Finite sample properties of the QMLE in the ACD-ECOGARCH(1,1) model

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Abstract

Recently The ACD-ECOGARCH$(1,1)$ model was introduced in [7]. In their approach they use an exponential autoregressive conditional duration model to describe the dependence structure in durations of ultra-high-frequency financial data. The innovation process of the ACD model then defines the interarrival times of a compound Poisson process. This compound Poisson process is then the background driving Lévy process of an exponential continuous time GARCH$(1,1)$ process. The parameters of the latter process are estimated by means of quasi maximum likelihood estimation. In this paper we analyse the finite sample properties of this estimator. We conclude with an extended version of the empirical application in [7].

1 Introduction

The fundamental characteristic of tick-by-tick, or also called ultra-high-frequency, data is the irregular spacing of the observation times. This feature prevents the application of standard discrete time econometric models to analyse such kind of data. The reason is of course that in such models the durations between two observations are assumed to be constant. Thus new econometric methods have to be developed for the analysis of ultra-high-frequency data. In doing so one has to deal with several problems. One such problem is that the random durations seem not to be independent but show an autoregressive dependence structure given the past observations. Therefore Engle and Russel [9] introduce the autoregressive conditional duration model (ACD) to describe such a behaviour. Based on the ACD model there were several extensions of the GARCH process developed to model irregularly sampled financial time series. Here we have to mention the ACD-GARCH model of [11] and the work of [8]. Both approaches are summarised and compared in [20]. The authors also propose a further specification of a GARCH model for irregularly spaced data, which incorporates the advantages of the previous two models. Grammig and Wellner [13] extend the UHF-GARCH model of [8] by modelling the interdependence of intraday volatility and trading intensity. All of these models are based on the discrete time weak or strong GARCH process. A different way to model tick-by-tick data is to assume the existence of an underlying continuous time model. Such an approach was developed in [19]. They specify a discrete time approximation of the continuous time GARCH$(1,1)$ process (COGARCH) defined in [15], which is suitable for irregularly spaced observation times. This idea can be extended to other continuous time GARCH or stochastic volatility model as long as the approximation has a tractable form. We refer to [17] and the references therein for an overview of continuous time approximations of GARCH and stochastic volatility models.

Bollerslev et al. [4] report that a model, which is applied to high-frequency financial data, has to be able to describe the so called leverage effect. A continuous time model with a tractable

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discretisation, which further incorporates a leverage effect, is the exponential COGARCH process recently introduced by [14]. However the exponential COGARCH as well as the other approaches based on continuous time models can not directly deal with a dependence structure in the durations between observations. Therefore Czado and Haug [7] combined the ACD model and the exponential COGARCH$(1,1)$ process, abbreviated to ECOGARCH$(1,1)$ in the following, to address both problems, the dependence structure in the durations and the leverage effect. They presented also a quasi maximum likelihood estimator (QMLE) for the parameters of the ECOGARCH process. In this paper we will analyse the finite sample properties of the proposed estimator.

The paper is now organised as follows. In Section 2 we recall ACD-ECOGARH$(1,1)$ model. The QMLE of [7] will be introduced in Section 3. The finite sample properties of the QMLE are then analysed in Section 4 under different simulation scenarios. An illustrative data example is presented in Section 5.

2 ACD-ECOGARCH(1,1) model

We assume to have ultra-high-frequency observations $P_{T_1}, \dots, P_{T_n}$ of an asset log-price at transaction times T_i, $i = 1, \dots, n$. The observed durations ΔT_i will be modelled by an exponential ACD(p,q) model, $p, q \in \mathbb{N}$, as introduced in [9], i.e.

$$\Delta T_i = \psi_i \Delta t_i \,,$$

where

$$\psi_i = \mathbb{E}(\Delta T_i | \Delta T_{i-1}, \dots \Delta T_1) = \omega + \sum_{j=1}^{p} \alpha_j \Delta T_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j} \,.$$

We have to assume an exponential distribution for the innovations Δt_i since they will be the interarrival times of the driving compound Poisson process L of the ECOGARCH$(1,1)$ process, which is defined as follows:

Let L be the compound Poisson process (CPP)

$$L_t = \sum_{k=1}^{J_t} Z_k, \quad t > 0, \qquad L_0 = 0 \,,$$

where $(J_t)_{t \geq 0}$ is an independent Poisson process with intensity $\lambda > 0$ and $(Z_k)_{k \in \mathbb{N}}$ is an i.i.d. sequence independent of J. The jump sizes Z_k are assumed to have a symmetric distribution function $F_{0,1/\lambda}$ with mean 0 and variance $1/\lambda$. Then the ECOGARCH$(1,1)$ process G is defined as the stochastic process satisfying,

$$G_t = \int_0^t \sigma_{s-} dL_s = \sum_{k=1}^{J_t} \sigma_{t_k -} Z_k \,, \quad t > 0, \quad G_0 = 0 \,,$$

where $(t_k)_{k \in \mathbb{N}}$ are the jump times of L. The log-volatility process $(\log(\sigma_t^2))_{t \geq 0}$ is an Ornstein-Uhlenbeck process with state space representation

$$\begin{aligned} \log(\sigma_t^2) &= \mu + b_1 X_t \\ X_t &= e^{-a_1 t} X_0 + \int_0^t e^{-a_1 (t-s)} dM_s \,, \quad t > 0 \,, \end{aligned}$$

nd parameters $\mu, a_1, b_1 \in \mathbb{R}$. Here $X_0 \in \mathbb{R}$ is independent of the driving CPP L and

$$M_t = \sum_{k=1}^{J_t} [\theta Z_k + \gamma |Z_k|] - \gamma \lambda K t \,,$$

with $K = \int_{\mathbb{R}} |x| F_{0,1/\lambda}(dx)$, is a zero mean CPP with parameters $(\theta, \gamma) \in \mathbb{R}^2 \setminus \{\mathbf{0}\}$.

The above assumption implies that the observations G_{t_i} of the ECOGARCH$(1,1)$ process are given by $G_{t_i} := P_{T_i}\,,\; i = 1, \ldots, n\,.$

