This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

Transceiver Design in Multiuser MISO Systems with Limited Feedback

Mario H. Castañeda, Israa Slim and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München, Munich, Germany
Email: {maca,issl,jono}@nws.ei.tum.de

***Abstract*—It is well known that the availability of M transmit antennas at the base station enables to serve K single-antenna users ($K \leq M$), under the assumption of perfect *channel state information* (CSI) at the base station. However, in practical *frequency division duplex* (FDD) systems, the channel knowledge available at the base station is not perfect since it is obtained through a limited feedback in the uplink consisting of B bits per user. Part of the B feedback bits can be employed to quantize the *channel direction information* (CDI), while the rest can be used to quantize the *channel magnitude information* (CMI) of each user. In this work we address the transceiver design of a multiuser MISO system based on the minimum mean square error criterion and with transmit CSI obtained through a limited feedback composed of quantized CDI and CMI. We treat the case of $K \leq M$ users with different average channel gains and based on our findings we conclude that for uncorrelated channels it is optimum to employ all the B bits for quantizing the CDI under the assumption that the base station knows the statistics of the CMI of the users.**

I. Introduction

Deploying M transmit antennas at the *base station* (BS) permits to serve K single-antenna users ($K \leq M$) assuming perfect CSI at the base station. Nonetheless, perfect CSI is not a realistic assumption in several scenarios as in an FDD system. Due to the lack of reciprocity between the uplink and downlink in FDD systems, transmit CSI for the downlink is obtained through limited feedback in the uplink consisting of B bits per user [1]. The K users must first, however, estimate their downlink channel with a common downlink pilot.

With the estimated channels at the users, there are several strategies that have been studied in the literature, to convey transmit CSI to the BS with limited feedback of the K users. For instance, several works on limited feedback in multiuser systems [2], [3], [4], [5] consider relaying a quantized version of the *channel direction information* (CDI), i.e. the normalized estimated channel vector of each user. Zero-forcing beamforming based on quantized CDI with uniform power allocation among the users has been considered in [2] and [3], while MMSE beamforming has been analyzed in [4]. The MMSE criterion is also considered in [5] but under the assumption that the users have multiple receive antennas. Another possible approach for the limited feedback is to quantize the entire estimated channel vector according to a mean-square error criterion employing a generalized Lloyd vector quantizer as it is done in [6], where the tranmission strategy was a MMSE based precoding. A third feasible method is to quantize separately the CDI of the estimated channel and the *channel magnitude information* (CMI), i.e. the channel norm of the estimated channel, as described for instance in [7] and [8].

In contrast to the previous works, we address the transceiver design of a multiuser MISO system based on the *minimum mean square error* (MMSE) criterion and with transmit CSI obtained through a limited feedback composed of quantized CDI and CMI. The first approach of solely quantizing the CDI is also included as a special case. After each user has estimated its downlink channel, each of the K users compute their estimated CDI and CMI. The estimated CDI is quantized using the *random vector quantization* (RVQ) scheme [9] with B_{CDI} bits, while the estimated CMI is quantized using a *Lloyd Max Quantizer* (LMQ) with B_{CMI} bits. The whole feedback message consists of $B = B_{\text{CMI}} + B_{\text{CDI}}$ bits, which are then relayed back to the BS in the uplink. Hence, the BS has only access to a quantized version of the estimated CDI and CMI. The transceiver is designed based on this quantized CDI and CMI in order to minimize the sum MMSE of the users.

We treat the case of $K \leq M$ users with different average channel gains and given a limited feedback of B bits, we are interested in knowing how many bits should be employed to quantize the CDI and the CMI, i.e. to determine B_{CDI} and B_{CMI} in order to minimize the MMSE. Our results show that for uncorrelated channels and $K \leq M$ it is optimum for each user to employ all the B bits for quantizing the CDI. This holds under the assumption that the base station also knows the CMI statistics of the users. To this end, this paper is organized as follows. The system model and the cases with perfect CSI and perfect CDI are presented in Section II. Section III and Section IV discuss the transceiver design based on quantized CDI with the CMI statistics and based on quantized CDI and CMI, respectively. Simulation results are shown in Section V and the paper is concluded in Section VI.

II. System Model and Perfect CDI

We consider a multiuser MISO downlink with a BS equipped with M antennas in a single isolated cell transmitting to K i.i.d. single-antenna users with $K \leq M$. Denote the independent zero-mean transmitted signals with variance σ_s^2 for the K users as $\boldsymbol{s} \in \mathbb{C}^K$ with covariance matrix $\boldsymbol{R}_s = \sigma_s^2 \mathbf{I}_K$. Define also the channel from the BS to user k as $\boldsymbol{h}_k \in \mathbb{C}^M$ and the channel matrix $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_K]^{\mathrm{T}} \in \mathbb{C}^{K \times M}$. We assume the elements of $\boldsymbol{h}_k \ \forall\, k$ to be i.i.d. zero-mean complex

Gaussian random variables with variance γ_k^2 for $k = 1, \ldots, K$, i.e. user k has basically an average channel gain γ_k with Rayleigh fading with unit-variance. With a linear precoder $\boldsymbol{P}$ at the BS and a transmit power contraint tr $\left(\boldsymbol{P}\boldsymbol{R}_s\boldsymbol{P}^{\mathrm{H}}\right) \leq P_{\mathrm{DL}}$, where P_{DL} is the available transmit power at the BS, the estimated signal at the K users is given by

$$\hat{\boldsymbol{s}} = g\left(\boldsymbol{H}\boldsymbol{P}\boldsymbol{s} + \boldsymbol{n}\right), \tag{1}$$

where g is a scalar receive filter applied at each user since there is no cooperation among the single-antenna users and $\boldsymbol{n} \in \mathbb{C}^K$ is the collection of the zero-mean AWGN experienced at each user with covariance matrix $\boldsymbol{R}_n = \sigma_n^2\mathbf{I}_K$. Under the assumption of *perfect CSI*, i.e. $\boldsymbol{H}$ is known at the BS, g and $\boldsymbol{P}$ can be computed in order to minimize the system *mean square error* (MSE) E $\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_2^2\right]$ subject to the power constraint, i.e.

