Pilot Coordination for Large-Scale Multi-Cell TDD Systems

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***Abstract*—Pilot contamination limits the performance of a multi-cell time division duplex system with a large number of base station antennas. We study the potential benefits of coordination during the training phase and we propose efficient algorithms for practical systems. Our derivations are based on results from asymptotic analysis and the practical relevance is demonstrated by simulations with realistic system parameters.**

I. Introduction

Recently, there has been an increasing interest in cellular networks with a large number of base station antennas. This so called massive MIMO concept promises high gains with very simple signal processing methods [1], [2].

The high number of antennas makes channel estimation and feedback very costly in a frequency division duplex (FDD) system. Thus, most works on this topic assume time division duplex (TDD) systems, where the estimation of the channel takes place in an uplink training phase [1], [3]–[5]. That is, the resources spent on pilots depend on the number of served users, but not on the number of antennas at the base station.

For the very high antenna gains in these systems, the performance is severely degraded by channel estimation errors due to inter-cell interference in the training phase, so called pilot contamination. It can be shown that this interference ultimately limits the performance for uncoordinated base stations with a very high number of antennas and with favorable propagation conditions, i.e., independently distributed channel coefficients for each antenna [1], [3], [6]–[9]. A few methods have been proposed to tackle the contamination issue in the uncoordinated case [4], [9]–[11].

In this work, we study the coordination of pilots in the uplink training phase. Previous work on this subject has been done in [12], where a straightforward greedy algorithm is proposed, based on the channel estimation error as performance metric. This approach is based on a specific spatial channel model that leads to low rank channel covariance matrices. In contrast to this work, we use results from the asymptotic analyses in [3] and [5] to formulate a combinatorial network utility maximization (NUM) problem with respect to the coordination strategy. Thus, our approach can handle arbitrary covariance matrices and we can show an improved performance even when the covariance matrices are scaled identities. We analyze possible benefits from pilot coordination by an optimal algorithm based on exhaustive enumeration and provide efficient algorithms for training coordination in practical systems.

II. System Model

We consider a cellular network with L base stations, where each base station has M transmit antennas and serves K single antenna users. The number of base station antennas is significantly larger than the number of simultaneously served users per base station, i.e., $K \ll M$. We further assume that the communication system is in TDD mode and that channel reciprocity holds.

We consider a block fading channel model. Let $\boldsymbol{h}_{ijk} \in \mathbb{C}^M$ denote the vector of complex channel gains from user k in cell j to all antennas of base station i in one coherence block. These vectors are pairwise statistically independent and each vector channel is Gaussian distributed with zero mean and covariance matrix $\boldsymbol{R}_{ijk} \in \mathbb{C}^{M\times M}$. For ease of notation, we collect the channel vectors of all K users in cell j to the base station i as columns of the matrix $\boldsymbol{H}_{ij}$.

Let ρ_{tr} denote the effective training SNR and T_{tr} the number of available pilot symbols, i.e., the available number of orthogonal pilot sequences. Under the assumptions that the training takes place simultaneously in all cells and the reception is synchronized, the received training signals at base station i are given by

$$\boldsymbol{W}_i = \sqrt{\rho_{\text{tr}}} \sum_{j=1}^{L} \boldsymbol{H}_{ij}\boldsymbol{D}_j + \boldsymbol{N}_i \quad \in \mathbb{C}^{M\times T_{\text{tr}}} \tag{1}$$

where the orthonormal rows of $\boldsymbol{D}_j \in \mathbb{C}^{K\times T_{\text{tr}}}$ contain the pilot sequences for all K users in cell j and the entries of $\boldsymbol{N}_i$ are assumed to be i.i.d. complex Gaussian distributed with zero mean and unit variance.