3 Estimation in the ACD-ECOGARCH(1,1) model

To estimate the parameters in our model we will follow a two step estimation strategy. In a first step the MLE $\widehat{\boldsymbol{\vartheta}}_n^d$ of the parameter $\boldsymbol{\vartheta}^d \in \mathbb{R}^{p+q+1}$ of the ACD(p,q) model will be computed as described in [9]. In our example in Section 5 we consider the cases $p = q = 1$ and $p = q = 2$. Given the observed durations ΔT_i, $i = 1, \ldots, n$, and the MLE $\widehat{\boldsymbol{\vartheta}}_n^d$ we can compute the fitted innovations

$$\widehat{\Delta t_i} = \frac{\Delta T_i}{\widehat{\psi}_i}\,, \qquad i = 1, \ldots, n\,,$$

where $\widehat{\psi}_i = \widehat{\omega}_n + \sum_{j=1}^{p} \widehat{\alpha}_{j_n} \Delta T_{i-j} + \sum_{j=1}^{q} \widehat{\beta}_{j_n} \widehat{\psi}_{i-j}$. In the following we will denote them by Δt_i for ease of notation. Hence after the first estimation step the data is given by the pairs

$$\{(G_{t_i}^{\Delta t_i}, \Delta t_i)\,,\; i = 1, \ldots, n\}\,,$$

where

$$G_{t_i}^{\Delta t_i} := G_{t_i} - G_{t_{i-1}}\,.$$

Since we assume to observe G at n consecutive jump times $0 = t_0 < t_1 < \cdots < t_n$, the state process X of the log-volatility process has the following autoregressive representation

$$\begin{aligned} b_1 X_{t_i} &= b_1 e^{-a_1 \Delta t_i} X_{t_{i-1}} + \sum_{k=J_{t_{i-1}}+1}^{J_{t_i}} b_1 e^{-a_1 (t_i - t_k)} [\theta Z_k + \gamma |Z_k|] - \gamma\lambda \int_{t_{i-1}}^{t_i} b_1 e^{-a_1 (t_i - s)} K ds \\ &= b_1 e^{-a_1 \Delta t_i} X_{t_{i-1}} + b_1 \theta Z_i + b_1 \gamma \left(|Z_i| - \frac{\lambda K}{a_1} (1 - e^{-a_1 \Delta t_i}) \right). \end{aligned} \tag{3.1}$$

Here we used the fact $J_{t_{i-1}} + 1 = J_{t_i} = i$. This implies that the left-hand limit $\log(\sigma_{t_i-}^2)$ of the log-volatility process at the jump times $0 = t_0 < t_1 < \cdots < t_n$ is given by

$$\log(\sigma_{t_i-}^2) = \mu + b_1 e^{-a_1 \Delta t_i} X_{t_{i-1}} - b_1 \gamma \frac{\lambda K}{a_1} (1 - e^{-a_1 \Delta t_i})\,. \tag{3.2}$$

In [14], Proposition 3.1, it is shown that the leverage effect depends on the sign of θb_1. To identify θ as the leverage parameter we will set b_1 equal to one in the following. The observations of the ECOGARCH process are then

$$G_{t_i} = \sum_{k=1}^{J_{t_i}} \sigma_{t_k-} Z_k = G_{t_{i-1}} + \sigma_{t_i-} Z_i\,,$$

which implies that the return at time t_i is equal to $G_{t_i}^{\Delta t_i} := \sigma_{t_i-} Z_i\,.$

Given the data $\{(G_{t_i}^{\Delta t_i}, \Delta t_i)\,,\; i = 1, \ldots, n\}$, we know aim at estimating the remaining unknown parameters $\boldsymbol{\vartheta}^g := (a_1, \theta, \gamma, \mu, \lambda, K) = (\boldsymbol{\vartheta}, \lambda, K)$ in our model. But equation (3.2) contains an identifiability problem. The constant term in (3.2) is given by $\mu^* := \mu - \gamma \frac{\lambda K}{a_1}$. In the quasi maximum likelihood approach, which we will take, only the constant term μ^* is identifiable and not μ, K and λ. Because of that we will estimate the rate λ given only the jump times $t_1, \ldots, t_n$ of the compound Poisson process. K will either be approximated by $\widehat{K}_n := (\frac{\pi}{2} \widehat{\lambda}_n)^{-1/2}$, which is motivated by the fact that $K = (\frac{\pi}{2}\lambda)^{-1/2}$ in case $F_{0,1/\lambda}$ is a normal distribution or

K and ϑ are estimated simultaneously. The consequences of both strategies concerning the estimators of μ and K are analysed in Section 4.3.

To derive a contrast function, which we can maximise with respect to the unknown parameters, we followed [8] by assuming the joint density of the data conditional on past information is given by

$$\rho_{(\lambda,\vartheta)}((G_{t_i}^{\Delta t_i}, \Delta t_i)|\mathbf{G}_{i-1}^{\Delta}, \mathbf{\Delta}_{i-1}) = \rho^{g}_{(\lambda,\vartheta)}(G_{t_i}^{\Delta t_i}|\Delta t_i, \mathbf{G}_{i-1}^{\Delta}, \mathbf{\Delta}_{i-1})\rho^{d}_{\lambda}(\Delta t_i|G_{t_i}^{\Delta t_i}, \mathbf{G}_{i-1}^{\Delta}, \mathbf{\Delta}_{i-1})$$

where $\mathbf{G}_k^{\Delta} = (G_{t_1}^{\Delta t_1}, \dots, G_{t_k}^{\Delta t_k})$, $1 \le k \le n$ and $\mathbf{\Delta}_k$ is defined analogously.

Since the durations Δt_i are the interarrival times of the compound Poisson process L, their distribution is independent of the current and past returns. Hence we write the log-likelihood in the following form

$$\log \rho_{(\lambda,\vartheta)}(\mathbf{G}_n^{\Delta}, \mathbf{\Delta t}_n|X_0) = \sum_{i=1}^{n} \left(\log \rho^{g}_{(\lambda,\vartheta)}(G_{t_i}^{\Delta t_i}|\mathbf{G}_{i-1}^{\Delta}, \mathbf{\Delta t}_i) + \log \rho^{d}_{\lambda}(\Delta t_i|\mathbf{\Delta t}_{i-1})\right),$$

where $\mathbf{G}_k^{\Delta} = (G_{t_1}^{\Delta t_1}, \dots, G_{t_k}^{\Delta t_k})$, $1 \le k \le n$, and $\mathbf{\Delta t}_k$ is defined analogously.

Because of the above mentioned reasons the rate λ of the Poisson process J is estimated by maximising only $\sum_{i=1}^{n} \log \rho^{d}_{\lambda}(\Delta t_i|\mathbf{\Delta}_{i-1})$. In particular this means that we perform in the second step of our estimation procedure again a two-step estimation. In the first step the rate λ can be estimated given only the interarrival times $\mathbf{\Delta}_i$ of the Poisson process J. The MLE of λ is given by

$$\widehat{\lambda}_n := \frac{n}{\sum_{i=1}^{n} \Delta t_i}.$$

To estimate the remaining parameter ϑ in the second step we replace λ by $\widehat{\lambda}_n$ in $\rho^{g}_{(\lambda,\vartheta)}$. Since the conditional distribution of the returns is unknown we will follow a quasi maximum likelihood approach in the second estimation step. Therefore we will use instead of

$$\log \rho_{(\widehat{\lambda}_n,\vartheta)}(\mathbf{G}_n^{\Delta}|\mathbf{\Delta}_i, X_0) = \sum_{i=1}^{n} \log \rho^{g}_{(\widehat{\lambda}_n,\vartheta)}(G_{t_i}^{\Delta t_i}|\mathbf{G}_{i-1}^{\Delta}, \mathbf{\Delta}_i, X_0)$$

the Gaussian quasi log-likelihood

$$\ell_{(\widehat{\lambda}_n,\vartheta)}(\mathbf{G}_n^{\Delta}|\mathbf{\Delta}_i, X_0) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{n}\left(\log(\sigma^2_{t_i-}/\widehat{\lambda}_n) - \frac{(G_{t_i}^{\Delta t_i})^2}{\sigma^2_{t_i-}/\widehat{\lambda}_n}\right) \tag{3.3}$$

as contrast function.

