Institut für Informatik
der Technischen Universität München
Lehrstuhl für Informatik XVIII

ALPS – Design and Analysis of a Robust Iterative Combinatorial Auction Format

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Vollständiger Abdruck der von der Fakultät für Informatik der Technischen Universität München zur Erlangung des akademischen Grades eines

Doktors der Naturwissenschaften (Dr. rer. nat.)

genehmigten Dissertation.

Vorsitzender:		Univ.-Prof. Dr. J. Schlichter
Prüfer der Dissertation:		
	1.	Univ.-Prof. Dr. M. Bichler
	2.	Univ.-Prof. M. Beetz, Ph.D.

Die Dissertation wurde am 10.12.2008 bei der Technischen Universität München eingereicht und durch die Fakultät für Informatik am 19.05.2009 angenommen.

Orach Boris Grigorjevich

Abstract

In a combinatorial auction (CA) several heterogenous items are traded simultaneously, they can be distributed between several winners, and the bidders can submit indivisible all-or-nothing “bundle” bids on groups of items. The idea of using CAs for capturing economies of scale and scope and thus achieving better economic results on complex markets was first suggested in 1982 in the context of allocating airport slots. For several years the concept was not considered practical because of its combinatorial complexity, but it was eventually picked up by the US Federal Communication Commission (FCC) as a promising tool for conducting spectrum auctions, where bidders have strong preferences to win licenses for geographically adjacent regions. However, it took the FCC over 15 years of research and testing until the first combinatorial spectrum auction was conducted in 2008.

In the meantime, combinatorial auctions were discovered by frontier professionals in industrial purchasing. There are a couple of published cases which demonstrate the high potential of this new technology for procurement applications, among them two finalists and one winner of the prestigious practitioner INFORMS Edelman Award in the past six years.

However, there are no standard solutions with adequate support for combinatorial auctions available to date. Only the recent advances in computer science and the optimization theory made their application possible, and they still require significant research and engineering work. In particular, there is little understanding regarding the applicability of various existing CA formats, and regarding their robustness in cases where bidders do not follow theoretically optimal strategies.

In this context, our goal was to suggest a practical and robust combinatorial auction format which delivers good results for various types of bidder valuations and strategies. Such a design shall be based on a thorough analysis and

comparison of existing mechanisms. In particular, it was not clear whether CAs based on linear or non-linear prices present the preferable approach.

We have chosen computational experiments to be our main research tool. The game-theoretical approach, which has been used extensively to model single-item auctions, has only limited applicability in the context of combinatorial auctions due to their high strategic complexity. Furthermore, there are strong indications that the bidders fail to act rationally in their exponential strategic space. Experimental economics, which is another proven approach to the studying of market mechanisms, has delivered only very limited results to date, due to the high complexity and cost of laboratory experiments with CAs.

Computational experiments allowed us to systematically test and compare many combinatorial auction designs under different valuations and different bidder behavioral models. We could also measure their sensitivity with respect to different parameters. To achieve reliable results, our experiments are based on a broad range of economically motivated value models and bidding agents with different behavior, based both on theoretical assumptions and on our observations in the laboratory. Overall, this thesis summarizes the results of over 50'000 auctions.

The main contribution of this work is the ALPS linear-price-based iterative combinatorial auction (ICA) format. It demonstrates high allocative efficiency of over 98% in our experiments, as well as very good robustness in cases when the bidders do not follow the theoretically optimal strategy. It has several practice-oriented features which further improve its performance. The *dynamic minimum increment* can halve the auction duration without sacrificing the efficiency. The *surplus eligibility* rule can mitigate the negative effect of activity rules in the auction. While our results achieve high efficiency values on average, we have identified and described cases where linear price CAs are not efficient. There are a few remedies, such as the proxy phase in the Clock-Proxy auction or after-market negotiation on unsold items.

During development of the new CA design, we run a thorough comparison of existing formats along various criteria. In particular, this work is the first detailed benchmark of two big ICA families: linear-price and non-linear price designs.

An important result of our work is the MarketDesigner platform for combinatorial auctions, which was a significant investment, and is a joint effort together with several colleagues and many students. It is currently being used for laboratory experiments and pilot projects with industry partners.

Zusammenfassung

In einer kombinatorischen Auktion werden mehrere heterogene Güter gleichzeitig versteigert; es sind mehrere Gewinner möglich; und die Bieter können untrennbare alles-oder-nichts Bündelgebote auf Gruppen von Gütern abgeben. Auf komplexen Märkten können solche Auktionen Verbund- und Skaleneffekte adressieren und dadurch bessere ökonomische Ergebnisse erzielen. Das Konzept wurde zum ersten Mal im Jahr 1982 im Zusammenhang mit der Terminalzeitplanung auf Flughäfen vorgeschlagen. Nachdem die kombinatorischen Auktionen zuerst wegen ihrer exponentiellen Komplexität als impraktikabel galten, wurden sie von der US Federal Communication Commission (FCC) zum Versteigern von Frequenzlizenzen vorgeschlagen, wo die Bieter starke Präferenzen für benachbarte Regionen haben. Es hat aber über 15 Jahren gedauert, bis das FCC die erste kombinatorische Frequenzauktion im Jahr 2008 durchgeführt hat.

Inzwischen wurden die kombinatorischen Auktionen vom Industrieeinkauf entdeckt. Das große Potenzial der neuen Technologie im Einkauf wurde durch einige Publikationen bewiesen, unter denen befinden sich zwei Finalisten und ein Gewinner des angesehenen INFOMRS Edelman Award in den letzten sechs Jahren.

Dennoch gibt es keine standardmäßigen Lösungen für kombinatorische Auktionen. Lediglich die jüngsten Entwicklungen der Informatik und Optimierungstheorie haben ihre praktischen Anwendungen ermöglicht, und sie erfordern immer noch wesentliche Forschungs- und Entwicklungsarbeit. Insbesondere gibt es wenig Wissen sowohl bezüglich der Anwendbarkeit von verschiedenen kombinatorischen Auktionsformaten, als auch bezüglich deren Robustheit gegenüber suboptimaler Bietstrategien.

In diesem Zusammenhang, das Ziel dieser Arbeit war, ein praktisches und robustes kombinatorisches Auktionsformat vorzuschlagen, das für verschiedene

Wertigkeiten und Bietstrategien gute Ergebnisse zeigt. Der Vorschlag soll auf gründlicher Analyse und Vergleich von bekannten Auktionsformaten basieren. Speziell, es war nicht klar ob kombinatorische Auktionen mit linearen oder nichtlinearen Preisen vorzuziehen sind.

Zum Hauptwerkzeug unserer Forschung haben wir Simulationen gewählt. Die Spieltheorie, die zur Analyse von üblichen Auktionen oft benutzt wird, ist wegen der hohen Komplexität von Spielerstrategien in kombinatorischen Auktionen nur begrenzt anwendbar. Außerdem gibt es Anzeichen dafür, dass die Bieter sich in solch komplexen Strategieräumen nicht rationell verhalten. Die andere bekannte Forschungsmethode - die experimentelle konomie hat bis heute auf dem Gebiet, wegen hoher Komplexität und Kosten von Experimenten mit kombinatorischen Auktionen, nur sehr begrenzte Ergebnisse hervorgebracht.

Mittels Simulationen konnten wir viele kombinatorische Auktionsformate unter verschiedenen Wertigkeits- und Bieterverhaltensmodellen systematisch testen und vergleichen. Ebenfalls konnten wir ihre Sensitivität bezüglich unterschiedlicher Parameter messen. Um verlässliche Ergebnisse zu bekommen, verwenden unsere Simulationen viele unterschiedliche Wertigkeitsmodelle mit ökonomischem Hintergrund und verschiedene Bieteragenten, die beides auf theoretischen Annahmen und auf Beobachtungen im Labor basieren. Insgesamt haben wir in Rahmen dieser Arbeit über 50'000 Auktionen analysiert.

Der Hauptbeitrag dieser Arbeit ist ALPS: ein iteratives, linear-Preis basiertes kombinatorisches Auktionsformat. Es zeigt in unseren Simulationen eine hohe allokative Effizienz von über 98auch eine gute Robustheit gegenüber suboptimaler Bieterstrategien. ALPS hat auch einige praxisorientierte Eigenschaften. Das dynamische Preisinkrement (*dynamic minimum increment*) kann die Auktionslänge halbieren, ohne dass die allokative Effizienz fällt. Die *surplus eligibility* Regel kann die negative Wirkung von Aktivitätsregeln in der Auktion reduzieren. Obwohl ALPS im Durchschnitt eine sehr hohe allokative Effizienz zeigt, haben wir Fälle ermittelt und beschrieben, bei denen kombinatorische Auktionen mit linearen Preisen mangelnde Ergebnisse haben. Es existieren verschiedene Verbesserungsmöglichkeiten, zum Beispiel eine zweite Proxy-Phase, oder eine Nachverhandlung mit unverkauften Gütern.

Während der Entwicklung von ALPS haben wir einen gründlichen Vergleich von bekannten kombinatorischen Auktionsformaten bezüglich unterschiedlichster Kriterien gemacht. Insbesondere, diese Arbeit ist der erste Vergleichstest von zwei großen Auktionsfamilien mit linearen und nichtlinearen Preisen.

Ein wichtiges aber auch aufwändiges Ergebnis dieser Arbeit ist die Market-

Designer Plattform zur Durchführung von kombinatorischen Auktionen. Sie entstand in Zusammenarbeit mit einigen Kollegen und vielen Studierenden, und wird auch weiter in Laborexperimenten und Pilotprojekten mit Industriepartnern verwendet.

Acknowledgments

First of all, I am thankful to Professor Martin Bichler for the possibility of working on this research project at his Chair of Internet-based Information Systems at the TU München. With your excellent guidance and constant support, not only I completed this thesis, but also look happily at the last four years of my life. I learned a lot, and it was fun. Thank you Martin!

This thesis was a part of a bigger research project and I was lucky to have great colleagues around me. I am grateful to Christian Hass, Alexander Pikovsky, Tobias Scheffel, Stefan Schneider, and Georg Ziegler for invaluable cooperation and feedback which have contributed in a fundamental way to my work. I would also like to thank Professor Michael Beetz for co-supervising my dissertation.

I would like to thank all former students who contributed to our research project: Dmitri Alexeev, Piotr Grobelny, Maximilian Hötzl, Alexandra Klimova, Alexander Klitni, Bernd Laqua, Peter Laubmeier, Yang Liu, Vladimir Neykov, Hernan Raffi, Boris Saydakov, Frank Schmaus, Timo Terletzki, Tetyana Zhartovska.

Finally I want to express my thanks to all the important people in my life who supported and tolerated me during this long project, in the first place my sister Dasha and my parents.

All errors, idiocies and inconsistencies remain my own.

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Chapter 1

Introduction

Are you sitting comfortably?
Then I'll begin.

Julia Lang 1921- : Listen with Mother (BBC radio programme for children, 1950-82)

Society increasingly depends on redistribution of values of all kind. A couple of hundred years ago it was normal to buy food, clothes and some basic utensils. Today our "shopping list" includes airline tickets, books, cars, phone bills, insurances, various services, etc. and it will probably grow even longer in the future. A similar development is typical for companies. They tend to concentrate on their core business, while outsourcing almost everything else. People, companies, and whole nations need to exchange results of their activities for other goods and services which they cannot produce efficiently themselves – they need mechanisms for setting up *trades*.

The very first barter trades were conducted around 10'000 years ago, presenting the first attempt to find a solution for the problem of redistributing values in the society. A drastic improvement was made in China around 3'500 years ago with the introduction of money. Since the invention of the wheel, this was the most influential innovation in world history.

The next logical question was how to set the correct price for the subject of trade, which would satisfy all parties. Roughly a thousand years later, 500 BC that is, a simple and elegant model was found which is used until today: auctions.

Auctions can determine the price of a trade dynamically, depending on the offer and demand volumes. Ideally, an *equilibrium* outcome is reached, where neither party wants to change the result in any legitimate way, by changing the price or allocation of the goods. The price-finding properties of auctions are well known and have been successfully used in many situations:

- To sell perishable goods within volatile markets, where fast and efficient trade is important. Well-known examples include Japanese fish markets and flower auctions in the Netherlands.
- To sell scarce or exceptional goods like arts, rare wine, etc. It is especially difficult to set an optimal fixed price for such goods.
- To sell goods with unknown and unpredictable value. For example the actual value of a mining license depends on the amount of minerals located in the area, which can be only very approximately predicted beforehand.
- To handle markets with high competition, where demand is significantly higher then supply.

Practical application of an auction mechanism for conducting a trade requires certain additional effort from both auctioneer and bidders, compared to simply setting a fixed price. Therefore, the auctions are usually used only if the potential improvement in the economical results of the trade is higher than the incurred additional costs. This effect is studied in Transaction Cost Economics (Williamson, 1998). For example, it makes little sense to auction sugar in a supermarket, since the potential savings are minimal and there is usually no competition. However it does make sense to auction off sugar delivery contracts for a big confectioner company, since the potential savings are significant and the market is competitive (Hohner et al., 2003).

Whenever auction mechanisms are not applicable, alternative possibilities to organize a market are possible. Fixed price is normally used in situations with low competition. Lottery is a good solution in cases where equal opportunity must be preserved, which is sometimes the case in assignment of values in the public sector.

Modern IT systems have significantly reduced the effort of setting up an auction. Auctioning off a 3-year-old personal computer was a significant effort ten years ago and was hardly feasible. Today it takes 15 minutes to put it on eBay. Even though the monetary value of a 3-year-old computer is much lower today than it was ten years ago, more people will auction it off nowadays.

IT growth not only supported the spread of the traditional auctions, but also assisted in the development of new auction formats for complex markets, which are the subject of this thesis. In many practical cases the market does not include just a single good, but rather several dependent, similar or completely different items. Possible extensions of the plain single-item auction are illustrated in Figure 1.1. In a *combinatorial auction* (*CA*), which is the subject of this thesis, several heterogenous goods are sold simultaneously. In a ***multi-unit auction*** many homogenous goods are traded, and the total quantity can be split between several winners. Goods in a ***multi-attribute auction*** have multiple attributes, as opposed to a single price attribute. Bidders are requested to specify values for each attribute, which can include product properties as well as conditions of the transaction.

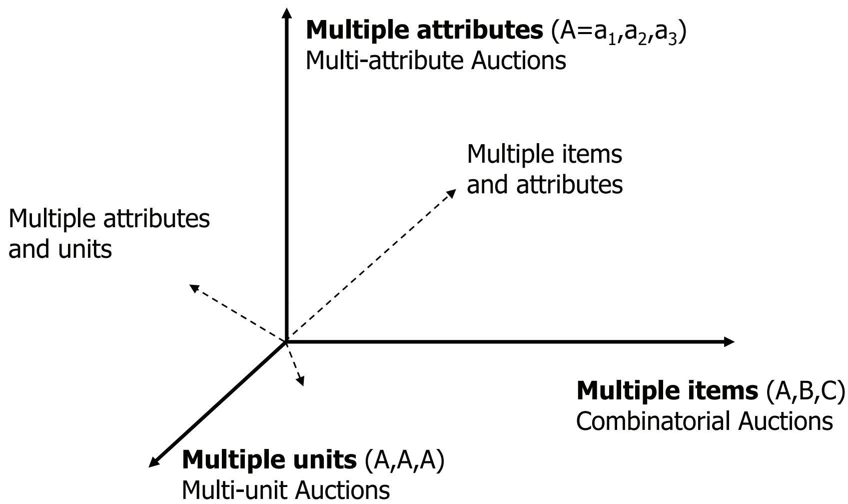


FIGURE 1.1: Multidimensional Auctions (Bichler et al., 2002)

Combinatorial auctions are used for handling complex markets with several heterogenous items, when bidders value groups of items differently from the sum of corresponding individual item valuations:

- Valuations can be ***superadditive***, when a group of items is valued higher than the sum of the individual item valuations. Imagine that cinema tickets are sold in an auction. If you go to the cinema with a good friend, you will pay 8 € for each single ticket, but will probably agree to pay more then 16 € if you can get two adjacent seats.

- Valuations can be ***subadditive***, when a constellation of items is valued lower than the sum of separate item valuations. Think about purchasing a TV-set on an auction, where several devices are sold simultaneously. For example, you agree to pay 1500 € for a Sony and 1600 € for a Panasonic. However you will probably agree to pay only significantly less then 3100 € (= 1500 € + 1600 €) for both of them together, since you need only one new TV-set.

The microeconomic theory uses terms ***complements*** and ***substitutes*** to refer to items with superadditive and subadditive valuations correspondingly. Both types of valuations can be mixed in the same market. In some situations these dependencies are extreme, for example if you purchase tickets for connecting flights. In other cases the dependencies are weaker, for example when caused by shipping costs.

MUC	
A	7:00
B	7:20
C	7:40
D	8:00

CDG	
E	9:00
F	9:20
G	9:40
H	10:00

FIGURE 1.2: Example of a Combinatorial Market

Figure 1.2 illustrates a scenario where application of a combinatorial auction can be advantageous. Imagine you represent an airline, and you want to service a flight from Munich (MUC) to Paris (CDG). You need time slots at both airport terminals. In this simplified version each airport has only one terminal, and it sells time slots of 20 minutes, which are necessary for takeoff and landing of the plane. These time slots are sold on an auction as items A . . . H. The flight takes two hours. From your analysts you know, that optimal arrival time for your passengers is 9:00. Therefore, you would agree to pay 20T € for items

A and E together. Alternatively, you would agree to pay 18T € for B and F together. You might also agree to pay 15T € for A and F together since your plane will have to stay additional 20 minutes in the air, and you arrive later. Note that A alone, like also any other single item alone, is not worth you anything. The same is true for pairs like A and C (both in the same airport) and B and E (the plane won't make it that fast).

Setting fixed prices in such situation is a very difficult task. The demand has significant intraday fluctuations, since most airlines prefer to have slots early in the morning and in the evening, to serve the lucrative business customers. During the day the demand can significantly drop. Similarly there are strong seasonal fluctuations. The valuations can also vary significantly across airlines. Therefore using an auction in this case can help to find better prices, and increase revenues. However traditional auctions will be problematic for the bidder in this case.

The example illustrates the fundamental ***exposure problem*** of the bidder in such markets. If the slots are sold individually in consecutive English auctions, you will have a problem bidding for example for the slot A. Without knowing the final prices of slots E and F, you will have to speculate with your highest bid on A. If other two slots become too expensive at the end, you will incur losses even if A alone was relatively cheap.

The US Federal Communication Commission (FCC) faced a similar problem while auctioning off spectrum licenses to telecom providers. Bidders in such auctions have strong preferences to win licenses for adjacent areas. The FCC has been using the ***Simultaneous Multiround Auction*** (***SMR***), which is basically comprised of simultaneous English auctions on each good (Milgrom, 2000). However, the SMR auction does not solve the problem completely. In our example, if the prices for A and E are 8T € each, you are not sure whether to bid or not. You might get A, but the price for E will rise above 12T €, and you will make losses. After years of research and considerations, in 2008 the FCC started using combinatorial auctions to address this issue.

Combinatorial auction are designed to solve the exposure problem. Their principal idea is to allow for ***bundle bids***, which connect an indivisible subset of goods and a price. In the above example, a combinatorial auction would allow the bidder to submit a bundle bid for A and E together for 20T €, ensuring that he will either receive both items for at most 20T €, or nothing. The exposure problem, which is an issue in every market with superadditive valuations, is therefore solved.

When bidders have subadditive valuations, they face the ***overflow problem***: they are interested in several different items, but does not want to get all of them at the end. In the last example, you might submit bundle bids ({A,E}, 20T €) and ({B,F}, 18T €), but that should not mean that you are ready to purchase all four items for 38T €. Combinatorial auctions can address the overflow problem too, however selection of a proper bidding language is essential in this case (see Section 2.3.1).

1.1 Literature Overview

Even though auctions have been in practical use for centuries, the academic community addressed this topic only recently. Interestingly enough, already the first investigation on auctions (Vickrey, 1961) together with later work of the author earned the 1996 Nobel Prize in Economics[1] "for fundamental contributions to the economic theory of incentives under asymmetric information". William Vickrey introduced the concept of *independent private valuations*, which is used for majority of the work on combinatorial auctions today, and presented several special cases of the *Revenue Equivalence Theorem* (Theorem 1), which has an important place in the modern auction theory. His results were extended by Clarke (1971) and Groves (1973). They constructed the unique Vickrey-Clarke-Groves (VCG) auction, which remains a reference point for many modern auction designs for the reasons described in Section 2.3.4.

Several important contributions to game-theoretical aspects of the auction theory deal with different classes of bidder valuations. Wilson (1969) introduced and analyzed the model of ***common valuations***, where goods have the same, but unknown value for every bidder. Milgrom and Weber (1982) extended his results by introducing and analyzing the ***affiliated valuation*** model, which represents a general case, with private valuations and common valuations being its two extreme instances.

Another Nobel Prize in Economics for an auction-related research went in 2007 to Roger Myerson[2] "for having laid the foundations of mechanism design theory". While the previous works concentrated on equilibrium analysis and comparison of existing auction formats, Roger Myerson started development of the theory, which can characterize equilibrium outcomes of all auction mechanisms

[1] together with James A. Mirriees.

[2] together with Leonid Hurwicz and Eric Maski.

in a general case, and to design auctions with certain desired properties, for example revenue- or efficiency-maximizing mechanisms. His results are based on the assumption of risk-neutral bidders with independent and identically distributed valuations.

Myerson (1979) formulates the ***revelation principle***. It states that for any auction mechanism, there is an equivalent ***direct revelation mechanism***, in which the bidders simultaneously and truthfully report their private valuations, and then the allocation and payments are determined by the auctioneer based on this information. The expected payoffs for each bidder and the auctioneer are equal for both auction formats.

(Myerson, 1981) formulates and proves another principal contribution:

Theorem 1 (Revenue Equivalence Theorem)**.**
In every auction mechanism, where the highest bidder wins and bidders with zero bids have zero payments, the auctioneer's expected revenue depends only on bidder's valuations and not on the payment function.

Consequently, it is sufficient to study only those mechanisms which provide incentives for truthful bidding (*incentive compatible*, see Definition 4) and are risk-free (*individually rational*, see Definition 3), and, without loss of generality, transfer the obtained results to other auction designs. Nevertheless, the required assumptions are tough and the obtained results must be interpreted with caution, more on this in Section 2.2.

The first remarkable publication on combinatorial auctions is the Rassenti et al. (1982), suggesting a sealed-bid combinatorial auction for the allocation of airport takeoff and landing slots, similar to our example in Figure 1.2. Additionally to economists, who study auctions as efficient market mechanisms, this idea soon attracted researchers from other fields:

- Bundle bidding adds combinatorial complexity and brings in topics from the optimization theory.
- Expressiveness of bidder languages and algorithmic aspects have attracted the attention of computer scientists.

Combinatorial auctions face several computationally hard problems. The *Combinatorial Allocation Problem* (*CAP*) can be interpreted as a weighted set packing problem (SPP) (Lehmann et al., 2006), which is NP-hard. Calculation of *ask prices* in some auction formats utilizes complex algorithms, too. Good overview of the problem is presented in de Vries and Vohra (2003) and Sandholm (2006). Jones and Koehler (2005) suggested an approximating heuristic for the CAP. Interesting insights into the CAP structure and alternative possibilities of conducting an ICA can be found in Adomavicius and Gupta (2005).

Several researchers considered ***simplified mechanisms*** (?), which put restrictions on the set of acceptable bids and thus reduce complexity of the allocation problem. Polynomial-time algorithms for restricted cases of CAP have been suggested in Rothkopf et al. (1998), Carlsson and Andersson (2004) and Goeree and Holt (2008). Distributing the computational load from the auctioneer to all auction participants was addressed by Kelly and Steinberg (2000) and Fan et al. (2003).

The interest in *iterative* designs for combinatorial auctions (ICA) started growing after two pioneering contributions. Milgrom and Weber (1982) showed for single item auctions that iterative auctions perform better then sealed-bid mechanisms in scenarios with affiliated valuations. Cramton (1998) presented other practical arguments related to multi-unit and combinatorial auctions, advocating the use of "ascending" auctions. Another virtue of iterative combinatorial auctions is that bidders are not required to reveal their true preferences on all possible bundles in one shot, as it is assumed in the VCG design (Ausubel and Milgrom, 2006b). Further arguments for iterative formats, both theoretical and practical, are given in Porter et al. (2003). A comprehensive and up-to-date survey of iterative combinatorial auctions is given in Parkes (2006).

Bidders in an auction communicate their preferences to the auctioneer using a *bidding language* (Nisan, 2006). This is a central component for every combinatorial auction design, since the auctioneer can reliably find a good allocation only having sufficient and precise information about valuations of all bidders. On the other side, bidders must analyze and express exponentially big space of possible bundles, which is often infeasible (?). We will continue discussion on bidding languages in Section 2.3.1, where we also review other relevant publications.

A promising possibility to help the bidders with orientation in the exponentially big space of possible bids in a CA is to provide them with adequate support tools. Hoffman et al. (2005) describe a domain-specific bidder aid tool for FCC spectrum auctions, which helps bidders locate optimal bids after they configure it by expressing their preferences. Such configuration options can include logical relationships between items, and spectrum-specific preferences, for example those related to minimum population coverage, bandwidth, and budget. Adomavicius and Gupta (2005) suggest to provide bidders not only with ask prices in each round, but with precise information about how much they have to bid to become a winner in the next round, and how high they have to bid to have any chance to be a winner at all. Sandholm and Boutilier (2006) argue that the auctioneer can do better than requesting bids on all possible bundles, and incrementally and proactively ask bidders only for the relevant information, until an optimal allocation is found.

A seminal contribution to the field of combinatorial auctions was made by Bikhchandani and Ostroy (2002). They connected the optimization theory-based interpretation of the combinatorial auctions to the game-theoretical aspects of equilibrium and optimal strategies by applying the duality concept of the linear optimization theory. Their results answer an important for the ICA design question of sufficient price format, differentiating between linear (or item) prices, non-linear prices, and personalized (or discriminatory) prices (see also Section 2.4.2). de Vries et al. (2007) demonstrate in detail how this theory can be applied to auction design. A whole family of non-linear price auction designs emerged based on the results of Bikhchandani and Ostroy: Ausubel and Milgrom (2002); de Vries et al. (2007); Mishra and Parkes (2004, 2007); Parkes and Ungar (2000); **?**.

Although exact linear prices exist only for very restricted types of valuations (Kelso and Crawford, 1982), they have many positive properties. Such prices are easy to understand for bidders in comparison to the non-linear ask prices, where the number of prices to communicate in each round is exponential in the number of items. Linear prices give good guidance to the bid formation for new entrants and for losing bidders, who can use them to compute the price of any bundle even if no bids were submitted for it so far. This motivated several ICA designs with approximated linear prices: Bichler et al. (2009); Day (2004); Kwasnica et al. (2005); Kwon et al. (2005); Porter et al. (2003); Wurman and Wellman (2000). Finally, some approaches suggest ICAs without price information, but provide bidders with other feedback: Hohner et al. (2003); Kelly and Steinberg (2000).

Combinatorial auctions, and especially iterative combinatorial auctions, offer the mechanism designer a much wider design space, compared to single-item auctions, simultaneously increasing the strategy space of bidders. Various ask price calculation schemes, bidder decision support tools, activity and bid increment rules make it extremely complicated to admit much theoretical analysis at a greater level of detail (Rothkopf, 2007a). Furthermore, experimental studies demonstrate that bidders do not follow the often assumed best-response bidding strategy. According to a study of CAs in transportation by Plummer (2003), out of the 644 carriers, only about 30 percent submitted bundle bids at all. This group of carriers submitted between two and seven lane combinations and the vast majority of the bundles were small, containing between two and four lanes. Apart from the novelty of CAs and the complexity of knowing their valuations over all possible bundles, the bidders face the bundle selection problem from an exponential number of possible bundles. Even in simple scenarios and with adequate supporting tools, the bidders fail to follow the theoretically optimal best-response strategy (**?**). This "trembling hand" phenomena was first described and studied by Selten (1975).

Therefore, experimental studies are important to understand performance of combinatorial auctions in real-life settings, when bidders either cannot follow the optimal strategy for computational or cognitive reasons, or deliberately behave differently. For the same reasons of extensive design space, laboratory experiments with combinatorial auctions are difficult to design correctly, they are costly, and are therefore restricted to a few treatment variables. As of today, only a handful of reports on laboratory experiments with combinatorial auctions exist, each of them addressing a very focused question: Banks et al. (2003); Chen and Takeuchi (2009); Kazumori (2005); Kwasnica et al. (2005); Porter et al. (2003); **?**. Computational experiments, where the role of bidders is taken by software agents, allow to conduct more auctions and to address more questions. However, they require significant investment into the software. Aside from our work and related publications (Bichler et al., 2009; Schneider et al., 2010; Shabalin et al., 2007, 2006), we are aware only about the contribution by An et al. (2005), which uses computational experiments to study impact of bidding strategies on revenue distribution in a combinatorial auction.

Finally we want to direct readers' attentions towards the excellent book Cramton et al. (2006), which summarizes all key aspects of the current state in the combinatorial auction research and practice.

1.2 Applications in Industrial Procurement

Field applications of combinatorial auctions deserve a separate attention, as practice in this case preceded theory. Advances in the computer science and the optimization theory made their growing acceptance in the last ten years possible. However, there are no standard solutions with adequate support for combinatorial auctions on the market of e-sourcing tools until now. All known applications of combinatorial auctions required significant research and engineering work. We believe that this situation will change, and combinatorial auctions, together with other optimization-based tools, will become a widespread and important e-sourcing instrument. This development is envisioned by major analytical companies, too (AberdeenGroup, 2007; Gartner, 2008).

The high potential and growing importance of new optimization-based technologies in procurement is best proven by the fact, that auction-related projects were twice finalists and once won the prestigious INFORMS Edelman Award, which rewards outstanding practical applications of management science and operations research:

- Hohner et al. (2003) described an application of new auction formats at Mars, Inc. supported by the IBM Research. Their new sourcing solution enabled buyers to incorporate iterative auction mechanisms into strategic sourcing negotiations. The results were not only 6% savings compared to traditional auctions, but also improvement of critical relationships between Mars, Inc. and its suppliers, who highlighted the benefits of time, efficiency, transparency, and fairness in using the new sourcing tools.

- Metty et al. (2005) won the 2005 Edelman Award with a project, which Motorola, Inc. launched to rebuild its sourcing process. The new approach was based on modern optimization tools and combined innovative bidding, online negotiations and scenario-based optimization analysis. The sourcing professionals use the new functionality to identify the optimal award strategy under different scenarios while considering constraints such as parts qualification status, supplier count and capacity. The captured savings comprise 4-7% in sourcing spend, in addition to significant reduction of process cost and complexity.

- Procter & Gamble rebuilt its procurement organization using “expressive competition” (Sandholm and Begg, 2006), which allows suppliers to

> make electronic offers expressing rich forms of capabilities and efficiencies. At the same time, the buyer can formulate constraints and preferences regarding the outcome of the sourcing event. Optimization-based tools are then used to find the optimal allocation, resulting in 14.3% in recommended savings for Procter & Gamble. The authors also report improved relationships with suppliers, because expressive competition generates a win-win between them and the buyer.

Constant public interest is generated by the US Federal Communications Commission (FCC) projects on combinatorial spectrum auctions (Banks et al., 2003; FCC, 2002). Further list of CA applications includes trading of truckload transportation (Caplice, 2006; Plummer, 2003), bus routes (Cantillon and Pesendorfer, 2006), milk for school meals (Epstein et al., 2002).

1.3 Research Objectives

A few known practical applications of combinatorial auctions provide an empiric evidence that this mechanism has a significant economical potential. However its applications are still far from being mainstream, primarily because every practical scenario requires careful analysis and significant implementation effort.

The mission of this work is to create a good[3] combinatorial auction format, which will work well for a wide range of bidder valuation types and bidding strategies. This task is solved by iteratively benchmarking different auction designs, and then formulating and testing new auction rules, which shall improve the auction outcome. We introduce the notion of *robustness* of an auction format with respect to suboptimal bidding strategies, and compare various designs based on this property.

The ALPS ICA format is the result of this work. In our experiments, it has high indices in performance and robustness. Ultimately, we expect to see the evolution of standard software components and standard designs for combinatorial auctions that work well in a wide variety of bidder valuations and bidding strategies.

Because of a high number and broad variety of combinatorial market parameters, it is not likely that a single CA design will dominate in future. Various

[3] The comparison criteria are defined in the following chapter.

auction formats have different properties, like for example non-linear price auctions and pseudo-dual auctions (Section 2.4.3) . Therefore, another important goal of this work was to compare known combinatorial auction formats and describe their applicability for specific combinatorial markets.

An important side result of this work is the creation of the MarketDesigner platform, an extendable environment for conducting computational and laboratory experiments with combinatorial auctions. The project itself is not completed with this thesis. We use the MarketDesigner platform to conduct laboratory experiments and to run pilot projects with industry partners. We have established contacts and are working on cooperations with leading researches in this field. Research of new combinatorial auction rules and designs will continue.

1.4 Research Approach

To analyze and compare different combinatorial auction designs we use computational experiments. We have created several models which define bidders' valuations based on economically motivated scenarios. Furthermore, different bidding agents, implemented in software, define behavior of auction participants under various assumptions, ranging from idealistic bidders with unlimited computational power to simplest agent who does not use bundle bids at all. Different auction designs and parameters are tested using combinations of these models.

Considering the research field of traditional single-item auctions, there are primarily two approaches.

- Game-theoretical analysis has been extensively used to study all known single-item auctions, both sealed and iterative.
- Experimental economics is used to detect and study phenomena which are difficult to formalize.

Experimental results show that the usual assumptions of the game theory are often not met. In particular, the assumption of independent private valuations seems to present a problem even in laboratory settings where it is explicitly given in bidders' valuations. Apparently human agents fail to act as straightforward revenue maximizers; their behavior is influenced by many other cognitive and psychological factors (Rothkopf, 2007a).

Game-theoretical analysis of combinatorial auctions is even harder because their design space is substantially broader than that of the single-item auctions Wurman et al. (2001). Equilibrium analysis has been performed for personalized non-linear price combinatorial auctions under strong assumptions on bidders' behavior (Parkes, 2006). However our results indicate that performance of non-linear price auctions can significantly degrade when these assumptions are not met (Section 6.5). For iterative CAs with linear prices, which have high practical potential (Ausubel et al., 2006; Bichler et al., 2009; Dunford et al., 2007), such analysis is still an open question. Furthermore, the space of bidding strategies in combinatorial auctions can be very large (Anandalingam et al., 2005; Sureka and Wurman, 2005), which requires additional assumptions on the bidders' behavior, including assessment of their cognitive abilities.

Therefore, "those who will accept no analysis other than a game theoretical one are often left with no realistic answer" (Rothkopf, 2007a) – game-theoretical analysis alone cannot provide reliable results for the field of combinatorial auctions.

Laboratory experiments are another important instrument in the research of combinatorial auctions, especially in the view of the difficulties of the game-theoretical approach. However, because of their high complexity and cost, their results are very limited.

In our work we use computational experiments as the primary research tool. It allows us to test various auctions under different settings, with different valuation models and different bidder behavior models, and thus to explore potential auction designs and analyze the virtues of various design options. Not only are we able to compare auction formats and settings, but we also can measure the sensitivity of different auction designs with respect to various parameters. An important precondition was the creation of the MarketDesigner software platform for the computational experiments, which was a significant investment.

To obtain reliable results, we base our computational experiments on a broad range of valuations and bidding strategies. The valuations are built using different economically motivated scenarios, including those from the Combinatorial Auction Test Suite (CATS) (Leyton-Brown et al., 2000). The implemented bidding strategies range from theoretically optimal best-response bidders with unlimited computational capacity to simplest agents with linear complexity.

Several strategies are based on our empirical observations of bidders' behavior in laboratory experiments with combinatorial auctions (Scheffel et al., 2009; ?).

Partial results of this work have been published or submitted for publication. In particular, Bichler et al. (2009) will appear in the INFORMS Information System Research (ISR) journal; Schneider et al. (2010) is submitted to another renowned operational research journal.

This thesis has the following structure:

- Chapter 2 introduces the relevant theoretical concepts.
- Chapter 3 gives an overview of the combinatorial auction formats, which are addressed in our research.
- Chapter 4 describes the new ALPS/ALPSm auction format, which was created based on our experiments.
- In Chapter 5 we give a detailed description of our experimental framework.
- Chapter 6 presents the results of the computational experiments. For each set of settings, we describe the research question and setup, demonstrate and then discuss the obtained results.
- Chapter 7 concludes by summarizing the results of our research and giving an outlook on the future work.

Chapter 2

Theoretical Background

> These students are so stupid... I am explaining it for the third time and already start understanding it myself, and they still don't get it.
>
> — Unknown

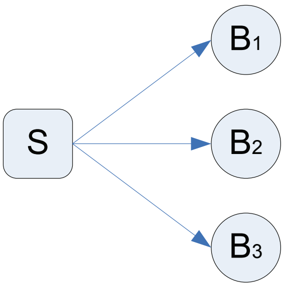
Auctions have been used to conduct trades for centuries, and have long proven their virtue. Their theoretical understanding however only goes back 50 years. The relevant academic fields include the game theory with its applied branch, the auction theory, and the decision theory. Combinatorial auctions have additionally attracted the attention of researchers in the fields of optimization theory, information systems, and computer science.

In this chapter we review the theoretical background necessary for the further presentation of our work. We will first discuss several key points about auctions in general, and then concentrate on combinatorial auctions. The order of presentation is incremental, which in some cases results in interleaving and mixing of topics from different research fields. To keep the presentation concise we will skip proofs, but provide literature references where ever necessary.

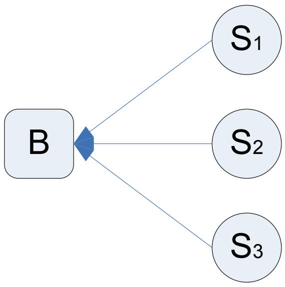
2.1 Why Auctions?

The purpose of an auction, and of any trade in general, is redistribution of goods within some closed system of agents. Depending on their nature, three auction types are possible, as shown in Figure 2.1. Here S indicates sellers, B indicates buyers, and arrows show the direction of good transfer during the trade. In a ***forward*** or ***sell*** auction several bidders-buyers compete for the good(s) sold by a single auctioneer-seller. In a ***reverse*** or ***buy*** auction several bidders-sellers compete for the right to sell their good(s) to a single auctioneer-buyer. The latter case is frequently named ***procurement auction*** after its common application case in industrial procurement, where several potential suppliers compete for a contract.

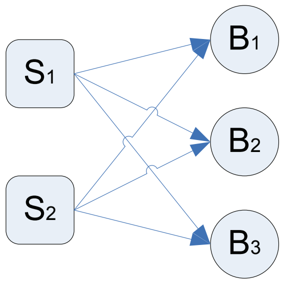
From the theoretical perspective, forward and reverse auctions are very similar, and we will consider only forward auctions for the rest of this thesis, with an exception of Section 2.6, where we review the few essential differences.



(a) Forward (Sell) Auction



(b) Reverse (Buy) Auction



(c) Double Auction

FIGURE 2.1: Classification of Auctions

The ***double auction***, which is presented in Figure 2.1(c), is a generalization of both previous scenarios, where competition was present only on one side of the market. In the case of a double auction, there are multiple sellers and multiple buyers, and participants on both sides race for a deal. Furthermore, a single party can participate on both sides of the trade at the same time, being seller and buyer simultaneously under different conditions, or for different goods. For these reasons, the auctioneer in double auctions is usually a third person and cannot be associated with one of the participants, like the seller in a forward auction. Double auctions are widely used, for example, by stock markets and other exchanges. They have further important distinctions which make their mechanics significantly different from the first two auction types. For the rest of this work, we leave double auctions out of the attention.

At this point we introduce several definitions. Even though the definitions in this section are valid for any kind of trade, we will use auction-related terminology and call trade participants *auctioneer* and *bidders* rather than using generic *seller* and *buyers*. This shall help the reader to make a smoother transition to the next sections, which concentrate on auctions.

As mentioned in Chapter 1, combinatorial auctions deal with several heterogenous items. Let $\mathcal{K}$, $|\mathcal{K}| = m$ denote the item set. Symbols $k \in \mathcal{K}$ and $l \in \mathcal{K}$ denote a specific item. We call any item subset $S \subseteq \mathcal{K}$ or $T \subseteq \mathcal{K}$ a ***bundle***, or a ***package***, which can also be empty. The symbol $\mathcal{I}$, $|\mathcal{I}| = n$ denotes the set of bidders with individual bidders denoted as $i \in \mathcal{I}$ or $j \in \mathcal{I}$.

The motivation for the bidder i to compete in the market for the bundle S is his ***valuation*** $v_i(S)$. Intuitively, the valuation $v_i(S)$ defines how much the bidder i can earn if he gets the bundle S. Consequently, this is the highest price which the bidder is willing to pay for the bundle. We assume that each bidder has some valuation for every possible bundle, whereby some valuations can be zero. The set of valuations for all bundles $\mathcal{V}_i = \{v_i(S)\}\ \forall\ S \subseteq \mathcal{K}$ is the ***value model*** of the bidder i.

The ***allocation*** X is a tuple $(S_1, \ldots, S_n)$, which describes how the goods are distributed between bidders after the trade. The allocation is ***feasible*** if it satisfies all restrictions set by the auctioneer. In particular, the allocated bundles must be non-intersecting (possibly empty): $\forall\ i, j : S_i \cap S_j = \emptyset$. Some items can remain unallocated: $\cup_{i \in \mathcal{I}} S_i \subseteq \mathcal{K}$. The auctioneer can potentially define other restrictions on feasible allocations, like minimum number of winners, maximum number of items per winning bidder, etc. (Section 2.3.3).

The set of all feasible allocations is denoted by $\mathcal{X}$. When we talk about iterative auctions, we distinguish between a ***provisional allocation*** at some intermediate point during the auction and the ***final allocation*** at the end of the auction.

The term ***price*** can have several meanings in the context of auctions. Whenever ambiguities are possible, we will explicitly state the price type as one of the following:

- The ***bid price*** $p_{bid,i}(S)$ is the price suggested by the bidder i for the bundle S at some point of time in the auction.
- The ***pay price*** $p_{pay,i}(S)$ is the price paid by a winning bidder i for the purchased bundle S at the end of the auction. Depending on the auction format, the pay price can be lower than the corresponding bid price, but never higher: $p_{pay,i}(S) \leq p_{bid,i}(S)$. Unless explicitly stated otherwise, we assume ***pay-as-bid*** auctions with $p_{pay,i}(S) = p_{bid,i}(S)$ for winning

bidders and $p_{pay,i}(\emptyset) = 0$ for losing bidders. In such cases we sometimes write just "price" to refer to both pay price and bid price.

- The ***ask price*** $p_{ask,i}(S)$ predefines, depending on the auction format, the lowest limit or the precise value for new bid prices.

2.1.1 Efficiency and Revenue Distribution in a Trade

Before we take a close look at the reasons that make auctions attractive as a market clearing mechanism, let us define what is a good trade and how we can measure quality of the trade outcome. Intuitively, a better trade brings more revenue to its participants. Let us calculate what trade participants gain from the trade.

The ***bidder revenue*** (also ***bidder payoff*** or ***bidder utility***) $\pi_i(S, \mathcal{P}_{pay})$ is the financial result of a bidder i who receives a bundle S. It is the difference between his valuation for the bundle and the price he pays for it:

$$\pi_i(S, \mathcal{P}_{pay}) = v_i(S) - p_{pay,i}(S)$$

Note that the revenue of a bidder who does not win anything is zero.