III. Channel Estimation

If we reuse the same pilot sequences in all cells, i.e., $\boldsymbol{D}_j = \bar{\boldsymbol{D}}\ \forall j$, and correlate the received training signals with the pilots, we obtain the estimate due to $\bar{\boldsymbol{D}}^{\mathsf{H}}\bar{\boldsymbol{D}} = \mathbf{I}$,

$$\boldsymbol{Y}_i = \boldsymbol{W}_i \frac{1}{\sqrt{\rho_{\text{tr}}}} \bar{\boldsymbol{D}}^{\mathsf{H}} = \sum_{j=1}^{L} \boldsymbol{H}_{ij} + \frac{1}{\sqrt{\rho_{\text{tr}}}} \tilde{\boldsymbol{N}}_i \tag{2}$$

at base station i, that coincides with the least squares (LS) estimate of the channels $\boldsymbol{H}_{ii}$, since the noise at the base station antennas is white. Because of the orthonormal rows in $\bar{\boldsymbol{D}}$, the transformed noise matrix $\tilde{\boldsymbol{N}}_i = \boldsymbol{N}\bar{\boldsymbol{D}}^{\mathsf{H}}$ still has i.i.d entries with zero mean and unit variance.

We note that, even if we reuse the same pilot sequences in each cell, the assignment of the pilots to the users influences the channel estimation. The assignment can be modeled by $\boldsymbol{D}_j = \boldsymbol{P}_j \boldsymbol{D}$, where $\boldsymbol{D} \in \mathbb{C}^{T_{\text{tr}} \times T_{\text{tr}}}$ is a unitary matrix containing the pool of orthonormal pilot sequences and $\boldsymbol{P}_j \in \{0,1\}^{K \times T_{\text{tr}}}$ is a matrix which describes the assignment of pilots to the users in cell j with $\boldsymbol{P}_j \boldsymbol{P}_j^{\mathsf{T}} = \mathbf{I}$.

The modified expression for the LS estimates is given by

$$\boldsymbol{Y}_i = \boldsymbol{W}_i \frac{1}{\sqrt{\rho_{tr}}} \boldsymbol{D}^{\mathsf{H}} \boldsymbol{P}_i^{\mathsf{T}} = \boldsymbol{H}_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{L} \boldsymbol{H}_{ij} \boldsymbol{P}_j \boldsymbol{P}_i^{\mathsf{T}} + \frac{1}{\sqrt{\rho_{\text{tr}}}} \tilde{\boldsymbol{N}}_i. \tag{3}$$

Now the channel estimates still depend on the assignment matrices $\boldsymbol{P}_j$, even if we have $T_{\text{tr}} = K$, i.e., we reuse the same T_{tr} pilot sequences in every cell.

Also note that we usually have $\boldsymbol{D} = \mathbf{I}$ in many practical systems and the pilot sequences represent the different orthogonal time-frequency blocks used for training.

For a more convenient notation of the following expressions, we let $\mathcal{K}_l$ denote the set of all users (i,k), i.e., user k in cell i, which are assigned to pilot l and let $\mu(i,k)$ denote the pilot assigned to user (i,k).

If the second order information $\boldsymbol{R}_{ijk}$ of the channel vectors is available, the channel estimation can be improved by using the minimum mean squared error (MMSE) estimator. The MMSE estimates are given by (e.g., [5])

$$\hat{\boldsymbol{h}}_{iik} = \boldsymbol{R}_{iik} \left(\frac{1}{\rho_{\text{tr}}} \mathbf{I} + \sum_{(j,m) \in \mathcal{K}_{\mu(i,k)}} \boldsymbol{R}_{ijm} \right)^{-1} \boldsymbol{y}_{ik} \tag{4}$$

where $\boldsymbol{y}_{ik}$ denotes the kth column of $\boldsymbol{Y}_i$ from (3). Following the notation introduced above, the set $\mathcal{P}_{\mu(i,k)}$ contains all users in the system that are assigned the same pilot as user k in cell i. The estimates are distributed as $\hat{\boldsymbol{h}}_{iik} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Phi}_{ik,ik})$ where

$$\boldsymbol{\Phi}_{ik,jm} = \boldsymbol{R}_{iik} \left(\frac{1}{\rho_{\text{tr}}} \mathbf{I} + \sum_{(c,n) \in \mathcal{K}_{\mu(i,k)}} \boldsymbol{R}_{icn} \right)^{-1} \boldsymbol{R}_{ijm}. \tag{5}$$

analogous to the definition in [5].

Using the MMSE estimate can improve performance significantly for certain assumptions on the channel covariances (cf. [12]) at the cost of additional computational complexity for the estimation process.

