Structured Subchannel Impulse Response Estimation for Filter Bank based Multicarrier Systems

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***Abstract*—In a Filter Bank based Multicarrier (FBMC) system employed in a highly frequency selective propagation channel each subchannel experiences a multi-tap impulse response (IR). In this contribution we present an efficient method of estimating the IR of the propagation channel experienced by each subchannel, the so-called structured channel estimation. The structured subchannel IR estimation allows us to isolate the propagation channel from the transmit and receive filters. As a consequence the number of parameters to be estimated is reduced as well as the length of the training sequence.**

I. Introduction

The physical layer of future wireless communications systems is expected to provide even higher data rates compared to current schemes. Multicarrier (MC) based systems have shown to be the best choice for this requirement because of their many advantages. The basic multicarrier principle to divide the frequency spectrum into many narrow subchannels is not new, but only in the last decade a widespread use in practical systems could be observed, e.g. Local Wireless Networks, DSL and recently Long Term Evolution (LTE). There are many classes of MC systems, but the Cyclic Prefix based Orthogonal Frequency Division Multiplexing(CP-OFDM) is certainly the most investigated and explored one. It offers the advantage of efficient implementation and channel estimation/equalization become trivial tasks. As a result of the insertion of some redundancy (CP), the broadband frequency selective propagation channel becomes a frequency flat subchannel in each subcarrier and only one tap per-subchannel is necessary. The drawbacks of CP-OFDM compared to other MC modulation schemes include a loss in spectral efficiency, as a consequence of the CP insertion, a higher level of out-of-band radiation, since the subcarriers have a sinc-like frequency behavior, and a higher sensitivity to narrowband interferers when the synchronization is not perfect, because the low attenuation of the side-lobes results in an undesired overlap of the subchannels.

CP-OFDM is based on the general MC concept of modulated transmultiplexers (TMUX), which are composed of exponentially modulated analysis and synthesis filter banks, what we call FBMC systems. Maximally decimated filter banks are of particular interest. Instead of using a rectangular window for pulse shaping, a finite impulse response (FIR) prototype filter that has a longer impulse response than the symbol period, i. e. the number of filter coefficients is higher than the number of subchannels M, is modulated by complex exponentials to form each subchannel. Because of their longer length, the filters can be more concentrated in the frequency domain and the subchannels are shaped to overlap only with the adjacent ones. The prototype filter is also chosen to fulfill the Nyquist Intersymbol Interference (ISI) criterion, so that its impulse response has zero crossings at the symbol period T. But it is known from filter bank [1] and communication theory [2] that, in a complex modulated and critically sampled TMUX, if the input signals are complex and in order to achieve the perfect reconstruction or ISI- and ICI-free conditions, the real and imaginary parts of the input signals must be staggered by $T/2$, resulting in the so called Offset Quadrature Amplitude Modulation (OQAM).

The problem of propagation channel equalization in FBMC systems is still an active research topic [3], [4]. The solutions that depend only on the output signals of each subchannel [5], [6] present lower complexity than solutions that take into account multiple subcarriers and allow a further independent processing of the different subchannels. In this way per-subchannel equalizers work like single carrier (SC) equalizers for OQAM modulated symbols, but with the difference that Interchannel Interference (ICI) is present. Since noise cannot be considered white at the output of a filter with bandwidth smaller than the sampling frequency, this has to be considered in the equalizer design. Furthermore, in an FBMC system with OQAM input symbols the equalizer can be inserted in front of the de-staggering, leading to a fractionally spaced equalizer (FSE) working at a rate of $2/T$, where $1/T$ is the symbol rate.

Most of the channel equalization methods considered in the recent literature assume that the channel impulse response or its frequency response is known at the receiver side. Of course this is only possible if they were estimated beforehand. The issue of channel estimation for FBMC systems has been covered in some contributions but it has not been depleted. In [7] the authors consider the case where the channel experienced inside one subchannel is not frequency selective and can be described by one tap, like in traditional CP-OFDM. They further investigate the issue of training symbol design when a training block is transmitted before the data block. The authors of [8] consider the channel selectivity inside one subchannel, but they estimate samples of the subchannel frequency response and then they can approximate the subchannel impulse response.

