

## A MODIFIED ANT COLONY DAMAGE IDENTIFICATION ALGORITHM FOR NOT WELL- SPACED FREQUENCY SYSTEMS.

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**Parole chiave:** Danno, Algoritmi euristici, Segnale analitico, ACORL

**Abstract.** *Damage identification is of primary concern in many fields of civil engineering. Usually the damage is detected from the variation of structural response induced. When the damage level is very low, incipient damage, this variation is hardly seen. In the present work is studied the case of not well spaced frequency systems. Identification problem is formulated as a minimum problem of a functional expressed in term of damage parameters. The minimum problem is solved by heuristic algorithm, ACORL.*

**Sommario.** *L'identificazione del danno è di primario interesse in molti campi dell'ingegneria civile. In genere la presenza del danno è osservabile dalla variazione della risposta strutturale ma quando i livelli di danno considerati sono molto bassi tale variazione è di difficile valutazione. Nel presente lavoro viene esaminato il caso della risposta di sistemi strutturali caratterizzati da frequenze molto vicine. Il problema dell'identificazione viene riformulato in un problema di minimo di un funzionale dipendente dai parametri di danno. Il problema di minimo viene risolto utilizzando un algoritmo di tipo euristico denominato ACORL.*

### 1 INTRODUCTION

Damage identification at early stages is of primary interest in many practical engineering problems. The numerical techniques usually adopted are based on comparison between the real response and a numerical model assumed representative of the structural system. Response in time domain and in frequency domain is the most common choice in

order to analyse the structural response. For systems characterised by low level of damage this choice is usually considered not suitable due to the minimum difference between damaged and undamaged response and a different representation of structural response is used. In [1], [2], [3] the analytical signal is chosen and is showed that is very sensitive to small variation of damage. Use of analytical signal needs Hilbert Transform, being the analytical signal a complex signal whose real part is the signal itself and the imaginary part is the Hilbert transform of signal. But due to the characteristics of analytical signal itself for multi degrees of freedom is necessary to use techniques like EMD [4], [5] or labeled filtered modal responses [2]. These techniques are not suitable for not well-spaced frequency systems. To overcome these difficulties in present work a recently proposed heuristic algorithm [6], [7] is used, ACORL. The optimisation algorithm is used considering the response of a three-storey shear-beam type building having closed frequencies. The response is characterised by acceleration response and by the phase of analytical signal.

## 2 DAMAGE IDENTIFICATION

Generally speaking, the determination of the output of the model is called direct problem. From every direct problem it is possible to identify at least two inverse problems :

a) causation problem: given the model and an assigned output find the input.

b) model identification, parameters identification: given an assigned input and output find the model or if, as in the present case, the model is assumed to be known find the parameters from which the model depends.

For a problem to be well-posed, in the Hadamard sense, it must meet the following criteria:

- (1) For each data set, there exists a solution.
- (2) The solution is unique.
- (3) The solution depends continuously on the data.

If a problem does not meet one or more of these criteria the problem is considered to be ill-posed. Even if the direct problem is well posed, in the Hadamard sense, there is no guarantee that the parameters identification inverse problem is well posed too. As a matter of fact the damage parameters identification inverse problem, in terms of experimental measurement, lacks, in general, of two of the three criteria for being well-posed: there is not a unique solution, and furthermore the solution does not depend continuously on the data.

The first drawback, lack of uniqueness, means that there is a multitude of solutions; the second one means that small errors in measurements may cause large errors in the solution.

The inverse problem can be formulated as an optimization problem. Let us consider nexp experiments and let be  $s_j$  the experimental measurements obtained at some ndat assigned instants of time  $t_j$  :

$$\tilde{s}_i \in \mathbb{R}^{ndat}; \quad i = 1, 2, \dots, nexp \quad (1)$$

$$\tilde{s}_i \equiv (\tilde{x}_j, t_j); \quad j = 1, 2, \dots, ndat$$

and let be  $k$  the npar-dimensional vector of the unknown parameters and  $si$  the numerical results computed for the experiment  $i$  using the assigned model with parameters  $k$ :

$$\begin{aligned}
 \mathbf{s}_i(\mathbf{k}) &\in \mathbb{R}^{n\text{dat}}; \quad i = 1, 2, \dots, n\text{exp} \\
 \mathbf{s}_i(\mathbf{k}) &\equiv (s_j(\mathbf{k}), t_j); \quad j = 1, 2, \dots, n\text{dat} \\
 \mathbf{k} &\in \mathbb{R}^{n\text{par}}
 \end{aligned} \tag{2}$$

