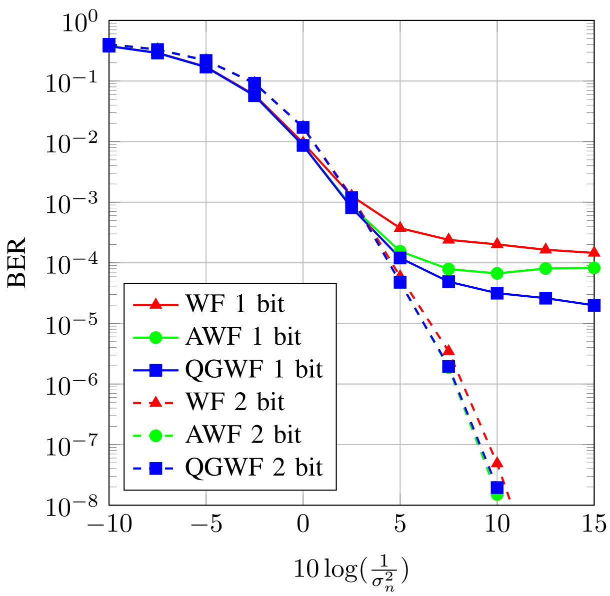

Fig. 5. BER simulation for $M = 2, N = 16$.

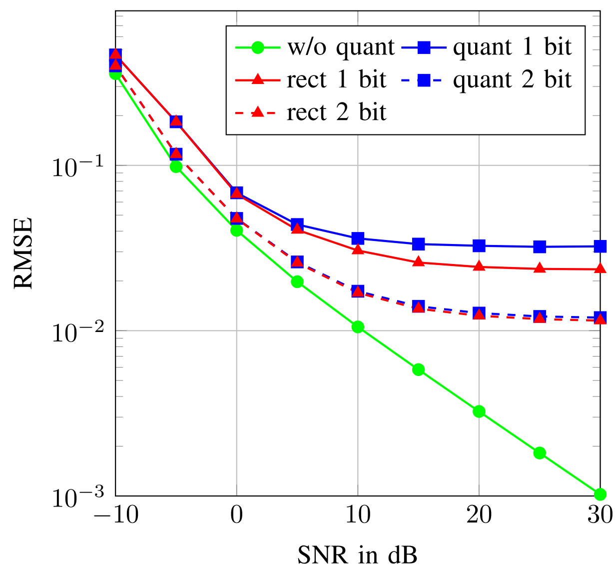

Fig. 7. MSE in estimating θ , bits = 1, M = 2, θ = 10°.

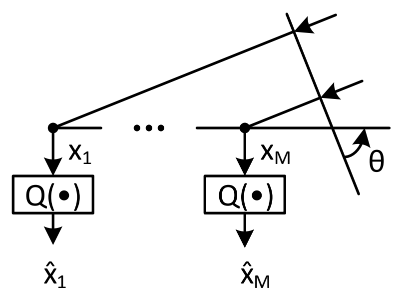

Fig. 6. System model AoA estimation.

regime. If we compare the results for 1-bit (Figure 5) and 2-bit resolution it is obvious, that the performance of the AWF and the QGWF are almost the same in the case of a 2-bit quantizer. Further simulations showed that the performance of the AWF and the QGWF is similar for 2 bit or higher resolution. From this it is concluded that the proposed approach is only capable of improving the performance for a 1-bit quantizer. This relates to the fact that the perturbation of the covariance matrix was by far strongest for a 1-bit quantizer.

C. Blind Angle of Arrival estimation with quantized Signals

A system model for blind Angle of Arrival (AoA) estimation is shown in Figure 6. Here a planar wavefront arrives with an angle of θ at a uniform linear array of antennas. The transmitted signal is called s and is not known by the receiver. The signal and the quantized signal from the antenna i are called x_i and $\hat{x}_i = Q(x_i)$, where $Q(\cdot)$ is the quantization operator. The vector $\boldsymbol{x}$ contains the combined stacked signal from all antennas. Since we assume that the bandwidth of the signal is small compared to the carrier frequency, we can model the time shift between the signals of adjacent antennas as a phase shift ϕ. Subspace based blind AoA estimation algorithms rely on estimating ϕ and therefore θ by first estimating the covariance matrix $\boldsymbol{R_{xx}}$ and then relating it to ϕ. Since we can only observe $\boldsymbol{R_{\hat{x}\hat{x}}}$ which is perturbed by the quantization. We could now use the presented method to relate the covariance matrix of the quantized signal $\boldsymbol{R_{\hat{x}\hat{x}}}$ to the covariance matrix to the input signal before the quantizer $\boldsymbol{R_{xx}}$.

D. Simulation Results of AoA estimation

In the simulation we used the ESPRIT algorithm [16] to estimate the angle θ from a covariance matrix. The transmitted signal s was chosen to be a circular symmetric Gaussian signal. The MSE between θ and the estimated $\hat{\theta}$ is taken as the performance measure. The angle is estimated with the covariance matrices of the signal without quantization (w/o quant) $\boldsymbol{R_{xx}}$, the quantized signal (quant) $\boldsymbol{R_{\hat{x}\hat{x}}}$ and the covariance matrix that was reconstructed with the proposed method (rect). As we see in Figure 7 the performance by using the reconstructed covariance matrix is much better in the case of a 1-bit quantization. A simulation with 2-bit resolution in Figure 7 shows only a moderate improved MSE.

V. Conclusion

The introduced method was able to improve the performance of a correlation based parameter estimators by treating the quantization as an important part of the system. The simulation results for the chosen algorithm showed that it is necessary to consider the perturbation of the correlation due to quantization, especially for very low resolution.

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