On Matching Strategies for Wireless Receivers

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Abstract—Power matching and noise matching are well known established approaches, though they are not the only strategies for designing an optimal receiver. If the best SNR is the design goal, then obviously noise matching is the choice, while for obtaining the highest signal level, power matching has to be chosen. In fact the matching two port between the antenna and a commercial low noise amplifier (LNA) for both strategies turns out to be quite different. While for power matching all the power available from the antenna will be absorbed and amplified by the LNA, for noise matching a possibly considerable part of the available power will be reflected. The SNR at the output of the LNA will be superior in the latter case, but at the expense of a reduced signal level. But an ADC at the end of the analog part of the receiver needs a certain signal level to work properly. Therefore a compromise between the two general purpose strategies, which will be called "Sensitivity matching", may be the optimum approach.

I. INTRODUCTION

In the case of transmit amplifiers, it seems clear that *power matching* is the strategy one should follow in the design of matching networks, for one usually wants to extract all the generator's available power. To achieve efficiencies higher than 50% nowadays voltage matching is used for example in class C high power RF amplifiers.

For the case of a receive amplifier, the situation is more complicated. On the one hand, it seems that signal to noise ratio (SNR) is the all-important figure of merit for the receiver [1]. If this is so, then one should clearly follow the *noise matching* strategy in designing the matching network, rather than power matching. On the other hand, the *absolute level* of the output signal (voltage or current) is also important, for the received and amplified signal has to be further processed by some kind of signal processing. If the signal level is too low, signal processing will not work anymore.

An immediately arising question is, therefore, why not go for power and noise matching at the same time? However, it has been shown long time ago [2], [3], that to design an amplifier which achieves power- and noise matching simultaneously while still providing reasonable gain, low noise figure and remaining stable, is next to impossible. This problem is caused by different mechanisms causing noise in a semiconductor, [4], [5].

If one cannot have noise- and power matching simultaneously, one may need to make a design decision which way to go. The fundamental trade off between information transfer i.e. minimum noise behaviour and power transfer has been ²Institute for Information Technology Munich University of the Federal Armed Forces email: berthold.lankl@unibw.de

investigated in [6] for a simple LNA model. Alternatively, one can ask what would be the optimum matching strategy which may well include noise and power matching as extreme cases. In this paper, we look into these problems.

To elaborate, the analog signal is usually first converted into a digital signal by means of an analog to digital converter (ADC). The ADC requires its input signal to be within a certain range for proper operation. Because signal levels may be changing in time, one usually employs an automatic gain control (AGC) which follows the low-noise amplifier (LNA). The purpose of the AGC is to keep the root mean square (RMS) of its output signal roughly constant by changing the amplification factor. However, the AGC has got a maximum amplification such that proper operation requires a *certain minimum signal level* coming out of the LNA [7]. One therefore has to ensure that, at the *output of the* LNA, both the

- signal to noise ratio, and
- the signal level

stay above some defined thresholds. While the latter's value is determined by the requirement of the ADC and the maximum amplification of the AGC, the minimum signal to noise ratio is determined by factors such as the modulation alphabet and channel code.

II. CHOOSING THE MATCHING STRATEGY

Let us call U_{\min} the minimum RMS voltage that must appear at the output of the LNA. It is more convenient, however, to use the normalized quantity:

$$\alpha = \frac{U_{\min}^2}{4kT\Delta fR_A},\tag{1}$$

which brings the minimum required signal level at the output of the LNA in relation to the strength of the noise signal received by the antenna. Herein, k is the Boltzmann constant, T is the antenna noise temperature, while Δf and R_A are the signal bandwidth and the real-part of the antenna's output impedance, respectively. Therefore, the aforementioned two goals become in mathematical notation:

$$SNR \geq SNR_{min}$$
 (2)

$$\frac{\mathrm{E}\left[|u_{\mathrm{out}}|^{2}\right]}{4\mathrm{k}T\Delta f R_{\mathrm{A}}} \geq \alpha, \tag{3}$$

where ${\rm SNR}_{\rm min}$ and α are given specifications. Per definition,

$$SNR = \frac{SNR_{av}}{NF}$$
(4)

where NF is the *noise figure* of the LNA and SNR_{av} is the *available* SNR at the antenna port:

$$SNR_{av} = \frac{E[|u_0|^2]}{4kT\Delta fR_A},$$
(5)

with u_0 being the complex envelope of the antenna's opencircuit *desired signal* voltage. Substituting (5) into (4), the condition (2) can be rewritten as:

$$\frac{\mathrm{E}\left[|u_0|^2\right]}{4\mathrm{k}T\Delta fR_{\mathrm{A}}} \geq \mathrm{SNR}_{\mathrm{min}} \cdot \mathrm{NF}.$$
 (6)