$$\{\boldsymbol{P}, g\} = \underset{\{\boldsymbol{P},g\}}{\operatorname{argmin}} \ \mathrm{E}\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_2^2\right] \quad \text{s.t.} \quad \mathrm{tr}\left(\boldsymbol{P}\boldsymbol{R}_s\boldsymbol{P}^{\mathrm{H}}\right) \leq P_{\mathrm{DL}}, \tag{2}$$

where the expectation is taken over $\boldsymbol{s}$ and $\boldsymbol{n}$. The solution to (2) is [10]

$$g = \sqrt{\frac{\mathrm{tr}\left(\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \xi\mathbf{I}_M\right)^{-2}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_s\boldsymbol{H}\right)}{P_{\mathrm{DL}}}} \tag{3}$$

$$\boldsymbol{P} = \frac{1}{g}\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \xi\mathbf{I}_M\right)^{-1}\boldsymbol{H}^{\mathrm{H}}, \tag{4}$$

where

$$\xi = \frac{\mathrm{tr}\left(\boldsymbol{R}_n\right)}{P_{\mathrm{DL}}} = \frac{K\sigma_n^2}{P_{\mathrm{DL}}}. \tag{5}$$

In addition, the MMSE with perfect CSI, i.e. given $\boldsymbol{H}$ is

$$\mathrm{MMSE}_{\mathrm{p}} = \xi\,\mathrm{tr}\left(\left(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} + \xi\mathbf{I}_K\right)^{-1}\boldsymbol{R}_s\right). \tag{6}$$

Now let us consider as a reference the case when the users have access to their perfect CSI but the BS has access only to the perfect CDI and to the CMI statistics of the users. The perfect CDI consists of $\frac{\boldsymbol{h}_k}{\|\boldsymbol{h}_k\|_2}$ $\forall k$. Define $\boldsymbol{H}_{\mathrm{n}} = \left[\frac{\boldsymbol{h}_1}{\|\boldsymbol{h}_1\|_2}, \cdots, \frac{\boldsymbol{h}_K}{\|\boldsymbol{h}_K\|_2}\right]^{\mathrm{T}}$ such that $\boldsymbol{H} = \boldsymbol{B}\boldsymbol{H}_{\mathrm{n}}$ with the perfect CMI (channel norm) of the users on the diagonal of $\boldsymbol{B} = \mathrm{diag}\left([\|\boldsymbol{h}_1\|_2, \cdots, \|\boldsymbol{h}_K\|_2]\right)$[1].

In order to match the channel observed by the BS, each user k multiplies its received signal with $\frac{1}{\|\boldsymbol{h}_k\|_2}$ besides a scalar filter g_{n}, such that the resulting channel $\boldsymbol{B}^{-1}\boldsymbol{H} = \boldsymbol{H}_{\mathrm{n}}$ is inherently known at the BS. The multiplication with $\frac{1}{\|\boldsymbol{h}_k\|_2}$ represents a normalization with each user's instantaneous CMI. Let us denote the precoder and the scalar receiver with perfect CDI and with the multiplication of $\frac{1}{\|\boldsymbol{h}_k\|_2}$ at each user k as $\boldsymbol{P}_{\mathrm{n}}$ and g_{n}, respectively. The estimated signal is given as

$$\hat{\boldsymbol{s}}_{\mathrm{n}} = g_{\mathrm{n}}\boldsymbol{B}^{-1}\left(\boldsymbol{B}\boldsymbol{H}_{\mathrm{n}}\boldsymbol{P}_{\mathrm{n}}\boldsymbol{s} + \boldsymbol{n}\right) = g_{\mathrm{n}}\left(\boldsymbol{H}_{\mathrm{n}}\boldsymbol{P}_{\mathrm{n}}\boldsymbol{s} + \boldsymbol{B}^{-1}\boldsymbol{n}\right), \tag{7}$$

since $\boldsymbol{B}^{-1} = \mathrm{diag}\left(\left[\frac{1}{\|\boldsymbol{h}_1\|_2}, \cdots, \frac{1}{\|\boldsymbol{h}_K\|_2}\right]\right)$. The *overall* scalar receiver for each user k is actually $\frac{g_{\mathrm{n}}}{\|\boldsymbol{h}_k\|_2}$ which is based on the CDI and CMI, while $\boldsymbol{P}_{\mathrm{n}}$ is based only on the CDI and on the statistics of the CMI. The effective noise $\boldsymbol{B}^{-1}\boldsymbol{n}$ has covariance matrix $\sigma_n^2\mathrm{E}\left[\boldsymbol{B}^{-2}\right] = \frac{\sigma_n^2}{M-1}\boldsymbol{\Gamma}^{-2}$ because

$$\mathrm{E}\left[\boldsymbol{B}^{-2}\right] = \frac{1}{M-1}\boldsymbol{\Gamma}^{-2}, \tag{8}$$

since the diagonal of $\boldsymbol{B}^{-2}$ contains the inverse of i.i.d chi-squared random variables with $2M$ degrees of freedom each with variance $\frac{\gamma_k^2}{2}$ and where $\boldsymbol{\Gamma} = \mathrm{diag}\left([\gamma_1, \cdots, \gamma_K]\right)$. $\boldsymbol{P}_{\mathrm{n}}$ and g_{n} are computed in order to minimize the MSE E $\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}_{\mathrm{n}}\|_2^2\right]$ subject to the power constraint, where the expectation is taken over $\boldsymbol{s}$ and the effective noise $\boldsymbol{B}^{-1}\boldsymbol{n}$, by which we also average over the CMI of the users. The effective noise in this case is still uncorrelated with the normalized channel since the CMI and the CDI are independent [9]. With the aid of Lagrangian multipliers, the solution to the optimization problem with perfect CDI and the normalization with the CMI at each user k results in