IV. Large System Results

In general, rate expressions for systems with imperfect channel state information are difficult to handle. Thus, we base our algorithms on large system results which lead to simple analytical expressions. These large system rates are accurate for a sufficiently large number of base station antennas.

A. Reverse Link

In the reverse link, or uplink, we use the results from [5] for $M, K \to \infty$ and $M/K = \alpha$ adapted for arbitrary pilot assignment. For MMSE channel estimation and a simple matched filter, the asymptotic signal to interference and noise ratios (SINRs) are given in (8) at the top of the next page.

If we further assume $M \gg K$, i.e., $\alpha \to 0$, this can be simplified to

$$\gamma_{ik}^{\text{ul}} = \frac{\beta_{ik,ik}^2}{\sum_{\substack{(j,m) \in \mathcal{K}_{\mu(i,k)} \\ (j,m) \neq (i,k)}} |\beta_{ik,jm}|^2} \tag{6}$$

where $\beta_{ik,jm} = \lim_{M \to \infty} \frac{1}{M} \operatorname{tr}(\boldsymbol{\Phi}_{ik,jm})$.

We note that the interference and thus the achievable SINRs can be influenced by the pilot assignments $\mu(i,k)$.

B. Forward Link

The matched filter expression for the forward link in [5] is given for a joint normalization of the transmit power for all users. We alter this expression by considering a per stream normalization with potential power allocation to the streams instead of the joint normalization in [5].

The resulting asymptotic expression is given in (9) where ρ_{ik} denotes the fraction of the total downlink power at base station i allocated to user k in the same cell. The user m in the last expression in the denominator is the user for which $\mu(i,k) = \mu(j,m)$, i.e., the user in cell j which is assigned to the same pilot as user (i,k). If there is no such user in cell j, there is no interference from base station j to user (i,k) due to pilot contamination.

With $M \gg K$ the expression simplifies to

$$\gamma_{ik}^{\text{dl}} = \frac{\rho_{ik} \beta_{ik,ik}}{\sum_{\substack{j \neq i \\ m: \mu(i,k) = \mu(j,m)}} \rho_{jm} |\beta_{jm,ik}|^2 / \beta_{jm,jm}}. \tag{7}$$

V. Assignment Problem

In the following, we will formulate a NUM problem based on the asymptotic SINR expressions presented in the previous section. Combined optimization with respect to both, the pilot assignment and the downlink power allocation is out of the scope of this paper and we assume equal power allocation to all users, i.e., $\rho_{ik} = \rho_{\text{dl}}/K \; \forall i, k$.

The general NUM problem formulated with respect to the assignment matrices $\boldsymbol{P}_i$ is given by

$$\max_{\boldsymbol{P}_1, \ldots, \boldsymbol{P}_L \in \{0,1\}^{K \times T_{\text{tr}}}} U(\boldsymbol{r}_{\text{ul}}, \boldsymbol{r}_{\text{dl}}) \quad \text{s.t.} \quad \boldsymbol{P}_i \boldsymbol{P}_i^{\mathsf{T}} = \mathbf{I} \; \forall i \tag{10}$$

where U denotes the network utility function and $\boldsymbol{r}_{\text{ul/dl}} \in \mathbb{R}^{KL}$ is a vector of the achievable rates $r_{ik}^{\text{ul/dl}} = \log_2(1 + \gamma_{ik}^{\text{ul/dl}})$ of all users in the system.

The NUM problem is a combinatorial optimization problem and as such hard to solve optimally. Note that the problem can be relaxed by removing the integer constraint on the assignment matrices. This is equivalent to directly optimizing with respect to the pilot sequences $\boldsymbol{D}_j$. However, the resulting problem is non-convex and additionally there are several practical difficulties with allowing arbitrary orthogonal pilot sequences, such as feeding back the pilot sequences to the users. For these reasons, we focus on efficient suboptimal solutions to the combinatorial problem formulated in (10).