The ***auctioneer revenue*** (also ***auctioneer payoff*** or ***auctioneer utility***) $\Pi(X, \mathcal{P}_{pay})$ is the sum of all pay prices:

$$\Pi(X, \mathcal{P}_{pay}) = \sum_{i \in \mathcal{I}} p_{pay,i}(S_i)$$

Now we can calculate the total revenue of all agents in the auction with a final allocation X and final pay prices $\mathcal{P}_{pay}$:

$$\Pi(X, \mathcal{P}_{pay}) + \pi_{all}(X, \mathcal{P}_{pay}) = \\ = \sum_{i \in \mathcal{I}} p_{pay,i}(S_i) + \sum_{i \in \mathcal{I}} (v_i(S_i) - p_{pay,i}(S_i)) = \sum_{i \in \mathcal{I}} v_i(S_i) \tag{2.1}$$

This formula implies certain assumptions, for example that the situation of the bidder who does not win anything does not change ($v_i(\emptyset) = 0$) and that the auctioneer's own valuation for the goods is zero. We will review and discuss these and other assumptions on value models in Section 2.2.2.

We make two important observations from (2.1):

- The total revenue after the trade does not depend on the prices which winners pay for their goods. It depends only on the final allocation.
- There is a tradeoff between the auctioneer revenue and the revenue of all bidders.

Because of this tradeoff, there cannot be a general definition of a "good" trade outcome. There is a political question of what outcome is most desirable in a particular case. A private auctioneer will probably want to increase his own revenue, whereas a government organization might want a fair distribution of the profit between the auctioneer and the bidders to preserve the healthy market in a longer-term perspective.

We will adopt the approach which is often found in literature, which states that a better trade has higher total revenue, calculated over all trade participants according to (2.1). Since there is a limited number of possible allocations in every trade, there is also the upper limit on the total revenue achieved in the auction. An allocation X^*, which corresponds to the maximum possible total revenue, is called an ***efficient allocation***. Note that an efficient allocation is not necessarily unique.

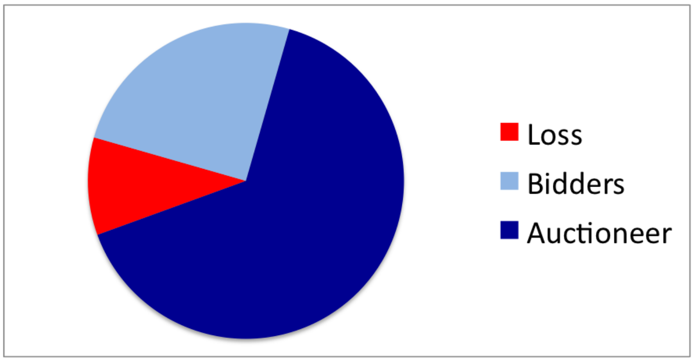


FIGURE 2.2: Efficiency and Revenue Distribution in a Trade

An important measurement for the trade outcome describes what part of this potential maximum revenue is in fact achieved, and how this revenue is distributed between auctioneer and bidders, as illustrated in Figure 2.2.

The whole pie corresponds to the maximum possible revenue. It is divided into three parts. The *Loss* is inevitably gone because the auction failed to find an efficient allocation. The rest is split between the auctioneer and bidders.

We use the term ***allocative efficiency*** (or simply ***efficiency***) to refer to the ratio of the overall revenue in the achieved allocation X to the maximum possible overall revenue of an efficient allocation X^*:

$$E(X) := \frac{\Pi(X, \mathcal{P}_{pay}) + \pi_{all}(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} \in [0, 1]$$

We can transform $E(X)$ into a form free of the pay prices $\mathcal{P}_{pay}$:

$$E(X) := \frac{\sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i(S) v_i(S)}{\sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i^*(S) v_i(S)} \in [0, 1] \tag{2.2}$$

We define the ***auctioneer utility share*** or ***auctioneer revenue share*** as the part of the maximum possible overall revenue which stays with the auctioneer:

$$\begin{aligned} R(X) := & \frac{\Pi(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} = \\ & = \frac{\sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i(S) p_{pay,i}(S)}{\sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i^*(S) v_i(S)} \qquad \in [0, E(X)] \subseteq [0, 1] \end{aligned}$$

and, similarly, the ***bidder utility share*** or ***bidder revenue share*** as the part obtained by all bidders:

$$\begin{aligned} U(X) := & \frac{\pi_{all}(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} = \\ & = E(X) - R(X) \qquad \in [0, E(X)] \subseteq [0, 1] \end{aligned}$$

Note that we need full information about bidders' valuations to calculate efficiency and revenue distribution. This is possible in the laboratory, but rarely in the field, where real valuations are seldom revealed by bidders, and often are not even precisely defined.

Revenue distribution between auctioneer and bidders is an important characteristic of an auction design. Auctioneer revenue and bidder revenue depend both on the final allocation X and on the pay prices $\mathcal{P}_{pay}$. As already mentioned, there is a tradeoff: Increasing bidder revenue will decrease auctioneer revenue to the same amount, provided the allocation does not change.

Often the selection of the auction mechanism is the prerogative of the auctioneer, who might prefer to choose revenue-maximizing mechanisms. However such a decision will raise the issue of strategic bidding. If bidders know that the selected auction design favors the auctioneer in revenue distribution, they will have incentives to ***shade*** their bids (report bid prices below their actual valuations). The auction efficiency will consequently suffer since the final allocation will be selected based on false preferences. Setting prices, which motivate bidders to truthful reporting of their valuations, is a key problem in the auction theory, and we will discuss it in detail in Section 2.4.1.

2.1.2 The First Example

We use the example in Figure 2.3 to illustrate several important auction properties before diving into the theory. In this scenario, a single seller offers a single good **$** to three potential buyers A, B and C. Each buyer has an internal valuation for the good, which in our case is 50 €, 70 € and 100 € for the bidders A, B and C respectively.

Consider two cases presented in Figure 2.3 which describe possible trade outcomes. In Case 1, bidder C purchases the good for the price of 30 €, and the overall revenue is 100 €. In Case 2, bidder B gets the good for 30 €, and the overall revenue is 70 €. Now consider what happens if the price changes from 30 € to 40 €. Note that the overall revenue does not change if the price changes.

In this straightforward example it is easy to find an efficient allocation — we have to select the bidder with the highest valuation. This simple approach would work in most practical cases too, and even in the case of combinatorial auctions - if the valuations of the bidders were publicly known. The problem

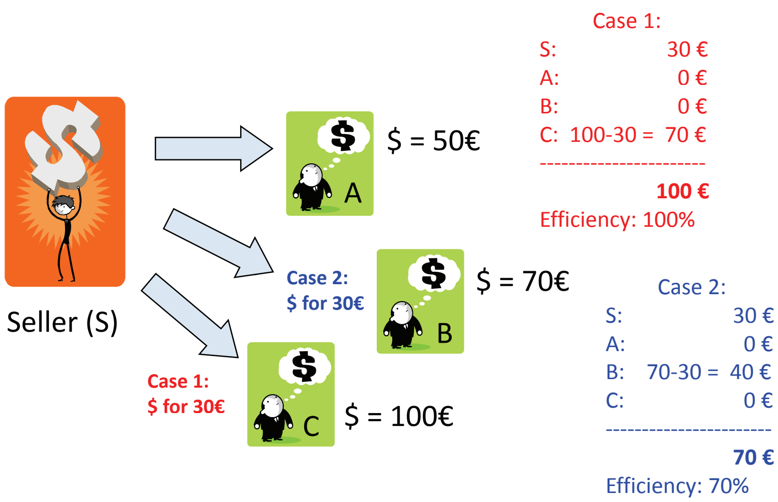


FIGURE 2.3: Example of a Trade

is that the bidders are usually reluctant to reveal their valuations. If asked to report them, they can try to speculate and understate their true valuations if they know that their statement is somehow connected with the price they will have to pay.

Having bidders reporting their valuations truthfully is essential. Without it, a trade will always be a lottery to some extent, and its efficiency cannot be guaranteed. This is where auctions come to play. Consequently, a successful auction design satisfies two properties:

- It motivates the bidders to tell the truth about their valuations.
- It achieves an efficient outcome *with respect to* the reported valuations.

These objectives are usually achieved by finding a suitable price which satisfy all parties in the auction, even when the trade is completed and the results are published. Consider again our example in Figure 2.3.

- If the price is too low, for example 30 €, several bidders would want to have the good. Consequently, those bidders who do not receive the good after the trade will not be happy with the outcome.

- If the price is too high, for example 120 €, none of the bidders will be interested in purchasing the good. This is an unsatisfactory outcome for the auctioneer.
- When the price lies between 70 € and 100 €, only bidder C is willing to pay for the good. Therefore this is the optimal range for the price. However only when the price is equal to $(50 + \epsilon)$ € for some very small ϵ will bidder C be really satisfied with the deal. If the price is higher, he will be motivated to shade his valuation.

Following these considerations, the auction shall terminate with the price of $(50 + \epsilon)$ € if bidders communicate information about their valuations truthfully. Simultaneously, this property is also the motivation for the bidders to bid truthfully. The popular ***English auction***, where bidders cry out new increasing prices, belongs to this class, as it will terminate as soon as the bidder C bids $(50 + \epsilon)$ €.

The ***Vickrey auction*** (Vickrey, 1961) - also called a *second-price sealed-bid* auction - has the same property. The ***sealed-bid*** property means that each bidder submits his bids (or a single bid in the single-item case) only once without knowing what other participants are bidding. The highest bid wins, but the pay price of the winning bid is equal to the second-highest bid, which reflects the ***second-price*** property. The following considerations are important:

- Pay price for the winning bidder is not affected by the bidder himself. In particular, he has no incentives to understate his valuation.
- There is no incentive to bid more than your own valuation, as such behavior is connected with a risk of paying more than this valuation and thus incurring losses. If a bidder overstates his valuation and then luckily wins with a positive revenue, bidding exactly his valuation would always result in the same outcome, but without risking potential losses.

From these considerations it follows[1] that the Vickrey mechanism has the desired properties. It achieves an outcome where neither bidder wants to change his bid after the auction, if he has reported his preferences truthfully. At the same time, this property motivates the bidders to bid their true valuations,

[1]For formal proof consult Vickrey (1961) or Ausubel and Milgrom (2006b).

independently of what other participants are doing. The bidder in this case is said to have a *dominant strategy*, which will be covered in more detail in Section 2.2.2.

The traditional English auction achieves the same result. The bidder will continue to improve his bid either until he wins in the auction when the last competitor stops bidding, or until someone else submits a bid which is higher than his valuation. It is said to be ***strategically equivalent*** to the Vickrey auction.

There exists a series of other single-item auction formats:

- ***Dutch auction***. In a Dutch auction the auctioneer begins with a high ask price which is lowered until some participant "raises hand" and accepts it. That participant pays the last announced price. Note that the bidder who has the highest valuation has an incentive to speculate and to raise his hand not precisely at his valuation, but lower. How much lower is a difficult question. It depends on valuations of other bidders, which must be estimated. Strategically, the Dutch auction is equal to the *first-price sealed-bid* auction. This auction format is convenient when it is important to auction goods quickly, since a sale never requires more than one bid. The Dutch auction is named after its best known example, the Dutch flower auctions.

- ***Chinese auction***. This is an interesting mechanism where every bidder pays his bid to the auctioneer (an ***all-pay auction***), and the winner is selected randomly with probability proportional to one's bid. Chinese auctions are used to model political elections or patent races, in which the chance of winning is seen in proportion to the amount spent.

- ***Japanese auction***. This format is similar and strategically equivalent to the English auction. The auctioneer regularly raises the current price. Participants must signal at every price level their willingness to stay in the auction and pay the current price. The auction terminates when only one bidder indicates his willingness to stay in. This auction format is also known as the ***button auction*** – every player holds the button as long as he agrees to pay the current price, the auction terminates when all bidders but one release the button.

2.2 Some Game Theory

Now we make an excursus into relevant aspects of game theory, which studies behavior of selfish ***agents***, or ***players*** in a ***game***. A game is defined as a process where two or more players interact following certain rules which restrict a set of possible actions, or ***strategies***, for every player. A game has a set of possible ***outcomes*** which depend on strategies which bidders choose to pursue. Each outcome has a certain ***payoff*** for each player, which is defined by his ***utility function***, and which selfish players try to maximize.

An auction is a game in this respect, and the game theory has been an important tool for modeling and understanding auctions, both traditional single-item and combinatorial. The notion of the game outcome corresponds in this case to the final revenue distribution in the auction and is characterized by the ***payoff vector*** $(\Pi, \pi) = (\Pi, \pi_1, \dots \pi_n)$, which includes in this case the payoff of the auctioneer and each bidder.

In Section 2.1.1 we assumed that the bidder's payoff is equal to the difference between the bidder's valuation and the price he pays for the bundle: $\pi_i(S, \mathcal{P}_{pay}) := v_i(S) - p_{pay,i}(S)$, and we will discuss this assumption in more detail in Section 2.3. However we do not need this assumption for the game-theoretical concepts, which are the subject of this section.

Within the game theory, it is often assumed that each player knows all other participants, strategies which they can choose, and their payoffs. Whenever these assumptions are not met, we have a ***Bayesian game***, where the information about characteristics of other players is incomplete.

Several related theories are helpful in the auction research, too.

- The ***general equilibrium theory*** studies games with a large number of players. In particular, it is used in microeconomics theory to model markets, trades, consumption and production.
- The ***decision theory*** deals with one person games, or games of a single player against nature. It focuses on a player's preferences and the formation of his beliefs and decisions, especially in scenarios with risky or uncertain alternatives.
- The ***mechanism design theory*** complements the game theory by exploring how different rules influence the game outcome. Ultimately its

mission is to design such rules that the players have incentives to guide the game to the desired outcome.

Below we introduce important concepts from this family of theories.

2.2.1 Game Types

A game can be ***cooperative*** (also ***coalitional***), or ***non-cooperative***. In a cooperative game players can form ***coalitions*** $C_I, I \subsetneq \mathcal{I}$ by closing binding contracts with each other. The ***grand coalition*** $C_{\mathcal{I}}$ is the coalition containing all players in the game. Usually communication between players is allowed in cooperative games, but not in noncooperative ones.

A closely related concept is the ***transferable utility***, which can be assumed in a cooperative game. The utility is transferable if one player can transfer part of his utility to another player without loss. In such cases payoffs are calculated not for individual players but for whole coalitions. This is known as the *coalitional value function*:

Definition 1. *The* ***coalitional value function*** $w(C_I)$ *is defined as the maximum overall revenue that can be generated by the bidders contained in the coalition:*

$$w(C_I) := \max_{X=(S_1,\ldots,S_n)\in\mathcal{X}} \sum_{i\in I} v_i(S_i)$$

for any coalition C_I *of players. In an auction, every meaningful coalition will include the auctioneer, since no game is possible without his participation. The coalitional value function of the grand coalition corresponds to efficient allocations in the auction.*

Both cooperative and non-cooperative games can be used to model auctions. If certain properties hold for the cooperative case, the auction design is better, since the desired result will be achieved even if the bidders communicate and try to engage in agreements (coalitions).

An important concept in this respect is the ***core***.

Definition 2. *The set of* ***core payoffs*** *is defined as*

$$Core\,(\mathcal{I}, w) = \left\{ (\Pi, \pi) : \Pi + \sum_{i\in\mathcal{I}} \pi_i = w(C_{\mathcal{I}}) \text{ and } \forall\, I \subset \mathcal{I} : w(C_I) \le \Pi + \sum_{i\in I} \pi_i \right\}$$

In a cooperative game, the core of the game is a set of payoff vectors which are efficient in the grand coalition and no coalition can earn more by separating from the grand coalition. For a payoff vector (Π, π) which is not in the core, a coalition $C_I, I \subsetneq \mathcal{I}$ exists, with a coalitional value $w(C_I)$ which is strictly higher than the total payoff (Π, π). The coalition C_I has an incentive to break from the game and redistribute $w(C_I)$ between them, which makes each member strictly better off. The outcome, characterized by the non-core payoff vector (Π, π), is said to be ***blocked*** by the coalition C_I.

For an auction, core outcomes are essential to ensure that neither coalition of bidders wants to separate and review results of the auction together with the auctioneer after the results are announced. As shown below in Section 2.3.4, other important auction properties require core outcomes too.

A game can be ***simultaneous*** (also ***direct mechanism***), or ***sequential*** (also ***dynamic***). In a simultaneous game each player makes his decisions unaware of the actions of the other players. In a sequential game, players have some knowledge about earlier actions of other players, and can adjust their moves according to this information. Sealed-bids auctions correspond to simultaneous games, and iterative auctions – where bidders receive some information between their bids – correspond to sequential games.

2.2.2 Game Outcomes

A central task of game theory is to characterize possible outcomes of the game, assuming that each bidder behaves ***rationally*** – tries to maximize his own payoff.

In the terminology of the game theory, the desirable efficient outcome of an auction is ***Pareto efficient***, or ***Pareto optimal***. An outcome is Pareto efficient if there is no other outcome that makes every player at least as well off and at least one player strictly better off. In other words, a Pareto optimal outcome cannot be improved without hurting at least one player.

An interaction of all rational players in the game results in an ***equilibrium*** outcome. A successful auction design shall terminate in an equilibrium which has two properties:

- It shall be Pareto optimal; that is, correspond to an efficient allocation.

- It shall be stable, which means that the bidders who deviate from their equilibrium strategies have strictly worse payoffs, and bidders who do not change their strategies have no better strategy in the new circumstance.

Nash equilibrium is a set of strategies where no player can do better by unilaterally changing his strategy. Nash equilibrium strategies are a natural assumption for selfish bidders in an auction, which makes this concept highly relevant for the auction theory. Since the Nash equilibrium focuses on an individual's preferences given that the others stay the same, there can be Nash equilibria which become unstable if bidders are able to form coalitions. There can be several Nash equilibria in a game.

A stronger concept is the ***dominant strategy***, which is the best possible strategy for a player *regardless* of what other players are doing. Auctions with dominant strategies are attractive since they make it unnecessary for bidders to guess or otherwise learn actions of other bidders – they are called ***strategy-proof***.

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

FIGURE 2.4: Payoff Matrix of the Prisoner's Dilemma Game

Several important properties of the equilibria concept are illustrated by the famous ***prisoner's dilemma***, presented in Figure 2.4 in the canonical form. Each player has a possibility either to *Cooperate* or *Defect*; numbers in the table give payoffs of each player for each of the four possible outcomes. We can observe that:

- Each bidder has a dominant strategy of *Defecting* independently of what the other party is doing.
- As always in cases where each player has a dominant strategy, the corresponding dominant strategy equilibrium coincides with the single Nash equilibria.
- The dominant strategy equilibrium (*Defect, Defect*) is not Pareto optimal, since the (*Cooperate, Cooperate*) outcome is better for each player.

- Consequently, Nash equilibrium is not necessarily a core outcome.

An ***ex post equilibrium*** of a Bayesian game is a profile of strategies which are optimal for each bidder given any types of other bidders. Intuitively that means that the bidder will not want to change his behavior after the game finishes and he is told about the strategies of other bidders. The ex post equilibrium is stronger than the Nash equilibrium of a Bayesian game, but weaker than the dominant-strategies equilibrium.

For market-like games with prices, a ***Walrasian equilibrium*** defines a vector $(X, \mathcal{P})$, where $\mathcal{P}$ is a price vector for each good and X is a feasible allocation which satisfies each bidder given prices $\mathcal{P}$. That means that none of the bidders would want to change the allocation if the prices remain fixed to $\mathcal{P}$. Every Walrasian equilibrium has the core property, but not vice versa. However, under some assumptions, as the number of market participants grows to infinity, the core expands and tends to a set of Walrasian equilibria, a result known as the Edgeworth conjecture.

An important question is whether the game rules stimulate participants to participate and to tell the truth about their profiles, or rather to withdraw from the game or try to get a better payoff by playing ***strategically*** and reporting incorrect or incomplete information.

Definition 3. *A game is* ***individually rational*** *if players expect to gain higher utility from participating in the game than from avoiding it.*

Definition 4. *A game is* ***incentive compatible*** *if a player is better off when he truthfully reveals any private information the mechanism asks for.*

There are different degrees of incentive compatibility. In a *strategy-proof* game, truth-telling is a dominant strategy. A weaker case is when truth-telling is a Bayes-Nash equilibrium: It is best for each participant to tell the truth, provided that others are also doing so. However, simultaneous games are ex post incentive compatible if and only if they are dominant-strategies incentive compatible.

Two theorems reveal important relations between the above concepts.

Theorem 2 (First Fundamental Theorem of Welfare Economics)**.**
Every Walrasian equilibrium is Pareto efficient.

The theorem is subject to strong assumptions that players are rational, markets are complete, there are no externalities and information is perfect. Such assumptions are obviously unrealistic. The reasonable interpretation of the theorem, which is important for the auction design, is that an efficient market (price-based game) is *possible*, and the source of inefficiencies is not the distributed nature of the market, but rather something else.

Theorem 3 (Second Fundamental Theorem of Welfare Economics)**.**
Every efficient allocation can be supported by some set of prices.

We define *supporting* prices formally in Section 2.4.1. For now, we give the theorem the following interpretation, related to the auction design: It is *possible* to construct a price-based efficient auction. However, the assumptions of the Second Theorem are even stronger than those of the First Theorem. The actors' preferences must be convex, which corresponds to the *goods are substitutes* condition in the terminology of combinatorial auctions (Definition 12). The questions which remain open at this point are what is the sufficient price format for an efficient combinatorial auction, and how can such prices be calculated. The answers are presented in Section 2.4.2.

The game theory is elegant and concise. However, its practical applicability for auctions is often criticized (Rothkopf, 2007a), and laboratory experiments indicate that it fails to model auctions precisely enough. Its strong assumptions are probably the reason:

- Payoffs of bidders are known and fixed.
- Risk neutrality, meaning that the expected payoff is treated in the same way as the actual payoff.
- All players behave rationally. They understand and seek to maximize their own payoffs and do not care about other bidders' payoffs. They are flawless in calculating actions which increase their payoffs.
- The rules of the game are common knowledge. Each player knows all rules, understands them, knows all other players and their possible strategies.

These assumptions are obviously too strong, especially for the modeling of combinatorial auctions. Truly private independent valuations, which are implied by these assumptions, are extremely rare. Exponential size of bidders' valuations and exponentially large strategy space mean that the assumption of rational behavior is too optimistic.

2.3 Combinatorial Auctions

From this point on, we concentrate on ***combinatorial auctions*** (***CA***) – auctions which have the following distinguishing features:

- Several heterogenous items are traded simultaneously.
- The auction can have several winners, whereby each item is assigned to one winner at most.
- Bidders can submit bids for indivisible bundles – the *bundle bids*.

Bidders in the auction are usually modeled as selfish agents, who have fixed valuations $v_i(S)$ for every possible bundle $S \subseteq \mathcal{K}$ and try to maximize their payoffs $\pi_i(S, \mathcal{P}_{pay})$. The following presentation is based on a number of assumptions which define the behavior of bidders in an auction. These assumptions are typical for the auction literature (Parkes, 2006), albeit not free of critique (Rothkopf, 2007a):

- ***Independent private valuations***, meaning that $\mathcal{V}_i$ does not depend on valuations of other bidders in any way. Furthermore, the valuations cannot change during the auction, as the bidders learn more about valuations of other bidders.
- ***Free disposal***, meaning that every bidder can dispose of any item at no cost: $v_i(S) \leq v_i(T)\ \forall\ S \subset T$.
- Consequently, $v_i(S) \geq 0\ \forall\ S$ and the assumption of ***normalization*** $v_i(\emptyset) = 0$ holds.
- ***Quasi-linear bidder utilities*** with $\pi_i(S, \mathcal{P}_{pay}) := v_i(S) - p_{pay,i}(S)$ and $\pi_i(\emptyset, \mathcal{P}_{pay}) := 0$. This implies that the bidders have no budget constraints.
- The auctioneer has no own value for the items; his utility is the sum of all pay prices.

2.3.1 Bidding Languages

To find an efficient allocation, the auctioneer needs sufficient information about bidder's value models. The ***bidding language*** of an auction is a formally defined communication protocol which bidders use to represent and transmit their preferences concerning traded goods to the auctioneer. The process of collecting this information itself is called ***preference elicitation***.

Being trivial in the case of single-item auctions, the preference elicitation quickly becomes problematic in the context of combinatorial auctions, since bidders need to calculate, formulate, and transfer to the auctioneer information on exponentially many $2^{|\mathcal{K}|} - 1$ bundles. A suitable bidding language is important in this respect; its selection is driven by the following criteria:

- The bidding language shall be expressive. It shall allow the bidder to represent his valuations precisely and completely, for every possible value model.
- It shall be simple and intuitive for bidders. As with most language design tasks, there is a tradeoff between simplicity and expressiveness.
- The bidding language shall be suitable for formal processing by the auctioneer. An algorithm must exist for selecting a revenue-optimizing subset of bids from the complete set of bids submitted by all bidders in the auction.

Early work on combinatorial auctions paid little attention to the issue of bidding languages. The common approach was to allow the bidder to submit a set of plain bids, which combine a bundle of items and a bid price:

Definition 5. *An* ***atomic bid*** $b_i(S) = (S, p_{bid,i}(S))$ *is a tuple consisting of a bundle* S *and a bid price* $p_{bid,i}(S)$ *submitted by the bidder* i*. A set of atomic bids is* ***overlapping*** *if at least one item is included in more than one bid.*

Further rules define which combinations of atomic bids can win simultaneously, in case the bidder submits several atomic bids. The two following interpretations are the most common:

- The ***additive-OR*** (***OR***) bidding language allows the bidder to win any non-overlapping combination of his atomic bids.

- The ***exclusive-OR*** (***XOR***) bidding language implies that the bidder can win at most one of his atomic bids.

Historically, most existing CA designs use either OR or XOR bidding languages, and some auction formats can use both. These bidding languages are universal, and can be used in any application domain. For these reasons we concentrate in our work only on the auctions which use one of these two bidding languages. Since we do not consider more complex bidding languages in the following presentation – except for the rest of this chapter – we will often use the term *bid* instead of *atomic bid*.

The OR bidding language is less expressive than the XOR bidding language. A value model {({A},10 €), ({B}, 10 €), ({A,B}, 15 €)} can be described using the XOR bidding language with three atomic bids which directly correspond to the valuations. The OR bidding language, however, is insufficient in this case. If the same three atomic bids are submitted, the auction engine will always prefer two single-item atomic bids ({A},10 €) and ({B}, 10 €) to the bundle atomic bid ({A,B}, 15 €). The bidder will have to pay 20 € for both items and consequently lose 5 €.

Intuitively, the OR bidding language is insufficient whenever a bidder has subadditive valuations or budget constraints. The XOR bidding language, on the other hand, is fully expressive, meaning that it can be used to express every possible value model. Its negative side is that if a bidder has only additive or weakly complementary valuations, and is interested in an arbitrary subset of items in any combination, he will have to submit exponentially many XOR bids in situations where just a few OR bids would suffice.

Several researchers suggest more sophisticated bidding languages for combinatorial auctions. The proposed improvements target primarily two objectives:

- Improved usability of the bidding language, which allows the bidders to represent their value models in a more laconic form compared to the XOR bidding language. The bidder must communicate an exponential amount of information to fully describe his valuations in the worst case (Nisan, 2006). However it is often possible, especially in domain-specific applications, to use the knowledge which the auctioneer and the bidders share about the value model structure and achieve much more compact representation.

- Mitigating the complexity of searching for a revenue-maximizing allocation, which is generally NP-hard. This fact presents a practical problem for combinatorial auctions, since the problem can become infeasible already for a moderate number of items and bidders. However, if the bidders are restricted in their bids to a certain set of bundles, it is possible to develop algorithms which find a revenue-maximizing allocation in a polynomial time (Carlsson and Andersson, 2004; Goeree and Holt, 2008; Rothkopf et al., 1998). It is important to note that the NP-completeness is not a problem of the XOR bidding language by itself, and any combinatorial auction with a fully expressive bidding language will face the same issue.

One approach to the construction of a more versatile bidding language is to allow for more complex logical operations on individual atomic bids. Sandholm (2002) suggests two-level nesting formats: OR-of-XOR and XOR-of-OR, which shall provide more compact bid-terms and simplify the translation of bidder preferences into bids. Obviously both bidding languages are fully expressive, since they present a generalization of the XOR language. Nisan and Ronen (2001) introduces the OR* bidding language, an OR language with *dummy items* added to bids, which must be exclusive. If, as in the above example, a bidder has sub-additive valuation for the bundle {A,B} and can use the OR* bidding language, he submits bids ({A,X}, 10 €), ({B,X}, 10 €), ({A,B}, 15 €) where X is a dummy item, which eliminates the described problem. The OR* bidding language is both fully expressive and compact. Nisan (2006) provides an extensive analysis and comparison of these three, and other similar constructs. Boutilier and Hoos (2001) suggest the $\mathcal{L}_{\mathcal{GB}}$ language, which allows the use of the combinatorial $k - of$ operator applied to a set of atomic bids.

All bidding languages listed above have a straightforward LP formulation (see Section 2.3.2 below) for the cost of giving the bidder only very basic possibilities for expressing his bids. Several other approaches suggest more compact and expressive bidding languages which require, however, non-trivial LP formulations for the problem of calculating a revenue-maximizing allocation. On the other hand, the compactness can be an effective approach to reducing computational times caused by the exponential growth of possible combinations. One example of such a "complex-bid" language is the ***Matrix Bidding Language*** (***MBL***). It was introduced by **?** with the intention of compacting the bid expressions. The authors showed that the MBL is equally expressive, and in some cases even more compact, than the $\mathcal{L}_{\mathcal{GB}}$ language.

Another approach described by Cavallo et al. (2005) generalizes the idea of two-level nested logical connectives. In contrast to the OR-of-XOR bidding language, there are no more limitations on the number of nested levels. The structure of a bid in this language can be viewed as a tree where internal nodes describe the relationship between lower-level nodes and individual items are represented by the leaves. The proposed ***Tree-Based Bidding Language*** (***TBBL***) features several novel options. One of the most interesting of them is the ability to handle ask-items and bid-items in a single bid. Expression of preferences for both buying and selling goods complies with the requirements for double-sided auctions and with an even broader vision of combinatorial exchanges (Parkes et al., 2005). TBBL also uses an interesting method for the expression of monetary valuations. The bid as a whole and also every node in the tree can have its own “added value”. A multi-unit extension for the TBBL language was suggested and implemented within the scope of the MarketDesigner project (Alexeev, 2008).

2.3.2 Combinatorial Allocation Problem (CAP)

An *efficient allocation* (see Section 2.1.1 for the formal definition) in a combinatorial auction can be found by solving the ***Combinatorial Allocation Problem*** (***CAP***), also called the ***Winner Determination Problem*** (***WDP***):

$$\max_{X=(S_1,\ldots,S_n)\in\mathcal{X}} \sum_{i\in\mathcal{I}} v_i(S_i) \qquad (\textbf{CAP}) \tag{2.3}$$

Under assumptions introduced in the previous section, the CAP (2.3) can be represented in the form of an integer linear program (ILP) (2.4). It uses binary decision variables $\{x_i(S)\}$, $x_i(S) \in \{0;1\}$ where $x_i(S) = 1$ means that the bidder i gets exactly the bundle S. The objective function maximizes the sum of valuations of the winning bundles, and thus maximizes the overall revenue. The first set of constraints guarantees that at most one bundle can be allocated to each bidder, as required by the fully expressive XOR bidding language. Without these constraints, the auctioneer allows for OR bids. Note that each bidder can be assigned either OR or XOR bidding language independently of other bidders. The second set of constraints ensures that each item is sold at most once.

$$\max \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) v_i(S) \qquad (\textbf{\textit{CAP-I}}) \qquad (2.4)$$

s.t.

$$\sum_{S \subseteq \mathcal{K}} x_i(S) \leq 1 \qquad \forall\, i \in \mathcal{I}$$

$$\sum_{S : k \in S} \sum_{i \in \mathcal{I}} x_i(S) \leq 1 \qquad \forall\, k \in \mathcal{K}$$

$$x_i(S) \in \{0; 1\} \qquad \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K}$$

The CAP-I (2.4) is NP-hard if bidders submit a number of bundle bids that is less than some polynomial function of the number of items m. When bids are submitted on all bundles, which is rarely realistic, and certain other restrictions are met, Rothkopf et al. (1998) provide a polynomial algorithm for solving the CAP-I.

Another less obvious complication is that the CAP-I (2.4) requires valuations of every bidder and for every bundle, if we use it to find an efficient allocation directly. However, the bidders do not necessarily communicate their valuations in submitted bids truthfully and completely. To emphasize this fact, in the case when the allocation is calculated using bids rather than valuations, we call it a ***revenue-maximizing*** rather than *efficient* allocation.

2.3.3 Allocation Rules

The LP-based interpretation of the combinatorial auction allows the addition of arbitrary ***allocation rules*** to the market. Simply by adding new constraints to the CAP-I (2.4), the auctioneer can specify, for example, that there must be no more than 10 and no less than 3 winners; that a certain bidder i shall not receive more than 10% of the overall market volume; that items k and l shall be won by different bidders, etc.

Another attractive feature is that the cost of each allocation rule applied by the auctioneer can be explicitly quantified. After calculating the revenue-maximizing allocation with and without the rule, the difference in the objective function value gives the cost of enforcing the allocation rule in the auction. The auctioneer can then decide whether he is willing to pay the additional cost and insist on the constraint – the process is called ***scenario navigation*** in the literature (Boutilier et al., 2004). In such cases the negotiation process cannot be called "an auction" any more since the market rules are not a priori fixed and can be changed by the auctioneer dynamically. Bidders might want to adopt their strategies accordingly.

Another interesting possibility is to use bidder-side allocation constraints, thus creating a more expressive bidding language. For example, bidders could define conditional dependencies between their bids, conditional discount offers, capacity or budget constraints, etc.

Allocation rules add complexity to the optimization problems solved by the auctioneer and affect the running time correspondingly (Kalagnanam et al., 2001). Especially dynamic manipulation of the rules in the process of the auction can require adequate algorithms with appropriate hardware and software support.

Another issue is that allocation rules, especially when they are changed during the auction, can significantly reduce the quality of the feedback which ask prices provide for the bidders (Klimova, 2008). How price calculation algorithms must be modified, or which additional information must be communicated to bidders to better reflect the market situation in cases where allocation rules are used, is still an open question in auction research.

The MarketDesigner framework provides the auctioneer with a broad set of allocation rules (Klimova, 2008) and with tools for scenario navigation (Neykov, 2007).

Now, as we understand the task which combinatorial auctions are designed for better, we can start looking at possible solutions.

2.3.4 Vickrey-Clarke-Groves (VCG) Auction

The single-item Vickrey auction described in Section 2.1 has an important property: Truthful bidding is a dominant strategy which leads the auction to an efficient outcome. From the game-theoretical point of view this is the best possible mechanism, if auction efficiency is our desired property.

The Vickrey auction was generalized by Clarke (1971) and Groves (1973) to a generic competitive process, which includes the concept of a combinatorial auction as a special case. The ***Vickrey-Clark-Groves*** (***VCG***, or ***generalized Vickrey***) auction has the same wonderful properties: Truthful bidding is a dominant strategy which guides the auction to an efficient allocation. This is achieved by refunding bidders the increase in the overall revenue caused by their bids. In other words, each bidder pays the social opportunity cost of winnings, rather than the full bid price of the submitted bid.

In a VCG auction, the bidders report their valuations $v_i(S)$ on all bundles $S \subseteq \mathcal{K}$ to the auctioneer, who determines a revenue-maximizing allocation by solving the CAP-I (2.4). Winners pay their bid prices reduced by ***VCG discounts***, which are calculated as $w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus i})$:

$$p_{pay,i}(S) = p_{bid,i}(S) - \big(w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus i})\big) \tag{2.5}$$

Unfortunately, the theoretically beautiful VCG design has several drawbacks, which make its practical application problematic (Ausubel and Milgrom, 2006b; Rothkopf, 2007b). As of today, there are no documented cases of application of the VCG auction in the field. At the same time the first-price sealed-bid combinatorial auction, where winning bidders pay exactly what they bid, was used as a model for some auctions in practice (Elmaghraby and Keskinocak, 2002), even though its strategic complexity for bidders is high.

The following Example 1 illustrates some problems of the VCG auction[2].

Example 1. *Problems of the VCG CA format.*

In a market with two goods A and B the bidder b_1 wants to have both goods for at most 2, and both bidders b_2 and b_3 are interested in any single item, and are ready to pay for it at most 2.

Valuations	A	B	AB
Bidder b_1	0	0	2
Bidder b_2	2*	2	2
Bidder b_3	2	2*	2

An efficient outcome, marked in the table with asterisks, is to sell the good A to the bidder b_2 and the good B to the bidder b_3. Both bidders receive discounts on their bid price, which are equal to the difference between the efficient allocation value, which is four in our case, and an efficient allocation value, calculated without the bidder, which is 2. Therefore each bidder has to pay $(2-(4-2)) = 0$ and receives his good for free. The auction revenue is 0.

[2]Taken from Ausubel and Milgrom (2006b)

The potential revenue deficiency is the biggest issue with the VCG auction format. As it is the auctioneer who usually decides on the selected market format, there are few arguments to defend an auction which, even with sufficient competition, can result in low or zero revenue. Note that even if the auctioneer were to abstain from the idea of conducting a combinatorial auction at all and sell both goods as a bundle in an English auction, he would probably receive a revenue of 2.

There are further problems with the VCG format:

- Closely related to the above low revenue issue is the monotonicity problem. Removing the bidder b_3 from the auction in the above example will *increase* the auctioneer's revenue from 0 to 2. Similarly, adding new bidders to an auction might *reduce* revenue, which is counterintuitive.

- The VCG auction is vulnerable to ***collusion*** even by losing bidders. Imagine that bidders b_2 and b_3 are ready to pay not 2 but only 0.5 for a single license. The allocation accordingly changes in favor of the first bidders. Now if bidders b_2 and b_3 both bid 2 instead of 0.5, they receive items for nothing. An associated problem is ***shill bidding***, where losing bidders might try to use additional false bidding identities to improve their position in the auction.

- The auctioneer must solve an additional reduced CAP for each bidder to calculate his discount. This makes the VCG auction significantly more computationally complex than for example the first-price sealed bid auction.

- Each bidder must report his complete value model by submitting $2^{|\mathcal{K}|} - 1$ bids. This task quickly becomes overwhelming for bidders, and also results in large input sizes for the CAP. In practice bidders will be likely to skip their low-valued bundles and implicitly report zero valuation on them. As the auctioneer revenue in the VCG auction depends not only on winning bids, but also on the losing bids, this can be a problem and further reduce the auctioneer's revenue.

- There are certain problems of privacy and trust with the VCG auctions. Often bidders are reluctant to reveal their true valuations to the auctioneer since they are afraid that they will be pressed to pay more than they should. The auctioneer must be trusted to calculate prices fairly, or a trusted third party is required. Fairly speaking, this particular issue can be handled using modern cryptographic protocols (Brandt, 2003).

Even though the above problems have been preventing the practical use of the VCG auction, it remains an important theoretical construct. It is used as a reference point for numerous auction formats, and helps to explain many theoretical issues regarding combinatorial auctions.

All the VCG problems discussed above can be roughly grouped into two classes:

- The VCG outcome is not necessarily in the core. In Example 1, the coalition of the seller and the bidder b_1 blocks the efficient allocation.
- Each bidder must submit bids for every bundle in one shot. This requirement can quickly become overwhelming since the amount of possible bundles is exponential in the number of items. Therefore the bidders might decide to bid only on bundles which have high valuations and skip low-valued bundles, implicitly bidding zero on them. Even though this problem is relevant for every sealed-bid auction, it is especially urgent for the VCG auction, since its results depend not only on the winning bids, but on the “best” losing bids too.

2.4 Iterative Combinatorial Auctions (ICAs)

A common approach to addressing many problems of the VCG format and other sealed-bid CA designs is the ***iterative combinatorial auction*** (***ICA***). A typical iterative auction uses a *tâtonnement* mechanism (Figure 2.5), which repeatedly collects bids from bidders, analyzes them, and communicates back a set of ask prices and (optionally) further information about the auction status. Such information can include the provisional allocation, the list of bids submitted by other bidders, the number of active bidders and/or bids, etc.

The process of gradually increasing prices until the demand equals supply (*tâtonnement*), when designed properly, results in a *Walrasian equilibrium*, which is Pereto optimal and therefore efficient. Simultaneously, the result is a core outcome. Certainly both efficiency and core properties hold only with respect to the reported preferences. Proper ask price calculation, which roughly means calculating minimum prices, can provide strong incentives for bidders to bid truthfully. However, the dominant truth revealing is not possible, since it would require VCG prices. This is an important dilemma in the CA design, which is addressed in more detail below in Section 2.4.2.1.

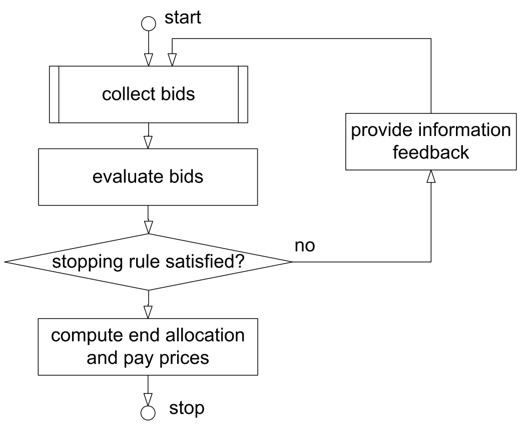


FIGURE 2.5: Flowchart of an Iterative Combinatorial Auction

Prices and other feedback in an ICA can help bidders to find the most attractive goods and bundles without evaluating and submitting bids for all possible valuations. This is important both because there are exponentially many bundles, and because bidders might be reluctant to lay all valuations open. The problem of eliciting and submitting the bids "has emerged as perhaps the key bottleneck in the real-world application of combinatorial auctions. Advanced clearing algorithms are worthless if one cannot simplify the bidding problem facing bidders" (Parkes, 2006). The problem is known as the ***Preference Elicitation Problem*** (***PEP***). ICAs are to date the most promising way of addressing the PEP.

Although there is no formal proof, there is strong evidence that iterative auctions perform better than sealed-bid designs when the assumption of private valuations, which is in fact very strong for practical applications, is not met (Elmaghraby and Keskinocak, 2002; Milgrom and Weber, 1982). "Experience in both the field and laboratory suggest that in complex economic environments iterative auctions, which enhance the ability of the participant to detect keen competition and learn when and how high to bid, produce better results than sealed bid auctions" (Porter et al., 2003).

An iterative CA is usually conducted in ***rounds***. Bidders receive new information about the auction state (prices, etc) only at the beginning of the round, and this state remains unchanged during the whole round. New provisional allocation and ask prices are calculated between rounds. All bids within the same round normally conform to the same requirements (ask prices, activity rules, etc.). Although some designs clear rounds after each bid (Adomavicius and Gupta, 2005), usual practice is to have rounds of a certain duration. This is done mainly for the following reasons. More arguments can be found in (Cramton, 1998).

- Rounds allow the construction of effective and concise activity rules, which require bidders to remain competitive in each consecutive round. This prevents "sniper" behavior when bidders just monitor the auction until it nearly finishes and submit their bids at the very end. If used by many bidders, this can prevent correct demand elicitation and render the auction to a lottery. This will be looked at in Section 4.1.3.
- Calculating new ask prices after each new bid can cause significant fluctuations. Accumulated over some period of time and from many bidders, new bids will lead to more balanced prices and better reflect the market situation.
- Clearing a round often requires significant computational effort to determine the new provisional allocation and ask prices. Doing it after each bid can present a technical problem and negatively impact responsiveness of the auction.