$$g_{\mathrm{n}} = \sqrt{\frac{\mathrm{tr}\left(\left(\boldsymbol{H}_{\mathrm{n}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{n}} + \xi_{\mathrm{n}}\mathbf{I}_M\right)^{-2}\boldsymbol{H}_{\mathrm{n}}^{\mathrm{H}}\boldsymbol{R}_s\boldsymbol{H}_{\mathrm{n}}\right)}{P_{\mathrm{DL}}}} \tag{9}$$

$$\boldsymbol{P}_{\mathrm{n}} = \frac{1}{g_{\mathrm{n}}}\left(\boldsymbol{H}_{\mathrm{n}}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{n}} + \xi_{\mathrm{n}}\mathbf{I}_M\right)^{-1}\boldsymbol{H}_{\mathrm{n}}^{\mathrm{H}}, \tag{10}$$

where

$$\xi_{\mathrm{n}} = \frac{\sigma_{\mathrm{n}}^2\,\mathrm{tr}\left(\mathrm{E}\left[\boldsymbol{B}^{-2}\right]\right)}{P_{\mathrm{DL}}}. \tag{11}$$

The MMSE given the users' CDI $\boldsymbol{H}_{\mathrm{n}}$, i.e. averaging over the noise and the users' CMI $\boldsymbol{B}$, results in

$$\mathrm{MMSE}_{\mathrm{n}} = \xi_{\mathrm{n}}\sigma_s^2\,\mathrm{tr}\left(\left(\boldsymbol{H}_{\mathrm{n}}\boldsymbol{H}_{\mathrm{n}}^{\mathrm{H}} + \xi_{\mathrm{n}}\mathbf{I}_K\right)^{-1}\right). \tag{12}$$

We point out, that this MMSE is just employed as an intermediate step in order to compute afterwards the average MMSE, which is obtained by averaging $\mathrm{MMSE}_{\mathrm{n}}$ over $\boldsymbol{H}_{\mathrm{n}}$.

III. Quantized CDI: $B = B_{\mathrm{CDI}}$

In an FDD system, the BS obtains transmit CSI for the downlink through the limited feedback of B bits. To this end, each user k obtains a minimum mean square error estimate of its downlink channel using a common pilot of length $T_{\mathrm{DL}} \geq M$ which is emitted from the base station. Since the transmit power at the base station is allocated over M antennas, we have that the error variance is given as [11]

$$\sigma_{e_k}^2 = \frac{\gamma_k^2}{1 + \gamma_k^2\frac{P_{\mathrm{DL}}}{M\sigma_n^2}T_{\mathrm{DL}}}, \tag{13}$$

where P_{DL} is the transmit power at the BS and σ_n^2 is the variance of the AWGN at the users. Denote the estimated channel of user k as $\hat{\boldsymbol{h}}_k \in \mathbb{C}^M$ and $\hat{\boldsymbol{H}} = \left[\hat{\boldsymbol{h}}_1, \cdots, \hat{\boldsymbol{h}}_K\right]^{\mathrm{T}} \in \mathbb{C}^{K\times M}$. The channel from all users can be written as

$$\boldsymbol{H} = \hat{\boldsymbol{H}} + \boldsymbol{E}, \tag{14}$$

where $\boldsymbol{E} = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_K]^{\mathrm{T}} \in \mathbb{C}^{K\times M}$, $\boldsymbol{e}_k$ is the estimation error vector when user k estimates its channel. The elements of $\boldsymbol{e}_k$ are zero-mean complex Gaussian random variables with variance $\sigma_{e_k}^2$ while the elements of $\hat{\boldsymbol{h}}_k$ are zero-mean complex Gaussian random variables with variance $(1 - \sigma_{e_k}^2)$.

[1]The operator diag($\boldsymbol{x}$) with $\boldsymbol{x} \in \mathbb{C}^K$ results in a $K \times K$ matrix with all zeros except on the diagonal, which contains the elements of $\boldsymbol{x}$. If $\boldsymbol{X} \in \mathbb{C}^{K\times K}$, diag($\boldsymbol{X}$) returns a vector with the elements of the diagonal of $\boldsymbol{X}$.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

The MMSE estimate $\hat{\boldsymbol{h}}_k$ has the property that $\mathrm{E}\left[\hat{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{e}_k\right] = \boldsymbol{0}$ and furthermore due to the independence assumption among the users, we have that $\mathrm{E}\left[\hat{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{e}_j\right] = \boldsymbol{0}\ \forall j,k$, such that $\mathrm{E}\left[\hat{\boldsymbol{H}}^{\mathrm{H}}\boldsymbol{E}\right] = \boldsymbol{0}$.

After estimating its channel, user k normalizes its estimated channel $\hat{\boldsymbol{h}}_k$ in order to obtain $\hat{\boldsymbol{h}}_{k,\mathrm{n}} = \frac{\hat{\boldsymbol{h}}_k}{\|\hat{\boldsymbol{h}}_k\|_2}$, which represents an estimate of the CDI with $\|\hat{\boldsymbol{h}}_k\|_2$ as an estimate of the CMI. We treat first the case when all the limited feedback is employed for quantizing the estimated CDI, i.e. $B = B_{\mathrm{CDI}}$. Each user k then quantizes its estimated CDI, i.e. the *normalized* channel $\hat{\boldsymbol{h}}_{k,\mathrm{n}}$, with B bits, which are then relayed back to the BS. For the quantization, we consider *random vector quantization* (RVQ) [9] by taking the minimum chordal distance as follows

$$\hat{\boldsymbol{h}}_{k,\mathrm{q}} = \underset{\boldsymbol{t}_{k,j}\in\mathcal{C}_k}{\mathrm{argmax}}\ |\boldsymbol{t}_{k,j}^{\mathrm{H}}\hat{\boldsymbol{h}}_{k,\mathrm{n}}|, \tag{15}$$