$$\gamma_{ik}^{\text{ul}} = \frac{\left(\frac{1}{M}\,\mathsf{tr}(\boldsymbol{\Phi}_{ik,ik})\right)^2}{\frac{1}{\rho_{\text{ul}}M}\frac{1}{M}\,\mathsf{tr}(\boldsymbol{\Phi}_{ik,ik}) + \frac{1}{M}\sum_{j,m}\frac{1}{M}\,\mathsf{tr}(\boldsymbol{R}_{ijm}\boldsymbol{\Phi}_{ik,ik}) + \sum_{\substack{(j,m)\in\mathcal{K}_{\mu(i,k)}\\(j,m)\neq(i,k)}}|\frac{1}{M}\,\mathsf{tr}(\boldsymbol{\Phi}_{ik,jm})|^2}. \tag{8}$$

$$\gamma_{ik}^{\text{dl}} = \frac{\rho_{ik}\frac{1}{M}\,\mathsf{tr}(\boldsymbol{\Phi}_{ik,ik})}{\frac{1}{\rho_{\text{dl}}M} + \frac{1}{M}\sum_{j,m}\rho_{jm}\,\mathsf{tr}(\boldsymbol{R}_{jik}\boldsymbol{\Phi}_{jm,jm})/\,\mathsf{tr}(\boldsymbol{\Phi}_{jm,jm}) + \sum_{\substack{j\neq i\\ m:\mu(i,k)=\mu(j,m)}}\rho_{jm}\frac{1}{M}|\mathsf{tr}(\boldsymbol{\Phi}_{jm,ik})|^2/\,\mathsf{tr}(\boldsymbol{\Phi}_{jm,jm})} \tag{9}$$

VI. Algorithms

A. Exhaustive Enumeration

To get an idea of the potential benefits of coordination, we solve the NUM problem in (10) optimally by exhaustive enumeration of all possible pilot assignments. For one cell, the number of possible assignments is

$$\prod_{k=0}^{K-1}(T_{\text{tr}} - k) = \frac{T_{\text{tr}}!}{(T_{\text{tr}} - K)!}. \tag{11}$$

Note that we can fix the assignment of one cell without affecting the performance. The total number of possible assignments is thus

$$\left(\frac{T_{\text{tr}}!}{(T_{\text{tr}} - K)!}\right)^{L-1}. \tag{12}$$

For larger systems, the enumeration of all possible assignments quickly becomes computationally intractable. Thus, we need efficient algorithms to manage the training coordination.

B. Degradation Based Greedy Assignment

The first greedy algorithm we introduce is based on a degradation measure as proposed in [13]. At each iteration of the algorithm, we have a set of users which are already assigned to pilots and a set of free users which still have to be assigned. Initially, the users in one cell are assigned randomly, while all other users are free.

The first step in each iteration is to calculate the utilities that result from adding each of the free users to the set of assigned users for each possible pilot. Then for each user calculate the degradation, i.e., amount of utility that is lost, when the user only gets the second best pilot.

The user which has the highest degradation, i.e., the user which is most sensitive to the current assignment, is then assigned to its best pilot.

To calculate the utilities for the assigned users the utility function has to be separable, i.e.,

$$U(\boldsymbol{r}) = \sum_{(i,k)} U_{ik}(r_{ik}). \tag{13}$$

Each of the assigned users is in one of the sets $\mathcal{K}_1, \ldots, \mathcal{K}_{T_{\text{tr}}}$ and the partial utility is given by

$$\tilde{U}(\mathcal{K}_1, \ldots, \mathcal{K}_{T_{\text{tr}}}) = \sum_{(i,k)\in\bigcup_p \mathcal{K}_p} U_{ik}(r_{ik}) \tag{14}$$

where the rates r_{ik} are calculated using the assignments $\mathcal{K}_p$.

Formally we have the following steps. Let $\mathcal{F}$ denote the set of unassigned users. For each unassigned user $(i,k) \in \mathcal{F}$ Calculate the optimal pilot

$$p_{ik}^{\star} = \underset{p\in\mathcal{P}_i}{\mathsf{arg\,max}}\,\tilde{U}(\mathcal{K}_1, \ldots, \mathcal{K}_p \cup \{(i,k)\}, \ldots, \mathcal{K}_{T_{\text{tr}}}) \tag{15}$$

and degradation measure

$$\begin{aligned} d_{ik} &= \tilde{U}(\mathcal{K}_1, \ldots, \mathcal{K}_{p_{ik}^{\star}} \cup \{(i,k)\}, \ldots, \mathcal{K}_{T_{\text{tr}}}) \\ &\quad - \underset{p\in\mathcal{P}_i, p\neq p_{ik}^{\star}}{\mathsf{arg\,max}}\ \tilde{U}(\mathcal{K}_1, \ldots, \mathcal{K}_p \cup \{(i,k)\}, \ldots, \mathcal{K}_{T_{\text{tr}}}) \end{aligned} \tag{16}$$

where $\mathcal{P}_i$ denotes the set of still available pilots in cell i.

