Market Liquidity Risk
- Dissertation -

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Abstract

Market liquidity risk is the potential loss, because assets cannot be sold at the price previously thought. Although evidence suggests that liquidity effects are significant, they often remain neglected in practical risk management. One of the reasons is the limited scientific research in the area of liquidity risk measurement.

This thesis provides an up-to-date overview on market liquidity risk research. It covers all aspects of market liquidity that are relevant to risk management as well as existing liquidity risk models.

The empirical analysis is based on weighted spread, a relatively new liquidity measure, which can be extracted from the limit order book of electronic exchanges. A unique, representative sample of weighted spread allows to provide estimates on the effect of order size on liquidity costs, as well as the dynamics, distributional characteristics and cross-sectional structure of this liquidity measure.

The thesis also proposes two new liquidity risk models. The modified liquidity risk model introduces a new way to account for the non-normality in liquidity with the help of the Cornish and Fisher (1937)-approximation. The empirical net-return model based on weighted spread analyzes the use of the weighted spread liquidity measure in risk measurement.

Both models are tested empirically in daily data. The modified liquidity risk model implemented with the bid-ask-spread proves to be superior to the standard model of Bangia et al. (1999). Common backtests by Kupiec (1995) demonstrate that risk is forecasted with much higher precision when non-normality is taken into account via the proposed Cornish-Fisher approximation.

With the help of the empirical net-return model, I find that liquidity risk strongly increases with the size of the position. The impact of liquidity on risk is significant - even at 10-day horizons. Liquidity risk models neglecting this effect must necessarily underestimate total risk. Further, the correlation between liquidity and return is significant and reduces the liquidity impact by about 50 % compared with the standard assumption of perfect correlation. These results are robust to change in risk measure, effects of time variation as well as portfolio diversification.

A final test runs a performance benchmark of nine different liquidity risk models implementable in daily data, including the new propositions. I find that available data is the main driver of model preciseness. Models with extensive data from the limit order book generally outperform. My

new propositions, modified add-on with weighted spread and empirical net-return with weighted spread as well as Giot and Grammig (2005), are all recommendable. The first model delivers precise results most consistently. If only transaction data are available, the model by Cosandey (2001) performs best. With bid-ask-spread data the proposed modified add-on model with bid-ask-spread achieves superior results.

Overall, this thesis underlines the usefulness of the weighted spread measure in liquidity risk modeling. If the analyzed structure of liquidity costs, i.e. non-normality as well as increase with order size, is properly integrated, the preciseness of risk forecasts can be greatly improved. The new model contributions prove to be particularly helpful in practical risk management.

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List of Abbreviations

Abbreviation	*Definition*
ASX	Australian Stock Exchange
BIS	Bank for International Settlement
bp	basis points
Coef.	Coefficient
c.p.	compare
e.g.	exempli gratia
et al.	et alienes
ES	expected shortfall
etc.	et cetera
EWMA	exponential weighted moving average
f./ff.	following
FSE	Frankfurt Stock Exchange
GARCH	Generalized Autoregressive Conditional Heteroscedasticity model
i.e.	id est
Log.	Logarithm
L-VaR	Liquidity-adjusted Value-at-Risk
Mio.	million
MQV	Minimum quotation volume
n/a	not available / applicable
OLS	Ordinary Least Squared
OTC	Over-the-counter
p./pp.	page/pages
thsd.	thousand
TSE	Toronto Stock Exchange
VaR	Value-at-Risk
VIF	Variance inflation factor
vs.	versus
XLM	Xetra Liquidity Measure

List of Symbols

Symbol	*Definition*
a	Quoted / minimum ask price
b	Quoted / minimum bid price
β	Regression coefficient
C	Regression constant
γ	skewness (modified models)
$D(q)$	Delay costs of order quantity q in percent of mid-price
d	Number of days with losses in excess of VaR-estimate
ϵ	error term
€	Euro
i	stock i
κ	excess kurtosis (modified models)
κ	Price elasticity with respect to volume (Angelidis and Benos (2005) model)
$\kappa(q)$	Correlation factor at size q (empirical net return model)
$L(q)$	Liquidity cost of order quantity q in percent of mid-price
$\gamma(q)$	Liquidity impact of size q (empirical net return model)
MV	Market value in million Euro
M	Magnitude of exceedances
μ	Mean
N	Number of traded shares in the market
n	$= q/P_{mid}$, position size in number of shares
P	Mid-price = middle of the bid ask-spread (in regression)
P_{mid}	Mid-price
P_{TA}	Price of a transaction
perc	Percentile of a distribution
$PI(q)$	Price impact costs of order quantity q in percent of mid-price

Symbol	*Definition*
Φ	Size-independent liquidity cost per share (Angelidis and Benos (2005) model)
q	Order quantity in currency units
Q	Traded volume in currency units
R	Continuous mid-price return (in regression)
r	Continuous mid-price return
RSIGMA10	10-day backward, rolling variance (in regression)
$rnet$	Net return = mid-price return net of liquidity costs
S	Bid-ask-spread in percent of mid-price
$S(q)$	Spread of order quantity q (Francois-Heude and Van Wynendaele (2001))
SCOM	Sample average half bid-ask-spread in basis points (in regression)
$stat$	Cross sectional statistic
σ	Standard deviation
t	time
$T(q)$	Direct trading costs of order quantity q in percent of mid-price
θ	Degree of information asymmetry (Angelidis and Benos (2005) model)
θ	Price impact per share traded (all other models)
VO	Market trading volume in number of traded shares (in regression)
vol	Trading volume in currency units
v	Order quantity of single limit order
$WS(q)$	Weighted spread of order quantity q
x	Risk factor
z_α	Normal-distribution percentile at confidence $1-\alpha$
$\hat{z}_\alpha$	Empirically estimated distribution percentile at confidence $1-\alpha$
$\tilde{z}_\alpha$	Cornish-Fisher-estimated distribution percentile at confidence $1-\alpha$
$z_{t,\alpha}$	T-distribution percentile at confidence $1-\alpha$

1 Introduction

1.1 Delimitation and relevance of topic

Liquidity has lately received much attention in the academic world and in practice.[1] In reality, a stock position cannot be bought or sold without cost or delay in execution. The most important cost is the spread, the difference between the achievable transaction price and the fair price of a stock. This spread serves as important measure of the liquidity of an asset. Moreover, if volume traded in the stock is not large enough, the investor has to delay his trade, which induces further costs. From an investor perspective, the liquidity of an asset can be measured by the total cost required to trade a position in an asset.

In general, the term 'liquidity' is used in three different settings.[2] First, liquidity can designate the liquidity of a firm, also called solvency. From the corporate perspective, solvency is the net liquidity of assets and liabilities. Liquidity of the liability side is also called 'funding liquidity'. Second, liquidity is a characteristic of an asset, also called 'asset liquidity' or 'market liquidity' depending on whether the balance sheet or the market is in focus. From an investor's perspective it describes the marketability or ease of trading an asset[3]. Third, liquidity is also used from a monetary perspective and addresses the liquidity of the whole economy. This thesis addresses issues of market and asset liquidity, which have more recently been brought into focus.

[1] For this section, cp. Stange and Kaserer (2008a,c,b); Ernst, Stange and Kaserer (2008, 2009).

[2] Extended from Jorion (2007), p. 334.

[3] Cp. also Longstaff (1995).

Many recent crises have been liquidity crises. The two large hedge fund breakdowns of LTCM in 1998 and Amaranth Advisors in 2006 were mainly caused, because they took positions that were too large to be liquidated without substantial price impact.[4] In the recent sub-prime crises of 2007/08 banks around the world were troubled by funding liquidity shortages and had to liquidate assets to reduce risk exposure. Stock prices slumped because many funds were forced to sell-off positions due to margin calls and fund outflows.

The regulators are alert and the Basel II committee has already published several reports and guidelines on liquidity in recent months. Banks are requested to "use appropriately conservative assumptions about the marketability of assets" and "incorporate liquidity costs, benefits and risks in the internal pricing, performance measurement and new product approval process for all significant business activities"[5]. Still, a BIS survey among banks revealed, that market liquidity remains the single risk factor across all asset classes, that is not easily captured.[6]

Today, the most popular tool to measure, control and manage financial risk within corporations and financial institutions is the Value-at-Risk (VaR) concept.[7] VaR measures the worst expected loss over a given horizon and a certain confidence level. In most institutions the standardized VaR-methodology is used to determine capital requirements.[8]

One often criticized downside of the traditional VaR-model is its inability to capture liquidity risk, because its computation generally relies on market prices.[9] Due to the neglect of liquidity risk the real risk of an institution is generally underestimated.[10] In this context, liquidity risk, more specifically market liquidity risk, can be understood as the difficulty or cost of trading assets in crises. Market liquidity risk has

[4] Cp. Jorion (2007).
[5] Cp. Basel committee (2008), p. 6 and p. 9.
[6] Cp. Basel committee (2005), p. 10.
[7] Cp. Dowd (2001), pp. 4-5.
[8] Cp. Basel Committee on Banking Supervision (1996).
[9] Cp. Jorion (2007) p. 333.
[10] Cp. Bangia et al. (1999); Stange and Kaserer (2008c) and others.

to be distinguished from funding risk, which is the potential shortfall of meeting liabilities and having sufficient cash available.

Market liquidity risk has already acquired a great deal of attention. During the last few years several academic papers have been written on the consideration of liquidity risk in the VaR framework. The proposed solutions can be classified into two groups: The first one focuses on indirect risk measures by determining price quantity functions from transaction data. In this stream, the approaches of Cosandey (2001), Jarrow and Protter (2005a), Berkowitz (2000b), Jarrow and Subramanian (1997) and Almgren and Chriss (2000) are widely cited. In contrast, the second group makes use of direct liquidity cost measures such as the bid-ask-spread or the order-size-dependent weighted spread. For instance Bangia et al. (1999), Francois-Heude and Van Wynendaele (2001) and Giot and Grammig (2005) propose models that can be classified into this latter category.[11]

A major issue in all liquidity risk models is the assumed distributional properties of asset returns and liquidity measures. For reasons of simplicity most often either a normal or empirical distribution is used. Since the distributions of continuous asset returns and liquidity costs are often skewed and leptocurtic or platycurtic, inappropriate normality assumptions necessarily lead to incorrect risk estimates. The use of empirical distributions might also be suboptimal, because large data sets are required and historical distributions might poorly proxy for the future.

In other risk management contexts, non-normal distributions have already been addressed. Zangari (1996) and Mina and Ulmer (1999) suggested and analyzed the Cornish and Fisher (1937)-approximation as method to account for the non-normality in the case of derivatives. Favre and Galeano (2002) propose to apply this method to hedge fund risk, Lee (2007) use it in the context of real estate asset allocation.

Another major issue is the precise integration of the price impact of order size, i.e. the fact that liquidity costs rise with the position size

[11] Detailed description in section 2.2.

traded. Simple models such as Bangia et al. (1999) neglect price impact, other models try to approximate it, for example Cosandey (2001), Jarrow and Protter (2005a) and Berkowitz (2000b). More recently, a new price impact measure has been brought into the discussion. Irvine et al. (2000) suggested to use the price impact implicit in the limit order book of exchanges, a measure also called weighted spread. While weighted spread has been used by Giot and Grammig (2005) to analyze intraday variation, the aspects of size impact as such and the mechanics of precise liquidity integration have not been addressed.

Both issues, distributional assumptions and precise integration of price impact of order size have not been resolved yet.

1.2 Research questions and contribution

This thesis aims to clarify, how liquidity risk can be precisely integrated into a risk measurement framework. It concentrates on two issues, the problem that liquidity is non-normally distributed, and the precise measurement of price impact. Specifically, I address the following research questions:

1. How can the Cornish Fisher approximation account for non-normality in liquidity risk?
   a) How can a liquidity risk model on the basis of the Cornish Fisher approximation be set up?
   b) Does this approach yield more precise results than existing methods?
2. How can weighted spread as price impact measure be precisely integrated into a risk management framework?
   a) In which situations is weighted spread a valid liquidity measure?
   b) What are the distributional properties of weighted spread?

   c) Is the liquidity risk impact of order size substantial enough to justify the use of this type of data - especially at longer forecast horizons, where the importance of liquidity declines?

   d) What is the effect of liquidity-return correlation on risk measurement, a commonly discussed issue?

3. How do liquidity risk models compare empirically with respect to their preciseness?

   a) Which model preforms best and can be recommended?

   b) Are there structural differences in the precision of risk forecasts on the basis of data used, position size or market segment?

The hypothesis implicit in these research questions will be empirically tested in a large sample of daily stock data.

The relevance of these questions is quite apparent. The Cornish-Fisher method has been helpful in other areas of risk management and is therefore a promising candidate for liquidity risk measurement. The use of the weighted spread as liquidity cost measure is relatively new in risk management and its structure and precise application have not been clarified yet.

An empirical comparison of liquidity risk models has not been conducted in academia so far. Its results will help to judge which models should be used in practice. Comparative backtests also identify which simplifying model assumptions have the largest distorting effects, which can provide impetus for promising directions of future model development.

1.3 Structure of analysis

Chapter 2 provides an overview of the existing literature on liquidity and liquidity risk, which includes an overview of all relevant aspects of market liquidity. It also surveys existing liquidity risk models and discusses their explicit and implicit assumptions. In chapter 3, I present a description of the data set used for the empirical analysis as well as a discussion of the characteristics of the new liquidity measure, weighted spread. On this basis, chapter 4 proposes two new liquidity risk models. The modified add-on model is a new approach to account for non-normality in liquidity risk. The net return approach with weighted spread suggests a framework to analyze the importance of weighted spread as risk measure as well as the question of precise measurement of price impact. Chapter 5 contains the empirical test of the newly suggested liquidity risk models and a comparison with other approaches. Chapter 6 summarizes, concludes and outlines questions for further research.

2 Background and existing literature

This chapter provides an overview of the existing literature on market liquidity and its risk. The discussion of market liquidity risk requires an understanding of the characteristics of liquidity itself that are relevant from a risk perspective. Section 2.1 clearly defines liquidity, describes its characteristics, and surveys existing market liquidity measures.

Based on an understanding of market liquidity, section 2.2 turns to market liquidity risk. It provides a general liquidity risk definition followed by detailed descriptions of existing liquidity risk models. I also analyze these existing models from a theoretical perspective and clarify their explicit as well as implicit assumptions. The section concludes with a model overview.[1]

2.1 Background on market liquidity

2.1.1 Definition of market liquidity

Market liquidity can be defined as the cost of trading an asset relative to fair value.[2] Fair value is set at the middle of the bid-ask-spread, the mid-price. This has the advantage that it is most objective, but the disadvantage, that the fair, fundamental value fluctuates heavily, which is slightly less intuitive.

[1] For this chapter, cp. Stange and Kaserer (2008b).
[2] Cp. Dowd (2001), p. 187 ff. and Buhl (2004); Amihud and Mendelson (2006).

I distinguish three components of liquidity cost $L_t(q)$ in percent of the mid-price for an order quantity q at time t[3]

$$L_t(q) := T(q) + PI_t(q) + D_t(q) \tag{2.1}$$

where $T(q)$ are direct trading costs, $PI_t(q)$ is the price impact vs. mid-price due to the size of the position, $D_t(q)$ are delay costs if a position cannot be traded immediately.

Direct trading costs comprise exchange fees, brokerage commissions and transaction taxes. They are also called explicit transaction costs, because they are known beforehand and time invariant, i.e. deterministic.[4] The *price impact* is the difference between the achieved transaction price and the mid-price.[5] They result from imperfectly elastic demand and supply curves for an asset at a specific point in time, which makes the price impact increase with the size traded.

Liquidity costs increase with order size for two reasons. First, investors have heterogeneous expectations with respect to the fair value of an asset and are subject to capital restrictions. They are therefore willing to trade only a limited quantity at their own prespecified price. When trading a small position, a trader is likely to find a counterparty which is willing to exchange the full position at or close to the trader's fair value expectation. The larger the position to be traded, the more counter-parties have to be found. The achievable transaction price falls. Compared to the trader's fair value expectation, the liquidation cost rises with the size of the position. Second, liquidity costs are also a price for immediacy. An immediate transaction at a certain price is essentially an American option paired with an exchange.[6] The option component comprises the

[3] This closely follows Amihud and Mendelson (2006), but additionally differentiates by the size of the position. Compare also similar in Aitken and Comerton-Forde (2003); Torre (1997).

[4] Also cp. Loebnitz (2006), p.18 f.

[5] Similarly Demsetz (1968) defines transaction cost as the price concession needed for an immediate exchange of an asset into money (p.35). This is also called market impact.

[6] Cp. Chacko et al. (2008).

right to receive a certain amount of shares at order execution with the current market price as strike. This optionality has an immanent value, which depends on price volatility and the order size relative to expected transaction volume, because this determines the future liquidity of the position for the buyer. Due to these two components, price impact cost can be expected to rise with the size of the position.

Delay costs comprise the cost for searching a counterparty and the cost imposed on the investor due to bearing risk, because prices and price impact cost might change during the delay.[7] For many assets, like most stocks and bonds on an exchange, search costs are negligibly small, but costs of additional risk during delay can remain substantial.

Because liquidity costs increase with size, a trader faces a possible trade-off between price impact cost and delay. He can save on price impact cost by deliberately delaying parts of the transaction. But then he has to face delay risk for the delayed portion of the position. This deliberate delay is optimal if the savings on price impact costs exceed the additional delay cost. These strategies are analyzed in the literature on optimal trading strategies.[8] As a consequence, there are two types of delay, forced and deliberate. *Forced delay* occurs if a position can currently not be traded in the market. *Deliberate delay* occurs if the trader does not want to trade a certain position strategically, because he expects savings on total liquidation costs.

Relation to other liquidity definitions Above cost definition takes a practical, concrete investor's perspective and can integrate other definitions in the literature. In my view, it also provides a suitable framework to integrate the multitude of available perspectives and makes liquidity a less elusive concept.

[7] Almgren (2003) calls price impact risk "trading enhanced risk".

[8] Cp. for example Bertsimas and Lo (1998); Almgren and Chriss (1999, 2000); Almgren et al. (2005); Almgren (2003); Subramanian and Jarrow (2001) and the discussion in section 2.2.2.4.

The most often cited dimensions of liquidity are tightness, depth, resiliency and immediacy.[9] They can be easily understood in above cost framework. *Tightness*, "the cost of turning a position around in a short time", corresponds to the sum of direct trading costs T and price impact costs PI. *Depth*, "the size of an order flow innovation required to change prices a given amount", is the quantity q transactable at a specific price impact PI, i.e. $PI^{-1}(q) = q(PI)$. *Resiliency*, "the speed with which prices recover from a random, uninformative shock", is the mean reversion speed of liquidity cost after a shock, i.e. the time dimension of liquidity cost. *Immediacy*, the time between order submission and settlement, directly corresponds to the delay time of the delay cost component D. Thus, all four dimensions can be analyzed in the cost framework introduced above.

In the cost framework, liquidity is the effect a transaction has on an investor. The importance of other, more indirect liquidity measures like transaction volume, zero trading days, depth, etc.[10] can be much better understood from a cost perspective. If a liquidity aspect results in high liquidity costs in economic downturns, it will have a large effect on asset prices. The cost perspective provides the economic explanation for the validity of many liquidity measures.[11]

Kempf (1999) defines liquidity in more abstract terms and cites the dimensions price and time. Price directly corresponds to cost, but time should - in above view - also be converted into a cost component via delay costs. While time is a more direct aspect of liquidity, its conversion into cost make it more concrete from an investor's perspective. Longstaff (1995) defines liquidity as "the ease of trading an asset", which is similarly abstract and needs to be broken down into more tangible aspects as suggested above.

[9] Cp. Kyle (1985), p. 1361 for the first three dimensions and the citations and Black (1971), p.30 for the latter. Tightness is also sometimes called 'width' or 'breadth'.
[10] Cp. Datar et al. (1998); Liu (2006); Bekaert et al. (2007); Goyenko et al. (2008) and others.
[11] Cp. Stange and Kaserer (2008a), p.4.

2.1.2 Overview of important aspects

2.1.2.1 Degrees of market liquidity

Liquidity is a continuous characteristic. Hence, assets can have different degrees of liquidity.[12] The liquidity degree is determined by the type of the asset, the size of the position and the liquidation horizon. It is useful to distinguish at least four categories of liquidity degrees as illustrated in figure 2.1 on the following page. They are closely related to the magnitude of liquidity costs and require substantially different treatment.

If an asset is 'fully liquid' any position in the asset can be immediately traded without a cost. Cash is the primary example. For practical purposes, liquidity adjustments to its value are not necessary. An asset can be called 'continuously tradable' when most positions can be traded albeit with a cost. A good example are limit order books of developed stock markets. The determination of the costs of trading is the main issue from a liquidity perspective. If liquidity deteriorates further, the asset becomes 'disruptively tradable', i.e., it can be traded from time to time. While market price provides an indicator for the fair value of the asset, delay and its incorporation into liquidity measures is a major issue - in addition to trading costs. A good example are over-the-counter markets of exotic bonds. Finally, an asset is 'illiquid' if no position size can be traded. Market prices are thus non-observable and value has to be determined by intrinsic methods. Rare art or currently collateralized debt obligations can be considered illiquid.

Not only the type of the asset, but also the size of the position determines the degree of liquidity. In most cases, it is the position size relative to the prevailing trading volume, that determines the degree of liquidity, which also shows the relation between asset and market liquidity. Is the position size much larger than traded volume, we can expect significant trading delay. The asset position is only interruptedly tradable. If it is

[12] Cp. also discussion in Stange and Kaserer (2008a), p. 4f.

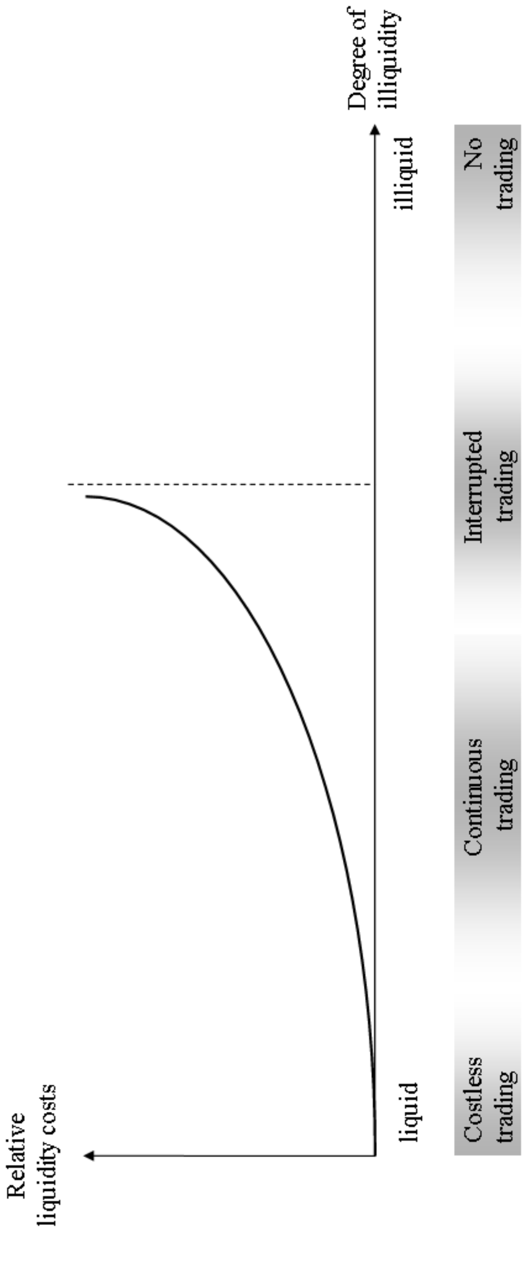

Figure 2.1: Degrees of market liquidity

Figure 2.1 illustrates the different degrees of market liquidity and the resulting liquidity categories.

too large, it might even be illiquid in the short term due to the lack of counterparties.

The liquidation horizon is another determinant of a position's liquidity degree. A security might be illiquid in the short term because of a lack of counterparties, but interruptedly tradable at longer liquidation horizons. If an asset is held to maturity, then, obviously, liquidity costs are zero and irrelevant, because they are a transaction feature.

2.1.2.2 Characteristics of market liquidity

When measuring market liquidity, ex-ante *committed* liquidity and possible *hidden* liquidity have to be distinguished.[13] The advantage of market organization on the basis of order books lies in the fact, that more liquidity is ex-ante committed and transparent to market participants.

The price impact component of asset liquidity can be described in a price-quantity diagram, which collects all potential counterparty orders with their order size and their willingness to pay. In case of committed liquidity in a limit order book, these are limit orders. These counterparty orders, if sorted by best price construct the buy- or sell-price function. The cost of liquidity of a round-trip[14] can be then described by a price-quantity function, which is the difference between the buy- or sell-price function and the mid-price as displayed in figure 2.2 on the next page. The trader buys at the buy price function and sells at the sell price function. The difference between the two is the liquidity cost from the transaction.

For small orders, not larger than the quote depth, this cost of a round-trip corresponds to the bid-ask-spread. For larger orders the liquidity cost of a round-trip is the weighted spread between the buy- and sell-side functions up to the traded quantity. The spread of the individual limit orders are weighted with their respective limit order quantity. In

[13] Cp. Irvine et al. (2000).
[14] I.e. buying and immediately selling a position.