The inverse problem can be assumed coincident with the following constrained optimisation problem:

$$\begin{aligned}
 \min_{\mathbf{k} \in \mathbb{R}^{n\text{par}}} \quad & \sum_{i=1}^{n\text{exp}} D(\mathbf{s}_i(\mathbf{k}) - \tilde{\mathbf{s}}_i) \\
 \text{subject to} \\
 k_m^{\min} \leq k_m \leq k_m^{\max} \quad & m = 1, 2, \dots, n\text{par}
 \end{aligned} \tag{3}$$

Where  $D$  represent a measure of the distance between the experimental results  $\mathbf{s}_i$  and the numerical results  $\tilde{\mathbf{s}}_i$ . Without lack of generality, it is also assumed that the parameters vector  $\mathbf{k}$  belongs to the hypercube with faces  $k_i = \min_i$ ,  $k_i = \max_i$ .

In present work it is assumed that no model errors are present, namely the only source of errors are due to the measuring devices, hence a relative measure of distance between experimental and numerical results is used. The optimisation problem could then be stated as:

$$\begin{aligned}
 \min_{\boldsymbol{\alpha} \in \mathbb{R}} \quad & J_\eta(\boldsymbol{\alpha}) = \frac{\int_{T_{\text{ini}}}^{T_{\text{fin}}} [\eta^{\text{th}}(\boldsymbol{\alpha}, t) - \eta^{\text{ex}}(t)]^2 dt}{\int_{T_{\text{ini}}}^{T_{\text{fin}}} [\eta^{\text{ex}}(t)]^2 dt}
 \end{aligned} \tag{4}$$

Where equation [3] is extended to the interval  $T_{\text{ini}}-T_{\text{fin}}$  and  $\eta^{\text{th}}$  is an assigned signal feature. Choice of different signal features gives different optimisation problems and for small damage identification problem has already pointed out that the phase of analytical signal gives good results.

### 3 ACORL ALGORITHM

Ant Colony Optimization (ACO) was introduced [8] for combinatorial optimisation problems. Using the natural metaphor of foraging behaviour of ants, ACO belongs to the brother class of swarm intelligence algorithms. In the search of food scout ants leave the nest for exploring the ambient, when an ant find the food return to the nest depositing a pheromone trail. The pheromone attract other ants and, due to the evaporation of the pheromone itself, shorter paths become more appealing. The key aspect of this behaviour is the lack of direct communication between ants and the lack of a coordinator. For optimisation problems this approach gives the possibility to use only informations based on the actual

value of the function itself, hence without using information on the derivative, using a population of solutions that by indirect communication search the optimal, or sub-optimal, point in the parameters space. An extension to the original ACO algorithm was proposed [9] to take into account problems defined in continuous domains (ACOR) and, recently, the authors have proposed an extension to the ACOR algorithm particularly suited for problems with many local optima (ACORL) [4], [5].

In what follows the main steps of ACORL are briefly outlined, more details are available in [4]. Create an archive T of k solutions,  $T=\{x_1, x_2, \dots, x_k\}$ . Where  $x_r = [x_{r1}, x_{r2}, \dots, x_{rN}]$ . Order the solutions of the archive T according to their objective function value. Given a decision variable  $x_i$ ,  $i=1, \dots, N$ , an ant constructs a solution by performing N construction steps. At construction step i, the ant chooses a value for the variable  $x_i$ . At this construction step, only the information related to the i-th dimension is used. For each solution a weight is calculated as:

$$\omega_l = (qk\sqrt{2\pi})^{-1} \exp(-(l-1)^2/2(qk)^2) \quad (5)$$

which essentially defines the weight  $\omega$  to be a value of the Gaussian function with argument r, mean 1 and standard deviation qk, where q is a parameter of the algorithm. When q is small, the best-ranked solutions are strongly preferred, and when it is large, the probability becomes less dependent on the rank of the solution.

All the components  $x_{ri}$  for  $i=1$  to  $N$  of the chosen r-th solution in the following steps are perturbed following the Levy distribution. Levy distribution is characterised by four parameters: the scale parameter,  $\sigma$ , the skewness parameter,  $\beta$ , the shift parameter  $\mu$  and the  $\alpha$  parameter. The scale parameter is defined by

$$\sigma_l^i = \xi \sum_{r=1}^k |s_l^j - s_r^i| / (r-1) \quad (6)$$

where  $\xi$  is a parameter user-defined in the algorithm ranging from 0 and 1. The higher the value of this parameter the slower the convergence speed.