The selected user is then given by

$$(i^{\star}, k^{\star}) = \underset{(i,k)\in\mathcal{F}}{\mathsf{arg\,max}}\, d_{ik} \tag{17}$$

and is assigned to its optimal pilot

$$\mathcal{K}_{p_{i^{\star},k^{\star}}^{\star}} \leftarrow \mathcal{K}_{p_{i^{\star},k^{\star}}^{\star}} \cup \{(i^{\star}, k^{\star})\} \tag{18}$$

$$\mathcal{P}_{i^{\star}} \leftarrow \mathcal{P}_{i^{\star}} \backslash \{p_{i^{\star},k^{\star}}^{\star}\} \tag{19}$$

$$\mathcal{F} \leftarrow \mathcal{F} \backslash (i^{\star}, k^{\star}). \tag{20}$$

These steps are repeated until all users are assigned, i.e., $\mathcal{F} = \emptyset$.

C. Variance Based Greedy Assignment

The degradation based greedy algorithm still needs a lot of SINR evaluations for each assignment. To further reduce complexity, we propose another greedy algorithm, where we use a heuristic to select the most sensitive user in a given iteration. Namely, we select the unassigned user with the worst average channel condition since this user is most likely to be affected by inter-cell interference. Thus, we avoid the costly selection process of the degradation based algorithm and only have to search for the optimal pilot for the selected user.

D. Position Based Assignment

Another possible coordination strategy is based on the observation that, with a simple geometric path-loss model, weak users generate a large amount of interference in neighboring cells, while strong users generate a small amount of interference. This motivates a coordination strategy which is only based on the positions of the users and that can be applied in each cell separately.

Let us first assume we have $T_p = K$, a one dimensional Wyner network where the cells are sequentially indexed with $i = 1, \ldots, L$ and a sufficiently large number of uniformly distributed users. The covariance matrices are scaled identities, where the scaling factor is calculated with a simple distance based path-loss model

$$\sigma_{ijk}^2 = \sigma_0^2 d_{ijk}^{-\alpha} \tag{21}$$

where d_{ijk} denotes the distance of user k in cell j to base station i and α is the pathloss factor and σ_0^2 is a constant normalization factor.

Now we sort the users in each cell based on their distance to the serving base station. In the odd cells, we assign pilot 1 to the strongest user down to pilot K for the weakest one. In the even cells the assignment is done the other way round.

If we consider a cell edge user, the corresponding interfering users in the two neighboring cells are approximately in the cell center, i.e., the interfering users have twice the distance to the serving base station compared to the cell edge user.

Let σ_u^2 denote the scaling factor of the user being served, and σ_i^2 the scaling factor of the interfering users. Using (6) we get the following asymptotic uplink SIR

$$\gamma_u^{\mathrm{ul}} = \frac{\frac{\sigma_u^4}{1/\rho_{\mathrm{tr}}+2\sigma_i^2}}{2\frac{\sigma_u^2\sigma_i^2}{1/\rho_{\mathrm{tr}}+2\sigma_i^2}} = \frac{\sigma_u^2}{2\sigma_i^2} = 2^{(\alpha-1)} \tag{22}$$

where we use the above approximation that $d_i = 2d_u$. In the uncoordinated case, the worst case SINR is approximately $1/2$ if the two interfering users of the neighboring cells are close to the cell under consideration.

In a regular two dimensional network with hexagonal cells, the approach has to be modified in a way that results in three different compatible assignment strategies instead of two. Then, we can ensure that there are no two neighboring cells which use the same assignment strategy. The algorithm is described in Algorithm 1. The basic idea is to divide the cells into G disjoint sets where none of the cells in one set are neighbors. Now, if the cells in one set put a weak user on a certain pilot, the cells in the other groups should put a strong user on the same pilot. That is, for every weak user we need $G-1$ strong users.