Since the volatility is unobservable, (3.3) can not be evaluated numerically. Therefore we need an approximation of the state process X, which together with (3.2) gives estimates of $\sigma^2_{t_1-}, \dots, \sigma^2_{t_n-}$. Based on equation (3.1) and given ϑ and $\widehat{\lambda}_n$ estimates of the state process X are computed by

$$\begin{aligned} \widehat{X}_{t_i}(\vartheta, \widehat{\lambda}_n) &= e^{-a_1 \Delta t_i}\widehat{X}_{t_{i-1}}(\vartheta, \widehat{\lambda}_n) + \theta\frac{G_{t_i}^{\Delta t_i}}{\widehat{\sigma}_{t_i-}(\vartheta, \widehat{\lambda}_n)} + \gamma\frac{|G_{t_i}^{\Delta t_i}|}{\widehat{\sigma}_{t_i-}(\vartheta, \widehat{\lambda}_n)} \\ &\quad -\gamma\frac{\widehat{\lambda}_n\widehat{K}_n}{a_1}(1 - e^{-a_1\Delta t_i}), \end{aligned} \tag{3.4}$$

$i = 1, \dots, n$, where $\widehat{K}_n := (\frac{\pi}{2}\widehat{\lambda}_n)^{-1/2}$.

The recursion needs a starting value $\widehat{X}_0$. We set $\widehat{X}_0$ equal to the mean value of the stationary distribution of X, which is zero.

Recursion (3.4) together with expression (3.2) provides then estimates of the volatility given by

$$\widehat{\sigma}^2_{t_i-}(\boldsymbol{\vartheta},\widehat{\lambda}_n) := \exp\left(\mu + e^{-a_1\Delta t_i}\widehat{X}_{t_{i-1}}(\boldsymbol{\vartheta},\widehat{\lambda}_n) - \gamma\frac{\widehat{\lambda}_n\widehat{K}_n}{a_1}(1-e^{-a_1\Delta t_i})\right), \quad i=1,\dots,n.$$

Based on the approximation of the volatility we define the quasi log-likelihood function for $\boldsymbol{\vartheta}$ given the data $(\mathbf{G}_n^\Delta, \boldsymbol{\Delta}_n)$ and the MLE $\widehat{\lambda}_n$ by

$$L(\boldsymbol{\vartheta}|\mathbf{G}_n^\Delta,\boldsymbol{\Delta}_n,\widehat{\lambda}_n) := -\frac{1}{2}\sum_{i=1}^{n}\left(\log(\widehat{\sigma}^2_{t_i-}(\boldsymbol{\vartheta},\widehat{\lambda}_n)) + \frac{(G_{t_i}^{\Delta t_i})^2}{\widehat{\sigma}^2_{t_i-}(\boldsymbol{\vartheta},\widehat{\lambda}_n)/\widehat{\lambda}_n}\right). \tag{3.5}$$

Maximising the quasi log-likelihood function (3.5) with respect to $\boldsymbol{\vartheta}$ over the parameter space $\Theta := \mathbb{R}_+ \times \mathbb{R}^3$ yields QML estimates

$$\widehat{\boldsymbol{\vartheta}}_n := \operatorname{argmax}_{\boldsymbol{\vartheta}\in\Theta} L(\boldsymbol{\vartheta}|\mathbf{G}_n^\Delta,\widehat{\lambda}_n) \tag{3.6}$$

of $\boldsymbol{\vartheta}$. As a byproduct we get a parametric estimator of the volatility. If we first determine the QMLE $\widehat{\boldsymbol{\vartheta}}_n$ in (3.6) then we can substitute $\widehat{\boldsymbol{\vartheta}}_n$ into (3.4) and get estimates

$$\widehat{\sigma}^2_{t_i-}(\widehat{\boldsymbol{\vartheta}}_n^g) := \exp\left(\widehat{\mu}_n + e^{-\widehat{a}_{1_n}\Delta t_i}\widehat{X}_{t_{i-1}}(\widehat{\boldsymbol{\vartheta}}_n,\widehat{\lambda}_n) - \widehat{\gamma}_n\frac{\widehat{\lambda}_n\widehat{K}_n}{\widehat{a}_{1_n}}(1-e^{-\widehat{a}_{1_n}\Delta t_i})\right)$$

of the volatility at the jump times $t_1,\dots,t_n$ based on $\widehat{\boldsymbol{\vartheta}}_n^g = (\widehat{a}_{1_n},\widehat{\theta}_n,\widehat{\gamma}_n,\widehat{\mu}_n,\widehat{\lambda}_n,\widehat{K}_n)$.

The performance of the estimator $\widehat{\boldsymbol{\vartheta}}_n^g$ in small samples is now analysed in the following section. The limiting properties of $\widehat{\boldsymbol{\vartheta}}_n^g$ are not available up to now.

4 Simulation study

In [9] it was shown that the ACD model can be estimated with standard GARCH software by taking the square root of the durations as dependent variable. Hence there are a number of results concerning the finite sample properties of maximum likelihood type estimators in ACD models, like e.g. [12] and [16]. On the other hand there are rather few results about finite sample properties of maximum likelihood type estimators in EGARCH models. We like to mention the recent investigation of [21] who compared the performance of maximum likelihood and Whittle estimators in EGARCH models. Therefore we will concentrate in our simulation study on the QMLE of the ECOGARCH parameters. In all of the following simulation cases we will consider a compound Poisson ECOGARCH$(1,1)$ observed at all jump times t_i. In each simulation we condition on the observed number of jumps n. This of course implies that the observation time interval $[0,t_n]$ will be different for each simulation. The estimates will be computed for 1000 independent replications in each case. We compare the estimation results for $n = 500, 1500, 3000$ and 5000 jumps.

In all of the following cases we have taken the parameter a_1 equal to 0.1, the intensity λ equal to 1 and the mean μ of the log-volatility process will be equal to -3. An intensity of 1 is chosen, since in our applications in Section 5 we will take as durations between the jump times the fitted innovations of an exponential ACD(p,q) model.

The leverage parameter θ and γ will vary over the examples. In most of the cases θ will be -0.1 and γ will be equal to 0.4, i.e. we model the leverage effect as observed in stock price data. If $-\gamma < \theta < 0$, this corresponds to the case where a positive shock in the return data increases the log-volatility process less than a negative one of the same magnitude. The last example will illustrate the case where a positive shock in the log-price process increases the log-volatility process more than a negative one of the same size and we denote it as non leverage case. For a more detailed discussion of the leverage effect see also Section 3.1 in [14].