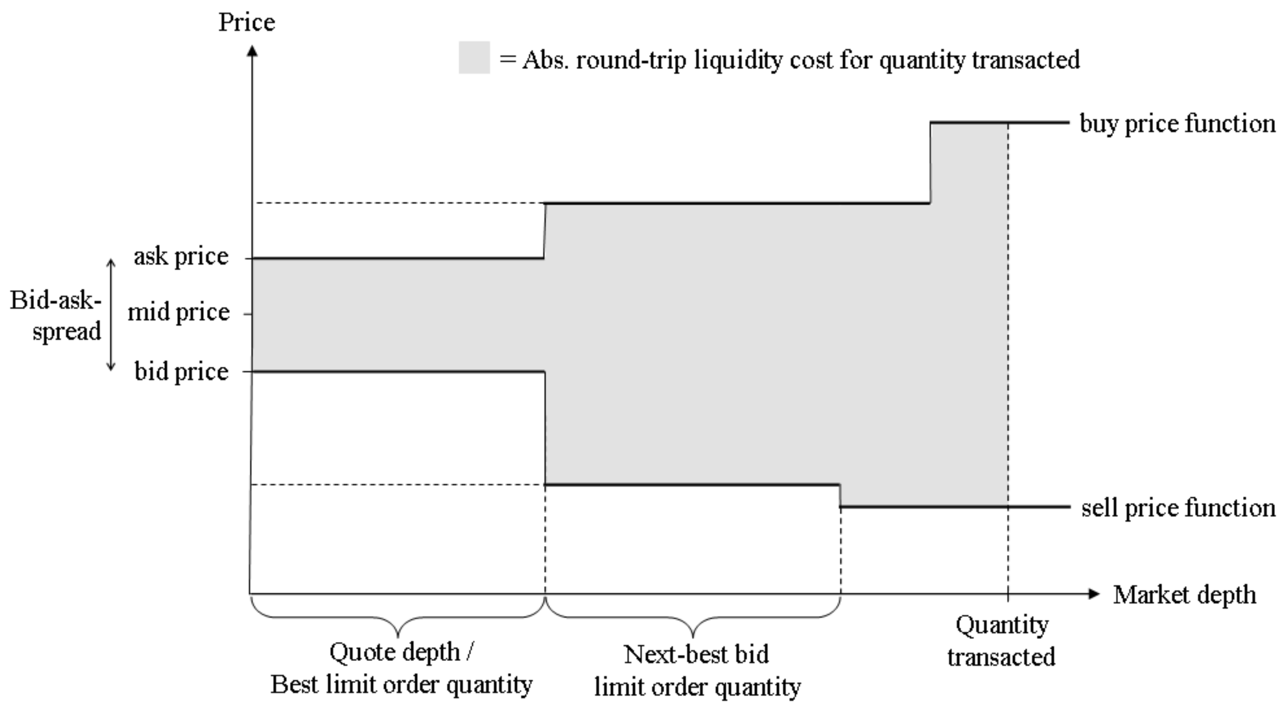

Figure 2.2: Price quantity function

Figure 2.2 illustrates the price impact function as difference between the buy-price and the sell-price function.

general, this weighted spread is called 'price impact'.[15] Because the limit order book only measures committed liquidity, due to hidden liquidity, transactions can and do occur inside the bid-ask-spread. Therefore the commonly used quoted-spread as well as the weighted spread measure ex-ante committed liquidity.

Up to the quote depth, liquidity costs are sometimes called *exogenous* and beyond *endogenous*.[16] It is argued, that bid-ask-spread up to the quote depth is exogenous, because it is common to all market participants while the weighted spread is endogenous depending on the individual trader's position. I believe that this argument is imprecise with respect to the structure of liquidity costs. The whole price impact curve is exogenously given, because it is determined by the market. This is also true beyond the quote depth. The size of the trade (endogenously) determines the point on the curve valid for a specific trade. In this way, the bid-ask-spread is also endogenous - determined by a very small specific trade position. As a consequence, the cost itself is neither exogenous nor endogenous - at any size - but can be decomposed into an exogenous price-quantity curve and an endogenous point on this curve.

A possible cause for this misleading distinction is the usual graphical representation, which shows a flat price impact curve similar to the display above, but a continuous increases of liquidity cost beyond the spread. This falsely implies that liquidity costs would be structurally different beyond the spread.

Above graphical display necessarily neglects the temporal dynamics of liquidity. Important is the distinction between temporary and permanent price impact.[17] *Temporary price impact* is the portion of the price impact, that will dissipate over time and is closely related to the notion of resiliency.[18] It is driven by order imbalances when trades are purely motivated by liquidity needs. Temporary price impact might also

[15]A detailed description will be provided in section 3.1.1.1.
[16]Cp. Bangia et al. (1999), p.68 f., also in Jorion (2007), p. 336 or Bervas (2006), 3.
[17]Holthausen et al. (1987) first introduced this setup.
[18]Cp. section 2.1.1..

occur under information asymmetries, if the market reacts on perceived informational content, i.e. it occurs due to adverse effects. *Permanent price impact* is the portion of the price impact that will permanently move mid-prices. In an efficient market, the permanent part is directly related to the true informational content of the trade. Measurement of temporary and permanent price impact separately is still difficult.[19]

2.1.3 Existing liquidity cost measures

How can one measure the cost of trading a position? Academic literature has brought forward a multitude of cost measures. Starting with Roll (1984) and Amihud and Mendelson (1986), many papers have analyzed variants of the bid-ask-spread, data which is easily available. But this measure neglects that spread differs for different order sizes. Only small positions, smaller than the bid-ask-depth, can be traded at such a cost.

Larger positions incur larger costs, the price impact (of the position's size). Initially the price impact was measured with proxies.[20] The problem with estimating liquidity cost ex-post from transaction prices is to distinguish between the informational and the liquidity component in the price change. Pastor and Stambaugh (2003) used a method based on price change with subsequent reversals, but were not able to distill stock-specific liquidity measures.

More recently, a direct method of measuring size-specific spread has been used. When order book data is available, the price of instant liquidity for a position of a certain size can be extracted as weighted spread from the limit order book. Under the assumption that a position is transacted as a market-order against available limit-orders, the difference between the realized price and the mid-point of the bid-ask-spread measures the price impact of the trade due to liquidity. As this is an ex-ante measure of committed liquidity, informational effects of a trans-

[19] Cp. Amihud (2002); Pastor and Stambaugh (2003), who try to extract temporary price impact from prices.

[20] Cp. for example Kyle (1985); Amihud (2002); Brennan and Subrahmanyam (1996).

action cannot play a role here. Exchanges increasingly use transparent, electronic limit order books, for example the London Stock Exchange, the Nasdaq, the Frankfurt Xetra, the Euronext or the Australian Stock Exchange. They also start to make these weighted spread data available to researchers and practitioners. Hence, the above method of calculating liquidity becomes more generally applicable.

Several papers have already used this new method of measuring liquidity costs as displayed in table 2.1. Irvine et al. (2000) use the cost of a round trip for trades of various sizes as a liquidity measure, which they compare to quoted and effective spread. Empirically, they show that the measure is correlated with other measures of liquidity and that it predicts the number of trades of a certain size. Coppejans et al. (2001) employ a similar measure to analyze the relation between market liquidity, returns and volatility in an intraday sample. They reveal a large inter-temporal variation and show that liquidity is concentrated on certain points in time. Coppejans et al. (2004) discuss the stochastic dynamics of liquidity with a measure similar to the cost of a round trip and find a negative relation to volatility and a high degree of resiliency, i.e. high mean reversion speed of liquidity prices after shocks. Domowitz et al. (2005) employ the cost of round trip to analyze liquidity commonality and show that market liquidity and returns can remain uncorrelated because they are caused by different economic forces. While liquidity is driven by liquidity supply and demand (i.e. cross-correlation between limit and market orders), returns are driven by correlation in order flow (i.e. order direction and size). Gomber et al. (2004) extract weighted spread from the limit order book to show that resiliency is generally high after liquidity shocks and public information has negligible impact on liquidity. They also show that large transactions are timed on periods with high liquidity.

Paper	Topic	Size of sample
Irvine et al. (2000)	Comparison of several measures	263 (TSE), 2 months, 1996
Buhl (2004)	Liquidity in risk management	10 stocks (SMI), 3 months, 2002
Giot and Grammig (2005)	Liquidity risk	3 stocks (DAX), 3 months, 1999
Francois-Heude and Van Wynendaele (2001)	Liquidity risk	1 stocks (CAC40), 4 months, 1999
Gomber et al. (2004)	Resiliency of liquidity	21 stocks (DAX and others), 1 month, 2002
Beltran-Lopez et al. (2006)	Principle component analysis of the order book	30 stocks (DAX), 3 months, 2004
Coppejans et al. (2001)	Market liquidity and volatility	Swedish futures, 7 months, 1995-96
Biais et al. (1995)	Liquidity supply and demand	40 stocks (CAC40), 1 month, 1991
Domowitz et al. (2005)	Liquidity and order flow	19 stocks (ASX), 10 months, 2000

Table 2.1: Overview of empirical papers on order-size-differentiated liquidity

2.2 Existing market liquidity risk models

2.2.1 General definition of market liquidity risk

Traditional risk measurement assumes that liquidity costs can be neglected if the liquidation horizon is long enough.[21] Therefore, there is no adjustment for liquidity costs in many practical market valuation models: Liquidity cost is assumed to be zero and positions to be liquidated at mid-prices.

Liquidity risk can generally be defined as the potential loss due to time-varying liquidity costs. Several empirical papers have already shown that liquidity risk is a substantial risk component, already when only cost at the bid-ask-spread level is accounted for. Bangia et al. (1999) find underestimation of total risk by 25-30% in emerging market currencies in daily Value-at-Risk. Le Saout (2002) estimates that the bid-ask liquidity component can represent over 50% of total risk for illiquid stocks. Lei and Lai (2007) reveal a 30% total intraday risk contribution by liquidity in small-price stocks.

Also, the adjustment for the full price impact cost - beyond the spread - is significant. Francois-Heude and Van Wynendaele (2001) find a 2-21 % contribution of price impact in one stock. Giot and Grammig (2005) show that 30-minute liquidity-adjusted VaR is 11-30 % for three stocks. Angelidis and Benos (2006) estimate that liquidity risk constitutes 11 % of total VaR in low capitalization stocks.

Time horizon The time horizon in the VaR framework is usually the time required to orderly liquidate an asset. It is differentiated between asset classes but usually assumed constant within one asset class such as stocks.[22]

There is an important conceptual distinction to be made when defining 'horizons' in the liquidity risk management framework. The *reaction*

[21] Cp. Jorion (2007), p. 333.
[22] Cp. for example Jorion (2007), p. 24.

horizon is the time until management takes a decision vis-a-vis the liquidation of an asset, while the *liquidation horizon* is the period during which the position is liquidated. Although this distinction is usually neglected, it has important consequences. Usually, the horizon is used as a forecast period. Based on this information a decision is taken now, i.e. the reaction horizon is zero and the liquidation horizon is equal to the forecast period. Although the position is said to be orderly liquidated during the liquidation horizon, its worst value is calculated for the end of the liquidation horizon, which is logically inconsistent but conservative.

When directly adjusting for liquidity risk, it is possible to be more precise and logically consistent. However, 'horizon' then has to be distinguished into above aspects.

2.2.2 Model overview and evaluation

The choice of liquidity risk model strongly depends on the purpose as well as the type of asset position in question. In the following, I will look at models for regular risk measurement, which are not necessarily suitable for stress testing. If intraday forecasts are not aimed for or the integration of intraday data is too computational intensive, several models based only on intraday data are ruled out.

For the choice of an appropriate liquidity risk model, assets on the balance sheet have to be categorized according to the following three criteria: General degree of market liquidity, typical size of a position and data availability.

What is the general liquidity degree of the asset? If the asset is continuously traded, liquidity cost models are in focus, if it is only traded with large interruptions, models incorporating execution delay have to be applied. If the asset is illiquid, i.e. generally not traded, value has to be determined with internal models. The same is true, if data is hardly available or of limited quality, e.g. in some over-the-counter markets. Internal value models and possible liquidity adjustments therein are outside the focus of this thesis.

How large is the typical position size relative to traded volume? If sizes are relatively small, models which neglect the price impact of position size can be applied, i.e. models based on bid-ask-spread data. If sizes get larger, these models are naturally imprecise. If positions are especially large, like block holdings, even models which incorporate price impact will loose precision.

What type of data is available? The precision of the price impact measurement depends directly on the amount of data available. On the basis of spread data, price impact is generally neglected. On the basis of transaction data, price impact approximations are possible.[23] With limit order book data, price impact can be quite precisely estimated. The type of data determines the liquidity measure than can be used.

In the following, I will introduce relevant liquidity risk models and indicate, which assumptions are made and when they can be applied. I want to emphasize at this point, that my discussion is based on my very own interpretation of the liquidity risk models, because many aspects I point out are only implicit in the model structure and not explicitly discussed by the original authors. If possible, I used my own, consistent notation to allow for better comparisons between the different models.

2.2.2.1 Models based on bid-ask-spread

Add-on model based on bid-ask-spread: Bangia et al. (1999) Bangia, Diebold, Schuermann and Stroughair (1998, 1999) include time-varying, empirical bid-ask-spreads into a parametric Value-at-Risk

[23] Cp. also Erzegovesi (2002), p. 9 ; Torre (1997) argues that large costs cannot be observed because trades at such cost are not executed and transaction data is most sparse in illiquid assets where expected price impact are largest.

(VaR). Transaction price is modeled as mid-price with an add-on for the bid-ask-spread,

$$P_{mid,t+1} = P_{mid,t}exp(r_{t+1}) - \frac{1}{2}P_tS_{t+1} \tag{2.2}$$

where P_{mid} is the middle of the bid-ask-spread, r is the continuous mid-price return between t and t+1 and S is the time-varying bid-ask-spread. Relative liquidity-adjusted total risk (L-VaR) is then the sum of the mean-variance-estimated price-risk percentile and the empirically-estimated spread percentile.

$$L - VaR = 1 - exp(z_\alpha\sigma_r) + \frac{1}{2}P_{mid}\left(\mu_S + \hat{z}_\alpha\sigma_S\right) \tag{2.3}$$

where σ_r is the volatility of the continuous mid-price return and μ_S and σ_S are the mean and volatility of the bid-ask-spread. z_α is the α-percentile of the normal distribution, $\hat{z}_\alpha$ is the empirical percentile of the spread distribution. As spread is not normally distributed, it is not possible to take percentiles from theoretical distribution tables. Therefore, Bangia et al. take the percentile of the empirical spread distribution, which ranges - in their 99% case - between 2.0 and 4.5, which is partially far away from 2.33, the 99% cut-off of the normal distribution.

Bangia et al. (1999) also address the problem of moving from single asset to portfolio VaR. They argue, that aggregating single asset L-VaRs by using the spread covariance matrix is of dubious value, because spreads are non-normally distributed. Instead, they suggest to aggregate single asset's price risk in a more traditional way and then deduct a weighted average spread from the portfolio VaR. Single currency and portfolio L-VaRs are calculated as illustration in their paper. Other empirical applications of their model include Mahadevan (2001), Lei and Lai (2007) and Roy (2005).

The great advantage of the methodology of Bangia et al. is the low data requirement. Spread data is available at all frequencies for most assets, often also in over-the-counter (OTC) markets. It is also quickly implementable, because the liquidity-adjustment can be simply added to existing price risk measures.

The greatest drawback is the neglect of price impact, the fact that only small order sizes can be traded at the spread and liquidity costs quickly increase with order size. As consequence, liquidity risk can be heavily underestimated for large positions.

Further, their add-on approach is logically inconsistent, because spread is calculated on the current mid-price and not on the crises mid-price, which is however easily correctable.[24] Bangia et al. also make the assumption of perfect tail correlation between spread and price, i.e. they assume that worst liquidity costs and lowest prices occur simultaneously. Because tail correlations can be much lower in reality, this technical assumption probably overestimates liquidity risk.[25]

Another problem is the estimation of the spread distribution. As stated in their paper, spreads are often far from normal, because regime-switching leads to multi-modality and because trending creates skewness and fat tails.[26] Accounting for non-normality by using empirical percentiles remains difficult, because this requires longer time series as a basis for estimation, which might themselves exhibit structural breaks with several modi. Structural breaks might especially occur in crises. These distributional properties make further underestimation of liquidity risk highly likely.

Although, the Bangia, Diebold, Schuermann and Stroughair (1999)-model suffers from several imprecisions, it is one of the few models of choice, when data is scarce, especially on transaction volumes or transactions. I recommend to keep the add-on approach under the assumption of perfect correlation, because this (partially) compensates the tendency

[24]Critique noted and corrected by $L - VaR = 1 - exp(\alpha\sigma_r) \times 1/2(\mu_S + \tilde{\alpha}\sigma_S)$ in Loebnitz (2006), p.71 f.

[25]Cp. critique in Francois-Heude and Van Wynendaele (2001); Angelidis and Benos (2006); Jorion (2007).

[26]Cp. discussion of the distributional characteristics of spread in section 3.3.4.

to underestimate due to the neglect of position size and the empirical approximation of percentiles.

2.2.2.2 Models based on volume or transaction data

Transactions regression model: Berkowitz (2000) Berkowitz (2000a,b) estimates the liquidity price impact from past trades. While controlling for the influence of other risk factors, price impact is measured from the time-series of trades in a linear regression.

$$P_{TA,t+1} = P_{mid,t} + C + \theta N_t + x_{t+1} + \epsilon_t \tag{2.4}$$

where $P_{TA,t+1}$ is the transaction price at time $t+1$, N_t is the number of shares sold, θ is the regression coefficient, x_{t+1} is the effect of risk factor changes on the mid-price, C is a constant and ϵ_t the error term of the regression. The regression coefficient θ acts as liquidity measure and can be seen as the absolute return due to changes in volume, i.e. the absolute liquidity cost per share traded.

To construct a liquidity-adjusted risk measure in a convenient way, Berkowitz assumes that liquidity and other risk factors are independent from each other, which is equivalent to zero liquidity-return correlation. They also build on Bertsimas and Lo (1998), who show that under linear price impact an optimal execution strategy within a horizon of h days is to liquidate $\frac{1}{h}$th of the portfolio each day during the liquidation period. Similar to equation (2.4), price then follows

$$P_{TA,t+1} = P_{mid,t} + x_{t+1} - \theta \frac{N_t}{h} \tag{2.5}$$

Risk can then be derived from the general probability distribution. The choice of concrete risk measurement (numerical, simulation, parametric) is left to the reader.

The advantage of the Berkowitz-approach is the integration of price impact of order size beyond the bid-ask-spread. While being more computationally extensive through the regression methodology, it only uses transaction data for the liquidity measurement, which is available in many markets. However, intraday data are required to calculate the price impact cost from single trades. Otherwise, the estimation can get very approximate.

The liquidity measure used in their approach, however, is quite imprecise. In general, it closely resembles the liquidity measure of Amihud (2002). Berkowitz additionally controls for risk factor changes in his empirical regression. One problem is, that θ can become positive or negative, which is counter-intuitive as size should always lead to a price discount. Further research should empirically verify in how far this measure proxies for real liquidity cost.

Also the liquidity concept as such has to be criticized. Berkowitz assumes linear, non-time-varying price impact, which is clearly not the case and most likely underestimates liquidity risk impact. The assumption of zero liquidity-return correlation in his risk estimates leads to further underestimation, because, empirically, positive correlations can be observed.[27] Further, as will be discussed at the beginning of section 2.2.2.4, I doubt that an optimal trading strategy applied above is as such a suitable approach in crises situation. A correction is however simple, because traded volume does not have to be divided by the liquidation horizon.

Overall, Berkowitz (2000a,b) provides an approach to integrate price impact of order size into a risk framework, but liquidity measurement remains highly approximate.

Crises transactions regression model: Jarrow and Protter (2005)

Jarrow and Protter (2005a) use a framework which is very similar to Berkowitz (2000a). Price impact is also measured in a regression from transaction data. However, they do not explicitly control for other risk

[27] See empirical analysis in section 5.2.3.

factors and only take a sample of crises transactions to derive a crises price impact coefficient.

$$log\left(\frac{P_{TA,t+1}}{P_{TA,t}}\right) = \left(\mu_{r_t} - \frac{1}{2}\sigma^2_{r_t}\right) + \theta_c(N_{t+1} - N_t) + \epsilon_t \qquad (2.6)$$

where μ_{r_t} and $\sigma^2_{r_t}$ are continuous mean and variance of the mid-price return, θ_c is the crises price impact coefficient and N_t is the number of shares traded at time t.[28] The restriction to crises introduces time-variation into the price impact which is neglected by Berkowitz. The additional, relative liquidity component in a VaR when selling a position immediately in crises can then be calculated as

$$VaR_L = 1 - \theta_c N \qquad (2.7)$$

where N is now the trader's quantity to be traded.[29]

The advantage of Jarrow and Protter (2005a) is the integration of time-varying price impact, because the crises coefficient approximates the distribution percentile of liquidity cost. The crises specific coefficient also implicitly accounts - at least in approximation - for the liquidity-return correlation in crises. Similar to the Berkowitz critique, this type of empirical liquidity measure remains generally highly approximate. Running the regression in crises periods only might, however, severely shrink the sample, which further reduces the validity of the liquidity estimate θ. Therefore, their approach is overall of similar value than Berkowitz (2000b).

Volume-based price impact: Cosandey (2001) Cosandey (2001) proposes a simple framework to estimate price impact from volume data. The price is a function of the number of shares traded, $P = Q/N$, where

[28] To keep notation consistent, I used the Greek letters from Berkowitz (2000b), which carry different meaning than the original Greeks in Jarrow and Protter (2005a).

[29] To simplify, I neglect that in the original paper the position is only partially liquidated.

Q is the (constant) quantity of money traded and N is the number of shares traded. Under the assumption, that traded amount of money Q is independent of a single trade, price including the impact of trading ΔN shares can be simply estimated as

$$P_{mid,t}(\Delta N) = \frac{Q}{N + \Delta N} = P_{mid,t} \times \frac{N}{N + \Delta N} \tag{2.8}$$

where the number of traded shares N is assumed to be constant over time. The trade fully increases the number of shares traded in the market. The price impact is thus assumed to be linearly related to relative traded volume. Relative liquidity-adjusted total risk can then be calculated as

$$L - VaR(\Delta N) = perc\left(r_{t+1} \times \frac{N}{N + \Delta N}\right) \tag{2.9}$$

where $perc$ determines the percentile from simulated distributions. The effect of mid-price change and order size is jointly modeled.

Cosandey (2001) already addresses his shortcoming of the linearity of the price impact function in (2.8) and proposes to model it as

$$P_{mid,t}(\Delta N) = P_{mid,t} \times \left(\frac{N}{N + \Delta N}\right)^{\frac{1}{a}} \tag{2.10}$$

where a is the - possibly time-varying - curvature parameter, but leaves its measurement to future research.

The approach of Cosandey offers a major improvement over Bangia et al. (1999), because the price impact of order size is accounted for. While the important determinant of order size is integrated, the integration of price impact remains simple and has very few data and computational requirements. Volume data are available for many markets and a large range of frequencies. However, not only single transaction data, as in Berkowitz (2000a) or Jarrow and Protter (2005a) are required, but the overall market volume. The linear implementation is simple and straight forward.

At the same time, the linearity of the price impact in the standard specification is one main source of imprecision. Empirically, price impact is shown to be concave, which makes a linear functional form overestimate liquidity risk for large order sizes.[30] Curvature parameters in this functional specification are difficult to measure, which makes this problem hard to solve in this setup.

The second reason for imprecision is the assumption, that the amount of trading in the market, N, does not vary over time. The dynamics of trading volume in crises might significantly alter the picture. The much cited 'flight-to-liquidity' effect can introduce complicated mechanics, because liquid assets improve in liquidity while illiquid assets deteriorate.[31] If this is consistently the case, the liquidity risk of more illiquid positions will be underestimated, which should be a major concern. As a conservative solution, trading volume can be assumed to dry up in crises, e.g. by assuming that trading volume falls to the lowest percentile of the volume distribution. But if this suggestion more precisely captures liquidity effects in reality is unclear. Overall, neglect of time variation is a problem difficult to solve.

Further, liquidity is assumed constant between stocks apart from differences in trading volume. However, section 3.3.5.1 will show, that liquidity cost also greatly vary with market capitalization. Integration of this fact might possibly capture flight-to-liquidity effects but requires further research.

In summary, Cosandey offers a framework, which can integrate price impact in a simple way, especially in markets where data availability is limited.

Structurally implied spread: Angelidis and Benos (2006) Angelidis and Benos (2005, 2006) develop an implied liquidity cost model from structural considerations, i.e. liquidity is traced to its underlying

[30] Cp. section 3.3.
[31] Cp. Longstaff (2004).

drivers. They combine an inventory model of a market maker with a fundamental model of information asymmetry. This yields an implied spread, where the impact of traded volume depends on the degree of information asymmetry and the price elasticity with respect to volume and a volume-independent minimum cost component.

$$L = \sqrt{N_t}(\theta + \kappa) + \Phi \tag{2.11}$$

where N_t is the absolute number of total shares traded, θ is the degree of information asymmetry, κ is price elasticity with respect to volume and Φ is the size-independent cost per share. The Greek letters are estimated from intraday data with a Generalized Method of Moments.