where each user k has a different codebook $\mathcal{C}_k$ consisting of 2^B unit-norm random beamforming vectors $\boldsymbol{t}_{k,j}\in\mathcal{C}_k$, which is also available at the BS. We assume a different codebook for each user, since otherwise there exists a non-zero probability that two or more users feed back the same channel vector. Denote $\hat{\boldsymbol{H}}_{\mathrm{q}} = \left[\hat{\boldsymbol{h}}_{1,\mathrm{q}},\cdots,\hat{\boldsymbol{h}}_{K,\mathrm{q}}\right]^{\mathrm{T}} \in \mathbb{C}^{K\times M}$. After *error-free* feedback of the B bits from each user, the BS would have access to $\hat{\boldsymbol{H}}_{\mathrm{q}}$ as transmit CSI. Note that $\hat{\boldsymbol{h}}_{k,\mathrm{q}}$ represents a quantized version of the estimated *channel direction information* of user k. Let us denote $c_k = \hat{\boldsymbol{h}}_{k,\mathrm{q}}^{\mathrm{H}}\hat{\boldsymbol{h}}_{k,\mathrm{n}} \in \mathbb{C}$ with $|c_k|\le 1$ since

$$|c_k| = \|\hat{\boldsymbol{h}}_{k,\mathrm{q}}\|_2\|\hat{\boldsymbol{h}}_{k,\mathrm{n}}\|_2\cos\theta_k = \cos\theta_k, \tag{16}$$

since $\hat{\boldsymbol{h}}_{k,\mathrm{q}}$ and $\hat{\boldsymbol{h}}_{k,\mathrm{n}}$ have unit norm and where θ_k is the angle between $\hat{\boldsymbol{h}}_{k,\mathrm{q}}$ and $\hat{\boldsymbol{h}}_{k,\mathrm{n}}$ as depicted exemplary in Figure 1.

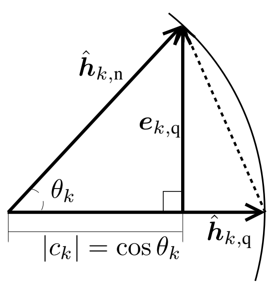

Fig. 1. Quantized and Estimated CDI

As pointed out in [12], quantizing a unit norm vector according to the chordal distance as in (15), produces a phase uncertainty, i.e. $\arg(c_k) \neq 0$. Nevertheless, as it will be shown later on, this phase uncertainty can be removed at the receiver since the receiver of user k has access to c_k and hence, there is no need to feed back the phase $\arg(c_k)$ to the BS as it is suggested in [12].

With $c_k = \hat{\boldsymbol{h}}_{k,\mathrm{q}}^{\mathrm{H}}\hat{\boldsymbol{h}}_{k,\mathrm{n}}$, we can express the normalized estimated channel in terms of the quantized channel as

$$\hat{\boldsymbol{h}}_{k,\mathrm{n}} = c_k\hat{\boldsymbol{h}}_{k,\mathrm{q}} + \boldsymbol{e}_{k,\mathrm{q}}, \tag{17}$$

such that $\hat{\boldsymbol{h}}_{k,\mathrm{q}}$ and the quantization error $\boldsymbol{e}_{k,\mathrm{q}}$ are orthogonal, i.e. $\hat{\boldsymbol{h}}_{k,\mathrm{q}}^{\mathrm{H}}\boldsymbol{e}_{k,\mathrm{q}} = 0$ as shown in Figure 1. This implies that

$$\mathrm{diag}\left(\hat{\boldsymbol{H}}_{\mathrm{q}}^{\mathrm{H}}\boldsymbol{E}_{\mathrm{q}}\right) = \boldsymbol{0}, \tag{18}$$

which has to be considered when designing the transceiver. This means that $\boldsymbol{e}_{\mathrm{q},k}$ lies in the nullspace of $\hat{\boldsymbol{h}}_{k,\mathrm{q}}^{\mathrm{H}}$ and is uniformly distributed in the $M-1$ dimensional nullspace. In addition, we have that [4]

$$\|\boldsymbol{e}_{k,\mathrm{q}}\|_2^2 = \sin^2\theta_k. \tag{19}$$

Denote $\boldsymbol{E}_{\mathrm{q}} = [\boldsymbol{e}_{1,\mathrm{q}},\cdots,\boldsymbol{e}_{K,\mathrm{q}}]^{\mathrm{T}}$, $\boldsymbol{C} = \mathrm{diag}([c_1,\cdots,c_K])$ and $\hat{\boldsymbol{B}} = \mathrm{diag}\left(\left[\|\hat{\boldsymbol{h}}_1\|_2,\cdots,\|\hat{\boldsymbol{h}}_K\|_2\right]\right)$. Using (17) and collecting the estimated CDI of all users, we have

$$\hat{\boldsymbol{H}}_{\mathrm{n}} = \boldsymbol{C}_k\hat{\boldsymbol{H}}_{\mathrm{q}} + \boldsymbol{E}_{\mathrm{q}}, \tag{20}$$

and the channel $\boldsymbol{H}$ from (14) can be rewritten as

$$\boldsymbol{H} = \hat{\boldsymbol{B}}\boldsymbol{C}\hat{\boldsymbol{H}}_{\mathrm{q}} + \hat{\boldsymbol{B}}\boldsymbol{E}_q + \boldsymbol{E}. \tag{21}$$

In order to match the channel observed at the BS, each user k multiplies its received signal with $\frac{1}{c_k\|\hat{\boldsymbol{h}}_k\|_2}$ besides the scalar filter g_{q}. Multiplying with $\frac{1}{c_k}$ removes the phase uncertainty discussed before, since $\arg\left(\frac{1}{c_k}\right) = -\arg(c_k)$. The estimated signal of the K users is given by

$$\hat{\boldsymbol{s}}_{\mathrm{q}} = g_{\mathrm{q}}\,\boldsymbol{C}^{-1}\hat{\boldsymbol{B}}^{-1}(\boldsymbol{H}\boldsymbol{P}_{\mathrm{q}}\boldsymbol{s}+\boldsymbol{n}) = g_{\mathrm{q}}\left(\hat{\boldsymbol{H}}_{\mathrm{q}}\boldsymbol{P}_{\mathrm{q}}\boldsymbol{s}+\boldsymbol{n}_{\mathrm{q}}\right), \tag{22}$$

with the effective noise

$$\boldsymbol{n}_{\mathrm{q}} = \boldsymbol{C}^{-1}\boldsymbol{E}_q\boldsymbol{P}_{\mathrm{q}}\boldsymbol{s} + \boldsymbol{C}^{-1}\hat{\boldsymbol{B}}^{-1}\boldsymbol{E}\boldsymbol{P}_{\mathrm{q}}\boldsymbol{s} + \boldsymbol{C}^{-1}\hat{\boldsymbol{B}}^{-1}\boldsymbol{n}. \tag{23}$$

The scalar receiver at user k is actually $\frac{g_{\mathrm{q}}}{c_k\|\hat{\boldsymbol{h}}_k\|_2}$ which is based not only on the estimated CDI but also on the estimated CMI and c_k.