The innovations Z_i will be t-distributed with ν degrees of freedom. The influence of the heavy-tailedness of the distribution of the innovations on the estimation results will also be analysed. Therefore we will consider the three cases $\nu = 6, \nu = 9$ and $\nu = \infty$.

Since we assume $\mathbb{E}(Z_1) = 0$ and $\mathbb{V}\mathrm{ar}(Z_1) = 1/\lambda$ the innovations are defined as

$$Z_i := \sqrt{\frac{\nu - 2}{\nu\lambda}}\varepsilon_i\,,$$

where $(\varepsilon_i)_{i\in\{1,\dots,n\}} \sim$ i.i.d. t_ν.

We compute the empirical mean ($\widehat{\text{mean}}$), relative bias ($\widehat{\text{rbias}}$), and root mean square error ($\widehat{\text{RMSE}}$) for all parameter estimates based on 1000 independent replications.

4.1 Estimation results for different sample sizes

First we want to analyse the influence of an increasing sample size. Therefore we estimated the parameters based on $n = 500, 1500, 3000$ and 5000 jumps. In each case the underlying t-distribution of the innovations had 6 degrees of freedom. Boxplots of the parameter estimates $\widehat{\alpha}_{1_n}, \widehat{\theta}_n, \widehat{\gamma}_n$ and $\widehat{\mu}_n^*$ are shown in Figure 1. The corresponding statistics are summarised in Table 2 in the Appendix.

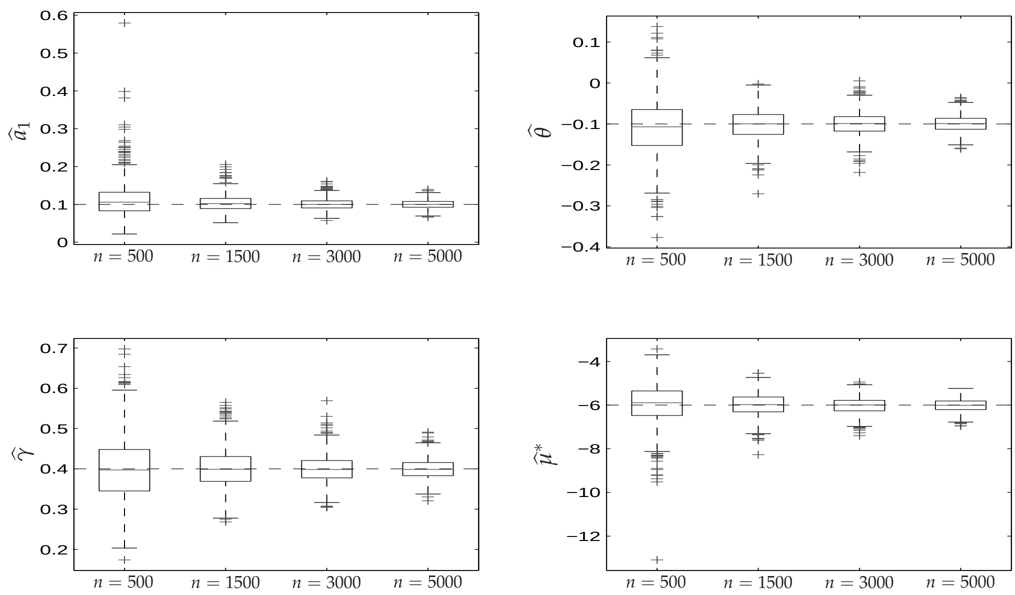

Figure 1: Boxplots of parameter estimates $\widehat{\alpha}_{1_n}^{(k)}, \widehat{\theta}_n^{(k)}, \widehat{\gamma}_n^{(k)}$ and $\widehat{\mu}_n^{*(k)}$, $k = 1, \dots, 1000$, for $\nu = 6$ and sample sizes $n = 500, 1500, 3000$ and 5000.

The relative bias of $\widehat{a}_1$ and $\widehat{\theta}$ decreases with an increasing sample size, whereas for the remaining parameters the relative bias does not change over the different sample sizes. As expected the RMSE of all estimates decreases with the increasing sample size, which can also be seen in Figure 1. From the boxplot of $\widehat{\mu}_n^*$ we recognise that μ^* can indeed be estimated without a systematic bias. This is not the case for μ, which can be seen from the corresponding statistics in Table 2.

For the last example we take a sample size n of 3000 and chose the parameter θ and γ such that $0 < \theta < \gamma$. In particular we set $\theta = 0.1$ and $\gamma = 0.2$. This means that a positive shock in the return data increases the log-volatility process more than a negative one. The remaining parameters are the same as before. We obtain similar results as in the leverage case. The corresponding statistics are shown in the last four rows of Table 2.

The goodness of fit of our estimation method is further investigated by an analysis of the fitted innovations for the case of $n = 3000$ jumps. The fitted innovations are given by $\widehat{Z}_i := G_{t_i}^{\Delta t_i}/\widehat{\sigma}_{t_i-}$, $i = 1, \ldots, n$. Since our innovations are t-distributed with a standard deviation of $1/\sqrt{\lambda}$, we expect the empirical mean $\overline{Z} := \frac{1}{n}\sum_{i=1}^{n} \widehat{Z}_i$ of the fitted innovations to be close to zero, the empirical standard deviation $\left(\frac{1}{n-1}\sum_{i=1}^{n}(\widehat{Z}_i - \overline{Z})^2\right)^{1/2}$ close to $1/\sqrt{\lambda} = 1$ and the empirical skewness close to zero. For all three quantities we computed $\widehat{\text{mean}}$ and $\widehat{\text{RMSE}}$ over all 1000 replications. The results are reported in Table 1 and indicate an adequate fit.

$\nu = 6$	mean($\widehat{Z}_i$)	std($\widehat{Z}_i$)	skewness($\widehat{Z}_i$)
$\widehat{\text{mean}}$	0.0002	0.9998	-0.0017
$\widehat{\text{RMSE}}$	0.0179	0.0091	0.2271

Table 1: Estimated mean and RMSE for the mean, standard deviation and skewness of the fitted innovations based on 1000 replications.

Under the assumption of a correctly estimated volatility the fitted innovations are a white noise series, in particular the innovations and also the squared innovations should be uncorrelated. The correlation of the squared innovations was checked by performing a Ljung-Box test (cf. [18]). The test statistic is given by

$$Q = n(n+2)\sum_{k=1}^{m} \frac{\widehat{\rho}_{\widehat{Z}^2}(k)^2}{n-k}$$

where $\widehat{\rho}_{\widehat{Z}^2}(k)$ is the empirical autocorrelation function of the the squared fitted innovations at lag k for one replication, and asymptotically χ^2-distributed with m degrees of freedom under the null hypothesis of no correlation. The number of lags m taken into account to compute the statistic was set equal to $\sqrt{n}$ (cf. Section 9.4 in [5]). The null hypothesis of no correlation was rejected 83 times out of 1000 simulations at the 0.05 level, which is higher than expected. The empirical mean of the 1000 p-values was equal to 0.60, which shows that a majority of the test statistics has a rather large p-value confirming the hypothesis of no correlation. In the next section we will consider the case of normally distributed innovations. There the null hypothesis was rejected 45 times out of 1000 simulations, which is in line with the 0.05 level of test.