In the end, we sort the users again by channel quality and assign to the pilots sequentially. If the pilot index is a multiple of G plus the group index of the cell, we assign the weakest unassigned user to this pilot. Otherwise, the strongest unassigned user is used.

If the number of available pilots is actually larger than the number of users per cell, we can further reduce the interference for the weakest users by leaving the corresponding pilots unassigned in the other cells.

Algorithm 1 Positioned based assignment for cell i

```
Require: σ_{ii,k} ≤ σ_{ii,k+1} where σ_{ii,k} = E[h^H_{iik} h_{iik}]
Require: number of groups: G
Require: group of cell i: g_i
Require: number of users per cell: K
Require: number of pilots: T_tr
  w ← 1                                   Weakest unassigned user
  s ← K                                   Strongest unassigned user
  o ← T_tr − K
  for p = 1 : T_tr do
    if (p − 1) mod G = g_i − 1 then
      μ(i, w) ← p
      w ← w + 1
    else if o > 0 then
      o ← o − 1
    else
      μ(i, s) ← p
      s ← s − 1
    end if
    if w > s then
      break
    end if
  end for
```

VII. Results

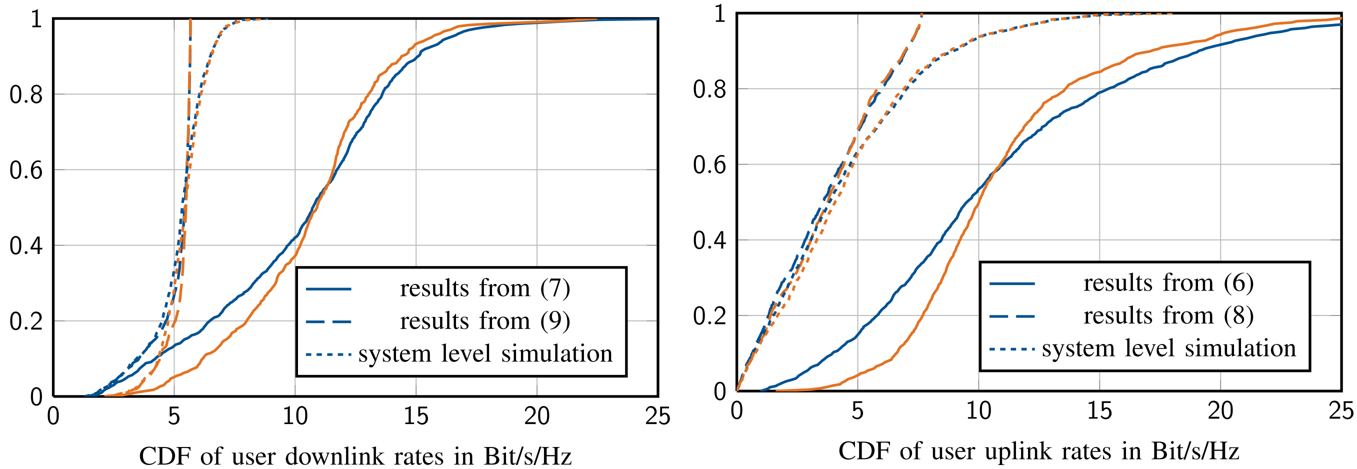
To demonstrate the potential benefit of coordinated training, we first consider a cellular system with $L = 3$ cells, where each cell serves $K = T_{\mathrm{tr}} = 5$ users. The number of antennas is set to $M = 200$, the cell radius is $r = 1.6\,\mathrm{km}$. The users are uniformly distributed within the cells and the channel covariance matrices are scaled identities. The scaling factor is based on a geometric fading coefficient which is calculated with the simple distance based path-loss model in (21) with a path-loss factor of $\alpha = 3.8$. For the simulations, we consider the uplink and downlink rates separately.

In Fig. 1, we see the empirical cumulative distribution functions (CDF) of the users' rates, for the uncoordinated case (blue) and for the optimal coordination based on the proportional fair utility (orange). In general, the coordination increases the performance of weak users. We can see that the asymptotic expressions from (8) and (9), which incorporate the intra-cell interference, are very accurate for the weak users and slightly pessimistic for the strongest users, while the expressions for $M \gg K$ are way off for the uplink case.