We are now interested in computing $\boldsymbol{P}_{\mathrm{q}}$ and g_{q} which minimize the MSE based on $\hat{\boldsymbol{H}}_{\mathrm{q}}$ and on the statistics of the estimated CMI, i.e. $\hat{\boldsymbol{B}}$, which is the information available at the BS. The optimizaton problem reads as

$$\{\boldsymbol{P}_{\mathrm{q}}, g_{\mathrm{q}}\} = \underset{\{\boldsymbol{P}_{\mathrm{q}}, g_{\mathrm{q}}\}}{\mathrm{argmin}}\ \mathrm{E}\left[\|\boldsymbol{s}-\hat{\boldsymbol{s}}_{\mathrm{q}}\|_2^2\right]\ \text{ s.t. } \mathrm{tr}\left(\boldsymbol{P}_{\mathrm{q}}\boldsymbol{R}_s\boldsymbol{P}_{\mathrm{q}}^{\mathrm{H}}\right) \le P_{\mathrm{DL}}, \tag{24}$$

where the expected value is taken over $\boldsymbol{s}$, $\boldsymbol{n}$, $\boldsymbol{E}$, $\hat{\boldsymbol{B}}$, $\boldsymbol{C}$ and $\boldsymbol{E}_{\mathrm{q}}$ since only $\hat{\boldsymbol{H}}_{\mathrm{q}}$ is known at the BS. Note that we do not consider the feedback of $\cos\theta_k$ as in [13]. The solution to (24) can be derived with the aid of Lagrangian multipliers in a similar fashion as for the quantized CDI case in [4], where it is assumed that the users have the same average channel gain. Due to lack of space we do not show the derivation here. The optimum g_{q} and $\boldsymbol{P}_{\mathrm{q}}$ read as:

$$g_{\mathrm{q}} = \sqrt{\frac{\mathrm{tr}\left(\left((1-\kappa)\hat{\boldsymbol{H}}_{\mathrm{q}}^{\mathrm{H}}\hat{\boldsymbol{H}}_{\mathrm{q}} + \xi_{\mathrm{q}}\mathbf{I}_M\right)^{-2}\hat{\boldsymbol{H}}_{\mathrm{q}}^{\mathrm{H}}\boldsymbol{R}_s\hat{\boldsymbol{H}}_{\mathrm{q}}\right)}{P_{\mathrm{DL}}}} \tag{25}$$

$$\boldsymbol{P}_{\mathrm{q}} = \frac{1}{g_{\mathrm{q}}}\left((1-\kappa)\hat{\boldsymbol{H}}_{\mathrm{q}}^{\mathrm{H}}\hat{\boldsymbol{H}}_{\mathrm{q}} + \xi_{\mathrm{q}}\mathbf{I}_M\right)^{-1}\hat{\boldsymbol{H}}_{\mathrm{q}}^{\mathrm{H}}, \tag{26}$$

where

$$\kappa = \frac{\mathrm{E}\left[\tan^2\theta_k\right]}{M-1} \tag{27}$$

$$\xi_{\mathrm{q}} = K\kappa + \left[\cos^{-2}\theta_k\right]\mathrm{tr}\left(\mathrm{E}\left[\hat{\boldsymbol{B}}^{-2}\right]\left(\boldsymbol{R}_{\mathrm{e}} + \frac{\sigma_n^2}{P_{\mathrm{DL}}}\mathbf{I}_K\right)\right),$$

978-1-4244-5638-3/10/$26.00 ©2010 IEEE

with $\mathrm{E}\left[\tan^2\theta_k\right] \approx \frac{\mathrm{E}\left[\sin^2\theta_k\right]}{1-\mathrm{E}\left[\sin^2\theta_k\right]}$ and $\mathrm{E}\left[\sin^2\theta_k\right]$ given as [2]

$$\mathrm{E}\left[\sin^2\theta_k\right] = 2^B \,\mathrm{Beta}\left(2^B, \frac{M}{M-1}\right). \tag{28}$$

Furthermore, we have $\boldsymbol{R}_e = \mathrm{diag}\left(\left[\sigma_{e_1}^2, \cdots, \sigma_{e_K}^2\right]\right)$ and $\mathrm{E}\left[\hat{\boldsymbol{B}}^{-2}\right] = \left(\boldsymbol{\Gamma}^2 - \boldsymbol{R}_e\right)^{-2}$, where we observe that $\mathrm{E}\left[\hat{\boldsymbol{B}}^{-2}\right]$ depends on both the average channel gain γ_k of the users, which can be obtained via measurements in the uplink, and on the estimation error variance $\sigma_{e_k}^2$ of the users, which can be computed at the BS assuming the knowledge of the fading statistics of the users.

The resulting MMSE given the quantized estimated CDI $\hat{\boldsymbol{H}}_\mathrm{q}$, i.e. averaging over $\boldsymbol{n}$, $\boldsymbol{E}$, $\boldsymbol{E}_\mathrm{q}$, $\hat{\boldsymbol{B}}$, and $\boldsymbol{C}$, results in

$$\mathrm{MMSE_q} = \sigma_s^2 \,\mathrm{tr}\Big(\left(\xi_\mathrm{q}\mathbf{I}_K - \kappa\hat{\boldsymbol{H}}_\mathrm{q}\hat{\boldsymbol{H}}_\mathrm{q}^\mathrm{H}\right) \times \left(\left(1-\kappa\right)\hat{\boldsymbol{H}}_\mathrm{q}\hat{\boldsymbol{H}}_\mathrm{q}^\mathrm{H} + \xi_\mathrm{q}\mathbf{I}_K\right)^{-1}\Big), \tag{29}$$

which is a function of $\hat{\boldsymbol{H}}_\mathrm{q}$ and B. This MMSE represents solely an intermediate step in order to compute the average MMSE, which is obtained by averaging $\mathrm{MMSE_q}$ over $\boldsymbol{H}_\mathrm{q}$.