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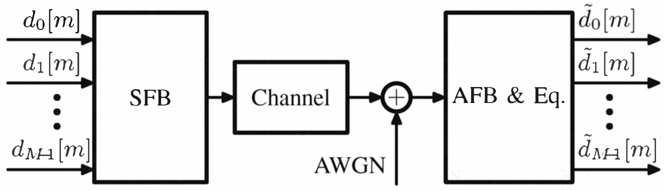

Figure 1. FBMC System Overview

They also consider a joint time and frequency synchronization.

In this work we approach the problem of estimating the subchannel impulse response by isolating the propagation channel viewed in one subcarrier from the effects of the transmit and receive filters. The total impulse response resulted from the convolution of transmit, receive filters and propagation channel is very long. This results in a high number of variables to be estimated. As a consequence also more training symbols need to be transmitted, reducing the spectral efficiency. In this contribution we first show how much training is necessary to estimate the long subchannel impulse response. Then we employ the so-called structured channel estimation, a method that is already known for single carrier systems and we extend here for multicarrier symbols. We can show that this dramatically reduces the number of variables to be estimated, allowing shorter training symbols and better spectral efficiency.

II. FBMC System and Subchannel model

A general overview of the FBMC system model is illustrated in Fig. 1. Here the filter banks are employed in a transmultiplex configuration [9]. At the transmitter a synthesis filter bank (SFB) performs a frequency division multiplexing (FDM) of the complex data symbols $d_k[m]$ into parallel subchannels of rate $1/T$. At the receiver, an analysis filter bank (AFB) separates the data from the single subchannels. In our model we include a frequency selective channel and an AWGN source between the SFB and the AFB. We consider here an exponentially modulated filter bank in both SFB and AFB. This means that only one prototype low-pass filter has to be designed and the other subfilters are obtained by modulating it as follows

$$h_k[l] = h_0[l] \exp\left(\mathrm{j}\,\frac{2\pi}{M}k\left(l - \frac{P-1}{2}\right)\right), \quad l = 0, \ldots, P-1, \tag{1}$$

where $h_0[l]$ is the impulse response of the prototype filter with length P and M is the total number of subcarriers. The prototype is a Nyquist-like filter with a roll-off factor $\rho \leq 1$ and as a consequence only contiguous subcarriers overlap in the frequency domain and non-contiguous subcarriers are separated by the good stop-band behavior. For example, a Root Raised Cosine filter (RRC) with length $P = KM + 1$ can be used, where K is the time overlapping factor that determines how many blocks of symbols superpose each other. K should be kept as small as possible in the first place to allow a low complexity and in the second place to reduce the spreading of the symbols in the time domain, specially in mobile environments.

