OPTIMUM CHIP PULSE SHAPE DESIGN FOR TIMING SYNCHRONIZATION

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ABSTRACT

In this work a systematic methodology is presented to design optimum chip pulse shapes for DS-CDMA systems for timing synchronization. A nonlinear bi-objective problem with additional constraints is established. The derived optimum chip pulse shapes show significant improvement in timing synchronization performance.

Index Terms— chip pulse shape, DS-CDMA, timing synchronization, time-delay estimation, GNSS

1. INTRODUCTION

Chip pulse shape design for timing synchronization with DS-CDMA systems and in particular for Global Navigation Satellite Systems (GNSS) shall provide minimum error in time-delay estimation while also achieving robustness in its estimation. Maximizing the synchronization accuracy can be attained by minimizing the Cramer-Rao lower bound (CRLB) for time-delay estimation. Robustness in the estimation of the time-delay, thus tracking and acquisition robustness can be achieved by limiting the absolute value of the sidelobes of the autocorrelation function of the signal. Furthermore, bandwidth efficiency is an important objective, as well as maximizing time concentration of the chip pulse shape and providing a fast decay in time domain. Multiple access interference (MAI) and interchip interference (ICI) need to be controlled to maintain system performance [1, 2] and especially for GNSS, we need to account for spectral separation to other signals. Thus, not only intra system MAI but also inter system MAI between different GNSS which transmit signals in the same frequency band has to be considered [3]. In GNSS binary offset carrier (BOC) signals [4] are used to accomplish spectral separation of different services of different GNSS. BOC signals are a practical solution, but they sacrifice bandwidth efficiency in terms of a low time-bandwidth product in order to achieve acceptable inter system MAI. A low time-bandwidth product results to large chip durations, low symbol rates, and finally low data rates.

In this work a systematic approach to design optimum strictly band-limited chip pulse shapes for DS-CDMA systems for timing synchronization is established. Timing synchronization with GNSS is considered. The proposed methodology makes it possible to formulate the problem of designing optimum chip pulse shapes in terms of achieving a trade-off between timing synchronization accuracy and time concentration of the chip pulse shape while accounting for acquisition and tracking robustness, as a tractable nonlinear multi-objective optimization problem. Additional constraints can be introduced to the problem in order to take into account further properties. Especially, inter system MAI and a fast decay of the chip pulse in time domain (smooth cut-off) is considered. This methodology is based on the prolate spheroidal wave functions (PSWF) [5], which enable to transform the primal variational problem into a dual, tractable parametric optimization problem.

2. SYSTEM MODEL

We assume coherent downconversion of the radio frequency signal to baseband. The received DS-CDMA baseband signal is given by

$$y(t) = \sqrt{P}\, c(t-\tau) + n(t), \tag{1}$$

where P denotes the signal power, $c(t)$ is the pseudo random (PR) sequence, τ is the time-delay, and $n(t)$ is white Gaussian noise with two-sided power spectral density $N_0/2$. Thus, the PR sequence is given by

$$c(t) = \sum_{k=-\infty}^{\infty} d_k\, \delta(t-kT_c) * h(t) = \sum_{k=-\infty}^{\infty} d_k\, h(t-kT_c), \tag{2}$$

where $h(t)$ denotes the chip pulse shape which is not necessarily restricted to be time-limited to only one chip interval T_c. The PR sequence is a binary, zero-mean wide-sense cyclostationary (WSCS) sequence with $\{d_k\} \in \{-1, 1\}$ and has period $T = N_d T_c$. $N_d \in \mathbb{N}$ denotes the number chips of the PR sequence $c(t)$. The autocorrelation of $c(t)$ can be given by

$$R_c(\varepsilon) = \int_{-\frac{T}{2}}^{\frac{T}{2}} c(t)\, c^*(t+\varepsilon)\, dt = \int_{-\infty}^{\infty} |H(f)|^2 \mathrm{e}^{\mathrm{j}2\pi f\varepsilon}\, df, \tag{3}$$

where $H(f)$ denotes the Fourier transform of the chip pulse shape $h(t)$ and the PR sequence is assumed to be random.