This liquidity measure is then integrated into relative VaR as add-on similar to the quoted spread in Bangia et al. (1999).

$$L - VaR = VaR + \left[(\theta + \kappa)\sqrt{N_t^{\alpha'}} + \Phi\right] \tag{2.12}$$

where VaR is mid-price risk and $N_t^{\alpha'}$ is the top α' percentile of traded volume.

Angelidis and Benos assume, that the individual position size of a trader dissipates in the volume of the market and does not increase total traded volume as long as the position size is smaller than traded volume. This is the opposite extreme to Cosandey (2001), who assumed, that the trader's volume fully increases traded volume. Angelidis and Benos take a less conservative approach. On the other hand, the assumption that liquidity cost is calculated for the top percentile of traded volume, probably captures the volume increase in the case of liquidation implicitly.

Angelidis and Benos (2006) provide a new approach of liquidity modeling by tracing liquidity cost to its underlying determinants. This allows to estimate liquidity even in markets, where other liquidity cost estimations are not available. However, their approach requires intraday data and heavy computations to get estimates for the structural coefficients.

For practical purposes the main question is, if the structural model is correct. If main liquidity effects are not captured, liquidity estimates will be strongly biased. I would hypothesize for example, that volume elasticity strongly varies over time, which is not captured. This might substantially influence results if these effects are of large magnitude. Also, the degree of information asymmetry can be expected to change over longer periods. Therefore, this model is probably most useful when calculating intraday risk.

The second critique addresses the mechanics of integrating liquidity into the VaR-approach. As discussed above, adding liquidity risk to price risk assumes perfect price-liquidity correlation, which might overestimate risk. Since the dynamics of volume are not fully researched yet, it is unknown if the assumption of increased volume in crises is really valid and if it is safe to assume, that the trader's position disappears in the generally increased market volume without additional impact.

Overall, Angelidis and Benos (2006) provide an interesting intraday model of liquidity risk, but relies on a large amount of intraday data as well as some strong structural assumptions. Testing the validity of the structural approach or empirically verifying the real dynamics of traded volume in crises could take this line of research to the next level.

2.2.2.3 Models based on limit order book data

Price impact from limit orders: Francois-Heude and Van Wynendaele (2001) Francois-Heude and Van Wynendaele (2001) estimate price impact of order size by using information from the limit order book. They suggest to estimate the price impact for a certain order size by interpolating the price impact function from the best five limit order quotes made available by the Paris Stock Exchange. This estimation of the spread $S(q)$ for a specific positions size q makes their approach quite precise, at least for smaller order sizes.

Relative liquidity-adjusted total risk is then calculated in the following intraday model

$$L - VaR(q) = 1 - exp(-z_\alpha \sigma_r)\left(1 - \frac{\bar{S}(q)}{2}\right) + \frac{1}{2}\left(S(q) - \bar{S}(q)\right) \quad (2.13)$$

where z_α is the normally distributed mid-price return percentile and σ_r the standard deviation of the mid-price return distribution. $\bar{S}_t(q)$ is the average spread in the market for order quantity q and $S_t(q)$ is the spread of the asset. Market spreads are subtracted from worst mid-prices. However, as market average spread and individual asset spread might differ, the second term tries to correct for this difference.

Because it seems logically inconsistent that the correction term is multiplied with current and not with worst mid-prices, I suggest to modify the risk term into

$$L - VaR(q) = P_{mid,t} \times \left[1 - exp(-z_\alpha \sigma_r)\left(1 - \frac{S_t(q)}{2}\right)\right] \quad (2.14)$$

which is simpler, more consistent and does not require average market spread data.

Still, time variation of liquidity is not accounted for in the Francois-Heude and Van Wynendaele (2001)-model, but could be similarly implemented as in Bangia et al. (1999) using mean and variance of the spread distribution. This would, however, require the estimation of liquidity cost distributions for all order sizes.

This approach generally requires intraday data to estimate the price impact function, which restricts its application to risk estimation at intraday frequencies. Also, the type of data described above needs to be available. A suitable degree of precision is restricted to order sizes that are not too large, because extrapolation much beyond the fifth limit order quote is approximate.

Overall, it is difficult to judge whether the increased preciseness through integration of price impact or the lacking time-variation dominate in a specific situation. If the approach of Francois-Heude and Van Wynendaele (2001) is used, I would suggest to integrate time-variation in a suitable way.

Price impact from weighted spread: Giot and Gramming (2005)
In order to address price impact, Giot and Grammig (2005) extend the idea of Bangia et al. (1999) by using spread data beyond the spread depth. They assume, that the position is immediately liquidated as market order against limit orders in the limit order book. Liquidity costs can then be calculated as the average weighted spread of those limit orders necessary to liquidate a certain position size. In this way, the liquidity costs of different order sizes can be extracted from the limit order book.

In detail, price impact is calculated as

$$WS_t(q) = \frac{a_t(n) - b_t(n)}{P_{mid,t}} \tag{2.15}$$

where WS is weighted spread in percent and q is the size of the position in mid-price value. $a_t(n)$ is the weighted ask price of trading n shares calculated as

$$a_t(n) = \frac{\sum_i a_{i,t} n_{i,t}}{n}$$

with $a_{i,t}$ being the ask-price and $n_{i,t}$ being the ask-volume of individual limit orders. Individual limit orders add-up to the size of the position, i.e. $\sum_i n_i = n = q/P_{mid}$. $b_t(n)$ is defined analogously as

$$b_t(v) = \frac{\sum_i b_{i,t} n_{i,t}}{v}$$

where $n_{i,t}$ being the ask-price and $n_{i,t}$ being the ask-volume of individual limit orders.

The liquidity measure defined above can be used to calculate the net return, return net of liquidity cost at time t over horizon h as

$$rnet_t(h,q) = r_t(h) \times (1 - \frac{WS_t(q)}{2}) \tag{2.16}$$

where $r_t(h)$ is the h-period mid-price return at time t. Net return including price impact is then integrated in a parametric, intraday VaR-framework. Relative liquidity-adjusted total risk over horizon h is estimated by using tails of the student distribution as

$$L - VaR(h,q) = 1 - exp\left(\mu_{rnet(h,q)} + z_{t,\alpha}\sigma_{rnet(h,q)}\right) \tag{2.17}$$

where $\mu_{rnet,t}$ is the mean and $\sigma_{rnet,t}$ is the volatility of net returns, while allowing for diurnal variation of spreads and time-varying clustering of return volatility by modeling conditional heteroskedasticity.[32] $z_{t,\alpha}$ is the α-percent percentile of the student distribution.

The main advantage of using weighted spreads is the precise modeling of the price impact of positions size. However, a precise definition of the situations where weighted spread is a valid liquidity measure is still missing.

Time variation and non-normality is accounted for by using the parametric specification. While it is possible, that the assumption of the t-distribution is a source of imprecision, this would need empirical testing. A further advantage is the modeling of net-return instead of separating mid-price return and liquidity cost, because the correlation between return and liquidity cost does not have to be explicitly modeled. Total risk is measured when the combination of mid-price return and liquidity cost are lowest.

Unfortunately, this method requires a transparent limit order book market such as the London Stock Exchange, the NASDAQ, the Deutsche Börse Xetra or the Euronext. If weighted spread data have to be manually calculated from the full intraday limit order book, the method is highly computationally intensive due to the large amount of data. However, some exchanges, like the German Xetra, provide weighted spread data, which can be integrated into a risk framework with limited computational requirements.[33]

[32] For details please refer to the original paper.
[33] Available as Xetra Liquidity Measure (XLM).

Overall, the weighted spread approach allows for highly precise integration of liquidity risk including price impact of order size - if limit order book data is available.

2.2.2.4 Theoretical models

General remarks In addition to the models analyzed so far, a different class of models has been suggested by academia in the context of liquidity risk measurement. As discussed in section 2.1.1, optimal trading strategies try to find an optimal balance between price impact costs and delay cost by delaying parts of a transaction. They are very helpful in determining a valid liquidity cost estimate when liquidating a large stock position in normal situations.

I only provide a short overview, because I believe that in risk management the usefulness of these strategies is limited for three reasons.[34] First, I doubt that optimal trading strategies are suitable approach from a risk perspective in general. They assume, that there is enough time to delay portions of a trade, which is rather unrealistic in a crises situation. Calls on margin accounts and strong expected momentum enforce a fast liquidation, leaving little room for patient optimal delay. If we assume a 10-day forecast horizon and a crises occurs on day one, does a trader really wait the nine remaining days to liquidate the position? Second, even if there is enough time, optimization parameters must be stable enough to yield an optimized result. Otherwise, it might be that the optimized trading strategy yields worse results than by trading as quick as possible. This is especially the case, if a position is to be liquidated due to informational advantage with respect to the further development of a crises.[35] Third, optimal trading strategies are usually based on a large amount of parameters that are difficult or impossible to estimate in practice. The more aspects are mathematically integrated, the more difficult and possi-

[34] More detailed discussions of these theoretical models can be found in Erzegovesi (2002), Loebnitz (2006) and Jorion (2007).

[35] This translates into high permanent vs. temporary price impact.

bly unstable is the implementation. All of the model suggestions have yet failed to demonstrate that they can be empirically applied in real crises data.[36] To prove the validity of optimal trading strategies, empirical estimation procedures need to be developed and it needs to be shown, that the analytical optimal strategies are stable in crises situations. I believe that optimal trading strategies have their greatest validity when trying to liquidate block holdings in normal market situations, but have limited applicability in risk management.

Nevertheless, for sake of completeness, I provide a brief overview. Papers with optimal trading strategies usually assume some form of price impact function and a particular structure of the temporal dynamics. I will highlight those two main characteristics for each model to clarify the differences.

Model overview Lawrence and Robinson (1995) include liquidation costs, delay costs, which are measured as risk exposure during liquidation, and hedging costs into a net sales value. Risk is then measured as the maximum net sales price when setting the liquidation horizon in an optimal way. Unfortunately, the problem of liquidity cost measurement and its dynamics is left to be specified by the reader. It seems, that liquidity costs are measured as constant bid-ask-spread only, i.e. price impact and time variations are neglected. The general critique on optimal trading strategies applies as discussed above. In addition, it can be doubted that maximizing expected proceeds and neglecting potential shortfall due to proceed variance is a suitable way from a risk perspective. Also, using an unbounded liquidation horizon is a questionable procedure in crises. Therefore, their approach can only serve as a very general framework for analyzing the problem.

Jarrow and Subramanian (1997)/Subramanian and Jarrow (2001) include liquidity cost and execution delay in an optimized framework maximizing liquidation proceeds within a given horizon. They assume that

[36] Cp. also critique in Bangia et al. (1999), p. 69.

liquidity costs are non-decreasing with order size and that trading has economies of scale, i.e. that liquidating the full position at once is always optimal. Liquidity-price correlation is assumed to be zero. The trader is treated as risk neutral. Under these assumptions, an analytically optimal solution is derived. Unfortunately the framework must place heavy restrictions on reality to find an analytical solution. If the optimal liquidation strategy is optimal in real data remains to be seen. The critique on optimal trading strategies in general and on the neglect of proceed variance analogously applies. How the parameters used in the optimization are to be empirically estimated will have to be developed.

Almgren and Chriss (2000) construct an optimal trading strategy within a given liquidation horizon. They decompose liquidation cost into a temporary and a permanent component and construct a liquidity-adjusted VaR by minimizing VaR itself. This approach is extended in Almgren (2003) by including non-linearity in the price impact. However, the question of measuring these parameters remains unsolved in both papers. This especially concerns the magnitude and functional form of permanent and temporary price impact as well as the duration of the temporary price impact. If time-variation of liquidity is incorporated, distributional estimations are also necessary.[37] Concerns with respect to the validity of optimal trading strategies in crises as such apply.

Hisata and Yamai (2000) also construct an optimal trading strategy by minimizing the cost of liquidation, also including normally-distributed permanent and temporary price impact. They determine the optimal holding period at constant sales speed by maximizing expected sales proceeds with a penalty for proceed variance. Liquidity risk then is the price impact variance under the condition, that the sales strategy is optimized. Several variations as well as portfolio considerations are discussed. Unfortunately, the paper also fails to specify how to empirically estimate

[37] Almgren et al. (2005) present a calibration procedure based on internal trade data. This is, however, less helpful when trades are sparse for certain assets in general or the specific institution.

the parameters used in the framework.[38] Several assumptions that are required to technically find an analytical solution, might not be robust in reality. Also using an unbounded liquidation horizon is questionable as discussed above.

Dubil (2003) analyzes the optimal execution strategy between delaying parts of a position and the price impact. Liquidation costs are also decomposed into a permanent and a temporary component. He optimizes the liquidation horizon by maximizing the total VaR of the transaction when assuming a constant liquidation speed, i.e. when price impact is linear. Above critique on optimization strategies, unbounded horizon optimization in particular, as well as empirical parameter estimation applies.

Engle and Ferstenberg (2007) optimize the sales trajectory within a given horizon to maximize expected proceeds with a penalty for proceed variance. Similar to Almgren and Chriss (2000), they assume that permanent and temporary price impact can be measured and solve this theoretical problem, but fail to address how these parameters can be estimated.

This line of research will proceed quickest to practical implication, if two questions are addressed. It needs to demonstrate the empirical estimation technique for the multitude of parameters and prove if or under which circumstances optimal trading strategies yield superior results in crises situations compared with instant liquidation. In the end, integration of many aspects might not be the best way because implementation and result stability are relevant aspects as well.

2.2.3 Synopsis

Liquidity risk measurement has to take two problematic steps: Measurement of liquidity and integration of the measure into a risk framework.

[38]The numerical illustration takes important parameters such as temporary price impact recovery and permanent price impact coefficient as given or sets them to zero.

The measurement technique is closely connected to the data available. The preciseness should increase the more information is used in determining the price impact curve. The correct risk integration technique is generally a balance between simplicity and applying suitable, non-distorting assumption. Table 2.2 summarizes the traceable models based on these criteria. While this provides a theoretical indication, which models should be most suitable, the ultimate test must be empirical. This empirical comparison will be conducted in section 5.3.

Model	Short description	Relevant assumptions	Strengths	Weaknesses
Bangia et al. (1999)	• L. measured with bid-ask-spread • Parametric worst liquidity cost added to price risk • Non-normality accounted for through empirical percentiles	• Position size without significant influence • Liquidity cost and price perfectly correlated	• Only spread data required • Simple add-on to existing risk measures	• Underestimation for larger order sizes • Overestimation because correlation less than perfect and spread calculated based on current price • Regime switching (multi-modality) of spread is neglected
Cosandey (2001)	• L. measured as position size relative to traded shares • L. adjustment from worst market price	• No time variation of l. • Price impact linear to relative traded shares • Further l. differences neglected	• Market volume data required and available at all frequencies • Accounts for price impact of order size	• Price impact approximation • Underestimation because deterioration of liquidity in crises neglected
Berkowitz (2000) Jarrow and Protter (2005)	• L. measured from transaction prices, Berkowitz controlling for other risk factor changes	• Cost precisely extractable from transaction data • Linear price impact • Time variation and correlation issues solved by Jarrow, Protter (2005)	• Accounts for price impact of order size	• Intraday data on single transaction prices and volumes required • Price impact only very approximate
Francois-Heude and Van Wynendaele (2001)	• Quantity adjusted L. measured from best five limit-orders • Current market avg. liquidity costs added to price risk with ad-hoc add-on of difference between market and individual liquidity	• No time variation of l. • Liquidity cost and price perfectly correlated	• Accounts for price impact of order size • More precise price impact measurement than when extracted from transaction prices	• Intraday data of best limit orders required • Price impact approximation most precise for small sizes only • Underestimation because deterioration of liquidity in crises neglected • Somewhat arbitrary spread-adjustment leads to overestimation
Angelidis and Benos (2006)	• L. measured by estimating a model of structural liquidity determinants • Worst liquidity cost added to price risk	• Specific structural model • Volume increase in crises • Liquidity cost and price perfectly correlated	• Partially accounts for price impact of order size if larger than worst market volume	• Intraday data required • Assumptions empirically not verified • Overestimation because correlation less than perfect • Complex, time-consuming estimation of parameters
Giot and Gramming (2005)	• L. measured by weighted spread in limit order book for a specific position size • Risk modeled based on net return	• Worst case perspective (because immediate liquidation) OR Efficient market for liquidity	• Accounts for price impact of order size • Precise liquidity measure • Correlation correctly accounted for	• Only applicable in limit order book markets • If weighted spread data not provided by exchange, intraday data of full limit order book required • Possible overestimation if instant liquidation highly suboptimal

Table 2.2: Overview of traceable models integrating liquidity risk

3 Description and analysis of data sample

This chapter describes the type of data used in the liquidity risk models in chapter 4, and in the empirical analysis of chapter 5. Section 3.1 provides an overview of the data types used in the analysis. In particular, it defines weighted spread as liquidity measure, analyzes under which assumptions it can be used and describes the weighted spread data set. This is followed by short descriptive statistics in section 3.2, which provide a useful background on general market conditions in the sample period. In section 3.3, I provide a detailed empirical analysis of weighted spread as liquidity measure.[1]

3.1 Data types

3.1.1 The weighted spread liquidity measure

3.1.1.1 Definition

I define weighted spread liquidity measure from the cost perspective outlined in 2.1.1. Similarly to Giot and Grammig (2005) in section 2.2.2.3, I define weighted spread as the liquidity cost of a round-trip of size q when liquidating against the limit order book.

The weighted bid-price $b_t(n)$ for selling n number of shares is calculated as

$$b_t(n) = \frac{\sum_i b_{i,t} n_{i,t}}{n} \tag{3.1}$$

[1] For this chapter, cp. Stange and Kaserer (2008a).

where $b_{i,t}$ and $n_{i,t}$ are the bid-prices in Euro and bid-volumes of individual limit orders at time t sorted by price priority. Individual limit order volume add up to n shares, $\sum_i n_i = n$. The weighted ask-price $a_t(n)$ is calculated analogously. Weighted spread is then calculated in basis points (bp) as a function of predefined order sizes q

$$WS(q) = \frac{a_t(n) - b_t(n)}{P_{mid}} \times 100 \tag{3.2}$$

where P_{mid} is the mid-price of the quoted (minimum) spread and $q = n \times P_{mid}$ is the size of the position measured in Euro-mid-price value.

3.1.1.2 Range of applications

Under which assumptions can the weighted spread liquidity measure be validly applied? The following lists the necessary assumptions with respect to position size and type of asset.

First, I assume that direct trading costs are zero, $T(q) = 0$. For very large or institutional traders in developed markets, T(q) can generally be considered negligible. On the Xetra system of the Deutsche Börse, for example, institutional traders pay only around 0.5 bp as transaction fee.[2] Transaction cost T(q) can also be neglected if time variation of liquidity is of major interest.

The second characteristic concerns data availability. Because I focus on the price impact of a specific position size, this type of price impact data needs to be available. This is most probably true in markets with an electronic limit order book, where limit order book data is made available, such as the London Stock Exchange, the NASDAQ, the Frankfurt Xetra or the Euronext.

Third, I look at assets positions, which are continuously tradable during crises.[3] This means, that no (or very few) zero trading days occur and the position size is not larger than market depth. This is a close

[2] Cp. Deutsche Boerse (2008), p.6 ff.
[3] Cp. categorization of liquidity degrees in section 2.1.2.1.

approximation for most stocks, which have no or very few zero trading days. Therefore, investors are not forced to delay the execution of a transaction and costs from forced delay are zero. Scanning the sample data of 160 German stocks over 5.5 years (6/2002 to 1/2008) shows that this assumption is less restrictive than it first seems. Even for less continuously traded stocks in my sample, trading gets continuous during market turmoils. Zero trading days seem to occur mainly in calmer market periods. I hypothesize that tumbling market prices attract traders, who want to liquidate positions or to stop loss via limit orders, which ensures continuous trading. However, I leave a rigorous analysis of this aspect to future research.

Fourth, I assume that deliberate, strategic delay has no significant benefit, i.e. I assume that positions can be equally good instantly liquidated against the limit order book.[4] So, I neglect any (potential) effect of optimal trading strategies, which balance the increased price risk of delay against reduced liquidity cost by trading smaller quantities.[5]

In my view, this is a reasonable assumption in four cases. When I take the worst case perspective of impatient traders, a common risk assumption, potential benefits are consciously neglected.

Benefits are also non-existent, if informational content of the trade is too high. The trader wants to trade immediately on an informational advantage, which would be revealed by trading more slowly or which would dissolve over time. Adverse informational effects are also possible, i.e. trading more slowly could have price effects because the market assumes informational advantage, which is not present in reality.[6]

Immediate liquidation is fair, too, if liquidity prices are efficient and a traders risk aversion is greater or equal to that of the market. If liquidity costs are too high, liquidity providers will enter with limit orders, because

[4] This also neglects liquidation via limit instead of market orders as well as up-floor or over-the-counter trading.

[5] Cp. section 2.2.2.4.

[6] Technically expressed as high permanent price impact rendering optimal trading strategies useless.

liquidity costs, i.e. their profits, will compensate for the additional risk during the delay until the limit order is executed. If liquidity costs are too low, market orders and withdrawn limit orders will deplete the order book, because nobody is willing to take price risk during delay. In this case, marginal gain from lower liquidity costs by delaying a transaction balances the marginal loss due to higher price risk.

Finally, optimal trading strategies might not be feasible in times of market stress,[7] because the optimization parameters are not stable or strategic trading is not always possible.

If there is no forced or deliberate delay, delay cost are zero ($D(q) = 0$) and as a consequence, total liquidity cost can be fairly measured with the price impact from immediate execution.

$$L(q) = PI(q)$$

The first two assumptions are generally less critical. Although the latter two assumptions place restrictions on the range of applications, the discussion shows, that the approach is still valid in a large variety of situations, especially if markets are fairly liquid, positions are not too large and a worst case perspective is of interest.

3.1.1.3 Data sample

I have obtained liquidity data from the Xetra system of the Frankfurt Stock Exchange, which covers the bulk of stock transactions in Germany. Deutsche Börse is among the top 10 largest stock exchanges in the world and Xetra is its electronic trading platform. Trading can be conducted from 9 a.m. to 5.30 p.m. and starts with an opening auction. It is interrupted by an intraday auction around 1 p.m. and ends with a closing auction. Between auctioning times, trading is continuously possible. An electronic order book collects all limit and market orders from market

[7] A point raised in Jarrow and Protter (2005a), p.9.

participants. Orders in the order book will be matched based on price and time priority.

In general, the limit order book is anonymous, but transparent to all participants. However, traders can also submit hidden, "iceberg" orders to trade large volumina, where traded volume is only revealed up to a certain size and a similar order of equal size will be initiated once the first limit order is transacted. Market makers post bid- and ask quotes up to a prespecified minimum quotation volume.

The Xetra system automatically calculates the Xetra Liquidity Measure (XLM) from the visible and invisible part of the limit order book, i.e. including "iceberg" orders. XLM is a weighted spread measure, calculating the cost of immediate execution of a round-trip order of a specific size q compared to its fair value as defined in equation (3.2).

$$XLM(q) = WS(q) \tag{3.3}$$

Gomber and Schweickert (2002) provide some further theoretical background.

My sample consists of 5.5 years of daily data (July 2002 to January 2008) for all 160 stocks in the four major German stock indices (DAX, MDAX, SDAX, TecDAX). The DAX contains the 30 largest publicly listed companies in Germany (by free-float market volume), the MDAX the subsequent 50 largest[8] and the SDAX the following 50 largest. The TecDAX, introduced during the sample period on 24.03.2003, comprises the 30 largest technology stocks. In total, I therefore cover a market capitalization of approximately € 1.2 trillion, which represents the largest part of the market capitalization in Germany.[9] As far as I know, this is the most representative sample on weighted spread available to academia so far.

I received XLM data for all days, where a stock was included in one of the above indices. Daily values are calculated by Xetra as the equal-

[8] MDAX contained 70 stocks before 24.03.2003 and 50 stocks thereafter.

[9] Values as of 1/2008.

weighted average of all available by-minute data points.[10] I break my total sample into four sub-samples, each containing the stocks of one major index.

With the data items above, I proceeded as follows. Liquidity costs L(q) were calculated from a transaction perspective. As a per-transaction figure has much more practical meaning, than a per-round-trip figure, I assume that the order book is symmetrical on average, i.e. the liquidity cost for buying and selling are equal. Therefore, I can calculate the price impact per transaction under the assumptions outlined in section 3.1.1.2 as

$$L(q) = PI(q) = \frac{XLM(q)}{2} \tag{3.4}$$

It is important to note that this measure captures the committed part of liquidity only, while there might possibly be additional hidden liquidity in the market. Since I assume a worst case, however, where I transact immediately against the order book, there is no time for additional (hidden) liquidity to enter the market. This type of measure acts as an upper bound to liquidity cost, because it only measures part of the liquidity supply.[11]

Liquidity costs were provided for each stock for 10 out of the 14 volume classes q of € 10, 25, 50, 75, 100, 150, 250, 500, 750, 1.000, 2.000, 3.000, 4.000 and 5.000 thousand. Volume classes for DAX stocks went up to € 5.000 thsd., but excluded € 10, 75, 150 and 750 thousand. Stocks in the other indices had liquidity costs for all volume classes up to € 1 million.