The shift parameter,  $\mu$ , is the value of the i-th parameter of the base solution itself ( $x_{ri}$ ). The skewness parameter is set to 0, namely no dissymmetry of the probability density function about the shift value  $\mu$ . The fourth parameter  $\alpha$  is a control parameter set by the user and its value ranges between 0 and 2. So the i-th parameter is newly determined. The same procedure is repeated for all the N parameters. At the end, once the solution is entirely constructed, it is evaluated and if better than any of the solutions in T, it is included into the archive set T. From what was said above, if the used defined parameter  $\alpha$  is set to 2, the ACORL coincides with ACOR. The ACORL pseudo-code is reported in the following table.

**Algorithm 1** ACOR<sub>L</sub> Pseudocode

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Random creation of the solutions archive of size k
Choice of  $\xi, q, \alpha, \mu$ 
while not(termination) do
    for z=1 to m do
        Choice of one solution from the archive using (2)
        for all parameter (Ant construction) do
            Calculate standard deviation  $\sigma_l^i$  using (4)
            Modify the i-th parameter in the following way:
             $x^i = x^i + S_\alpha(\sigma_l^i, 0, 0)$ 
        end for
        Evaluation of the new solution
    end for
    Archive update
end while

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Figure 1: ACORL Pseudocode.

### 3 NUMERICAL EXPERIMENTS

Let us consider a 3 degrees of freedom shear type structure, Fig. 2, the chosen structural parameters are reported in table 1. In order to simulate damage the damaged stiffness at each floor is defined as  $k_i(1-\gamma_i)$  where  $\gamma_i$  represent the damage coefficient, i.e.  $\gamma=1$  full damage,  $\gamma=0$  pristine structure. For each combination of  $\gamma_i$  is possible to obtain the response of the structure. The natural frequencies of the structure are not well spaced and traditional methods to isolate single modal component from the recorded signal are not suitable. The first test was conducted considering the phase of the analytical signal computed at the third floor. The analytical signal is the complex signal defined as

$$z(t) = x(t) + i\hat{x}(t) \quad \hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau \quad (7)$$

where  $x(t)$  is the signal considered and  $\hat{x}(t)$  its Hilbert transform

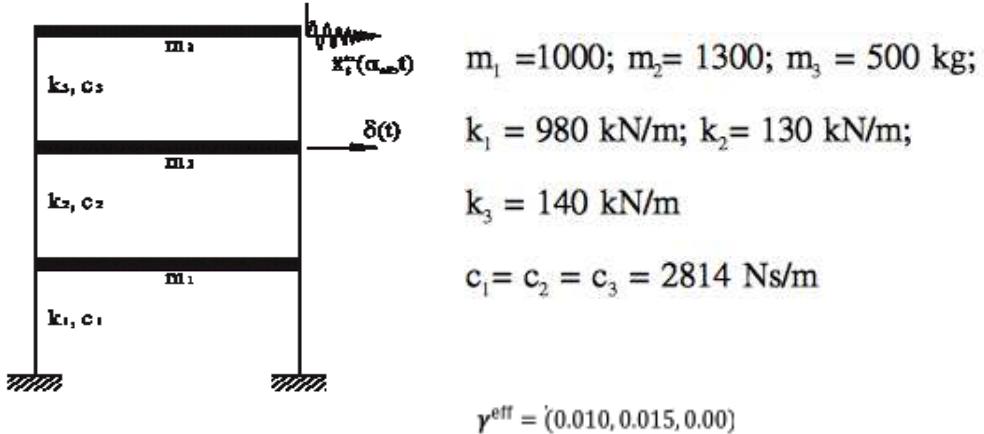


Figure 2: Shear type structure and structural parameters.

The analytical signal can also be expressed in terms of the amplitude function  $A(t)$  and the phase function  $\eta(t)$ . The optimisation algorithm has been implemented with  $q=0.1$ ,  $csi=0.8$ ,  $alfa=1.8$ , and a population size  $k=50$  considering as signal the phase of analytical signal. The algorithm was stopped when 10.000 function evaluations were performed. In fig. 3 are reported the mean, median, best and worst case of functional along 30 independent runs. The algorithm succeeds in locate the global minimum consistently with little dispersion between independent runs. Sub-optimal minima founds, like in the worst case, are really near to the optimal one. Fig. 4 reports a typical trajectory of best solution along a single optimisation run in the plane  $\gamma_1, \gamma_2$ .