4.2 Estimation results for different jump distributions

In all the previous examples the degrees of freedom of the jump distribution were equal to six. Now we want to analyse the influence of an increasing number of degrees of freedom. We consider the three cases $\nu = 6, \nu = 9$ and $\nu = \infty$. The parameter ϑ is set equal to $(0.1, -0.1, 0.4, -3)$ and we observe $n = 1500$ jumps. Boxplots of the estimated $\widehat{\vartheta}_n^{(k)}$, $k = 1, \ldots, 1000$, can be seen in Figure 2. In contrast to the last section we plotted here a boxplot of $\widehat{\mu}_n$, since the relative bias of $\widehat{\mu}_n$ for an increasing number of degrees of freedom should be analysed. As expected the relative bias as well as the RMSE of $\widehat{\vartheta}$ decrease with increasing

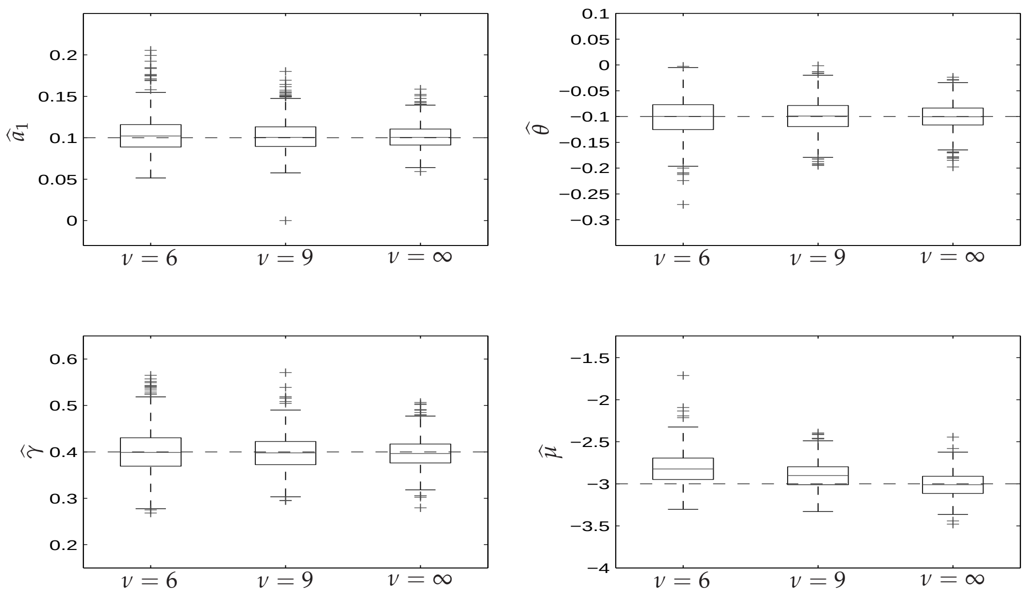

Figure 2: Boxplots of parameter estimates $\widehat{\vartheta}_n^{(k)}$, $k = 1, \ldots, n$, for $\nu = 6$, $\nu = 9$ and $\nu = \infty$.

degrees of freedom. The relative bias of $\widehat{\mu}$ for example reduces to 0.0039 in the normal case compared to a value of -0.0616 in the case of six degrees of freedom. This effect is also expected since for the normal case there is no bias introduced in the estimation of K through $(\frac{\pi}{2}\widehat{\lambda}_n)^{-1/2}$.

4.3 Joint QMLE of all parameters

In this simulation example the parameter K is estimated along with ϑ by means of quasi maximum likelihood estimation. The parameter ϑ is the same as in the first example in Section 4.1. The estimation is based on $n = 1500$ jumps and the jump distribution has six degrees of freedom. Therefore the parameter K is equal to 0.75. The estimation results are compared with those from Section 4.1. Therefore we plotted in Figure 3 boxplots of $\widehat{\vartheta}_n^{(k)}$, $k = 1, \ldots, 1000$,

for both cases. In the bottom row of Figure 3 one can also see the boxplot of $\widehat{K}_n^{(k)}$ along with the boxplot of our previous estimate $(\frac{\pi}{2}\widehat{\lambda}_n)^{-1/2}$. When we estimate ϑ and K by QML we observe a slight increase in the relative bias and RMSE of $\widehat{\vartheta}_n$. For $\widehat{\mu}_n$ the two statistics take on values of 0.1004 and 0.6349. The corresponding values for $\widehat{K}_n$ are -0.0999 and 0.1475. On the right hand side of the bottom row boxplots of the estimators $\widehat{\mu}^*$ of $\mu^* = \mu - \frac{\gamma\lambda K}{a_1}$ are shown for both estimation strategies. In this example μ^* is equal to -6. One can observe that the constant term μ^* in equation (3.2) can be estimated without bias in each case. Identification of μ and K in μ^* nevertheless is not possible. Therefore we will set in the following $\widehat{K}_n$ again equal to $(\frac{\pi}{2}\widehat{\lambda}_n)^{-1/2}$.

4.4 Empirical characteristics

Finally some empirical characteristics of the volatility process will be presented. In particular we computed for each set of parameters the empirical mean $\overline{\sigma^2}$, variance $s^2(\sigma^2)$ and 99% quantile of the volatility process. Further we estimated the correlation $\mathrm{Corr}(\Delta G_{t_i}, \sigma^2_{t_i})$, which

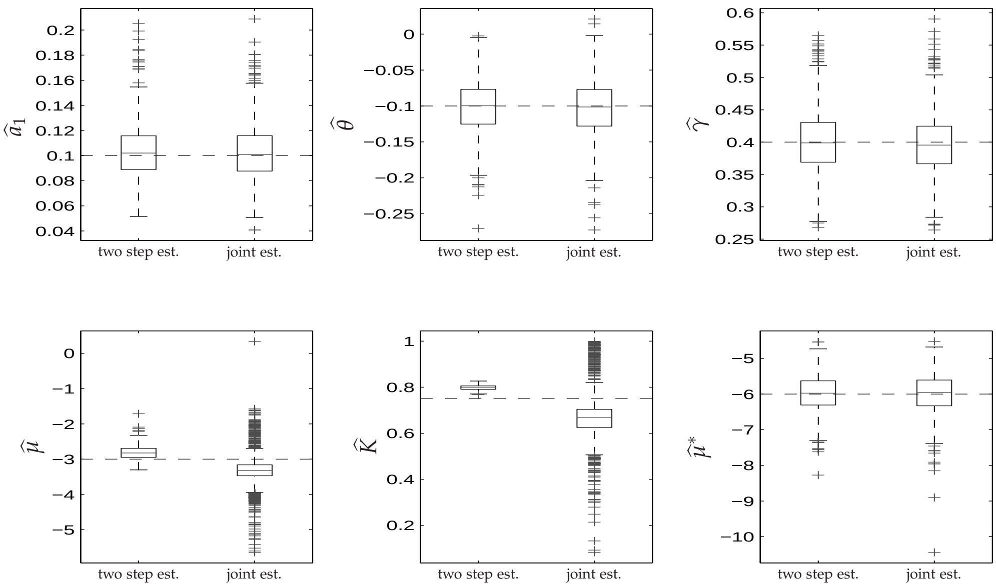

Figure 3: Boxplots of parameter estimates $\widehat{\vartheta}_n^{(k)}$ and $\widehat{K}_n^{(k)}$, $k = 1, \ldots, 1000$, for both estimation strategies for K.