The complex envelope, u_{out} , of the LNA's output voltage consists of both signal and noise parts. Assuming noise and signal to be uncorrelated, their variances add up:

$$\mathbf{E}\left[|u_{\text{out}}|^2\right] = \mathbf{E}\left[|u_0|^2\right] \cdot |A|^2 + 4\mathbf{k}T\Delta f R_{\text{A}} \cdot \mathbf{NF} \cdot |A|^2, \quad (7)$$

where A is the voltage gain between the output of the LNA and the open-circuit voltage of the antenna. Substituting (7) into (3), the latter can be rewritten as:

$$\frac{\mathrm{E}\left[|u_0|^2\right]}{4\mathrm{k}T\Delta fR_{\mathrm{A}}} \geq \frac{\alpha}{|A|^2} - \mathrm{NF}.$$
(8)

Taking (6) and (8) together into account, it follows that

$$\frac{\mathrm{E}\left[|u_0|^2\right]}{4\mathrm{k}T\Delta fR_{\mathrm{A}}} \geq \max\left(\mathrm{SNR}_{\mathrm{min}}\cdot\mathrm{NF} \; ; \; \frac{\alpha}{|A|^2} - \mathrm{NF}\right) \quad (9)$$

must hold for proper receiver operation. Note that NF and $|A|^2$ depend on the matching strategy:

- NOISE MATCHING: NF is minimized for the prize of reduced gain, $|A|^2$,
- POWER MATCHING: $|A|^2$ is maximized for the prize of increased noise figure, NF.

Which matching strategy should one use then? Answer: the one for which

$$\max\left(\mathrm{SNR}_{\min}\cdot\mathrm{NF}\ ;\ \frac{\alpha}{|A|^2}-\mathrm{NF}\right) \tag{10}$$

is *minimum*. If we do not care for the signal level at all (by setting $\alpha = 0$), we see that noise matching is the winner. Yet, power matching will take the lead when α is increased large enough, i.e., for a large enough minimum signal level specification. The optimum choice between noise- and power matching strategies, therefore, depends on the pair

$$($$
 SNR $_{\min}, \alpha)$,

while the pairs $(NF, |A|^2)$ for noise- and power matching, respectively, depend on the design of the LNA circuit. In the following, we give an illuminating example. Suppose that:

Strategy	noise figure, NF	gain, $ A ^2$
Noise matching	$1.58, (1.99 \mathrm{dB})$	$4.75, (6.77 \mathrm{dB})$
Power matching	$2.51, (4.00 \mathrm{dB})$	$9.65, (9.84 \mathrm{dB})$

With noise matching, the amplifier has a 2dB better noise figure than with power matching. However, this comes at the price of the signal amplification being about 3dB smaller. The



Figure 1: Regions of optimality of noise-matching and power matching as special cases of sensitivity matching. In the area in the middle, sensitivity matching is different from both powerand noise-matching.

optimum choice between noise- and power matching for this case is shown in Fig. 1 for different values of SNR_{min} and α .

<u>EXAMPLE</u>: Suppose the ADC requires an RMS input voltage of 1V. The AGC tries to keep this value constant by adjusting the amplification of the LNA output voltage. The maximum amplification shall be given by 100 dB. The minimum required signal level at the output of the LNA is, therefore, $10 \,\mu$ V. Assuming a half-wavelength dipole antenna with a noise temperature of 300 K and a signal bandwidth of 1 MHz, we obtain a value of $\alpha \approx 83$, or about 19 dB. It then turns out from (10) that noise matching is preferable to power matching if the minimum required SNR is larger than about 8 dB.