I had to exclude 408 (<0.01% of total) observations, where liquidity data were available outside the volume class structure described above. As these values were available for connected periods of less than seven days, I assume that the automatic calculation routine of the Xetra computer was extended during trial periods. This procedure ensures that

[10] For liquid volume classes this comprises a maximum of 1,060 measurements during continuous trading.

[11] Cp. also Irvine et al. (2000), p.4f.

liquidity estimates remain representative. In total, my sample contains 1.8 million weighted-spread observations for the 1.424 trading days.

3.1.2 Price, bid-ask-spread and volume data sample

For each stock I define the following variables in addition to L(q):

- P: mid-point of the bid-ask quote at day closing in Euro
- S: quoted bid-ask-spread at day closing relative to the mid-point in bp
- MV: market value at day closing in million Euro
- VO: trading volume in number of traded shares

Data for all items were obtained from Thomson Financial Datastream. Three stocks could not be included in the analysis due to missing XLM or Datastream data.[12] This left 99.9% of the total 323,953 stock-days[13] in the sample. I also had to adjust mid-price data P, because Datastream carries forward price data even if no transaction took place. I removed all price data at days, when no transaction volume was recorded. Data for market value MV and transaction volume VO were used as provided by Datastream.

Quoted spread S measures the minimum ex-ante liquidity cost. While XLM is standardized by size category, quoted spread is not. The largest order size tradable at the quoted spread, i.e. the spread depth, differs between stocks and changes over time. Spread measures different economic aspects for stocks which are covered by a market maker and for those stocks without coverage. Therefore spread depth differs between, but also within those categories.

[12] Procon Multimedia (in SDAX between 10/2002 and 03/2003) and Medisana (in SDAX between 12/2002 and 03/2003). Data could not be obtained for Sparks Networks (in SDAX between 06/2004 and 12/2005), because it was not available in Datastream anymore.

[13] 383 stock-days excluded.

On Xetra, market maker coverage is required only for illiquid stocks - as defined by past XLM and order book volume criteria.[14] On 31.01.2008, 35% of my sample had coverage.[15] In DAX and MDAX only one stock was covered, in SDAX 86% of the stocks were covered.[16] In the case with coverage spread is the quoted spread of the market maker. Spread depth can be freely selected by the market maker above the Xetra-regulated minimum, called minimum quotation volume (MQV), which varies depending on stock liquidity as measured by past-XLM. According to my data, minimum quotation volume for covered stocks was € 17.338.

In cases without coverage, spread is the minimum spread available in the order book. It corresponds to the order size of the limit order with the best price at a particular moment, which is naturally non-standardized. While the Xetra MQV is valid for liquid, non-covered stocks as well, the average minimum was € 27, i.e. non-existent for practical purposes. Spread depth for non-covered stocks therefore varies even more widely.

Two aspects should be kept in mind when comparing spread and the XLM liquidity measure. First, spread for covered stocks is likely to follow other dynamics, since the size of the spread has Xetra-regulated upper bounds.[17] In contrast, XLM liquidity prices result from free supply and demand behavior. Second, there is potential overlap between spread and the XLM. 51 stocks in my sample had minimum quotation volume above € 10.000, 4 stocks between € 25.000 and € 30.000 (mostly in SDAX and TecDAX). As a consequence, XLM measured quoted spread in small volume classes q of € 10 and 25 thousand for these 51 stocks. While no historic data on MQV is available, it is safe to assume that this was valid over the whole sample period.

[14]Market makers are called 'Designated Sponsors' on Xetra.
[15]Data taken from Deutsche Börse (2008).
[16]While historic data was not available, it is plausible that similar differences existed during the whole sample period.
[17]Cp. Deutsche Boerse (2007), p. 5, 9.

Market segment overview	II/2002	2003	2004	2005	2006	2007	1/2008	Total period[a]
Average continuous period return [b]								
DAX	-52%	24%	6%	27%	20%	22%	-15%	6%
MDAX	-23%	39%	15%	25%	25%	-1%	-12%	12%
SDAX	-36%	35%	11%	28%	29%	4%	-14%	10%
TecDAX	n/a	52%	3%	26%	24%	32%	-25%	23%
Total	-35%	24%	10%	26%	24%	11%	-15%	8%
Average period return volatility (annualized) [c]								
DAX	64%	41%	22%	19%	23%	25%	51%	30%
MDAX	54%	39%	28%	26%	30%	35%	59%	35%
SDAX	65%	47%	35%	31%	36%	38%	58%	40%
TecDAX	n/a	54%	42%	31%	38%	44%	71%	42%
Total	60%	44%	32%	27%	32%	36%	59%	37%
Average free-float market capitalization in million Euro								
DAX	15,217	14,615	17,983	20,350	24,357	29,949	29,325	21,008
MDAX	1,043	1,330	1,940	2,537	3,734	3,797	3,121	2,453
SDAX	106	235	320	393	500	775	640	418
TecDAX	n/a	725	863	898	995	1,221	1,204	955
Total	3,639	3,483	4,319	4,998	6,154	7,379	7,009	5,160
Average daily transaction volume in thsd. Euro								
DAX	93,500	94,399	98,037	119,563	165,833	250,835	351,793	144,040
MDAX	1,384	2,297	4,035	6,242	11,034	18,243	22,351	7,557
SDAX	36	160	237	514	958	2,129	2,081	780
TecDAX	n/a	1,813	2,345	2,308	4,769	7,946	11,430	4,052
Total	20,431	19,543	20,268	25,206	35,797	54,891	75,739	31,020

Table 3.1: Market conditions during sample period

a. annualized; b. Includes dividend returns, because price series are adjusted for corporate capital actions; c. volatility estimated from daily cont. returns and annualized with $\sqrt{250}$; All values equal-weighted.

3.2 Market background in sample period

As background to the empirical analysis, table 3.1 summarizes market conditions during the sample period. Markets were bullish in the largest part of the sample period. I also captured the downturns in the second half of 2002 and the first month of 2008. Due to beginning and end of period declines, overall return was rather average at 8% p.a.. Naturally, market capitalization increased similar to returns. Average market capitalization is several times larger in the DAX than in all other indices. MDAX contained the second largest average market capitalization stocks. Volatility exhibited a similar, but reversed pattern than returns. Consequently, my sample is rather positively biased.

Daily transaction volume increased strongly during the sample period, which is already a plausible indicator for improving liquidity. Transaction volume was largest in the DAX. Transaction volume in the other indices were several magnitudes smaller. Contrary to the general trend, transaction volume in the TecDAX remained rather steady after its initiation in 2003 and exhibits a level slightly lower than the MDAX. SDAX transaction volume was again several times smaller than in MDAX or TecDAX. The high diversity in transaction volumes underlines the representativeness of the sample.

3.3 Empirical analysis of weighted spread

3.3.1 Motivation

As such a large sample of weighted spread has never been available to academia before, I conduct an empirical analysis of weighted spread as liquidity measure. Section 3.3.2 presents representative weighted spread estimates of the impact of order size on liquidity cost, which have not been available so far due to short sample restrictions. Sections 3.3.3 and 3.3.4 analyze the time development and distributional characteristics of weighted spread, which are important to evaluate the importance of the weighted spread in the risk management context. As outlined in section 2.2, several liquidity risk models neglect the impact of order size on liquidity cost. Therefore, this analysis will allow to judge the importance of this assumption. Section 3.3.5 makes cross-sectional comparisons to provide more insight into the structure of weighted spread.

3.3.2 Descriptive statistics

I start with looking at detailed descriptive statistics of liquidity cost L(q), which will serve as representative reference for practice and provide some structural insight.

From an economic perspective, it is difficult to aggregate liquidity cost by absolute order size across stocks. It can be argued that, for example, liquidating a € 100.000 position in a large-cap stock is not comparable to the same position in a small cap stock, as the position in the large cap stock represents a much smaller part of the market value and should therefore be more liquid and have consequently less liquidity costs. A similar argumentation goes for the Euro-position in relation to the prevailing transaction volume in the market. A position size relative to the market value of the stock and prevailing transaction volume would be more comparable across stocks.

While I do not want to empirically investigate into this argument further in this section[18] and to keep the provided statistics as simple as possible, I choose not to generate new relative size categories. I also want to avoid reducing the generality of results by using a specific method for re-categorizing liquidity data. To still account for the argument above, my distributional statistics will not be calculated on liquidity data aggregated across all stocks, but I calculate stock-specific distribution statistics and present their cross-sectional mean and median. As reference, I included spread in the distributional analysis. Because the order class of spread differs widely between stocks, I designated this order class as "min".

I calculate the cross sectional averages for a specific sub-sample over a specific period. Table 3.2 shows average liquidity cost over the whole sample period by index and order size. The first columns present average liquidity costs for different order sizes. The min-column contains the bid-ask-spread estimate for the minimum order size, the following columns

[18] Refer to 3.3.5.1 for a more detailed analysis.

Avg. liquidity cost (in bp)		Order size (in thsd. Euro)																Size
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	Mean	0.09	n/a	6.06	7.10	n/a	9.25	n/a	16.07	28.54	n/a	56.10	97.50	116.90	136.91	153.60	56.75	28.28 ***
	Median	0.05	n/a	6.09	7.01	n/a	8.86	n/a	14.56	24.01	n/a	49.98	67.91	97.67	120.89	154.43	23.81	25.08 ***
	Std. Dev.	0.11	n/a	2.68	3.56	n/a	5.42	n/a	11.57	23.28	n/a	46.63	97.39	83.42	87.35	91.56	76.14	19.88 ***
	Availability	99%	n/a	100%	100%	n/a	100%	n/a	100%	100%	n/a	100%	98%	94%	90%	85%	97%	-0.02 **
MDAX	Mean	0.29	27.08	34.09	42.36	50.65	59.39	77.62	109.80	160.77	194.36	217.26	n/a	n/a	n/a	n/a	88.19	43.72 ***
	Median	0.17	20.32	25.11	33.30	40.27	48.40	68.18	100.24	155.58	189.58	205.36	n/a	n/a	n/a	n/a	52.80	43.81 ***
	Std. Dev.	0.37	38.84	60.12	63.92	55.84	51.31	58.04	71.17	86.30	99.90	105.71	n/a	n/a	n/a	n/a	92.70	12.99 ***
	Availability	100%	100%	100%	99%	98%	97%	95%	92%	81%	71%	62%	n/a	n/a	n/a	n/a	90%	-0.08 ***
SDAX	Mean	0.64	81.87	97.29	127.58	156.12	179.68	209.48	251.68	326.19	397.71	460.23	n/a	n/a	n/a	n/a	172.57	82.41 ***
	Median	0.41	61.87	81.20	109.15	134.32	150.80	201.72	233.58	293.86	342.42	412.83	n/a	n/a	n/a	n/a	133.12	75.62 ***
	Std. Dev.	0.88	100.29	82.88	94.51	111.74	145.43	181.29	166.41	137.31	199.11	247.46	n/a	n/a	n/a	n/a	159.55	29.71 ***
	Availability	98%	97%	91%	86%	81%	77%	69%	56%	33%	20%	13%	n/a	n/a	n/a	n/a	62%	-0.19 ***
TECDAX	Mean	0.26	31.68	41.92	58.87	77.32	96.61	128.30	174.31	245.30	291.33	331.65	n/a	n/a	n/a	n/a	125.20	68.82 ***
	Median	0.20	30.82	38.85	53.61	70.05	86.73	120.43	173.01	227.30	255.28	311.84	n/a	n/a	n/a	n/a	75.72	63.27 ***
	Std. Dev.	0.22	17.56	26.60	44.01	64.15	84.86	102.59	109.12	112.08	124.42	136.73	n/a	n/a	n/a	n/a	123.57	27.35 ***
	Availability	99%	100%	100%	100%	100%	99%	98%	92%	73%	56%	44%	n/a	n/a	n/a	n/a	86%	-0.12 ***
All	Mean	0.36	n/a	48.59	62.43	n/a	87.60	n/a	130.62	164.15	n/a	192.90	n/a	n/a	n/a	n/a	108.63	40.82 ***
	Median	0.19	n/a	30.22	39.03	n/a	59.12	n/a	103.05	153.73	n/a	161.66	n/a	n/a	n/a	n/a	69.23	39.96 ***
	Std. Dev.	0.58	n/a	67.15	78.98	n/a	107.83	n/a	129.46	135.74	n/a	168.71	n/a	n/a	n/a	n/a	122.48	26.61 ***
	Availability	98%	n/a	97%	95%	n/a	92%	n/a	82%	68%	n/a	51%	n/a	n/a	n/a	n/a	82%	-0.12 ***

Table 3.2: Liquidity cost by index and size

Discrete liquidity cost, average over sample period; equal weighting in categories; TecDAX from start 2003/03 only; Min-column measures spread for the minimum order size, all-column is average over all standardized order sizes, i.e. without minimum; Size impact is the coefficient in 10^2 of log order-size in a regression of the distribution statistic on log order-size including an intercept; * indicates 10% confidence level, ** 5% and *** 1% of being different from zero based on two-tailed test.

the cost estimates for higher order sizes according to weighted spread, equations (3.2) and (3.4). I report the cross-sectional mean, median and standard deviation in each sub-sample. Availability is available data in percentage of the theoretical maximum. As the sample comprises 1.424 trading days, the maximum possible number of observations per volume class is 42.720 for the DAX, 74.900 for the MDAX, 71.200 for the SDAX and 37.170 for the TecDAX.

In the last column of table 3.2 I specifically estimated the impact of doubling order size in absolute basis points on liquidity costs and in percentage points on availability. This is done with an ordinary-least-squared (OLS) regression. The specification for each statistic $stat(q)$ is $stat(q) = C + ln(q) + \epsilon_q$ with C being the constant intercept. Statistics of the minimum order size/spread do not enter the calculation, because corresponding minimum order size is not available.[19]

Between 6/2002 and 1/2008, investors had to pay between 0.09 bp and 460 bp on average for buying or selling a stock position, which already shows that liquidity costs varies largely between order sizes and can reach substantial amounts. While in the DAX average liquidity costs start at a negligible 0.09 bp for the minimum order size, they reach over 100 bp when trading a position larger than of € 3 million. Liquidity costs at the smallest order size of € 10 and 20 thsd. respectively is several times the level of the spread. As many (institutional) investors rarely trade positions lower than € 10 thsd., the spread is therefore insufficient as liquidity estimate.

Comparing average and median liquidity level between different indices, the DAX was the most liquid on average, followed by the MDAX, TecDAX and then the SDAX. A similar result shows when looking at the size impact on liquidity. The size impact was statistically significant at the 1 %-level in all indices, smallest in the DAX and largest in the SDAX. When doubling order size, liquidity costs in the DAX increase by

[19] OLS regression with availability has limited validity, because the statistic is distributed between 0 and 1 and is non-normal, but has been included for sake of completeness.

an absolute 28.28 bp in the average stock. In the SDAX liquidity impact in the average stock was almost three times as high at 82.41 bp.

Median liquidity was lower than the mean in all order sizes, which reveals a right-skewed liquidity cost distribution for all sizes and all indices. Size impact in the median is very similar to the impact in the average. The dispersion of liquidity cost across stocks is of a similar order of magnitude as the liquidity level and increases with order size. Liquidity variation seems to be closely connected to liquidity level.

Generally, as order size increases, availability decreases, which is underlined by the statistically significant size-impact statistic.[20] This is due to the fact already mentioned above that larger orders could not be transacted against the limit order book for all stocks. Availability of spread was in some cases slightly below 100 %, because Datastream did not provide data for all stock-days. For small order classes up to € 25 thousand, over 90 % of all stock positions could be instantly liquidated. However, in the SDAX, for example, availability drops down to 13 % of all stocks for the volume class of € 1 million. In the DAX, even large orders can be continuously executed against the limit order book as availability is below 90 % for the largest volume class of € 5 million only. The pattern of availability for the TecDAX underlines the conclusion above that the TecDAX is much more liquid than the SDAX. Comparing the TecDAX with the MDAX with respect to availability, the MDAX is only very slightly more liquid in order sizes above € 500 thousand.

The TecDAX was created in March 2003. Therefore, TecDAX numbers are based on the mean from 3/2003 to 1/2008, in contrast to the rest of the sample, which ranges from 7/2002 to 1/2008. Statistics for the comparable sample (3/2003 to 1/2008) are shown in table 3.3. They are similar in relative magnitude between the indices and structure.

[20] Because spread data in the min-column comes from a different source than the liquidity data, availability between these two is not directly comparable.

Avg. liquidity cost (in bp since 24/3/03)		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	Mean	0.06	n/a	4.92	5.67	n/a	7.18	n/a	11.79	19.67	n/a	37.85	67.54	91.03	111.10	127.27	46.99	66.42 ***
	Median	0.04	n/a	5.09	5.83	n/a	7.34	n/a	11.49	19.05	n/a	35.81	64.83	96.06	106.61	111.87	20.21	64.73 ***
	Std. Dev.	0.07	n/a	1.85	2.34	n/a	3.39	n/a	6.75	12.87	n/a	26.86	50.46	56.72	62.31	70.60	58.70	75.29 ***
	Availability	99%	n/a	100%	100%	n/a	100%	n/a	100%	100%	n/a	100%	99%	97%	94%	90%	98%	
MDAX	Mean	0.20	18.63	23.02	30.35	38.01	45.67	61.36	91.35	145.83	182.26	207.95	n/a	n/a	n/a	n/a	78.41	57.34 ***
	Median	0.14	15.58	18.56	24.34	30.85	38.55	53.84	79.60	126.95	169.64	197.27	n/a	n/a	n/a	n/a	42.36	60.56 ***
	Std. Dev.	0.20	12.04	16.29	22.53	28.22	33.37	43.06	59.14	84.20	97.77	105.49	n/a	n/a	n/a	n/a	84.73	50.61 ***
	Availability	98%	100%	100%	100%	100%	100%	100%	99%	91%	82%	72%	n/a	n/a	n/a	n/a	94%	
SDAX	Mean	0.47	60.20	80.62	112.83	139.46	162.69	195.38	245.75	324.65	396.40	459.60	n/a	n/a	n/a	n/a	160.79	45.09 ***
	Median	0.36	52.27	67.84	97.56	125.18	150.78	184.46	227.33	293.86	342.42	412.83	n/a	n/a	n/a	n/a	121.94	46.04 ***
	Std. Dev.	0.39	34.10	55.01	69.41	77.18	86.10	97.16	130.71	137.11	198.68	247.50	n/a	n/a	n/a	n/a	134.53	39.46 ***
	Availability	99%	99%	99%	96%	92%	88%	79%	64%	38%	23%	15%	n/a	n/a	n/a	n/a	69%	
TECDAX	Mean	0.26	30.83	40.77	57.26	75.28	93.87	124.54	170.17	241.78	289.84	326.97	n/a	n/a	n/a	n/a	122.72	55.07 ***
	Median	0.20	28.51	38.64	53.61	70.05	86.73	120.43	173.01	227.30	255.28	288.88	n/a	n/a	n/a	n/a	73.87	54.36 ***
	Std. Dev.	0.22	17.18	26.17	43.37	63.24	83.93	101.26	108.03	112.80	125.08	137.66	n/a	n/a	n/a	n/a	122.76	45.15 ***
	Availability	99%	100%	100%	100%	100%	99%	98%	92%	73%	56%	44%	n/a	n/a	n/a	n/a	86%	
All	Mean	0.27	n/a	40.77	55.79	n/a	80.87	n/a	125.47	160.37	n/a	188.51	n/a	n/a	n/a	n/a	100.84	42.95 ***
	Median	0.17	n/a	25.88	33.62	n/a	50.75	n/a	93.51	133.15	n/a	163.86	n/a	n/a	n/a	n/a	61.91	53.36 ***
	Std. Dev.	0.30	n/a	44.51	60.42	n/a	85.15	n/a	120.25	138.09	n/a	173.51	n/a	n/a	n/a	n/a	111.43	36.65 ***
	Availability	98%	n/a	100%	99%	n/a	96%	n/a	87%	73%	n/a	54%	n/a	n/a	n/a	n/a	86%	

Table 3.3: Liquidity cost by index and size since 24/03/2003

Discrete liquidity cost, average over sample period; equal weighting in categories; Min-column measures spread for the minimum order size, all-column is average over all standardized order sizes, i.e. without minimum; Size impact is the coefficient in 10^2 of log order-size in a regression of the distribution statistic on log order-size including an intercept; * indicates 10% confidence level, ** 5% and *** 1% of being different from zero based on two-tailed test.

All in all, the discussion shows that liquidity costs can be substantially underestimated when looking at spread only. The impact of size is quite substantial, especially in stocks with smaller market capitalization.

3.3.3 Time dynamics

To provide a first picture on the different behavior of liquidity at different position sizes over time, I calculated pairwise-sample correlations between spread and liquidity at larger order sizes as presented in table 3.4. Correlation between spread and liquidity in the rest of the order book are relatively low below 65%, correlations between adjoining measures of L(q) are very close to one. Correlations drop to 30 to 40% when looking at correlations between liquidity of very small and of very large sizes. While correlations continuously drop as the difference between order sizes gets larger, there is an increase in correlation between the volume class of € 750 thsd. and € 1 million. This is due to the fact that the sample at € 1 million is dominated by DAX stocks. DAX stocks have generally higher correlations as is shown in table 3.5 on page 57, which explains the increase between € 750 thsd. and € 1 million in the full sample.

This correlation analysis is an indicator that liquidity behaves very differently across order sizes. Liquidity cost at the left side of the order book, like spread, are a poor proxy for the liquidity cost of larger position.

Figure 3.1 shows the daily development of liquidity cost L(q), averaged over all order sizes and cross-sections during the whole sample period by index. Detailed mean estimates are provided in table 3.6. While the equal average over all available volume classes is somewhat arbitrary, because it is strongly influenced by the selection of volume classes, it nevertheless gives a picture of the general liquidity trend. While the underlying stocks change over time as stocks move in and out of indices, the effect on the index mean should be negligible. The average can be

	Spread	L(10)	L(25)	L(50)	L(75)	L(100)	L(150)	L(250)	L(500)	L(750)	L(1000)	L(2000)	L(3000)	L(4000)	L(5000)
Spread	1.00	0.64	0.64	0.59	0.50	0.50	0.44	0.44	0.41	0.27	0.37	0.46	0.45	0.41	0.41
L(10)		1.00	0.88	0.81	0.73	0.70	0.65	0.59	0.56	0.48	0.50				
L(25)			1.00	0.92	0.82	0.79	0.70	0.67	0.67	0.52	0.65	0.78	0.73	0.69	0.67
L(50)				1.00	0.95	0.89	0.78	0.72	0.69	0.55	0.68	0.81	0.77	0.73	0.72
L(75)					1.00	0.97	0.88	0.79	0.65	0.57	0.62				
L(100)						1.00	0.95	0.86	0.72	0.58	0.71	0.83	0.79	0.75	0.74
L(150)							1.00	0.91	0.72	0.65	0.66				
L(250)								1.00	0.90	0.87	0.78	0.86	0.81	0.77	0.77
L(500)									1.00	0.95	0.87	0.88	0.83	0.79	0.78
L(750)										1.00	0.95				
L(1000)											1.00	0.89	0.76	0.66	0.61
L(2000)												1.00	0.93	0.83	0.78
L(3000)													1.00	0.95	0.91
L(4000)														1.00	0.97
L(5000)															1.00

Table 3.4: Correlation of liquidity costs across order size

Pairwise sample correlations between bid-ask-spread and weighted spread L(q) of different order sizes q in thsd. Euro.

	Spread	L(10)	L(25)	L(50)	L(75)	L(100)	L(150)	L(250)	L(500)	L(750)	L(1000)	L(2000)	L(3000)	L(4000)	L(5000)
Spread	1.00		0.59	0.57		0.54		0.52	0.51		0.37	0.46	0.45	0.44	0.43
L(10)															
L(25)			1.00	0.99		0.97		0.91	0.87		0.77	0.78	0.73	0.73	0.71
L(50)				1.00		0.99		0.95	0.90		0.80	0.81	0.77	0.78	0.76
L(75)															
L(100)						1.00		0.98	0.93		0.82	0.83	0.79	0.80	0.79
L(150)															
L(250)								1.00	0.97		0.85	0.86	0.81	0.83	0.81
L(500)									1.00		0.88	0.88	0.83	0.84	0.83
L(750)															
L(1000)											1.00	0.89	0.76	0.70	0.64
L(2000)												1.00	0.93	0.86	0.79
L(3000)													1.00	0.95	0.90
L(4000)														1.00	0.97
L(5000)															1.00

Table 3.5: Correlation of liquidity costs across order size for DAX
Pairwise sample correlations between bid-ask-spread and weighted spread L(q) of different order sizes q in thsd. Euro.