Bidders usually report their demand precisely at the ask prices; in some settings ***jump bids*** with bid prices strictly above ask prices are allowed. Such mechanisms actually deviate from the *tâtonnement* concept, but allow the auction to progress faster. However in such auctions bidders can attempt signaling, directed at staking out the bundles they are interested in, for example by using high jumps or encoding areas they are interested in in the price (Cramton et al., 2006). Information hiding (e.g. bid price rounding, setting fixed bid steps) can be used to limit the possibilities of signaling between bidders.

2.4.1 Equilibrium Prices in ICA

The key challenge in the iterative combinatorial auction design is the calculation of prices, which will guide the auction towards an efficient equilibrium outcome and provide high incentives for truthful reporting. In contrast to single-item auctions, pricing is not trivial when combinatorial bids are allowed.

A usual assumption is that bidders, if they do not know when the auction will terminate, will try to maximize their potential payoff in every round given current ask prices. This strategy is called ***best-response bidding*** (also ***myopic bidding***). In every round the bidder bids on his *demand set*:

Definition 6. ***Demand set*** *includes all bundles that maximize the bidder's utility at the given prices:*

$$D_i(\mathcal{P}_{pay}) := \left\{ S \subseteq \mathcal{K} : \pi_i(S, \mathcal{P}_{pay}) \geq \max_{T \subseteq \mathcal{K}} \pi_i(T, \mathcal{P}_{pay}) \text{ and } \pi_i(S, \mathcal{P}_{pay}) \geq 0 \right\}$$

In many pay-as-bid auctions, prices rise until an equilibrium outcome is reached where neither bidder wants to change the allocation by submitting any further bids and consequently increasing prices. At this point the auction will end. The ***competitive equilibrium*** (***CE***) condition formalizes this concept:

Definition 7. *Prices $\mathcal{P}_{pay}$ and allocation $X^* = (S_1^*, \ldots, S_n^*)$ are in competitive equilibrium if:*

$$\begin{aligned} \pi_i(S_i^*, \mathcal{P}_{pay}) &= \max_{S \subseteq \mathcal{K}} [\pi_i(S, \mathcal{P}_{pay}), 0] \quad \forall\, i \in \mathcal{I} \\ \Pi(X^*, \mathcal{P}_{pay}) &= \max_{X \in \mathcal{X}} \Pi(X, \mathcal{P}_{pay}) \end{aligned}$$

The allocation X^* is said to be ***supported by prices*** *$\mathcal{P}_{pay}$ in competitive equilibrium.*

Intuitively these two conditions mean that all participants are satisfied with the outcome:

- The first condition states that each bidder obtains the bundle which maximizes his payoff with respect to his exposed preferences. In the event that the bidder could have more revenue from winning another bundle, he would submit a corresponding bid and the auction would continue. Note that bundles S_i^* can also be empty. All losing bids will be below equilibrium prices, or the bidders will want to change the allocation: $x_i(S) = 0 \Leftrightarrow p_{ask,i}(S) > p_{bid,i}(S)$.
- The second condition states that the selected allocation X^* maximizes the auctioneer's revenue given the submitted bids. Pay prices of all winning bids will be equal to the equilibrium prices: $x_i(S) = 1 \Leftrightarrow p_{ask,i}(S) = p_{bid,i}(S)$.

2.4.1.1 Minimum Competitive Equilibrium Prices

Let us come back to the example in Figure 2.3. Every price between 70 € and 100 € inclusive, together with allocating the item **$** to the bidder C, is a CE outcome. However, the bidder C will have incentive to speculate and shade his true valuation if the final price is above $(70 + \epsilon)$ € for any significant $\epsilon > 0$.

Therefore, to support truthful bidding, the auction design shall minimize CE prices:

Definition 8 (Minimal CE Prices)**.** ***Minimal CE prices*** $\mathcal{P}_{pay}$ *minimize the auctioneer revenue* $\Pi(X^*, \mathcal{P}_{pay})$ *on an efficient allocation* X^* *across all CE prices.*

An important question is whether an auction with minimal CE outcome provides bidders with enough incentives for truthful bidding. A simple answer is that this is not the case, since the CE outcome is in the core, and any incentive compatible mechanism must result in the VCG outcome, which is not necessarily in the core.

A more elaborate answer requires the definition of cases when minimum CE outcome is in the core, and analysis of equilibrium strategies in an auction which results in a minimal CE outcome. We provide these details in Section 2.4.2.1.

2.4.1.2 Price Formats in Iterative Combinatorial Auctions

Before concentrating on the problem of calculating CE prices, we review possible price formats in combinatorial auctions. Different pricing schemes have been discussed in the literature, including linear, non-linear, and non-linear personalized prices (Xia et al., 2004).

Definition 9. *A set of ask prices* $\{p_{ask,i}(S)\}$ *is called* ***linear (additive)*** *if* $\forall\ i, S : p_{ask,i}(S) = \sum_{k \in S} p_{ask,i}(k)$

Definition 10. *A set of ask prices* $\{p_{ask,i}(S)\}$ *is called* ***anonymous*** *if* $\forall\ i, j, S : p_{ask,i}(S) = p_{ask,j}(S)$

In other words, the prices are *linear* if the price of a bundle is always equal to the sum of the prices of its items, and the prices are *anonymous* if the price of the bundle is the same for every bidder. Non-linear ask prices are also called ***bundle*** ask prices. Non-anonymous ask prices are also called ***discriminatory*** or ***personalized*** ask prices. By combining these two characteristics four settings for ask prices can be built:

- a set of linear anonymous prices $\mathcal{P}_{ask} = \{p_{ask}(k)\} \ \forall \ k \in \mathcal{K}$
- a set of linear personalized prices $\mathcal{P}_{ask} = \{p_{ask,i}(k)\} \ \forall \ i \in \mathcal{I}, k \in \mathcal{K}$
- a set of non-linear anonymous prices $\mathcal{P}_{ask} = \{p_{ask}(S)\} \ \forall \ S \subseteq \mathcal{K}$
- a set of non-linear personalized prices $\mathcal{P}_{ask} = \{p_{ask,i}(S)\} \ \forall \ i \in \mathcal{I}, S \subseteq \mathcal{K}$

Linear personalized prices have hardly been considered in the context of combinatorial auctions, except for a special case of the ALPS auction format (Section 4.2.4). All three other price formats have been extensively discussed in literature.

2.4.1.3 Existence of Competitive Equilibrium Prices

Compared to single-item auctions, calculating CE prices in a combinatorial auction is not a trivial task. To start with, a simple Example 2 demonstrates that it is not always possible to find linear prices in a CA.

Example 2. *Linear CE prices do not always exist.*

There are 3 bidders and 3 items, bids are given by the following table.

Bids	*A*	*B*	*C*	*AB*	*BC*	*AC*	*ABC*
Bidder 1	*60*	*50*	*50*	***200****	*100*	*110*	*250*
Bidder 2	*50*	*60*	*50*	*110*	***200***	*100*	*255*
Bidder 3	*50*	*50*	***75****	*100*	*125*	***200***	*250*

The revenue-maximizing allocation, marked in the table with asterisks, is $x_1(AB) = 1, x_3(C) = 1$ *with the total revenue of* 275. *To support this allocation, the prices must be equal to the winning bids and be above every losing bid. If we take as an example two losing bids, which are marked in the table, we obtain the following conditions:*

$$\begin{array}{lcr} p_{ask}(A) + p_{ask}(B) & = & 200 \\ p_{ask}(C) & = & 75 \\ \hline p_{ask}(A) + p_{ask}(B) + 2p_{ask}(C) & = & 350 \end{array} \qquad \begin{array}{lcr} p_{ask}(A) + p_{ask}(C) & > & 200 \\ p_{ask}(B) + p_{ask}(C) & > & 200 \\ \hline p_{ask}(A) + p_{ask}(B) + 2p_{ask}(C) & > & 400 \end{array}$$

which is a contradiction. Consequently, no linear CE prices exist in this case.

Obviously the important question is what conditions must hold for linear CE prices to exist in a combinatorial auction. We introduce two definitions before clearing this question.

Definition 11. *Bidder's i valuations $\mathcal{V}_i$ satisfy* ***unit-demand*** *property when $v_i(S) = \max_{j \in S} \{v_{i,j}\}$ for all S, where $v_{i,j}$ is the value for item j in isolation. Intuitively this means that the bidder i generally demands only a single item.*

Definition 12. *Valuations $\mathcal{V}_i$ satisfy the* ***goods are substitutes (GAS)*** *(also* ***substitutes*** *or* ***gross substitutes****) property if for all linear price sets $\mathcal{P}_{pay}$, $\mathcal{P}'_{pay}$ such that $\mathcal{P}'_{pay} \geq \mathcal{P}_{pay}$ (component-wise), and all $S \in D_i(\mathcal{P}_{pay})$ there exists $T \in D_i(\mathcal{P}'_{pay})$ such that $\{k \in S : p_{pay,i}(k) = p'_{pay,i}(k)\}$.*

The *goods are substitutes* requirement on bidders' valuations is a strong restriction. It implies that a bidder will continue to demand items that do not change in price as the price on other items increases. In particular, it precludes any complementary values in the bidder's valuations.

Gul and Stacchetti (2000) address the question of different price formats in detail, and demonstrate that the GAS condition is sufficient for the existence of linear CE prices. This condition is also almost necessary for the existence of linear CE prices. Said precisely, the valuations which satisfy the goods are substitutes condition is the largest set containing unit-demand valuations for which the existence of linear CE prices can be established.

We have demonstrated that linear CE prices do not always exist. The same problem persists when we try to build anonymous CE prices. Example 3 illustrates that they do not always exist in the general case.

Example 3. *Anonymous CE prices do not always exist.*

There are 2 bidders and 2 items, bids are given by the following table (bids belonging to the revenue-maximizing allocation are marked with an asterisk):

Bids	*A*	*B*	*AB*
Bidder 1	*0*	*0*	*3**
Bidder 2	*2*	*2*	*2*

The revenue-maximizing allocation is $x_1(AB) = 1$ with the total revenue of 3. To support this allocation, anonymous prices $p_{ask}(A)$ and $p_{ask}(B)$ both have to be larger than the corresponding bid prices of the second bidder, which is

2. This implies that the auctioneer can get the total revenue of at least 4 by selling the items separately, which is larger than the total revenue of the current revenue-maximizing allocation. Consequently, no anonymous CE prices exist in this case.

There are substantially fewer results in the literature on conditions for the existence of anonymous CE prices. Again we need some definitions first.

Definition 13. *Bidder's i valuations $\mathcal{V}_i$ satisfy **supermodular preferences** property when for all $S, T \subseteq \mathcal{K}$,*

$$v_i(S) + v_i(T) \leq v_i(S \cup T) + v_i(S \cap T)$$

The supermodularity condition coincides with the *increasing returns* property (Gul and Stacchetti, 1999):

Definition 14. *Bidder's i valuations $\mathcal{V}_i$ satisfy **increasing returns** property when for all $S \subset T \subseteq \mathcal{K}$ and all $k \in \mathcal{K}$,*

$$v_i(T) - v_i(T \setminus \{k\}) \geq v_i(S) - v_i(S \setminus \{k\})$$

Definition 15. *Bidder's i valuations $\mathcal{V}_i$ are **single-minded** if he values only one particular bundle S. Correspondingly,*

$$\begin{aligned} v_i(T) &= 0, \quad \forall\ T \subsetneq S \\ v_i(T) &= v_i(S), \quad \forall\ T \supseteq S \end{aligned}$$

Definition 16. *Bidder's i valuations $\mathcal{V}_i$ are **safe** if each pair of bundles with positive value shares at least one item:*

$$\begin{gathered} (v_i(S) > 0, v_i(T) > 0) \Rightarrow (S \cap T \neq \emptyset) \quad \forall\ S, T \subseteq \mathcal{K} \\ or, equivalently: \\ (v_i(S) > 0, S \cap T = \emptyset) \Rightarrow (v_i(T) = 0) \quad \forall\ S, T \subseteq \mathcal{K} \end{gathered}$$

Parkes (2001) demonstrates that either supermodular preferences, single-minded bidders, or safe valuations are sufficient for the existence of anonymous (non-linear) CE prices. Obviously each of these conditions is a strong restriction on the bidder valuations. The question of the minimum requirement on the bidders' valuations which is necessary for existence of anonymous CE prices is still open.

We have demonstrated that neither linear nor anonymous pricing models are sufficient for constructing CE prices in a combinatorial auction. An intuitive – but still to be proven – assumption is that personalized non-linear prices shall be the sufficient construct. Another important question is how to calculate such prices – both CE and minimum CE, since we need the latter to conduct iterative combinatorial auctions where bidders have a motivation for truthful bidding. The next section answers these questions.

2.4.2 Bridging Game Theory and Optimization Theory

A single publication by Bikhchandani and Ostroy (2002) sheds light on many questions in the CA theory. The authors reveal parallels between iterative combinatorial auctions and the duality concept from the linear optimization theory. This helps to define a sufficient price format for CE prices in a combinatorial auction. Furthermore, the authors provide a basis for algorithms for calculating minimal CE prices and for building iterative CAs which converge to minimum CE prices under reasonable assumptions on bidders' behavior. We explain the main ideas of this work here.

As we know from Section 2.3.2, a combinatorial auction can be interpreted as an integer linear program (ILP). Variables of the dual formulation of a linear program (LP) can be used to measure cost of restrictions in the original LP. The idea of using dual variables for prices in a CA was first suggested by Rassenti et al. (1982). Since the second set of constraints in the CAP-I (Section 2.3.2) corresponds to the set of items in the auction, the dual variables can be interpreted as item prices.

However, the CAP-I is an ILP, and additional integrality constraints on its variables cause the ***duality gap***, meaning that the optimal solutions of primal and dual problems do not necessarily match. Consequently, the linear prices are imprecise (Example 2).

Bikhchandani and Ostroy (2002) suggest an alternative formulation for the CAP, the CAP-III (2.6). The additional variables δ_X indicate "weight" of every allocation $X \in \mathcal{X}$ in the resulted allocation. Dual variables are shown in parentheses.

$$
\begin{aligned}
&\max \sum_{i \in \mathcal{I}} \sum_{S \subseteq \mathcal{K}} v_i(S) x_i(S) && && (\textbf{\textit{CAP-III}}) \qquad (2.6)\\
&\text{s.t.} \\
&x_i(S) = \sum_{X : X_i = S} \delta_X && \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K} && (p_i(S)) \\
&\sum_{S \subseteq \mathcal{K}} x_i(S) = 1 && \forall\, i \in \mathcal{I} && (\pi_i) \\
&\sum_{X \in \mathcal{X}} \delta_X = 1 && && (\pi^s) \\
&0 \le x_i(S) && \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K} && \\
&0 \le \delta_X && \forall\, X \in \mathcal{X} &&
\end{aligned}
$$

The first constraint states that the weight of bidder i getting the bundle S is equal to the sum of weights over all allocations where bidder i gets the bundle S. The second constraint ensures that each agent receives at most one bundle. The third constraint ensures that the total weight of all selected allocations equals one. The last two constraints ensure that no allocation has negative weight, or assigns a bundle to a bidder with a negative weight.

Bikhchandani and Ostroy (2002) prove that the optimal solution of CAP-III is always integral and describes every feasible solution of the auction, even if the integrality constraints are omitted. Therefore, the duality concept can be applied without restrictions in this case.

Dual variables π_i can be interpreted as the bidder's i revenue. Dual variables $p_i(S)$, which correspond to the constraints $x_i(S) = \sum_{X:X_i=S} \delta_X$, represent personalized, non-linear prices of a bidder i for the bundle S. The variable π^s is the seller revenue. The dual problem has the following form:

$$\min \sum_{i \in \mathcal{I}} \pi_i + \pi^s \qquad (\textbf{\textit{CAP-III-dual}}) \qquad (2.7)$$

s.t.

$$\begin{aligned}
\pi_i + p_i(S) &\geq v_i(S) && \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K} && (x_i(S)) \\
\pi^s - \sum_{i \in \mathcal{I}} p_i(S_i) &\geq 0 && \forall\, X = (S_1, \ldots, S_n) \in \mathcal{X} && (\delta_X) \\
\pi_i, \pi^s, p_i(S) &\in \mathbb{R} && \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K} &&
\end{aligned}$$

The following theorem concludes:

Theorem 4 (Bikhchandani and Ostroy (2002))**.** *For an efficient allocation X^* there always exist personalized non-linear competitive equilibrium prices $\mathcal{P}_{pay}$. This is not always true for linear or anonymous prices.*

Another important observation by Bikhchandani and Ostroy (2002) is the parallel between the LP duality theorem (see Section 2.4.4) for the CAP-III and the concept of Competitive Equilibrium. They note that the satisfaction of the complementary slackness condition at the CAP-III optimum solution corresponds to the definition of the CE prices (Definition 7).

$$\begin{aligned}
x_i(S)\ [\pi_i - [v_i(S) - p_i(S)]] &= 0 \qquad \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K} \\
\delta_X[\pi^s - \sum_{i \in \mathcal{I}} p_i(S_i)] &= 0 \qquad \forall\, X = (S_1, \ldots, S_n) \in \mathcal{X}
\end{aligned}$$

The primal and dual solutions are optimal if and only if these conditions hold:

- $x_i(S) = 1$, the bidder i receives the bundle S, and $\pi_i = v_i(S) - p_i(S)$.
- $\delta_X = 1$, and the auctioneer maximizes his revenue and $\pi^s = \sum p_i(S_i)$.

The second important theorem summarizes these observations and gives a guideline for building efficient iterative CAs.

Theorem 5 (Bikhchandani and Ostroy (2002))**.** *An allocation X^* is supported in CE by some set of prices $\mathcal{P}_{pay}$ if and only if X^* is an efficient allocation.*

Therefore, to conduct an ICA, we select sufficiently low start prices and gradually increase them until there are no new bids. Provided the price format is sufficiently expressive, which means using non-linear personalized prices, such auction will converge to *minimal CE prices* and an efficient allocation. There are several auction designs which are based on these considerations; we discuss these in Section 2.4.4 and Section 3.3.

The results of Bikhchandani and Ostroy (2002) are fundamental, but it is still important to understand their limitations:

- Solving the CAP-III directly is impractical, since it adds exponentially many new variables and constraints to the original CAP-I formulation.
- Although personalized non-linear prices are universal and always exist, they have their drawbacks. These are discussed in Section 2.4.3.
- Auction mechanisms built on this theory assume best-response bidding, which is necessary for the auction to terminate at the minimal CE prices. This assumption is subject to two critique points:
  - Auction outcome with minimum CE prices loses the *dominant strategy* property of the VCG auction. The best-response bidding strategy leads in this case to an ex post Nash equilibrium only when valuations have the BSM property (Definition 18).
  - In combinatorial auctions, bidders face very complex problems of interpreting prices, selecting bundles, and determining bid prices. Even when the bidder sincerely tries to follow the best-response strategy, he might make mistakes and deviate from it.

It is very difficult to approach cognitive abilities of bidders and their risk profiles analytically. However it is possible to address the question of strategic complexity of auctions with minimum CE outcomes, and try to decide what is "better" for a CA design – achieving VCG or minimum CE prices. Again, Bikhchandani and Ostroy (2002) give an interesting insight into this dilemma. It states that there is an equivalence between the core of a coalitional game and the set of CE prices. All core outcomes can be priced, and all CE outcomes are in the core.

Theorem 6 (Bikhchandani and Ostroy (2002)). $(\Pi, \pi) \in Core(\mathcal{I}, w)$ *if and only if corresponding personalized non-linear CE prices exist.*

This result is particularly important, since many problems of the VCG auction stem from the fact that its outcome is not always in the core (Section 2.3.4).

So which goal is preferable for an ICA design? One with a VCG outcome and dominant strategies, or one with a minimum CE outcome given presumably strong, but not dominant best-response strategies? The next section analyzes this dilemma in detail.

2.4.2.1 Strategy-Proof or Core?

A strategy-proof auction format is an ultimate goal for any market designer. This property promises that the bidders will report their valuations truly, which is in turn indispensable for finding an efficient allocation. If bidders speculate, the auction will always remain a lottery to some extent. As we know from Section 2.3.4, only auctions with VCG outcomes guarantee this property.

Unfortunately, auctions with VCG outcomes suffer from several serious problems. They are consequences to the fact that the VCG outcome is not necessarily a core outcome, meaning that bidders might want to build coalitions, participate in the auction under several different identities simultaneously, or engage in shill bidding.

Auction designs which result in minimal CE prices are an interesting alternative to the VCG auction. They always terminate with core results. The price to pay for this is the loss of incentive compatibility. However, the intuitively strong arguments for best-response strategies are preserved. The best-response bidding is also an ex post Nash equilibrium when valuations have the BSM property (Definition 18).

A bidder's payment in the VCG mechanism is always less than or equal to his payment at any CE price. If VCG payments are not supported in any price equilibrium, coalition building can bring advantages for bidders, and truthful bidding is correspondingly not the dominant strategy any more.

An interesting question is when minimal CE prices coincide with VCG prices. Bikhchandani and Ostroy (2002) show that the *bidders are substitutes* (BAS) condition is necessary and sufficient to support VCG payments in competitive equilibrium.

Definition 17. *The* ***bidders are substitutes*** *condition (**BAS**) is satisfied if* $\forall I \subseteq \mathcal{I}$ *and* $\forall\, i \in I$*:*

$$w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus I}) \geq \sum_{i \in I} \left[w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus i}) \right]$$

Intuitively, the BAS condition requires that the marginal contribution of every set of bidders $I \subseteq \mathcal{I}$ exceeds the sum of the marginal contributions of the individual bidders $i \in I$.

However, even though the BAS condition is sufficient for the VCG outcomes to be in the core, it still does not guarantee that the best-response bidding in ascending price auctions will lead to the same outcome. Therefore, the dominant strategy property is still not given. Example 4 demonstrates such a scenario.

Example 4. *BAS is not sufficient for an ascending auction to terminate with VCG prices (Ausubel and Milgrom, 2006a).*

There are four goods and five bidders in the auction. The following table gives bidder's valuations. All valuations which are not explicitly listed in the table are zero.

Valuations	*AB*	*CD*	*BD*	*AC*
Bidder 1		***10****		
Bidder 2	*20*			
Bidder 3	***25****			
Bidder 4			*10*	
Bidder 5				*10*

The efficient allocation, marked in the table with asterisks, is $x_1(CD) = 1, x_3(AB) = 1$*. Vickrey payoff vector* $(20, 10, 0, 5, 0, 0)$ *corresponds to Bidder 3 paying* 20 *for his goods and Bidder 1 receiving his goods for 0. However, in an ascending auction the Bidder 1 is likely to pay a positive price for his bundle* $\{C, D\}$*.*

Ausubel and Milgrom (2006a) show that a stronger *bidder submodularity* condition (BSM) is required for iterative auctions with ascending prices[3] to terminate with VCG payments.

[3] Not for any iterative auction with ascending prices. The auction must still be properly constructed.

Definition 18. *The* ***bidder submodularity*** *condition (**BSM**) is satisfied if* $\forall I \subseteq I' \subseteq \mathcal{I}$ *and* $\forall i \in \mathcal{I}$*:*

$$w(C_{I\cup i}) - w(C_I) \geq w(C_{I'\cup i}) - w(C_{I'})$$

With respect to bidding strategies, the above statement means that the BSM condition is required for minimum CE prices to support VCG payments in a (properly constructed) ascending ICA, and only in this case straightforward bidding is a dominant strategy (de Vries et al., 2007). The BSM condition is quite strong and is often not given in realistic value models (Section 6.3 and Parkes (2001)).

de Vries et al. (2007) demonstrate an interesting connection between the existence of some (maybe non-linear and personalized) price equilibrium that supports the VCG outcome and the existence of a linear price equilibrium. The GAS condition (Definition 12) is sufficient and almost necessary for the BSM condition to hold. Precisely, if at least one bidder does not satisfy the GAS condition, then it is possible to construct GAS-compatible valuations for other bidders such that the overall valuation profile does not satisfy the BSM property.

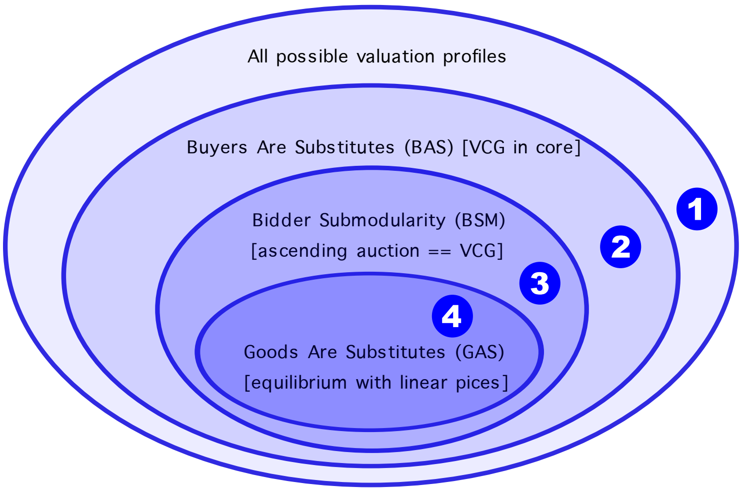
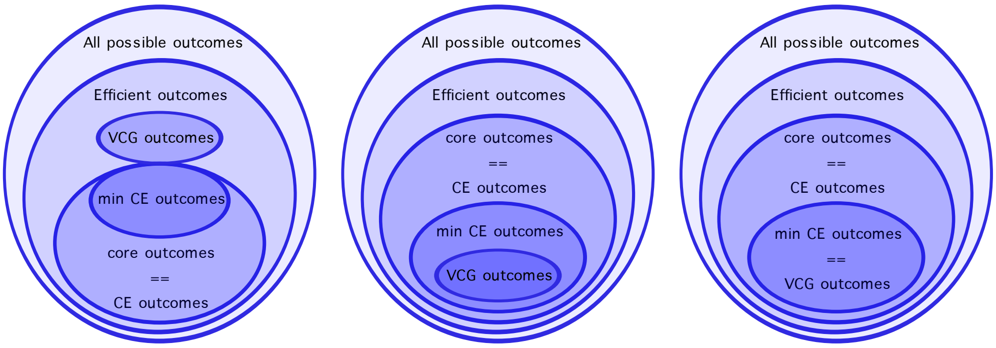


FIGURE 2.6: Classification of Valuations

We illustrate the relations between GAS (Definition 12), BAS (Definition 17), BSM (Definition 18) conditions and different auction outcomes in Figures 2.6 and 2.7.



(a) NOT BAS – (1) (b) BAS, NOT BSM – (2) (c) BSM – (3)

FIGURE 2.7: Classification of Auction Outcomes

Figure 2.6 shows how different conditions are related. The weakest property BAS is required for VCG outcome to be in the core, and consequently to be supported by some set of equilibrium prices. Note that the illustration is not made up to scale, and the whole area (2) occupies probably only a fraction of the universe (1) of all possible valuations, since the BAS property is rarely satisfied in practice (Section 6.3). A stronger BSM property is required for an auction with ascending (personalized, non-linear) prices to terminate with a VCG outcome. The even stronger GAS condition guarantees that an equilibrium outcome (not necessarily minimum-CE or VCG) is supported by a set of linear prices. It does not however guarantee that a particular linear-price auction design will find this equilibrium.

Figure 2.7 shows possible auction outcomes (in this case for an ascending auction with non-linear personalized prices) for valuations which satisfy different conditions. The three diagrams, left to right, correspond to areas (1), (2), and (3) in Figure 2.6. In the most general case (1), when the BAS property is not given, VCG outcomes are outside of the core. To illustrate that VCG prices represent a lower bound for minimum CE prices, both corresponding areas touch, but do not intersect. When the BAS property alone is given (2), the VCG outcomes are always in the core. However, the minimum CE outcomes which are found by ascending auctions with non-linear personalized prices, do not always correspond to the VCG outcomes. Finally, when the BSM condition is satisfied (3), the set of minimum CE outcomes shrinks and becomes equal to the set of VCG outcomes.

2.4.2.2 Iterative CAs with VCG Outcomes

Several suggestions for price-based ICA designs exist, which terminate with VCG prices for general (non-BSM) valuations, given best-response bidding (Mishra and Parkes, 2007; Parkes, 2006). These auctions are not precisely "ascending" since they calculate the pay prices using special algorithms, and they are not necessarily equal to the corresponding bid prices of the winning bidders.

Mishra and Parkes (2004) study the question of the price format which is necessary to calculate the VCG payments in an auction with arbitrary (non-BSM) valuations. They introduce the concept of ***universal competitive equilibrium*** (***UCE***) prices.

Definition 19 (Parkes (2006))**.** *Prices $\mathcal{P}$ are Universal Competitive Equilibrium (UCE) prices if:*

1. *Prices $\mathcal{P}$ are CE prices.*
2. *For every bidder i, the prices $\mathcal{P}_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$ are CE prices for $CAP(\mathcal{I} \backslash i)$, meaning they support some efficient allocation in $CAP(\mathcal{I} \backslash i)$.*

Intuitively, UCE prices $\mathcal{P}$ support some efficient allocation for the restricted CAP without bidder i with prices $\mathcal{P}_{-i}$, for every bidder i removed in turn. Thus, UCE prices are CE prices in the main economy and in every ***marginal economy*** *– an economy where one bidder is excluded. Note that UCE prices do not require that the same allocation is supported in every marginal economy. The prices must support* some *efficient allocation in each marginal economy.*

UCE prices always exist, for example $p_i = v_i$, for all bidders i, are UCE prices.

An important feature of the UCE prices is that they provide sufficient information to calculate the VCG outcome of the auction:

Theorem 7 (Parkes (2006))**.** *Given a UCE with prices $\mathcal{P}_{UCE}$ and efficient allocation $X^* = (S_1^*, \ldots, S_n^*)$, the VCG payment for bidder i is computed as:*

$$p_{VCG,i} = p_{UCE,i}(S_i^*) - [\Pi^*_{\mathcal{I}}(p_{UCE}) - \Pi^*_{\mathcal{I} \backslash i}(p_{UCE})]$$

where $\Pi^*_I(p) = max_{X \in \mathcal{X}}(p_i(S_i^*))$ for bidders $I \subseteq \mathcal{I}$

In the special case when the UCE prices are equal to valuations, this calculation is equivalent to the standard definition of VCG payments (2.5). In general, UCE prices are greater than the minimal CE prices because they must consider competition in the marginal economies in addition to the main economy. Minimal CE prices are universal if and only if the BAS condition holds.

Parkes (2006) demonstrates important parallels between price formats, problem of bidders' preference elicitation, and auction outcomes. In particular, the following two statements hold:

- A combinatorial auction achieves an efficient allocation if and only if the auction also finds a set of CE prices and the allocation is supported in the price equilibrium.
- A combinatorial auction realizes the VCG outcome if and only if the auction also finds a set of UCE prices and the allocation is supported in the price equilibrium of the main economy.

From the equivalence between the efficient outcome and the problem of discovering CE prices follows an uneasy fact that the worst-case informational complexity of any efficient CA, iterative or otherwise, is exponential in the number of items (Segal, 2006). Iterative CAs are designed to have better elicitation complexity in typical scenarios, while sealed-bid auctions must suffer the worst case every time.

2.4.3 Linear or Non-Linear Ask Prices?

At this point we know that only non-linear personalized prices are sufficient to support an efficient allocation in the general case. Does it mean that the non-linear and personalized prices are the superior format in every case? In particular, we want to compare two big families of non-linear price auctions and linear-price auctions.

Comparing personalized and anonymous prices is not so complicated. Personalized prices can require more computational resources to compute, store and communicate. However, this increase is linear in the number of bidders, at least for the known auction formats which support both personalized and anonymous prices. An important drawback of personalized prices is that they might be seen as unfair by bidders, since different bidders might need to bid different amounts for the same goods.

Note that personalized prices are required only if XOR bid language is used in the combinatorial auction. For the OR bid language, anonymous prices are always sufficient since it does not matter who wins the corresponding goods (see Section 2.3.1 for a discussion on bidding languages). We believe that in the cases where personalized prices are really necessary, it is possible to explain to the bidders the issues connected with this price format and prevent misinterpretations.

A more difficult task is to compare linear and non-linear price formats.

Non-linear price formats are clearly superior due to the fact that non-linear (personalized) CE prices always exist. Therefore, only approximated linear prices can be used in the general case. Most existing ICA designs with non-linear prices have a solid game-theoretical background, and provably lead to an efficient outcome when bidders follow the best-response bidding strategy.

The serious disadvantage of non-linear prices is their communication and cognitive complexity. The exponential number of prices quickly becomes infeasible to understand and analyze. It is particularly difficult to find new attractive bundles during the auction. Automated bidder support tools can help addressing this problem, but to provide effective help they must be tailored to the specific domain. This does not correspond to our aim of designing a *universally* applicable combinatorial auction design.

Linear CE prices do not exist in the general case. Consequently, approximation methods are usually used which aim to reflect the competition on the market in each auction round. In compensation to their imprecise character, linear price formats can offer interesting advantages. Most importantly, there are only as many prices in each round as there are items in the auction. Consequently, there are the following important advantages:

- Linear prices are easy to communicate, store and understand. Simplicity of the feedback given to bidders is very important in many practical application domains.
- Linear prices provide intuitive market overview.
- Linear prices can significantly help bidders to analyze the competition on the market and find new bundles *during the auction*. This can be the decisive factor in achieving high efficiency of the outcome (Kwon et al., 2005).

At the first glance, the existence of non-linear prices seem to be an important and decisive argument. However, prices in an iterative combinatorial auction always have only an indicative nature. Even the precisely determined personalized non-linear prices fully describe the market for the bidder i only under the *ceteris paribus* assumption, when there are no bids from other bidders in the same round. A single new bid can completely change the allocation, and previously losing bids may become winning bids. Therefore, even in the case of non-linear personalized prices, bidding according to the current price does not guarantee the bidder that he will be a provisional winner in the following round, let alone the complete auction.

Nice theoretical properties of non-linear price ICAs are based on strong assumptions on bidders' behavior. It includes assumptions on bidders' cognitive abilities, that they can understand and analyze the whole exponentially large set of prices in each round. Previous to our work, there were no results which describe what happens to such ICAs when these assumptions are not met.

Problems with linear prices due to their approximated nature can occur in two scenarios:

1. The calculated bundle price is too low. The bidder who submits a bid according to the price does not win it provisionally in the next round.
2. The calculated bundle price is too high. A bidder cannot submit a bid because the ask price is above his valuation, even though the valuation would allow him to win the bundle.

The first problem can be confusing for a bidder, but is not dramatic since the bidder usually does not know whether there were any bids by other bidders submitted in the same round. Consequently, this problem can delay the auction, but does not necessarily impact the efficiency of the outcome.

The second issue is more serious since it can result in degraded efficiency of the auction. ICAs with linear prices are usually designed to reduce possibilities of this phenomena (Section 2.4.5). Of course how imprecise the linear prices are, and to what extent this error can affect the auction outcome, is an important question. It is however very difficult to address analytically because of high algorithmic complexity and the large design space of the ICAs with linear prices. We analyze this question using computational experiments (Chapter 6).

2.4.4 ICAs with Non-Linear Ask Prices

Since a combinatorial auction can be seen as a procedure of solving the CAP ILP, it is appealing to apply advanced linear optimization algorithms to the auction scenario. We review below two such methods which are used by ICAs in our computational experiments. Both methods use the duality concept of the linear optimization theory. From the results of Section 2.4.2 it follows that such auctions require personalized non-linear prices.

To every LP we can construct a corresponding dual LP:

$$
\begin{array}{llll}
 & (primal) & & (dual) \\
\max & \sum_i c_i x_i & & \min \quad \sum_j b_j y_j \\
\text{s.t.} & & \Leftrightarrow & \text{s.t.} \\
 & \sum_i a_{ji} x_i \le b_j \quad \forall\, j & & \sum_j a_{ji} y_j \ge c_i \quad \forall\, i \\
 & x_i \ge 0 \qquad \forall\, i & & y_j \ge 0 \qquad \forall\, j
\end{array}
$$

In the case of combinatorial auctions, the corresponding interpretation for CAP-III and CAP-III-dual is given by (2.6) and (2.7) in Section 2.4.2. From the strong duality theorem (equality of the objective function values for both LP and dual LP in the optimum) follows:

Theorem 8 (The Complementary Slackness Condition)**.**
Suppose $\bar{x} = \{x_i\}$ and $\bar{y} = \{y_j\}$ are feasible solutions to a primal and the corresponding dual LPs respectively. Then $\bar{x}$ and $\bar{y}$ are optimal if and only if the following two conditions are satisfied:

$$
\begin{array}{ll}
\forall\, j & \left(\sum_i a_{ji} x_i \; - \; b_j \right) y_j = 0 \\
\forall\, i & \left(\sum_j a_{ji} y_j \; - \; c_i \right) x_i = 0
\end{array}
$$

Furthermore, we review the *subgradient algorithm* and the *primal dual algorithm* as they have been used to motivate and construct several auction formats, described in detail in Section 3.3. Both algorithms use the above *complementary slackness condition* to check optimality of the current solution, but use different approaches to iteratively improve the feasible solution in the direction of the optimum.

2.4.4.1 Subgradient Algorithm

The ***subgradient algorithm*** solves a restricted linear program by relaxing its constraints and transforming the problem into an unrestricted linear program using Lagrange multipliers. The linear program

$$
\begin{aligned}
& Z = \max \sum_{i=1}^{n} c_i x_i \\
& \text{s.t.} \\
& \qquad \sum_{i=1}^{n} a_{ji} x_i = b_j \qquad\qquad \forall\, j \in \{1, ..., m\} \\
& \qquad\qquad x_i \geq 0 \qquad\qquad \forall\, i \in \{1, ..., n\}
\end{aligned}
$$

is relaxed by pulling first $m' \leq m$ constraints into the objective function with Lagrange multipliers Θ_j:

$$
\begin{aligned}
& Z(\Theta) = \max \sum_{i=1}^{n} c_i x_i + \sum_{j=1}^{m'} \Theta_j (b_j - \sum_{i=1}^{n} a_{ji} x_i) \\
& \text{s.t.} \\
& \qquad \sum_{i=1}^{n} a_{ji} x_i = b_j \qquad\qquad \forall\, j \in \{m'+1, ..., m\} \\
& \qquad\qquad x_i \geq 0 \qquad\qquad \forall\, i \in \{1, ..., n\}
\end{aligned}
$$

By duality theorem, $Z = min_{\Theta \geq 0} Z(\Theta)$. The optimization problem is therefore reduced to finding $\Theta^* \in arg\ min_{\Theta \geq 0} Z(\Theta)$. This is done iteratively:

$$\Theta^{t+1} = \Theta^t + \Delta_t(Ax^t - b_j) \ \ \forall\ j$$

where Δ_t is a positive *step size* and x^t is an intermediate solution at the iteration t. If $\sum_i a_{ji} x_i^t > b_j$ and the constraint j is violated, then $\Theta_j^{t+1} > \Theta_j^t$, increasing the penalty for the violated constraint. If $\sum_i a_{ji} x_i^t < b_j$, then Θ_j^t is decreased in the next iteration.

In the context of combinatorial auctions, the subgradient algorithm is implemented by the iBundle ICA format (de Vries et al., 2003; Parkes, 1999).

For constant step sizes, the subgradient algorithm converges within some distance of the optimal solution. Parkes (1999) calculates the corresponding range for the iBundle auction. In our computational experiments, the parameters of value models and the minimum increment are selected to ensure precise convergence.

Since Lagrange multipliers Θ_i are not monotonous, prices in the auction interpretation are not necessarily increasing and additional care must be taken to ensure a strictly ascending auction, as for example in the iBundle format. Because of the approximation-based approach and ability to correct errors of the subgradient algorithm, we can expect that ICAs which are based on it will be more robust to suboptimal bidding. In particular, the efficiency of the outcome will not deteriorate if the bidders report only part of the demand set in each round. Our results support this conjecture (Section 6.5).

2.4.4.2 Primal Dual Algorithm

The ***primal dual algorithm*** uses the complementary slackness conditions both to check the solution for optimality and to adjust the current solution. Before applying it, we will convert the primal LP into the equation-based form:

$$\begin{array}{llll} & (primal) & & (dual) \\ \max & \sum_i c_i x_i & & \min \ \sum_j b_j y_j \\ \text{s.t.} & & \Leftrightarrow & \text{s.t.} \\ & \sum_i a_{ji} x_i = b_j \quad \forall\ j & & \sum_j a_{ji} y_j \geq c_i \quad \forall\ i \\ & x_i \geq 0 \qquad \forall\ i & & \end{array}$$

in this form, the *complementary slackness conditions*, which must be satisfied in the optimum, are reduced to:

$$\forall\, i \quad \left(\sum_j a_{ji} y_j \; - \; c_i \right) x_i = 0 \tag{2.8}$$

The algorithm first chooses a feasible dual solution. In the auction interpretation, we can usually start with the solution where all prices are zero. Let $I = \{i : \sum_j a_{ji} y_j > c_i\}$. From (2.8) follows that we need to check whether the primal LP has a feasible solution with $x_i = 0 \;\forall\, i \notin I$. We do it by solving the restricted primal LP:

$$
\begin{array}{lll}
 & (\textit{restricted primal}) & \\
\max & -\sum_j z_j & \\
\text{s.t.} & & \\
 & \sum_i a_{ji} x_i + z_j = b_j & \forall\, j \\
 & x_i \geq 0 & \forall\, i \\
 & z_j \geq 0 & \forall\, j \\
 & x_i = 0 & \forall\, i \in I
\end{array}
$$

The restricted-primal LP is always feasible since $\bar{b} > 0$, for example $\bar{x} = 0$ and $\bar{z} = \bar{b}$. The current solution $\bar{x}$ and $\bar{y}$ is optimal if the solution of the restricted primal LP is zero. If not, the restricted dual LP is formed:

$$
\begin{array}{lll}
 & (\textit{restricted dual}) & \\
\min & \sum_j b_j \lambda_j & \\
\text{s.t.} & & \\
 & \sum_j a_{ji} \lambda_j \geq 0 & \forall\, i \notin I \\
 & \lambda_j \geq -1 & \forall\, j
\end{array}
$$

If $\bar{\lambda}$ is a feasible solution (not necessarily optimal) for the restricted dual LP, which has a negative objective function value, then $\forall\ \Delta > 0 \Rightarrow \sum_j b_j(y_j + \Delta\lambda_j) \leq \sum_j b_j y_j$ will improve (reduce) the objective function of the dual problem. To ensure that the dual LP remains feasible, we require $\sum_j a_{ji}(y_j + \Delta\lambda_j) \geq c_i \,\forall\, i$. Every iteration of the primal dual algorithm applies the maximally possible adjustment that yields a feasible dual, until optimality:

- Choose a feasible dual solution and do until optimality:
  - Build the restricted-primal problem and check optimality.
  - If not optimal, build the restricted-dual LP and adjust the dual solution as described above.

de Vries et al. (2007) use the primal dual algorithm to construct the dVSV ICA format. Authors claim that an auction which is based on the primal dual algorithm can converge faster than a format based on the subgradient algorithm. The price for this possibility is the reduced robustness, since the algorithm requires that the complete demand set is reported by every bidder in each round (Section 6.5).

2.4.5 ICAs with Linear Ask Prices

Although linear prices supporting the revenue-maximizing allocation are only possible in very restricted cases (Section 2.4.1.3), their virtues (Section 2.4.3) have motivated several auction designs which perform surprisingly well in the laboratory. Even the US Federal Communications Commission (FCC) has been examining their use for spectrum auctions, and finally used a linear-price auction in the field (FCC, 2002; Goeree and Holt, 2008). We review in this work two approaches to calculating linear prices: ***Pseudo-dual prices*** and per-item *tâtonnement*.