This is due to the large ratios of the geometric fading coefficients between cell center and cell edge users, that can be as high as 40dB in the considered scenario. To cancel out intra-cell interference for cell edge users with a simple matched filter, the number of antennas has to exceed the ratio between the strongest interferer and the considered cell edge user. That is, we need tens of thousands of antennas to reach the asymptotic results of (6), which is unrealistic. This means that pilot contamination is not actually the limiting factor in this case.

However, if we reduce the intra-cell interference, e.g., by not serving users with highly different channel gains simultaneously, by adding power allocation on the user side or by using a more sophisticated filter, pilot contamination beomes a significant factor again. To stay focused on the problem of pilot coordination, we consider only the downlink in the following.

In the downlink, we do not have the same problem, i.e., high intra-cell interference for weak users, because the spatial streams of all users of a base station are transmitted with equal power following our assumptions. However, the asymptotic



(a) Downlink

(b) Uplink

Fig. 1. CDF of user rates: coordinated (orange) vs. uncoordinated (blue)

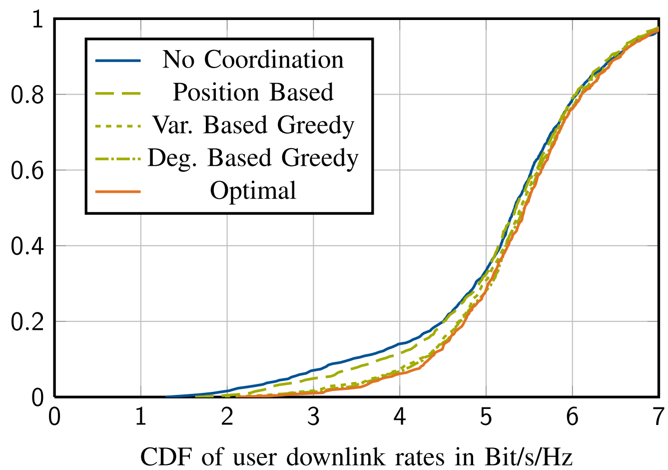


Fig. 2. User rates for $L = 3, K = 5$

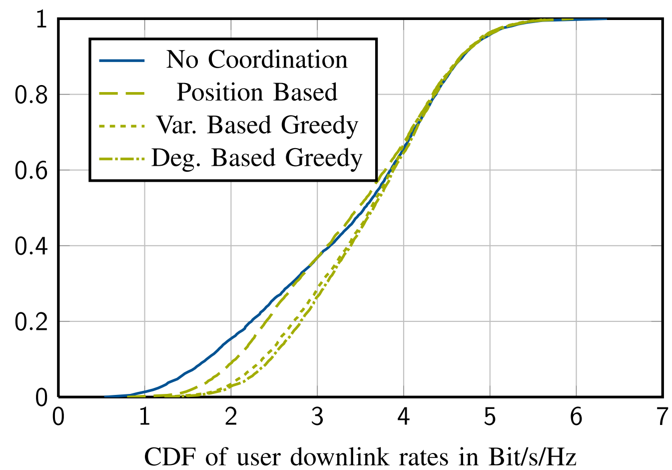


Fig. 3. User rates for $L = 21, K = 10$

results for $M \gg K$ are still too optimistic for stronger users. There is only a small gain in the average rate for the coordinated case (around 4%), but we see that especially the weak users benefit a lot from the coordination. For the results shown in Fig. 1, we have a gain of around 40% for the fifth percentile.

In Fig. 2, the simulation results are depicted for all pilot coordination algorithms in the scenario considered above. The performance of the greedy algorithms is close to optimal while the position based method shows smaller gains.

In the following, we compare the proposed efficient algorithms for a larger system with 21 cells in a wrap around configuration. The other system parameters stay the same. The greedy algorithms use the asymptotic expression in (9) and the proportional fair utility.

In Fig. 3, we present the results for $K = 10$, where the number of available pilots is the same as the number of users. Note that the rates of the stronger users are only slightly affected by the coordination, i.e., the interference due to pilot contamination is not a dominating factor for those users. In this scenario, coordination with the variance based greedy algorithm leads to a gain of around 50% for the fifth percentile.