should be negative in the leverage case (see also Section 3.1 in [14] for details). This is done for $\nu = 6$ and $\nu = \infty$ degrees of freedom of the jump distribution. The results are shown in Table 3.

The estimated correlation is negative in all of the leverage cases and positive for the non-leverage case ($\theta = 0.1, \gamma = 0.2$). We further observe a slightly increased variance and greater quantiles in case of $\nu = 6$, which seems reasonable.

5 An illustrative example

As an illustration of the potential usefulness of the ACD-ECOGARCH model we present an extended version of the data example in [7]. The model will be applied to General Motors tick-by-tick data, which was extracted from the Trade and Quote database released by the NYSE. The time period under consideration spans four weeks starting form 6th of May 2002 until the 31st of May. Due to the Memorial Day there was no trading on the 27th of May at the NYSE. Only transaction between 9:30am and 4:00pm are considered. If equal transaction times T_i occurred, the corresponding trades are combined to a single trade at an average price. We omitted consecutive zero returns if they occurred at the beginning or end of the trading day. On average we have about 1960 observations per day.

The data will be analysed on a daily basis to get insight about varying parameter values over the observation period. Since durations in ultra-high-frequency data are characterised by an intraday seasonality, as e.g. reported in [3], [9] or [22], we diurnally adjusted them at first. For that purpose we fitted a cubic smoothing spline to the durations of each day of the week. The diurnally adjusted durations are then computed by dividing each durations with the corresponding smoothing spline value. If we would aim at estimating one model for the whole data set, then the overnight durations have to be adjusted as e.g. explained in [2]. Typically the volatility also shows a deterministic time-of-day effect, see e.g. [8]. We therefore computed diurnally adjusted returns by dividing each returns with the corresponding value of a cubic smoothing spline fitted to the absolute returns. The resulting smoothing splines are shown in Figure 4. The volatility smoothing splines for Wednesday and Thursday show

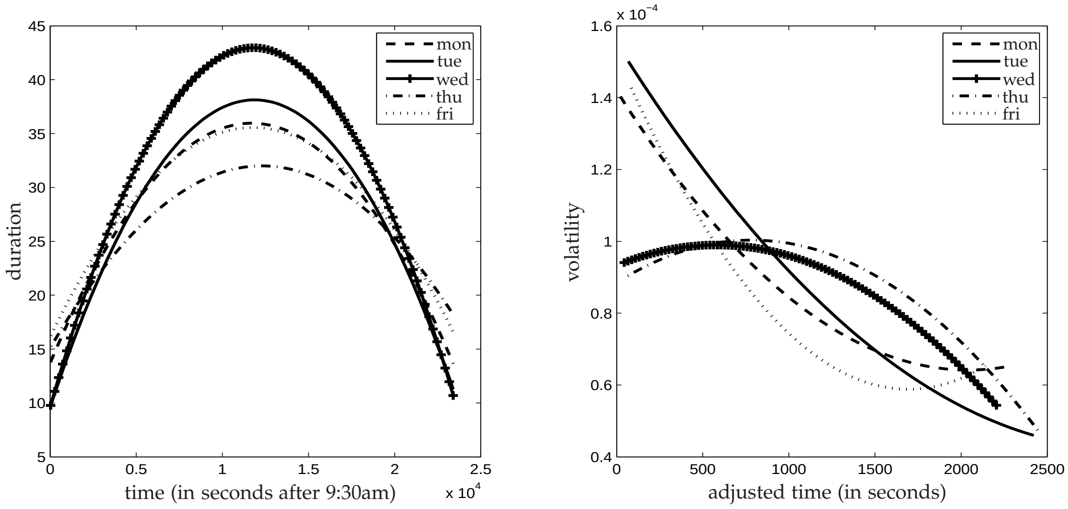

Figure 4: Cubic smoothing splines of durations (*left*) and absolute returns (*right*).

a rather atypical behaviour of slightly increasing during the first half of the trading day and decreasing afterwards. The shapes of the remaining splines are conform with results reported in the literature. Further we have to take into account a market microstructure noise on this fine level. To address this problem we will follow [8], by considering mid quotes, which are the average of the last bid and ask quote just before the trade, as our price data. In particular this means, if we have observation points $T_1, \ldots, T_n$, then the log-price P_{T_i} is given by

$$P_{T_i} = \frac{1}{2}\left(\log(b_{T_i-}) + \log(a_{T_i-})\right), \qquad i = 1, \ldots, n,$$

where b_{T_i-} (a_{T_i-}) denotes the last bid (ask) quote just before or at time T_i.

However one has to be aware that this choice of price measure reduces the econometric issues of bid ask bounce and price discreteness but it does not eliminate these problems as mentioned by [8]. More sophisticated models or approaches in dealing with market microstructure noise can be found e.g. in [1] or [10] and references therein.

Autocorrelation in the diurnally adjusted durations was tested through a Ljung-Box test with 15 degrees of freedom. The hypothesis of no correlation is rejected for all 19 days. An ACD model was estimated for each day. We just considered the cases $p, q \in \{1, 2\}$ and chose p and q such that the test statistic of the Ljung-Box test with 15 degrees of freedom applied to the fitted innovations $\widehat{\Delta} t_i = \frac{\Delta T_i}{\widehat{\psi}_i}$, $i = 1, \ldots, n$, was minimal. The hypothesis of no correlation in the fitted innovations could not be rejected at the 0.05 significance level on each of the 19 days. Except for two days, where we fitted an ACD$(1,1)$ model, an ACD$(2,2)$ model was utilised. The sums over the estimated coefficients vary mainly between 0.6 and 0.9, but significantly smaller values such as 0.18 are also obtained for two of the days. The full set of estimated parameters is given in Table 4. Given the fitted innovations we define through $t_0 = 0, t_i = t_{i-1} + \widehat{\Delta} t_i$, $i = 1, \ldots, n$, the observations of the ECOGARCH process as $G_{t_i} = P_{T_i}$, $i = 1, \ldots, n$.