III. OPTIMUM MATCHING STRATEGY

In general, neither noise- nor power matching are optimum, though. The optimum matching strategy would be to ensure that both the minimum required SNR and the minimum required signal level at the output of the LNA are met with the smallest possible antenna desired signal voltage. For a given tuple (SNR_{min}, α), the optimum matching network, therefore, is obtained by solving

$$\min_{\text{matching network}} \max\left(\text{SNR}_{\min} \cdot \text{NF} \; ; \; \frac{\alpha}{|A|^2} - \text{NF}\right). \quad (11)$$

As we will show later, both power- and noise matching are solutions of (11) but for different requirements (α , SNR_{min}). This is displayed in Figure 1. Note that NF and $|A|^2$ depend on the matching network and the amplifier circuit. A *joint design* of the amplifier and its matching network therefore looks promising.

A. Modeling

To be able to find the solution for (11) we have to establish NF and |A| as a function of the parameters of the LNA and of the antenna. A circuit model is needed to derive these functions, see Fig. 2.



Figure 2: Circuit model for antenna, lossless reciprocal matching twoport and LNA.

The antenna with $Z_G = R_G + jX_G$ and u_0 together with the matching twoport can be replaced by a transformed source with

$$Z'_G = \frac{X_{12}^2}{R_G + j(X_{11} + X_G)} + jX_{22},$$
 (12)

and

$$u'_{0} = u_{0} \frac{jX_{12}}{R_{G} + j(X_{11} + X_{G})},$$
(13)

as shown in Fig. 3. Analysis leads to

$$\left|\frac{u_{out}}{u_0}\right|^2 = |A|^2 = \mu^2 \frac{R'_G}{R_G} \cdot \left|\frac{Z_L}{Z_L + Z'_G}\right|^2 \tag{14}$$

NF = 1 +
$$\frac{\beta}{4kT\Delta f R'_G} (R_N^2 (1 - |\rho|^2) + |Z'_G - \rho R_N|^2),$$
 (15)

where $\beta = \mathrm{E}[|i_N|^2]$,

$$R_N = \sqrt{\frac{E[|u_N|^2]}{E[|i_N|^2]}}$$
(16)

and

$$\rho = \frac{E[u_N i_N^*]}{\sqrt{E[|u_N|^2]E[|i_N|^2]}}$$
(17)

have to be known. A measurement setup to determine R_N and ρ has been described in [8].

B. Problem Reformulation

First we reformulate (11) using an intermediate variable t as follows:

$$\min t \quad \text{s.t.} \ c \cdot \text{NF} \le t, \ \frac{\alpha}{|A|^2} \le t, \tag{18}$$

where $c = \text{SNR}_{\min} + 1$. To accomodate the constraints the Lagrangian functional

$$L(t, \lambda_1, \lambda_2) = t + \lambda_1 (c \cdot NF - t) + \lambda_2 \left(\frac{\alpha}{|A|^2} - t\right) \quad (19)$$

with $\lambda_1, \lambda_2 \ge 0$ has to be minimized with respect to t and maximized with respect to λ_1 and λ_2 . The KKT conditions lead to

$$\frac{\partial L}{\partial t} = 1 - \lambda_1 - \lambda_2 = 0 \tag{20}$$

$$\frac{\partial L}{\partial \lambda_1} = c \cdot NF - t = 0 \tag{21}$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{\alpha}{|A|^2} - t = 0.$$
(22)

Next we have to compute the derivatives

$$\frac{\partial L}{\partial R'_G} = 0, \ \frac{\partial L}{\partial X'_G} = 0.$$
 (23)

For X'_G and R'_G we get the solution

$$X'_G = \gamma \Im\{\rho\} R_N - (1 - \gamma) X_L \tag{24}$$

$$R'_{G} = \sqrt{\gamma R_{N}^{2} (1 - \Im\{\rho\}^{2}) + (1 - \gamma) R_{L}^{2} + \frac{\gamma (1 - \gamma) (\Im\{\rho\} R_{N} + X_{L})^{2}}{\gamma (1 - \gamma) (\Im\{\rho\} R_{N} + X_{L})^{2}}}, \quad (25)$$

where we combine the to Lagrangian multipliers λ_1 and λ_2 into one parameter γ

$$\gamma = \frac{\frac{\lambda_1 c\beta}{4kT\Delta f}}{\frac{\lambda_1 c\beta}{4kT\Delta f} + \frac{\lambda_2 \alpha R_G}{\mu^2 |Z_L|^2}}.$$
(26)