3. STATEMENT OF THE PROBLEM

The objective of this work is to establish a design methodology to systematically derive optimized chip pulse shapes for DS-CDMA systems for timing synchronization, which are strictly band-limited, thus $H(f) = 0\ ,\ |f| > B$. The optimization is performed with respect to maximizing timing synchronization accuracy on the one hand, and on the other hand with respect to maximizing time concentration of the chip pulse shape within a desired interval of chip duration $[-T_c/2, T_c/2]$. Additional constraints can be considered. Maximization of timing synchronization accuracy can be accomplished by minimizing the CRLB for the time-delay τ. Maximizing the time concentration of the chip pulse shape is directly related with the absolute value of the sidelobes of the chip pulse shape and its autocorrelation function $R_c(\varepsilon)$. The maximum of the absolute value of the sidelobes of $R_c(\varepsilon)$ is given by

$$\underset{i\in\mathbb{N}}{\forall} |\nu_i| \leq \kappa, \tag{4}$$

where ν_i denote the value of $R_c(\varepsilon)$ at the sidelobes, besides the global maximum of $R_c(\varepsilon)$ at $\varepsilon = 0$, and $\kappa \in [0, 1]$. The higher κ, the less robust time-delay estimation, and thus tracking and acquisition becomes.

Furthermore, for a given spreading gain of the DS-CDMA system, a small time-bandwidth product ϱ is desired in order to ensure bandwidth efficiency. Additionally, MAI and ICI need to be controlled. Here, intra system MAI, inter system MAI, and ICI are to be considered for the signal design.

Minimizing the CRLB of the time-delay and maximizing time concentration are two conflicting tasks and thus only a trade-off between these two objectives can be achieved for a fixed time-bandwidth product ϱ. Such problems are called multiple-objective problems [6], where it is only possible to improve one objective at the cost of the other.

3.1. Synchronization Accuracy

The variance of the time-delay estimation error σ_τ^2 of any unbiased estimator is lower bounded by the CRLB [7]

$$\sigma_\tau^2 \geq \frac{B_n}{\frac{P}{N_0}\,4\pi^2}\,\frac{\int_{-\infty}^{\infty} |H(f)|^2\,df}{\int_{-\infty}^{\infty} f^2\,|H(f)|^2\,df}, \tag{5}$$

where B_n denotes the equivalent noise bandwidth of the generic estimator.

The first objective of our trade-off is to minimize σ_τ^2. This minimization is subject to the constraint

$$\int_{-B}^{B} |H(f)|^2\,df = 1. \tag{6}$$

Instead of minimizing (5) considering (6) we can maximize the second moment of the power spectrum $\int_{-\infty}^{\infty} f^2\,|H(f)|^2 df$, where it is straight forward to show that

$$\int_{-B}^{B} f^2\,|H(f)|^2 df \leq \int_{-B}^{B} B^2\,|H(f)|^2 df \leq B^2, \tag{7}$$

subject to (6). Thus, $|H(f)|^2 = \frac{1}{2}\left(\delta(f-B)+\delta(f+B)\right)$ maximizes $\int_{-\infty}^{\infty} f^2\,|H(f)|^2 df$ and the pulse shape $h(t)$ results to either $h(t) = \cos(2\pi Bt)$ or $h(t) = \sin(2\pi Bt)$. This denotes the analytical solution of the first objective of the trade-off where maximum synchronization accuracy in terms of minimizing the CRLB is achieved.

3.2. Time Concentration

The second objective of our trade-off is to maximize the time concentration of $h(t)$ within the interval $[-T_c/2, T_c/2]$. This also achieves better acquisition and tracking robustness by minimizing the sidelobes of $h(t)$ and consequently the sidelobes of $R_c(\varepsilon)$ (low κ).