IV. Quantized CDI and Quantized CMI

Now we will assume that part of the limited feedback is employed to convey information about the CMI to the BS, i.e. $B = B_\mathrm{CDI} + B_\mathrm{CMI}$. We assume that each user k quantizes $\|\hat{\boldsymbol{h}}_k\|_2$ using a LMQ codebook with 2^{B_CMI} quantization levels, which is also available at the BS. Hence, we have that

$$\hat{\boldsymbol{B}} = \hat{\boldsymbol{B}}_\mathrm{q} + \hat{\boldsymbol{B}}_\mathrm{e}, \tag{30}$$

where $\hat{\boldsymbol{B}}_\mathrm{q}$ is a diagonal matrix with the quantized CMI's of the users on the diagonal and $\hat{\boldsymbol{B}}_\mathrm{e}$ is a diagonal matrix with the quantization error of the users' CMI. In addition, the users quantizes the estimated CDI employing the remaining $B_\mathrm{CDI} = B - B_\mathrm{CMI}$ bits of the limited feedback, using RVQ as before

$$\hat{\boldsymbol{h}}'_{k,\mathrm{q}} = \underset{\boldsymbol{t}'_{k,j}\in\mathcal{C}'_k}{\mathrm{argmax}}\ |\boldsymbol{t}'^{,\mathrm{H}}_{k,j}\hat{\boldsymbol{h}}_{k,\mathrm{n}}|,$$

where each user k has a different codebook $\mathcal{C}'_k$ consisting of 2^{B_CDI} unit-norm random beamforming vectors $\boldsymbol{t}'_{k,j} \in \mathcal{C}'_k$, which is a smaller codebook than $\mathcal{C}_k$. We have employed "$'$" to denote the variables when the estimated CDI is quantized with $B_\mathrm{CDI} < B$, in order to distinguish them from the variables when the estimated CDI is quantized using all the available feedback bits, i.e. $B_\mathrm{CDI} = B$.

Similarly to the previous section, we assume each user k multiplies its received signal with $\frac{1}{c'_k}$ besides the scalar filter g'_q. Nevertheless, contrary to when only the CDI is quantized, in this case no normalization with the CMI $\|\hat{\boldsymbol{h}}_k\|_2$ is performed since the BS has access to a quantized version of the estimated instantaneous CMI. The scalar receiver at user k is actually $\frac{g'_\mathrm{q}}{c'_k}$. The estimated signal of the K users is given by

$$\hat{\boldsymbol{s}}'_\mathrm{q} = g'_\mathrm{q}\,\boldsymbol{C}'^{,-1}\left(\boldsymbol{H}\boldsymbol{P}'_\mathrm{q}\boldsymbol{s} + \boldsymbol{n}\right) = g'_\mathrm{q}\left(\hat{\boldsymbol{B}}\hat{\boldsymbol{H}}'_\mathrm{q}\boldsymbol{P}'_\mathrm{q}\boldsymbol{s} + \boldsymbol{n}'_\mathrm{q}\right), \tag{31}$$

with the effective noise

$$\boldsymbol{n}'_\mathrm{q} = \boldsymbol{C}'^{,-1}\hat{\boldsymbol{B}}\boldsymbol{E}'_q\boldsymbol{P}'_\mathrm{q}\boldsymbol{s} + \boldsymbol{C}'^{,-1}\boldsymbol{E}\boldsymbol{P}'_\mathrm{q}\boldsymbol{s} + \boldsymbol{C}'^{,-1}\boldsymbol{n}, \tag{32}$$

where $\hat{\boldsymbol{B}}$ is given as in (30). We now consider the $\boldsymbol{P}'_\mathrm{q}$ and g'_q which minimize the MSE based on $\hat{\boldsymbol{H}}'_\mathrm{q}$ and $\hat{\boldsymbol{B}}_\mathrm{e}$, which are known at the BS:

$$\left\{\boldsymbol{P}'_\mathrm{q}, g'_\mathrm{q}\right\} = \underset{\left\{\boldsymbol{P}'_\mathrm{q}, g'_\mathrm{q}\right\}}{\mathrm{argmin}}\ \mathrm{E}\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}'_\mathrm{q}\|_2^2\right] \ \text{s.t.}\ \mathrm{tr}\left(\boldsymbol{P}'_\mathrm{q}\boldsymbol{R}_s\boldsymbol{P}'^{,\mathrm{H}}_\mathrm{q}\right) \le P_\mathrm{DL}, \tag{33}$$

where the expected value is taken over $\boldsymbol{s}$, $\boldsymbol{n}$, $\boldsymbol{E}$, $\boldsymbol{C}'$, $\boldsymbol{E}'_\mathrm{q}$ and over the CMI quantization error $\hat{\boldsymbol{B}}_\mathrm{q}$ given $\hat{\boldsymbol{H}}'_\mathrm{q}$ and $\hat{\boldsymbol{B}}_\mathrm{q}$, which are known at the BS. The solution to (33) can be derived in a similar way as for (24):

$$g'_\mathrm{q} = \sqrt{\frac{\mathrm{tr}\left(\left((1-\kappa')\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\boldsymbol{\Lambda}\hat{\boldsymbol{H}}'_\mathrm{q} + \xi'_\mathrm{q}\mathbf{I}_M\right)^{-2}\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\hat{\boldsymbol{B}}_\mathrm{q}\boldsymbol{R}_s\hat{\boldsymbol{B}}_\mathrm{q}\hat{\boldsymbol{H}}'_\mathrm{q}\right)}{P_\mathrm{DL}}}$$