Now we have three different cases to distinguish:

$$\lambda_{1} = 0, \ \lambda_{2} = 1, \ t = \frac{\alpha}{|A|^{2}} \Rightarrow \text{power matching}$$

$$R'_{G} = R_{L}, \ X'_{G} = -X_{L}, \ \gamma = 0$$

$$\lambda_{1} = 1, \ \lambda_{2} = 0, \ t = c \cdot NF \Rightarrow \text{noise matching}$$

$$R'_{G} = R_{N}\sqrt{1 - \Im\{\rho\}^{2}}, \ X'_{G} = \Im\{\rho\}R_{N}, \ \gamma = 1$$

$$\lambda_{1}, \ \lambda_{2} \neq 0, \ t = cNF = \frac{\alpha}{|A|^{2}} \Rightarrow \text{sensitivity matching}$$

$$0 < \gamma < 1. \quad (27)$$

In the case of sensitivity matching, we get a specific value for $\gamma \in (0,1)$ from setting $cNF = \frac{\alpha}{|A|^2}$, which we plug in (24) and (25) to arrive at a second order equation for γ and therefore an optimum value Z'_G in closed form.



Figure 3: LNA model driven by a signal source. The loading effect has been taken into account by the VCV's gain μ .

A T-circuit, shown in Fig. 4 seems to offer a reasonable topology for an implementation transforming Z_G to Z'_G . For there



Figure 4: Impedance matching network

are two degrees of freedom, we need only two reactive elements. Fig. 5 shows a possible circuit topology. The output



Figure 5: Simple topology

impedance of the matching network terminated by the source should be equal to Z'_G

$$\frac{jX_2R_G - X_2(X_G + X_1)}{R_G + j(X_G + X_1 + X_2)} = R'_G + jX'_G.$$
 (28)

Solving this system leads to:

$$X_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
(29)

with the following variables a, b, c

$$a = R'_G \tag{30}$$

$$b = 2X_1 X_G R'_G \tag{31}$$

$$c = R'_G(R_G^2 + X_G^2) - R_G(R'_G^2 + X'_G^2)$$
(32)

on the other hand we have

$$X_2 = \frac{X'_G R_G + (X_G + X_1) R'_G}{R_G - R'_G}.$$
 (33)

Note that $b^2 \ge 4ac$ is assumed. Otherwise, an alternative topology with $X_1 = 0$ and $X_3 \neq 0$ has to be chosen.

IV. DESIGNING THE SYSTEM

For the realization, the A-band, which is identical to the former VHF band, was chosen. At frequencies around 100 MHz, lumped elements with a quality factor of 100 and even more are available. The matching network should be connected to the amplifier via an SMA connector. Power matching, noise matching and other designs could be plugged in without soldering and changing the amplifier itself. Each matching network can be set up on its own circuitboard.

A. Amplifier design

The amplifier is a two stage common emitter circuit. It is a broadband amplifier, optimized for linearity, designed to work from 20 MHz to 1.5 GHz, which uses the low noise transistor BFT66 in its input stage. Figure 6 shows the schematic of its first stage. Here, a 15Ω emitter resistor and between base and collector 680Ω and 1nF are used for feedback. This causes lower gain, a higher noise figure but improves linearity and bandwidth very much. For bias a 15V source is used, a 475Ω resistor limits the collector current to 13 mA and a 100nFceramic capacitor blocks the RF.

If the collector current is increased to achieve better linearity the noise figure will rise, too [9]. If the base emitter diode current increases, R_N will move to lower values and more shot noise will be produced. Usually input stages of measurement



Figure 6: Amplifier 1st stage

equipment like spectrum analyzers are optimized in a similar way. This kind of amplifier fits best to show the trade off. To keep its IP3 high the 2.7 W RF power transistor BFQ34 is used, collector to base and emitter feedback are also implemented. In the 2nd stage, due to the higher signal power a much stronger feedback is implemented. By its biasing network, the collector current is adjusted to 50 mA.



Figure 7: Amplifier 2nd satge

B. Setting the optimization goal

Having designed a two stage LNA let us now set the required values both for SNR and the RMS voltage at the output of the amplifier. The necessary mimimum SNR is determined by the modulation format and the code, while the minimum amplifier output voltage $U_{\rm min}$ is determined by the required signal level at the ADC and the available AGC in between. As an example, we have chosen an SNR_{min} of 10 dB and a $U_{\rm min}$ of 7.16 $\mu V_{\rm RMS}$. All the parameters we need to compute α according to (1) are summarized in Table I.