It has been shown in [5] that for any time-bandwidth product $\varrho = T_c B$

$$\int_{-T_c/2}^{T_c/2} |h(t)|^2 dt \leq \chi_0(\varrho), \tag{8}$$

subject to (6). Here, $\chi_0(\varrho)$ is the eigenvalue of the function $\psi_0(\varrho, t)$. The function $\psi_0(\varrho, t)$ has the largest eigenvalue of the PSWF [5]. Hence, the other extremal solution of our trade-off between maximum time concentration and maximum synchronization accuracy can be given in closed form solution by $h(t) = \psi_0(\varrho, t)$.

3.3. Smooth Cut-Off

Considering a real physical system a smooth cut-off of $H(f)$ at $\pm B$ is desirable, where $H(f)$ is strictly band-limited to $[-B, B]$ and $H(f)$ is assumed to be continuous in $[-B, B]$. Thus, we define the constraint

$$H(f) = 0\ ,\ |f| = B. \tag{9}$$

A smooth cut-off provides a faster asymptotic decay of $h(t)$ for $t \mapsto \pm\infty$ and thus this leads to less time support which is needed in signal generation.

3.4. Interchip Interference (ICI), Intra and Inter System Multiple Access Interference (MAI)

Following, [8, p.23 et seq.], and [1–3] we consider ICI, intra system MAI (MAI-A) and inter system MAI (MAI-R) as interference components with zero mean. In general ICI and both MAI-A and MAI-R are dependent on the propagation characteristics of the transmitted signal. We consider U users (e.g. visible GNSS satellites) with $u = 1, \ldots, U$ and power P_u causing MAI-A. Further, we assume that V users of another system (e.g. visible satellites of a different GNSS) with $v = 1, \ldots, V$ and power P_v are causing MAI-R. The received signal of another system in the same frequency band has power spectrum density (PSD) $\Phi_R(f)$. Thus, the ratio of the signal to noise ratio (SNR) with respect to the signal to interference plus noise ratio (SINR) can be given as [8, p.23 et seq.]

$$\begin{aligned}\Delta\mathrm{SNR} = 1 + &\underbrace{\frac{P}{N_0} 2T_c \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \left[\int_{-\infty}^{\infty} |H(f)|^2\,\cos(2\pi l T_c f)\,df\right]^2}_{\text{ICI}} \\ &+ \underbrace{\sum_{u=1}^{U} \frac{P_u}{N_0} \int_{-B}^{B} |H(f)|^4\,df}_{\text{MAI-A}} \\ &+ \underbrace{\sum_{v=1}^{V} \frac{P_v}{N_0}\ \int_{-\infty}^{\infty} |H(f)|^2\,\Phi_R(f)\,df}_{\text{MAI-R}}\,. \end{aligned}\tag{10}$$

In our problem at hand we will take into account ICI and MAI-A, but we will not introduce them to the optimization. However, we will introduce constraints in order to limit MAI-R with respect to a design parameter $\mu \in \mathbb{R}^+$

$$\int_{-B}^{B} |H(f)|^2\,\Phi_R(f)\,df \leq \mu. \tag{11}$$

4. NONLINEAR BI-OBJECTIVE PROBLEM

We can establish a nonlinear bi-objective optimization problem with several constraints. We apply the weighting method [6]. The weighting method aggregates the multiple objectives linearly into a single objective function. Weights are applied to derive a weighted sum of the different objective functions. The first objective function is to minimize the CRLB (5), thus to maximize $\int_{-B}^{B} f^2 |H(f)|^2 \, df$ subject to $\int_{-B}^{B} |H(f)|^2 \, df = 1$. The other objective of the trade-off is to maximize (8). The objective functions can be normalized with respect to B^2 and $\chi_0(\varrho)$ as shown in (7) and (8), respectively. We also introduce a constraint to achieve a smooth cut-off following (9) and further constraints in order to limit MAI-R given in (11).