Avg. liquidity cost		**Order size (in thsd. Euro)**															
(in bp)		**Min**	**10**	**25**	**50**	**75**	**100**	**150**	**250**	**500**	**750**	**1000**	**2000**	**3000**	**4000**	**5000**	**All**
DAX	**II/2002**	0.26	n/a	13.96	17.08	n/a	23.96	n/a	46.99	91.04	n/a	187.50	320.53	350.82	396.82	442.47	162.40
	2003	0.15	n/a	9.36	11.51	n/a	15.82	n/a	29.29	54.66	n/a	120.40	211.23	252.03	286.11	311.67	116.90
	2004	0.06	n/a	4.99	5.79	n/a	7.34	n/a	12.01	20.10	n/a	39.01	79.65	116.46	146.57	168.50	56.18
	2005	0.05	n/a	3.80	4.28	n/a	5.24	n/a	8.09	12.65	n/a	21.36	38.85	58.54	77.57	96.65	32.24
	2006	0.04	n/a	4.10	4.50	n/a	5.33	n/a	7.97	12.39	n/a	20.61	34.96	49.74	66.67	83.02	28.81
	2007	0.04	n/a	3.83	4.27	n/a	5.21	n/a	8.14	12.72	n/a	20.48	33.61	46.12	60.19	75.75	27.01
	01/2008	0.05	n/a	4.88	5.58	n/a	7.06	n/a	11.64	18.81	n/a	30.61	50.58	72.49	100.43	128.15	42.95
	All	0.09	n/a	5.99	7.04	n/a	9.22	n/a	16.10	28.56	n/a	56.79	97.51	116.90	136.30	153.17	60.15
MDAX	**II/2002**	0.72	85.00	110.60	134.10	157.38	179.16	229.36	301.14	415.67	452.24	460.99	n/a	n/a	n/a	n/a	189.32
	2003	0.47	43.70	56.48	73.38	93.12	112.71	151.13	215.18	292.46	342.23	365.64	n/a	n/a	n/a	n/a	141.79
	2004	0.22	21.52	26.38	34.89	43.98	53.62	75.14	116.36	197.07	242.91	275.82	n/a	n/a	n/a	n/a	92.87
	2005	0.17	14.74	18.03	23.29	28.49	33.71	44.38	66.64	124.13	169.20	198.75	n/a	n/a	n/a	n/a	66.69
	2006	0.13	11.87	14.02	18.01	22.22	26.52	35.18	52.63	98.00	137.95	171.71	n/a	n/a	n/a	n/a	57.78
	2007	0.13	10.79	12.89	16.76	20.79	24.78	32.49	47.05	82.13	117.52	152.18	n/a	n/a	n/a	n/a	51.39
	01/2008	0.18	15.69	19.17	25.68	32.86	40.10	54.21	80.90	146.36	211.70	279.26	n/a	n/a	n/a	n/a	89.70
	All	0.29	28.48	35.91	45.01	54.53	63.79	82.60	112.75	160.87	192.79	215.90	n/a	n/a	n/a	n/a	90.37
SDAX	**II/2002**	1.83	243.35	327.98	484.71	692.59	923.62	1,248.33	952.52	126.58	93.04	70.88	n/a	n/a	n/a	n/a	394.86
	2003	1.03	131.92	174.85	231.88	287.69	334.10	397.62	495.91	621.85	577.60	515.66	n/a	n/a	n/a	n/a	253.75
	2004	0.57	80.39	113.41	175.49	222.11	265.32	310.33	391.68	438.16	521.72	680.02	n/a	n/a	n/a	n/a	203.29
	2005	0.41	52.83	67.97	95.29	122.77	146.23	184.89	232.44	312.66	411.80	484.35	n/a	n/a	n/a	n/a	141.50
	2006	0.36	44.58	55.53	74.84	95.14	116.68	157.01	225.06	326.41	403.38	458.06	n/a	n/a	n/a	n/a	141.34
	2007	0.28	30.02	37.46	49.90	62.93	76.48	105.06	164.50	285.09	367.26	439.58	n/a	n/a	n/a	n/a	131.04
	01/2008	0.36	42.62	54.31	76.35	100.64	127.68	181.92	300.79	495.65	630.40	736.20	n/a	n/a	n/a	n/a	194.33
	All	0.64	80.80	95.90	125.21	151.96	174.88	203.74	248.41	326.41	398.92	464.20	n/a	n/a	n/a	n/a	169.94
TecDAX	**II/2002**		n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	
	2003	0.43	54.93	74.38	105.84	143.45	183.54	241.96	320.12	376.24	411.95	469.00	n/a	n/a	n/a	n/a	187.05
	2004	0.30	36.98	50.07	72.96	98.70	125.57	169.21	230.80	312.85	349.55	349.32	n/a	n/a	n/a	n/a	141.58
	2005	0.23	26.80	34.99	47.49	60.09	73.41	100.09	145.12	235.59	293.06	321.63	n/a	n/a	n/a	n/a	107.33
	2006	0.20	22.80	29.07	40.05	51.04	61.95	83.95	128.53	222.66	285.76	336.10	n/a	n/a	n/a	n/a	109.40
	2007	0.16	18.26	23.30	31.61	39.87	48.04	63.91	93.82	169.25	227.18	273.38	n/a	n/a	n/a	n/a	91.89
	01/2008	0.23	28.23	35.44	48.27	62.17	76.01	103.09	158.04	293.98	354.04	431.06	n/a	n/a	n/a	n/a	141.20
	All	0.26	30.83	40.77	57.26	75.28	93.87	124.54	170.17	241.78	289.84	326.97	n/a	n/a	n/a	n/a	122.72
All	**II/2002**	1.00	n/a	123.84	135.02	n/a	167.27	n/a	201.32	232.66	n/a	259.79	n/a	n/a	n/a	n/a	202.75
	2003	0.58	n/a	82.33	104.57	n/a	146.37	n/a	209.01	222.72	n/a	238.91	n/a	n/a	n/a	n/a	164.37
	2004	0.31	n/a	53.66	78.64	n/a	113.00	n/a	153.37	173.69	n/a	179.78	n/a	n/a	n/a	n/a	118.36
	2005	0.23	n/a	33.94	46.39	n/a	69.15	n/a	108.20	140.92	n/a	164.43	n/a	n/a	n/a	n/a	85.48
	2006	0.20	n/a	27.95	37.34	n/a	57.20	n/a	107.06	148.33	n/a	178.73	n/a	n/a	n/a	n/a	83.89
	2007	0.16	n/a	20.82	27.56	n/a	41.62	n/a	84.79	140.03	n/a	190.64	n/a	n/a	n/a	n/a	77.18
	01/2008	0.22	n/a	30.52	41.98	n/a	68.01	n/a	148.67	231.24	n/a	280.95	n/a	n/a	n/a	n/a	118.80
	All	0.36	n/a	48.59	62.38	n/a	87.37	n/a	130.25	163.60	n/a	192.25	n/a	n/a	n/a	n/a	108.36

Table 3.6: Liquidity costs by index, year and order size
Table 3.6 provides detailed mean liquidity cost estimates (bid-ask-spread and weighted spread) by index, year and order size.

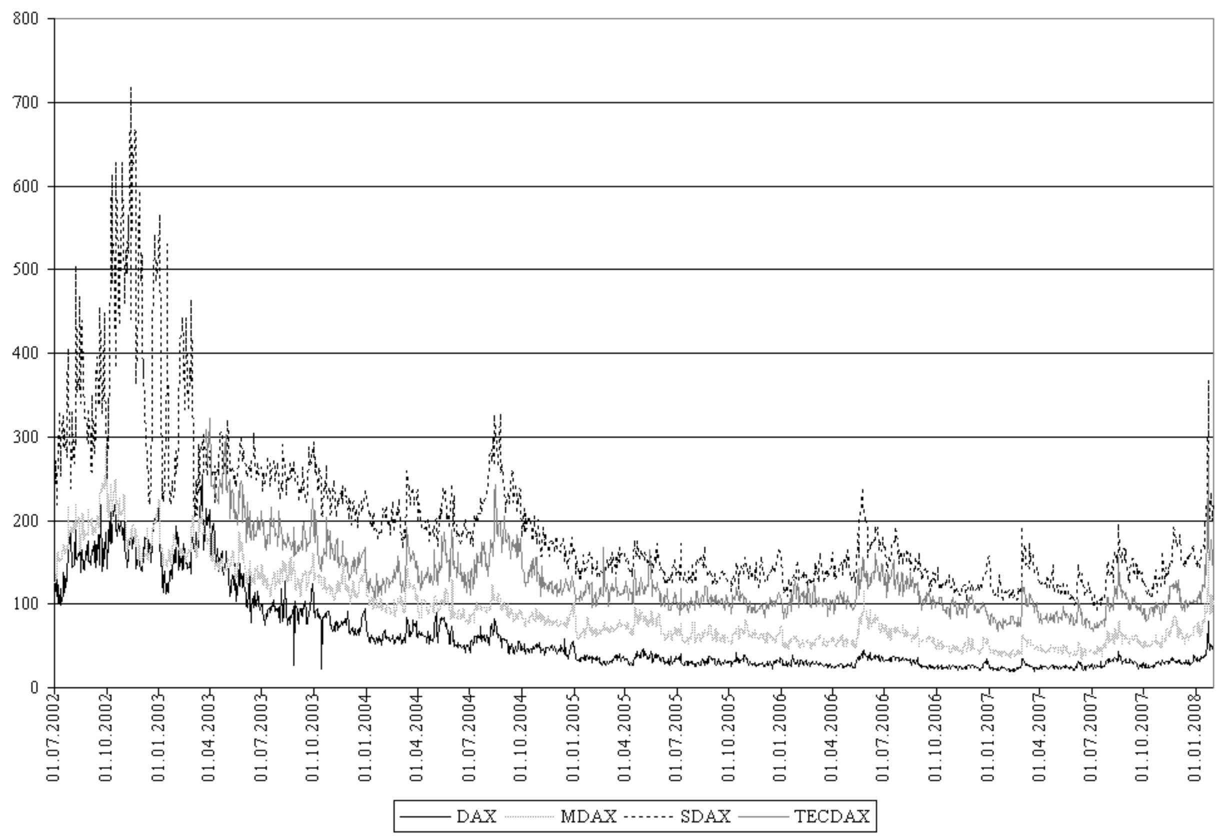

Figure 3.1: Development of average liquidity cost by index

Figure 3.1 shows the development of the cross-sectional mean of all order sizes by day over the sample period. As noted before, TecDAX was created in 3/2003.

interpreted as expected liquidity cost when trading a random position in the specific index.

All index averages have experienced a strong decline in the last 5.5 years with a recent strong increase in 1/2008. In a first phase from 7/2002 to 3/2003 liquidity was highly volatile and showed side-way movement. This phase corresponds to the end of the collapsing high-tech bubble. From 3/2003 on, liquidity steadily declined, interrupted by short, but substantial increases. Most of these increase spikes can be tracked to major disturbances at the stock market. Liquidity cost increased around August 2004 after the publication of low earnings forecasts in technology stocks and during the stock market crash of May 2006, which spilled over from emerging-market exchanges. The recent sub-prime crises is also apparent in the data. Upward spikes can be observed during the crash in February/March 2007 after bankruptcy declarations of sub-prime lenders, in August 2007, where the influence of sub-prime on bank portfolios became known, especially on the German IKB bank, and most recently during the crash of January 2008 after equity shortages of major banks around the world.

Increases occur over short periods of time, while decreases take place over calm periods of slightly positive market conditions. This asymmetry skews the distribution of liquidity changes to the right. The general negative trend explains the slight positive skewness in liquidity level distributions. Index means move relatively synchronous, while changes in liquidity seem to be connected to the liquidity level and are thus much less pronounced in the less liquid indices.

To investigate into the time variation of liquidity costs by size, I first look at the variation of availability over time. Figure 3.2 reveals that availability has strongly increased, especially in larger sizes. In 100% of the stocks, the volume classes of € 25 and € 100 thousand was tradable in recent months. Tradability of € 1 million strongly improved from around 30% of the stocks in 2002 to above 60% lately. Therefore, sample size increases with time for larger volumes. The availability picture by

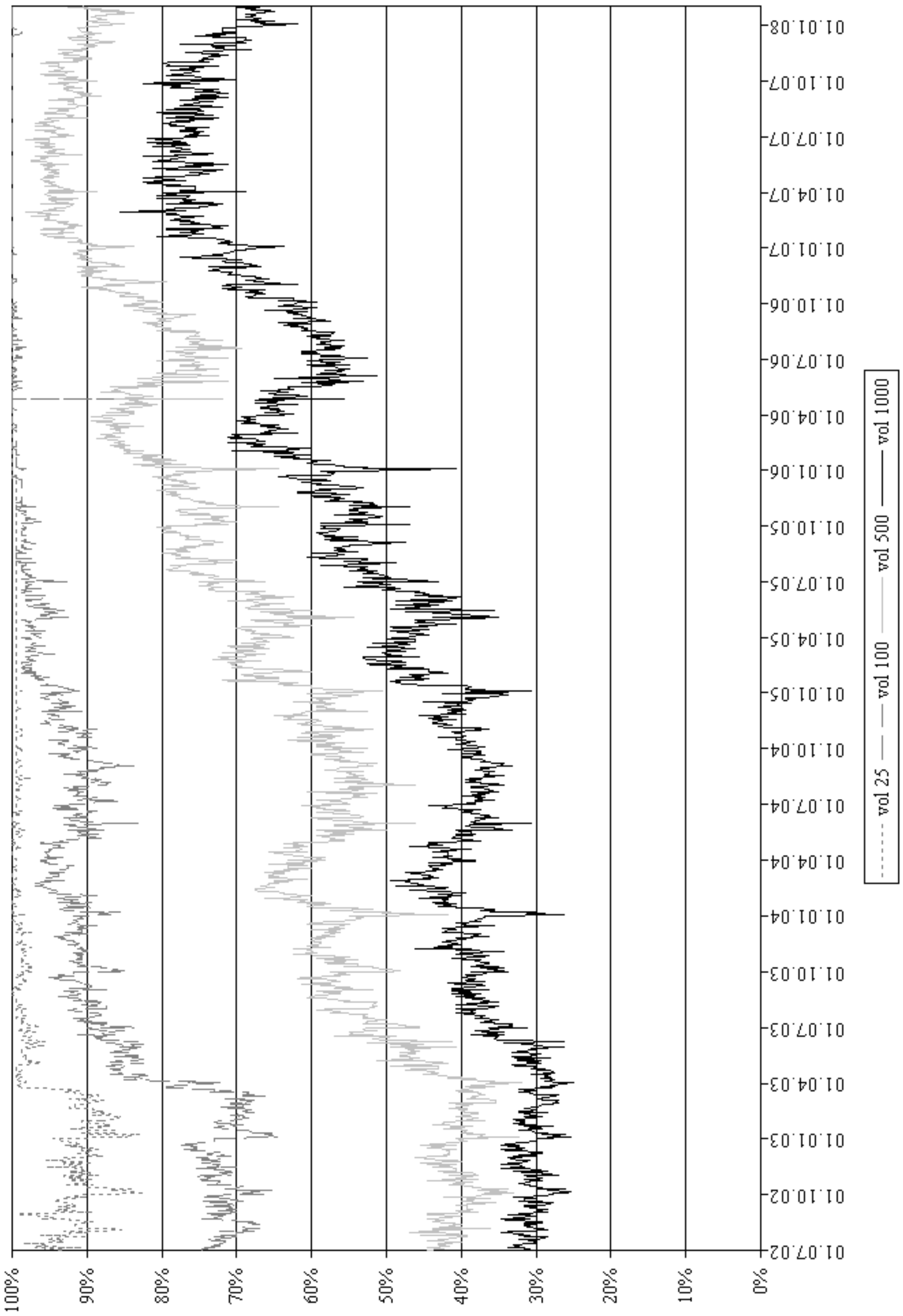

Figure 3.2: Development of average availability by selected volume classes

index shows a similar development as displayed in figure 3.3. The sharp increase of availability in the MDAX on 24.03.2003 was caused by the reduction in number of stocks in this index from 70 to 50 stocks, as mentioned earlier.

Due to the changing sample, I observe two contrary effects. As liquidity improves, liquidity costs fall. At the same time, larger stock positions become tradable. Availability in these order sizes increase. The successive inclusion of comparatively illiquid stocks with high liquidity cost drives up the average. As a consequence, the development of average liquidity cost will not be representative for the development of liquidity cost for a specific stock position. Non-constant sample average are upward biased over time, especially in larger order sizes, where availability strongly increases. Figure 3.3 reveals that the upward bias is especially present in illiquid indices such as the SDAX.

To measure the development of liquidity cost for a specific stock position, I constructed a constant sample and recalculated the average liquidity cost over time. I included only those stocks and sizes, which were available at least 97% of the sample period.[21] The caveat of this type of analysis is that only very liquid stocks are included in the average and the average is taken on a less-representative fraction of the market. To make different order sizes more comparable, I indexed liquidity cost levels on the July 2002 mean.

Results in figure 3.4 show that liquidity costs have decreased across all order sizes. Absolute reduction is larger for bigger positions, but relative decline was more similar across sizes. Relative reduction was larger in smaller order sizes over the whole sample period. Spread declined by 80 %, weighted spread of a € 25 thsd. position by about 50 %. In contrast, liquidity in larger volumes have been brought up to near high, historic levels in the recent crises. Larger volumes seem to be affected more strongly in crises. This effect can be interpreted as another vari-

[21] I chose 97 % availability as cut-off, because it provided a good balance between non-distorted results and excluding too many stocks from the analysis.

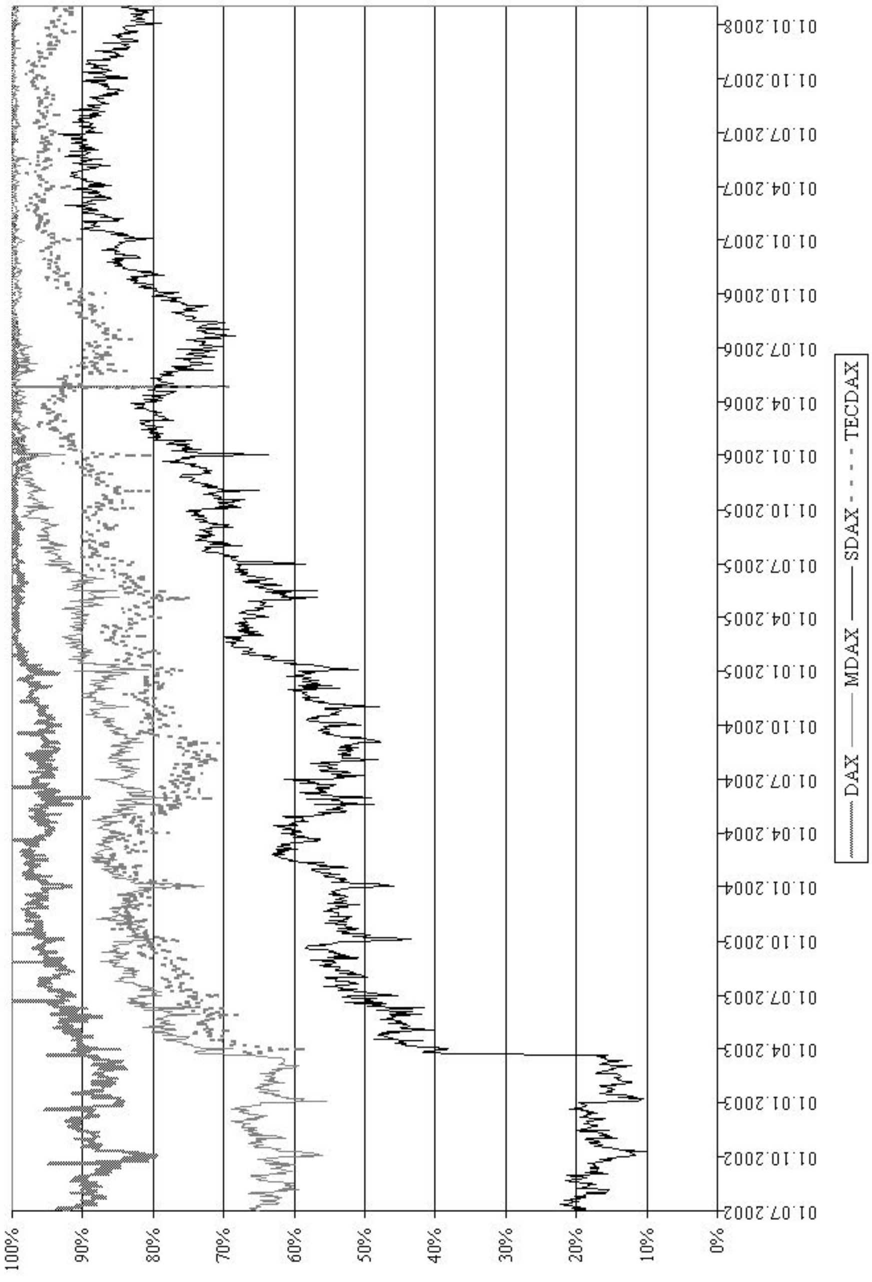

Figure 3.3: Development of average availability by index

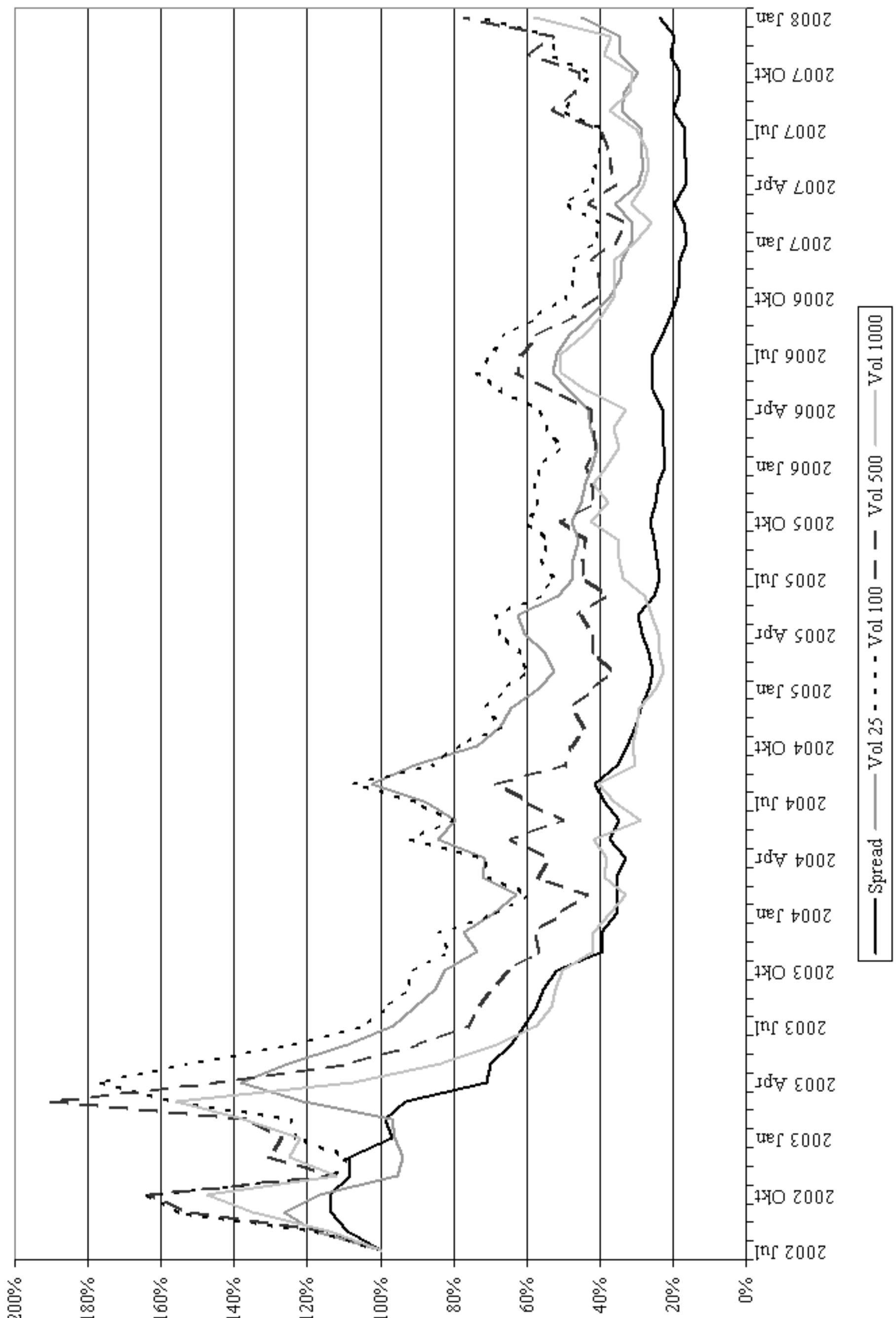

Figure 3.4: Development of average liquidity cost by selected volume classes (constant sample, indexed)
Monthly average of stocks with availability over 97% in volume class; values are indexed on 07/2002 mean.

ant of flight to liquidity, where stocks positions that are liquid remain relatively liquid in crises, while less liquid stock positions suffer more.

The discussion shows that the dynamics of liquidity are similar in the general direction across order sizes. However, the absolute magnitude of change is different. Absolute improvement has been greatest in larger sizes. I have also revealed different crises behavior, where I uncovered a flight-to-liquidity asymmetry between the liquidity of small and large order sizes. This is a strong indication that liquidity risk will increase strongly with increasing position size. Applying time dynamics from liquidity measures of small positions such as the spread will be inappropriate for capturing the dynamics of the liquidity deeper in the order book.

3.3.4 Distributional characteristics

Since I have access to a very representative sample, I will dedicate some time and space to the distributional characteristics of weighted spread. The analysis of the distributional characteristics is useful for several applications, for example in risk measurement and management, in asset pricing models or in theoretical models to assume appropriate liquidity processes.

As the selection of reported volume classes is arbitrary, it is important not to calculate aggregate distribution statistics across order classes. Fineness of the reported classes would directly impact distributional characteristics. I therefore present all distributional statistics separate for each order size. This also allows to investigate the impact of order size on the liquidity distribution.

