Feasibility Conditions for Interference Alignment over Sub-carriers

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Motivation

Challenges in mobile networks

- Increasing number of users
- Demand for high throughput
- Especially cell-edge users suffer from interference

⇒ Spectral efficiency needs to be improved and interference must be treated

Interference alignment is one possible strategy.

Feasibility of interference alignment

- Depends on the scenario, the signaling space and the number of symbol extensions
- Linear interference alignment over two sub-carriers is usually infeasible [1]

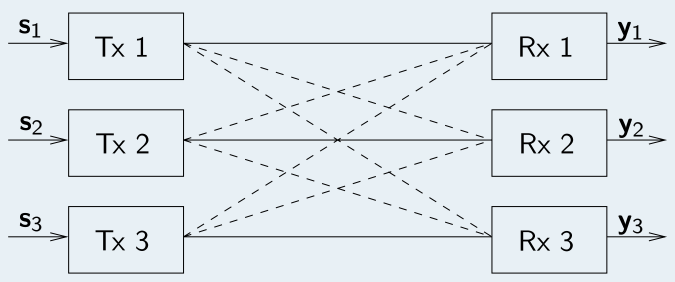
Our contribution

- Conditions on the channels for which two sub-carriers make interference alignment feasible are derived
- For line-of-sight channels these conditions can be fulfilled by choosing the sub-carrier spacing carefully and we even achieve the upper bound on the sum rate of interference alignment arbitrary closely

System Model

Symmetric Interference Channel with

- K transmitter-receiver pairs
- Single antenna nodes
- Two orthogonal sub-carriers
- Perfect channel knowledge of the complete network at all nodes



Received signal at receiver i:

$$\mathbf{y}_i = \underbrace{\mathbf{U}_i^\dagger \mathbf{H}_{ii} \mathbf{V}_i \mathbf{s}_i}_{\text{Useful Signal}} + \underbrace{\sum_{\substack{k=1 \\ k \neq i}}^{K} \mathbf{U}_i^\dagger \mathbf{H}_{ik} \mathbf{V}_k \mathbf{s}_k}_{\text{Interference}} + \underbrace{\mathbf{U}_i^\dagger \mathbf{z}_i}_{\text{Thermal Noise}}$$

Noise vector $\mathbf{z}_i$ is proper complex AWGN with variance σ_i^2.

Power constrains:

- Precoder: $\|\mathbf{V}_k\|^2 = 1$
- Receive beamforming filter: $\|\mathbf{U}_i\|^2 = 1$

Channel Models

General model

$$\mathbf{H}_{ik} = \begin{bmatrix} h_{ik}^{(1)} & 0 \\ 0 & h_{ik}^{(2)} \end{bmatrix} = \begin{bmatrix} \left|h_{ik}^{(1)}\right| e^{-j\angle h_{ik}^{(1)}} & 0 \\ 0 & \left|h_{ik}^{(2)}\right| e^{-j\angle h_{ik}^{(2)}} \end{bmatrix},$$

where $h_{ik}^{(l)}$ is short for $h_{ik}\left(f^{(l)}\right)$

Line-of-sight model

$$\mathbf{H}_{ik}^{\mathrm{LoS}} = |h_{ik}| \begin{bmatrix} e^{-j2\pi f^{(1)}\tau_{ik}} & 0 \\ 0 & e^{-j2\pi f^{(2)}\tau_{ik}} \end{bmatrix},$$

with time delay τ_{ik} and sub-carrier frequencies $f^{(1)}$ and $f^{(2)}$

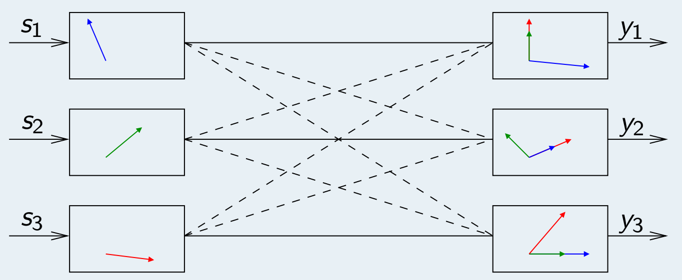
Interference Alignment over Two Sub-Carriers

For two sub-carriers and three Tx-Rx pairs degrees-of-freedom are bounded by 3/2 (one stream is transmitted between each pair) [1].