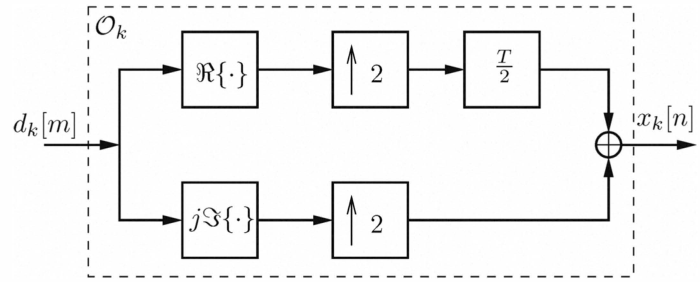

Figure 2. O-QAM staggering for odd sub carrier

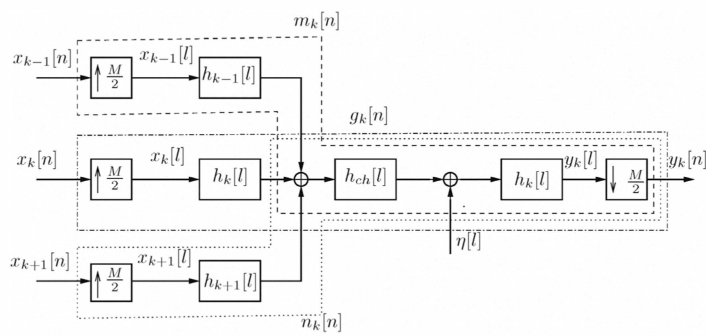

Figure 3. Subchannel model

Since the prototype filter is longer than the number of subchannels M, and in order to maintain the orthogonality between all of them and for all time instants, the complex input symbols $d_k[m]$ need to have their real and complex parts staggered by $T/2$ resulting in an OQAM modulation scheme [1], [10]. The OQAM staggering for even indexed subchannels is depicted in Fig. 2. In odd indexed subchannels the delay of $T/2$ is located at the lower branch with purely imaginary symbols. At the receiver the OQAM demodulation or destaggering is performed by applying a flow-graph reversal [11] in Fig. 2, substituting the up-samplers by down-samplers and exchanging the blocks $\Re\{\cdot\}$ and $j\Im\{\cdot\}$.

After the OQAM staggering the signals of the different subchannels are up-sampled by $M/2$, filtered and added. A broadband signal is then generated and after some eventual further digital processing, it is digital-to-analog (DA) converted to a baseband signal that will be then upconverted to RF and transmitted. At the receiver side the RF signal is brought to baseband, filtered and then analog-to-digital (AD) converted. The digital received signal is then filtered by the different analysis filters to generate the subchannel signals.

Since we have assumed that only contiguous subchannels overlap in the frequency domain, we can construct the discrete-time subchannel model shown in Fig. 3. The inputs $x_k[n]$ are the OQAM staggered symbols and the received subchannel signals $y_k[n]$ still have to be equalized, since a frequency selective propagation channel is assumed, and OQAM de-staggered before the QAM symbols are demodulated. As a consequence, in the subcarrier model the input and output sampling rate is $2/T$. We have assumed here perfect time and frequency synchronization. In other words, no time or frequency shifts (Carrier Frequency Offset or Doppler shift/spread) are present. A more realistic model would involve this and some other issues that are out of the scope of the present contribution.

We collect N output symbols of the k-th subchannel in an observation vector $\mathbf{y}_k[n] \in \mathbb{C}^N$ and express it as a function of the input signals of subchannels $k-1$, k and $k+1$, of the three impulse responses resulting from the convolution of the transmit with the receive filters and with the propagation channel, and of a thermal noise term. This leads us to the definition

$$\mathbf{y}_k[n] = \mathbf{G}_k\mathbf{x}_k[n] + \mathbf{M}_k\mathbf{x}_{k-1}[n] + \mathbf{N}_k\mathbf{x}_{k+1}[n] + \mathbf{\Gamma}_k\boldsymbol{\eta}[l], \quad (2)$$

where we have employed the convolution matrices $\mathbf{G}_k$, $\mathbf{M}_k$, $\mathbf{N}_k \in \mathbb{C}^{N\times N+Q-1}$, with $Q = \lceil\frac{2(P-1)+L_{\text{ch}}}{M/2}\rceil$, composed by the Q-long impulse responses $g_k[n]$, $m_k[n]$ and $n_k[n]$. $\mathbf{\Gamma}_k \in \mathbb{C}^{N\times(P+\frac{M}{2}N)}$ is obtained by taking each $\frac{M}{2}$-th row of the convolution matrix constructed with the analysis filter impulse response $h_k[l]$. This is the reason why the vector $\boldsymbol{\eta}[l] \in \mathbb{C}^{(P+\frac{M}{2}N)}$ is defined in the high sampling rate $\frac{1}{MT}$. The subchannel model in (2) was first proposed by [6].

The frequency selective propagation channel will introduce ISI and ICI in the received symbols. To reduce the bit error rate after the demodulation of the received symbols an equalizer has to be introduced before the OQAM de-staggering. A per-subchannel linear MMSE or DFE MMSE equalizer can be employed for this matter [5], [6]. But before those equalizers can be designed a channel estimation has to be performed and this topic will be covered in the next section.

We should mention here that there are efficient realizations for the subchannel filtering operation. In this case, the modulation of the prototype filter to generate each subfilter is performed with the aid of the Fast Fourier Transform. In order to perform the filtering at a lower sampling rate, the polyphase decomposition [9] of the prototype filter is also employed. Those efficient realizations can be found, for example, in [1].