Table 3.7 presents distributional statistics on liquidity cost and absolute liquidity cost change in bp. The size-impact statistic reveals that there is a statistically significant size-impact not only on the liquidity level, but also on its variance, skewness and kurtosis. Variance seems to be closely connected to the level of liquidity. The cost mean and also

Liquidity cost (in bp)

Distribution statistic	Cross-section statistic	Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
		Order size (in thsd. Euro where applicable)																
Mean	Mean	1.06	48.94	48.59	62.38	92.62	87.37	130.38	130.25	163.60	249.10	192.25	97.51	116.90	136.30	153.17	108.36	17.76 **
	Median	0.48	32.38	30.22	39.03	67.61	59.12	104.77	103.05	153.73	216.98	161.66	67.91	97.67	120.89	154.43	69.23	18.74 **
Median	Mean	0.58	40.89	39.20	48.57	70.26	65.35	98.54	98.31	127.62	202.42	151.00	50.09	61.85	76.07	90.46	81.87	8.67
	Median	0.35	26.94	24.74	31.28	49.50	43.11	73.43	71.16	107.43	173.07	120.69	41.29	53.25	65.91	81.57	50.02	9.66 *
Variance[a]	Mean	0.66	34.03	38.08	68.53	145.03	159.11	233.70	228.40	269.32	429.36	426.32	349.77	407.01	436.76	487.97	194.64	77.50 ***
	Median	0.00	3.87	3.57	5.92	20.52	16.91	81.19	84.92	162.11	283.40	172.44	62.08	106.68	171.06	184.68	23.46	31.81 ***
Skewness	Mean	2.99	1.96	2.41	2.47	2.61	2.86	3.06	2.92	3.01	3.30	3.23	3.22	3.94	3.65	3.75	2.80	0.27 ***
	Median	2.07	1.65	1.79	1.92	2.24	2.24	2.62	2.44	2.48	2.19	2.43	2.54	2.80	2.50	2.58	2.18	0.14 ***
Kurtosis	Mean	31.90	13.65	23.07	18.98	17.39	22.87	23.42	22.75	23.71	32.38	27.43	24.86	44.18	35.14	37.39	22.93	3.58 ***
	Median	8.65	6.58	6.74	7.87	9.75	10.21	13.63	10.88	11.75	10.43	11.37	11.14	11.74	10.98	12.11	9.53	0.75 ***
Mean - Median	Mean	0.48	8.05	9.39	13.82	22.36	22.02	31.84	31.94	35.99	46.69	41.26	47.42	55.05	60.23	62.71	26.49	9.09 ***
	Median	0.09	4.72	4.95	6.99	13.65	12.15	25.04	23.58	31.40	41.08	37.66	28.71	45.06	53.81	49.52	14.32	8.08 ***

Absolute change in liquidity cost (in bp)

Distribution statistic	Cross-section statistic	Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
		Order size (in thsd. Euro where applicable)																
Mean	Mean	0.00	0.03	-0.01	-0.03	-0.14	-0.13	-0.18	-0.17	-0.20	-0.06	0.00	-0.08	-0.16	-0.27	-0.35	-0.10	-0.03
	Median	0.00	-0.02	-0.02	-0.02	-0.04	-0.02	-0.06	-0.03	-0.03	-0.11	-0.02	-0.05	-0.08	-0.12	-0.17	-0.03	-0.02
Median	Mean	0.00	-0.07	-0.05	-0.17	-0.29	-0.17	-0.23	-0.03	-0.02	0.53	0.40	0.10	0.28	0.24	0.09	-0.04	0.08 **
	Median	0.00	-0.02	-0.01	-0.04	-0.11	-0.02	-0.11	0.01	0.01	0.18	0.13	0.06	0.16	0.11	0.08	-0.01	0.03 ***
Variance[a]	Mean	0.07	23.07	18.82	32.12	59.12	62.16	115.24	109.57	144.23	261.86	245.74	110.00	176.26	151.93	168.03	94.03	29.94 ***
	Median	0.00	1.27	1.10	2.09	6.92	7.42	27.94	42.87	64.45	144.82	75.64	15.24	31.11	54.52	66.68	9.55	11.55 **
Skewness	Mean	0.05	-0.01	-0.01	-0.01	0.23	0.05	0.01	-0.07	-0.31	-0.27	-0.06	-0.03	-0.12	-0.66	-0.34	-0.05	-0.07
	Median	0.01	0.05	0.02	0.04	0.15	0.01	-0.03	-0.05	-0.13	-0.10	-0.04	0.01	-0.13	-0.34	-0.31	0.00	-0.05
Kurtosis	Mean	27.26	19.09	34.33	37.14	30.36	43.26	37.27	41.54	43.13	42.57	49.63	64.40	78.86	73.49	79.45	40.20	8.72 ***
	Median	7.97	8.38	9.67	12.31	15.69	16.86	18.99	18.61	18.01	15.91	20.04	20.97	22.50	24.39	22.45	15.21	2.25 ***
Mean - Median	Mean	0.00	0.10	0.04	0.14	0.16	0.04	0.05	-0.14	-0.18	-0.59	-0.41	-0.18	-0.43	-0.50	-0.43	-0.06	-0.11
	Median	0.00	-0.01	0.00	0.01	0.04	-0.01	-0.03	-0.06	-0.04	-0.41	-0.16	-0.08	-0.24	-0.25	-0.23	-0.02	-0.05

Table 3.7: Distributional characteristics of liquidity

The "Min"-column contains the distribution statistic of the half-spread for a minimum order size, other order size columns contain the statistics for weighted spread L(q) and dL(q); Size impact is the coefficient of log order-size in an OLS-regression of the distribution statistic on log order-size including an intercept; a. Values in 10^2; ***, ** and * indicate 1%, 5% and 10% confidence level of being different from zero based on a two-tailed test.

the variance at the spread level are much lower. Otherwise, the distribution of the spread behaves similar to the distribution at the € 10 thsd. volume class.

Looking at absolute liquidity changes removes the skewness, which reveals that trend is a major cause of the skewness. The negative mean and median reflect the overall negative trend in the sample period. The trend seems to be increasing with size, but only in the median stock. The absolute value of the trend is very small, below 0.5bp per day on average. But variance is large so changes in liquidity cost can be quite significant at certain times. There has been no overall trend in the spread. Even when trend is removed, the distribution remains heavily fat-tailed. Kurtosis also strongly increases with order size.

In order to create a distribution that is more closely normally distributed, I take the logarithm of absolute liquidity in basis points.[22] Table 3.8 shows that this removes most of the kurtosis and skewness. Distributions are now by tendency much more normal. Kurtosis is almost removed, while some skewness remains in the data. While the economic interpretation is more difficult, this conversion is helpful in statistical applications, for example mean-variance estimations. Size impact remains intact and statistically significant for practically all statistics at the 1-5% level.

To analyze the remaining kurtosis in more detail from an economic point-of-view, I concentrate on outliers as potential source. To identify outliers, I calculate standardized z values of log liquidity log(L(q)) by subtracting the monthly mean and dividing by the monthly standard deviation. Scanning of situations with absolute z-values above 3 (0.4% of all observations), reveals four types of outlier situations, which all present variants of market imperfections.

First, some records of L(q) exceed 100% (46 observations), i.e. transaction cost exceed the price. This could be due to data punching errors

[22]Please note that I take logarithm of liquidity cost in basis points, i.e. in 10^{-4}, not in decimal.

Distribution statistic	Cross-section statistic	Order size (in thsd. Euro where applicable)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
Mean	**Mean**	-1.16	3.39	3.18	3.39	3.99	3.68	4.36	4.11	4.38	5.16	4.56	3.89	4.17	4.37	4.52	3.92	0.19 ***
	Median	-1.10	3.31	3.25	3.48	4.00	3.83	4.38	4.31	4.72	5.18	4.77	3.94	4.32	4.47	4.61	4.01	0.21 ***
Median	**Mean**	-1.12	3.34	3.11	3.31	3.92	3.59	4.29	4.02	4.29	5.13	4.46	3.66	3.94	4.15	4.32	3.83	0.16 **
	Median	-1.05	3.29	3.21	3.44	3.90	3.76	4.30	4.26	4.68	5.15	4.79	3.72	3.98	4.19	4.40	3.91	0.17 **
Variance	**Mean**	0.86	0.30	0.31	0.34	0.39	0.39	0.44	0.44	0.45	0.40	0.45	0.61	0.60	0.59	0.57	0.40	0.05 ***
	Median	0.78	0.26	0.26	0.29	0.34	0.33	0.38	0.36	0.36	0.33	0.37	0.50	0.54	0.55	0.52	0.33	0.05 ***
Skewness	**Mean**	-0.33	0.22	0.40	0.44	0.36	0.50	0.39	0.48	0.45	0.22	0.49	0.90	0.89	0.83	0.81	0.44	0.09 ***
	Median	-0.36	0.29	0.45	0.50	0.40	0.56	0.41	0.51	0.49	0.23	0.52	0.91	0.92	0.85	0.93	0.49	0.09 ***
Kurtosis	**Mean**	3.64	3.30	3.29	3.28	3.36	3.41	3.46	3.39	3.45	3.63	3.87	3.72	3.60	3.45	3.47	3.42	0.06 ***
	Median	3.14	3.01	3.07	3.08	3.19	3.14	3.21	3.19	3.19	3.26	3.24	2.99	3.08	2.98	3.06	3.13	0.00
Mean - Median	**Mean**	-0.03	0.05	0.07	0.08	0.07	0.09	0.07	0.09	0.09	0.04	0.10	0.23	0.23	0.22	0.21	0.09	0.03 ***
	Median	-0.05	0.04	0.07	0.07	0.05	0.08	0.06	0.08	0.08	0.02	0.07	0.20	0.20	0.22	0.20	0.07	0.03 ***

Table 3.8: Distributional characteristics of log liquidity

The "Min"-column contains the distribution statistic of the half-spread for a minimum order size, other order size columns contain the statistics for log weighted spread log(L(q)); Size impact is the coefficient of log order-size in an OLS-regression of the distribution statistic on log order-size including an intercept; ***, ** and * indicate 1%, 5% and 10% confidence level of being different from zero based on a two-tailed test.

or due to highly asymmetrical order books, where limit orders on the ask-side in the depth of the book are much larger than 200% of the mid-price. If the limit order book is highly asymmetrical, my estimation procedure for a per-transaction liquidity cost in equation (3.4) produces economically meaningless results. It is also very plausible that liquidity prices were inefficient in these situations. I removed these meaningless records from further analysis.

Second, outliers occur after large changes in trading volume, i.e. either if trading volume was very large on that day or on the day before. My explanation is that large trading volume consumes limit orders and will lead to large liquidity cost if resiliency for this particular stock is low. In this case, new limit orders do not refill the order book quickly enough. As a consequence, not all situations with exceptionally high volume exhibit large liquidity cost, but only those where resiliency was low.

Third, outliers occur after large price returns, because limit orders are fixed and do not necessarily adjust quickly to changing mid-prices. This is another type of low resiliency.

Fourth, outliers can be identified near the maximum order book depth as measured by the maximum volume class available in the liquidity data. This is also consistent with the fact that kurtosis increases with order size. The higher the order size, the more stocks in the sample have reached their maximum depth. In these cases, it is plausible that the price priority rule does not lead to efficient liquidity prices, because single or very few limit orders determine liquidity cost. Because it is implausible that large, single orders underestimate liquidity cost, because this would generate losses to the liquidity provider, a reduction of the number of limit orders will inflate liquidity cost and cause outliers.

In summary, the distributional analysis revealed that applications should use log versions of liquidity and respect liquidity trends that are inherent in the data. Despite the trends, daily fluctuations seem to be random over the longer term.

3.3.5 Cross-sectional comparisons

3.3.5.1 The role of relative order size

In this section, I want to follow up on the hypothesis that order size relative to market value and transaction volume is much more comparable across stocks than absolute order size. As argued in section 3.3.4, this is plausible using common sense. But it is also backed by analogous application of existing theory on the bid-ask-spread.

A market maker quoting the bid-ask spread and a trader initiating a limit order face a very similar situation.[23] A bid-ask-quote or a limit order commit to trade a certain quantity at a certain price. Both liquidity providers will want to get compensated for bearing two risks. First, they have to bear unwanted inventory risk that the price moves against them, e.g. through new, favorable information, while the limit order is in the order book. Second, they have to protect themselves against adverse information risks that traders only trade against limit orders when they are better informed. Liquidity costs, which are returns for liquidity providers, therefore compensate for price risk (i.e. inventory risk), informational asymmetry and possibly, in addition, the fixed cost for providing liquidity.[24] These risks get relatively more important for larger order sizes as capital restrictions aggravate the situation of the trader.

To analyze the impact of order size in the light of above consideration, I use the following ordinary least squared (OLS) regression specification in a pooled panel. It also mirrors my assumption that liquidity cost depend on relative order size. Sub-index t indicates time and super-

[23] This has been modeled for example by Rosu (2003) and Beltran et al. (2005).

[24] Cp. Grossman and Miller (1988) and the overview in Stoll (2000).

index i the stock. Formulation in elasticities allows for smooth statistical properties.[25]

$$log(L_t^i(q)) = C + \sum_j \beta_j z_{jt}^i + \sum_{k=1}^{4} \alpha_k log(L_{t-k}^i(q)) + \epsilon_t \qquad (3.5)$$

$L(q)$ is liquidity cost to be explained. C is a constant capturing the fixed cost liquidity level. I use different combinations of explanatory variables z_j. I included four lags of log liquidity to remove autocorrelation in the error term. ϵ_t is the time-varying error term. The main dependent variables z_j are as follows:

- $log(q_t^i)$ is the log of the size of the position in thsd. Euro,
- $log(VO)$ is the log of the trading volume in thsd. number of stocks,
- $log(MV)$ is the log of market value of the stock in million Euro,
- R is the continuous mid-price return of the day in percent,
- $log(\sigma_R)$ is the log of the daily return volatility in percent, which I measure with the 10-day backward looking, moving volatility.
- $log(P)$ is the log of the price level of the day in Euro.

Position size q is included to estimate the size impact. It proxies for the importance of capital restrictions. Transaction volume VO is a good proxy for low inventory risk due to higher participation in trading a particular stock. If transaction volume increases, the time until a limit order is executed is reduced, which in turn reduces unwanted price risk. Market value MV is a good proxy for both low inventory risk due to low price risk and low adverse information risk. High market value stocks experience higher coverage by analysts and traders. This reduces information asymmetries. In total, the same position in a high market value

[25]Cf. discussion in section 3.3.4.

Variance inflation factors (VIFs)	
Intercept	Constant
Log(x)	**5.36**
Log(VO)	**5.24**
Log(MV)	**5.27**
R	1.00
log(RSIGMA10)	1.50
Log(P)	2.52
Log(L(x)_t-1)	**18.24**
Log(L(x)_t-2)	**22.33**
Log(L(x)_t-3)	**22.28**
Log(L(x)_t-4)	**17.84**

Table 3.9: Variance inflation factors for combined specification

and high transaction volume stock should experience lower liquidity cost due to lower risks.

Continuous return R controls for market conditions and is also a proxy for increased trading and thus reduces inventory risk through shorter delay. Return volatility σ_R directly captures inventory risk and is also a control for market conditions. Price level P captures the fix cost of liquidity provision as low price stock require a higher liquidity cost percentage if fix costs exist.

I will have two main lines of regression specification. One includes market value as determinant and the other includes return volatility and price level. A combined specification leads to high multicollinearity as can be seen from the variance inflation factors (VIFs) in table 3.9 of the combined regression. I assume that this is caused because market value acts as proxy for differences in risk and will be correlated with the other risk factors. While the first specification line investigates into my hypothesis of order size, relative to market value and transaction volume, being a determinant of liquidity cost, the second specification analyzes liquidity cost when more finely accounting for differences in stock characteristics. I also employ different time-specific intercepts besides the constant intercept to account for time variation.

Table 3.10 shows results of the specification with market value. Model 1.0 reveals regression results with constant intercept. Coefficients are reported in percent. All variables are statistically significant at the 1% level. Adjusted-R^2 is high, the Durbin-Watson statistic indicates that autocorrelation has been successfully removed with four lags of the dependent variable.

Coefficient signs are as expected. Order size q is positively related to liquidity costs. Increases in market value MV and transaction volume VO decrease liquidity cost as does price return R. Liquidity is very persistent as can be seen from the high coefficients of the lagged variables. Standardized coefficients (reported in 10^4) reveal that return is, by far, the most influential factor. Increasing returns by one percentage point increases liquidity cost by 1.03 %. Order size and market volume are more important than transaction volume.

Interesting is the absolute value of the coefficients. When order size, market value and transaction volume is proportionally increased, liquidity cost remain approximately constant.[26] This confirms my hypothesis that relative order size, i.e. order size relative to transaction volume and market value, is a decisive category when comparing liquidity across stocks and time. It is also a practical rule of thumb. The error of this rule of thumb remains below 1.5% between specifications.

Results are robust when controlling for time variation in liquidity cost with yearly intercepts in models 1.1 or even finer, quarterly intercepts in model 1.2. Only coefficient levels vary very slightly. There is, however, high multicollinearity in the latter specification as revealed by the variance inflation factors at the bottom of the table, which distort results.

Time varying intercepts reveal that the descriptive results of section 3.3.3 must be differentiated. Liquidity levels improved over the last years,

[26] With an error of only 0.74% (=5.27% - 3.15% - 2.85%).

	Model 1.0			Model 1.1			Model 1.2			Model 1.3		
	Coef.	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**
Log(q)	5.27 ***	(0.00)	0.12	6.27 ***	(0.00)	0.17	6.43 ***	(0.00)	0.17	7.33 ***	(0.00)	0.19
Log(VO)	-3.15 ***	(0.00)	-0.05	-3.40 ***	(0.00)	-0.05	-3.46 ***	(0.00)	-0.05	-3.00 ***	(0.00)	-0.04
Log(MV)	-2.85 ***	(0.00)	-0.07	-3.67 ***	(0.00)	-0.10	-3.79 ***	(0.00)	-0.11	-2.86 ***	(0.00)	-0.07
R	-102.67 ***	(0.01)	-103.36	-99.16 ***	(0.01)	-98.87	-97.27 ***	(0.01)	-96.68	-99.43 ***	(0.01)	-98.22
Const. intercept	44.56 ***	(0.00)	10.42							53.69 ***	(0.00)	12.80
Log(SCOM(t))										5.63 ***	(0.00)	0.13
Intercept 2002				60.23 ***	(0.00)	17.96						
Intercept 2003				56.60 ***	(0.00)	15.54						
Intercept 2004				53.34 ***	(0.00)	14.01						
Intercept 2005				51.09 ***	(0.00)	12.87						
Intercept 2006				52.06 ***	(0.00)	13.33						
Intercept 2007				52.57 ***	(0.00)	13.53						
Intercept 2008				59.14 ***	(0.00)	17.40						
Intercept 2002 Q3							62.65 ***	(0.00)	19.88			
Intercept 2002 Q4							61.02 ***	(0.00)	19.20			
Intercept 2003 Q1							60.06 ***	(0.00)	18.62			
Intercept 2003 Q2							58.65 ***	(0.00)	17.51			
Intercept 2003 Q3							58.20 ***	(0.00)	16.73			
Intercept 2003 Q4							56.26 ***	(0.00)	15.87			
Intercept 2004 Q1							55.34 ***	(0.00)	15.68			
Intercept 2004 Q2							54.72 ***	(0.00)	15.43			
Intercept 2004 Q3							55.43 ***	(0.00)	15.42			
Intercept 2004 Q4							53.58 ***	(0.00)	14.53			
Intercept 2005 Q1							52.31 ***	(0.00)	13.98			
Intercept 2005 Q2							52.56 ***	(0.00)	13.99			
Intercept 2005 Q3							52.14 ***	(0.00)	13.76			
Intercept 2005 Q4							52.82 ***	(0.00)	13.98			
Intercept 2006 Q1							52.61 ***	(0.00)	14.03			
Intercept 2006 Q2							55.65 ***	(0.00)	15.22			
Intercept 2006 Q3							53.02 ***	(0.00)	14.31			
Intercept 2006 Q4							52.60 ***	(0.00)	13.85			
Intercept 2007 Q1							52.58 ***	(0.00)	13.97			
Intercept 2007 Q2							52.74 ***	(0.00)	14.05			
Intercept 2007 Q3							54.85 ***	(0.00)	14.87			
Intercept 2007 Q4							55.69 ***	(0.00)	15.08			
Intercept 2008 Q1							60.65 ***	(0.00)	18.14			
Log(L(q)_t-1)	50.20 ***	(0.00)	4.47	49.62 ***	(0.00)	4.42	49.50 ***	(0.00)	4.41	48.81 ***	(0.00)	4.31
Log(L(q)_t-2)	16.13 ***	(0.00)	1.44	15.80 ***	(0.00)	1.41	15.74 ***	(0.00)	1.41	15.47 ***	(0.00)	1.37
Log(L(q)_t-3)	11.39 ***	(0.00)	0.98	11.09 ***	(0.00)	0.95	11.05 ***	(0.00)	0.95	10.88 ***	(0.00)	0.93
Log(L(q)_t-4)	13.69 ***	(0.00)	1.04	13.25 ***	(0.00)	1.01	13.21 ***	(0.00)	1.01	12.98 ***	(0.00)	0.98
No. of obs.	1,772,853			1,772,853			1,772,853			1,764,198		
Adj. R-squared	0.95			0.95			0.95			0.95		
Durbin-Watson stat.	2.01			2.00			2.00			2.01		
Schwarz crit.	0.39			0.39			0.39			0.37		

Variance inflation factors (VIFs)				
Intercept	constant	yearly	quarterly	constant
Log(q)	3.77	4.29	4.38	4.32
Log(VO)	2.58	2.65	2.68	2.58
Log(MV)	4.62	**5.02**	**5.09**	4.61
R	1.00	1.00	1.01	1.00
SCOM(t)				2.05
Log(L(q)_t-1)	**18.13**	**18.24**	**18.26**	**18.26**
Log(L(q)_t-2)	**22.62**	**22.65**	**22.66**	**22.61**
Log(L(q)_t-3)	**22.61**	**22.64**	**22.64**	**22.59**
Log(L(q)_t-4)	**17.99**	**18.07**	**18.08**	**18.00**

Table 3.10: Regression results on relative order size

Dependent variable is log(L(q)), which is log liquidity cost of order size q in bp, q is order size in thsd. Euro, MV is market value in million Euro, VO is transaction volume in thsd. stocks, RSIGMA10 is the 10-day backward rolling volatility, P is the mid-price, R is the cont. mid-price return, SCOM is the average log half-spread at time t.

Heteroskedasticity consistent coefficient errors and covariances (White (1980)) used; standard errors in brackets; ***, ** and * indicate significance at 1%, 5% and 10% level; Coef.* contains coefficients standardized by coefficient variance over dependent variable variance in 10^4.

but almost reached past levels in the recent crises when accounting for improved market values and transaction volumes.

In model 1.3, I use the prevailing spread level as daily intercepts. Spread level is measured as the average daily half-spread across all stocks SCOM, also dubbed liquidity commonality. When finely accounting for time variation, results remain unchanged.

I now turn to the second main specification, which precludes market value MV but includes return volatility RSIGMA10 and price level P to control for stock characteristics in a more differentiated way. Table 3.10 shows regression statistics. Model 2.0 has been specified with constant intercept. The regression shows no autocorrelation and high adjusted-R^2. This specification is slightly preferable as shown with the lower Schwarz criterion compared to models 1.x.

All effects work in the expected direction. Liquidity cost is negatively related to transaction volume, price return and price level. It is positively correlated with order size and mid-price return volatility. Return keeps its dominant role and the coefficient is very similar to prior specifications of 1.x. In contrast, transaction volume VO takes a more important role. Increase of transaction volume by 100% decreases liquidity by 7.65 % in model 1.0 compared to 3.15 % in model 2.0. Return volatility's (RSIGMA10) influence is smallest. Absolute order size q has higher coefficients when more finely controlling for differences in stocks.

Effects are again robust when accounting for time variation in the various forms in models 2.1 to 2.3. Time coefficients show that time patterns are similar to the models 1.x, but more robust here because there is no multicollinearity.