The parameter ϑ^g of the ECOGARCH$(1,1)$ process is then estimated as explained in Section 3. The estimated parameter values suggest that we have a leverage effect, which is the case if $\widehat{\theta}_n < 0$, on 9 of the 19 days. On these days we observe different types of leverage effects.

All calculations are done using MATLAB 7.6

We have the case that a positive jump to the log-price increases the log-volatility less than a negative one ($-\widehat{\gamma}_n < \widehat{\theta}_n < 0$), the case that a negative jump in the price process decreases the log-volatility less than a positive one ($\widehat{\gamma}_n < \widehat{\theta}_n < 0$) and also the case that a negative jump in the log-price processes increases while a positive one decreases the log-volatility ($\widehat{\theta}_n < -|\widehat{\gamma}_n|$). From equation (3.2) we see that mostly long durations will decrease the volatility as long as $\widehat{\gamma}_n$ is positive, which is the case for 17 of the 19 days. The parameter a_1 reflects strong dependence in the log-volatility process for most of the days by taking on values between 0.0074 and 0.1577. However we also have two days with almost no correlation since $\widehat{a_1}_n$ is equal to 3.9915 and 4.9336 on those days. The estimated parameters $(\widehat{a_1}_n, \widehat{\theta}_n, \widehat{\gamma}_n)$ along with bootstrapped standard errors on days with strong persistence in the log-volatility are shown in Figure 5. Due to the assumptions in the exponential ACD model the jump rate λ should be one, which is confirmed by estimates $\widehat{\lambda}_n$ being close to one. Estimated parameter values for μ can be found in Table 5 along with the other estimated parameters.

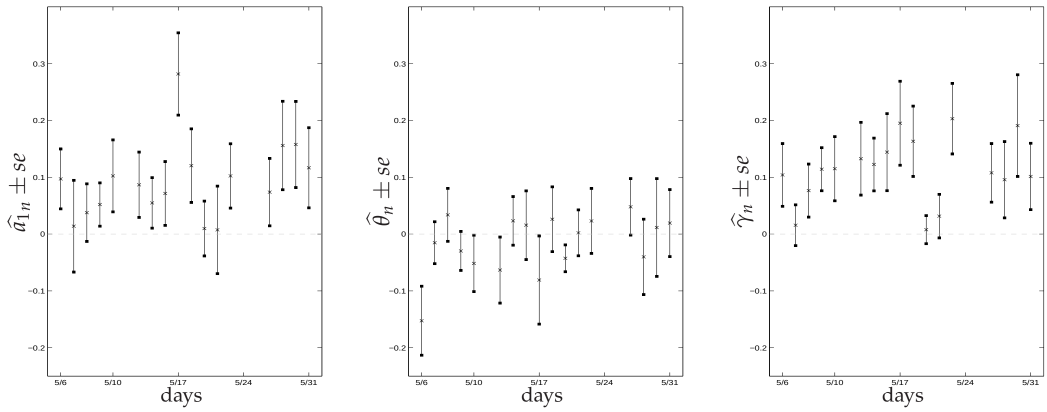

Figure 5: Estimated parameters $(\widehat{a_1}_n, \widehat{\theta}_n, \widehat{\gamma}_n)$ together with bootstrapped standard errors on those days with strong persistence in the log-volatility.

Given the parameter estimate $\widehat{\vartheta}_n^g$ we are able to estimate the volatility, which allows us to compute the fitted innovations $\widehat{Z}_i = G_{t_i}^{\Delta t_i}/\widehat{\sigma}_{t_i-}$. Due to our assumptions there should be no correlation in the squared fitted innovations. Therefore we performed a Ljung-Box test for the squared fitted innovations $\widehat{Z}_i^2$ on each day. The hypothesis of no correlation is rejected at the 0.05 significance level only on May 15th. Over the remaining days the average p value is equal to 0.76. The degrees of freedom df were chosen such that $df \approx \sqrt{n}$.
Mentionable is now that we obtained a suitable fit for most of the days although there is no direct dependence between the volatility and the observed durations in our model. The log-volatility process (3.1) depends only on the i.i.d. sequence of innovations $(\Delta t_i)_{i=1,\ldots,n}$ and not on the observed or conditions durations. Therefore the dependence structure in the durations does not influence the volatility process, which is in contrast to the results in [8], [11] and [20].

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We computed 999 residual-based bootstraps. For details on residual-based bootstrap see e.g. [6] and references therein.

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Appendix

	$\widehat{a}_1$	$\widehat{\theta}$	$\widehat{\gamma}$	$\widehat{\mu}$	$\widehat{\lambda}$
$\boldsymbol{\vartheta}$, λ	0.1000	-0.1000	0.3000	-3.0000	1.0000
n=500					
$\widehat{\text{mean}}$	0.1121	-0.1084	0.3998	-2.8379ï¿$\frac{1}{2}$	1.0007
$\widehat{\text{median}}$	0.1060	-0.1070	0.3973	-2.8377ï¿$\frac{1}{2}$	1.0009
$\widehat{\text{rbias}}$	0.1207	0.0837	-0.0004	-0.0540	0.0007
$\widehat{\text{RMSE}}$	0.0462	0.0691	0.0775	0.3621	0.0463
n = 1500					
$\widehat{\text{mean}}$	0.1037	-0.1011	0.4012	-2.8151	1.0006
$\widehat{\text{median}}$	0.1021	-0.0997	0.3988	-2.8231	1.0003
$\widehat{\text{rbias}}$	-0.0369	0.0109	0.0029	-0.0616	0.0006
$\widehat{\text{RMSE}}$	0.0215	0.0368	0.0469	0.2668	0.0263
n=3000					
$\widehat{\text{mean}}$	0.1008	-0.0999	0.3992	-2.8144	1.0001
$\widehat{\text{median}}$	0.1001	-0.0989	0.3984	-2.8220	1.0002
$\widehat{\text{rbias}}$	0.0077	-0.0010	-0.0020	-0.0619	0.0001
$\widehat{\text{RMSE}}$	0.0148	0.0278	0.0339	0.2313	0.0185
n=5000					
$\widehat{\text{mean}}$	0.1006	-0.0995	0.3996	-2.8125	0.9997
$\widehat{\text{median}}$	0.1001	-0.0992	0.3986	-2.8138	0.9995
$\widehat{\text{rbias}}$	0.0059	-0.0047	-0.0011	-0.0625	-0.0003
$\widehat{\text{RMSE}}$	0.0115	0.0200	0.0255	0.2141	0.0144
$\boldsymbol{\vartheta}$, λ	0.1000	0.1000	0.2000	-3.0000	1.0000
n=3000					
$\widehat{\text{mean}}$	0.1030	0.1017	0.2000	-2.9063	1.0006
$\widehat{\text{median}}$	0.1005	0.1008	0.2004	-2.9068	1.0006
$\widehat{\text{rbias}}$	0.0302	0.0167	0.0001	-0.0312	0.0006
$\widehat{\text{RMSE}}$	0.0221	0.0245	0.0280	0.1273	0.0176

Table 2: Estimated mean, median, relative bias and RMSE for $\widehat{\boldsymbol{\vartheta}}$ and $\widehat{\lambda}$ based on 1000 replications, where the jump distribution has 6 degrees of freedom.