III. Subchannel IR Estimation

In this work we assume that a per-subchannel equalizer is employed and as a consequence a per-subchannel estimator is sufficient for the equalizer design. We further assume that known training sequences are used as the input of all subchannels of interest and also at their contiguous subchannels. To get the subchannel model for the subchannel estimation we can rewrite (2) by stacking the total IR into a vector and the transmit signal in a Hankel matrix. We get then

$$\mathbf{y}_k[n] = \mathbf{X}_k[n]\mathbf{g}_k + \mathbf{X}_{k-1}[n]\mathbf{m}_k + \mathbf{X}_{k+1}[n]\mathbf{n}_k + \mathbf{\Gamma}_k\boldsymbol{\eta}[l], \quad (3)$$

where $\mathbf{y}_k[n] \in \mathbb{C}^N$, $\mathbf{g}_k$, $\mathbf{m}_k$, $\mathbf{n}_k \in \mathbb{C}^Q$ and $\mathbf{X}_k[n] \in \mathbb{C}^{N\times Q}$. We further collect the inputs of the three adjacent subchannels in a bigger data matrix $\mathbf{S}_k[n] = [\mathbf{X}_k[n]\mathbf{X}_{k-1}[n]\mathbf{X}_{k+1}[n]] \in \mathbb{C}^{N\times 3Q}$ and the impulse responses in a longer subchannel vector $\mathbf{f}_k = [\mathbf{g}_k^T\mathbf{m}_k^T\mathbf{n}_k^T]^T \in \mathbb{C}^{3Q}$ containing the three IR. Then the received signal can be written as

$$\mathbf{y}_k[n] = \mathbf{S}_k[n]\mathbf{f}_k + \mathbf{\Gamma}_k\boldsymbol{\eta}[l]. \quad (4)$$

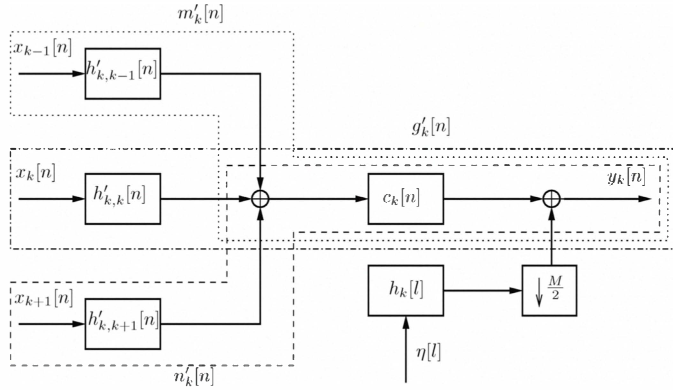

Figure 4. Modified subchannel model for structured approach

The Least squares estimator of the overlapping subchannel IR is given by

$$\mathbf{f}_k[n] = (\mathbf{S}_k[n]^H\mathbf{S}_k[n])^{-1}\mathbf{S}_k[n]^H\mathbf{y}_k[n]. \quad (5)$$

In the derivation of (5) we did not take into account any previous estimation of the adjacent subchannel IR. This means that if $\mathbf{f}_{k-1}[n]$ or even only $\mathbf{n}_{k-1}$ was previously calculated, it can be seen that $\mathbf{m}_k = \mathbf{n}_{k-1}$ and only part of $\mathbf{f}_k[n]$ needs to be estimated. We should also point out that the IR's $g_k[n]$, $m_k[n]$ and $n_k[n]$ are in general very long, because the prototype filter is also very long. Of course that depending on the design of the prototype, the taps with a considerable magnitude may be concentrated in time, allowing us to ignore the low magnitude taps and approximate the total IR with a shorter one. But still there will be always information about the prototype filter embedded in the vector to be estimated. In the next subsection we will show how to remove any information about the prototype filter of the overall subchannel to be estimated and isolate its IR component resulting from the frequency selective propagation channel.