In order to transform the primal variational problem into a dual parametric optimization problem we use an adequate set of strictly band-limited orthonormal basis functions. Due to their special properties (cf. Subsection 3.2) we propose to use the PSWF $\psi_m(\varrho, t)$ [5] and we define the expansion

$$h(t) = \sum_{m=0}^{M-1} x_m \, \psi_m(\varrho, t) \ \text{ and } \ H(f) = \sum_{m=0}^{M-1} x_m \, \Psi_m(\varrho, f), \tag{12}$$

where $\Psi_m(\varrho, f)$ denotes the Fourier transform of $\psi_m(\varrho, t)$ and $x_m \in \mathbb{R}$ are the expansion coefficients. In particular, the PSWF have the very interesting property of being orthonormal in $]-\infty, \infty[$ and also being orthogonal in the finite interval $[-T_c/2, T_c/2]$.

Now, we can formulate the parametric nonlinear bi-objective optimization problem as a weighted sum of two quadratic forms with the weight $w \in [0, 1]$:

$$\max_{\mathbf{x}} \left\{ \mathbf{x}^{\mathrm{T}} \left(\frac{w}{B^2} \, \mathbf{S}(\varrho) + \frac{(1-w)}{\chi_0(\varrho)} \, \mathbf{T}(\varrho) \right) \mathbf{x} \right\}, \tag{13}$$

$$\text{s.t. } \ ||\mathbf{x}||_2^2 = 1, \tag{14}$$

$$\mathbf{G}^{\mathrm{T}} \, \mathbf{x} = \mathbf{0}, \tag{15}$$

$$\text{and } \ w \in [0, 1]. \tag{16}$$

Here,

$$\mathbf{S}(\varrho) = \begin{bmatrix} s_{0\,0} & \cdots & s_{0\,N-1} \\ \vdots & \ddots & \vdots \\ s_{M-1\,0} & \cdots & s_{M-1\,N-1} \end{bmatrix} \in \mathbb{R}^{M \times M}, \tag{17}$$

where $s_{m\,n} = \int_{-\infty}^{\infty} f^2 \, \Psi_m(\varrho, f) \, \Psi_n^*(\varrho, f) df$, $n = 0, \ldots, N-1$, and

$$\mathbf{T}(\varrho) = \operatorname{diag}\{\boldsymbol{\chi}(\varrho)\} \ \in \ \mathbb{R}^{M \times M}. \tag{18}$$

Further, $\boldsymbol{\chi}(\varrho) = [\chi_0(\varrho), \ldots, \chi_{M-1}(\varrho)]^{\mathrm{T}} \in \mathbb{R}^{M \times 1}$ and $\mathbf{x} = [x_0, \ldots, x_{M-1}]^{\mathrm{T}} \in \mathbb{R}^{M \times 1}$ denotes the decision vector. The constraint matrix $\mathbf{G}$ is defined as

$$\mathbf{G} = \ [\mathbf{f}_R \ \mathbf{f}_I \ \mathbf{q}_1 \ \ldots \ \mathbf{q}_d] \ \in \ \mathbb{R}^{M \times (2+d)}, \tag{19}$$

with

$$\mathbf{f}_R = [\mathrm{Re}\{\Psi_0(\varrho, B)\}, \ldots, \mathrm{Re}\{\Psi_{M-1}(\varrho, B)\}]^{\mathrm{T}} \ \in \mathbb{R}^{M \times 1}, \tag{20}$$

and

$$\mathbf{f}_I = [\mathrm{Im}\{\Psi_0(\varrho, B)\}, \ldots, \mathrm{Im}\{\Psi_{M-1}(\varrho, B)]^{\mathrm{T}} \ \in \mathbb{R}^{M \times 1}, \tag{21}$$

which achieve a smooth cut-off. $\mathbf{q}_1, \ldots, \mathbf{q}_d$ denote the $d \in \mathbb{N}$ dominant eigenvectors of $\mathbf{M}(\varrho)$, where

$$\int_{-B}^{B} |H(f)|^2 \, \Phi_R(f) \, df = \mathbf{x}^{\mathrm{T}} \, \mathbf{M}(\varrho) \, \mathbf{x} \leq \mu, \tag{22}$$

with

$$\mathbf{M}(\varrho) = \begin{bmatrix} m_{0\,0} & \cdots & m_{0\,N-1} \\ \vdots & \ddots & \vdots \\ m_{M-1\,0} & \cdots & m_{M-1\,N-1} \end{bmatrix} \in \mathbb{R}^{M \times M}, \tag{23}$$

where $m_{m\,n} = \int_{-B}^{B} \Psi_m(\varrho, f) \, \Psi_n^*(\varrho, f) \, \Phi_R(f) df$. MAI-R is limited by choosing d such that (22) is fulfilled with respect to a defined $\mu \in \mathbb{R}^+$.