Parameter	Value
Т	290 K
k	$1.38 \cdot 10^{-23} \frac{\text{VAs}}{\text{K}}$
Δf	100 kHz
$Z_G = R_A$	$50 \ \Omega$
U_{\min}	$7.16 \ \mu V_{ m RMS}$
α	640
SNR_{\min}	10 dB

Table I: Source parameters

This finally leads to $\alpha = 640$. Therefore our optimization goal is to provide a source to the LNA such that, both the SNR_{min} of 10 dB and the necessary $\alpha = 640$ can simultaneously be achieved with the lowest possible voltage u_0 .

V. REALIZATION

In order to verify the proposed matching strategy, a real amplifier is built in a EMI shielded box. To measure the parameters a spectrum analyzer, a networkanalyzer and two ultra low noise power supplies are used. Between our amplifier and the spectrum analyzer a extreme low noise preamplifier is used. The measurement results are summarized in Table II.

Parameter	Value
μ	6.3
β	$2.6 * 10^{-22} A^2$
R_N	$20 \ \Omega$
Z_L	$153 - j26 \Omega$
ρ	-0.9 - j0.127

Table II: Amplifier parameter

From the parameters given in Table II we can design a matching twoport for the proposed sensitivity matching and compare the results with the well established power and noise matching:

$$Z'_{G} = Z^{*}_{L} = (153 + j26)\Omega : \text{ power matching } \gamma = 0$$

$$Z'_{G} = R_{N}(\sqrt{1 - \Im\{\rho\}^{2}} + j\Im\{\rho\}) = (19.84 - j2.54)\Omega :$$

noise matching $\gamma = 1$

$$Z'_{G} = (50.1 + j6 \cdot 10^{-5})\Omega : \text{ sensitivity matching } \gamma = 0.911.$$
(34)

The matching twoports for power and noise matching are shown in Fig. 8 and 9, while for sensitivity matching it turns out that the matching network collapses to just a through connection. This is a very desirable situation, where no reactive components are needed $(X_1 \rightarrow 0\Omega, X_2 \rightarrow \infty \Omega)$, no losses associated with these reactances degrade the performance and no bandwith limitation is introduced by the matching network.

A. Matching strategies

The amplifier has an input impedance of $Z_L = (153 - j26)\Omega$. For power matching Z_L is transformed to $Z_G = 50\Omega$ as Fig. 8 shows. For noise matching the amplifier input must



Figure 8: Power matching

be connected to an impedance of $Z'_G = (19.84 - j2.54)\Omega$ as shown in Fig. 9.



Figure 9: Noise matching

According to our amplifier design, sensitivity matching now is quite simple: There is no matching twoport necessary, the antenna is directly connected to the LNA.

VI. MEASUREMENT

The well established Y factor method will be used to verify our results. For this method of measurement an uncorrelated calibrated diode noise source with an ENR (Excess Noise Ratio) of 6.76 dB is connected via each matching network to the input of a spectrum analyzer via a further auxiliary low noise amplifier. When its power supply is turned off, the source produces noise like a 50 Ω resistor at $T_c = 290K$. The reverse biased source diode will produce avalanche noise, with the power supply turned on. In this case the noise level at 100 MHz is equal to a 50 Ω resistor at $T_h = 1375K$. With the reference temperature $T_0 = 290K$ ENR can be defined as:

$$ENR = \frac{T_h - T_c}{T_0}.$$
(35)

The amplifier and the noise source are fed with a low noise power supply. On the circuitboards of noise source and amplifier, there are additional low pass filters and each housing is shielded to prevent electromagnetic disturbance. This also called hot cold method is done for each kind of matching, the value N_h (hot noise) at the amplifier's output port appears with

Matching	Hot	Cold
Power	-144.16 dBm/Hz	-148.03 dBm/Hz
Sensitivity	-144.54 dBm/Hz	-149.10 dBm/Hz
Noise	-144.70 dBm/Hz	-149.61 dBm/Hz

Table III: Measurement results

the noise source supply turned on, N_c (cold noise) with the supply turned off. Y is the delta between hot and cold output power value of each matching strategy of Table III. According to Friis [10], gain G here is defined by the ratio of output and input power of an amplifier. If matched, the source will only provide half of its open voltage:

$$G = \frac{|A|^2}{4} \tag{36}$$

$$Y = \frac{N_X + kT_h \Delta fG}{N_X + kT_c \Delta fG} = \frac{N_h}{N_c}$$
(37)