A. Structured Subchannel IR Estimation

To perform the structured subchannel IR estimation we have first to modify the subchannel model. The idea is to model the propagation channel viewed from the receiver side at each subchannel as a narrow-band FIR filter with a short IR and represent it in a lower sampling rate, namely the double of the symbol rate or $2/T$. For this we first convolve the transmit filters $h_{k-1}[l]$, $h_k[l]$ and $h_{k+1}[l]$ with the receive filter $h_k[l]$. Than we down-sample by $M/2$ the resulting impulse responses and obtain $h'_{k,k-1}[n]$, $h'_{k,k}[n]$ and $h'_{k,k+1}[n]$. As a consequence the narrow-band propagation channel experienced by subchannel k has a short length IR that we call here $c_k[n]$. The three new overall impulse responses for the contiguous subchannels are $g'_k[n]$, $m'_k[n]$ and $n'_k[n]$. This new subchannel model is illustrated in Fig. 4.

We can now express the output of one subchannel as

$$\mathbf{y}_k[n] = \mathbf{X}'_k[n]\mathbf{g}'_k + \mathbf{X}'_{k-1}[n]\mathbf{m}'_k + \mathbf{X}'_{k+1}[n]\mathbf{n}'_k + \mathbf{\Gamma}_k\boldsymbol{\eta}[l], \quad (6)$$

where $\mathbf{g}'_k$, $\mathbf{m}'_k$ and $\mathbf{n}'_k$ are vectors containing the IR $g'_k[n]$, $m'_k[n]$ and $n'_k[n]$ that result from the convolution between the down-sampled sub-filters and the narrowband propagation channel. It is important to note here the difference to equation (3). In that model the impulse responses $g_k[n]$, $m_k[n]$ and $n_k[n]$ were obtained after the convolution between transmit filters, broadband propagation channel and receive filter, and then down-sampling the resulting IR's by $M/2$.

It is clear that the components of vectors $\mathbf{g}'_k$, $\mathbf{m}'_k$ and $\mathbf{n}'_k$ depend on the narrowband propagation channel model, but also on the known transmit and receive filters. If we separate both impulse responses we can write $\mathbf{g}'_k = \mathbf{H}'_{k,k}\mathbf{c}_k$, $\mathbf{m}'_k = \mathbf{H}'_{k,k-1}\mathbf{c}_k$ and $\mathbf{n}'_k = \mathbf{H}'_{k,k+1}\mathbf{c}_k$, where $\mathbf{c}_k \in \mathbb{C}^L_k$ is a vector containing the IR $c_k[n]$. As a consequence, the received signal is now given by

$$\begin{aligned}\mathbf{y}_k[n] &= (\mathbf{X}'_k[n]\mathbf{H}'_{k,k} + \mathbf{X}'_{k-1}[n]\mathbf{H}'_{k,k-1} \\ &\quad + \mathbf{X}'_{k+1}[n]\mathbf{H}'_{k,k+1})\mathbf{c}_k + \mathbf{\Gamma}_k\boldsymbol{\eta}[l] \\ &= \mathbf{S}'_k\mathbf{c}_k + \mathbf{\Gamma}_k\boldsymbol{\eta}[l]. \end{aligned} \tag{7}$$

We can note that the length L_k of $\mathbf{c}_k$ is a channel estimator design parameter. L_k can be different for different subchannels depending on how frequency selective the propagation channel is for a certain frequency range. The estimation of $\mathbf{c}_k$ is called in some references as structured channel estimation [12].

The Least-Squares solution to estimate the subchannel IR is then given by

$$\hat{\mathbf{c}}_k = (\mathbf{S}'^H_k\mathbf{S}'_k)^{-1}\mathbf{S}'^H_k\mathbf{y}_k[n]. \tag{8}$$

IV. Simulation Results

We have evaluated the performance of the channel estimation methods for the OQAM FBMC system by performing Monte Carlo simulations. We have assumed a bandwidth of 10 MHz and a sampling rate of 11.2 MHz. As channel model we have chosen the power delay profile of the ITU Vehicular B ($L_{\mathrm{ch}} = 228$) in an static environment, i.e. without any Doppler effects. An FBMC system with $M = 1024$ subcarriers and with a near perfect reconstruction prototype was employed with $K = 4$. We have simulated both structured and full subchannel IR estimation. As training sequences we have used part of the data QPSK sequence transmitted in each subchannel, i.e. no special training sequence design was employed. In other words, the training sequences were pseudo-random sequences of symbols taken from a QPSK alphabet.