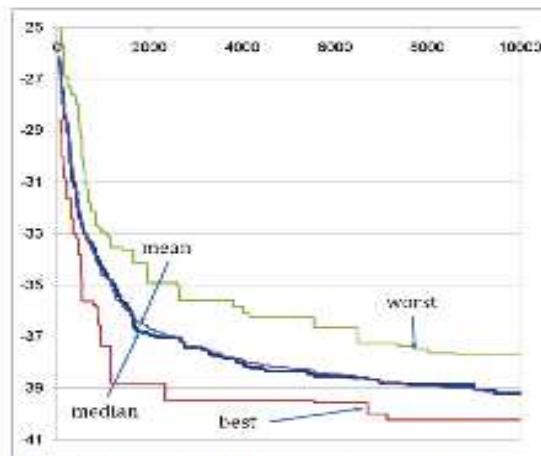


Figure 3: Evolution of solution.

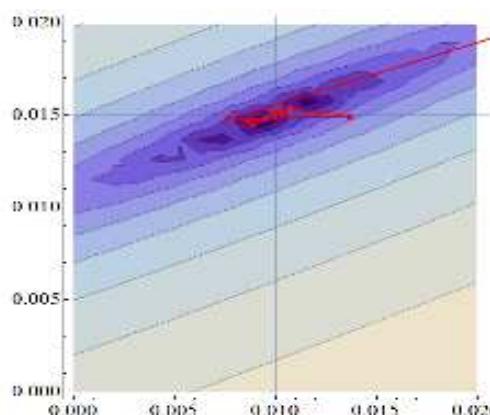


Figure 4: Typical trajectory in parameters space.

Second test was conducted considering not the analytical signal but the acceleration itself. Such a choice gives an optimisation problem more difficult to treat with traditional techniques but avoid the computational work associated to the analytical signal. The parameters used for the ACORL algorithm are the same of the precedent case but the algorithm was stopped after 1000 function evaluations. In Fig. 5 are reported the mean, median, best and worst case of functional along 30 independent runs. In Fig. 6 the evolution of damage parameters along a typical run is showed. Also in this case the results are really good. After few function

evaluations the value of parameters are really near to the searched ones and very little dispersion is present between different runs.

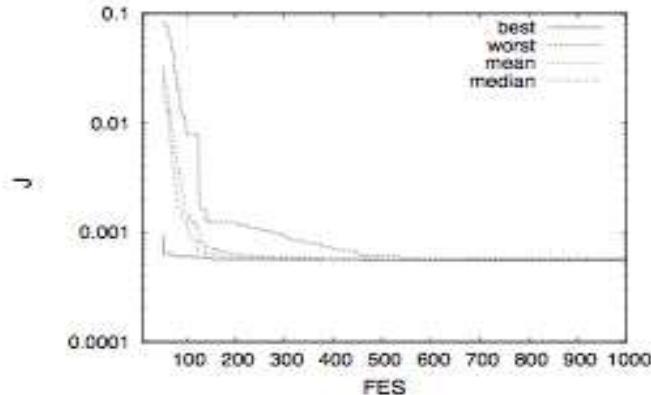


Figure 5: Evolution of solution.

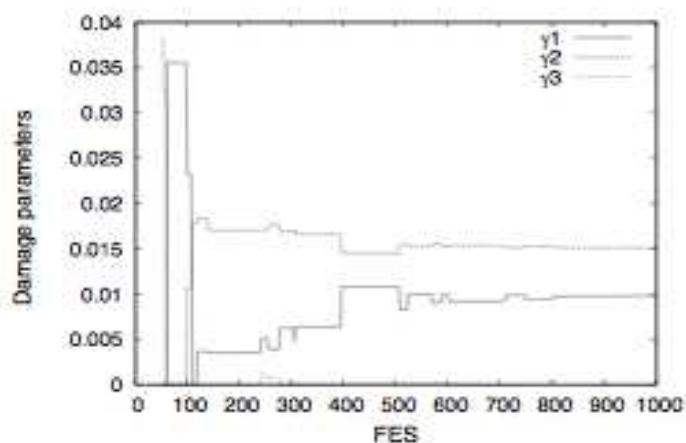


Figure 6: Evolution of damage parameters.

#### 4 CONCLUSIONS

In this paper a method for incipient damage identification based on heuristic algorithm ACORL was used. The robustness of chosen algorithm gives the possibility to overcome usual difficulties associated to this kind of approach. Numerical results on 3 degrees of freedom system are very accurate using both phase of analytical signal and acceleration as signal feature.

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