In summary, I have shown that order size is a significant determinant of liquidity cost, even when controlling for different stock characteristics and time variation. I can also safely conduct that relative order size is a much better category for comparing liquidity across cross-sections than absolute order size depending on the question at hand. Liquidity of an absolute order size might be of interest when holding a similar position

	Model 2.0			Model 2.1			Model 1.2			Model 1.3		
	Coef.	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**	**Coef.**	**Stdev.**	**Coef.'**
Log(q)	9.72 ***	(0.00)	0.28	10.13 ***	(0.00)	0.30	10.21 ***	(0.00)	0.30	10.66 ***	(0.00)	0.31
Log(VO)	-7.65 ***	(0.00)	-0.17	-7.98 ***	(0.00)	-0.19	-8.04 ***	(0.00)	-0.19	-7.13 ***	(0.00)	-0.16
R	-95.79 ***	(0.01)	-90.75	-93.29 ***	(0.01)	-87.52	-91.23 ***	(0.01)	-85.60	-94.72 ***	(0.01)	-88.97
log(RSIGMA10)	4.39 ***	(0.00)	0.09	3.98 ***	(0.00)	0.09	3.86 ***	(0.00)	0.09	3.96 ***	(0.00)	0.09
Log(P)	-9.24 ***	(0.00)	-0.32	-9.96 ***	(0.00)	-0.37	-10.09 ***	(0.00)	-0.37	-8.41 ***	(0.00)	-0.29
Const. intercept	117.14 ***	(0.01)	43.79							114.99 ***	(0.01)	42.78
Log(SCOM(t))										4.22 ***	(0.00)	0.10
Intercept 2002				121.81 ***	(0.01)	47.10						
Intercept 2003				119.60 ***	(0.01)	45.18						
Intercept 2004				117.32 ***	(0.01)	44.19						
Intercept 2005				115.82 ***	(0.01)	43.35						
Intercept 2006				117.91 ***	(0.01)	44.57						
Intercept 2007				119.78 ***	(0.01)	45.72						
Intercept 2008				127.91 ***	(0.01)	54.47						
Intercept 2002 Q3							123.11 ***	(0.01)	49.20			
Intercept 2002 Q4							120.64 ***	(0.01)	47.62			
Intercept 2003 Q1							122.15 ***	(0.01)	48.60			
Intercept 2003 Q2							119.17 ***	(0.01)	46.81			
Intercept 2003 Q3							119.76 ***	(0.01)	46.35			
Intercept 2003 Q4							117.91 ***	(0.01)	44.89			
Intercept 2004 Q1							118.68 ***	(0.01)	45.81			
Intercept 2004 Q2							116.52 ***	(0.01)	45.14			
Intercept 2004 Q3							117.48 ***	(0.01)	45.20			
Intercept 2004 Q4							115.71 ***	(0.01)	44.23			
Intercept 2005 Q1							115.96 ***	(0.01)	44.37			
Intercept 2005 Q2							115.12 ***	(0.01)	43.76			
Intercept 2005 Q3							115.52 ***	(0.01)	43.94			
Intercept 2005 Q4							115.64 ***	(0.01)	43.92			
Intercept 2006 Q1							117.15 ***	(0.01)	44.67			
Intercept 2006 Q2							119.84 ***	(0.01)	46.25			
Intercept 2006 Q3							117.26 ***	(0.01)	45.00			
Intercept 2006 Q4							117.28 ***	(0.01)	44.99			
Intercept 2007 Q1							118.89 ***	(0.01)	45.83			
Intercept 2007 Q2							117.85 ***	(0.01)	46.36			
Intercept 2007 Q3							120.29 ***	(0.01)	46.44			
Intercept 2007 Q4							120.58 ***	(0.01)	46.51			
Intercept 2008 Q1							128.03 ***	(0.01)	54.56			
Log(L(q)_t-1)	47.14 ***	(0.00)	4.38	46.85 ***	(0.00)	4.35	46.74 ***	(0.00)	4.34	46.44 ***	(0.00)	4.30
Log(L(q)_t-2)	14.90 ***	(0.00)	1.38	14.76 ***	(0.00)	1.36	14.72 ***	(0.00)	1.36	14.57 ***	(0.00)	1.34
Log(L(q)_t-3)	10.19 ***	(0.00)	0.90	10.06 ***	(0.00)	0.89	10.06 ***	(0.00)	0.89	9.99 ***	(0.00)	0.88
Log(L(q)_t-4)	12.09 ***	(0.00)	0.96	11.95 ***	(0.00)	0.95	11.97 ***	(0.00)	0.95	11.80 ***	(0.00)	0.93
No. of obs.		1,582,762			1,582,762			1,582,762			1,574,913	
Adj. R-squared		0.95			0.95			0.95			0.95	
Durbin-Watson stat.		1.98			1.97			1.97			1.98	
Schwarz crit.		0.34			0.34			0.34			0.33	

Variance inflation factors (VIFs)				
Intercept	constant	yearly	quarterly	quarterly
Log(q)	4.80	4.94	4.97	5.00
Log(VO)	4.51	4.67	4.69	4.61
R	1.00	1.01	1.01	1.00
log(RSIGMA10)	1.49	1.57	1.60	1.51
Log(P)	2.20	2.40	2.42	2.26
SCOM(t)				2.07
Log(L(q)_t-1)	**18.45**	**18.49**	**18.51**	**18.49**
Log(L(q)_t-2)	**22.56**	**22.57**	**22.57**	**22.53**
Log(L(q)_t-3)	**22.50**	**22.51**	**22.51**	**22.46**
Log(L(q)_t-4)	**17.97**	**17.99**	**18.00**	**17.95**

Table 3.11: Regression results on detailed stock characteristics

Dependent variable is l(q), which is liquidity cost of order size q in bp, q is order size in thsd. Euro, MV is market value in million Euro, VO is transaction volume in number of stocks, RSIGMA10 is the 10-day backward rolling volatility, P is the mid-price, R is the continuous mid-price return, SCOM is the average log half-spread at time t. Heteroskedasticity coefficient errors and covariances (White (1980)); standard errors in brackets; ***, ** and * indicate significance at 1%, 5% and 10% level; Coef.* contains coefficients standardized by coefficient variance over dependent variable variance in 10^4.

in different stocks. Liquidity of a relative order size will be more suitable when investing in a certain fraction of a company or when predicting liquidity cost across stocks. The rule-of-thumb of constant liquidity costs for relative order size (position relative to market value and transaction volume) is quite robust across specifications and has an approximation error of below 1.5 %. The interrelations are astonishingly stable, which might provide an indication, that they are driven by fixed structures yet to be analytically described.

3.3.5.2 Day-of-the-week and holiday effects

Chordia et al. (2001) have found a day-of-the-week effect in the quoted bid-ask-spread. Quoted spread is found to decline from Monday to Friday and be significantly lower next to holidays. I retest this hypothesis on the liquidity cost of different order sizes by including weekday dummies and dummies for days before and after holidays in my regression specification. However, in contrast to Chordia et al. I control for all stock characteristics. Table 3.12 on the next page shows the results. In all my specifications Monday and Fridays have significantly higher liquidity costs. Monday is the least liquid day of the week with liquidity cost around 5% higher than average, Tuesday is the most liquid day. Liquidity then continually deteriorates from Tuesday until the end of the week. Days adjoining holidays are similarly illiquid than start and end of the week.

This contrasts to Chordia et al., because I find Monday to be similarly illiquid than Fridays when looking at position size relative to transaction volume and market capitalization or relative to transaction volume alone. Investors should know that relative position size is more expensive to trade on Mondays, Fridays and on days adjoining holidays.

	Model 5.2			Model 5.3			Model 6.2			Model 6.3		
	Coef.	Stdev.	Coef.*	Coef.	Stdev.	Coef.*	Coef.	Stdev.	Coef.*	Coef.	Stdev.	Coef.*
Log(q)	6.38 ***	(0.00)	0.17	7.30 ***	(0.00)	0.19	10.13 ***	(0.00)	0.30	10.58 ***	(0.00)	0.31
Log(VO)	-3.42 ***	(0.00)	-0.05	-2.96 ***	(0.00)	-0.04	-7.98 ***	(0.00)	-0.19	-7.06 ***	(0.00)	-0.16
Log(MV)	-3.79 ***	(0.00)	-0.11	-2.87 ***	(0.00)	-0.07						
R	-98.20 ***	(0.01)	-97.88	-100.20 ***	(0.01)	-99.16	-91.57 ***	(0.01)	-85.97	-94.96 ***	(0.01)	-89.21
log(RSIGMA10)							3.84 ***	(0.00)	0.09	3.92 ***	(0.00)	0.08
Log(P)							-10.03 ***	(0.00)	-0.37	-8.36 ***	(0.00)	-0.29
Monday	5.11 ***	(0.00)	0.28	5.32 ***	(0.00)	0.29	4.29 ***	(0.00)	0.25	4.49 ***	(0.00)	0.26
Tuesday	-0.63 ***	(0.00)	-0.03	-0.50 ***	(0.00)	-0.03	-0.83 ***	(0.00)	-0.05	-0.75 ***	(0.00)	-0.04
Thursday	2.42 ***	(0.00)	0.13	2.41 ***	(0.00)	0.13	2.17 ***	(0.00)	0.12	2.17 ***	(0.00)	0.12
Friday	3.51 ***	(0.00)	0.19	3.38 ***	(0.00)	0.18	3.05 ***	(0.00)	0.17	2.95 ***	(0.00)	0.17
Before/after holiday	2.99 ***	(0.00)	0.70	3.07 ***	(0.00)	0.71	5.71 ***	(0.00)	1.54	5.49 ***	(0.00)	1.48
Const. intercept				51.53 ***	(0.00)	12.39				112.56 ***	(0.01)	42.22
Log(SCOM(t))				5.62 ***	(0.00)	0.13				4.22 ***	(0.00)	0.10
Intercept 2002 Q3	60.27 ***	(0.00)	19.21				120.76 ***	(0.01)	48.63			
Intercept 2002 Q4	58.65 ***	(0.00)	18.54				118.36 ***	(0.01)	47.10			
Intercept 2003 Q1	57.64 ***	(0.00)	17.98				119.73 ***	(0.01)	48.02			
Intercept 2003 Q2	56.33 ***	(0.00)	16.89				116.88 ***	(0.01)	46.24			
Intercept 2003 Q3	55.95 ***	(0.00)	16.18				117.53 ***	(0.01)	45.85			
Intercept 2003 Q4	53.97 ***	(0.00)	15.32				115.63 ***	(0.01)	44.38			
Intercept 2004 Q1	53.06 ***	(0.00)	15.13				116.41 ***	(0.01)	45.29			
Intercept 2004 Q2	52.51 ***	(0.00)	14.89				114.31 ***	(0.01)	44.60			
Intercept 2004 Q3	53.13 ***	(0.00)	14.87				115.19 ***	(0.01)	44.66			
Intercept 2004 Q4	51.33 ***	(0.00)	14.01				113.45 ***	(0.01)	43.71			
Intercept 2005 Q1	50.14 ***	(0.00)	13.50				113.72 ***	(0.01)	43.86			
Intercept 2005 Q2	50.29 ***	(0.00)	13.48				112.92 ***	(0.01)	43.26			
Intercept 2005 Q3	49.86 ***	(0.00)	13.26				113.25 ***	(0.01)	43.42			
Intercept 2005 Q4	50.61 ***	(0.00)	13.49				113.39 ***	(0.01)	43.41			
Intercept 2006 Q1	50.35 ***	(0.00)	13.53				114.93 ***	(0.01)	44.18			
Intercept 2006 Q2	53.46 ***	(0.00)	14.71				117.63 ***	(0.01)	45.75			
Intercept 2006 Q3	50.76 ***	(0.00)	13.80				115.00 ***	(0.01)	44.50			
Intercept 2006 Q4	50.35 ***	(0.00)	13.35				115.02 ***	(0.01)	44.48			
Intercept 2007 Q1	50.36 ***	(0.00)	13.49				116.66 ***	(0.01)	45.35			
Intercept 2007 Q2	50.46 ***	(0.00)	13.53				115.52 ***	(0.01)	45.73			
Intercept 2007 Q3	52.57 ***	(0.00)	14.35				118.01 ***	(0.01)	45.93			
Intercept 2007 Q4	53.39 ***	(0.00)	14.56				118.31 ***	(0.01)	46.01			
Intercept 2008 Q1	58.13 ***	(0.00)	17.55				125.77 ***	(0.01)	54.70			
Log(L(q)_t-1)	49.48 ***	(0.00)	4.41	48.75 ***	(0.00)	4.31	46.77 ***	(0.00)	4.35	46.45 ***	(0.00)	4.31
Log(L(q)_t-2)	16.04 ***	(0.00)	1.43	15.76 ***	(0.00)	1.40	14.97 ***	(0.00)	1.38	14.83 ***	(0.00)	1.37
Log(L(q)_t-3)	11.14 ***	(0.00)	0.96	10.97 ***	(0.00)	0.94	10.15 ***	(0.00)	0.90	10.07 ***	(0.00)	0.89
Log(L(q)_t-4)	12.87 ***	(0.00)	0.98	12.66 ***	(0.00)	0.96	11.66 ***	(0.00)	0.93	11.50 ***	(0.00)	0.91
No. of obs.	1,770,606			1,761,951			1,580,593			1,572,744		
Adj. R-squared	0.95			0.95			0.95			0.95		
Durbin-Watson stat.	2.00			2.01			1.98			1.98		
Schwarz crit.	0.38			0.36			0.33			0.33		

Variance inflation factors (VIFs)				
Intercept	constant	yearly	quarterly	quarterly
Log(q)	4.38	4.32	4.98	**5.01**
Log(VO)	2.68	2.59	4.71	4.63
Log(MV)	**5.09**	4.61		
R	1.01	1.00	1.01	1.00
log(RSIGMA10)			1.60	1.51
Log(P)			2.43	2.26
SCOM(t)		2.05		2.07
Log(L(q)_t-1)	**18.37**	**18.37**	**18.62**	**18.59**
Log(L(q)_t-2)	**22.77**	**22.72**	**22.68**	**22.64**
Log(L(q)_t-3)	**22.75**	**22.70**	**22.61**	**22.57**
Log(L(q)_t-4)	**18.17**	**18.09**	**18.10**	**18.05**

Table 3.12: Day-of-the-week and holiday effect

Dependent variable is log(L(q)), which is log liquidity cost of order size q in bp, q is order size in thsd. Euro, MV is market value in million Euro, VO is transaction volume in thsd. stocks, RSIGMA10 is the 10-day backward rolling volatility, P is the mid-price, R is the cont. mid-price return, SCOM is the average half-spread at time t. Heteroskedasticity consistent coefficient errors and covariances (White (1980)) used; standard errors in brackets; ***, ** and * indicate significance at 1%, 5% and 10% level; Coef.* contains coefficients standardized by coefficient variance over dependent variable variance in 10^4.

3.3.6 Synopsis

Based on a representative sample of weighted spread for over 320 thousand stock-days, I analyzed the impact of size on liquidity cost, its variation and generally its distributional characteristics. My main finding is that the impact of order size on liquidity is substantial and cannot be neglected. Easily available bid-ask-spread data can only poorly proxy for cost level and its variation in larger position sizes.

Average liquidity costs varied greatly between order sizes and stocks, strongly increasing with order size up to 460 bp. DAX was the most liquid with the lowest cost, followed by MDAX, TecDAX and than SDAX. Even in the DAX, liquidity cost surpassed 100 bp for order sizes larger than € 2 million. The possibility of being able to liquidate a position against the order book also strongly declined with size and showed a similar cross-sectional rank than the cost level. Availability was > 90% for small sizes, but dropped to 13 % for € 1 million in the SDAX.

Liquidity strongly improved over the last 5.5 years. Liquidity costs continuously decreased during calm, positive market periods. Sudden increases occurred at stock market crashes such as the events of the subprime crises in 2007 and 2008. These spikes are especially pronounced in larger order sizes. The fact that illiquid, large order sizes suffered worse than liquid, small order sizes, presents another aspect of the flight-to-liquidity asymmetry. Trading against the order book was increasingly possible over the sample period. Availability of limit order book increased to 100 % in small orders below € 100 thousand across all indices. DAX and MDAX of any size were almost 100 % tradable in recent months.

Distributional characteristics of liquidity costs differ greatly between order sizes. Not only do mean liquidity costs increase with order size, so does its variance. In the last 5.5 years, liquidity experienced a steady decline. Outliers due to inefficient liquidity prices generate fat tails in the liquidity distribution, especially in large order sizes.

I also investigated into the fact that the liquidity of absolute-Euro order sizes shows very different behavior across stocks. My explanation is that absolute order size is not very comparable across stocks. I show that order size relative to market volume and prevailing transaction volume has very stable liquidity cost across stocks and time. Liquidity of relative order size is therefore much better measure in cross-sectional analysis and can act as a rule-of-thumb in comparisons.

In summary, my main conclusions is that liquidity strongly differs across sizes. An impact of size is traceable in distributional characteristics and liquidity dynamics. The empirical evidence presented here can provide new impetus into theoretical modeling of liquidity. In addition, it has impact on practical applications, where liquidity cost and its variation play a role, especially on risk management.

4 New liquidity risk models

In this chapter, I suggest two new approaches to model liquidity risk. The modified add-on approach in section 4.1 provides a new, alternative framework to account for non-normality in liquidity risk.[1] In section 4.2 I propose a framework to analyze the integration of the weighted spread liquidity measure in liquidity risk measurement.[2]

4.1 Modified add-on model

Many studies show that the assumption of normally distributed returns is rejected for most financial time series, including those for individual stocks, stock indices, exchange rates, and precious metal prices. Specifically, continuous returns for these financial assets have empirical distributions which are leptocurtic relative to the normal distribution and in many cases skewed. Bollerslev (1987), for example, finds leptocurtosis in monthly S&P 500 returns, while French et al. (1987) report skewness in daily S&P 500 returns. Engle and Gonzales-Rivera (1991) find excess skewness and kurtosis in small stocks and in exchange rates.[3]

The argument of non-normality equally holds for liquidity costs. In section 3.3.4 I analyze the distributional properties of liquidity costs and showed that they are heavily skewed and fat tailed. Bangia et al. (1999)

[1] For this section, cp. Ernst, Stange and Kaserer (2008).
[2] For this section, cp. Stange and Kaserer (2008c).
[3] Cp. also Mandelbrot (1963), Fama (1965), Theodossiou (1998).

and Giot and Grammig (2005) account for non-normality in the context of liquidity risk management.

In the following, I outline a simple model of liquidity-adjusted risk based on Bangia et al. (1999) as a basis for discussion. Applicability to other risk models is discussed later. I then propose an adaption based on the Cornish-Fisher expansion which is a technique to correct the percentiles of a standard normal distribution for non-normality.[4]

4.1.1 Liquidity risk approach

I use the straight forward liquidity risk model of Bangia, Diebold, Schuermann and Stroughair (1998, 1999), which has been surveyed in section 2.2.2.1, to show how the risk calculation proceeds. Bangia et al. include time-varying bid-ask-spreads into a parametric Value-at-Risk. They also assume that liquidity costs of a transaction can be measured with the bid-ask-spread. They assume that continuous mid-price returns are normally distributed. Relative liquidity risk can then be calculated as the mean-variance estimated price-return percentile and the empirically estimated spread percentile.[5]

$$L - VaR = 1 - exp(z_\alpha \times \sigma_r) \times \left(1 - \frac{1}{2}\left(\mu_S + \hat{z}_\alpha(S) \times \sigma_S\right)\right) \qquad (4.1)$$

where σ_r is the volatility of the continuous mid-price return assuming zero daily mean returns, μ_S and σ_S are the mean and volatility of the spread - all over the chosen horizon. z_α denotes the percentile of the

[4] Mina and Ulmer (1999) investigate four possible methods to compute the Value-at-Risk for non-normally distributed assets: Johnson transformation, Fourier method, partial Monte-Carlo and Cornish-Fisher expansion. They find that Cornish-Fisher is simple to implement in practice, fast and traceable while the other three approaches requires a much larger implementation effort, but have higher precision for extreme distributions.

[5] I slightly deviate from the original model and incorporate the improvement suggested by Loebnitz (2006) as I deduct the worst half spread from the worst price and not from the current mid-price, which is conceptually more consistent. Notation is my own.

normal distribution for the given confidence level and $\hat{z}_\alpha(S)$ is the empirically estimated percentile of the spread distribution.[6] By estimating the percentile empirically, Bangia et al. avoid distortions from the non-normality in spreads, which they show to be highly present in several currencies.

The model by Bangia et al. represents an intuitively plausible and simple way to incorporate liquidity risk into a conventional Value-at-Risk framework. Data requirements are manageable as mid-price data and spread information are usually easily accessible. Another merit of this model is the additivity of price risk and liquidity risk which facilitates implementation in practice. There is no need to modify existing programs for determining VaR. The only necessary system change is to compute the cost of liquidity and add it to the existing VaR-figure.

Despite its appeal, the model has been subject to criticism in the literature. As extensively discussed and empirically analyzed in section 3.3, the assumption of perfect correlation between mid-price return and liquidity costs leads to distortions. In addition, the model does not account for the price impact of order size, i.e. the fact that liquidity costs strongly increase with the size of the order traded. Further, price risk is assumed as normally distributed and the use of empirical percentiles might not sufficiently capture the non-normality of the future spread distribution.

The following approach addresses this issue of non-normality, which is also present in other modeling solutions. Giot and Grammig (2005), section 2.2.2.3, assume a t-distribution in order to adjust for fat-tails in net returns, i.e. returns net of order-size-adjusted weighted spread. A t-distribution might, however, be similarly misleading than a normal distribution.

[6] The empirical percentile is calculated as $\hat{\alpha}_S = (\hat{S}_\alpha - \mu_S)/\sigma_S$, where $\hat{S}_\alpha$ is the percentile spread of the historical distribution.

4.1.2 Cornish-Fisher expansion

A normal distribution is fully described by its first two moments, mean and variance. Higher centralized moments like skewness and excess-kurtosis are zero. However, if the distribution is non-Gaussian higher moments will also determine loss probabilities. For this reason it is not accurate to use standardized percentiles of a normal distribution for the calculation of L-VaR of non-normally distributed returns. Cornish and Fisher (1937) have been the first to modify the standardized percentiles of a normal distribution in a way that higher moments are accounted for. Their technique is based on the following coherence: If any distribution is fitted by making the first n moments of the fitted and actual distributions agree, it is possible to calculate the percentiles of the fitted distribution and to regard these as approximations to the corresponding percentiles of the actual distribution. Basically the fitted percentiles are functions of the n fitted moments.[7]

Cornish and Fisher (1937) obtain explicit polynomial expansions for standardized percentiles of a general distribution in terms of its standardized moments and the corresponding percentiles of the standard normal distribution.[8] Their proceeding is widely known as Cornish-Fisher expansion. The corresponding formula approximates percentiles of a random variable based on its first few cumulants.[9] Using the first four moments (mean, variance, skewness and kurtosis), the Cornish-Fisher expansion for approximate $\alpha-$percentile $\tilde{z}_\alpha$ of a standardized random variable is calculated as[10]

$$\widetilde{z}_\alpha \approx z_\alpha + \frac{1}{6}(z_\alpha^2 - 1) * \gamma + \frac{1}{24}(z_\alpha^3 - 3z_\alpha) * \kappa - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha) * \gamma^2 \quad (4.2)$$

[7] Cp. Johnson and Kotz (1994), p. 63f.
[8] Cp. Johnson (1978), p. 537.
[9] The cumulants of a distribution are closely related to its moments and can be informally thought of as standardized moments. For a detailed definition of cumulants see Cornish and Fisher (1937).
[10] Cp. Mina and Ulmer (1999), p. 6.

where z_α is the α- percentile of a $N(0,1)$ distribution, γ denotes the skewness and κ the excess-kurtosis of random variable.[11] Note that in case of a normal distribution, skewness γ and excess-kurtosis κ are equal to zero, which leads to $\widetilde{z}_\alpha = z_\alpha$. The approximate $\alpha-$percentile $\tilde{z}_\alpha$ can now be used in a classic Value-at-risk approach.

4.1.3 Definition of liquidity-adjusted risk

Substituting z_α and $\hat{z}_\alpha(S)$ from equation (4.1) with the modified percentile $\tilde{z}_\alpha$ from (4.2) I obtain the following modified VaR estimate

$$L-VaR = 1 - exp(\mu_r + \tilde{z}_\alpha(r) \times \sigma_r) \times \left(1 - \frac{1}{2}\left(\mu_S + \tilde{z}_\alpha(S) \times \sigma_S\right)\right) \quad (4.3)$$

where $\tilde{z}_\alpha(r)$ is the percentile of the return distribution accounting for its skewness and kurtosis and $\tilde{z}_\alpha(S)$ and the corresponding spread distribution percentile.

This approach constitutes a simple parametric approach accounting for mean, variance, skewness and kurtosis of the underlying non-normal distributions. Although skewness and kurtosis are also difficult to estimate it induces less heavy data requirements than any ad-hoc or empirical approach and might possibly more accurately determine the distribution of future returns. However, the expansion is, after all, only a proxy for the real distribution. Therefore, if the real distribution is not sufficiently described by the first four moments or those moments cannot be estimated with sufficient accuracy, this method yields false risk estimates.[12]

[11] The skewness of y is computed from historical data over n days as $\gamma = \frac{1}{n}\sum_{t=1}^{n}(y_t - \bar{y})^3/\hat{\sigma}^3$ with $\bar{y}$ being the expected value and $\hat{\sigma}$ the volatility of y. The excess kurtosis for y is $\kappa = \frac{1}{n}\sum_{t=1}^{n}(y_t - \bar{y})^4/\hat{\sigma}^4 - 3$.

[12] Cp. also Zangari (1996), p. 10f.

While I applied the Cornish-Fisher approximation to the basic spread model of Bangia et al. (1999), analogous use in other liquidity models such as the weighted spread model in Giot and Grammig (2005) is also easily feasible. Section 5.1 will proceed with an empirical test of this new model suggestion.

4.2 Empirical net-return model with weighted spread

The model of Giot and Grammig (2005) as described in section 2.2.2.3 used weighted spread to calculate intraday liquidity-adjusted risk. As developed in section 2.2, two issues will be addressed. Section 4.2.1 develops a new liquidity-risk model based on weighted spread which correctly accounts for correlation between liquidity cost and mid-price return. The empirical, instead of parametrical specification allows to avoid distributional assumptions such as the t-distribution in Giot and Grammig (2005). Section 4.2.2 develops a decomposition which allows to extract liquidity and correlation effects separately.