The defined nonlinear bi-objective optimization problem results to an eigenvalue problem with respect to the trade-off introduced by $w \in [0, 1]$, as the matrices $\mathbf{S}(\varrho)$, $\mathbf{T}(\varrho)$, and $\mathbf{M}(\varrho)$ are positive semi-definite. Thus, the complete Pareto optimal set can be easily generated.

5. PARETO OPTIMAL FINAL SOLUTIONS

We consider a design example in the E1/L1 band for GNSS with a time-bandwidth product $\varrho = 1$ where the chip duration is $T_c = 488.76$ ns and the bandwidth $B = 2.046$ MHz. A maximum ΔSNR, ΔSNR_{max} using (10) is considered. Following [3] we assume that $P_u = -154$ dBW for all u which is the maximum power defined for the Galileo Open Service, that $U = 11$ which is the maximum number of visible Galileo satellites which contribute to MAI-A, that $P_v = -153$ dBW for all v which is the maximum power defined for GPS C/A code signal, that $V = 12$ which is the maximum number of visible GPS satellites which contribute to MAI-R, that $\Phi_R(f)$ to be the PSD of GPS C/A signal , and that $N_0 = -204$ dBW/Hz. In order to assess the timing synchronization accuracy we define the CRLB-I as a lower bound which considers noise plus ICI, maximum MAI-A, and maximum MAI-R:

$$\tilde{\sigma}_\tau^2 \geq \sigma_\tau^2 \cdot \Delta\text{SNR}_{max}. \tag{24}$$

In order to achieve a limited MAI-R we only introduce one dominant eigenvector ($d = 1$) of $\mathbf{M}(\varrho)$ to the constraint matrix $\mathbf{G}$. We will see in the following, that this yields acceptable levels of MAI-R for all the derived final solutions. With a more restrictive choice of the design parameter μ and thus larger d, further reduction of MAI-R could be achieved.

We restrict ourself to design $h(t)$ with even symmetry and we consider the PSWF with m even and $M = 40$, which provides sufficient precision for deriving the Parteo optimal set and the final solutions. Out of the Pareto optimal set we select the final solutions OPT(0.3) and OPT(0.5) with $\kappa = 0.3$ and $\kappa = 0.5$, respectively.

We compare the performance of the final solutions to a BOC(1,1) (Manchester bi-phase) signal [4] with $\kappa = 0.5$ which provides limitation of MAI-R with respect to the given $\Phi_R(f)$, but only achieves $\varrho = 2$. BOC signals are a practical solution in order to limit MAI-R, however they have a decreased bandwidth efficiency in terms of a higher time-bandwidth product. The final solutions OPT(0.3) and OPT(0.5) have $\varrho = 1$ and thus provide double the chip rate and consequently double the data rate than realizable with a BOC(1,1) signal while having the same bandwidth.

In Table 1 ΔSNR_{max}, ICI_{max}, MAI-A_{max}, and MAI-R_{max} are given for the final solutions and a BOC(1,1) signal. We observe that the BOC(1,1) signal has a significantly higher ΔSNR_{max} than the final solutions OPT(0.3) and OPT(0.5). ICI in general is negligible with respect to MAI-A and MAI-R.

$h(t)$	OPT(0.3)	OPT(0.5)	BOC(1,1)
ICI$_{max}$	0.0038	0.0098	0.0003
MAI-A$_{max}$	0.29	0.40	0.49
MAI-R$_{max}$	0.27	0.17	0.30
ΔSNR$_{max}$	1.56 (1.93 dB)	1.58 (1.97 dB)	1.79 (2.52 dB)

Table 1. ICI$_{max}$, MAI-A$_{max}$, and MAI-R$_{max}$ for $h(t)$.