The first approach uses the duality theory applied directly to the linear relaxation of the CAP-I (2.4):

$$\min \sum_i p(i) + \sum_k p(k) \qquad (\textbf{\textit{CAP-DLP}})$$

s.t.

$$p(i) + \sum_{k \in S} p(k) \geq v_i(S) \qquad \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K}$$

$$p(i), p(k) \geq 0 \qquad \forall\, i \in \mathcal{I}, k \in \mathcal{K}$$

The values of the dual variables $p(k)$ quantify the monetary cost of not awarding the item to whom it has been provisionally assigned. Integrality constraints on the variables of the primal CAP-I mean that the linear prices are imprecise. Different approaches have been suggested to minimize the negative effect of these distortions on the auction outcome (Bichler et al., 2009; Kwasnica et al., 2005; Kwon et al., 2005; Rassenti et al., 1982).

If we omit the constraints which are responsible for the XOR bidding language, the pseudo-dual prices can be calculated using the following scheme:

$$\min_{p_k, \delta_l} \{\max\{\delta_l\}, \max\{p_k\}\} \tag{2.9}$$

s.t.

$$\sum_{k \in S^l} p_k + \delta_l \geq b(S^l) \qquad \forall\, b(S^l) \in L$$

$$\sum_{k \in S^w} p_k = b(S^w) \qquad \forall\, b(S^w) \in W$$

$$p_k \geq 0$$

$$\delta_l \geq 0 \qquad \forall\, b(S^l) \in L$$

where L and W represent sets of provisionally losing and winning bids correspondingly. This price calculation scheme reflects requirements on supporting prices formulated for the equilibrium condition with minimal CE prices (Definition 8). The first set of constraints states that all losing bids must be below new prices; the second set of constraints sets all winning bids to be equal to the new prices. Since linear prices do not necessarily exist, variables δ_l are introduced to ensure that a feasible solution is always found.

Note that the objective function is not specified precisely. This is where different linear price formats vary (Chapter 3). Generally we try to minimize the prices to approximate the minimal CE prices, and simultaneously reduce the deviations δ_l. Additional requirements are possible, for example reduction of price non-monotonicity between rounds.

Another approach to using linear prices in an ICA is the per-item *tâtonnement* used by the *Combinatorial Clock* (*CC*) auction format (Section 3.2).

2.5 More Performance Measurements

Besides the allocative efficiency and revenue distribution which were introduced in Section 2.1.1, we use several other measurements to compare different auction designs. The same values are also used to quantify robustness of auction formats against different factors. In this section we formalize these measurements.

2.5.1 Speed of Convergence

All other things being equal, short auctions are preferable to long ones. We quantify an auction's ***convergence speed*** in rounds. This approach is preferable to calculating elapsed time, which will be highly dependent on the bidder's computational capacity and communication speed. Furthermore, our implementation is not optimized for performance and the LP solving library, which we use, was selected for its free availability and not for performance.

The duration of an iterative auction is influenced mainly by the following factors:

- Minimum increment, which defines how fast the ask prices are advancing.
- Strength of the activity rules.
- Distance between the ask prices at the beginning of the auction, as set by the auctioneer, and the valuations of the bidders, which roughly correspond to the ask prices at auction end.

The minimum increment, when set too high by the auctioneer, will force bidders to increase bid prices in large steps and can result in decreased auction efficiency. Therefore, auctioneers face a trade-off between an appropriately high minimum increment to achieve a reasonable progress of the auction, and an appropriately low minimum increment to not negatively impact the efficiency of the auction. An interesting possibility to address this dilemma is to use *dynamic minimum increment*, which starts high and decreases as the competition level in the auction goes down. We suggest and implement such rule within the ALPS auction format (Chapter 4).

Similarly, there is a tradeoff in the selection of appropriate activity rules. Tough activity rules will enforce aggressive bidding and thus decrease auction duration. However, they might at the same time prevent bidders from submitting potentially winning bids and thus negatively impacting the auction efficiency.

2.5.2 Price Monotonicity

Linear prices in iterative combinatorial auctions are not necessarily monotonous. As an auction progresses and ask prices rise, bidders might shift their initial preferences to other items, which in turn causes demand reduction on some items. Falling ask prices are necessary to reflect this phenomena, and to achieve an efficient outcome (Section 6.2). Strongly fluctuating prices, however, can stimulate strategic behavior when bidders withhold their bids in the hope that prices will fall. Furthermore, they can be confusing for bidders.

We introduce the price monotonicity measurement as a discrete function $m(T) : \mathbb{N} \to \mathbb{R}_0^+$ which correlates the sum of all positive differences and the sum of all negative differences:

$$m = \frac{\sum_{t=1}^{T} \sum_{k \in \mathcal{K}} \Delta e_{t,k}}{\sum_{t=1}^{T} \sum_{k \in \mathcal{K}} \Delta p_{t,k}}$$

Figure 2.8 illustrates the calculation. The positive price deltas $\Delta p_{t,k}$ and the negative price deltas $\Delta e_{t,k}$ for each item k are summed up over all rounds. The degree of an auction's price monotonicity is measured as a relation of the sum of error deltas to the sum of positive deltas. This calculation yields a monotonicity measure in the interval $m \in [0, 1]$. A monotonicity error of 0 corresponds to a fully monotonic function, whereas a value of 1 indicates the maximum possible monotonicity error.

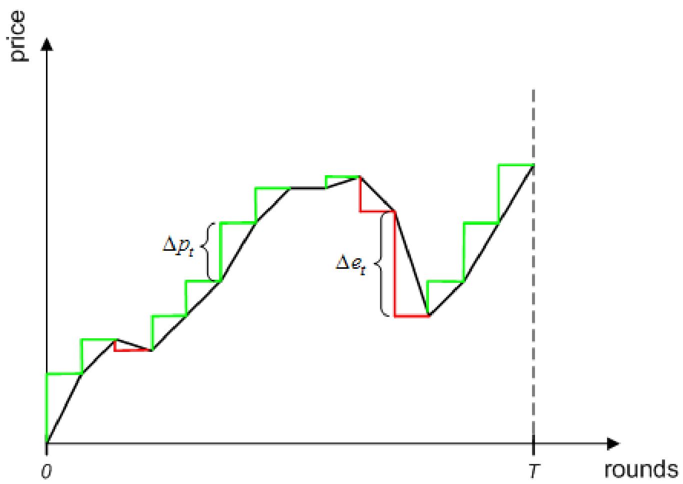


FIGURE 2.8: Calculation of Price Monotonicity for a Single Item

2.5.3 Robustness

There are many reasons why the bidders in an ICA might fail to follow the best-response bidding strategy:

- To follow the best-response strategy, a bidder needs to determine his demand set from an exponential number of possible bundles given current ask prices. This might be impossible due to cognitive restrictions, but there may also be strategic reasons, since the best-response bidding is not a dominant strategy.
- The assumption of independent private values is rarely held in practice, and bidders tend to review their valuations through the bidding process.
- Bidders can use jump bidding, which is not possible under the assumption of best-response bidding. Jump bidding can easily be eliminated by corresponding activity rules, which however would also result in increased auction duration. Therefore many designs allow for jump bidding, and many bidders use it. Isaac et al. (2007) describe jump bidding as taking place in a large proportion of FCC spectrum auctions (up to 44% of the bids with a 5% bid increment) as well as in the 3G spectrum auctions in the U.K.

An important quality of an auction design in this respect is to deliver efficient results not only with ideal best-response bidders, but also when bidders deviate from the theoretically optimal behavior, and across of different types of bidder's valuation. Therefore we introduce a notion of ***robustness***, which is a qualitative attribute describing the stability of an auction format under not ideal conditions. In this work we test robustness of several auction formats by running computational experiments with various suboptimal bidding strategies.

2.6 Reverse Combinatorial Auctions

In a ***reverse combinatorial auction*** there is a single buyer (auctioneer) and a set of sellers (bidders) who compete for the right to sell goods to the auctioneer. This scenario is omnipresent in industrial procurement. The reverse CA mirrors the forward auction and is very similar to it mathematically: The auctioneer always selects the cheapest combination of bids, and the prices are reducing from round to round. However, there are several important differences, which are the topic of this section.

First of all, the CAP-I (2.4) needs some modifications:

$$\begin{aligned}
&\max \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} -x_i(S) v_i(S) \equiv \\
&\equiv \min \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) v_i(S) && (\textbf{\textit{CAP-I-REV}}) \qquad (2.10)\\
&\text{s.t.}\\
&\sum_{S \subseteq \mathcal{K}} x_i(S) \le 1 && \forall\, i \in \mathcal{I}\\
&\sum_{S: k \in S} \sum_{i \in \mathcal{I}} x_i(S) \ge 1 && \forall\, k \in \mathcal{K}\\
&x_i(S) \in \{0; 1\} && \forall\, i \in \mathcal{I}, S \subseteq \mathcal{K}
\end{aligned}$$

The objective function minimizes the auctioneer's payment. The second set of constraints is modified and requires that each item is purchased at least once.

An important issue in reverse combinatorial auctions is setting the start price. Setting all start prices in forward auctions to 0 guarantees that the initial allocation is feasible and the allocation which corresponds to the minimum CE prices is still reachable using ascending prices. To follow the analogy, start prices must be set to ∞ in reverse auctions, which is not practicable. Instead, a high enough start price must be chosen, which is not always easy without knowing bidders' valuations.

Another problem is connected with the second set of constraints in (2.10). Since each item must be purchased at least once, it can easily happen that a feasible allocation cannot be found. To solve this problem in MarketDesigner, we add artificial "auctioneer's" OR-bids to the CAP-I-REV, with bid prices at which the auctioneer can produce the items himself, or buy them on the free market. This guarantees that a feasible allocation always exists.

The last problem which we discuss here is the calculation of the allocative efficiency. The formula (2.2) cannot be applied directly:

- If all goods are allocated, the formula gives results in the interval $[1, \infty)$. The value 1 corresponds to an efficient allocation. If an allocation includes sub-optimal bids, the numerator increases, and the whole value increases correspondingly.
- If not all goods are allocated, the numerator decreases, and the whole value decreases correspondingly. The result can be below 1.

There are two negative factors – sub-optimal bids and non-allocated goods – which drag the measured value in opposite directions. Therefore, the obtained value cannot characterize the auction outcome.

To measure efficiency of a reverse CA we use a modified formulation:

$$E(X) := \frac{\sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i(S) v_i(S) + \sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{a \in \mathcal{A}} x_a(S) v_a(S)}{\sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i^*(S) v_i(S)} \in [1, \infty)$$

where $\mathcal{A}$ is the set of items which was allocated to the auctioneer, $x_a(S)$ are the corresponding allocation variables and $v_a(S)$ are the auctioneer's own valuations for these items. The obtained value lies in the diapason from 1 to ∞, with 1 corresponding to an efficient allocation. Note that the efficiency depends on the valuations $v_a(S)$ which can be arbitrary selected by the auctioneer. This issue must be addressed correctly when different auctions are compared.

Chapter 3

ICA Designs

Variety's the very spice of life.

William Cowper, 1785

This chapter describes combinatorial auction formats, which were implemented in the MarketDesigner platform and tested in computational experiments during our research. We start with a brief comparative overview and provide detailed information on every auction later on.

Table 3.1 lists all CAs implemented in the MarketDesigner platform, together with their most important characteristics. They form the following six groups:

1. Both sealed-bid designs, the VCG auction (Section 2.3.4) and the first-price sealed bid auction, give bidders only one chance to submit their bids. This does not imply tough time restrictions on submitting bids, but rather means that bidders must place all bids without receiving any intermediate information about the competition level and about the status of already submitted bids. As previously discussed (Section 2.4), this can become a prohibitively complex task for bidders in a combinatorial auction with a moderate number of goods already. Compared to the first-price auction, where winning bidders pay exactly the bid price, in a VCG auction bidders are charged only the opportunity cost of winning the corresponding bids. This makes truthful bidding the dominant strategy of the VCG auction, a unique property which distinguishes it from all other designs, even though it holds only under strong assumptions.

Design	Price structure	Bidding language	Price Update Method	Valuations	Outcome
Sealed 1st price	—	XOR	—	general	—
VCG	—	XOR	—	general	VCG
RAD	linear	OR	pseudo-dual	general	—
CC	linear	OR, XOR	*tâtonnement*	general	—
iBundle(2)	anonymous non-linear	XOR	greedy	general	—
				supermodular	min CE
iBundle(3)	personalized non-linear	XOR	greedy	general	min CE
				BSM	VCG
dVSV	personalized non-linear	XOR	undersupplied set of bidders	general	min CE
				BSM	VCG
CreditDebit	pers. non-lin.	XOR	undersupp. set	general	VCG
ALPS	linear	OR, XOR	pseudo-dual	general	—
ALPSm	linear	OR, XOR	pseudo-dual	general	—

TABLE 3.1: Auction Formats, Implemented in MarketDesigner Framework

2. The RAD auction format is a historically first solid concept of an iterative CA based on linear pseudo-dual prices. Designed with practical applicability in mind, it addresses several issues – like activity rules and auction duration – which are sometimes neglected in the theory, but are very important for the field applications. Since our ALPS auction format (Chapter 4) is based on this design, we review the RAD ICA format in greater detail later in this chapter.

3. The Combinatorial Clock (CC) auction format, another concept which uses linear prices, attracts by its simplicity. Unlike the complex price calculation algorithm of the RAD design, it uses a very straightforward and intuitive price update mechanism (*tâtonnement*). This is an important virtue of the CC auction design, since bidders usually have more confidence in a transparent and intuitive mechanism.

4. Iterative ascending CAs with non-linear prices are presented in our work by three different formats: Two versions of the iBundle auction – with anonymous and with personalized prices – and dVSV auction. An important property of iBundle(3) and dVSV formats, which use personalized prices, is that they guarantee, under the best-response assumption on bidders behavior, certain efficiency of the auction outcome. There are no comparable results for any of the CAs with linear prices yet.

5. The CreditDebit auction design uses a sophisticated multi-trajectory price update rule and achieves the VCG outcome for all types of valuations. Consequently, this is not a *pay-as-bid* auction. One of our tasks was to test how this complex design behaves when the assumptions on rational bidder behavior are violated.

6. An important result of this work is the ALPS/ALPSm ICA design, which therefore deserves a separate chapter, Chapter 4. It has its roots in the RAD design, but introduces a number of significant changes to the price calculation algorithm, activity and termination rules.

In the rest of this chapter we provide detailed descriptions of all listed in Table 3.1 ICA formats, with two exceptions. The sealed-bid VCG auction design, which occupies an important place in the theory of combinatorial auctions, was already described in Section 2.3.4. The whole next chapter, Chapter 4, is dedicated to the ALPS auction format.

We provide literature references to the documents which were used as the basis for the software implementation of these auction formats in the MarketDesigner framework. As the literature was ambiguous or incomplete in several cases, we describe in detail the relevant implementation aspects as necessary.

3.1 Resource Allocation Design (RAD)

The ***Resource Allocation Design*** (***RAD***) ICA format proposed by Kwasnica et al. (2005) is attractive because of its straightforward and clear bidding rules. It conceals most of the combinatorial complexity from the bidder and will be easy to participate in for any bidder who is familiar with usual single-item English auctions. Parts of the RAD mechanism have been considered for practical application by the USA Federal Communication Community (FCC) for conducting wireless spectrum auctions (FCC, 2002).

3.1.1 Price Calculation and Bidding Rules

RAD uses pseudo-dual anonymous linear ask prices (Section 2.4.5) to give feedback to the bidders during the auction. The price calculation algorithm attempts to find such linear ask prices $p_{ask}^{t+1}(k) \quad \forall\, k \in \mathcal{K}$ for the round $t+1$, which would support the provisional allocation calculated after the round t and therefore reflect the current competition in the market. RAD ask prices are calculated based on the bids $\{b^t(S) = (S, p^t_{bid}(S))\}$ from round t by solving the following LP (3.1) after the corresponding CAP-I (2.4) was solved and all round t bids $\mathcal{B}^t = \{b^t(S)\}$ were separated in two subsets of winning W^t and losing L^t bids:

$$\min_{p^{t+1}_{ask}(k), Z, \delta(S_l)} Z \tag{3.1}$$

s.t.

$$\begin{aligned}
\sum_{k \in S_w} p^{t+1}_{ask}(k) &= p^t_{bid}(S_w) && \forall\, b^t(S_w) \in W^t \\
\sum_{k \in S_l} p^{t+1}_{ask}(k) + \delta(S_l) &\geq p^t_{bid}(S_l) && \forall\, b^t(S_l) \in L^t \\
\delta(S_l) &\geq 0 && \forall\, b^t(S_l) \in L^t \\
\delta(S_l) &\leq Z && \forall\, b^t(S_l) \in L^t \\
p^{t+1}_{ask}(k) &\geq 0 && \forall\, k \in \mathcal{K}
\end{aligned}$$

The first constraint ensures that, for each winning bid, the sum of new item ask prices within the corresponding bundle is equal to the winning bid price. However there is still flexibility in setting individual item prices. The second constraint enforces all losing bids to be below prices, whereby the variables $\delta(S_l)$ represent adjustments which are necessary to find a feasible solution. The objective function tries to reduce distortions by minimizing the highest adjustment $\delta(S_l)$.

After the LP (3.1) is solved, distortion variables $\delta(S_l)$ for some losing bids $\hat{b}$ will be at their minimum. Let's denote the set of these bids as $\hat{L}^t$. However, it is still possible that distortion variables for other bids $b \notin \hat{L}^t$ are not at their minimum yet. Therefore, the LP (3.1) is solved not directly, but iteratively. The following algorithm is used by the RAD design to calculate the new ask prices $p^{t+1}_{ask}(k)$ which minimize every distortion variable $\delta(S_l)$ (Kwasnica et al., 2005):

1. Initialize $\hat{L}^t = \emptyset$.
2. Solve the LP (3.2) and determine the new value for Z.
3. Replace all variables $\delta(S_l)$ which are equal to Z with their numerical values $\delta(\hat{S}_l)$. Move the corresponding bids from L^t to $\hat{L}^t$.

4. Terminate if there are no free variables $\delta(S_l)$ left. Last values of variables $p^{t+1}_{ask}(k)$ represent the new linear ask prices.

5. Repeat from step 2.

$$
\begin{aligned}
& \min_{p^{t+1}_{ask}(k), Z, \delta(S_l)} Z && (3.2)\\
& \text{s.t.} \\
& \sum_{k \in S_w} p^{t+1}_{ask}(k) = p^t_{bid}(S_w) && \forall\, b^t(S_w) \in W^t \\
& \sum_{k \in S_l} p^{t+1}_{ask}(k) + \delta(\hat{S_l}) = p^t_{bid}(S_l) && \forall\, b^t(S_l) \in \hat{L}^t \\
& \sum_{k \in S_l} p^{t+1}_{ask}(k) + \delta(S_l) \geq p^t_{bid}(S_l) && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t \\
& \delta(S_l) \geq 0 && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t \\
& \delta(S_l) \leq Z && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t \\
& p^{t+1}_{ask}(k) \geq 0 && \forall\, k \in \mathcal{K}
\end{aligned}
$$

The resulting ask prices $p^{t+1}_{ask}(k)$ are the primary guideline for the bidders, as they formulate their bids in the round $t+1$. In the new round, bids on arbitrary bundles can be submitted. The bid price must be at least as high as the sum of current ask prices for the items which comprise the bundle, plus a fixed minimum increment which is added to every item contained in the bundle.

Provisionally winning bids from the round t are automatically resubmitted to the round $t+1$; all losing bids are removed from the auction.

The original RAD design uses the OR bidding language. However, in our computational experiments we mostly used RAD with the XOR bidding language to achieve better comparability with other designs, which primarily use the XOR bidding language. Furthermore, with sub-additive valuations, the OR bidding language presents the bidder with the *overflow problem*, which makes the bidding strategy more complex and dependent on the risk profile of the bidder. Behavior of human bidders will heavily depend on the scenario in this case, and is therefore difficult to generalize and implement in software.

Note that the individual ask prices are not necessarily monotonous during the auction. The value of the CAP-I objective function is non-decreasing between rounds because all provisionally winning bids are always retained in the auction. Consequently, the sum of all prices cannot decrease since it is exactly equal to the value of the CAP-I objective function. But individual item prices can still fall, which can be necessary to better reflect the competition in the market. However, such price fluctuations can irritate the bidder, and it is desirable to find a suitable compromise, of which several approaches have been discussed in the literature (Dunford et al., 2007; Shabalin et al., 2007). We will review possible solutions in Section 4.2.

3.1.2 Activity Rule

An important issue for any iterative auction is the bidder activity. An attractive strategy for most bidders is to wait for other participants to uncover the competition on the market, and then enter the bidding only when the auction nears its completion. Such behavior, when adopted by majority of the bidders in the auction, can delay the auction significantly, or result in arbitrary allocations when some bidders do not enter bidding at all or do not submit bids on all bundles of interest.

To prevent bidders from inactivity, the RAD design enforces the ***eligibility rule*** as an activity driver. A bidder's i eligibility e_i^t indicates the number of distinct items the bidder is allowed to bid on in the round t. In the first round, each bidder obtains maximum eligibility $e_i^1 = |\mathcal{K}|$, which is equal to the number of items in the auction. The eligibility e_i^t for the round $t > 1$ is determined by the number of distinct items the bidder was bidding on in the previous round $t-1$:

$$e_i^t = \sum_{k \in \mathcal{K}} \prod_{b_j \in \mathcal{B}^{t-1}} x_j(k), \qquad t > 1$$

For example, if a bidder submitted a single valid bid on the bundle {A, B, C} in the round t, he could try the following options in the round $t+1$:

{A, B}, {A, C}, {B, C}, {A}	- ok.
{E, F, G}	- ok.
{A, B}, {C, D}	- not allowed, four distinguished items.

Note that the eligibility is non-increasing in any two consecutive rounds. It forces the bidders to be proactive right from the first round, because once eligibility is lost, it can never be recovered. Following this rule, the bidders will gradually reduce their participation in the auction, and finally stop bidding only when the ask prices rise above their valuations.

3.1.3 Termination Rule

The termination rule of the RAD auction format is based on the eligibility concept. RAD distinguishes between ***bound eligibility*** eb_i^t and ***unbound eligibility*** eu_i^t, with $eb_i^t + eu_i^t = e_i^t$. The bound eligibility of a bidder is defined as the number of distinct items contained in the bidder's current winning bids. This eligibility is *bound* because winning bids are automatically resubmitted and will inevitably count for eligibility in the following round again. The remaining eligibility of the bidder is *unbound.* RAD defines its termination rule as follows:

$$\text{stop at the end of round } t \text{ if } \sum_{i \in \mathcal{I}} e_i^t \leq |\mathcal{K}|$$

This termination condition requires that the total eligibility, summed up over all bidders, is less or equal to the number of items in the auction. If this happens, all bidders will have only bound eligibility and no chance to change their provisionally winning items; that is, no bidder will be allowed to bid on items other than those which he is currently winning.

This definition works fine for single-item bidding, but does not guarantee termination in the case of bundle bidding. The prices can start to oscillate, and some bidders can retain unbound eligibility indefinitely long, preventing the auction from terminating (see Example 7). As a trivial solution to this problem, RAD proposes an additional stopping condition. The auction is terminated as soon as an equal allocation occurs in two subsequent auction rounds.

Results of our study indicate that the RAD format has some design problems. The most serious issue is its premature terminations and, consequently, inefficiencies. Furthermore, RAD prices are not always optimal. We illustrate these problems using examples and propose solutions in Chapter 4 as we describe ALPS, our improved ICA design.

3.2 Combinatorial Clock (CC)

Another ICA with anonymous linear ask prices is the ***Combinatorial Clock*** (***CC***) auction proposed by Porter et al. (2003). Among all CA designs discussed in this work, only the CC format supports multi-unit auctions, where several identical instances of each item are traded simultaneously and bidders can bid on partial quantities (see also the classification in Figure 1.1). Similarly to the RAD design, it has simple and straightforward bidding rules. But unlike RAD, which conceals behind simple linear prices a complicated pseudo-dual calculation mechanism, pricing rules in the CC design are very straightforward.

The CC auction starts with sufficiently low linear ask prices; zero is always suitable. Each bidder expresses desired bundles, including item quantities for each item, at the current ask prices. Jump bidding is generally not allowed; bidders can bid only at the current ask prices. After each round the prices for those items for which demand exceeds supply "tick" upwards by a fixed price increment, as schematically illustrated in Figure 3.1. After that the auction moves to the next round. Note that it is not necessary to solve the CAP at this point.

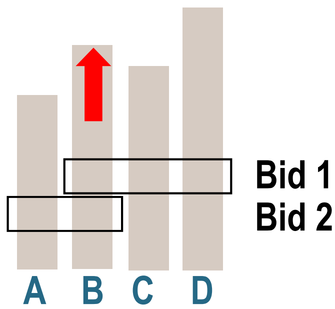


FIGURE 3.1: Price "Tick" in the Combinatorial Clock Auction

In the CC auction, all bids remain active throughout the auction. The bids which correspond to the current ask prices are called ***standing bids***, which include all bids from the current round t and those standing bids from the previous round $t-1$, for which the ask price did not change. In other words, a bid remains *standing* for the next round if there is no (more) competition on any of its items. A bidder is ***standing*** if he has at least one standing bid.

Porter et al. (2003) suggest for the CC design an activity rule which requires demand to be monotonous in quantity. As prices rise, quantities specified by bidders in their bids cannot increase. A more flexible activity rule allows bidders to shift quantities between items and requires monotonicity only in the aggregate quantity across all items (Ausubel et al., 2006).

The termination rule in the CC auction is tricky. In the simplest case, when in some round t the demand exactly equals supply, the auction terminates and the goods are allocated corresponding to the last round bids. If demand does not exceed supply, but is not exactly equal to supply for some goods, the CAP-I (2.4) is solved considering all bids submitted during the auction runtime. However, the resulting allocation cannot be declared as final in some cases. It is possible that some standing bidders are not included in it, which will be perceived by them as unfair. Furthermore, these bidders could potentially submit even higher bids, and thus increase the revenue.

Therefore, in the event that the CAP-I solution does not include some of the standing bidders, the auction continues. In this case, the ask prices are increased on those items which correspond to the non-standing winning bids.

Neither Porter et al. (2003) nor Ausubel et al. (2006) answer the question of the appropriate bidding language for the CC auction design. Porter et al. (2003) briefly mention that multiple bids of the same bidder can win simultaneously, whereas (Ausubel et al., 2006) does not discuss the bidding language question explicitly but, in our understanding, expects XOR bidding. In fact, the CC auction is capable of handling both OR and XOR bidding languages, even in the multi-unit case. However, the overall demand calculation has to be fine-tuned. With OR bidding, demands produced by different bids on the same item by the bidder have to be aggregated, as long as they do not exceed the total supply on that item. In contrast, in the XOR case only the maximum demand per bidder for every item has to be considered.

3.3 Non-Linear Price ICAs

Non-existence of precise linear prices (Section 2.4.1.3), and therefore the approximated nature of prices in all auctions discussed above, makes their theoretical analysis prohibitively complicated. Our experiments show that in some cases efficiency of linear-price auctions can be low even with theoretically optimal best-response bidding (Section 6.6), which is a serious drawback.

Bikhchandani and Ostroy (2002) created a basis for designing ICAs which can guarantee a certain efficiency level of the outcome. de Vries et al. (2007) show for the dVSV design that if all valuations and prices are kept integral, and a minimum increment of 1 is used in the auction format, the auction always terminates with an efficient solution, given best-response bidding. A whole family of similar ICAs is based on the strengthened CAP version – CAP-III (2.6). For complexity reasons, they do not try to solve it directly, but rather mimic the advanced LP solving algorithms (Section 2.4.4) in their price update rule. The initial dual feasible solution is gradually improved, until the complementary slackness condition is satisfied and optimality is therefore reached (de Vries et al., 2007).

This mathematically elegant approach is based on a strong assumption that all bidders follow the best-response strategy and always bid their *demand set* (Definition 6). There is a controversy regarding this assumption, which is necessary for achieving an efficient auction outcome. It is questionable at least for two reasons:

- To start bidding in the auction, a bidder must know his most valuable bundle and the second most valuable bundle. As the auction progresses, the bidder must linearly arrange all bundles according to their valuations. It is not clear whether bidders have sufficient computational and cognitive abilities to do that. An error in calculating valuations, made at the beginning of the auction, cannot be corrected during later rounds.
- The best-response is not a dominant strategy in this auction format. Even though the motivation to follow the best-response strategy seems to be high, bidders might try to bid strategically, especially if they have strong signals regarding valuations of other bidders.

Parkes and Ungar (2000) suggest that the best-response bidding can be enforced by keeping all losing bids in the auction and increasing their bid price automatically as the ask prices increase. Similarly, Ausubel and Milgrom (2006a) emphasize automated proxy agents, which receive valuations (bids) only once before the first round, and then participate in the auction on behalf of the bidder, adhering to the best-response strategy.

Both solutions essentially lead to a sealed-bid format, which means the bidders cannot really use prices to find new attractive bids, and are not allowed to make errors in their valuations through the whole auction.

All non-linear price ICAs use the XOR bidding language because of its full expressiveness.

The different non-linear price formats which are reviewed below distinguish primarily in the rule of selecting bundles and bidders whose prices are increased in each auction round.

3.3.1 iBundle and Ascending Proxy Auction

iBundle and ***Ascending Proxy Auction*** are essentially the same mechanism, described by Parkes (2001) and Ausubel and Milgrom (2006a) respectively. Given best-response assumption on bidders, they terminate with minimal CE prices and an allocation that is within $3\min\{|\mathcal{K}|, |\mathcal{I}|\}\Delta$ from an efficient solution, where Δ is the minimum increment (Parkes, 2001). Additionally, when the BSM condition is satisfied, the final bidder payments are equal to the VCG prices. By keeping valuations integral and setting $\Delta = 1$, we can guarantee that an efficient allocation is always found precisely (de Vries et al., 2007).

The iBundle auction design is actually a framework, incorporating several non-linear price designs with slightly different options. All of them calculate a provisional revenue maximizing allocation at the end of every round and increase the prices based on the bids of ***unhappy*** (non-winning) bidders. Three iBundle modifications differ in the price format:

- ***iBundle(3)*** maintains personalized non-linear prices throughout the auction. In every round the prices for every unhappy bidder are increased for every bundle on which he has submitted a bid.
- ***iBundle(2)*** uses only one set of (anonymous) non-linear prices during the auction. Prices are increased for each bundle an unhappy agent bid for, based on the best unsuccessful bid. The theoretical results regarding auction efficiency do not hold for this version of the iBundle ICA format since the anonymous ask prices are not fully expressive.
- ***iBundle(d)*** starts with anonymous non-linear ask prices for every bidder. In each round and for every bidder it is checked whether his newly submitted bids are *safe*, meaning that their bundles are mutually disjointed. Bidders with unsafe bids are switched to personalized prices from this point on and until the end of the auction.

In each round every bidder is provided with his ask prices and information about the bid he is currently winning, if any.

The anonymous version iBundle(2) works along the following steps:

- `ask prices := 0`
- `termination := false`
- `while` $\neg$ `termination`
  - `every bidder places his revenue maximizing bids`
  - `if (no new bids)`
    - `termination := true`
  - `calculate the current revenue maximizing allocation`
  - `if (every bidder receives a bundle)`
    - `termination := true`
  - `for all bids of unhappy bidders`
    - `new ask price:= bid price of highest losing bid +` Δ

The version with personalized prices, iBundle(3), differs from the anonymous version, iBundle(2), only by the price update rule. The price update for every bidder considers only bids of the bidder himself: In every round the personalized ask price for every unhappy bidder is raised by Δ for every bundle he bids for.

The *free disposal* condition must be enforced during every price update:

$$p_{ask}(S) \geq p_{ask}(T) \quad \forall \quad S \supseteq T$$

Winning bids are automatically resubmitted, and corresponding prices are not increased. When ask prices reach bidder's valuations, he signals it by submitting, together with his last bids, the ***empty bid*** – a bid on the empty bundle with a zero bid price. From this point, the bidder always wins at least his empty bid, and his prices are not increased any more. Therefore, the bidder can resubmit all his bids over and over again, retaining possibility of winning one of them in the event that a suitable allocation is found. From the best-response bidding assumption and the iBundle price update rule follows that every bidder must continue to demand each bundle he ever bid for the rest of the auction.

The iBundle auction design does not explicitly require resubmitting of losing bids, or enforce any other activity rules. However, its theoretical efficiency properties require such behavior which follows from the best-response assumption. Certainly an important question is what happens to the auction result when bidders do not follow this strategy. We address this question in our computational experiments (Section 6.5).

The iBundle design (except for iBundle(2)) does not allow for jump bids, and the bid price must be exactly equal to the ask price. A special option of ***last-and-final*** bids allows bidders to submit bids below the ask price $p^t_{ask}(S)$, but above the previous ask price $p^{t-1}_{ask}(S)$ for the bundle. The bidder is not allowed to submit any further bids on this bundle afterwards.

The auction terminates after calculating the provisional allocation in the round t if every bidder receives a bundle in it. The losing bidders in this case receive their empty bids for the zero price. Since in the iBundle auction the price is raised only for the losing bidders, an alternative termination rule is to close the auction when the same bids are submitted in two consecutive rounds.

3.3.2 dVSV Auction

The same as iBundle(3), the ***dVSV*** ICA design by de Vries et al. (2007) uses personalized non-linear prices. The price update mechanism however is substantially different. It is based on the concept of ***minimally undersupplied bidders***:

Definition 20 (Minimally undersupplied set of bidders)**.** *A set of bidders is minimally undersupplied if:*

- *In no efficient allocation all bidders receive a bundle of their demand set given the current prices.*
- *Removing one of the bidders forfeits this property.*

Like other non-linear price auctions, the dVSV design assumes best-response bidding. Additionally, dVSV requires bidders to have integer valuations on every bundle, uses only integer prices and a fixed price increment of 1. In each round, the bidders receive new ask prices but no provisional allocation. Jump bids are not allowed. The price update rule closely resembles the primal dual algorithm (Section 2.4.4.2), and always guarantees an efficient outcome – of course under the assumption of best-response bidding. Since valuations are integral and prices are always increased on all bundles of a bidder, the demand set of every bidder weakly increases from round to round. The same as with the iBundle design, a bidder in a dVSV auction uses empty bids to signal that he reached his valuations and will not increase the bid prices any more. The auction terminates when each bidder wins a bid (possibly his empty bid), and, consequently, there are no minimally undersupplied sets of bidders. The dVSV design does not explicitly require computing of a provisional allocation in every round, although our implementation does calculates it in order to find a minimally undersupplied set of bidders.

The dVSV auction executes the following algorithm:

- `ask prices := 0`
- `termination := false`
- `while` ¬ `termination`
  - `every agent places his revenue maximizing bids`
  - `if (no minimally undersupplied bidder sets)`
    - `termination := true`
  - `for all bidders in an arbitrary min. undersupplied set`
    - `for all bundles the bidder bids on`
      - `ask price ++`

To find a minimally undersupplied set of bidders is not a trivial task, and the authors (de Vries et al., 2007) do not suggest an algorithm to do it. We suggest our own algorithm. First we make the following observations:

- The simplest possibility to find a minimally undersupplied set of bidders is to find a single bidder who does not win any bids in any efficient allocation.

- In case this trivial case is not applicable and the minimally undersupplied set of bidders shall contain more than one bidder, their bids must compete in pairs, so that removing one bidder "frees" a bundle for at least one other bidder.

In our implementation, we search for a minimally undersupplied set of bidders by looking at bidders who submit competitive bids but do not win in a revenue-maximizing allocation, which simultaneously maximizes the number of winners:

- `determine an efficient allocation` $X*$ `that maximizes the number of winning bidders`
- `for all bidders` i `not receiving a bundle in` $X*$
  - `put` i `in the new set of candidates` C
  - `while not all bidders in` C `are flagged as considered`
    * `take a bidder` j `from` C `that is not flagged and flag him as considered`
    * `for all losing bids` l `of` j
      · `determine the set of winning bids` W `that overlap with` l
      · `if the sum of the prices over` W `is less or equal to the price of` l`, find a bidder who submitted one of the bids in` W `and is not in` C`, and add him to` C
  - `if` $|C| \geq 1$`, return` C

If a set C with $|C| \geq 1$ is found, it is minimally undersupplied by construction.

3.3.3 CreditDebit Auction

As already mentioned in Section 2.4.2.1, it is impossible to construct an ascending combinatorial auction with VCG outcomes for general valuations. Some newer approaches, such as the one by Mishra and Parkes (2004), overcome this limitation by relaxing the definition of ascending price auctions: The pay price for a winning bid is not necessarily equal to the bid price, but can be below it.

The ***CreditDebit*** format is an extension to the dVSV auction. It achieves the VCG outcome for general bidder valuations. The idea behind this design is to find a set of minimal UCE prices (Section 2.4.2.2) which can be used to calculate the VCG result. Another similar auction design called ***iBundle Extend and Adjust*** (***iBEA***) was suggested by Parkes and Ungar (2002). Best-response bidding is always an ex post Nash equilibrium in such auctions.

In addition to ask prices, the CreditDebit auction maintains a set of discounts which adjust the final payments towards VCG prices. These discounts are initialized with zero at the beginning of the auction, and incremented in every round for every bidder i by the difference between the revenue change in the main economy, which includes all bidders, and the marginal economy excluding the bidder i, since the final prices must be CE prices not only for the main economy, but also for all marginal economies.

Because of the complex algorithms used for calculating prices and payments, the CreditDebit auction is presumably even more sensitive to deviations from best-response bidding. This is one of the hypotheses which we verify in our computational experiments (Section 6.5). To mitigate this issue, authors suggest an incremental proxy design, which enforces the best-response bidding by controlling that the demand set of every bidder weakly increases after each round (Parkes and Ungar, 2000). However, this practically equates the design to a sealed bid auction and prohibits the central virtual of an iterative design – the incremental preference elicitation.

3.4 Hybrid Designs

Up to this point we have been talking about two big classes of iterative combinatorial auctions:

- ICAs with non-linear ask prices, which have nice theoretical property of delivering efficient results, but put strong assumptions on bidders' behavior. In particular, the bidders must be able to grasp and analyze exponentially many ask prices.
- ICAs with linear ask prices, which provide bidders with a better overview of the current market situation. However, the approximative nature of such linear ask prices practically prevents theoretical analysis of such mechanisms, and the auction result can potentially be very far from an efficient outcome (Section 6.6).

The idea of combining virtues of both ask price formats in one design seem obvious. There are several approaches which try do it, primarily through enhancing precise non-linear price auctions with better elicitation support, based on linear ask prices.

Kwon et al. (2005) suggest extending the iBundle auction with RAD-like pseudo-dual prices to enable "endogenous bidding", when bidders can more easily find new attractive bundles during the auction. The authors show that added support for linear prices produces allocations with efficiency at least as high as when bidding is restricted to a fixed set of packages determined before the start of the auction.

The ***Clock-Proxy Auction*** (Ausubel et al., 2006) was proposed in the context of the FCC spectrum auction design. It extends the Combinatorial Clock auction (Section 3.2) by a last-and-final ascending proxy auction round (Section 3.3.1).

The approach combines the simple and transparent preference elicitation mechanism of the CC auction with the efficiency of the ascending proxy auction. Linear ask prices are used during the first clock phase. When the clock phase finishes, all winning bids are automatically transferred to the proxy round. Additionally, the bidders have ability to feed the proxy with more bids, depending on their previous activity in the clock phase and auction activity rules. These bids must be higher than the final prices of the clock phase. After that, the proxy stage is executed, and its winners are declared to be the winners in the auction.

We do not address the Clock-Proxy auction format in our computational experiments since its two-stage design makes bidding strategies in such auctions complicated and difficult to model in software. Furthermore, the performance of the Clock-Proxy auction will depend to a large extent on the activity rules, limiting additional bids which bidders can submit in the proxy phase. These activity rules must be adopted to the selected scenario; it is difficult to formulate them universally.

However, our comparison of linear price auction formats can be used to propose a different first phase in such hybrid auctions. The first phase with linear prices can be easily replaced with a RAD or ALPS phase.

Chapter 4

The ALPS ICA Design

> Each problem that I solved became a rule, which served afterwards to solve other problems.
>
> Rene Descartes

The ***ALPS*** (Approximated Linear PriceS) ICA format is a result of an extensive engineering work. We have repeatedly analyzed drawbacks of existing CA formats, designed improved rules to address these problems, and benchmarked different solutions under various setups in computational experiments.

Several key decisions shaped the ALPS design:

- The primary design goal is the high allocative efficiency of the auction outcome.
- The auction shall provide bidders with linear ask prices for the reasons discussed in Section 2.4.3. Furthermore, the pseudo-dual "RAD-like" prices are preferable to the *tâtonnement* "CC-like" prices, since they can better reflect the competition dynamics on the market and are easier to follow for the bidder.
- The design shall be strategy-proof. In particular, the ask prices shall be minimized, which reduces incentives for bid shadowing by bidders.

- Appropriate activity rules shall ensure competitive bidding right from the beginning of the auction. The bidders shall not be able to engage in "sniper" behavior of waiting until other bidders reveal their preferences and submitting bids only at the last stages of the auction.
- The auction shall be robust and perform well for various bidding strategies and valuations, especially when the bidders deviate from the best-response bidding.
- The design shall be practical. In particular, the number of auction rounds shall not be too high.

Of all the auction designs, described in Chapter 3, the RAD format was selected as the basis, since its pseudo-dual linear price format seemed to be the most suitable for our goal. The RAD auction also suggests transparent and effective activity rules.

4.1 Problems of Existing Designs

In a set of computational experiments we identified cases where the auction process and the results were not optimal. These cases were analyzed to identify and generalize the causes of the negative performance. This section describes these problems, illustrating them with examples as necessary.

4.1.1 Suboptimal Prices

We have identified several problems with ask prices, calculated by the linear-price auction formats RAD and CC.

- The linear ask prices, as calculated by the RAD auction format, can in some cases be unnecessarily high (Example 5) or distorted (Example 6), and cause misinterpretation of the true picture of the competition on the market.
- During computational experiments we found that the CC ICA design usually terminates with higher final prices than other auction formats (Section 6.3.3). Human bidders who are aware of this phenomena are likely to bid strategically and to shade their bids.

- The RAD ask prices can have strong oscillations (Example 7), which can be confusing for bidders, and potentially cause strategic behavior when bidders withhold their bids in anticipation that prices might fall.

To understand problems of the RAD ask prices, let us look again at its price calculation mechanism (3.2). Note that the solution to the LP (3.1) is not necessarily unique, and there can be indefinitely many price sets, each formally satisfying the requirements on the pseudo-dual linear ask prices. However they do not provide the same feedback quality for the bidders. Specifics of the LP solvers, which are usually used to calculate the pseudo-dual linear ask prices, can cause suboptimal price sets to be selected. The following two examples illustrate the problem.

Example 5. *Unnecessarily high prices in the RAD auction.*

Consider an auction with three items A, B and C. In an intermediate round t there are four active bids, all submitted by different bidders:

Bids	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
Bidder 1	55^*						
Bidder 2			55^*				
Bidder 3				40			
Bidder 4						40	

Let us determine the ask prices for the round $t+1$ using the RAD price calculation scheme (3.2). The provisionally winning allocation is marked in the table by stars, $L = \emptyset$. After removing redundant inequalities the linear program (3.2) takes the following form:

$$\begin{aligned} &\min_{p(B),Y} \quad Y \\ &\text{s.t.} \\ &\qquad p(A) = 55 \\ &\qquad p(C) = 55 \\ &\qquad 55 \leq Y \\ &\qquad 0 \leq p(B) \leq Y \end{aligned}$$

There are inifinitely many solutions to this LP with $Y^* = 55$ and $p^*(B) \in [0, 55]$. A simplex-based LP solver will deliver a solution in one of the convex *polyhedron corners, which will be either* $\{p^*(B) = 55, Y^* = 55\}$ *or* $\{p^*(B) = 0, Y^* = 55\}$. *In the first case the RAD will fix all prices to* 55*, which would distort the bidder's understanding of the current demand for the item* B.