Degrees-of-Freedom

$$d = \lim_{\mathrm{SNR}\to\mathrm{inf}} \frac{R_{\mathrm{sum}}(\mathrm{SNR})}{\log \mathrm{SNR}}$$

Idea of interference alignment is to align interference in a subspace:



Conditions for Interference Alignment

Interference is aligned, if

$$\mathbf{u}_i^\dagger \mathbf{H}_{ik} \mathbf{v}_k = 0, \qquad \forall i \neq k$$
$$\left|\mathbf{u}_i^\dagger \mathbf{H}_{ii} \mathbf{v}_i\right| > 0, \qquad \forall i$$

Linear interference alignment seems to achieves degrees-of-freedom for diagonal channel matrices *for an asymptotically increasing number of symbol extensions (e.g. sub-carriers) only* [1].

Condition to make General Channel Feasible

Theorem 1

Three degrees-of-freedom over two sub-carriers are feasible for three user pairs with single antennas if the following condition on the channel coefficients holds

$$\frac{h_{1,2}^{(2)}}{h_{1,2}^{(1)}} \frac{h_{1,3}^{(1)}}{h_{1,3}^{(2)}} \frac{h_{2,3}^{(2)}}{h_{2,3}^{(1)}} \frac{h_{2,1}^{(1)}}{h_{2,1}^{(2)}} \frac{h_{3,1}^{(2)}}{h_{3,1}^{(1)}} \frac{h_{3,2}^{(1)}}{h_{3,2}^{(2)}} = 1.$$

For more Tx-Rx pairs many of similar conditions have to hold.

Condition to make Line-of-Sight Channel Feasible

Corollary 1

For line-of-sight channels (i.e. single-tap) Theorem 1 simplifies to

$$\Delta f = f^{(2)} - f^{(1)} = \frac{n}{\tau_{1,3} - \tau_{1,2} + \tau_{2,1} - \tau_{2,3} + \tau_{3,2} - \tau_{3,1}},$$

where $n \in \mathbb{Z} \setminus \{0\}$.

Choosing $n = 0$ violates the assumption of orthogonal sub-carriers.

Any non-zero multiple sub-carrier spacing of

$$\Delta f_{\min} = \frac{1}{\tau_{1,3} - \tau_{1,2} + \tau_{2,1} - \tau_{2,3} + \tau_{3,2} - \tau_{3,1}},$$

enables interference alignment.

For the special case

$$\tau_{1,3} - \tau_{1,2} + \tau_{2,1} - \tau_{2,3} + \tau_{3,2} - \tau_{3,1} = 0,$$

interference alignment is feasible for all sub-carrier spacings.

⇒ *Using sub-carrier spacing as an additional free variable enables linear interference alignment for line-of-sight channels.*

Effective Channel Amplitudes

For a sub-carrier spacing $n\Delta f_{\min}$ and precoders and receive filters, chosen such that interference aligns, the amplitudes of the resulting effective channels are

$$\left|\bar{h}_1\right| = |h_{1,1}| \left|\sin(\pi n \Delta f_{\min}(\tau_{2,1} - \tau_{1,1} + \tau_{1,3} - \tau_{2,3}))\right|$$
$$\left|\bar{h}_2\right| = |h_{2,2}| \left|\sin(\pi n \Delta f_{\min}(\tau_{2,3} - \tau_{2,2} + \tau_{1,2} - \tau_{1,3}))\right|$$
$$\left|\bar{h}_3\right| = |h_{3,3}| \left|\sin(\pi n \Delta f_{\min}(\tau_{3,2} - \tau_{3,3} + \tau_{1,3} - \tau_{1,2}))\right|.$$