In Figure 1 and Figure 2 the normalized pulse shape $h(t)$ and the normalized spectrum $|H(f)|^2$ are depicted for the final solutions OPT(0.3) and OPT(0.5). Also $\Phi_R(f)$ and the PSD of a BOC(1,1) signal are given.

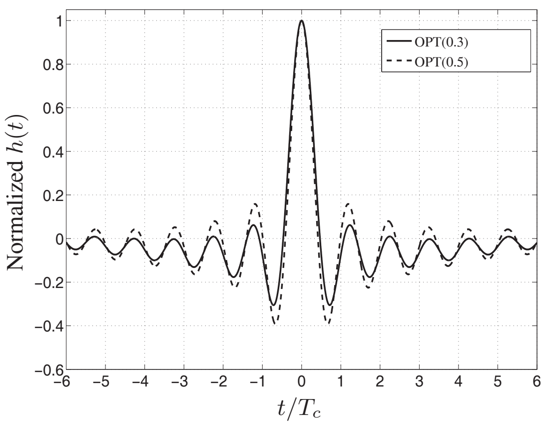

Fig. 1. Normalized $h(t)$.

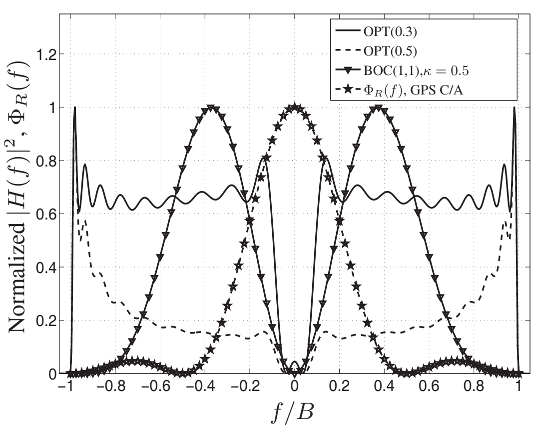

Fig. 2. Normalized $|H(f)|^2$ and $\Phi_R(f)$, GPS C/A.

Figure 3 depicts the CRLB-I as defined in (24) with $B_n = 1$ Hz for the selected final solutions and BOC(1,1). The final solutions OPT(0.3) and OPT(0.5) provide significantly increased synchronization accuracy while achieving larger spectral separation with $\Phi_R(f)$ than a BOC(1,1) signal.

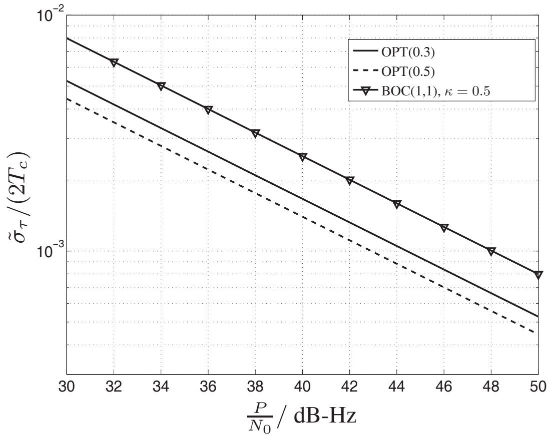

Fig. 3. CRLB-I with $B_n = 1$ Hz.

6. CONCLUSIONS

The proposed methodology for optimum chip pulse shape design provides a flexible and systematic approach in order to account for all important properties for timing synchronization. The complete Pareto optimal set of the defined nonlinear bi-objective problem can be easily derived by solving the resulting weighted sum of two quadratic forms in terms of an eigenvalue problem. With the proposed methodology bandwidth efficiency in terms of low time-bandwidth products can be accounted for. Thus, chip pulse shapes which are optimized for timing synchronization performance can be derived which provide significant service improvement for timing synchronization, in particular for GNSS.

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