To balance the prices, RAD design proposes maximizing the minimal price. However, if the solver finds the second solution $\{p^*(B) = 0, Y^* = 55\}$ *in our example, the algorithm will fix* $\hat{p}(A) = 55$ *and* $\hat{p}(C) = 55$*, and then yield* $p^*(B) = \infty$ *in the next iteration of the algorithm (3.2).*

Another important issue is the ask price balancing method. As Example 6 illustrates, RAD prices can distort the true picture of competition on the market.

Example 6. *Unbalanced prices in the RAD auction.*

Consider an auction with three items A, B, C and two currently active bids $b_1(ABC) = 160, b_2(A) = 70$*:*

Bids	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
Bidder 1							160*
Bidder 2	70						

The provisional winner is Bidder 1 and, again, $\hat{L} = \emptyset$. *The linear program (3.2) takes the following form:*

$$\begin{array}{rcl} & \min\limits_{p(A),p(B),p(C),Y} & Y \\ \text{s.t.} & & \\ p(A) + P(B) + P(C) & = & 160 \\ p(A) & \geq & 70 \\ 0 \leq p(A), p(B), P(C) & \leq & Y \end{array}$$

A simplex-based solver will normally yield one of the following two "corner" solutions: $\{p^*(A) = 70, p^*(B) = 20, p^*(C) = 70, Y^* = 70\}$ *or* $\{p^*(A) = 70, p^*(B) = 70, p^*(C) = 20, Y^* = 70\}$. *In both cases the RAD price calculating algorithm (3.2) will stop.*

Note that from the market point of view there is no reason for the item prices $p^*(B)$ *and* $p^*(C)$ *to be different. The desired outcome would be* $p^*(B) = p^*(C) = 45$*; item prices for* $p^*(B)$ *and* $p^*(C)$ *shall be balanced.*

4.1.2 Premature Termination of RAD Auctions

The termination rule plays a central role to an auction. The RAD design specifies two stopping rules (Section 3.1.3):

1. The eligibility-based rule stops the auction if every bidder has no unbound eligibility left. As illustrated by Example 7 below, this is not always sufficient to ensure auction termination.
2. The RAD auction is stopped if an identical provisional allocation is obtained in two consecutive rounds. As the same Example 7 shows, this rule can terminate the auction prematurely and consequently cause an inefficient allocation.

Example 7. *Premature termination of a RAD auction.*

Consider an auction with three items A, B and C, minimum increment of 2*, and two bidders, whose valuations are given in the table:*

Valuations	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
Bidder 1	10*						35
Bidder 2				32		32*	

The efficient outcome (marked with asterisks) would be to sell A to Bidder 1 and {B,C} to Bidder 2. Assume that the two following bids are active at some intermediate round $t-1$ during the auction:

Bids	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
Bidder 1							30.5*
Bidder 2				23			

The revenue-maximizing provisional allocation is marked with asterisks. The resulting ask prices $p^t_{ask}(k)$ are:

Prices	*A*	*B*	*C*
	11.5	11.5	7.5

Bidder 2 does not win in the provisional allocation. Since the ask prices are still below his valuations, he decides to submit another bid. He chooses between $p_{bid,2}(\{A,B\}) = (11.5 + 11.5 + 2 + 2) = 27$ *and* $p_{bid,2}(\{B,C\}) = (11.5 + 7.5 + 2 + 2) = 23$*. Since he has equal valuations for both bundles,* $\{B,C\}$ *is selected. The bids of the new round* t *are:*

Bids	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
Bidder 1							30.5*
Bidder 2						23	

The auction has two consecutive rounds $t-1$ *and* t *with the same provisional allocation. Consequently, the auction will terminate and assign all three items to* Bidder 1*. Obviously, this is not an efficient outcome. For* Bidder 2*, the auction termination comes unexpectedly, since he has been actively participating in bidding and was still ready to submit higher bids.*

Our computational experiments indicate that this situation is by far not exceptional. The poor performance of the RAD auction design (Section 6.3.1) is caused, in the first place, by premature termination of the auctions when some bidders were still ready to submit bids.

A naïve approach of removing the second termination rule, which is based on two consecutive equal allocations, solves the premature termination issue, but brings another problem. New ask prices in round $t+1$ *would be:*

Prices	*A*	*B*	*C*
	7.5	11.5	11.5

Obviously, the auction will oscillate from this point on, and will never terminate.

The termination rule of the RAD auction format must be modified to prevent premature terminations, but still to avoid oscillations and to ensure constant progress of the auction.

4.1.3 Activity Rules

An indispensable requirement for successful preference elicitation in an iterative auction is active participation of all bidders. Only the ask prices, which are calculated based on revealed valuations of all bidders, provide a true picture of the market competition.

However, active participation of all bidders in the auction is not given per se. On the contrary, bidders generally prefer to engage in ***sniping*** behavior and submit their bids only towards the end of the auction. Roth and Ockenfels (2002) show that sniping is a rational, gain-maximizing behavior, which is supported by evidence from many eBay auctions. Bidding in the last moments of the auction makes sense for at least two reasons:

- It avoids rising of the ask prices during the auction, when several bidders compete for provisional winning status.
- There is a good chance that the current provisional bidder will fail to outbid the sniping bid, even though his valuation could still allow it.

However there are objections to sniping, both from auctioneers and from non-sniping bidders:

- From the auctioneer's perspective, sniping reduces competition and leads to a lower final price, since some bidders may miss the opportunity to submit their bids at the end of the auction.
- Other bidders note that it is unfair to place bids where other bidders have no chance to analyze them and react.
- The auction can be significantly delayed if most bidders engage in sniping.

There are several known approaches to addressing this problem.

- A well-known solution, used by many single-item auction formats, is to automatically extend the auction after each placed bid. This approach is unpractical for combinatorial auctions. There are exponentially many bundles, and extending the auction after each possible bid can render it unbearably long.

- Another possibility is to terminate the auction randomly if there were no new bids within a certain period (or number of rounds). For example, the auction can terminate at any moment when 2 minutes after the last bid have passed, and the probability of auction termination is constantly increasing with time. This solution does not completely eliminate the incentive to use sniping bids, and can also lead to "random" allocations, where bidders did not submit some of the bids they intended to.

Note that neither of these solutions remove the incentive for sniping completely, since it can still be an advantage for bidders to wait "as long as possible" with their bids.

Most iterative combinatorial auctions use discrete rounds rather than continuous time. This allows the formulation of activity rules which can practically eliminate possibilities for sniping bidding. Introduced in the early FCC wireless spectrum auctions (Milgrom, 2000), such activity rules have since become standard for several combinatorial auction designs.

In many auction designs, including RAD, CC and ALPS, activity rule is given in the form of an *eligibility rule* (Section 3.1.2). The effect of the rule is intuitively clear. A bidder who bids on too few distinct items can never bid on a bigger bundle again. Thus, the bidders are effectively forced to bid aggressively right from the start of the auction.

However, this rule can be too restrictive and cause efficiency loss in some cases. In auctioning of transportation routes, for example, rising prices might force bidders to bid for a detour of the originally demanded shortest path. Since each section of a route is represented as an item in a transportation auction, bidding on a detour is equivalent to increasing the number of distinct items in the bidders' bids, which is impossible under the described eligibility rules.

Consequently, in this work we wanted to find a solution which allows a bidder to increase his eligibility, while at the same time retaining its role as an activity driver in the auction.

4.2 New Auction Rules Under Evaluation

After we have identified several problems of the RAD design, we suggest modifications which can address these issues and improve the auction results in other aspects. Most of these improvements are generic and can be applied universally to other ICA designs as well.

We implement in the MarketDesigner and test the following improvements:

- Properly minimizing and balancing ask prices, to avoid situations illustrated by Example 5 and Example 6.
- Reducing auction duration through dynamically increasing the minimum increment in situations where the competition is high enough.
- Using last-and-final bids (similar to iBundle, Section 3.3.1) to compensate for cases where the minimum increment is too high.
- Testing other possibilities for calculating the minimum increment.
- Improving the eligibility rule to prevent efficiency losses.
- Changing bid activity rules to accumulate more information about valuations and find a more efficient allocation.
- Improving price monotonicity in order to prevent confusion and strategic behavior of bidders.
- Changing termination rules to prevent problems of premature termination and oscillations (Example 7).

4.2.1 Balancing Ask Prices

The price calculation algorithm of the RAD design (3.2) pursues the following properties:

- New ask prices in the round $t+1$ shall support the provisional allocation from the previous round t; that is, all winning bids shall be equal to the new ask prices and all losing bids shall be below new ask prices. If such prices do not exist, they should be approximated as closely as possible.
- Among all price sets which satisfy the first requirement, the algorithm shall find the minimal prices in order to motivate straightforward bidding.

We add another requirement:

- The ask prices shall be balanced across items to better reflect the competition and be perceived as fair. Additionally, balanced ask prices will better mitigate the *threshold problem*. The threshold problem describes the situation where several small bidders try to outbid one bid on a big bundle, and need appropriate pricing information to support and coordinate their actions.

The RAD pricing algorithm (3.2) is therefore extended with additional steps which are necessary to further minimize and balance the newly calculated ask prices. Let us recall that for every losing bid $b^t(S_l)$ from the provisional allocation in round t we introduce a distortion variable $\delta(S_l)$, which measures the deviation of the round $t+1$ ask price from the bid price $b^t(S_l)$. Ideally, these distortions shall be minimal. Let $\hat{L}^t$ be the set of losing bids, for which the corresponding distortion variables cannot be further minimized. Here is the modified version of the price calculation algorithm, as used by the ALPS ICA design:

1. Initialize $\hat{L}^t = \emptyset$.
2. Minimize the maximum of all distortion variables $\delta(S_l)$ by solving the LP (4.1) (same as LP (3.2) in the RAD price calculation algorithm) and determine the new value for Z.

$$
\begin{aligned}
&\min_{p^{t+1}_{ask}(k), Z, \delta(S_l)} Z && (4.1)\\
&\text{s.t.}\\
&\sum_{k \in S_w} p^{t+1}_{ask}(k) = p^t_{bid}(S_w) && \forall\, b^t(S_w) \in W^t\\
&\sum_{k \in S_l} p^{t+1}_{ask}(k) + \delta(\hat{S_l}) = p^t_{bid}(S_l) && \forall\, b^t(S_l) \in \hat{L}^t\\
&\sum_{k \in S_l} p^{t+1}_{ask}(k) + \delta(S_l) \geq p^t_{bid}(S_l) && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t\\
&\delta(S_l) \geq 0 && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t\\
&\delta(S_l) \leq Z && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t\\
&p^{t+1}_{ask}(k) \geq 0 && \forall\, k \in \mathcal{K}
\end{aligned}
$$

3. Let $Z^*, \{\delta^*(S_l)\}, \mathcal{P}^* = \{p_{ask}^{t+1}(k)\}$ be the solution to LP (4.1) and let $L^* := \{b^t(S_l) : \delta_i^*(S) = Z^*\}$ be a set of all deviation variables which are equal to the new high bound. If $Z^* = 0$, we are done. Otherwise the RAD pricing algorithm would fix all distortion variables in L^* to Z^* and repeat step 2. However, as Example 5 demonstrates, there might still be room for improvement for some $\delta(S_l)$. First of all, it is possible that some distortion variables are fully independent of other variables, and can take any value in the interval $[0, Z^*]$. To minimize those, we restrict all distortion variables by Z^* and minimize their sum by solving the LP (4.2):

$$\min_{p_{ask}^{t+1}(k),\delta(S_l)} \sum_{b_i(S)\in L^*} \delta_i(S) \tag{4.2}$$

s.t.

$$\begin{aligned}
\sum_{k\in S_w} p_{ask}^{t+1}(k) &= p_{bid}^t(S_w) && \forall\, b^t(S_w) \in W^t \\
\sum_{k\in S_l} p_{ask}^{t+1}(k) + \delta(\hat{S_l}) &= p_{bid}^t(S_l) && \forall\, b^t(S_l) \in \hat{L}^t \\
\sum_{k\in S_l} p_{ask}^{t+1}(k) + \delta(S_l) &\geq p_{bid}^t(S_l) && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t \\
\delta(S_l) &\geq 0 && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t \\
\delta(S_l) &\leq Z^* && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t \\
p_{ask}^{t+1}(k) &\geq 0 && \forall\, k \in \mathcal{K}
\end{aligned}$$

If at least one of the distortion variables in L^* can be improved, this will be done by LP (4.2). We solve LP (4.2) iteratively as long as at least one distortion variable is improved, removing already minimized variables from L^*. After that we set $\hat{L}^t := \hat{L} \cup L^*$, fix all corresponding non-improvable distortion variables to their new numerical values ($\forall\, b_i(S) \in L^*$ set $\delta_i(\hat{S}) := \delta_i^*(S)$), and continue with LP (4.1).

4. At this point the set of all bids with positive distortion variables $\hat{L}$ is identified and fixed to corresponding numerical values. However, the prices themselves are still not necessarily unique and can be unbalanced, as in Example 6. To balance the prices, we use similar process to LP (4.1) and LP (4.2). Let $\hat{K}$ be the set of all items whose prices cannot be further

lowered. Initially let $\hat{K} = \emptyset$. First we minimize the maximum of all prices by solving the LP (4.3).

$$\begin{aligned}
&\min_{p_{ask}^{t+1}(k),Y} Y && (4.3)\\
&\text{s.t.}\\
&\sum_{k \in S_w} p_{ask}^{t+1}(k) = p_{bid}^{t}(S_w) && \forall\, b^t(S_w) \in W^t\\
&\sum_{k \in S_l} p_{ask}^{t+1}(k) + \delta(\hat{S_l}) = p_{bid}^{t}(S_l) && \forall\, b^t(S_l) \in \hat{L}^t\\
&\sum_{k \in S_l} p_{ask}^{t+1}(k) \geq p_{bid}^{t}(S_l) && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t\\
&p_{ask}(k) = \widehat{p_{ask}}(k) && \forall\, k \in \hat{K}\\
&p_{ask}(k) \geq 0 && \forall\, k \in \mathcal{K} \setminus \hat{K}\\
&p_{ask}(k) \leq Y && \forall\, k \in \mathcal{K} \setminus \hat{K}
\end{aligned}$$

5. Let $Y^*, \mathcal{P}_{ask}^*$ be the solution of LP (4.3) and let $K^* := \{k : p_{ask}^*(k) = Y^*\}$. If K^* contains more than one element, some of these prices may still be lowered or balanced. Therefore we next try to minimize the sum of all prices for items in K^* by solving the LP (4.4).

$$\begin{aligned}
&\min_{p_{ask}^{t+1}(k)} \sum_{k \in K^*} p_{ask}(k) && (4.4)\\
&\text{s.t.}\\
&\sum_{k \in S_w} p_{ask}^{t+1}(k) = p_{bid}^{t}(S_w) && \forall\, b^t(S_w) \in W^t\\
&\sum_{k \in S_l} p_{ask}^{t+1}(k) + \delta(\hat{S_l}) = p_{bid}^{t}(S_l) && \forall\, b^t(S_l) \in \hat{L}^t\\
&\sum_{k \in S_l} p_{ask}^{t+1}(k) \geq p_{bid}^{t}(S_l) && \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t\\
&p_{ask}(k) = \widehat{p_{ask}}(k) && \forall\, k \in \hat{K}\\
&p_{ask}(k) \geq 0 && \forall\, k \in \mathcal{K} \setminus \hat{K}\\
&p_{ask}(k) \leq Y && \forall\, k \in \mathcal{K} \setminus \hat{K}
\end{aligned}$$

6. If at least one of the prices in K^* can be lowered, this will be done by LP (4.4). We solve LP (4.4) iteratively as long as at least one price is lowered, removing already minimized prices from K^*. After that we set $\hat{K} := \hat{K} \cup K^*$, fix all corresponding prices to their new numerical values ($\forall\ k \in K^*$ set $\widehat{p_{ask}(k)} := p^*_{ask}(k)$), and continue with LP (4.3) unless $\mathcal{K} = \hat{K}$.

This version of the price calculation algorithm prevents the problems described by Example 5 and Example 6 from happening.

4.2.2 Dynamic Minimum Increment

The size of the minimum bid price increment in an auction presents a tradeoff between efficiency and reduced auction duration. Bigger price increments ensure quicker auction progress and termination. At the same time it can cause a decline of the auction efficiency, since bidders are more likely to get into a situation where the high minimum increment prevents them from submitting bids which could otherwise be a part of an efficient allocation.

A ***dynamic bid increment*** is a way of reducing the number of auction rounds without sacrificing too much on efficiency (Bapna et al., 2003, 2002; Hoffman et al., 2006). The specific of a combinatorial auction is that, at the same moment in time, the competition can have different levels on different items in the auction. Therefore, in our implementation, we calculate the minimum increment for each item and in each round separately, depending on the competition rather than using fixed minimum increments through the auction and for all items. Higher minimum increments while competition is high can advance the auction faster, while lower increments when only a few bidders are left shall guarantee fine price granularity. Consequently, this rule shall reduce auction duration without sacrificing the efficiency. Figure 4.1 illustrates the model for dynamic price increment calculation, which we adopted after several design iterations.

Compared to a single minimum increment value in the RAD auction format, the auctioneer must define several parameters in this case. Before the auction starts, the auctioneer sets an interval $[\Delta_{min}, \Delta_{max}]$ for the minimum bid increment. The parameter c defines the form of the non-linear part of the minimum increment function, and allows the use of smaller increment steps as

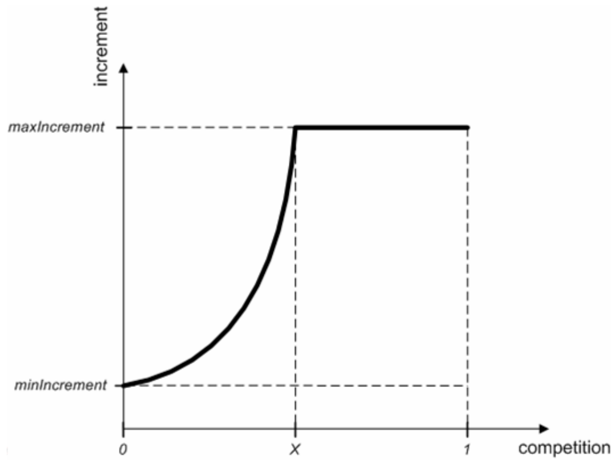


FIGURE 4.1: Dynamic Minimum Increment

competition is decreasing. Finally, the parameter $x \in [0, 1]$ defines the minimal competition level at which the Δ_{max} value is reached.

The current minimum increment Δ_k^t for the item k in the round t is determined in each round based on the total number of bidders n and n_k^{t-1}, the number of bidders who bid on the item k in the previous round $t-1$, using the following formula.

$$\Delta_k^t = \min\left(\Delta_{max}, \Delta_{min} + (\Delta_{max} - \Delta_{min}) * \left(\frac{n_k^{t-1}}{x * n}\right)^c\right)$$

Since the minimum increment Δ_k^t is calculated for each item k individually, the bidders receive a vector of current minimum increment values in each round, instead of using the same value for all rounds and all items. This does not necessarily complicate bidder's decisions. Since the minimum increment is added to every item in a bundle when calculating a new bid price, the current increment values can be automatically added to current item ask prices by the auction software, so that no additional complexity arises.

4.2.3 Bundle Minimum Increment

We test another approach to calculating the minimum increment in the auction. We apply the minimum increment to the bundle as a whole, and not to every item in the bundle, as defined by the RAD design. Suppose the minimum increment value is set to 5 and a bidder is interested in the bundle $\{A, B, C\}$. Under the RAD rules, the bidder would be requested to bid 15 above ask price, 5 for each item in the bundle. Using the ***bundle minimum increment*** rule, a single increment of 5 is applied to the whole bundle. Intuitively, this approach will favor bidders who bid for big bundles.

4.2.4 Old Bids Active

Generally speaking, having a higher number of bids available for the CAP implies better information about bidders' valuations, and consequently helps to find a more efficient allocation. Therefore we test in our computational experiments an intuitive idea of holding all ever submitted bids active through the auction – the ***old bids active*** rule. As a straightforward optimization, we keep active only the highest bid for each bidder and bundle.

The fact that a bidder bids on a bundle provides evidence that he is interested in the respective item combination at the given bid price. This interest shall a priori not decline over time. When ask prices are not monotonous, other bundles might become more valuable as the auction progresses. However, this shall not invalidate previously submitted bids, unless price fluctuations are strong. In case the rule is still seen as too restrictive by bidders, it is possible to make non-winning bids revocable explicitly.

Besides the expected positive effect on allocation efficiency, the rule also comes with a possible drawback. Keeping all bids active increases the size of the CAP. Hence, once the rule is active, the size of feasible auction scenarios, that is the manageable number of items and bidders, can get reduced.

The *old bids active* rule is used in connection with the *outbid old bids* rule to ensure termination of an ALPSm auction. (Section 4.2.9). In this case, the bidders have personalized non-linear prices for the bundles they already bid for, additionally to the pseudo-dual linear prices.

4.2.5 Last-and-Final Bids

This improvement is inspired by the iBundle auction format outlined in Parkes (2001). It shall reduce efficiency losses in situations where price increment steps are too high for some bidders. Bidders are allowed to bid below the required "ask price plus increment" amount if the following conditions are met:

- The new bid price is between the current ask price for the bundle (without the minimum increment) and the minimum bid price for the bundle (ask price plus minimum increment).
- The bid is explicitly marked as ***last-and-final***. For each bundle S and bidder i, only one last-and-final bid is allowed. No further bids, either normal or last-and-final, are accepted from bidder i for the bundle S.

A higher minimum increment reduces the auction duration, but is more likely to result in efficiency losses due to high price granularity. Last-and-final bids can help to find a better compromise since bidders are allowed to bid between increment steps. Therefore this rule can potentially yield a faster auction progress without sacrificing the efficiency. A further advantage of this rule is the perceived fairness on the bidder side, as they always have a possibility to bid up to their valuation.

4.2.6 Forced Price Monotonicity

Linear ask prices in the ALPS auction are not always monotonic. This can be confusing for bidders, and even stimulate strategic behavior, when bidders delay their bids in a hope that prices can fall.

We test a simple way to ensure that prices do not fall throughout the auction. For each item, we compare newly calculated prices in round t with previous $t-1$ round price. If the new price is lower, it is set to the level of round $t-1$.

4.2.7 Smoothed Anchoring

The pseudo-dual ask prices, calculated according to (2.9), are not necessarily unique, and give the auction designer a lot of flexibility. One of the possibilities is to derive prices which are as monotonic as possible from round to round. The technique of ***smoothed anchoring***, reviewed by the FCC (Dunford et al., 2007), takes this approach by applying exponential smoothing to calculation of the pseudo-dual prices.

In addition to the RAD pricing algorithm, the smoothed anchoring method minimizes the sum of squared deviations of the pseudo-dual prices between rounds, under the condition that the minimized sum of distortion variables remains constant at $\Omega^* = \sum_{b^t(S_l)\in L^t} \delta(S_l)$. This is accomplished by solving the following quadratic program.

$$\min \sum_{k\in\mathcal{K}} \left(p_{ask}^{t+1}(k) - \wp^t(k)\right)^2$$

s.t.

$$\sum_{k\in S_w} p_{ask}^{t+1}(k) = p_{bid}^t(S_w) \qquad \forall\, b^t(S_w) \in W^t$$

$$\sum_{k\in S_l} p_{ask}^{t+1}(k) + \delta(\hat{S}_l) = p_{bid}^t(S_l) \qquad \forall\, b^t(S_l) \in \hat{L}^t$$

$$\sum_{k\in S_l} p_{ask}^{t+1}(k) + \delta(S_l) \geq p_{bid}^t(S_l) \qquad \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t$$

$$\sum_{b^t(S_l)\in L^t} \delta(S_l) = \Omega^*$$

$$\delta(S_l) \geq 0 \qquad \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t$$

$$\delta(S_l) \leq Z \qquad \forall\, b^t(S_l) \in L^t \setminus \hat{L}^t$$

$$p_{ask}^{t+1}(k) \geq 0 \qquad \forall\, k \in \mathcal{K}$$

The value $\wp^t(k)$ is called ***smoothed price*** for item k in round t and is derived using the formula of exponential smoothing:

$$\wp^{t+1}(k) = \alpha \cdot p^{t+1}(k) + (1-\alpha)\cdot \wp^t(k)$$

The smoothing factor $\alpha \in [0, 1]$ determines how strong the impact of previous round prices is on $\wp^{t+1}(k)$. In the optimization of the above quadratic program, the values $\wp^t(k)$ are known and are treated as constants.

Similar to the ALPS price calculation, smoothed anchoring takes the minimized sum of distortion variables as the starting point. With reference to our modified price calculation algorithm described in Section 4.2.1, smoothed anchoring begins right after step 3, when all distortion variables are minimized and fixed to $\hat{\delta}_b$. Prices, however, are still not unique. Our approach continues by balancing both distortion variables (4.2) and prices (4.4), which shall also have a smoothing effect on prices. Smoothed anchoring, however, attempts to reduce the fluctuations by selecting those pseudo-dual prices that are as close as possible to the previous round's prices.

We did not include the *smoothed anchoring* approach in our analysis. Laboratory experiments with human bidders are necessary to measure the real effect of price monotonicity on the auction outcome. Furthermore, our modified pricing algorithm already delivers balanced prices, and the *smoothed anchoring* is likely to bring only a marginal improvement. Finally, we could not implement the *smoothed anchoring* algorithm, which requires optimization of a quadratic function, in the MarketDesigner framework, which supported only linear optimization.

4.2.8 Relaxed Eligibility

The RAD eligibility rule, which is an important and effective tool for assuring quick auction progress, can in certain cases cause problems. Especially when items in the auction significantly vary in price, and a group of items can play a replacement role for another item, bidders may want to replace a bid on a single expensive item with a bundle bid for a set of cheaper items. This is typically the case in transportation, when bidders give up bidding on the shortest route and start bidding on longer, but cheaper detours. The RAD eligibility rule will prohibit this. This issue is not only a potential source for allocative inefficiency, but can also be perceived as highly unfair and restrictive by the bidders.

We introduce a notion of ***surplus eligibility*** $e^t_{+,i}$, which allows the bidder i to extend his round t eligibility e^t_i and still stimulates competitive bidding. To retain the original purpose of enforcing activity in the auction, the size of the surplus eligibility is directly bound to the bidder's market activity in the auction so far. The surplus eligibility $e^t_{+,i}$ for each bidder is calculated by the auction in each round and is communicated to the bidders along with prices and provisional allocation. In round t a bidder is allowed to bid at maximum on as many distinct items as he bid on in the last round $t-1$, plus surplus eligibility:

$$e^t_i = e^t_{+,i} + \sum_{k \in \mathcal{K}} \prod_{b_j \in \mathcal{B}^{t-1}} x_j(k) \tag{4.5}$$

To determine the value $e^t_{+,i}$ we need to find a fair measure for a bidder's market activity. An important concern is to prohibit bidders from artificially simulating activity by submitting a lot of deliberately losing bids. We introduce the notion of ***bid volume*** of bidder i in round t.

$$\begin{aligned} rbv_i^t &= \sum_{k \in \mathcal{K}} maxbidprice_i^t(k) && \text{(Round Bid Volume)} \\ tbv_i &= \sum_{t=1}^{T} rbv_i^t && \text{(Total Bid Volume)} \end{aligned}$$

The function $maxbidprice_i^t(k)$ determines the maximum bid price for the single item k based on the bidder's i bids in round t. For each bid $b_i^t(S)$, the price for all $k \in S$ is determined by splitting the bundle bid price to individual items proportionally to item ask prices. For each item, the maximum over all bids value is taken. In other words, $maxbidprice_i^t(k)$ figures out how much item k is maximally worth to the bidder i in round t. Example 8 illustrates it:

Example 8. *Calculating* $maxbidprice_i^t(k)$.

Consider an auction with three items A, B and C and linear prices in round t respectively 10, 10, and 20. If bidder i submits a bid on the bundle $\{A, B, C\}$ for 50, the bid price is split proportionally to ask prices, resulting in values 12.5, 12.5, and 25 for A, B and C respectively. Let his second (and last) bid in the round t be 30 on the bundle $\{B, C\}$, which splits proportionally to ask prices as 10 for B and 20 for C. In this case, we obtain:

$$\begin{aligned} &maxbidprice_i^t(A) = 12.5 \\ &maxbidprice_i^t(B) = max(10, 12.5) = 12.5 \\ &maxbidprice_i^t(C) = max(20, 25) = 25 \end{aligned}$$

The ***total bid volume*** tbv_i equals to the sum of ***round bid volume*** rbv_i^t over all auction rounds and items, and represents the overall bid volume that bidder i generated in the auction so far. All bidders are ranked by their tbv_i in ascending order. The rank for bidder i, denoted by $r_i \in [1, |\mathcal{I}|]$, is the index of the position in the ordered sequence of this bidder's tbv_i. The surplus eligibility is then calculated as:

$$e^t_{+,i} = round\left(\frac{r_i}{|\mathcal{I}|} \cdot e_{+,max}\right)$$

The value $r_i/|\mathcal{I}|$ is scaled in $(0,1]$ and serves as an indicator for market activity of the bidder i. $e_{+,max}$ is the maximal surplus eligibility defined by the auctioneer. The fact that a bidder's activity is accumulated throughout the auction sets incentives for bidders to bid actively right from the start. Moreover, the ranking concept based on the tbv_i also ensures that all bidders are stimulated to bid better than others.

4.2.9 Termination Rule

We have previously illustrated problems with the RAD termination rule (Example 7). Now we suggest a two-step solution which overcomes the described problems.

1. We denounce the termination rule, which stops the auction after two equal allocations. To prevent premature terminations, where the auctions stops unexpectedly for some bidders who still could submit competitive bids, the auction terminates only if there are no new bids from any bidder in one round (***inactivity-based termination***).

2. Without further modifications to the termination rule, it would be possible that the auction starts oscillating (see Example 7) and never terminates. There are several possibilities to ensure auction progress. We test two versions:

    - The minimum increment is increased on each equal allocation and set back to its initial value as soon as the allocation changes. We call this rule ***increase minimum increment***.
    - Bidders are forced to outbid their previous bids on the same bundle, hence yielding monotonically increasing bundle bid prices for each single bidder. We call this rule ***outbid old bids***.

    If the losing bidder's valuation is high enough, each rule alone will eventually cause the allocation to change. Otherwise, the losing bidder will eventually stop bidding and the auction will end.

4.3 Putting ALPS Together

A central result of our work is the ALPS (Approximated Linear PriceS) auction design. We test in computational experiments all the improvements described in Section 4.2, and select the set of rules which bring the best results. The detailed test results are given in Section 6.2.

The ALPS auction design is based on pseudo-dual prices as with RAD, but contains the following modifications:

Improved prices: ALPS uses the improved pricing algorithm (Section 4.2.1), which achieves lower and better balanced prices compared to the original RAD algorithm. We found this to have a modest, but positive, impact on efficiency.

Termination rule: The termination rule has been adapted, since it is a potential cause of inefficiency in RAD. An auction terminates if there are no new bids submitted in the last round (the *inactivity-based termination*). To ensure auction progress and prevent oscillations, the ALPS design increases the minimum increment if the provisional allocation does not change in two consecutive rounds. After a new allocation is found, the minimum increment is reset to its initial value.

Surplus eligibility: Many auction scenarios suffer from the problem that the RAD eligibility rule does not allow for an increase in the number of distinct items a bidder is bidding on. In particular, in the transportation scenario it can become beneficial to bid on a longer detour during the course of an auction, when the direct connection becomes too expensive. We modify the RAD eligibility rule and allow active bidders to increase the number of items to bid on using *surplus eligibility* (Section 4.2.8).

We also formulate the ALPSm design (modified ALPS) with the following additional rules:

Old bids active: Provisional and final allocations are calculated based on all ever submitted in the auction bids. Losing bids are not removed from the auction after each round.

Termination rule: Every bidder has to outbid his old bids from previous rounds on the same bundle. The minimum increment remains constant throughout the auction, even if the allocation does not change in two consecutive rounds.

Our experiments showed that the ALPSm auction design has a better efficiency compared to the ALPS format, but increases the size of the CAP LP. Consequently, for large scenarios ALPS can be the tool of choice.

Two further modifications have a positive effect on the auction outcome, and can be used by both ALPS and ALPSm:

Dynamic minimum increment: Helps to reduce auction duration without sacrificing the efficiency. See Section 4.2.2 for implementation details and Section 6.2 for experiment results.

Last-and-Final bids: Allows the use of a bigger minimum increment and thus accelerates the auction without having a negative impact on the efficiency. This increases perceived fairness. See Section 4.2.5 for implementation details and Section 6.2 for experiment results.

ALPS and ALPSm auction formats can handle both OR and XOR bidding languages. Furthermore, we allow each bidder in the auction to use either OR or XOR bidding language independently of other bidders.

When the XOR bidding language is selected, the price calculation algorithm is modified and the losing bids of a corresponding winning bidder are not included in the price calculation algorithm. Since a XOR-bidder can only win one bundle at the most, his losing bids might keep prices of other items unnecessarily high, which conflicts with the goal of minimizing the ask prices.

The ALPS/ALPSm auction format has proven to be a successful design, which performs well under various conditions both in computational experiments and in the laboratory (Section 6.3; Scheffel et al. (2009); **?**). It also appeared to be robust against suboptimal and strategic bidding (Section 6.5).

Chapter 5

Experimental Framework

> All life is an experiment. The more experiments you make the better.
>
> Ralph Waldo Emerson

This chapter describes the framework for the computational experiments conducted in the course of our work. The framework is an integral part of the MarketDesigner platform which is being used for both computational and laboratory experiments. We describe below the functionality of the system; the implementation details can be found in (Laqua, 2006).

The framework for computational experiments consists of three main components (Figure 5.1).

- A *value model* defines valuations of every bidder in the auction for every possible bundle. It is independent of the selected auction format.
- A *bidding agent* implements a bidding strategy adhering to the value model which is assigned to him and to the restrictions of the selected auction design.
- An *auction processor* implements the auction logic, enforces auction protocol rules, and calculates allocations and ask prices.

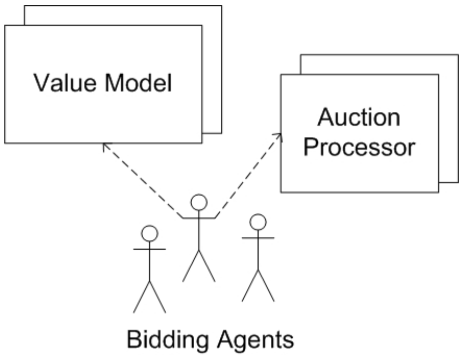


FIGURE 5.1: Architecture Overview of the Experimental Framework

At the same time, these software components play the role of different treatment variables in the computational experiments. Different types of value models, bidding agents (that is, strategies) and auction processors can be combined, which allows performing sensitivity analysis by running a set of computational experiments while changing only one component and preserving all other parameters. To compare the results, we use the following performance measures (Section 2.1.1 and Section 2.5):

- allocative efficiency;
- revenue distribution;
- price monotonicity;
- speed of convergence measured by number of auction rounds.

5.1 Value Models

The type of bidder valuations is an important treatment variable for the analysis of different auction formats. Performance of an auction format can depend significantly on properties of the valuations, particularly on the *bidder submodularity* (BSM) condition (Section 2.4.2.1) which often does not hold in practice.

Practical applications of combinatorial auctions are still very rare, and real-world CA data sets are hardly available. However, using realistic value models is important for achieving reliable comparison results. To ensure that our analysis gives reliable and upright results, a variety of value models with different structure and background is implemented in our framework:

- We have adopted the Combinatorial Auctions Test Suite (CATS) value models that have been widely used for the evaluation of winner determination algorithms (Leyton-Brown et al., 2000). They use economically motivated scenarios, and thus generate realistic data sets. For example, a transportation network, real estate lots, or an airport slot occupancy timetable provide the underlying rationale. Since the CATS library was designed originally for testing winner determination algorithms only, it was impossible to use it for testing of complete iterative combinatorial auctions directly. We have updated the implementation to generate complete valuations rather than set of bids, which put additional requirements on the consistency, since not every consistent set of bids represents a valid and consistent value model. Additionally, we allow for the full parametrization of value models using XML files, unlike the original CATS implementation, where many parameters are hard-coded in the source code.

- We have implemented the Pairwise Synergy value model described by An et al. (2005). It allows the free definition of the number of items in the auction and the complementarity degree between them, which makes this model perfectly suitable for sensitivity tests on various parameters. Furthermore, this value model allows subadditive valuations to be described, where the value of a bundle is lower than the sum of the individual item values comprising it.

- We have created a Tabular value model where the valuations can be manually defined for any combination of bundles. This value model proved to be helpful for testing small scenarios while identifying problems and describing various phenomena of different ICA designs.

All value models are constructed to meet the usual *free disposal* assumption (Section 2.3):

$$v_i(S) \leq v_i(T) \quad \forall\, S \subset T$$

meaning that bidders can discard any item free of charge.

5.1.1 Transportation

The ***Transportation*** value model uses the Paths in Space scenario from the Combinatorial Auction Test Suite (CATS) by Leyton-Brown et al. (2000). It is modeled by a nearly planar transportation graph in Cartesian coordinates, where each bidder is interested in securing a connection between two randomly selected vertices (cities). The items which are traded in the auction are the edges (routes) of the graph. The real-world examples which map to this scenario include truck routes, natural gas pipeline networks, communication network bandwidth allocation, and the rights to use railway tracks.

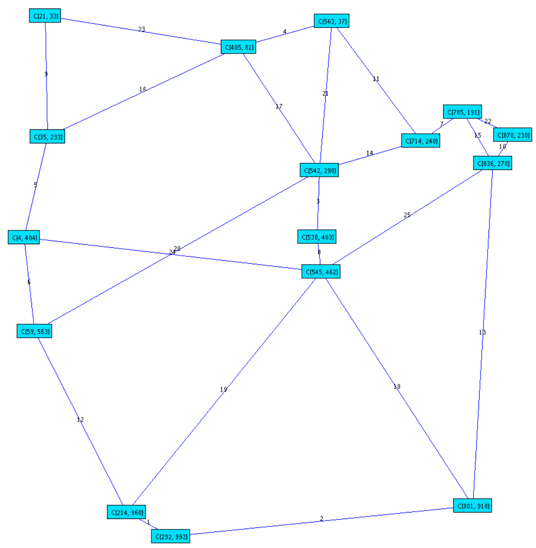


FIGURE 5.2: Transportation Value Model with 25 Items (Edges)

The bidder's valuation for a path is defined by the Euclidean distance between his two nodes multiplied by a utility factor, which is a random number drawn from the same normal distribution for every bidder. Consequently, only a limited number of bundles which represent paths between both selected cities are valuable for the bidder. Flexibility of the bidders can be controlled by changing the utility factor.

The relevant parameters for the Transportation value model are:

- The number of items (edges in the graph) m.

- The graph density ρ, which defines an average number of edges per city and is used to calculate the number of vertices as $(m * 2)/\rho$.
- The average utility factor u.
- The standard deviation σ of the random distribution, which describes the utility factor.

The items in the Transportation value model have the highest complementarity. The bidders are interested only in a set of items which represents a complete path connecting two selected cities. Any subset of such a path has zero valuation. Any extra item added to an already complete path does not increase the value of the bundle in any way.

5.1.2 Airports

The ***Airports*** value model is an implementation of the Matching scenario from CATS. It models the four busiest USA airports, for which the Federal Aviation Administration in fact auctions off takeoff and landing rights: La Guardia International, Ronald Reagan Washington National, John F. Kennedy International, and O'Hare International (Grether et al., 1989). For simplicity, each airport has the same predefined number of takeoff and landing time slots, and there is only one slot for each point in time available per airport.

Each airport is randomly assigned its utility value. Every bidder is interested in obtaining a pair of slots in two randomly selected airports. The maximum possible valuation for acquiring such a pair is defined as the sum of both individual airport utilities. This maximum is reached only when the landing time matches a certain value randomly selected for every bidder, and the flight time is at its possible minimum given the distance between the airports. The valuation is reduced if the landing time deviates from the ideal value, or if the time difference between takeoff and landing slots is longer than necessary. Similarly to the Transportation value model, the Airports value model has the strongest complementarities between items. Specific to it, every bidder is interested in a pair of items only.

The relevant parameters for the Airports value model are:

- The number of items m. Since every airport has the same number of time slots, it must be divisible by four.

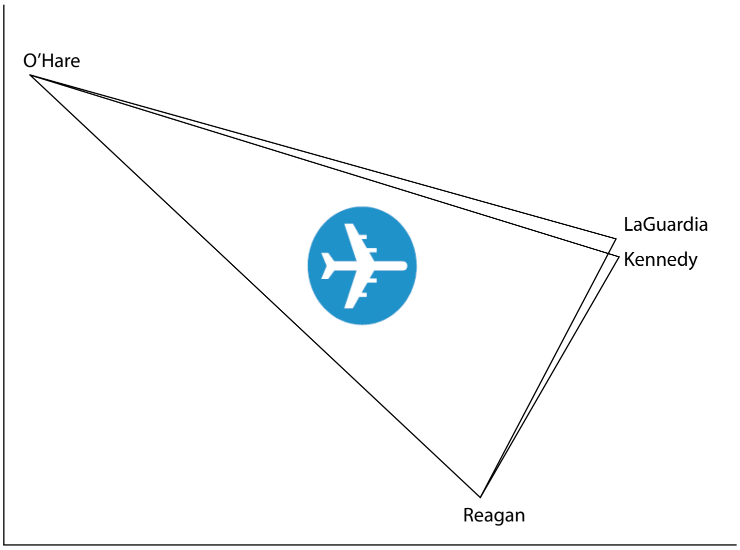


FIGURE 5.3: Airports Value Model: Four Largest USA Airports

- Minimum and maximum airport utilities $MinU$ and $MaxU$. The utility values for the individual airports AU_1 to AU_4 which are the same for every bidder are drawn from a uniform random distribution between $MinU$ and $MaxU$.
- The per-bidder utility deviation UD_i defines the maximum possible deviation from the common utility value, obtained as the sum of both takeoff and landing airport utilities $AU_t + AU_l$. The exact maximum valuation for every bidder is drawn randomly from the uniform distribution over the interval $[(AU_t + AU_l) - UD_i, (AU_t + AU_l) + UD_i]$.
- The maximum flight length L defines the range between the two most distant airports, Reagan and O'Hare, in flight hours.
- The early takeoff deviation ETD and the late takeoff deviation LTD define maximum allowed differences to the optimal takeoff time, in hours.
- The late landing deviation LLD defines the maximum possible landing delay compared to the optimal landing time, in hours. Early landing is not allowed. In the event that the plane departs earlier, it must wait in the air, and then an appropriate penalty on the valuation is calculated.

- The delay coefficient DC defines the penalty size for the case when the plane stays in the air longer than necessary.
- The late coefficient LC defines the penalty in the case of late landing.

5.1.3 Real Estate

The ***Real Estate*** value model is based on the Proximity in Space model from CATS. Items sold in the auction are the real estate lots k, which have valuations v_k drawn from the same normal distribution for each bidder. The bidders in such markets prefer to secure several adjacent pieces of real estate. Further real-life examples which can be described by this value model include auctioning of various area-bound licenses, like spectrum licenses in FCC auctions, or mining rights, where it is much cheaper for an oil company to drill in adjacent areas than in areas that are far from each other.