Upper Bound

The sum-rate of the proposed scheme for line-of-sight channels is upper bounded by

$$R_{\mathrm{sum}} \leq \sum_{\forall i} \log_2\left(1 + \frac{\left|\bar{h}_i\right|^2}{\sigma_i^2}\right).$$

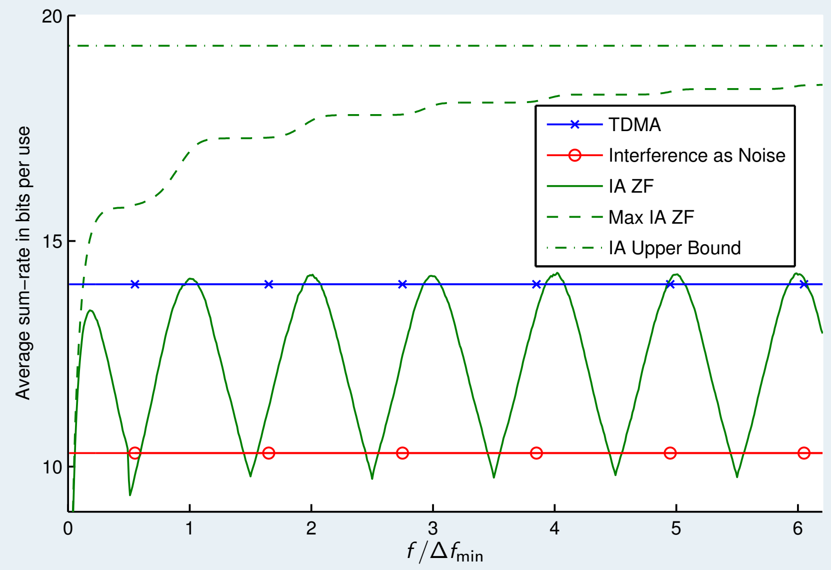
One can optimize the choice of $\Delta f = n\Delta f_{\min}$ within the available bandwidth to obtain the optimal rate.

Achievability of Upper Bound

For continuously and independently distributed delays the upper bound on the sum rate of the presented scheme is achieved arbitrarily closely for increasing bandwidth.

Proof outline:
We argue that for continuous delays there exist an n such that the product $n\Delta f_{\min}$ is arbitrary close to any number. Now we choose this number such that all sine expressions of the effective channel amplitudes are arbitrarily close to 1.

Simulation Results for Line-of-Sight Channels



- 3 transmitter-receiver pairs
- Distances $d_{i,k}$ are continuously and independently distributed
- Distances of the direct channels are $d_{i,i} \in [150m, 250m]$
- Distances of the cross channels $(i \neq k)$ are $d_{i,k} \in [250m, 350m]$.
- The delay is $\tau_{i,k} = c/d_{i,k}$, where $c = 3 \cdot 10^8 \mathrm{m/s}$
- The channel amplitude is $|h_{i,k}| = (1\mathrm{m}/d_{i,k})^\gamma$, with $\gamma = 3.76$

Benchmark schemes

- Treating *Interference as Noise*: Each transmitter transmit two streams in every time slot, while interference is treated as noise
- *TDMA*: Each transmitter transmits (without interference) two streams in every third slot, but with three times the power

Interference alignment (IA) curves

- *IA ZF*: Interference zero-forcing for the current subcarrier spacing (a least-squares solution is used where IA is infeasible)
- *Max IA ZF*: For each channel realization the maximal sum rate within the bandwidth equal to the x-axis' value is determined; next we take the average
- *IA Upper Bound*: Average of sum rate upper bounds

Conclusions

Summary:

- Conditions to make interference alignment feasible over two sub-carriers are derived
- Choosing the sub-carrier spacing carefully fulfills the conditions for line-of-sight channels and allows to achieve the upper bound

Outlook:

- Optimization of the sub-carrier spacing when maximizing SINR instead of zero-forcing the interference

References

▶ V. R. Cadambe and S. A. Jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

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