$$\boldsymbol{P}'_\mathrm{q} = \frac{1}{g'_\mathrm{q}}\left((1-\kappa')\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\boldsymbol{\Lambda}\hat{\boldsymbol{H}}'_\mathrm{q} + \xi'_\mathrm{q}\mathbf{I}_M\right)^{-1}\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\hat{\boldsymbol{B}}_\mathrm{q},$$

where

$$\kappa' = \frac{\mathrm{E}\left[\tan^2\theta'_k\right]}{M-1} \tag{34}$$

$$\boldsymbol{\Lambda} = \hat{\boldsymbol{B}}_\mathrm{q}^2 + \mathrm{E}\left[\hat{\boldsymbol{B}}_\mathrm{e}^2\,\middle|\,\hat{\boldsymbol{B}}_\mathrm{q}\right] \tag{35}$$

$$\xi'_\mathrm{q} = \kappa'\,\mathrm{tr}\left(\boldsymbol{\Lambda}\right) + \left[\cos^{-2}\theta'_k\right]\mathrm{tr}\left(\boldsymbol{R}_\mathrm{e} + \frac{\sigma_n^2}{P_\mathrm{DL}}\mathbf{I}_K\right) \tag{36}$$

where $\mathrm{E}\left[\hat{\boldsymbol{B}}_\mathrm{e}^2\,\middle|\,\hat{\boldsymbol{B}}_\mathrm{q}\right]$ indicates the CMI quantization error variance in the quantization interval which is represented by $\hat{\boldsymbol{B}}_\mathrm{q}$. $\mathrm{E}\left[\hat{\boldsymbol{B}}_\mathrm{e}^2\,\middle|\,\hat{\boldsymbol{B}}_\mathrm{q}\right]$ depends on the quantization levels of the LMQ and can be computed in closed form but it is not shown here due to lack of space. The resulting MMSE given the quantized estimated CDI $\hat{\boldsymbol{H}}'_\mathrm{q}$ and the quantized estimated CMI $\hat{\boldsymbol{B}}_\mathrm{q}$, i.e. averaging over $\boldsymbol{n}$, $\boldsymbol{E}$, $\boldsymbol{E}'_\mathrm{q}$, $\boldsymbol{C}'$ and over the CMI quantization error $\hat{\boldsymbol{B}}_\mathrm{e}$, results in

$$\mathrm{MMSE'_q} = \sigma_s^2\mathrm{tr}\Big(\left(\boldsymbol{\Lambda}^{\frac{1}{2}}\hat{\boldsymbol{H}}'_\mathrm{q}\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\boldsymbol{\Lambda}^{\frac{1}{2}} + \xi'_\mathrm{q}\mathbf{I}_K\right)^{-1} \times \left(\boldsymbol{\Lambda}^{\frac{1}{2}}\hat{\boldsymbol{H}}'_\mathrm{q}\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\boldsymbol{\Lambda}^{\frac{1}{2}} - \hat{\boldsymbol{B}}_\mathrm{q}\hat{\boldsymbol{H}}'_\mathrm{q}\hat{\boldsymbol{H}}'^{,\mathrm{H}}_\mathrm{q}\hat{\boldsymbol{B}}_\mathrm{q} + \xi'_\mathrm{q}\mathbf{I}_K\right)\Big), \tag{37}$$

which is a function of $\hat{\boldsymbol{H}}'_\mathrm{q}$, $\hat{\boldsymbol{B}}_\mathrm{q}$, B_CDI and B_CMI. This MMSE represents solely an intermediate step in order to compute afterwards the average MMSE, which is obtained by averaging $\mathrm{MMSE'_q}$ over $\hat{\boldsymbol{H}}'_\mathrm{q}$ and $\hat{\boldsymbol{B}}_\mathrm{q}$.

V. Simulation Results and Discussion

Let us now present some results for the transceiver design with quantized CDI and CMI. We assume $K = M = 4$ with a downlink training length $T_\mathrm{DL} = 4$ and recalling that $\xi^{-1} = \frac{P_\mathrm{DL}}{K\sigma_n^2}$, we set $10\log_{10}\xi^{-1} = 12$dB. In addition we assume the following average channel gains for the $K = 4$ users $\gamma_1 = 1, \gamma_2 = \sqrt{2}, \gamma_3 = \sqrt{3}$ and $\gamma_4 = 2$. Fig. 2 depicts the average user MMSE as a function of B for the case of quantized CDI with the statistics of the CMI and for the case of quantized CDI and CMI with different values of B_CMI and with the respective different values of $B_\mathrm{CDI} = B - B_\mathrm{CMI}$. As a reference we have also included the case with perfect CSI. It can be seen how the average MMSE of each of the quantized cases decreases linearly with increasing B and approaches the performance of the perfect CSI. However, one can also observe that it is optimum to employ all the B bits for quantizing the CDI.

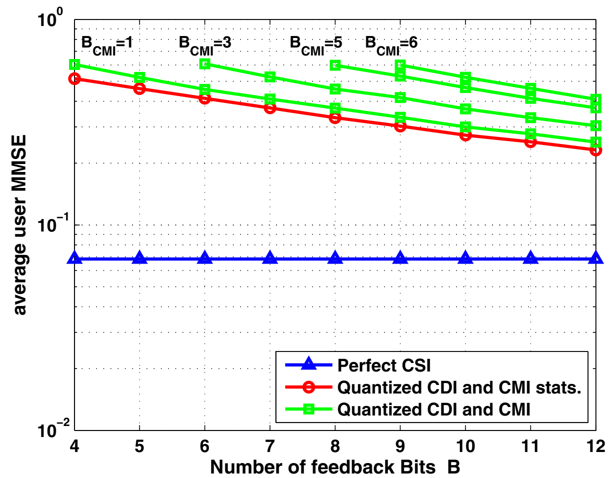

Fig. 2. Average user MMSE vs. B, $K = M = 4$, $10\log_{10}\xi^{-1} = 12$ dB.