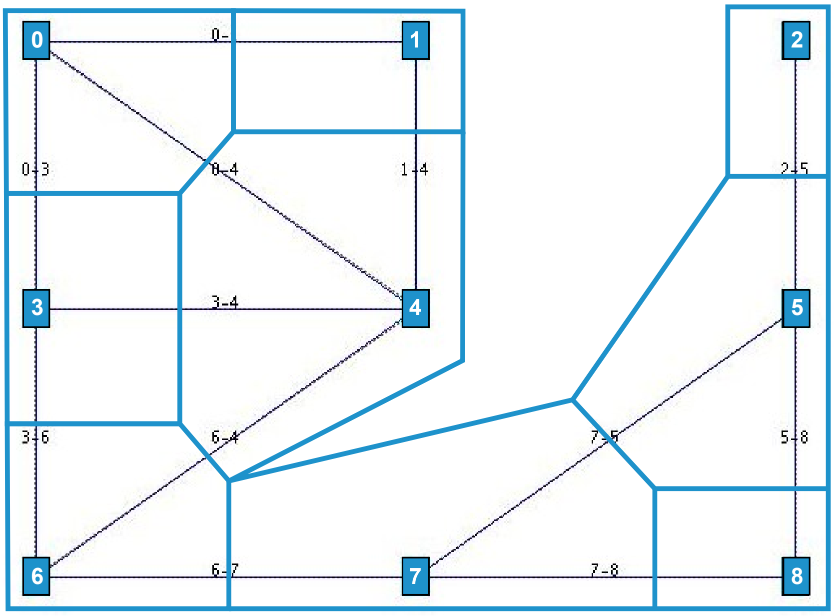


FIGURE 5.4: Real Estate Value Model with 9 Items

Complementarity in such markets arises from adjacency in the two-dimensional space. We model the valuations by placing all items in a grid graph, where the vertices can have horizontal, vertical and diagonal neighbors. The adjacency relationships $e_{ll'} \in \{0, 1\}$ between items l and l' are represented by edges which are randomly generated, whereby we take care that the diagonal edges do not intersect. An example graph with nine pieces of land property is shown in Figure 5.4. The dashed lines represent the adjacency relationships between lots, and the solid lines show possible property boundaries which would be represented by this graph. Note that exactly one vertex falls inside each piece of land property, and each two lots which are connected by a graph edge share a border.

The edge weights $w_{ll'} \in [0, 1]$ are generated randomly for every bidder, and they are used to determine bundle valuations of adjacent pieces of land:

$$v(S) = \left(1 + \sum_{e_{ll'}: l, l' \in S} w_{ll'}\right) \sum_{k \in S} v_k$$

Even though the bidders generally prefer to acquire as much real estate as possible, only adjacent lots have complementarity and increase the value of a bundle to be more than just a sum of the separate lot values.

The relevant parameters for the Real Estate value model are:

- Two parameters $SideX$ and $SideY$ define sizes for both sides of the land area, in lots.
- Two parameters $epHV$ and epD define probability of the existence of a horizontal/vertical and a diagonal neighborhood relationship between individual lots correspondingly.
- The neighborhood relations between lots are not necessarily relevant for each individual bidder. Therefore, the per-bidder parameter ep_i defines the probability that the edge from the "common" neighborhood graph is taken over to the individual graph of the bidder i. As a result, some neighborhood edges can be deleted from the individual graph of the bidder.
- Per-bidder parameters $lotPriceMean_i$ and $lotPriceDeviation_i$ define the normal distribution for drawing of lot valuations v_k for every bidder.
- Per-bidder parameters $weightMean_i$ and $weightDeviation_i$ define the normal distribution for drawing of edge weights $w_{ll'}$ in the neighborhood graph of every bidder individually. These weights define the degree of complementarity between connected lots.

5.1.4 Pairwise Synergy

The ***Pairwise Synergy*** value model is an implementation of the valuation calculation method described in An et al. (2005). This value model is useful for sensitivity analysis and comparison, since it describes a homogenous scenario where each combination of items is meaningful for every bidder, and the complementarity level among items is similar and easily configurable.

This value model can also describe subadditive valuations, where the value of a bundle is less than the sum of valuations for individual items comprising the bundle. It is also possible to define a mixed scenario, where some bundles have superadditive valuations and other bundles subadditive valuations.

The Pairwise Synergy value model has the following parameters:

- The number of items m.
- Two parameters per bidder, $minItemValue_i$ and $maxItemValue_i$, define the uniform distribution for sampling valuations of individual items v_k for all $k \in \mathcal{K}$ and for every bidder $i \in \mathcal{I}$.
- Two parameters per bidder, $maxSynergy_i$ and $minSynergy_i$, define the uniform distribution for generating a symmetric matrix of pairwise synergies between items $\{syn^i_{k,l} : k, l \in \mathcal{K}, syn^i_{k,l} = syn^i_{l,k}, syn^i_{k,k} = 0\}$, for every bidder $i \in \mathcal{I}$.

The valuation of a bundle S is calculated as:

$$v(S) = \sum_{k=1}^{|S|} v_k + \frac{1}{|S| - 1} \sum_{k=1}^{|S|} \sum_{l=k+1}^{|S|} syn_{k,l}(v_k + v_l)$$

The synergy value of 0 corresponds to completely independent items, and the synergy value of 1 means that the bundle valuation is twice as high as the sum of the individual item valuations.

5.2 Bidding Agents

A *bidding agent* implements a bidding strategy adhering to the given value model and to the restrictions of the specific auction design. Usually bidding agents in auctions are considered to be *selfish*, that is pursuing maximization of their own profit and neglecting all other factors. There are also many other usual assumptions of different strength (Section 2.2.2 and Section 2.3) which shape the theoretical understanding of auction mechanisms.

Under these assumptions and for auctions with VCG outcomes there exist a dominant strategy of truthful reporting. However, there is a substantial evidence that the assumptions are too strong, and that the bidders do not follow this strategy in practice. We discussed possible reasons for it in Section 2.4.2.

Therefore we conduct our computational experiments not only with idealized best-response bidders, but also with bidders who deviate from the theoretically optimal strategy to a different degree. We hereby achieve the following:

- Measuring auction results with theoretically optimal best-response bidders delivers upper bounds on the efficiency of the given ICA format.
- By using bidders who deviate from the best-response behavior, we can study the robustness of ICA formats against suboptimal bidder strategies.
- We can also group different types of bidders in the same auction, and study how different strategies perform relative to each other by comparing their revenue.

We implement several suboptimal bidding agents based on empirical observations of human bidders, which we made during laboratory experiments with combinatorial auctions (Scheffel et al., 2009; **?**). In these auctions, bidders were likely to bid on high-profit bundles, but did not follow the best-response strategy even in simple settings with adequate bidder support tools. In a non-linear price auction, the demand set of every active bidder constantly grows larger during the auction, but the amount of submitted bids did not increase from round to round for human bidders in this case.

5.2.1 BestResponse Bidders

The ***BestResponse*** bidder follows the best-response bidding strategy which is often assumed in game-theoretical analysis (Section 2.4.1). This bidder bids on all bundles that maximize his surplus, if he would win any of them at current prices, and only for these bundles (that is, his demand set $D_i(\mathcal{P}_{pay})$) (Definition 6).

Intuitively, the best-response strategy seems to be the obvious choice in case of an iterative auction. All non-linear price auction formats which guarantee an efficient auction outcome (Section 3.3) assume the best-response behavior of all bidders. Proxy agents which are suggested for many non-linear price auctions implement exactly the same strategy. However, the best-response bidding is not a dominant strategy in non-linear price auctions if BSM property is not given (Definition 18). For the linear-price auctions, there are no theoretical results regarding efficiency of different strategies at all. The bidder's choice is further complicated by the fact that he cannot determine whether the BSM property is given or not by looking at his own valuations alone. Furthermore, determining the demand set requires from the bidder advanced computational skills, which is the second and even more important critique point on the best-response assumption.

5.2.2 Powerset Bidders

The ***Powerset*** bidder evaluates all possible bundles in each round, sorts them by falling revenue given the current ask prices, and submits best x bids, where x is one of the bidder's strategy parameters. In contrast to the BestResponse bidder, the Powerset bidder can select not only the bundle(s) from his demand set and providing the maximum profit, but also less profitable bundles. Such behavior can be enforced by activity rules, when bidders have to submit more bids additionally to their demand set to preserve eligibility. Risk aversion can also explain overbidding: The bidders can submit suboptimal bids when they want to be sure to win at least something.

5.2.3 Random Bidders

The ***Random*** bidder is close to the Powerset bidder, and technically shares the same software implementation. This bidder evaluates all possible bundles in each round, sorts them by falling revenue given the current ask prices, and then submits x bids, randomly chosen out of the best y bids.

This behavior models real-world bidders with limited computational resources, who can generally find good bundles given current ask prices, but make minor errors in their calculations.

5.2.4 Preselect Bidders

If the bidders are restricted in time during the auction, they might select their most valuable bundles a priori, and stick to this selection throughout the auction. This might be a viable strategy in auctions with a large number of items. The ***Preselect*** bidder selects his l most valuable bundles before the auction. During the auction, the bidder follows the best-response strategy but bids only on the preselected bundles.

5.2.5 Level Bidders

The ***Level*** bidder models a speculative strategy that tries to exploit the termination rule of non-linear price auctions. These auctions terminate as soon as the new provisional allocation includes bids from every participating bidder. This termination rule can motivate bidders to submit more than just the demand set in the first round in the hope that some suitable allocation is found early and the auction terminates before the prices rise. The strategy is likely to be successful if adopted by many bidders in the auction and if the competition is not very high.

To model the Level bidder, we modify the original BestResponse bidder by lowering valuations of his best l bundles and setting all of them equal to the valuation of the l^{th} best bundle. Further on in the auction, the Level bidder follows the best-response strategy adhering to the modified valuations.

5.2.6 BestChain Bidders

The ***BestChain*** bidder is similar to the INT bidder in An et al. (2005). It implements a polynomial-time algorithm which builds "chains" of items by starting from a single item and then iteratively selecting the most valuable extension to already-existing bundle:

for each $k \in \mathcal{K}$
  1) Create a single-item bundle $B_k = \{k\}$
  2) Define $\alpha = argmax_{l \in \mathcal{K} \setminus B_k} AU(B_k \cup \{l\})$
  3) if $AU(B_k \cup \{\alpha\}) > AU(B_k)$
      then $B_k = B_k \cup \{\alpha\}$, goto 2)

Starting from each individual item $k \in \mathcal{K}$, the algorithm finds the next item l which provides a maximum increase in average per-item utility (AU) of the bundle given current prices. If the new average utility exceeds the previous value, the new item is added to the bundle and the process is continued until the average utility cannot be increased further. The algorithm generates at least m bids in $O(m^3)$ time.

The algorithm shall mimic behavior of a human bidder in homogenous value models where each combination of items is potentially interesting, like Pairwise Synergy and Real Estate. In value models with extreme complementarities, like Airports and Transportation, this bidding model is useless, since subsets of the valuable bundles have zero valuation. The bidder also requires item prices to construct chains, and cannot successfully participate in auctions where only non-linear prices are given.

5.2.7 Naïve Bidders

The ***Naïve*** bidder is an extreme case of a bidder who is completely ignorant of the bundle bidding. A Naïve bidder submits in each round singleton bids only for those items that would provide positive utility given current linear ask prices. In contrast to all other bidder types which can use any bidding language the Naïve bidder must use the OR bidding language to be able to win more than one item at all.

Since the optimal performance of non-linear price auctions requires best-response bidding, several authors demonstrated how the auction design can assist bidders in following it, or even enforce it. Ausubel and Milgrom (2006a) suggest using proxy agents which receive valuations once before the auction and participate in the auction adhering to the best-response strategy. Mishra and Parkes (2004) describe how the auctioneer can detect that bidders deviate from the best-response strategy, and even enforce it by automatically increasing bid prices on already submitted bids until the bidder signals that the ask prices have exceeded his valuations by submitting his empty bid.

Unfortunately, such tools effectively force the bidders to build a full picture of their valuations already before the auction, thus eliminating most virtues of an iterative mechanism. The bidders cannot use the ask prices during the auction to better understand competition on the market and to find new interesting bundles.

Furthermore, in some cases non-best-response bidding cannot be detected, and consequently prevented, by the auctioneer at all. The Level and Preselect bidders in our setup are the strategies which look as perfectly legitimate best-response bidding to the auctioneer.

5.2.8 Further Strategy Parameters

The strategy of a bidding agent in our computational experiments is further defined by a set of parameters which are universally applicable to all bidder types.

- The parameter *forgetBidProbability* defines the probability that a bidder "forgets" to submit a bid which was already prepared for submitting by following one of the strategies described above.
- The parameter *maxBundleSize* limits the maximum bundle size for every bid of a bidder. This parameter is important for value models where bigger bundles always have higher valuations. In case the bundle size is not limited, the biggest bundle is always the most valuable, and the combinatorial auction degenerates into a single-item auction for the complete bundle containing all items in the auction.
- The parameter *jumpBidIncrement* allows the definition of bidders who bid aggressively above current ask prices, but still always within their valuations.

Chapter 6

Results of Computational Experiments

Facts are the air of scientists.
Without them you can never fly.

Linus Pauling

This chapter describes and interprets the results of our computational experiments. In Section 6.1 we provide details on the experiment setup. The results themselves are grouped according to the following topics, each occupying its own section:

- Section 6.2 measures effects of the new improved auction rules which are suggested in Section 4.2. First each rule is evaluated separately, and then the subset of the most successful rules which comprises the ALPS/ALPSm auction design is tested again.
- Sections 6.3 and 6.4 analyze and compare performance and robustness of various linear-price auction designs and measure the effect of using different bidding strategies in the auction.
- Section 6.5 concentrates on robustness of non-linear price auctions in various settings, especially when the bidders do not follow the theoretically optimal best-response strategy. The results are benchmarked against linear-price ALPSm and CC designs.

- Section 6.6 illustrates simple examples and analyzes the cases when linear-price ICAs have low performance.

6.1 Settings

Most computational experiments in this work share the same set of settings for value models, bidding agents and auction formats.

The value model parameters for all auctions were selected to yield approximately the same total revenue of 200 in order to achieve better comparability of the measurements. Unless otherwise specified, we use value models with parameters given in Table 6.1. For a detailed description of individual parameters, consult Section 5.1.

Table 6.2 lists all instances of bidding agents. For a detailed description of their strategies consult Section 5.2.

Table 6.3 lists all auction settings. Unless specified otherwise, the minimum increment was set to 1 in all cases.

The following four sections, which describe the results of the computational experiments, are structured in the same format. First we present the research question which we target in the section. Then we describe the setup of the relevant experiments. The obtained results are presented in the form of tables and charts. Finally, the results are discussed and conclusions are made. To prevent overloading of the text with illustrations, additional charts were moved to the Appendix.

6.2 Improving the RAD Design

The ALPS/ALPSm auction design (Chapter 4) has the RAD format in its roots, but implements a set of improvements which address the RAD problems (Section 4.1) and further improve the auction results. During analysis of existing ICA designs we have formulated a list of new rules, which is given in Section 4.2. The objective of the computational experiments, described in this section, is to study the impact of these new and improved rules on the auction outcome and to formulate the optimal set of rules which have the

Name	Items	Bidders	Parameters and description
Transportation Small	25	15	With the edge density $\rho = 3.2$ we obtain 15 to 16 vertices in the graph. The average shipping utility $u = 2.2$. The standard deviation $\sigma = 0.25$.
Transportation Large	50	30	The edge density of $\rho = 2.9$ produces networks with ca. 34 cities. The average shipping utility $u = 2.2$. The standard deviation $\sigma = 0.25$.
Airports	84	40	21 time slots per airport. The minimum and maximum airport utilities $MinU = 2$ and $MaxU = 6$. The utility deviation $UD_i = 0.5$. The maximum flight length $L = 10$. The early takeoff deviation $ETD = 1$ and the late takeoff deviation $LTD = 2$. The late landing deviation $LLD = 2$. The delay coefficient $DC = 0.9$. The late coefficient $LC = 0.75$.
Real Estate 3x3	9	5	The individual item valuations have normal distribution with a mean of $lotPriceMean_i = 10$ and a standard deviation $lotPriceDeviation_i = 2$. The edge probabilities are $epHV = 0.9$ and $epD = 0.8$. Each edge is taken over to the individual graph of the bidder with a probability $ep_i = 0.9$. The edge weights have a mean of $weightMean_i = 0.5$ and a standard deviation of $weightDeviation_i = 0.3$. For the bidders in Real Estate 3x3 auctions, $maxBundleSize = 3$.
Real Estate 4x4	16	10	The individual item valuations have normal distribution with a mean of $lotPriceMean_i = 6$ and a standard deviation $lotPriceDeviation_i = 1.1$. The edge probabilities are $epHV = 0.9$ and $epD = 0.8$. Each edge is taken over to the individual graph of the bidder with a probability $ep_i = 0.9$. The edge weights have a mean of $weightMean_i = 0.5$ and a standard deviation of $weightDeviation_i = 0.3$. For the bidders in Real Estate 4x4 auctions, $maxBundleSize = 3$.
Pairwise Synergy High	7	5	The individual item valuations are uniformly drawn from the range $minItemValue_i = 0$ to $maxItemValue_i = 88$. The synergy values are between $minSynergy_i = 1.5$ and $maxSynergy_i = 2.0$. For the bidders, $maxBundleSize = 3$.
Pairwise Synergy Low	7	5	The individual item valuations are uniformly drawn from the range $minItemValue_i = 0$ to $maxItemValue_i = 195$. The synergy values are between $minSynergy_i = 0$ and $maxSynergy_i = 0.5$. For the bidders, $maxBundleSize = 3$.
Pairwise Synergy Zero	7	5	The individual item valuations are uniformly drawn from the range $minItemValue_i = 12$ to $maxItemValue_i = 28$. The synergy values are between $minSynergy_i = -0.5$ and $maxSynergy_i = 0.5$. For the bidders, $maxBundleSize = 3$.

TABLE 6.1: Value Model Settings

Name	Basis	Parameters and Description
BestResponse	**BestResponse**	Pure best-response bidder.
Powerset10	**Powerset**	Powerset bidder with number of bids per round $x = 10$.
3of10	**Random**	Submits random 3 bids from best 10 bids in each round.
5of20	**Random**	Submits random 5 bids from best 20 bids in each round.
Preselect20	**Preselect**	Chooses 20 most valued bundles before the auction, and bids only on them using the best-response strategy.
Level10	**Level**	Before the auction, lowers the valuation of the best 10 bundles to the level of the $10th$ best. Follows the best-response strategy afterwards.
Forgetful	**BestResponse**	A BestResponse bidder who has a 10% chance of forgetting to submit each bid.
BestChain	**BestChain**	Standard BestChain bidder.
Naïve	**Naïve**	Standard Naïve bidder.

Table 6.2: Bidding Agent Settings

largest positive impact on the auction outcome. This set of rules became the ALPS/ALPSm auction design.

We run experiments using five value models: Airports, Pairwise Synergy High, Pairwise Synergy Low, Transportation Large, and Transportation Small. The RAD+ auction format, which is the RAD auction with the new pricing algorithm and the new termination rule (Sections 4.2.1 and 4.2.9) is used as the basis for all comparisons. The original RAD format was unsuitable, since its performance was low due to premature terminations (Section 6.3.1). We activate each new rule individually and compare the auction performance with the RAD+ basis. For each setting, 70 auctions with Powerset10 bidders were instantiated. The Powerset10 bidders were selected because they have higher efficiency for the linear-price auctions with eligibility rules (Section 6.3.1). Finally, we select the set of rules which has the best effect on the auction outcome, declare them as the ALPS/ALPSm design, and repeat the experiments for this new format. Table 6.4 summarizes all experiment results for this section.

Name	Description
Sealed	VCG auction (Section 2.3.4).
RAD	RAD auction (Section 3.1).
RADne	RAD auction with disabled eligibility rule.
RAD+	RAD auction with ALPS price calculation and termination rules.
CC	Combinatorial Clock auction (Section 3.2).
CCne	Combinatorial Clock auction with disabled eligibility rule.
iBundle(2), iba	iBundle auction (Section 3.3.1) with anonymous prices.
iBundle(3), ibp	iBundle auction (Section 3.3.1) with personalized prices.
dVSV	dVSV auction (Section 3.3.2).
CreditDebit	CreditDebit auction (Section 3.3.3).
ALPS	ALPS auction (Section 4.3).
ALPSne	ALPS auction with disabled eligibility rule.
ALPSm	modified ALPS auction (Section 4.3).
ALPMSmne	modified ALPS auction with disabled eligibility rule.

TABLE 6.3: Auction Format Settings

6.2.1 Old Bids Active

The *old bids active* rule (Section 4.2.4) is clearly the most effective way to increase the auction efficiency out of all improvements which we have tested. We obtain roughly 99% allocative efficiency for each of the five selected value models. We explain this increase in efficiency by higher number of bids available for the winner determination in the late rounds. Another positive impact of this rule is the improved price monotonicity, since more losing bids are available for the ask price balancing.

There is also a small but significant reduction of the bidder's revenue share. The final prices are too high compared to the minimum CE prices. Being aware of this auction property, the real bidders can choose to speculate and never bid up to their valuations, which can negatively affect the auction's efficiency. This phenomena however strongly depends on the risk profile of the bidders and cannot be measured using our software bidding agents.

Value Model / ICA Design		Airports	PairSyn high	PairSyn low	Transp. large	Transp. small
RAD+	Efficiency	95.19%	96.56%	96.67%	94.61%	93.79%
	Rev. Auctioneer	80.44%	81.02%	79.41%	61.04%	58.48%
	Rev. Bidders	14.75%	15.54%	17.26%	33.57%	35.30%
	Rounds	58.49	50.07	51.96	31.39	41.19
	Monoton. Error	0.82	0.36	0.38	0.76	0.70
Old Bids Active	Efficiency	**98.83%**	**98.61%**	**99.30%**	**98.77%**	**98.92%**
	Rev. Auctioneer	**89.32%**	**86.66%**	**87.45%**	**67.52%**	**67.66%**
	Rev. Bidders	**9.51%**	**11.95%**	**11.85%**	**31.25%**	**31.26%**
	Rounds	**55.10**	50.84	**54.03**	**28.06**	41.41
	Monoton. Error	**0.76**	**0.12**	**0.11**	**0.72**	0.69
Last and Final	Efficiency	95.19%	96.17%	96.03%	94.02%	94.00%
	Rev. Auctioneer	**79.57%**	79.99%	78.98%	60.54%	58.95%
	Rev. Bidders	15.62%	16.18%	17.05%	33.48%	35.05%
	Rounds	**54.90**	**48.53**	**50.64**	30.60	40.21
	Monoton. Error	**0.78**	0.33	**0.31**	0.75	0.69
Dynamic Minimum Increment	Efficiency	94.60%	95.44%	96.67%	95.02%	92.76%
	Rev. Auctioneer	**79.06%**	81.56%	**81.99%**	62.48%	57.95%
	Rev. Bidders	15.55%	**13.38%**	**14.68%**	32.54%	34.81%
	Rounds	**46.69**	**17.70**	**16.10**	**17.57**	**24.54**
	Monoton. Error	**0.84**	0.39	0.36	**0.66**	**0.67**
Bundle Minimum Increment	Efficiency	94.45%	97.17%	97.53%	**92.34%**	91.53%
	Rev. Auctioneer	**79.55%**	**78.52%**	80.14%	60.07%	59.90%
	Rev. Bidders	14.90%	**18.65%**	17.39%	32.26%	**31.64%**
	Rounds	**67.46**	**45.73**	**38.13**	**40.30**	42.47
	Monoton. Error	**0.69**	0.40	0.36	**0.69**	**0.63**
Forced Price Monotonicity	Efficiency	**93.40%**	95.66%	96.20%	93.11%	**91.05%**
	Rev. Auctioneer	**77.55%**	**79.82%**	78.99%	60.54%	58.59%
	Rev. Bidders	**15.85%**	15.84%	17.21%	32.57%	**32.46%**
	Rounds	**56.09**	**45.74**	**49.23**	**29.06**	39.24
	Monoton. Error	**0**	**0**	**0**	**0**	**0**
Surplus Eligibility	Efficiency	95.23%	96.13%	97.23%	96.10%	**95.37%**
	Rev. Auctioneer	**81.39%**	**82.53%**	**82.24%**	**63.36%**	**63.09%**
	Rev. Bidders	**13.84%**	**13.59%**	**14.99%**	32.74%	**32.29%**
	Rounds	58.41	49.24	52.13	**29.81**	**38.30**
	Monoton. Error	**0.83**	0.35	**0.34**	**0.77**	0.71
None Eligibility	Efficiency	**96.63%**	97.00%	97.21%	**97.14%**	**95.60%**
	Rev. Auctioneer	**82.73%**	**83.91%**	**82.26%**	**65.99%**	**63.10%**
	Rev. Bidders	13.89%	**13.09%**	**14.95%**	**31.15%**	**32.50%**
	Rounds	58.86	49.36	51.47	**29.64**	**37.77**
	Monoton. Error	0.82	**0.31**	0.35	**0.78**	0.70
ALPS (no Old Bids Active)	Efficiency	95.35%	95.91%	97.45%	**97.34%**	**95.43%**
	Rev. Auctioneer	**82.53%**	**85.83%**	**84.76%**	**69.23%**	**63.83%**
	Rev. Bidders	**12.82%**	**10.08%**	**12.69%**	**28.11%**	**31.61%**
	Rounds	**42.76**	**14.39**	**15.10**	**18.01**	**23.69**
	Monoton. Error	0.82	0.32	**0.32**	**0.71**	**0.68**
ALPSm (with Old Bids Active)	Efficiency	**99.73%**	**99.81%**	**99.64%**	**99.26%**	**99.97%**
	Rev. Auctioneer	**92.77%**	**90.45%**	**90.96%**	**73.39%**	**75.01%**
	Rev. Bidders	**6.97%**	**9.35%**	**8.68%**	**25.89%**	**24.96%**
	Rounds	**38.80**	**12.84**	**14.74**	**15.86**	**28.91**
	Monoton. Error	**0.70**	**0.07**	**0.08**	**0.62**	**0.63**

TABLE 6.4: Performance of the New Auction Rules. Bold Text Indicates Significant Difference (Paired T-Test) Compared to RAD+

6.2.2 Last-and-Final Bids

This rule (Section 4.2.5) has only a negligible effect on most auction parameters. It can still be used to raise the perception of fairness which the bidders have towards the auction mechanism. On the other hand, it is questionable whether the rule is not too complex for the bidders to understand and use.

6.2.3 Dynamic Minimum Increment

The *dynamic minimum increment* (Section 4.2.2) is a very good instrument for reducing the auction duration. It is obvious that the increased minimum increment, other things being equal, will reduce the auction efficiency. Our question is whether the auction duration can be reduced using the dynamic minimum increment without significantly sacrificing the efficiency.

In Figure 6.1, we plot the allocative efficiency against the number of auction rounds for 40 samples of the Pairwise Synergy High value model, which compare RAD+ with a static increment of 5 and RAD+ with a dynamic minimum increment with minimal increment $\Delta_{min} = 1$, maximal increment $\Delta_{max} = 40$, curvature $c = 2$ and minimal competition level for Δ_{max}, $x = 1$ (all bidders). Table 6.5 shows the mean efficiency values and the number of rounds over all 40 auctions. Note that Table 6.4 presents the results for another set of samples, in particular that the static minimum increment used there is 1.

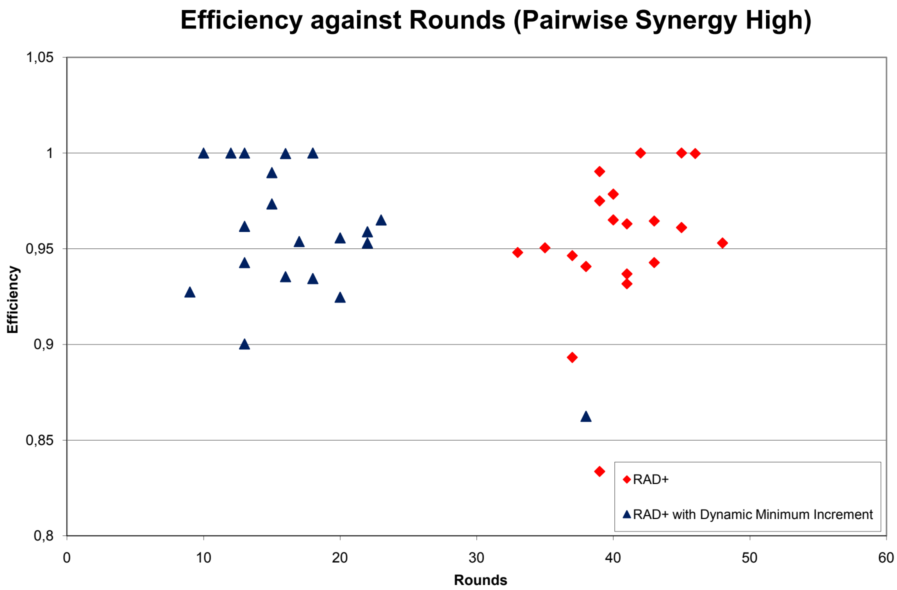


FIGURE 6.1: Effect of the Dynamic Minimum Increment

The experiment demonstrates that the number of auction rounds decreases considerably when we use the dynamic increment without the allocative efficiency being reduced. Similar results were obtained for the other value models.

	Efficiency	Number of Rounds
Static Minimum Increment	94.21%	39.7
Dynamic Minimum Increment	95.44%	17.7

TABLE 6.5: RAD+ with Static vs. Dynamic Minimum Increment

We can state that the dynamic minimum increment with properly selected parameters (which are not always trivial) can be an attractive option in practical applications.

6.2.4 Bundle Minimum Increment

The *bundle minimum increment* (Section 4.2.3) does not bring any improvements to the auction outcome. Our measurements do not show any trend across all value models in any parameter. Furthermore, the Bundle Minimum Increment rule has some undesired side effects.

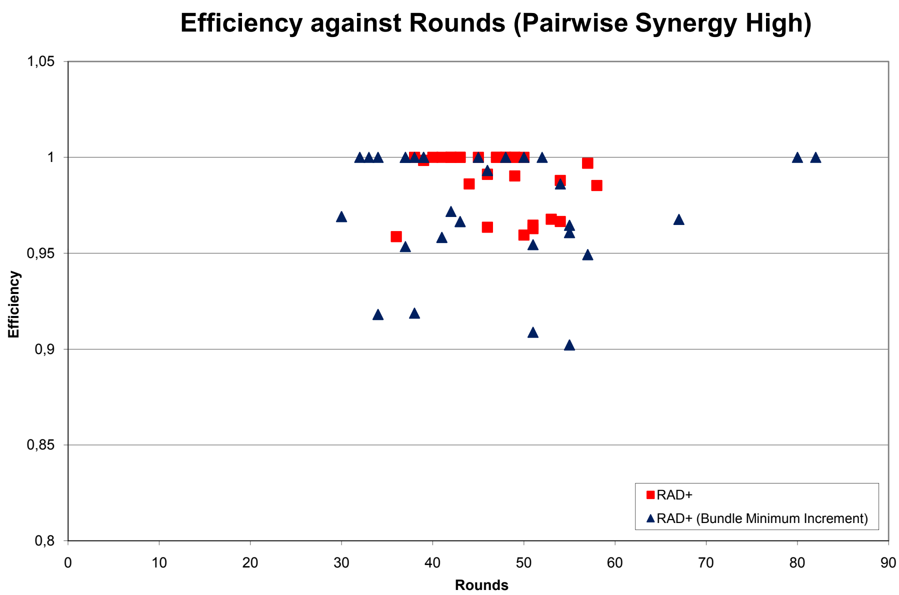


FIGURE 6.2: Effect of the Bundle Minimum Increment

We have monitored a significantly higher deviation between the samples for all measurements for RAD+ with the bundle minimum increment compared to the original RAD+ or ALPS auction formats. Figure 6.2 plots the efficiency against the auction duration for 30 auctions using the Pairwise Synergy High model for RAD+ and RAD+ with the *bundle minimum increment*. The number of rounds for RAD+ with the *bundle minimum increment* rule is dispersed between 30 and 82, whereby RAD+ stays in the range of 36 to 58 for the same set of samples. Also the auction efficiency is more scattered for RAD+ with the *bundle minimum increment* rule.

6.2.5 Forced Price Monotonicity

This rule (Section 4.2.6) makes prices fully monotonic. Another positive effect of the rule is a slight reduction of auction duration. However, the auction efficiency is reduced, in some cases significantly (Figure 6.3). This happens primarily because the ask prices changed by this rule do not provide a quality feedback for the bidders any more.

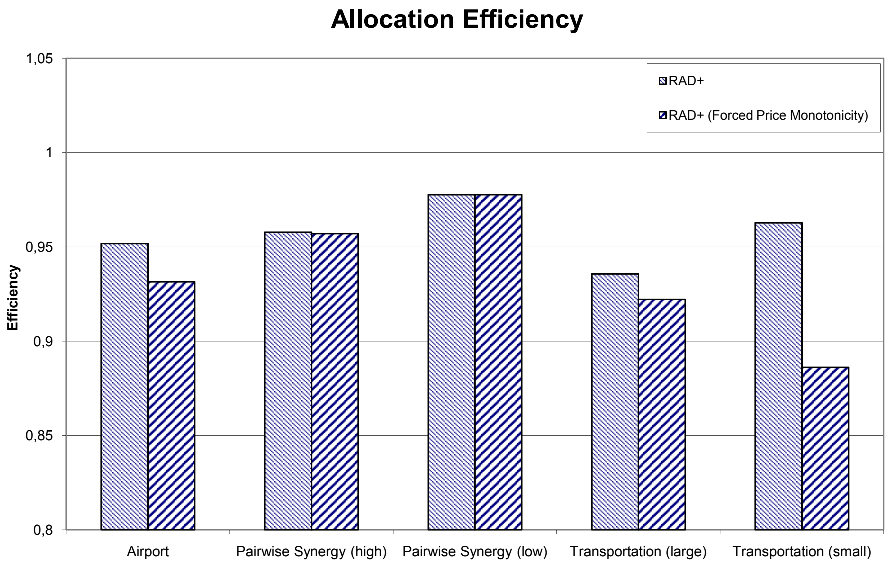


FIGURE 6.3: Effect of the Forced Price Monotonicity

6.2.6 Surplus Eligibility

The *surplus eligibility* rule (Section 4.2.8) relaxes limitations on the amount of different items the bidder can bid on, and consequently improves the efficiency. We have also included the *no-eligibility* option in our experiments, which allows for bids on any number of items independently of the bid history. This rule is not practical since the real bidders will misuse it and will not bid actively from the first auction rounds. Our software bidders however do bid competitively during the whole auction. The *no-eligibility* option gives the upper bound to the efficiency gain due to a relaxed eligibility rule.

We test the surplus eligibility rule with a maximum surplus eligibility $e_{+,max} = 5$. The results show an increased efficiency, significant in some cases. The largest share of the gained efficiency goes to the auctioneer; the bidders' revenue is even decreased in some cases. However, there is still a potential upwards for the *surplus-eligibility* rule, or some other replacement for the activity rule, of up to roughly 1%, to reach the *no-eligibility* level.

6.2.7 Combining New Auction Rules

The ALPSm auction format comprises the optimal set of rules which has the largest positive effect on the auction outcome. The following rules are included:

- The *new price calculation* algorithm avoids several pitfalls of the RAD price calculation by deriving lower and better balanced prices.
- The *new termination rule* stops the auction only when there are no new bids submitted within one round. This prevents premature terminations of the auction.
- The *old bids active* rule significantly boosts the auction efficiency. The bidders always have to outbid their own old bids on the same bundle which guarantees the auction progress and termination.
- The *last-and-final bids* rule has only a minor positive effect on the efficiency, but can increase bidders' fairness perception towards the auction.
- The *dynamic minimum increment* drastically reduces the number of auction rounds without sacrificing the efficiency.

- The *surplus eligibility* increases the auction efficiency.

The *forced price monotonicity* and the *bundle minimum increment* rules were discarded due to their negative impact on the auction efficiency.

The *old bids active* rule increases the size of the CAP problem which must be solved by the auction engine. This can be a serious practical issue, since the problem is NP-hard. Therefore, we suggest a second set of rules, where we omit the *old bids active* rule. We call this second setup the ALPS format. To ensure the auction progress and termination in this case, we increase the minimum increment in case the allocation does not change for two consecutive rounds. This forces the losing bidders to submit higher bids. After the allocation changes, the original minimum increment value is restored.

Already the ALPS format improves the auction results along many parameters significantly. The ALPSm setup performs even better and reaches 99% efficiency on average, over all value models. The auctioneer profits significantly from this efficiency increase; the bidders profit less. Both settings achieve a significant reduction of the auction duration and a better price monotonicity.

6.3 Comparing Linear-Price ICAs

In this section, we compare three linear-price auction designs: the Combinatorial Clock (CC) (Section 3.2), the Resource Allocation Design (RAD) (Section 3.1), and the Approximate Linear PriceS (ALPS/ALPSm) (Section 4.3). We also include in the comparison the RADne design, which is the RAD auction with disabled activity rules, in order to isolate the possible negative effect of activity rules on the allocative efficiency.

First we compare these linear-price ICA designs in idealized conditions and locate the performance problems that can be attributed to the auction rules and not to the bidding strategies. Therefore, we use only the theoretically optimal BestResponse bidder and another agent with high rationality, the Powerset10 bidder. In the linear-price auctions with eligibility-based activity rules, like those used by ALPS, RAD and CC, the bidders often cannot use the best-response strategy, because they must maintain their eligibility. Therefore, the bidders are likely to bid more than just the demand set in the first rounds, as the Powerset10 agents do.

Value Model \ ICA Format		ALPS	ALPSm	CC	RAD	RADne	VCG
Real Estate 3x3	Efficiency in %	96.5	98.81	97.13	69.9	71.21	100
16 auctions BAS	Rev. Auctioneer in %	67.75	82.5	86.56	10.11	10.37	84.2
	Rev. Bidders in %	28.75	16.31	10.57	59.79	60.84	15.8
	Rounds	532.98	760.83	400	46.95	47.15	1
Real Estate 4x4	Efficiency in %	96.84	99.82	96.24	76.13	76.09	100
1 auction BAS	Rev. Auctioneer in %	75.51	90.72	90.56	9.16	9.75	90.3
	Rev. Bidders in %	21.34	9.1	5.69	66.97	66.34	9.7
	Rounds	440.73	641.7	247.7	28.95	30.65	1
Pairwise Synergy Low	Efficiency in %	94.82	99.73	98.56	69.98	69.17	100
20 auctions BAS	Rev. Auctioneer in %	72.41	87.53	88.29	8.84	8.63	87.08
	Rev. Bidders in %	22.42	12.19	10.27	61.14	60.54	12.92
	Rounds	369.3	816	412.82	44.42	44.4	1
Pairwise Synergy High	Efficiency in %	92.8	99.64	99.87	72.66	71.99	100
15 auctions BAS	Rev. Auctioneer in %	76.28	87.97	89.18	9.82	9.6	87.5
	Rev. Bidders in %	16.52	11.68	10.69	62.84	62.4	12.5
	Rounds	354.65	656.38	338.48	41.8	41.67	1
Airports	Efficiency in %	97.27	99.81	97.95	90.09	90.56	100
0 auctions BAS	Rev. Auctioneer in %	52.01	53.81	67.9	28.26	30.45	42.33
	Rev. Bidders in %	45.26	46.01	30.04	61.83	60.11	57.67
	Rounds	671.55	186.47	93.47	23.3	27.5	1
Transportation Large	Efficiency in %	93.97	99.52	96.78	82.48	83.73	100
0 auctions BAS	Rev. Auctioneer in %	62.33	76.61	80.92	38.97	34.9	64.21
	Rev. Bidders in %	31.65	22.91	15.86	43.5	48.83	35.79
	Rounds	193.4	161.8	180.05	31.38	28.3	1
Transportation Small	Efficiency in %	98.26	99.78	97.73	82.98	81.31	100
0 auctions BAS	Rev. Auctioneer in %	54.79	59.54	65	21.96	17.93	48.32
	Rev. Bidders in %	43.48	40.23	32.74	61.02	63.38	51.68
	Rounds	409.32	327	314.62	66.17	51.1	1

TABLE 6.6: Revenue Distribution in Linear-Price ICAs with BestResponse Bidders

To avoid inefficiencies due to high bid increments, we set the increment to 0.1 for all auctions described in this section. Therefore, the average number of auction rounds is high in general. A minimum bid increment of 1 reduces the number of auction rounds by a factor of 10.

For the seven selected value models we create 40 auction instances with different valuations, and run them in all five auction formats, preserving the bidder valuations. Table 6.6 presents the average values of allocative efficiency, auctioneer's and bidders' revenue, and number of auction rounds, for every combination of settings, using BestResponse bidders. Table 6.7 contains the same information for Powerset10 bidders. The left-hand column indicates the value model and the number of auctions where the valuations fulfill the BAS property. As can be seen, in most cases BAS was not fulfilled. The results have the same pattern over all auction formats and value models within each of the two selected bidding strategies, but significantly differ between the bidding strategies. Powerset10 bidders bid more than just their demand sets, thus supplying the auctioneer with more information about their valuations

Value Model \ ICA Format		ALPS	ALPSm	CC	RAD	RADne	VCG
Real Estate 3x3	Efficiency in %	98.63	99.92	97.38	83.28	95.83	100
16 auctions BAS	Rev. Auctioneer in %	78.83	85.97	94.87	9.24	72.99	84.2
	Rev. Bidders in %	19.8	13.95	2.51	74.04	22.84	15.8
	Rounds	313.55	292.18	272.55	30.75	178.43	1
Real Estate 4x4	Efficiency in %	98.52	99.78	97.94	78.16	95.96	100
1 auction BAS	Rev. Auctioneer in %	87.07	92.52	95.98	10.86	77.4	90.3
	Rev. Bidders in %	11.45	7.26	1.97	67.3	18.56	9.7
	Rounds	219.75	181.8	184.05	26.15	120.85	1
Pairwise Synergy Low	Efficiency in %	98.7	99.9	99.02	83.57	94.99	100
20 auctions BAS	Rev. Auctioneer in %	80.67	89.66	96.98	20.27	39.78	87.08
	Rev. Bidders in %	18.03	10.24	2.04	63.3	55.21	12.92
	Rounds	346.52	354.93	352.3	68.58	127.2	1
Pairwise Synergy High	Efficiency in %	99.15	99.86	99.25	85.29	94.58	100
15 auctions BAS	Rev. Auctioneer in %	84.76	89.71	97.25	28.12	50.14	87.5
	Rev. Bidders in %	14.39	10.15	2.01	57.17	44.44	12.5
	Rounds	309.45	319.9	326.02	87.8	150.5	1
Airports	Efficiency in %	98.18	98.16	96.98	91.12	97	100
0 auctions BAS	Rev. Auctioneer in %	56.34	60.25	90.66	35.89	55.63	42.33
	Rev. Bidders in %	41.85	37.92	6.31	55.23	41.37	57.67
	Rounds	129.68	58.23	73.17	20.7	33.85	1
Transportation Large	Efficiency in %	95.93	99.48	98.29	83.54	89.39	100
0 auctions BAS	Rev. Auctioneer in %	67.07	78.42	90.14	41.14	57.27	64.21
	Rev. Bidders in %	28.86	21.06	8.15	42.4	32.13	35.79
	Rounds	87.1	85.65	139.47	25.82	23.9	1
Transportation Small	Efficiency in %	97.98	99.56	96.8	80.49	86.82	100
0 auctions BAS	Rev. Auctioneer in %	57.74	63.01	83.74	20.92	34.78	48.32
	Rev. Bidders in %	40.24	36.56	13.06	59.56	52.05	51.68
	Rounds	167.7	188.4	257.57	52.08	36.98	1

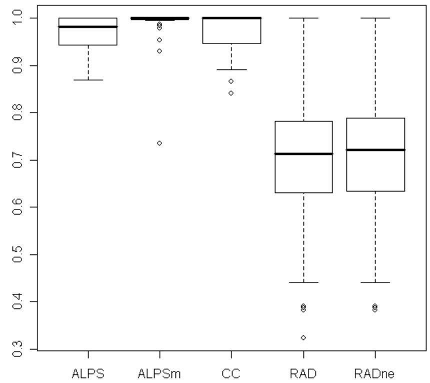
TABLE 6.7: Revenue Distribution in Linear-Price ICAs with Powerset10 Bidders

and generating more competition on the market.