$\nu = 6$	θ, γ	$\overline{\sigma^2}$	$s^2(\sigma^2)$	$\widehat{q}_{0.99}$	$\widehat{\text{Corr}}(\Delta G_{t_i}, \sigma^2_{t_i})$
	-0.10, 0.2	0.0636	0.0163	0.2193	-0.1485
	-0.18, 0.2	0.0681	0.0044	0.3005	-0.2120
	-0.30, 0.2	0.0831	0.1933	0.5338	-0.2615
	-0.10, 0.4	0.1022	0.0594	0.7977	-0.0937
	-0.18, 0.4	0.1109	0.0849	0.9216	-0.1569
	-0.30, 0.4	0.1442	0.9888	1.5152	-0.2151
	0.10, 0.2	0.0632	0.0017	0.2157	0.1476
$\nu = \infty$	θ, γ	$\overline{\sigma^2}$	$s^2(\sigma^2)$	$\widehat{q}_{0.99}$	$\widehat{\text{Corr}}(\Delta G_{t_i}, \sigma^2_{t_i})$
	-0.10, 0.2	0.0633	0.0015	0.2084	-0.1441
	-0.18, 0.2	0.0680	0.0031	0.2822	-0.2085
	-0.30, 0.2	0.0815	0.0116	0.4952	-0.2597
	-0.10, 0.4	0.0999	0.0244	0.7055	-0.0902
	-0.18, 0.4	0.1083	0.0433	0.8415	-0.1459
	-0.30, 0.4	0.1368	0.2569	1.3444	-0.2073
	0.10, 0.2	0.0633	0.0015	0.2076	0.1449

Table 3: Empirical mean, variance and 99% quantile of the volatility process with parameters $a_1 = 0.1, \mu = -3$ and $\lambda = 2$. The empirical correlation between the current jump in the log-price process and future volatility is given in the last column.

Date	$\widehat{\omega}$	$\widehat{\alpha}_1$	$\widehat{\alpha}_2$	$\widehat{\beta}_1$	$\widehat{\beta}_2$
06.05.02	0.1283	-0.0017	0.0210	1.5040	-0.8127
07.05.02	0.1440	0.0911	0.0407	-0.0170	0.5835
08.05.02	0.0529		0.1306	0.7378	
09.05.02	0.4184	0.1317	0.1107	-0.4870	0.3426
10.05.02	0.0505	0.0692	0.0241	0.1434	0.6404
13.05.02	0.3247	0.0514	0.0490	0.5774	-0.4942
14.05.02	0.1321	0.0892	0.1038	-0.1713	0.6794
15.05.02	0.0405	0.0524	0.0931	-0.0119	0.7729
16.05.02	0.1131	0.0905	0.1082	-0.1923	0.7434
17.05.02	0.0716	0.0494	0.0239	0.9628	-0.197
20.05.02	0.0905	0.0345	0.0420	0.6706	0.0728
21.05.02	0.0888	0.0462	0.0893	-0.0273	0.7013
22.05.02	0.0694	0.0920	0.0105	0.5491	0.2100
23.05.02	0.1768	0.0724	0.0616	-0.2586	0.7525
24.05.02	0.1434	0.0725	0.0965	-0.0582	0.6242
28.05.02	0.1191	0.0076	0.0987	0.5600	0.0713
29.05.02	0.1362	0.0889	0.1451	0.1449	0.3085
30.05.02	0.1777	0.1006	0.1149	-0.2278	0.5433
31.05.02	0.0964	0.0590		0.7065	

Table 4: Estimated parameters of the ACD model for the GM data over the time span 06.05.02-31.05.02.

Date	$\widehat{a}_1$	$\widehat{\theta}$	$\widehat{\gamma}$	$\widehat{\mu}$	$\widehat{\lambda}$
06.05.02	0.0971 (0.0527)	-0.1525 (0.0607)	0.1041 (0.0553)	1.8894 (0.2358)	1.0002
07.05.02	0.0139 (0.0807)	-0.0152 (0.0369)	0.0156 (0.0359)	2.0372 (0.3730)	0.9963
08.05.02	0.0377 (0.0507)	0.0338 (0.0466)	0.0766 (0.0467)	2.6524 (0.4785)	1.0000
09.05.02	0.0520 (0.0381)	-0.0297 (0.0343)	0.1142 (0.0379)	2.1216 (0.2727)	0.9988
10.05.02	0.1024 (0.0633)	-0.0516 (0.0496)	0.1151 (0.0565)	1.9692 (0.2500)	1.0018
13.05.02	3.9915 (1.8212)	-0.8347 (0.5182)	-0.9002 (0.6043)	1.1546 (0.1016)	1.0002
14.05.02	0.0868 (0.0575)	-0.0633 (0.0581)	0.1327 (0.0640)	0.8511 (0.3453)	0.9996
15.05.02	0.0549 (0.0445)	0.0233 (0.0428)	0.1225 (0.0464)	1.9026 (0.4311)	0.9753
16.05.02	0.0715 (0.0562)	0.0157 (0.0604)	0.1442 (0.0678)	2.0433 (0.4352)	1.0011
17.05.02	0.2818 (0.0724)	-0.0809 (0.0776)	0.1951 (0.0739)	1.9266 (0.2432)	0.9976
20.05.02	0.1204 (0.0648)	0.0261 (0.0570)	0.1633 (0.0619)	2.0354 (0.2546)	0.9918
21.05.02	0.0098 (0.0483)	-0.0426 (0.0236)	0.0078 (0.0247)	1.5622 (0.3508)	1.001
22.05.02	0.0074 (0.0770)	0.0022 (0.0404)	0.0316 (0.0383)	2.4512 (0.6781)	0.9984
23.05.02	0.1024 (0.0566)	0.0231 (0.0572)	0.2031 (0.0621)	2.2063 (0.2914)	1.0005
24.05.02	4.9336 (2.1456)	0.8747 (0.4857)	-0.5814 (0.4403)	1.4882 (0.1053)	0.9959
28.05.02	0.0738 (0.0593)	0.0480 (0.0498)	0.1078 (0.0516)	2.2197 (0.2848)	0.9984
29.05.02	0.1559 (0.0778)	-0.0401 (0.0663)	0.0957 (0.0672)	1.5319 (0.2994)	0.9999
30.05.02	0.1577 (0.0757)	0.0116 (0.0861)	0.1909 (0.0895)	2.0133 (0.3712)	0.9999
31.05.02	0.1167 (0.0705)	0.0194 (0.0590)	0.1014 (0.0584)	1.5649 (0.2560)	0.9996

Table 5: Estimated parameters of the ECOGARCH$(1,1)$ process for the GM data over the time span 06.05.02-31.05.02 with bootstrapped standard errors in parenthesis.