In Fig. 5 a comparison between the normalized mean squared error (NMSE) for both full and structured subchannel estimation are shown. The NMSE is defined as

$$\mathrm{NMSE} = \frac{||\hat{g}_k[n] - g_k[n]||^2}{||g_k[n]||^2}, \tag{9}$$

where $g_k[n]$ is the same as in model (2) and $\hat{g}_k[n]$ is its estimated version. In the case of the structured variant also two different lengths of the training sequence were considered. We can see that for the structured method with $N = 10$ and for the full subchannel estimation with $N = 75$ a similar performance is obtained. In the structured method only $L_k = 5$ parameters need to be estimated while in the full subchannel method $g_k[n]$ has length $Q = 17$. For the structured method with $N = 20$

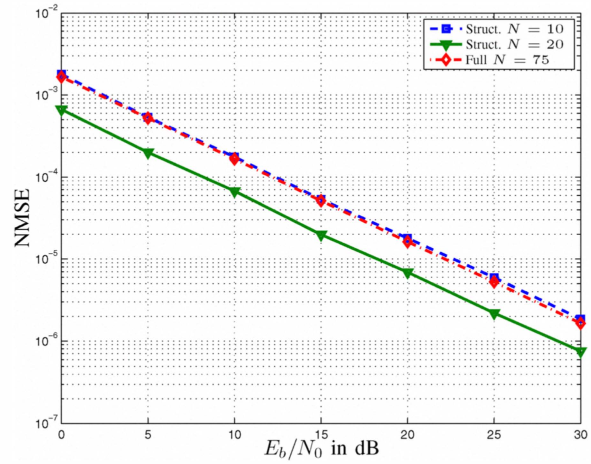

Figure 5. Comparison of NMSE for structured approach ($L_k = 5$) and full subchannel estimation, 1024 subchannels, ITU Veh. B

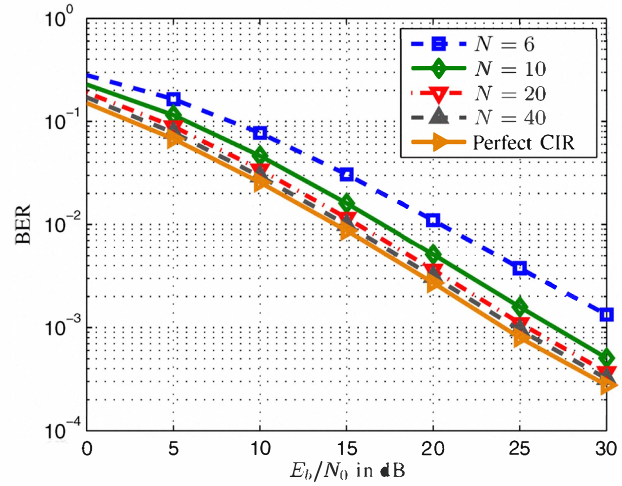

Figure 6. BER for structured approach, 1024 subchannels, $L_k = 5$, ITU Veh. B

a gain of almost $5dB$ in E_b/N_0 for the same NMSE can be observed. It is clear that the structured method allows a reduction in the number of variables to be estimated and as a consequence less training symbols are necessary to achieve a low estimation error.

The BER simulation results for the structured estimation are depicted in Fig. 6. In this case a linear MMSE equalizer was employed in each subchannel and for their calculation the estimated IR's were used. We can see how close the performance of the system can get to the perfect channel IR knowledge for different lengths of the training sequence.

V. Conclusion

We presented in this contribution a method for estimating the propagation channel IR that is experienced in each subcarrier of an FBMC system. With the use of the structured channel estimation we have seen that the number of parameters to be estimated is much lower than a full subchannel IR estimation. As a consequence a shorter training sequence per subchannel is necessary and the complexity of the channel estimator is reduced.

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