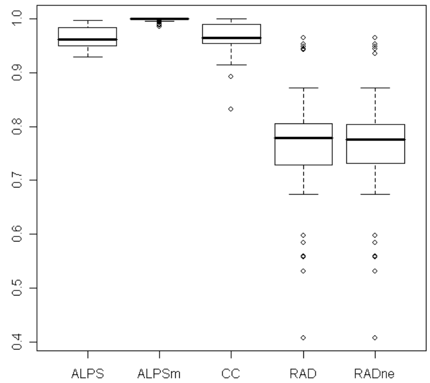
6.3.1 Efficiency

Overall, the efficiency in all value models using BestResponse bidders is very high and has the same pattern. The efficiency of auctions with Powerset10 bidders is even higher. Especially the RAD/RADne auction performs better with Powerset10 bidders. As can be seen from the increased number of auction rounds in this case, Powerset10 bidders reduce the chance of a premature auction termination, which is often a problem with the RAD design.

The ALPSm design has the highest average efficiency due to the *old bids active* rule. In the Pairwise Synergy High value model, there is no significant difference between the efficiency values of CC and ALPSm formats (t-test, p-value=0.79). With BestResponse bidders, the RAD design suffers from premature terminations. Also, omitting the eligibility rule (RADne) does not bring a significant improvement. With BestResponse bidders, the CC auction

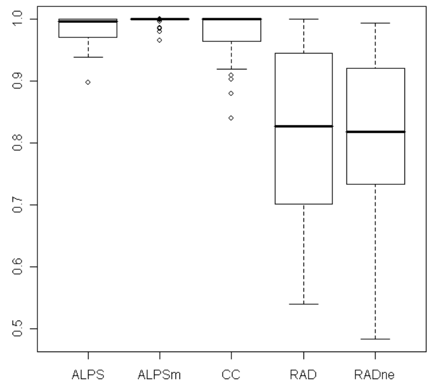


(a) Real Estate 3x3

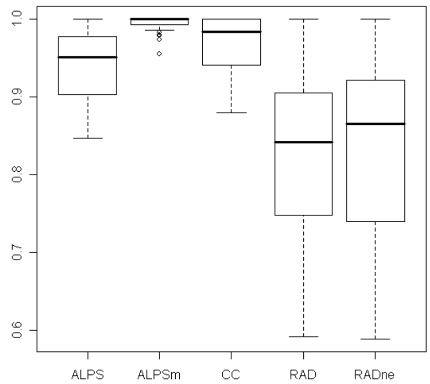


(b) Real Estate 4x4

FIGURE 6.4: Allocative Efficiency with BestResponse Bidders in Real Estate Value Models

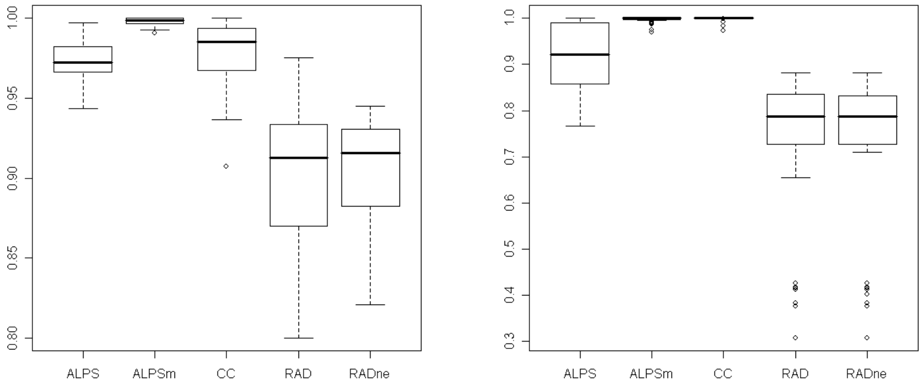


(a) Transportation Small



(b) Transportation Large

FIGURE 6.5: Allocative Efficiency with BestResponse Bidders in Transportation Value Models



(a) Airports (b) Pairwise Synergy High

FIGURE 6.6: Allocative Efficiency with BestResponse Bidders in Pairwise Synergy and Airports Value Models

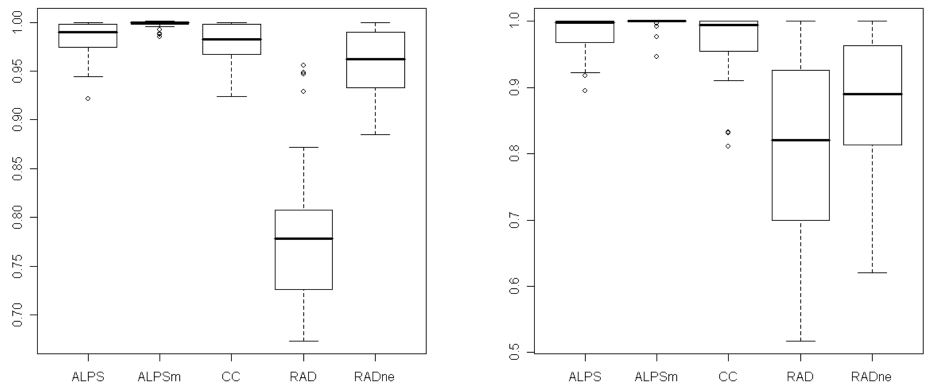
has a better efficiency than ALPS in all but two value models (Real Estate 4x4, Transportation Small). With Powerset10 bidders, they have similar efficiency.

Figures 6.4 to 6.6 show selected box plots for the efficiency of auctions with BestResponse bidders. We found a similar pattern for experiments with Powerset10 bidders (see Figure 6.7).

In a separate setup we analyze how the auction efficiency changes with increasing levels of synergy between items. We use the Pairwise Synergy value model with synergy levels increasing from 0 to 3 in 0.1 steps. The results are presented in Figure 6.8. Interestingly, the auction efficiency remains high for ALPS/ALPSm and CC auctions even in the case of high synergy values. Note that with a synergy value of 2.5 a bundle of items already has 3.5 times the value of the sum of its individual items.

6.3.2 Auction Duration

In auctions with BestResponse bidders, ALPSm has the highest number of auction rounds, except for Airports and Transportation value models. RAD often terminates prematurely, leading to a lower average number of auction rounds, but at the cost of much lower efficiency.



(a) Real Estate 4x4 (b) Transportation Small

FIGURE 6.7: Allocative Efficiency with Powerset10 Bidders in Real Estate and Transportation Value Models

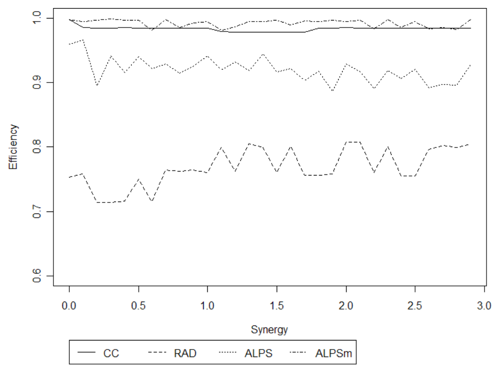
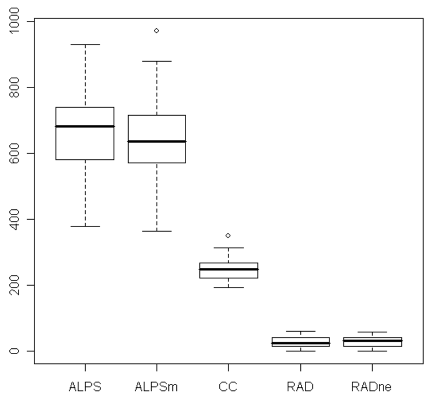
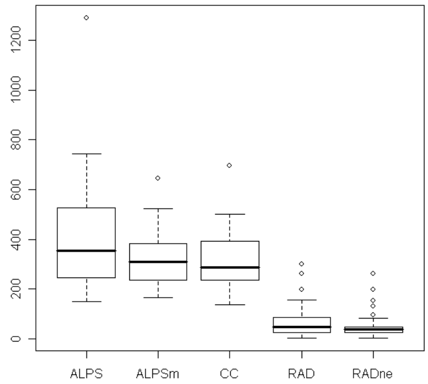


FIGURE 6.8: Dependency of the Allocative Efficiency from the Complementarity Level

Powerset10 bidders reduce the number of rounds significantly, except for the RAD auction, where the number of rounds even increases in some cases. This happens because Powerset10 bidders can somewhat mitigate premature terminations of RAD auctions. ALPSm has the highest reduction of rounds with Powerset10 bidders due to the *old bids active* rule, which helps to accumulate the information about bidders' valuations more quickly.



(a) Real Estate 4x4



(b) Transportation 25 Edges

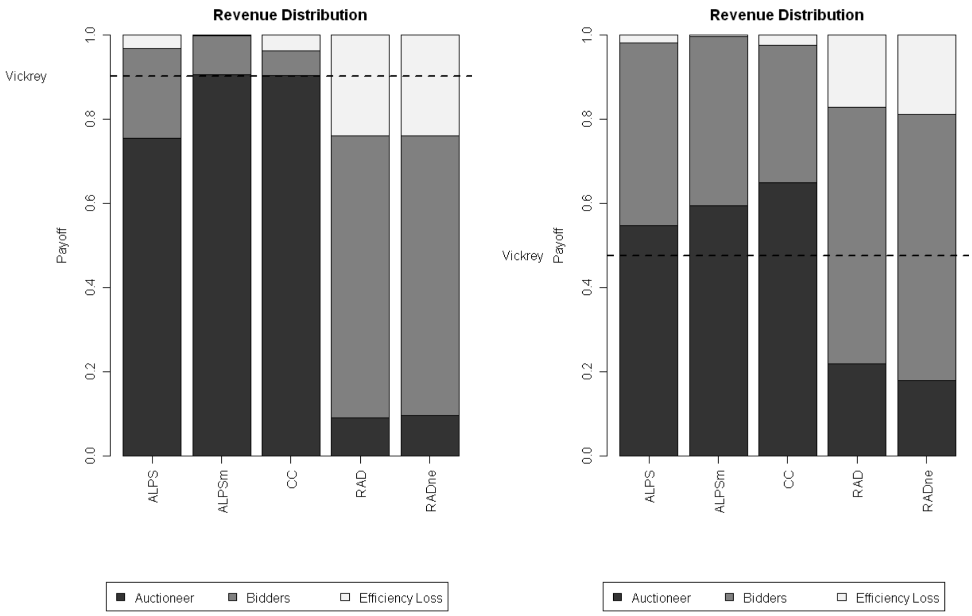
FIGURE 6.9: Average Number of Auction Rounds with BestResponse Bidders

6.3.3 Revenue Distribution

Our computational experiments indicate significant differences in revenue distribution between different auction designs. Again, we found similar patterns across different value models (Figure 6.10). An important observation is that the CC design has the highest average auctioneer revenue, followed by ALPSm. The dashed line in Figure 6.10 shows the average revenue distribution in the VCG auction. The level of VCG prices can serve as an indicator for competition in the auction, which was generally high.

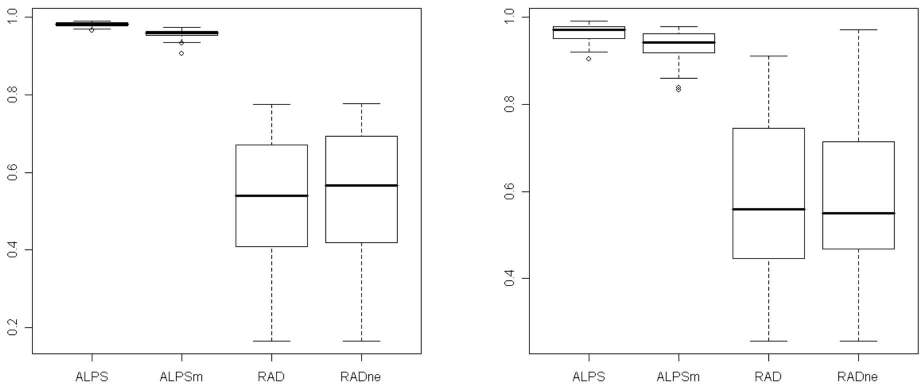
The bidders get less revenue in auctions with Powerset10 strategies than with BestResponse strategies. The strongest decline happens in CC auctions, followed by ALPS and ALPSm, which demonstrate only a moderate decrease of the bidders' revenue.

We run also experiments with little competition (for example, the Pairwise Synergy Low model with only 3 bidders), and found the final ALPS ask prices to be higher than the VCG prices, compared to auction instances with higher competition (Real Estate 3x3 with 5 or 7 bidders).



(a) Real Estate 4x4 (b) Transportation 25 Edges

FIGURE 6.10: Revenue Distribution with BestResponse Bidders



(a) Real Estate 4x4 (b) Transportation 25 Edges

FIGURE 6.11: Price Monotonicity in Linear-Price ICAs

6.3.4 Price Monotonicity

Figure 6.11 provides a box plot for price monotonicity values (Section 2.5.2) of ALPS, ALPSm, RAD, and RADne in the Real Estate 3x3 and the Transportation Small value models with BestResponse bidders. Lower price monotonicity in ALPS auctions can be attributed to the fact that they take more rounds than RAD and do not terminate prematurely.

6.4 Bidding Strategies in Linear-Price ICAs

In this section we analyze the impact of different bidding strategies on linear-price ICAs. As discussed in Section 2.4.2, the assumptions behind BestResponse or Powerset10 bidders are not always applicable. Due to the $2^{|\mathcal{K}|} - 1$ packages a bidder must decide on, it can be impractical for bidders to consider or even simply know valuations for the full range of packages that he can bid for. Therefore, the real-world bidders can use different types of bundling strategies. In order to achieve reliable and more general results, our analysis is based on a broad range of agents implementing various strategies, and different value models.

We use the ALPS auction format since it has very good allocative efficiency, and at the same time distributes the revenue between the auctioneer and the bidders fairer than the ALPSm format. We focus on bundle selection and assume that bidders bid the minimum price only, neglecting jump bids or similar phenomena. For each setting we create 40 auction instances with different random seeds.

All bidders use the fully expressive XOR bidding language with the notable exception of the Naïve bidder, who uses the OR language while bidding only on individual items. We analyze auctions with Real Estate 3x3 and Pairwise Synergy High value models, where a Naïve bidder can participate, but do not consider Airports and Transportation value models for this reason.

6.4.1 Auctions with Uniform Bidding Strategies

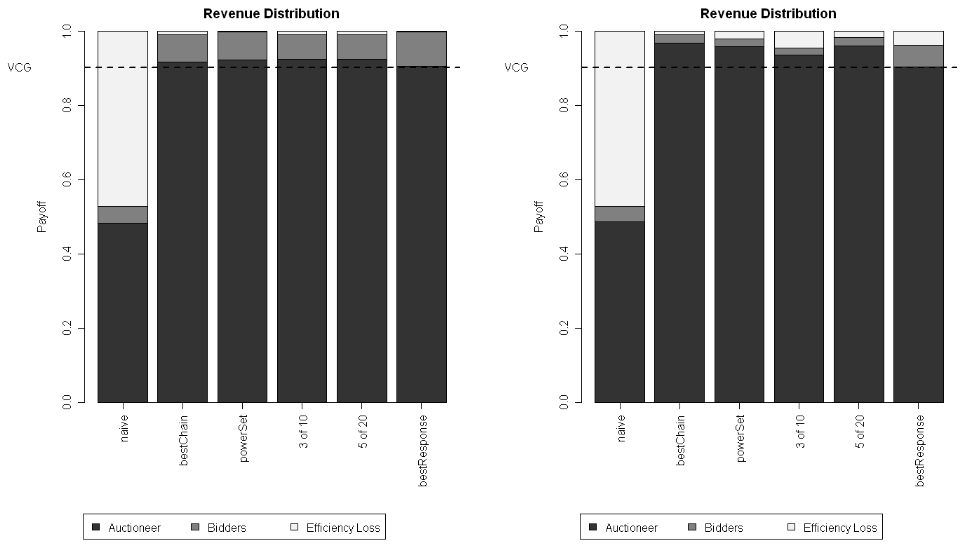
First we consider auctions where all bidders follow the same bidding strategy. Table 6.8 shows average values over 40 auctions for each setting and measurement.

Setup	Bidder Type	Naïve	BestChain	Powerset10	3of10	5of20	BestResponse
Real Estate 3x3	Efficiency in %	54.84	96.31	98.63	96.95	95.95	96.18
	Rev. Auctioneer in %	47.97	74.12	78.83	78.72	81	67.8
	Rev. Bidders in %	6.86	22.19	19.8	18.22	14.96	28.38
	Rounds	198.95	471	364.5	403.25	369.95	532.98
Real Estate 4x4	Efficiency in %	52.86	97.96	98.19	96.56	96.73	96.68
	Rev. Auctioneer in %	48.43	84.61	86.65	85.03	87.29	75.56
	Rev. Bidders in %	4.43	13.35	11.54	11.53	9.44	21.13
	Rounds	108.55	230.43	247.5	367.23	289.7	671.95
Pairwise Synergy Low	Efficiency in %	77.21	96.25	98.09	96.99	97.7	95.64
	Rev. Auctioneer in %	66.63	75.68	81.83	81.56	85.3	74.07
	Rev. Bidders in %	10.59	20.57	16.25	15.43	12.4	21.57
	Rounds	259.65	461.2	369.88	395.45	382.88	541.77
Pairwise Synergy High	Efficiency in %	36.53	96.61	98.61	96.55	97.98	93.6
	Rev. Auctioneer in %	31.53	78.62	83.25	82.19	85.91	76.47
	Rev. Bidders in %	5	17.99	15.36	14.36	12.06	17.14
	Rounds	116.35	380.32	335.8	351	342.05	466.18

TABLE 6.8: Uniform Bidding Strategies in ALPS ICA

The Naïve bidder only bids up to his item valuations and ignores synergetic valuations, which leads to the lowest efficiency, auctioneer and bidder revenue scores. The Powerset10 bidder comes out best in terms of efficiency and auctioneer revenue.

BestChain and Random (3of10, 5of20) bidders also achieve high levels of efficiency, since they are able to find fairly good bundles. Efficiency of auctions with these bidders is comparable; for example there is no significant difference



(a) Real Estate 4x4, ALPSm (b) Real Estate 4x4, CC Auction

FIGURE 6.12: Revenue Distribution for Uniform Bidding Strategies

between BestChain and Random 3of10 bidder in Real Estate 3x3 value model (t-test, p-value of 0.65).

From the bidder perspective, best-response bidding is the optimal strategy, which ensures that he gets only the most profitable bundles. The auctions with BestResponse bidders have, compared to all other auctions, significantly lower auctioneer revenue, except with Naïve bidders. We find the same pattern in ALPSm and CC auctions, and in all value models. Figure 6.12 illustrates the revenue distributions for Real Estate 4x4 value model in ALPSm and CC auctions.

The auctioneer revenue share in CC auctions is significantly higher than the VCG level (dashed line), which can motivate bidders to shade their bids in the real-life applications. ALPS auction distributes the revenue between the auctioneer and the bidders fairly.

6.4.2 Auctions with Different Bidding Strategies

Now we look at auctions where bidders with different strategies participate simultaneously. Every auction contains nine (for Real Estate 4x4 value model) or four (for all other value models) “base” bidders, and a last bidder with a different bundle selection strategy. For this last bidder, the average revenue over 40 ALPS auctions is calculated. For BestResponse base bidder, the results are shown in Table 6.9.

Setup \ Last Bidder Type		BestResponse	Powerset10	3of10	5of20	BestChain	Naïve
Real Estate 3x3	Efficiency in %	96.18	96.65	96.33	96.52	96.26	94.96
	Rev. Auctioneer in %	67.88	70.99	71.67	67.82	69.41	61.87
4 BestResponse	Rev. Bidders in %	28.30	25.67	24.66	28.7	26.84	33.09
plus one bidder	Rev. Last Bidder in %	3.785	4.844	4.708	6.041	5.392	0.5227
Real Estate 4x4	Efficiency in %	96.24	97.13	96.58	96.29	96.95	96.08
	Rev. Auctioneer in %	74.39	75.77	74.54	76.38	75.41	71.83
9 BestResponse	Rev. Bidders in %	21.85	21.36	22.05	19.91	21.54	24.25
plus one bidder	Rev. Last Bidder in %	1.314	2.558	2.562	2.187	2.269	0.2110
Pairwise	Efficiency in %	95.35	97.78	96.98	97.09	96.96	92.86
Synergy Low	Rev. Auctioneer in %	71.87	73.91	76.81	73.83	73.94	69.35
4 BestResponse	Rev. Bidders in %	23.48	23.87	20.17	23.26	23.02	23.51
plus one bidder	Rev. Last Bidder in %	4.826	7.928	5.679	6.908	5.487	1.533
Pairwise	Efficiency in %	92.04	94.17	92.9	93.88	93.98	86.37
Synergy High	Rev. Auctioneer in %	73.33	76.37	77.15	76.24	74.74	65.05
4 BestResponse	Rev. Bidders in %	18.71	17.8	15.75	17.64	19.24	21.32
plus one bidder	Rev. Last Bidder in %	3.191	5.076	4.569	5.577	5.487	0

Table 6.9: Sensitivity with BestResponse Base Bidders

Overall, the efficiency does not suffer significantly, since the majority of BestResponse bidders can keep it high. The Naïve bidding strategy came out worst. Interestingly, either the Powerset10, BestChain or Random strategies always perform better than the BestResponse strategy. A possible reason for this is the eligibility rules, which might prevent a BestResponse bidder from submitting optimal bundles at the end of the auction.

Setup \ Last Bidder Type		BestResponse	Powerset10	3of10	5of20	BestChain	Naïve
Real Estate 3x3	Efficiency in %	98.13	98.9	98.34	98.2	98.08	96.05
	Rev. Auctioneer in %	79.34	79.67	79.87	80.14	79.67	66.76
4 Powerset10	Rev. Bidders in %	18.79	19.23	18.47	18.06	18.41	29.29
plus one bidder	Rev. Last Bidder in %	1.522	3.237	3.263	2.809	2.341	0.05379
Real Estate 4x4	Efficiency in %	98.68	98.83	98.53	98.4	98.6	97.4
	Rev. Auctioneer in %	85.67	86.88	86.57	86.53	86.76	85
9 Powerset10	Rev. Bidders in %	13.01	11.95	11.95	11.88	11.83	12.40
plus one bidder	Rev. Last Bidder in %	0.4362	0.8017	0.6333	1.009	1.123	0.002506
Pairwise	Efficiency in %	98.42	99.6	98.3	98.84	99.25	96.33
Synergy Low	Rev. Auctioneer in %	80.57	83.78	83.85	84.06	84.62	78.41
4 Powerset10	Rev. Bidders in %	17.84	15.81	14.45	14.79	14.63	17.92
plus one bidder	Rev. Last Bidder in %	2.502	4.171	4.104	3.899	3.883	0.2604
Pairwise	Efficiency in %	98.17	99.06	98.55	99.01	98.36	95.88
Synergy High	Rev. Auctioneer in %	82.2	86.41	86.25	86.47	85.56	74.6
4 Powerset10	Rev. Bidders in %	15.97	12.65	12.29	12.54	12.80	21.27
plus one bidder	Rev. Last Bidder in %	1.949	3.336	2.876	3.106	2.757	0

Table 6.10: Sensitivity with Powerset10 Base Bidders

The same type of sensitivity analysis was repeated with respect to Powerset10 bidders. Table 6.10 summarizes the results. We can see that the results are similar to the setup where BestResponse bidders are taken for the basis. Again, it is advantageous for a bidder to use a non-best-response bundling strategy.

Finally we consider the setup where a single bundle bidder competes with nine (or respectively four) Naïve bidders. The results are shown in Table 6.11. The efficiency of these auctions decreases significantly compared to previous setups. This happens because the single bundle bidder can in most cases win his preferred bundle, independently of valuations of other Naïve bidders.

Setup \ Last Bidder Type		BestResponse	Powerset10	3of10	5of20	BestChain	Naïve
Real Estate 3x3	Efficiency in %	69.96	69.95	69.78	69.46	69.18	54.84
	Rev. Auctioneer in %	48.19	48.68	48.68	48.89	48.48	47.97
4 Naïve	Rev. Bidders in %	21.77	21.27	21.1	20.57	20.71	6.863
plus one bidder	Rev. Last Bidder in %	17.12	16.92	16.81	16.18	16.26	1.199
Real Estate 4x4	Efficiency in %	62.07	61.99	61.76	61.74	61.66	52.86
	Rev. Auctioneer in %	48.6	48.72	48.93	48.85	48.87	48.43
9 Naïve	Rev. Bidders in %	13.47	13.27	12.83	12.89	12.78	4.431
plus one bidder	Rev. Last Bidder in %	9.939	9.809	9.471	9.452	9.295	0.4877
Pairwise	Efficiency in %	85.13	85.1	85.08	85.08	84.66	77.15
Synergy Low	Rev. Auctioneer in %	67.63	68.34	68.46	68.46	68.28	67.62
4 Naïve	Rev. Bidders in %	17.5	16.76	16.61	16.62	16.38	9.535
plus one bidder	Rev. Last Bidder in %	10.96	10.55	10.57	10.61	9.984	1.827
Pairwise	Efficiency in %	61.96	61.97	61.97	61.87	60.5	36.50
Synergy High	Rev. Auctioneer in %	31.76	32.32	32.28	32.4	32.32	32.01
4 Naïve	Rev. Bidders in %	30.19	29.66	29.69	29.47	28.18	4.49
plus one bidder	Rev. Last Bidder in %	27.01	26.72	26.74	26.59	25.37	0.8595

TABLE 6.11: Sensitivity with Naïve Base Bidders

The BestResponse bidder receives higher revenue share than other bundle bidders on the background of Naïve bidders. This can be attributed to the fact that bidding more than just the demand set, like Powerset10, BestChain and Random bidders do, does not guarantee the winning of the most profitable bundle, and also can drive up prices higher than necessary.

In summary, from the perspective of a bidder who is interested in maximizing his own revenue, it is favorable to use bundle bidding. If all other bidders in the auction use the BestResponse or Powerset strategies, the bidder is better off using Powerset strategy. In contrast, if all other bidders bid naïvely, BestResponse strategy is slightly better than the Powerset strategy. Overall, the more bidders that use bundle bids, the better it is for the auctioneer.

An interesting result is that the auctions with Powerset10 bidders have the highest efficiency, while the best-response strategy is optimal from the bidder perspective. This happens because BestResponse bidders, unlike Powerset10 bidders, always receive bundles with the highest valuations. At the same time, Powerset10 bidders are less likely to be limited by eligibility rules, which results in better efficiency.

6.5 Non-Linear Price ICAs

In this section we concentrate on ICAs with non-linear prices: iBundle(2), iBundle(3), dVSV, and CreditDebit. Their remarkable property is that they, with the exception of the iBundle(2) format, can guarantee a certain level of allocative efficiency, under assumption of best-response bidding. We test the robustness of these mechanisms when this assumption is not given, and benchmark them against two ICAs with linear prices, ALPSm and CC.

Unless explicitly stated otherwise, each auction setup was repeated 50 times with different random seeds for value models and, where appropriate, bidding agents.

de Vries et al. (2007) show that if all valuations and prices are kept integral, and a minimum increment of 1 is used in the dVSV auction format, the demand set of every bidder weakly increases after a price adjustment. This also holds for iBundle(3), since it also increases the prices by a fixed increment for a set of bidders, for the bundles which correspond to the last round bids. Together with the termination rule based on complementary slackness, this guarantees that these auctions always terminate with a precisely efficient solution, given best-response bidding. To ensure comparability between auction formats, we use a fixed minimum increment of 1 and integer valuations for all auction formats.

6.5.1 Auctions with Uniform Bidding Strategies

First we assume that all bidders in the auction use the same strategy and analyze the auction results for different value models and bidding strategies. In particular, we want to investigate how the different non-linear price auctions behave when the bidders deviate from the best-response strategy.

The average performance metrics are summarized in Table 6.12 for Real Estate 3x3, in Table 6.13 for Pairwise Synergy High, and in Table 6.14 for Transportation Small value models. The values for efficiency, auctioneer and bidder revenue are given in percent. The Appendix A provides an intuitive graphical representation of the results.

Bidding Agent \ ICA Format		CreditDebit	dVSV	iBundle(3)	iBundle(2)	ALPSm	Clock	VCG
BestResponse	Efficiency	100.00	100.00	100.00	99.90	98.64	95.16	100.00
	Rev. Auctioneer	83.45	84.57	84.61	83.88	84.09	87.05	83.17
	Rev. Bidders	16.55	15.43	15.39	16.01	14.55	8.10	16.83
	Rounds	139.96	137.90	151.28	149.26	72.56	28.76	1.00
Forgetful	Efficiency	99.92	99.78	100.00	99.92	98.64	96.35	
	Rev. Auctioneer	81.73	85.28	84.73	83.88	84.11	88.02	
	Rev. Bidders	18.19	14.51	15.27	16.03	14.54	8.30	
	Rounds	550.54	545.30	329.68	237.20	72.26	29.52	
Level10	Efficiency	90.00	90.05	89.71	90.12	91.12	86.67	
	Rev. Auctioneer	72.10	72.29	72.34	71.55	72.25	76.46	
	Rev. Bidders	18.12	17.97	17.58	18.72	19.01	10.30	
	Rounds	82.38	82.00	133.22	130.96	128.88	26.06	
Powerset10	Efficiency	90.50	89.67	98.48	99.20	99.57	97.27	
	Rev. Auctioneer	23.10	71.53	82.93	82.14	87.65	94.09	
	Rev. Bidders	67.33	18.22	15.55	17.02	11.91	3.20	
	Rounds	1525.44	1519.58	979.18	283.46	24.94	25.30	
Preselect20	Efficiency	98.24	98.24	98.24	97.56	91.51	92.07	
	Rev. Auctioneer	76.26	79.55	79.27	78.69	76.35	84.65	
	Rev. Bidders	21.94	18.64	18.92	18.79	15.07	7.39	
	Rounds	147.18	141.32	146.34	145.62	61.76	28.32	
5of20	Efficiency	76.09	73.68	98.94	97.51	99.29	98.18	
	Rev. Auctioneer	22.11	52.16	82.03	83.55	87.88	94.19	
	Rev. Bidders	54.07	21.56	16.88	14.02	11.40	4.00	
	Rounds	3183.12	3058.88	1860.88	524.04	27.44	25.60	

TABLE 6.12: Robustness in Real Estate 3x3 Value Model

Interestingly, we found a similar pattern in the results for all value models. We repeated the same tests with Pairwise Synergy Low and Pairwise Synergy Zero value models, where the synergy level was lower and in some cases negative (subadditive valuations), and with Real Estate 4x4 value model. These modifications led to similar results.

Only the Transportation value model was different with respect to its better robustness against Preselect20 bidding. The main reason is the low number of bundles with significant competition, which is due to the underlying topology of transportation networks and the fact that only a few bundles are of interest to every bidder. For the same reason, Level10 bidders could successfully collude and significantly increase their payoff.

Below we describe the main findings for every type of the bidding strategy, for the case when all bidders in the auction follow it.

Bidding Agent \ ICA Format		CreditDebit	dVSV	iBundle(3)	iBundle(2)	ALPSm	Clock	VCG
BestResponse	Efficiency	100.00	100.00	100.00	100.00	99.65	99.53	100.00
	Rev. Auctioneer	89.63	90.53	90.46	90.47	89.81	92.83	89.60
	Rev. Bidders	10.37	9.47	9.54	9.53	9.85	6.69	10.40
	Rounds	204.44	202.96	154.96	154.82	68.70	31.68	1.00
Forgetful	Efficiency	99.81	99.65	100.00	99.99	99.65	99.56	
	Rev. Auctioneer	88.31	91.04	90.41	90.54	89.92	92.97	
	Rev. Bidders	11.50	8.63	9.59	9.45	9.73	6.58	
	Rounds	715.40	712.26	333.48	243.32	68.60	31.86	
Level10	Efficiency	98.40	98.41	98.48	98.47	97.21	94.46	
	Rev. Auctioneer	88.38	88.78	88.82	88.80	88.08	86.90	
	Rev. Bidders	10.03	9.64	9.67	9.68	9.14	7.59	
	Rounds	150.16	149.44	150.28	150.10	117.74	33.66	
Powerset10	Efficiency	96.01	95.91	99.16	99.55	99.75	99.51	
	Rev. Auctioneer	35.82	87.06	89.01	89.98	92.28	98.19	
	Rev. Bidders	60.18	8.85	10.15	9.57	7.47	1.31	
	Rounds	1353.50	1352.36	650.24	192.60	29.34	31.24	
Preselect20	Efficiency	85.80	85.80	85.80	85.80	82.75	85.21	
	Rev. Auctioneer	79.47	79.91	79.97	79.98	77.07	82.30	
	Rev. Bidders	6.35	5.90	5.84	5.84	5.70	2.91	
	Rounds	230.90	230.06	138.54	138.46	48.14	30.56	
5of20	Efficiency	85.33	85.40	98.70	98.15	99.40	99.35	
	Rev. Auctioneer	25.97	60.64	88.86	88.85	92.87	97.92	
	Rev. Bidders	59.37	24.78	9.84	9.30	6.53	1.41	
	Rounds	2551.94	2544.98	1261.50	338.50	31.86	31.34	

TABLE 6.13: Robustness in Pairwise Synergy High Value Model

6.5.1.1 BestResponse Bidder

Theory states that iBundle(3) and dVSV auctions always terminate with core results, given best-response bidding strategies. However, the BSM condition needs to be satisfied so that they also achieve VCG prices. Otherwise the final payments might be higher than the VCG prices. The CreditDebit auction calculates VCG discounts throughout the auction and is able to achieve VCG payments for general valuations and best-response bidding.

Our computational experiments with BestResponse bidders yield outcomes in line with the theory. All non-linear price auctions are efficient. iBundle(3) and dVSV achieve VCG outcomes only when the BAS condition is satisfied. When BAS is not satisfied, iBundle(3) and dVSV formats result in higher prices. In Transportation Small value model we observe cases where the prices are up to 250% higher than in the VCG auction (see Appendix A, Figure A.10), whereas in the Real Estate 3x3 and Pairwise Synergy High value models, the price increase is low (see Appendix A, Figures A.4 and A.7).

The CreditDebit auction always results in VCG prices, as the theory predicts. Clearly, this comes at a cost of eliciting all losing valuations throughout the

Bidding Agent \ ICA Format		CreditDebit	dVSV	iBundleP	iBundleA	ALPSm	Clock	VCG (tr)
BestResponse	Efficiency	100.00	100.00	100.00	99.99	99.55	99.48	100.00
	Rev. Auctioneer	56.13	66.93	65.43	65.36	65.92	77.10	55.49
	Rev. Bidders	43.87	33.07	34.57	34.63	33.60	22.41	44.51
	Rounds	216.78	205.18	78.66	78.02	32.24	29.64	1.00
Forgetful	Efficiency	99.47	99.60	99.93	99.97	99.55	99.58	
	Rev. Auctioneer	51.38	67.88	65.55	65.81	65.92	77.10	
	Rev. Bidders	48.10	31.76	34.35	34.15	33.60	22.48	
	Rounds	434.96	414.70	124.84	106.90	32.24	29.74	
Level10	Efficiency	84.95	85.06	83.64	84.01	83.96	84.56	
	Rev. Auctioneer	23.13	26.56	26.36	26.30	27.65	37.05	
	Rev. Bidders	61.92	58.46	57.31	57.70	56.19	47.36	
	Rounds	60.56	56.06	40.14	39.38	19.62	14.02	
Powerset10	Efficiency	91.33	91.73	97.28	97.26	99.78	97.39	
	Rev. Auctioneer	2.47	56.09	58.94	59.95	72.56	88.60	
	Rev. Bidders	88.80	35.52	38.19	37.20	27.18	8.82	
	Rounds	312.56	311.36	154.06	93.36	19.80	25.00	
Preselect20	Efficiency	99.80	99.80	99.80	99.80	99.52	99.24	
	Rev. Auctioneer	55.40	66.51	65.24	65.24	66.44	76.77	
	Rev. Bidders	44.39	33.28	34.56	34.56	33.06	22.50	
	Rounds	217.24	205.62	78.64	78.12	32.56	29.88	
5of20	Efficiency	84.14	84.50	96.24	96.10	99.75	97.73	
	Rev. Auctioneer	6.06	54.98	59.04	59.59	73.82	88.35	
	Rev. Bidders	78.26	29.75	37.10	36.43	25.90	9.33	
	Rounds	789.24	788.24	268.62	158.12	21.20	25.08	

Table 6.14: Robustness in Transportation Small Value Vodel

auction, and consequently 2-3 times more auction rounds than linear price auctions (see Section 6.5.3). iBundle(2) does not result in an efficient outcome for some instances, but these occasions are rare and the loss of efficiency is generally very low.

In the linear-price auctions (ALPSm and CC) the BestResponse bidder is less efficient than in the non-linear price auctions. Still, their efficiency is 95.16% and 98.64% on average for the Real Estate 3x3 value model, and even more than 99% for the Pairwise Synergy High and the Transportation Small value models. It is important to note that with the BestResponse bidder, we have cases with ALPSm and CC auctions where efficiency is as low as 70% in the Real Estate 3x3 value model (Section 6.6). If bidders follow the best-response strategy in ALPS and the CC auction, it can happen that they do not reveal certain valuations that would otherwise be part of the efficient solution (Bichler et al., 2009). In the presence of activity rules, the bidders are forced to bid on more than just their demand set. This has a positive effect on the robustness of the ALPSm format, as we will see when we discuss Powerset10 bidders.

6.5.1.2 Forgetful Bidder

Non-linear price auctions are fairly robust against Forgetful bidders, and the efficiency losses are low. Only the number of auction rounds increases significantly across all value models. For example, the CreditDebit auction takes on average 139.96 auction rounds in the Real Estate 3x3 value model with BestResponse bidders and 550.54 rounds with Forgetful bidders. Interestingly, the linear price auctions are hardly impacted at all compared to the best-response bidding strategy. The average number of auction rounds also remains almost the same.

6.5.1.3 Level10 Bidder

The Level10 bidder is collusive and intends to exploit the termination rule of non-linear price auctions. As expected, such bidding generally leads to an efficiency loss. In the Real Estate 3x3 value model, efficiency drops to around 90% in all auction formats and the auctioneer revenue is also significantly lower (t-test, $\alpha = 0.05$). In the Transportation Small value model, this strategy is very successful for bidders. Here, the Level10 bidder achieves a significantly higher revenue than a BestResponse bidder, albeit at the expense of efficiency, which drops to around 84% on average. For the Transportation Small value model, the competition is focused on a small number of items or legs in the transportation network and it is more likely that a valid allocation is found earlier when all bidders follow the Level10 bidding strategy.

In the CreditDebit auction, the high bidders' revenue can be explained by the fact that discounts are overestimated due to the bid shading in the Level10 strategy. In all auction formats, however, there are also instances in which the auctioneer gains more and the bidders gain less compared to the best-response bidding strategy. So, this strategy works only in an expected sense if all bidders adhere to it. It does not represent a stable equilibrium.

6.5.1.4 Powerset10 Bidder

This agent submits bids on ten bundles with the highest payoff in each round. For iBundle(2), iBundle(3), dVSV, and CreditDebit auctions, this strategy leads to a significant decrease in efficiency compared to the best-response strategy (t-test, $\alpha = 0.05$). For iBundle auctions the efficiency loss is lower than for dVSV and CreditDebit auctions. Both these formats are based on price calculation using a minimally undersupplied set, which appeared to be less robust against non-best-response bidding.

In contrast, the efficiency and auctioneer revenue share of ALPS auctions is equal or higher compared to the best-response strategy in all value models. The number of rounds is significantly reduced at the same time. The CC auction performs well in homogenous markets, modeled by Real Estate 3x3 and Pairwise Synergy High value models. Typically, these linear-price auctions are used with strong activity rules to encourage the revelation of many bundle preferences already in the early rounds of an auction, which might lead to a similar strategy with bidders in the field.

6.5.1.5 Preselect20 Bidder

The Preselect20 bidder also follows the best-response strategy, but only on the set of the 20 bundles with highest valuations. As a consequence, the auction cannot find efficient outcomes in instances where the omitted bundles are included in the efficient solution. In the Transportation Small value model this strategy has little effect on efficiency compared to best-response bidding, since there is only a small number of interesting bundles for every bidder in this case. In other value models we can see a significant decrease in all measurements.

6.5.1.6 5of20 Bidder

This Random agent bids on 5 of his 20 best bundles, modeling a bidder with bounded rationality. This leads to significant efficiency losses in all non-linear price auctions (see Figure A.6). We observe the highest efficiency loss for dVSV and CreditDebit auctions. The reason for the low revenue of CreditDebit auction is again that discounts are miscalculated if not all bundle bids are available at the end. In addition, the more complex price update rule is less robust against non-best-response bidding.

6.5.2 Auctions with Different Bidding Strategies

We conducted another set of auctions using Real Estate 3x3 and Transportation Small value models to measure the effect of one single bidder deviating from the best-response strategy while the rest adhere to it. For each setting, we ran 50 auctions using iBundle(3), iBundle(2) and ALPSm formats.

The results follow the same pattern over all three ICA formats and both value models. The allocative efficiency is not impacted, except that just a single Level10 bidder reduces the efficiency significantly. The Level10 bidder also suffers highest loss of 46% of his revenue, followed by the Preselect20 bidder, who has only a minor revenue loss of less than 5%. This indicates that the equilibrium which brings significant increase in revenue to Level bidders when all bidders follow this strategy is not stable. All other bidder types do not change the auction outcome significantly.

The results are visualized in Figure A.13 and Figure A.14 in Appendix A.

6.5.3 Auction Duration

CC auction has the lowest number of rounds in all treatments. Since iBundle(3), dVSV, and CreditDebit mechanisms elicit all losing valuations throughout the auction, the number of auction rounds can become very high for them. On average, non-linear price auctions take three times as many rounds as linear-price based auctions. In contrast to the theory, which expects a lower number of auction rounds in dVSV compared to iBundle(3), we observe an even higher number of auction rounds in dVSV. This happens because the minimally undersupplied set is not unique and in our simulations we use the smallest possible minimally undersupplied, which results in small price steps. For the same reason, non-best-response strategies cause the highest increase in rounds for dVSV and CreditDebit auctions, compared to other formats. The speed of convergence of these two formats can be improved by increasing prices on several disjunct minimally undersupplied sets in every round.

6.5.4 Impact of Competition Level

Auctions are expected to yield more auctioneer revenue if there is more competition. Table 6.15 presents results of different auction formats using Real Estate 3x3 value model and a varying number of bidders. Each number represents an average of 10 auctions with the same setting and different random seeds for the value model.