With the Y factor and the ENR the amplifier's additional noise is:

$$N_X = kT_c \Delta f G \left(\frac{ENR}{Y-1} - 1\right). \tag{38}$$

According to its definition the noise figure is then described by:

$$NF = \frac{kT_c\Delta fG + N_X}{kT_cBG} = \left(\frac{ENR}{Y - 1}\right)$$
(39)

The noise figure can be calculated, Table IV: The networkan-

Measurement
5.18 dB
4.07 dB
3.54 dB

Table IV: Noise figure

alyzer helps us getting the gain in a two port measurement. When noise matching an amplifier, a not negligible amount of

Matching	Measurement
Power	21.0 dB
Sensitivity	19.6 dB
Noise	17.2 dB
Table V: Gain	

signal energy is reflected form the amplifiers input, this causes a smaller gain, Table V. Power matching is also not an optimal strategy, here no energy is reflected in an ideal case but the related input noise floor is much higher, Table VI. Finally, we

Matching	Measurement
Power	-168.8 dBm/Hz
Sensitivity	-169.9 dBm/Hz
Noise	-170.4 dBm/Hz

Table VI: Noise floor

calculate the input sensitivity out of these results and see, that sensitivity matching provides us with the best sensitivity, see Table VII, i.e. we achieve our required SNR and signal level at the LNA output with a lower source voltage.

Matching	Measurement
Power	-158.8 dBm/Hz
Sensitivity	-160 dBm/Hz
Noise	-157.6 dBm/Hz

Table VII: Sensitivity

VII. CONCLUSION

A new strategy of matching was presented. To this end, we first developed a mathematical model. In a second step, we designed an amplifier and implemented the system first on MATLAB and spice to simulate it. By building up the real circuit and measuring the parasitics with a network analyzer and a calibrated noise source, the simulation model was improved. Finally power matching, noise matching and sensitivity matching have been shown. The theoretical results have been compared with data obtained from measurement results and found to agree well. The new kind of matching is an efficient method to increase sensitivity of a receiving system. The design strategy was based on a given LNA and a given source. An interesting direction for research is to design the LNA for a given source such that the matching twoport becomes as simple as possible. Another imprtant direction is to expand the methodology for matching multiports for multi antenna systems.

References

- C. Findeklee, "Array noise matching; generalization, proof and analogy to power matching," *Antennas and Propagation, IEEE Transactions on*, vol. 59, no. 2, pp. 452–459, 2011.
- [2] A. van der Ziel and J. W. Ero, "Small-signal, high-frequency theory of field-effect transistors," *Electron Devices, IEEE Transactions on*, vol. 11, no. 4, pp. 128–135, 1964.
- [3] W. Bruncke and A. van der Ziel, "Thermal noise in junction-gate fieldeffect transistors," *Electron Devices, IEEE Transactions on*, vol. 13, no. 3, pp. 323–329, 1966.
- [4] M. B. Das, "FET noise sources and their effects on amplifier performance at low frequencies," *IEEE Transactions on Electron Devices*, vol. 19, no. 3, pp. 338–348, Mar. 1972.
- [5] A. Podell, "A functional GaAs FET noise model," *Electron Devices*, *IEEE Transactions on*, vol. 28, no. 5, pp. 511–517, 1981.
 [6] F. Steiner, A. Mezghani, and J. Nossek, "Information theoretic analysis
- [6] F. Steiner, A. Mezghani, and J. Nossek, "Information theoretic analysis of concurrent information transfer and power gain," in *Circuits and Systems (ISCAS), 2012 IEEE International Symposium on*, May 2012, pp. 548–551.
- [7] M. Ivrlac and J. Nossek, "Toward a circuit theory of communication," *Circuits and Systems I: Regular Papers, IEEE Transactions on*, vol. 57, no. 7, pp. 1663–1683, 2010.
- [8] M. Ivrlac, B. Lehmeyer, J. Nossek, C. Hofmann, and B. Lankl, "Estimation of noise parameters in multi- antenna receivers using digitized samples," in WSA 2013 - 17th International ITG Workshop on Smart Antennas, 2013.
- [9] H. Nyquist, "Thermal Agitation of Electric Charge in Conductor," *Physics Review*, vol. 32, pp. 110–113, 1928.
- [10] H. T. Friis, "Noise figure of radio receivers," *Proc. IRE*, vol. 32, pp. 419–422, jul 1944.