We now consider the same scenario as before but we plot in Figure 3, the average user MMSE achieved with the quantization of only the CDI with the statistics of the CMI available at the BS, as a function of the average user SNR ξ^{-1} in dB. This is shown for different values of B. As a reference we have included also the cases of perfect CSI, perfect CDI and the MMSE achieved with the MMSE based precoding scheme presented in [6] which is based on quantized CSI, i.e. the entire estimated vector is quantized using a generalized Lloyd vector quantizer. The saturation due to the quantization, which has been documented in the literature, is also visible, since the quantization error κ does not decrease with increasing SNR. In addition, we point out that the proposed scheme outperforms the one from [6] for a given limited feedback of B bits. Furthermore, the average performance with perfect CDI case is not far from the perfect CSI case. This behavior is regardless of the number of antennas M and it indicates that for the case $K \leq M$, the instantaneous CMI does not help much which is in accordance with the result in Figure 2. This holds for the case of uncorrelated channels and under the assumption that BS knows the statistics of the users' CMI.

VI. Concluding Remarks

We have presented the transceiver design of a multiuser MISO system based on the MMSE criterion and with transmit CSI obtained through a limited feedback composed of quantized CDI and CMI. Given a limited feedback of B bits, our findings show that for uncorrelated channels it is optimum to dedicate all the B for the estimated CDI quantization. However, the BS must know the statistics of the CMI of the users, which consists of the average channel gain, which can be obtained from uplink measurements, and of the users' estimation error variance. This holds when $K \leq M$, i.e. no *channel-aware* scheduler is involved. As a future work, we plan to investigate what is the optimum partition between B_{CDI} and B_{CMI} when $K > M$, which implies user selection at the BS. Contrary to the case $K \leq M$, we posit that quantized instantaneous knowledge of the CMI would be required in order to exploit the multiuser diversity when $K > M$. The performance of non-linear schemes and the consideration of correlated channels are also matters of future interest.

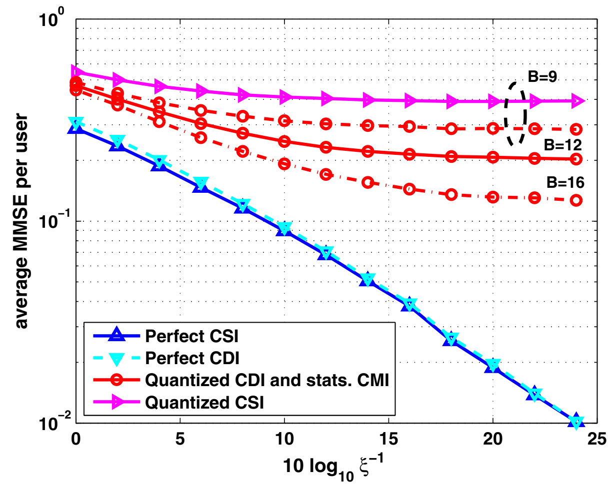

Fig. 3. Average user MMSE vs. $10\log_{10}\xi^{-1}$, $K = M = 4$.

References

[1] D. J. Love, R. W. Heath Jr., W. Santipach, and M. L. Honig, "What is the Value of Limited Feedback for MIMO Channels?", *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54-59, Oct. 2004.

[2] N. Jindal, "MIMO Broadcast Channels with Finite-Rate Feedback", *IEEE Transactions on Information Theory*, Vol. 52, No. 11, Nov. 2006.

[3] G. Caire, N. Jindal, M. Kobayashi and N. Ravindran, "Multiuser MIMO Downlink Made Practical: Achievable Rates with Simple Channel State Estimation and Feedback Schemes", *Arxiv preprint cs.IT/0710.2642*, 2008.

[4] M. Castañeda, Israa Slim, A. Mezghani and J. A. Nossek, "Transceiver Design in Multiuser MISO Systems with Imperfect Transmit CSI", in *ITG Workshop in Smart Antennas (WSA)*, Feb. 2010.

[5] M. N. Islam and R. Adve, "Transceiver Design using Linear Precoding in a Multiuser MIMO System with Limited Feedback", *Arxiv preprint cs.IT/1001.2307v1*, Jan. 2010.

[6] A. D. Dabbagh and D. J. Love, "Multiple Antenna MMSE based Downlink Precoding with Quantized Feedback or Channel Mismatch", *IEEE Trans. Commun.*, vol. 56, pp. 1859-1868, Nov. 2008.

[7] D. J. Ryan, I. B. Collings, I. V. L. Clarkson and R. W. Heath Jr., "Performance of Vector Perturbation Multiuser MIMO Systems with Limited Feedback", *IEEE Transactions on Communications*, Vol. 57, No. 9, pp. 2633-2644, Sep. 2009.

[8] B. Khoshnevis and W. Yu, "Limited Feedback Multi-Antenna Quantization Codebook Design-Part II: Multiuser Channels", *Arxiv preprint cs.IT/1003.2259*, Mar. 2010.

[9] C. K. Au-Yeung and D. J. Love, "On the Performance of Random Vector Quantization Limited Feedback Beamforming in a MISO System", *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, Feb. 2007.

[10] M. Joham, M., W. Utschick, J. A. Nossek, "Linear Transmit Processing in MIMO Communications Systems", *IEEE Transactions on Signal Processing*, Vol. 53, No. 8, Aug. 2005, pp. 2700 - 2712.

[11] B. Hassibi and B. M. Hochwald, "How much Training is needed in a Multiple-antenna Wireless Link?", *IEEE Trans. Inform. Theory*, vol. 49, pp. 951-964, Apr. 2003.

[12] M. Trivellato, S. Tomasin, and N. Benvenuto, "On Channel Quantization and Feedback Strategies for Multiuser MIMO-OFDM Downlink Systems", in *IEEE Trans. Commun.*, Sep. 2009

[13] M. Trivellato, F. Boccardi, and F. Tosato, "User Selection Schemes for MIMO Broadcast Channels with Limited Feedback," in *Proc. Vehicular Technology Conference* (VTC'07), Apr. 2007.