In Fig. 4, we show the results for an increased number of available pilots $T_{\text{tr}} = 15$. The position based assignment shows significant gains in this case, but we have to keep in mind that the additional pilot slots cannot be used for data and thus the total throughput also depends on the coherence interval [14].

VIII. Conclusion

We proposed several methods to coordinate the pilot assignment in a cellular network. System level simulations showed a performance improvement as result of the coordination, especially for weak users.

The position based assignment does not yield the best performance. However, as the necessary computational cost is little and the algorithm can be run independently on each

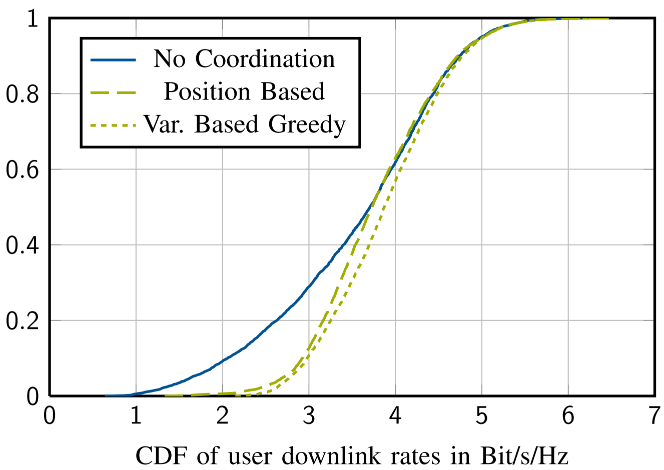


Fig. 4. User rates for $L = 21, K = 10, T_{\text{tr}} = 15$

base station, it is still worth considering. The greedy algorithms offer significant gains and show the potential of pilot coordination in larger systems.

References

[1] F. Rusek, D. Persson, B. K. Lau, E. Larsson, T. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: opportunities and challenges with very large arrays," *IEEE Signal Processing Magazine*, vol. 30, no. 1, pp. 40 –60, Jan. 2013.

[2] S. Mohammed and E. Larsson, "Per-antenna constant envelope precoding for large multi-user MIMO systems," *IEEE Transactions on Communications*, vol. 61, no. 3, pp. 1059–1071, 2013.

[3] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3590 –3600, Nov. 2010.

[4] J. Jose, A. Ashikhmin, T. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems," *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2640 –2651, Aug. 2011.

[5] J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?" *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 160 –171, Feb. 2013.

[6] B. Gopalakrishnan and N. Jindal, "An analysis of pilot contamination on multi-user MIMO cellular systems with many antennas," in *2011 IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jun. 2011, pp. 381 –385.

[7] H. Q. Ngo, T. L. Marzetta, and E. G. Larsson, "Analysis of the pilot contamination effect in very large multicell multiuser MIMO systems for physical channel models," in *2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2011, pp. 3464 –3467.

[8] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "The multicell multiuser MIMO uplink with very large antenna arrays and a finite-dimensional channel," *IEEE Transactions on Communications*, 2013.

[9] F. Fernandes, A. Ashikhmin, and T. L. Marzetta, "Inter-cell interference in noncooperative TDD large scale antenna systems," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 192 –201, Feb. 2013.

[10] A. Ashikhmin and T. Marzetta, "Pilot contamination precoding in multi-cell large scale antenna systems," in *2012 IEEE International Symposium on Information Theory Proceedings (ISIT)*, Jul. 2012, pp. 1137 –1141.

[11] R. R. Müller, M. Vehkaper, and L. Cottatellucci, "Blind pilot decontamination," in *International ITG Workshop on Smart Antennas (WSA)*, 2013.

[12] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 2, pp. 264 –273, Feb. 2013.

[13] C. Hellings, M. Joham, and W. Utschick, "Power minimization in parallel vector broadcast channels with zero-forcing beamforming," in *2010 IEEE Global Telecommunications Conference (GLOBECOM 2010)*, 2010, pp. 1–5.

[14] D. Neumann, A. Gründinger, M. Joham, and W. Utschick, "On the amount of training in coordinated massive mimo networks," in *Sensor Array and Multichannel Signal Processing Workshop (SAM)*, Jun. 2014, invited paper,in preparation.