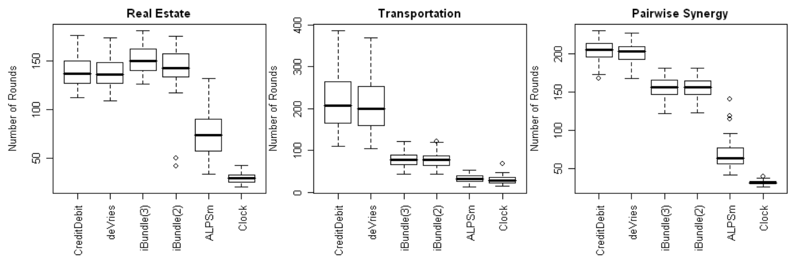


FIGURE 6.13: Average Number of Auction Rounds

Bidding Agent \ ICA Format		iBundle(2)	iBundle(3)	dVSV	CreditDebit	Clock	ALPS m	VCG (tr)
4 bidders	Efficiency in %	99.74	100.00	100.00	100.00	96.69	95.88	100.00
BAS fulfilled	Rev. Auctioneer	78.49	80.34	80.34	79.96	84.17	73.82	79.96
100 %	Rev. Bidders	21.25	19.66	19.66	20.04	12.52	22.06	20.04
	Rounds	195.84	201.46	83.40	83.40	35.66	103.06	1.00
5 bidders	Efficiency in %	99.94	100.00	100.00	100.00	96.52	95.06	100.00
BAS fulfilled	Rev. Auctioneer	84.48	84.94	84.94	83.16	88.09	77.27	83.16
90 %	Rev. Bidders	15.46	15.06	15.06	16.84	8.43	17.79	16.84
	Rounds	148.96	150.10	143.72	146.28	31.14	68.92	1.00
6 bidders	Efficiency in %	99.91	100.00	100.00	100.00	94.74	97.06	100.00
BAS fulfilled	Rev. Auctioneer	87.00	87.20	87.39	85.42	88.04	82.44	85.42
50 %	Rev. Bidders	12.91	12.80	12.61	14.58	6.69	14.62	14.58
	Rounds	132.58	133.42	207.10	209.62	30.68	61.86	1.00
7 bidders	Efficiency in %	99.89	100.00	100.00	100.00	94.35	96.98	100.00
BAS fulfilled	Rev. Auctioneer	88.29	88.61	88.79	86.38	87.94	84.45	86.38
40 %	Rev. Bidders	11.59	11.39	11.21	13.62	6.41	12.54	13.62
	Rounds	122.06	122.82	271.46	274.54	29.92	52.58	1.00

TABLE 6.15: Impact of Competition Level on Different ICAs

We observe the expected behavior in non-linear price auctions and in ALPSm design. The average revenue share in CC auctions decreases from 5 to 7 bidders. Linear price-based auctions and the iBundle design show a lower number of rounds with an increasing number of bidders. In contrast, the dVSV and CreditDebit auctions show a massive increase of rounds. This is explained by a different price update mechanism. The iBundle design, which increases prices for all unhappy bidders, generally increases more prices when the competition is higher. The dVSV and CreditDebit auctions, which increase prices for a minimally undersupplied set of bidders, can find a smaller minimally undersupplied set (typically with only one bidder) when the competition increases, and therefore increase prices for fewer bundles in each round.

6.5.5 Impact of BAS

We have discussed that if the BSM property is satisfied, non-linear price auctions lead to Vickrey prices (Section 2.4.2.1). Due to computational reasons, we have restricted ourselves to analyzing the somewhat weaker BAS condition only.

Revenue \ ICA Format	iBundle(2)	iBundle(3)	dVSV	CreditDebit	Clock	ALPSm
Min in % of VCG	98.86	100.00	100.00	100.00	102.40	86.68
Mean in % of VCG	99.87	100.00	100.00	100.00	107.51	97.25
Max in % of VCG	101.72	100.00	100.00	100.00	120.81	116.51

Table 6.16: Revenue in Real Estate Value Model with BAS Fulfilled

Revenue \ ICA Format	iBundle(2)	iBundle(3)	dVSV	CreditDebit	Clock	ALPSm
Min in % of VCG	96.55	100.52	100.52	100.00	95.71	84.56
Mean in % of VCG	102.48	103.31	103.30	100.00	111.39	98.61
Max in % of VCG	112.34	111.69	111.69	100.00	127.99	119.25

Table 6.17: Revenue in Real Estate Value Model with BAS Not Fulfilled

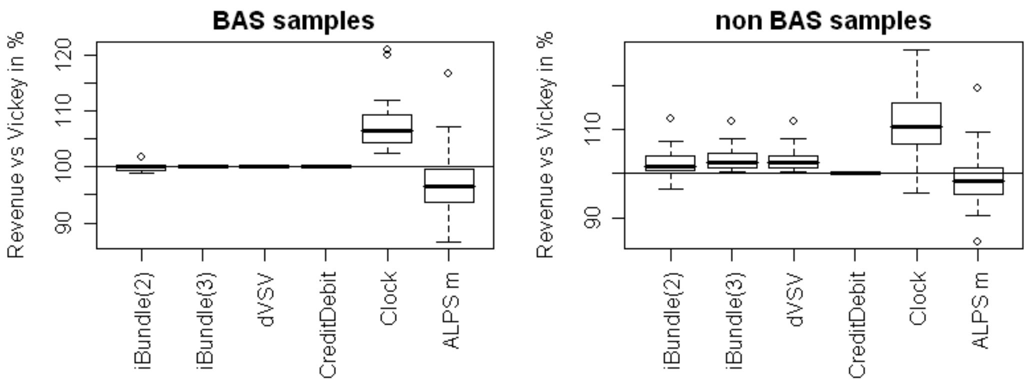


Figure 6.14: Impact of BAS on Revenue in Real Estate Value Model

We created a setup based on the Real Estate 3x3 value model, where the BAS condition was fulfilled in approximately half of the randomly generated instances, with all agents following the best-response strategy. The results are summarized in Tables 6.16 and 6.17, as well as in Figures 6.14. and 6.15.

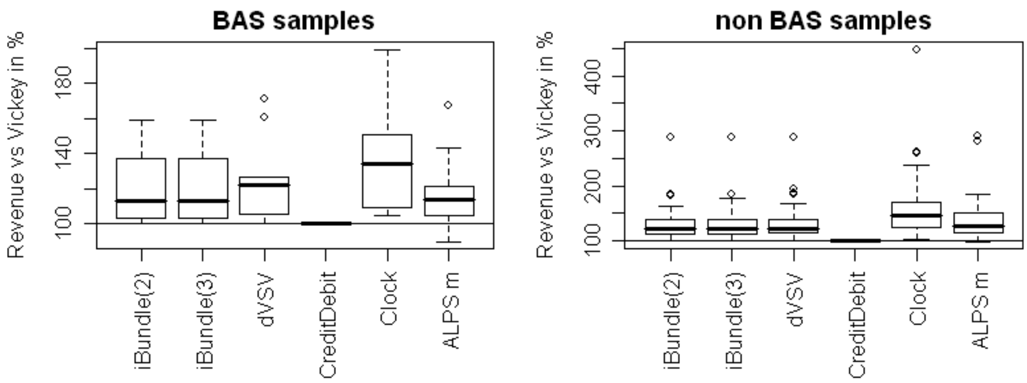


FIGURE 6.15: Impact of BAS on Revenue in Transportation Value Model

As expected, prices and consequently auctioneer revenue in non-linear price auctions is higher than the VCG outcome for samples without BAS property. For ALPSm auctions, there is no significant difference.

6.6 Inefficiences in Linear-Price ICAs

While the efficiency of linear-price ICAs in our experiments was generally high, it is important to understand those cases where the final allocation is not optimal. We have analyzed all instances of auctions in Real Estate and Pairwise Synergy value models where efficiency was particularly low (90% and below). We have focused on ALPSm and CC designs with disabled eligibility rules to isolate the negative impact of linear prices from inefficiencies due to activity rules. Here, only BestResponse bidders were used.

In all situations with an efficiency of less than 90% the auctioneer does not sell all items, as compared to an efficient allocation. These situations happen rarely in Real Estate value models, and even less so in Pairwise Synergy value models, as can be seen in Figures 6.4 and 6.6. Whenever all items are sold, the allocative efficiency is always higher than 98%. Two small examples in Tables 6.18 and 6.19 illustrate structural characteristics of valuations which can lead to inefficiencies in ICAs with linear prices and best-response bidding.

The example in Table 6.18 illustrates a scenario with three items A, B, C and four bidders. The efficient allocation is marked with asterisks. In this

Valuations	A	B	C	AB	AC	BC	ABC
Bidder1					9*		
Bidder2		2*					
Bidder3				10			
Bidder4						10	

Table 6.18: Example of Inefficiencies in ALPSm

example, the ALPSm design selects the bid of bidder 4 on bundle $\{B, C\}$ and leaves the item A unsold. The distinguishing property of these valuations is the set of mutually exclusive bundle valuations $\{A, B\}$ and $\{B, C\}$, none of which belongs to the efficient allocation. During the auction bidders 3 and 4 drive up the prices, which blocks other bidders from submitting their true valuations. Interestingly, the auction outcome in this case is sensitive to start prices. The ALPSm outcome was efficient for item start prices of 1.3 and 1.9, but was not efficient for all other values from 0 to 2.0 in 0.1 steps and 0.1 minimum bid increments. CC design was always efficient in this example.

Valuations	A	B	C	AB	AC	BC	ABC
Bidder1						20*	60
Bidder2	61*						
Bidder3				50	50		

Table 6.19: Example of Inefficiencies in CC

The second example in Table 6.19 illustrates a set of valuations where CC design leads to an inefficient allocation. It allocates the item A to bidder 2, while both items B and C remain unsold. Note that the high valuation of bidder 1 on the bundle $\{A, B, C\}$ dominates the bundle $\{B, C\}$. At the time when bidder 2 overbids him, the prices are already too high on all items, which prevents bidder 1 from submitting bids on the bundle $\{B, C\}$. Again, all bidders follow the best-response strategy in this case. ALPSm design always terminates with the efficient allocation in this example.

One possibility to mitigate the remaining inefficiencies in ALPS and CC designs is to ask bidders to submit single-item bids before the auction. However, this still does not guarantee 100% efficiency, since there might be no demand for some of the individual items. A better solution would be to auction off the goods that have been unsold in an after-market, which can be added to the auction transparently for the bidders.

Another alternative is the addition of a second phase with an Ascending Proxy Auction, as suggested in the Clock-Proxy auction (Ausubel et al., 2006), with suitable eligibility rules. Without eligibility rules, if the bidders can submit any additional bids to the final prices of the first stage, the auction can be always efficient with truthful bidders. However, both the eligibility rules are necessary to encourage active participation during the first linear-price auction phase. The impact of different eligibility rules on the allocative efficiency in a two-stage auction, and optimal bidding strategies in these auction designs, is still an open question in the auction theory.

Chapter 7

Conclusions and Future Work

> The science of today is the technology of tomorrow.
>
> Edward Teller

The main contribution of this thesis is the new ALPS iterative combinatorial auction (ICA) format, designed for practical applicability and robustness. It was created based on thorough analysis and comparison of existing CA designs, iteratively formulating and testing new auction rules which shall improve the auction outcome. In particular, this work is the first detailed benchmark of two big ICA families: linear-price and non-linear price designs. In our experiments, the ALPS format demonstrates high indices in performance and robustness. Ultimately, we expect to see an evolution of standard software components and standard designs for combinatorial auctions that work well in a wide variety of bidder valuations and bidding strategies.

The idea of using combinatorial auctions (CAs) for capturing economies of scale and scope and thus achieving better economical results in complex markets was first suggested by Rassenti et al. (1982) for allocating airport slots. Much publicity and academic attention have been attracted by the US Federal Communication Commission (FCC) spectrum multi-lot auctions (Goeree and Holt, 2008). Recently, several cases have demonstrated the applicability of combinatorial auctions in industrial procurement (Hohner et al., 2003; Metty et al., 2005; Sandholm and Begg, 2006). However, each of these cases required significant research and engineering work, and combinatorial auctions are still far from being a mainstream tool.

We have chosen computational experiments to be the main research tool in our work. The game-theoretical approach, which has been used extensively to model single-item auctions, has only limited applicability in the context of combinatorial auctions due to their high strategic complexity. Furthermore, there are strong indications that the bidders fail to act rationally in their exponential strategic space. Experimental economics, which is another proven approach to the studying of market mechanisms, has delivered only very limited results to date, due to the high complexity and cost of laboratory experiments with combinatorial auctions.

Computational experiments allowed us to systematically test and compare many combinatorial auction designs under different valuations and different bidder behavioral models. We could also measure their sensitivity with respect to different parameters. To achieve reliable results, our experiments are based on a broad range of economically motivated value models and bidding agents with different behavior, based both on theoretical assumptions and on our observations in the laboratory. Overall, this thesis summarizes outcomes of over 50'000 auctions. An important result of our work is the MarketDesigner platform for combinatorial auctions, which was a significant investment, and is a joint effort together with several colleagues and many students.

We decided to use for our design an iterative mechanism which can mitigate the exponential complexity of bidders' decisions using prices and other feedback. Furthermore, there is evidence that iterative formats perform better in markets with affiliated valuations (Milgrom and Weber, 1982; Porter et al., 2003).

An important decision was the selection of the price format for the new ICA. Non-linear personalized price format is required in the general case to calculate minimum competitive equilibrium prices, which are optimal from the game-theoretical perspective. Furthermore, only non-linear price auctions can guarantee – based on strong assumptions on bidders – a certain level of auction efficiency. However, using non-linear prices means supplying every bidder with exponentially many prices, which significantly complicates his decisions.

We have selected pseudo-dual linear prices for several reasons. There are only as many prices as there are items in the auction, which makes them easy to understand and use for bidders. Especially important is that the bidders can easily find new profitable bundles during the auction. The approximated nature of linear prices appeared not to be a big problem in the end, since even the non-linear prices are precise only under the *ceteris paribus* condition, when no other bidders are active in the auction.

Furthermore, our experiments demonstrate that linear-price ICAs have several advantages over non-linear price formats. They usually require only a fraction of rounds to terminate, compared to the non-linear price designs. Linear-price auctions are much more robust with respect to suboptimal bidding behavior. Several authors suggested proxy agents to mitigate this problem of non-linear price auctions which, however, effectively turns them into sealed-bid auctions, thus eliminating most advantages of an iterative design.

ALPS uses pseudo-dual rather than *tâtonnement*, like in Combinatorial Clock, linear prices. Our experiments indicate that pseudo-dual prices result in fairer revenue distribution, and consequently will provide better incentives for truthful bidding.

The ALPS ICA format has a high allocative efficiency of over 98% on average in every value model, provided that bidders reveal enough information about their valuations. It demonstrates highest robustness with respect to the bundle selection strategy compared to all other ICA formats in our benchmark. It has several practice-oriented features which further improve its performance. The dynamic minimum increment can halve the auction duration without sacrificing the efficiency. The surplus eligibility rule can mitigate the negative effect of activity rules in the auction.

While the ALPS format achieves high efficiency values on average, we have identified and described cases where linear price CAs are not efficient. There are a few remedies, such as the proxy phase in the Clock-Proxy auction or after-market negotiation on unsold items.

7.1 Future Work

The project itself is not completed with this thesis. We have been using the MarketDesigner platform to conduct laboratory experiments and to run pilot projects with industry partners. We have established contacts and are working on cooperations with leading researchers in this field. Research into new combinatorial auction rules and designs will continue.

From our robustness analysis we can see that the most critical point for the performance of a combinatorial auction is the ability of bidders to grasp the market situation and to find the most profitable bundles. Therefore, an important research direction is to improve support for bidders in combinatorial auctions. This includes the development of new bidding languages, which help bidders to express their valuations more easily.

Using the MarketDesigner software, we are currently planning and conducting laboratory experiments with students in order to study how the auction size affects the ability of bidders to make correct decisions. Based on empirical observations in the laboratory, we plan also to implement new software bidding agents and use them in further computational experiments.

Allocation rules which are often required in procurement auctions present another important research direction. An open question is how they impact iterative auctions, and how the prices and other feedback must be adopted in their presence.

Appendix A

Additional Charts on Robustness Analysis

This Appendix contains additional charts illustrating robustness of various ICA formats against strategic and suboptimal bidding. Figures A.1 to A.3 summarize the results on allocative efficiency and revenue distribution in a concise way. The black bars describe the auctioneer's revenue, the grey bars the bidders' revenue, and the white bars the lost efficiency. Based on three selected value models, we plotted six diagrams for six different auction formats. Each diagram describes the results for BestResponse bidders (BR), Forgetful bidders (FB), Powerset10 bidders (PS10), Random 5of20 bidders (5SUB20), and Preselect20 bidders (PRE20). A more detailed view in form of scatter plots can be found in Figures A.4 to A.12.

Figures A.13 and A.14 visualize the revenue distribution in auctions where one single bidder deviates from the best-response strategy, while all other bidders adhere to it. Here, the light grey bars correspond to the revenue of the single bidder who changes his strategy, and the dark grey bars to the cumulative revenue of all other bidders, who follow the best-response strategy in all auctions.

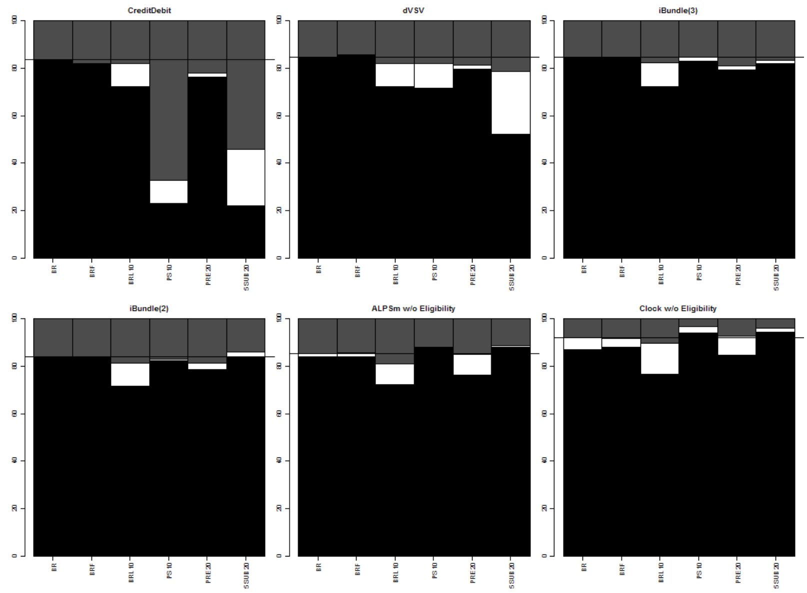


Figure A.1: Revenue Distribution in Real Estate Value Model

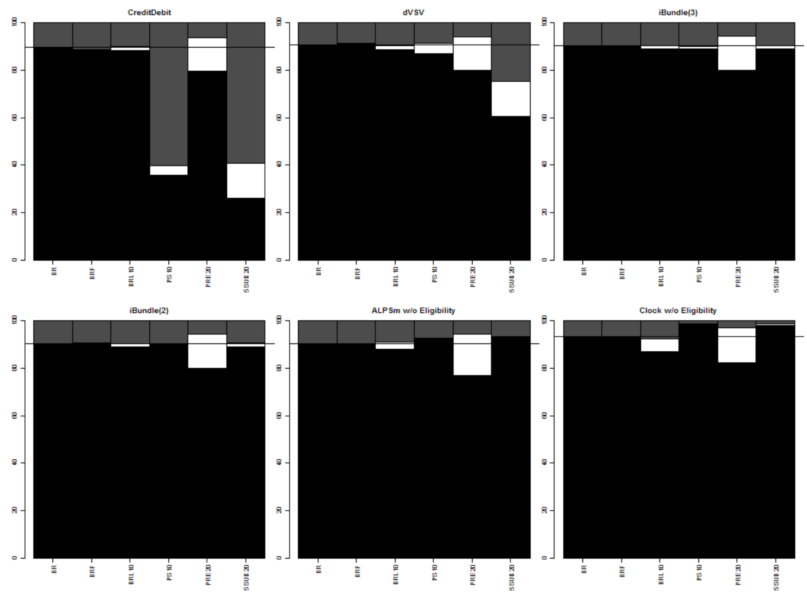


Figure A.2: Revenue Distribution in Pairwise Synergy Value Model

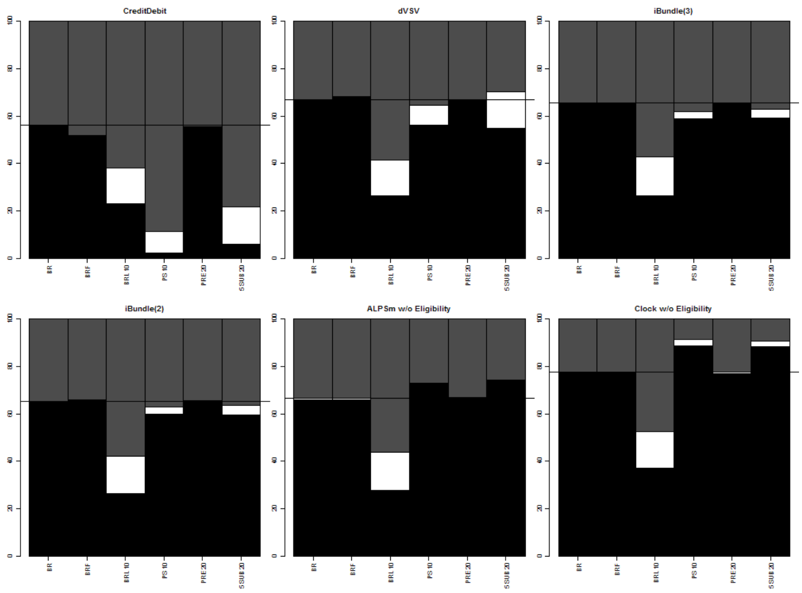
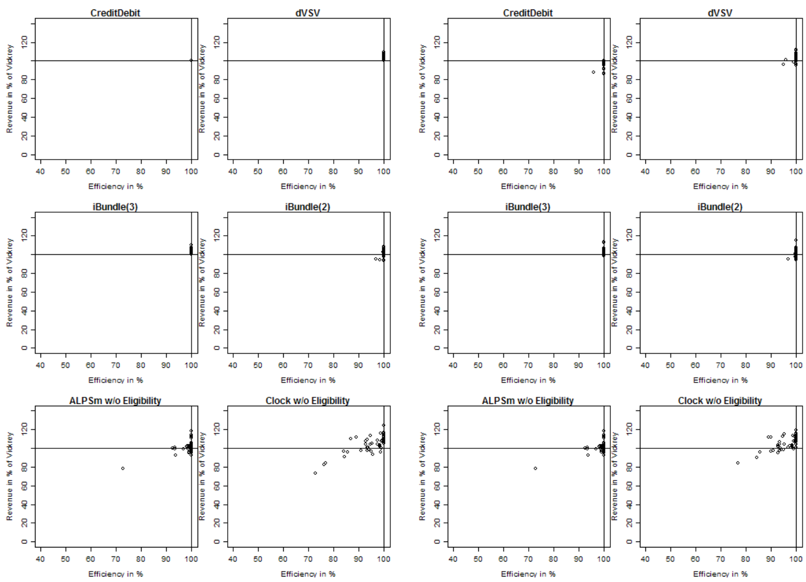
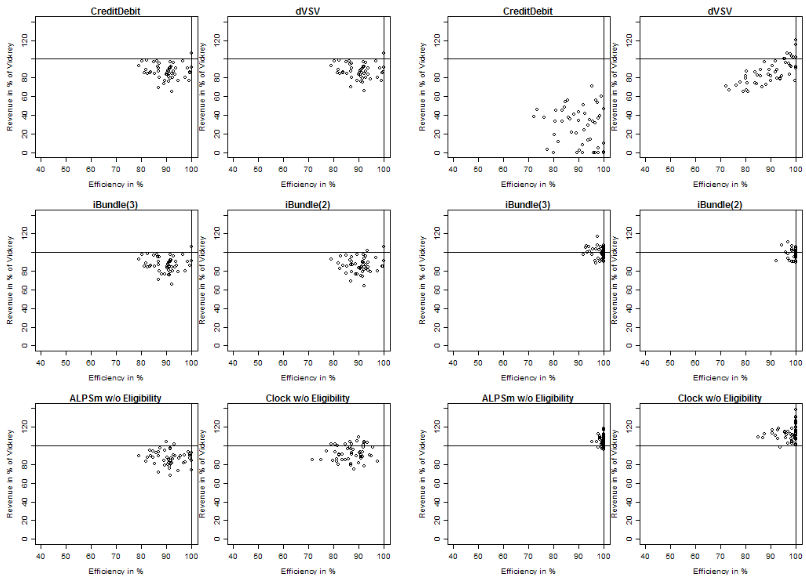


FIGURE A.3: Revenue Distribution in Transportation Value Models



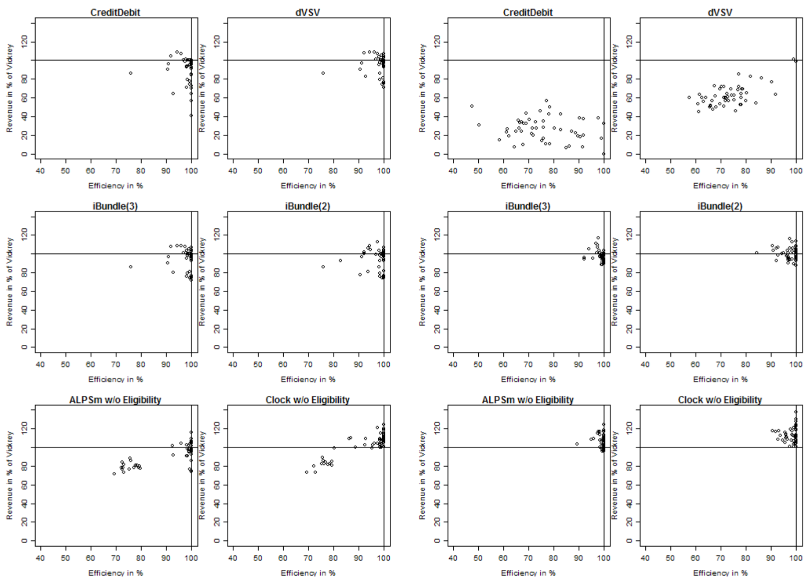
(a) BestResponse (b) Forgetful

FIGURE A.4: BestResponse and Forgetful Bidders in Real Estate VM



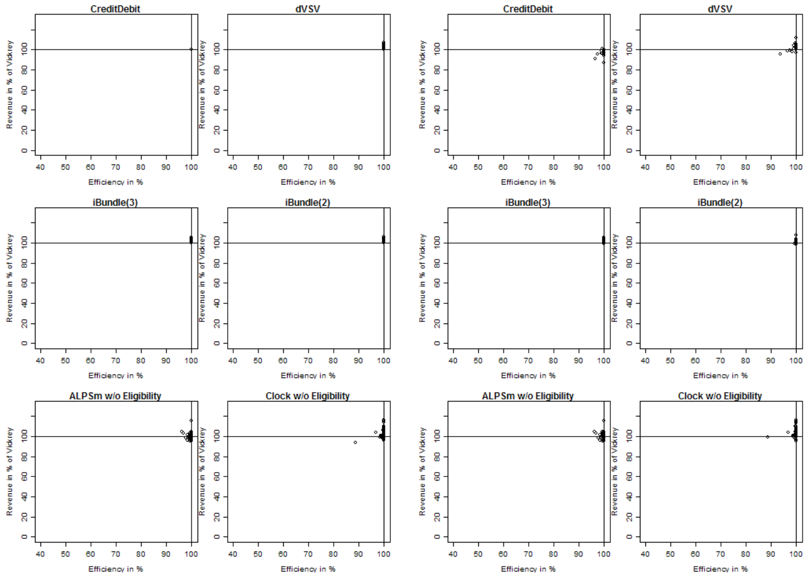
(a) Level10 (b) Powerset10

FIGURE A.5: Level10 and Powerset10 Bidders in Real Estate VM



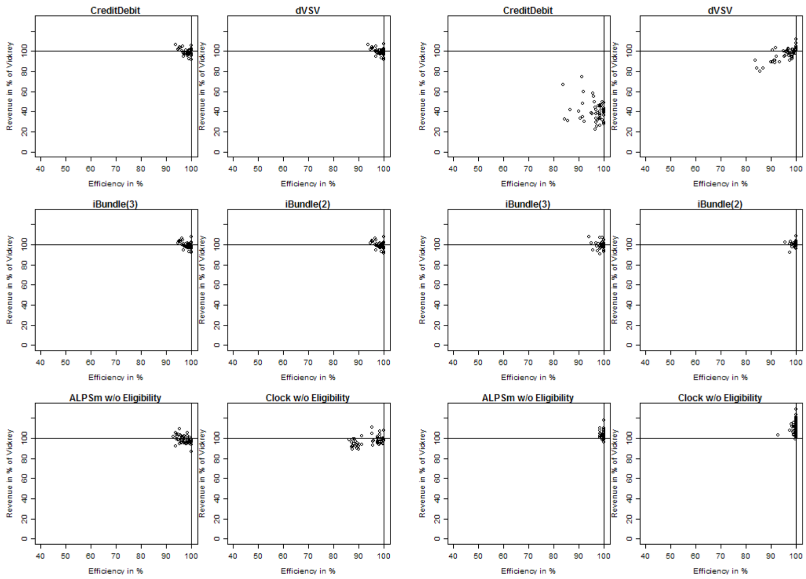
(a) Preselect20 (b) Random 5of20

FIGURE A.6: Preselect20 and 5of20 Bidders in Real Estate VM



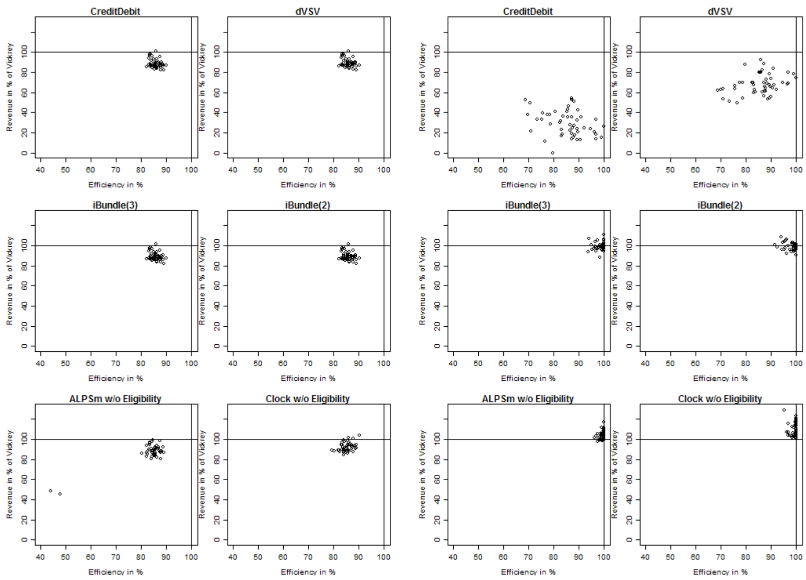
(a) BestResponse (b) Forgetful

FIGURE A.7: BestResponse and Forgetful Bidders in Pairwise Synergy VM



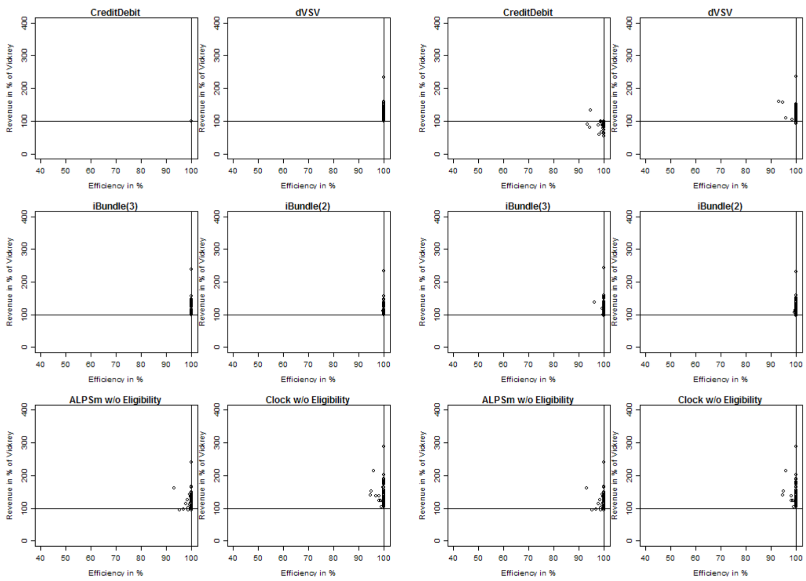
(a) Level10 (b) Powerset10

FIGURE A.8: Level10 and Powerset10 Bidders in Pairwise Synergy VM



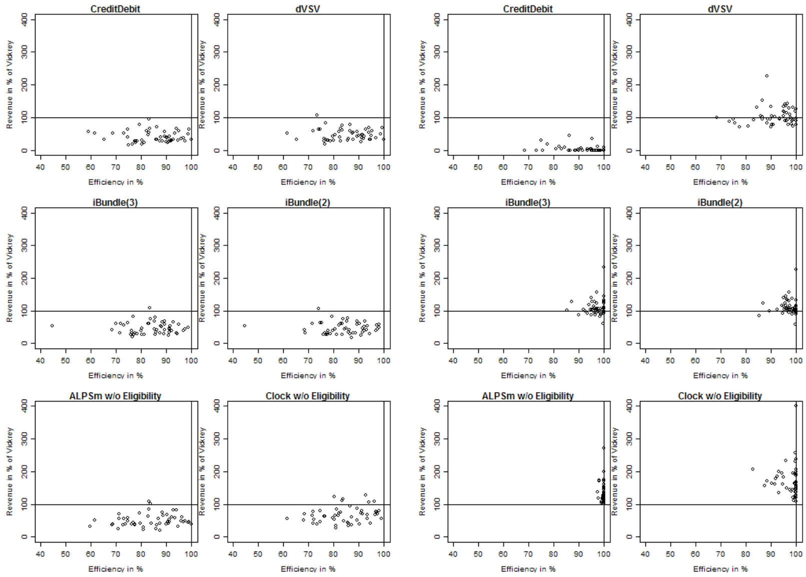
(a) Preselect20 (b) Random 5of20

FIGURE A.9: Preselect20 and 5of20 Bidders in Pairwise Synergy VM



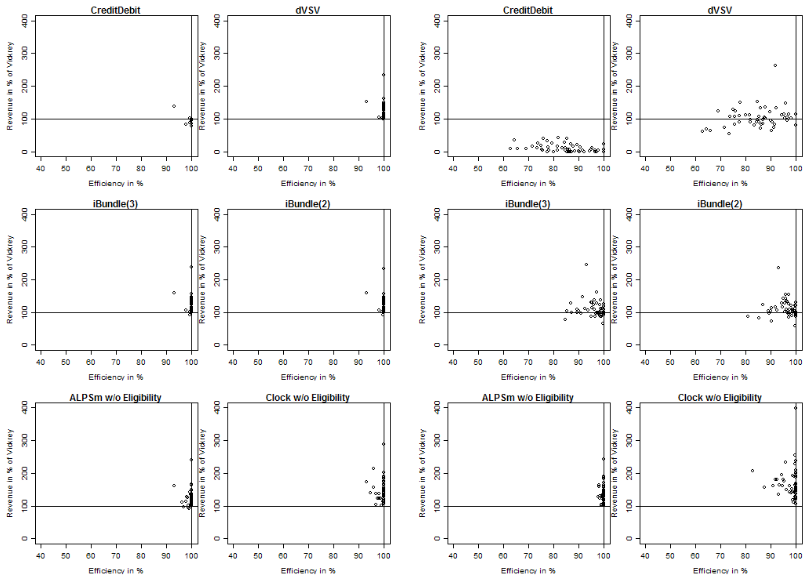
(a) BestResponse (b) Forgetful

FIGURE A.10: BestResponse and Forgetful Bidders in Transportation VM



(a) Level10 (b) Powerset10

FIGURE A.11: Level10 and Powerset10 Bidders in Transportation VM



(a) Preselect20 (b) Random 5of20

FIGURE A.12: Preselect20 and 5of20 Bidders in Transportation VM

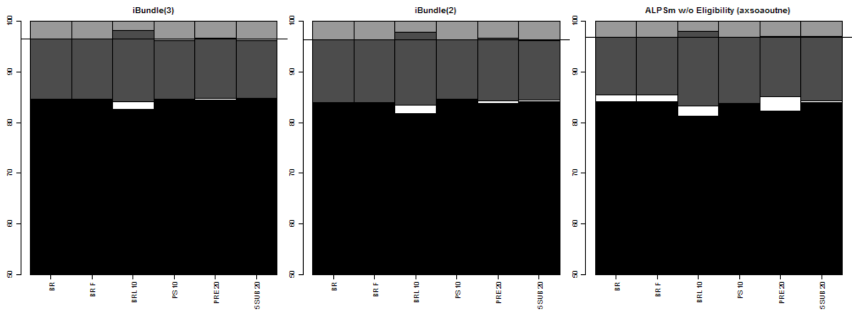


FIGURE A.13: Sensitivity to Bidding Strategies with BestResponse Base Bidders in Real Estate 3x3 Value Model

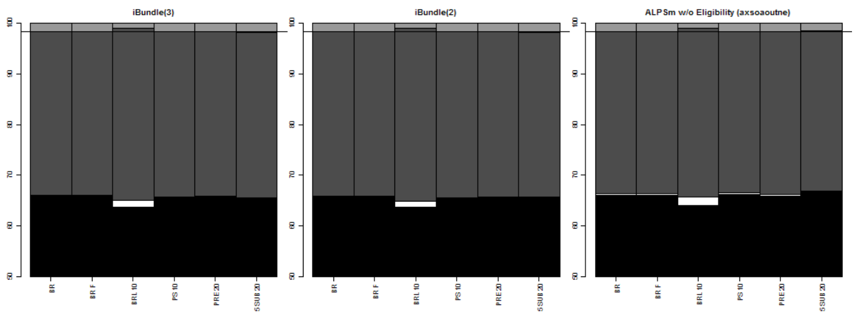


FIGURE A.14: Sensitivity to Bidding Strategies with BestResponse Base Bidders in Transportation Small Value Model

Appendix B

List of Symbols

$\mathcal{K}$ - set of items

$k \in \mathcal{K}$**, also** $l \in \mathcal{K}$ - item

m - number of items

$S \subseteq \mathcal{K}$**, also** $T \subseteq \mathcal{K}$ - subset of items (bundle, package)

$\mathcal{I}$ - set of bidders

$i \in \mathcal{I}$**, also** $j \in \mathcal{I}$ - bidder

$I \subseteq \mathcal{I}$ - subset of bidders

n - number of bidders

t - round number

$\mathcal{B}^t = \{b_i^t(S)\}$ - set of bids active after the round t

$b_i^t(S) = \{S, p_{bid,i}^t(S)\} \in \mathcal{B}^t$ - bid of the bidder i for the bundle S active after the round t

$p_{bid,i}^t(S) \in \mathcal{B}^t$ - bid price of the bid $b_i^t(S)$

$\mathcal{P}_{ask}^t = \{p_{ask,i}^t(S)\}$ **or** $\{p_{ask}^t(S)\}$ **or** $\{p_{ask}^t(k)\}$ - set of ask prices valid during the round t

$p_{ask,i}^t(S)$ - personalized bundle ask price for the bidder i and bundle S valid during the round t

$p^t_{ask}(S)$ - anonymous bundle ask price for the bundle S valid during the round t

$p^t_{ask}(k)$ - anonymous linear ask price for the item k valid during the round t

Δ^t - price increment valid during the round t or used for the price update from the round t to the round $t+1$

$v_i(S)$ - private valuation of the bidder i for the bundle S

$\mathcal{P}_{pay} = \{p_{pay,i}(S)\}$ - set of pay prices

$p_{pay,i}(S)$ - pay price for the bidder i and bundle S

$\pi_i(S, \mathcal{P}_{pay})$ - utility of the bidder i for the bundle S at the pay prices $\mathcal{P}_{pay}$

$\mathcal{X} = \{X\}$ - set of all possible allocations

$X = (S_1, \ldots, S_n) = \{x_i(S)\}$ - allocation where bidder i gets bundle S_i

$S_i \subseteq \mathcal{K}$ - bundle allocated to the bidder i

$x_i(S) \in \{0; 1\}$ - binary variable which determines, whether the bidder i becomes allocated exactly the bundle S

$\pi_i(X, \mathcal{P}_{pay})$ - utility of the bidder i for the allocation X at the pay prices $\mathcal{P}_{pay}$

$\pi_{all}(X, \mathcal{P}_{pay})$ - total bidder utility for the allocation X at the pay prices $\mathcal{P}_{pay}$

$\Pi(X, \mathcal{P}_{pay})$ - auctioneer revenue for the allocation X at the pay prices $\mathcal{P}_{pay}$

$X^* = (S^*_1, \ldots, S^*_n) = \{x^*_i(S)\}$ - efficient allocation

X^t - provisional allocation calculated on the basis of the bids active in the round t

W^t - set of provisionally winning bids in the allocation X^t

L^t - set of provisionally losing bids in the allocation X^t

$E(X) \in [0, 1]$ - allocative efficiency of the allocation X

$R(X) \in [0, E(X)]$ - auctioneer utility share in the allocation X

$U(X) \in [0, E(X)]$ - total bidder utility share in the allocation X

C_I - coalition consisting of the bidders $I \subseteq \mathcal{I}$ and the auctioneer

$w(C_I)$ - coalitional value function on the coalition C_I

(Π, π) - payoff vector

$Core\,(\mathcal{I}, w)$ - set of core payoffs

$\delta_i(S)$ - linear price compatibility distortion of the bid $b_i(S)$

$D_i(\mathcal{P}_{pay})$ - demand set of the bidder i at the prices $\mathcal{P}_{pay}$

e_i^t - eligibility of bidder i in round t

eb_i^t - bound eligibility of bidder i in round t

eu_i^t - unbound eligibility of bidder i in round t

$e_{+,i}^t$ - surplus eligibility of bidder i in round t

rbv_i^t - round bid volume of bidder i in round t

tbv_i - total bid volume of bidder i

Appendix C

List of Abbreviations

ALPS **A**pproximate **L**inear **P**rice**S**

AUSM **A**daptive **U**ser **S**election **M**echanism

BAS **B**idders **A**re **S**ubstitutes condition

BSM **B**idder **S**ub**m**odularity condition

CAP **C**ombinatorial **A**llocation **P**roblem

CA **C**ombinatorial **A**uction

CATS **C**ombinatorial **A**uction **T**est **S**uite

CC **C**ombinatorial **C**lock auction

CE **C**ompetitive **E**quilibrium

FCC **F**ederal **C**ommunication **C**ommission

GAS **G**oods **A**re **S**ubstitutes condition

IBIS Chair of **I**nternet-**b**ased **I**nformation **S**ystems at the Technische Universität München (Munich, Germany)

ICA **I**terative **C**ombinatorial **A**uction

ILP **I**nteger **L**inear **P**rogram

IS **I**nformation **S**ystems

ISR **I**nformation **S**ystem **R**esearch Journal

LP **L**inear **P**rogram

MBL **M**atrix **B**idding **L**anguage

NP **N**on/deterministic **P**olynomial time

OR additive-**OR** (bidding language)

PAUSE **P**rogressive **A**daptive **U**ser **S**election **E**nvironment

PEP **P**reference **E**licitation **P**roblem

RAD **R**esource **A**llocation **D**esign

SMR **S**imultaneous **M**ulti-**R**ound Design

TBBL **T**ree-**B**ased **B**idding **L**anguage

TUM **T**echnische **U**niversiät **M**ünchen

VCG **V**ickrey-**C**larke **G**roves mechanism

UCE **U**niversal **C**ompetitive **E**quilibrium

VM **V**alue **M**odel

WDP **W**inner **D**etermination **P**roblem

XOR e**x**clusive-**OR** (bidding language)

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