4.2.1 Definition of liquidity-adjusted risk

Before I turn to defining liquidity risk, I start with the definition of price risk. Standard risk statistics will be used to measure the impact of liquidity risk.

Price and return are described in the usual framework of

$$P_{mid,t} = P_{mid,t-\Delta t} \times exp(r_{t,\Delta t})$$

where P_{mid} is defined as the mid-price $P_{mid,t} = \frac{a_t+b_t}{2}$ with a_t and b_t being the (best) ask- and bid-price at time t respectively. $r_{t,\Delta t}$ is the Δt-period continuous mid-price return at time t, i.e., $r_{t,\Delta t} = ln(P_{mid,t}/P_{mid,t-\Delta t})$. I take a traditional approach from a value-at-risk (VaR) perspective and define price risk as the relative VaR at the $(1 - \alpha)$-percent confidence level over the horizon Δt

$$VaR_{price}^{\alpha,\Delta t} = 1 - exp(r_{t,\Delta t}^{\alpha}) \tag{4.4}$$

where $r_{t,\Delta t}^{\alpha}$ is the α-percentile of Δt-period return distribution. Consequently, VaR_{price} measures the maximum percentage loss over the period Δt with a confidence of $(1 - \alpha)$-percent.

Analogously, I measure total risk including liquidity risk. To calculate the impact of liquidity, I define the Δt-period *net return* in t as the sum of the continuous mid-price return and the liquidity discount converted to a continuous value, $l_t(q) = ln\,(1 - L_t(q))$.

$$rnet_{t,\Delta t}(q) = r_{t,\Delta t} + l_t(q) \tag{4.5}$$

Please note the difference of (4.5) to net-price returns.[13] Using net returns instead of net-price returns, I implicitly assume that the liquidity cost of entering a position has already been properly accounted for. If I used net-price returns, the implicit assumption would be that not only the liquidity cost of entering a position, but also the expected liquidity cost of the liquidation is properly accounted for already when entering it. I believe that my assumption is more realistic in practice.

Transaction price is calculated as

$$P_{net,t}(q) = P_{mid,t-\Delta t} \times exp(r_{t,\Delta t} + l_t(q)) \tag{4.6}$$

where $P_{net,t}(q)$ is the achievable transaction price.

The Δt-period *liquidity-adjusted total risk* is defined in a VaR-framework as the empirical α-percentile of the net-return distribution.

$$VaR_{total}^{\alpha,\Delta t}(q) = 1 - exp(rnet_{t,\Delta t}^{\alpha}(q)) \tag{4.7}$$

[13]I.e. $ln\,([P_{mid,t} \times (1 - L_t(q))]\,/\,[P_{mid,t-1} \times (1 - L_{t-1}(q))])$.

VaR_{total} is the maximum percentage loss due to mid-price risk and liquidation cost over the period Δt with a confidence of $(1 - \alpha)$-percent.

This specification covers the real dynamics of the net return on a certain stock position. It is practical but also more general than existing approaches in the following ways:

1. I use a more precise liquidity measure than most papers by covering more aspects of liquidity. Specifically, I account for the impact of order-size on liquidity. This extends the approach of Bangia et al. (1998, 1999), where liquidity costs of any order size is proxied for with the bid-ask-spread. The XLM measure can be assumed to be more precise than the ones used in Berkowitz (2000a), Francois-Heude and Van Wynendaele (2001) or Angelidis and Benos (2006), because it uses more and direct liquidity data.

2. As I take empirical percentiles instead of a parametric method, I avoid any distributional assumption, especially on liquidity cost, such as in Giot and Grammig (2005). My approach will capture non-normality of the distribution as well, which is made possible by my large sample size.

3. My approach takes percentiles of the net return distribution and does not treat price risk and liquidity separately. I look at the dynamics of net returns which combines the mid-price-return dynamics and liquidity cost dynamics. Instead of adding distribution percentiles of liquidity and price risk separately, I acknowledge that liquidity cost and mid-price might not be perfectly correlated. While it is possible that large liquidity discounts and low prices coincide, this must not be the case.

4.2.2 Risk decomposition

To uncover the structure of the liquidity impact, I decompose total risk into its components. I define relative liquidity impact $\lambda(q)$ as

$$\lambda(q) = \frac{VaR_{total}(q) - VaR_{price}}{VaR_{price}} \tag{4.8}$$

$\lambda(q)$ is the maximum percentage loss due to the liquidity in relation to price risk. It can be interpreted as the error made when ignoring liquidity. It is therefore a measure of the relative significance of liquidity in the risk management context. In addition, it can be used as a scaling factor with which price risk would need to be adjusted in order to correctly account for liquidity. I measure it relative to price risk, because absolute liquidity impact has little meaning by itself for this type of analysis.

In order to uncover the effect of tail correlation between liquidity and price, I define liquidity cost risk as the relative worst liquidity cost

$$VaR^{\alpha}_{liquidity}(q) = 1 - exp(l^{\alpha}_{t,\Delta t}(q)) \tag{4.9}$$

with $l^{\alpha}_{t,\Delta t}$ being the empirical percentile of the continuous liquidity discount. This is the maximum percentage loss due to liquidity cost at an $(1-\alpha)$-percent confidence level.

I can now apply a further decomposition of total risk and define the correlation factor $\kappa(q)$ as residual of

$$VaR_{total}(q) = VaR_{price} + VaR_{liquidity}(q) + \kappa(q) \times VaR_{liquidity}(q) \tag{4.10}$$

Naturally, this is just a further decomposition of the liquidity impact

$$\lambda(q) = \frac{VaR_{liquidity}}{VaR_{price}}(1 + \kappa(q)) \tag{4.11}$$

$\kappa(q)$ measures the tail correlation factor between mid-price return and liquidity cost, the proportion of liquidity risk, that is diversified away due to tail correlation. In this definition the correlation factor is always non positive, $\kappa(q) \leq 0$. If tail correlation is perfect, $\kappa(q)$ is zero and worst mid-prices and worst liquidity costs can be added to get total risk. This corresponds to the add-on approach of Bangia et al. (1999) in section

2.2.2.1. If there is some diversification between cost and price, $\kappa(q)$ will become negative.

The liquidity impact $\lambda(q)$ contains the following conceptual components. First, it contains the mean liquidity discount for the position of size q - in contrast to other approaches. This is suitable as positions are usually valued at mid-prices already neglecting mean liquidity costs. Second, it includes negative deviations from the mean cost as measured by volatility and higher moments. Third, possible diversification effects between price and liquidity are included and reduce liquidity risk. If liquidity cost and mid-prices have a less than perfect, negative tail correlation ($\kappa(q) < 0$), a liquidity risk estimate based on the α-percentile of the liquidity cost distribution as in (4.9) will be incorrectly higher than based on the net-return distribution as in (4.8).

5 Empirical analysis of liquidity risk models

Chapter 5 contains the empirical analysis of the suggested liquidity risk approaches. Section 5.1 analyzes the performance of the modified add-on model based on bid-ask-spread data.[1] In section 5.2, I analyze the question of precise weighted spread risk measurement.[2] In the last section 5.3, all existing and newly proposed models will be benchmarked.[3]

5.1 Modified add-on model

5.1.1 Motivation

To evaluate if the modified add-on model proposed in section 4.1 is not only a mere alternative but offers any improvement, I conduct an empirical backtest. Sections 5.1.2 and 5.1.3 describe the implementation in detail and provide some descriptive statistics on the magnitude of risk estimates. Section 5.1.4 contains a detailed empirical benchmark of the newly proposed model against the original specification of Bangia et al. (1999). I used the large representative set of spread data described in section 3.1.2.

[1]For this section, cp. Ernst, Stange and Kaserer (2008).
[2]For this section, cp. Stange and Kaserer (2008c).
[3]For this section, cp. Ernst, Stange and Kaserer (2009).

5.1.2 Implementation specification

For the risk estimation I chose a 1-day horizon and a 99 % confidence level, conforming to the standard Basel framework, to calculate daily risk forecasts. Mean continuous mid-price return in the Bangia model is set to zero. Spread means as well as returns means in the modified L-VaR model are estimated using a 20-day rolling procedure. I account for volatility clustering using a common exponential weighted moving average (EWMA) method. Volatilities are also calculated rolling over 20 days as

$$\sigma_t^2 = (1-\delta)\sum_{i=1}^{20} \delta^{i-1} r_{t-i}^2 + \delta^{20} r_{t-20}^2$$

with a weight δ of 0.94.[4] Skewness and excess-kurtosis are calculated as 500-day rolling estimates. I choose a very long estimation horizon, because moment estimates with the standard method gain significantly in accuracy with the length of the horizon. This approach therefore aims to evaluate the potential of the Cornish Fisher methodology. Other estimation procedures are available and might generate more precise results with smaller estimation samples.[5] This might be explored in future research.

Table 5.1 provides an overview of skewness and excess-kurtosis estimates. Continuous mid-price returns are only very slightly skewed. However, excess-kurtosis can become quite substantial with values of around 6, which is far from zero of the normal distributions. Spreads are heavily right-skewed and also exhibit fat tails. As exemplary illustration figure 5.1 shows the sample period histogram of the spread for Comdirect, an SDAX stock. It is clear, that the normal distribution hardly fits

[4] In correspondence with JP Morgan (1996) and practical implementation of the last term as squared return instead of squared volatility in Hull (2006), p.575. I neglected the GARCH model class, because it is less common in practice and has higher computational requirements.

[5] Cp. for example Sengupta and Zheng (1997); Joanas and Gill (1998); Kim and White (2004).

Return moment estimates		Index				
		DAX	MDAX	SDAX	TECDAX	All
Skewness	Mean	0.11	-0.06	0.13	0.23	0.08
	Median	0.10	0.09	0.18	0.22	0.14
	Std. Dev.	65.0%	140.1%	111.3%	97.0%	112.9%
Excess-Kurtosis	Mean	3.21	7.49	6.38	6.39	6.12
	Median	1.67	2.67	3.40	3.71	2.85
	Std. Dev.	785.2%	1668.0%	960.7%	955.2%	1221.2%

Spread moment estimates		Index				
		DAX	MDAX	SDAX	TECDAX	All
Skewness	Mean	1.90	1.23	1.22	1.26	1.37
	Median	1.78	1.22	1.06	1.23	1.27
	Std. Dev.	63.4%	69.4%	126.6%	75.4%	94.6%
Excess-Kurtosis	Mean	6.19	2.93	4.35	2.77	3.98
	Median	4.52	1.84	1.29	1.81	2.09
	Std. Dev.	647.4%	474.3%	1658.4%	803.0%	1060.6%

Table 5.1: Skewness and kurtosis estimates

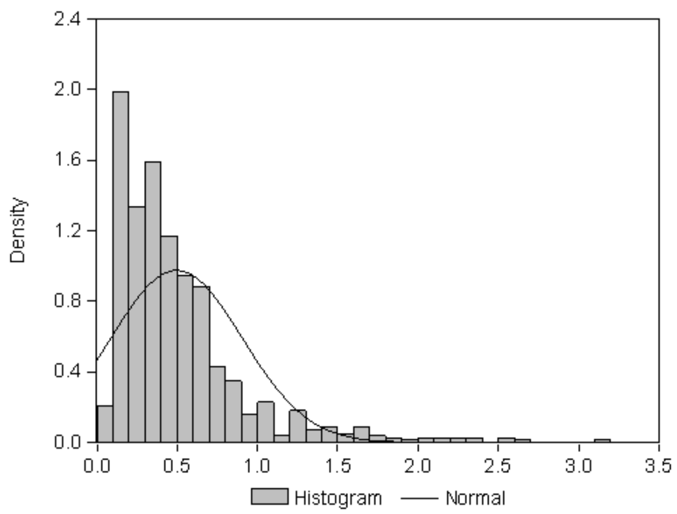

Figure 5.1: Sample period spread histogram and fitted normal distribution for the Comdirect stock

Emp. spread percentile estimate z(S)	**Index**				
	DAX	**MDAX**	**SDAX**	**TECDAX**	**All**
Mean	4.01	3.55	3.35	3.43	3.59
Median	4.02	3.66	3.40	3.30	3.72
Std. Dev.	22.8%	37.8%	63.8%	53.7%	52.2%

Table 5.2: Empirical 99 %-percentile estimates for the Bangia model

Cornish-Fisher percentile estimates z		**Index**				
		DAX	**MDAX**	**SDAX**	**TECDAX**	**All**
Return	**Mean**	-2.84	-3.39	-3.26	-3.29	-3.23
	Median	-2.60	-2.85	-2.84	-2.87	-2.79
	Std. Dev.	125.5%	209.6%	161.3%	157.3%	173.3%
Spread	**Mean**	3.67	3.17	3.08	3.10	3.23
	Median	3.48	3.09	2.97	3.08	3.13
	Std. Dev.	76.3%	65.2%	88.5%	42.2%	75.7%

Table 5.3: Cornish-Fisher 99 %-percentile estimates

the empirical distribution and that the distribution is right-skewed with skewness of 2.24 and fat-tailed with excess kurtosis of 6.97.

In the Bangia framework, I determined the empirical percentiles and calculated $\hat{z}_\alpha(s)$. Descriptive statistics are shown in table 5.2 and are similar to the range of results between 2 and 4.5 for currencies in the original paper. Average empirical 99 %-percentiles range from 3.35 to 4.01 with a mean of 3.69, which is far from the 2.33 for the normal distribution. Worst losses are much more probable than would be expected from the normal distribution.

For the modified L-VaR estimation, I calculated percentiles based on the Cornish-Fisher approximation (4.2). Statistics are shown in table 5.3. Estimates also deviate from the 2.33 expected from the normal distribution. The spread estimates are, however, substantially different from the empirical estimates of the original Bangia approach and in general slightly lower.

Risk estimates		DAX	MDAX	SDAX	TECDAX	All
Price Risk	**Mean**	4.14%	4.55%	5.30%	5.59%	4.81%
	Median	3.31%	3.97%	4.56%	4.89%	4.09%
L-VaR (Bangia et al.)	**Mean**	4.42%	5.25%	6.64%	6.29%	5.57%
	Median	3.51%	4.60%	5.78%	5.57%	4.80%
Modified L-VaR	**Mean**	4.48%	7.40%	8.11%	9.58%	7.47%
	Median	3.95%	5.78%	6.97%	6.94%	5.88%

Table 5.4: Risk estimates by index

5.1.3 Magnitude of liquidity risk

Table 5.4 shows the mean and median risk levels by index and year for each risk estimate. This allows to compare the magnitude of estimates when calculating normally distributed mid-price risk, liquidity-adjusted total risk of the original Bangia model (equation (4.1)) and the new modified L-VaR (equation (4.3)). Liquidity adjusted total risk estimated with the original Bangia model is naturally higher at an average 5.57 % than normally estimated price risk at 4.81 %, which neglects liquidity. The modified L-VaR provides the highest risk estimates with 7.47 % average daily VaR. As could be expected, SDAX and TecDAX are the indices with the highest overall risk level. The DAX has the lowest risk level, especially pronounced if liquidity risk is also taken into account.

Overall, neglecting liquidity risk leads to a severe underestimation of the total risk of an asset. The deviation between the Bangia method and the Cornish-Fisher method is largest for the less liquid indices SDAX and TecDAX. I will now analyze which liquidity adjustment is more precise.

5.1.4 Model preciseness

5.1.4.1 Backtesting framework

L-VaR models are only useful insofar as they predict risk reasonable well. Therefore I will evaluate their validity through a comparison between actual and predicted loss levels. If a model is perfectly calibrated, the percentage of days where losses exceed the VaR-prediction exactly matches the confidence level. If there are more exceedances than predicted, the model underestimates risk and too little regulatory capital is allocated to the position. However, too little exceedance, hence overestimation, leads to inefficient use of capital.[6] Since parameters are backward-estimated, the backtesting is, of course, out-of-sample.

Since I calculate L-VaR for a confidence level of 99 %, I expect the frequency of exceedances to equal 1 %. I use the standard test by Kupiec (1995) to determine if the realized frequency deviates from the predicted level of 1 % on a statistically significant basis.[7] Kupiec (1995) shows, that the probability of observing losses in excess of VaR on d days over the forecast period T at confidence α is governed by the binomial process[8]

$$P(d) = (1-\alpha)^{T-N}\alpha^{N}$$

The question if the realized frequency of losses in excess of VaR d/T is significantly different from the predicted α can be answered with the likelihood ratio (LR) test statistic

$$LR_{uc} = -2ln\left[(1-\alpha)^{T-N}\alpha^{N}\right] + 2ln\left[(1-d/T)^{T-N}(d/T)^{N}\right] \qquad (5.1)$$

which is chi-squared distributed with one degree of freedom under the null hypothesis that $\alpha = d/T$. Taking a confidence interval of 95% for the test statistic, the null hypothesis would be rejected for $LR_{uc} < 3.84$.[9] The test statistic will reject an L-VaR model if the actual exceedance frequency is significantly below 1 % (model overestimates risk) or significantly above 1 % (model underestimates risk).

[6] Cp. Jorion (2007) p. 140.
[7] Cp. Jorion (2007), p. 142.
[8] Cp. Kupiec (1995), p. 79.
[9] Please note that the choice of the confidence region for the test statistic is not related to the confidence level selected for the L-VaR-calculation, but merely refers to the decision rule to accept or reject the model.

Realized losses are calculated as realizable net return when liquidating the position

$$rnet_t = ln\left(\frac{P_t^{mid}}{P_{t-1}^{mid}}\right) + ln(1 - \frac{1}{2}S_t)$$

For my sample of stocks, I calculated the percentage of stocks where the realized loss frequency did not deviate from the predicted frequency on a statistically significant basis. For the fraction of stocks, where the Kupiec-statistic could not accept the VaR-approach, I further investigate the reason. Either the model has been rejected due to too many L-VaR-exceedances and overestimates risk or too few, i.e. it underestimates risk. I also determine the respective fraction of stocks with under- and overestimation.

In addition I will analyze the magnitude M of VaR-exceedances calculated as difference between realized and predicted loss

$$M = (-rnet_t - LVaR_t | rnet_t < -LVaR_t) \tag{5.2}$$

M can be seen as the unpredicted loss and characterizes the level of underestimation of the risk measure. It shows if the realized loss is only marginally or substantially larger than estimated, therefore being an additional indicator for the accuracy of a L-VaR-approach.

5.1.4.2 Backtesting results

Figure 5.2 shows the breakdown of stocks, where risk has been correctly, under- or overestimated. The risk measure is defined as incorrect, if the Kupiec statistic (equation (5.1)) produces a statistically significant deviation between risk forecast and return realization. The results demonstrate the vast improvement of the Cornish-Fisher parametrization over the original Bangia model. Risk has been correctly predicted for 83 % of the stocks with the modified L-VaR compared with 44 % with the empirical percentiles model of Bangia et al. The Bangia model seems to generally underestimate risk in all indices, although there is also slight

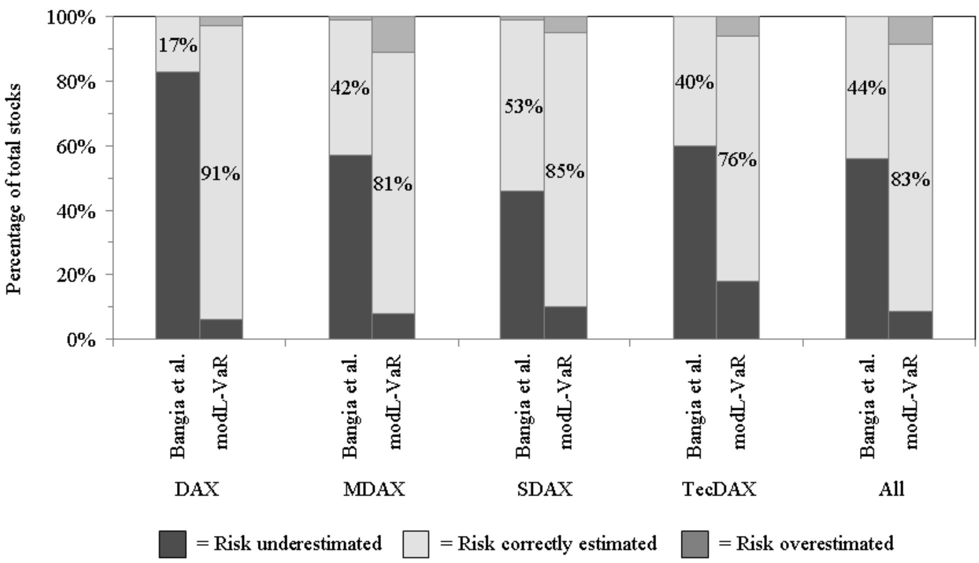

Figure 5.2: Comparison of acceptance rates by index

overestimation in the MDAX and SDAX as well. The modified L-VaR model produces around 8 % under- and 8 % overestimation across all stocks. The performance in the TecDAX is worst compared to all other indices. Especially from a regulatory perspective, the substantial underestimation by the Bangia model poses a significant problem. Risk does not seem to be adequately measured by the normal price distribution and empirical spread percentiles.

Table 5.5 shows the magnitude of exceedances M as calculated by equation (5.2). As shown already by the acceptance rate, the number of exceedances is much higher in the Bangia than in the modified L-VaR model. The level of exceedance is comparable between both models. There seem to a large number of small exceedances in the Bangia model, which slightly downward biases the mean exceedance for DAX and MDAX stocks. This is also underlined by the higher standard deviation in the Bangia model. The maximum exceedance can reach 90-100 % of the estimation, slightly higher for the modified L-VaR model than for the original Bangia. However, maximum exceedance is only higher in the

Magnitude of exceedances		Index				
		DAX	MDAX	SDAX	TECDAX	All
No.	Bangia et al.	653	891	670	458	2,672
	Modified L-VaR	70	115	249	78	512
Mean	Bangia et al.	1.07%	1.75%	2.45%	2.43%	1.88%
	Modified L-VaR	0.87%	1.14%	2.56%	3.37%	2.13%
Median	Bangia et al.	0.59%	0.87%	1.25%	1.18%	0.90%
	Modified L-VaR	0.41%	0.84%	2.24%	1.41%	1.31%
Max.	Bangia et al.	45.67%	50.72%	47.85%	87.09%	87.09%
	Modified L-VaR	33.22%	103.76%	33.11%	71.27%	103.76%
Std. Dev.	Bangia et al.	2.21%	3.51%	4.13%	5.47%	3.87%
	Modified L-VaR	1.03%	1.16%	2.07%	8.33%	3.71%

Table 5.5: Magnitude of exceedances

Risk correctly estimated	DAX	MDAX	SDAX	TECDAX	All
Sub-period II/2002 - I/2005					
L-VaR (Bangia et al.)	50.00%	62.50%	73.58%	67.50%	66.17%
Modified L-VaR	84.38%	85.39%	83.18%	85.00%	84.33%
Sub-period II/2005 - II/2007					
L-VaR (Bangia et al.)	33.33%	38.33%	51.25%	52.50%	45.07%
Modified L-VaR	93.75%	80.00%	83.54%	90.00%	85.31%

Table 5.6: Acceptance rate by sub-period

MDAX, in the other indices, the Bangia model surpasses the modified L-VaR. All in all, exceedances seem to be similar in both models.

While results already seem robust when looking at different indices, I will look at time sub-samples as further robustness test. I split the full period and calculated the percentage of stocks with correct risk estimation separately for each sub-period. While the absolute acceptance level of the Kupiec-statistic is less reliable, because the sample is smaller, the relative level between the two models is of interest. Results are shown in table 5.6. In both sub-periods, the modified L-VaR-model performs consistently better than the original Bangia model across all indices. This provides an indication, that the higher preciseness of the suggested model is not specific to the period used in the comparison.

Overall, the higher acceptance rate and the comparable level of exceedance magnitude make the modified L-VaR model highly superior to the original Bangia et al. specification. While these results are restricted to situations where positions can be liquidated at bid-ask-spread costs, I hypothesize that results similarly improve when using other, possibly more comprehensive liquidity risk measures.[10] Results might also be further improved if more sophisticated estimation techniques for skewness and kurtosis are incorporated. I leave this point for further research. Overall, backtesting results demonstrate the vast superiority of the suggested liquidity risk estimation technique based on a Cornish-Fisher approximation.

5.1.5 Synopsis

In this section I tested the newly proposed modified L-VaR, as well as a standard specification by Bangia et al. (1999) in a sample of daily frequency for 160 stocks over 5.5 years. The modified L-VaR proves to be highly superior. The Kupiec test statistic indicates that risk is correctly estimated for substantially more stocks. Accounting for non-normality via the Cornish-Fisher approximation provides significantly more accurate results than with the empirical method of Bangia et al..

While these results are restricted to situations where positions can be traded at the bid-ask-spread the method can be analogously applied to other liquidity measures such as weighted spread. As other liquidity cost measures like weighted spread are similarly non-normal, as has been discussed in section 3.3.4, I hypothesize that the proposed method is also superior in other liquidity risk approaches.

[10]This hypothesis will be tested in section 5.3

Index	Mean	Median	Std. Dev.	Obs.
DAX	16,3%	14,7%	4,4%	43.767
MDAX	17,2%	15,8%	8,1%	76.750
SDAX	19,5%	17,7%	7,3%	72.373
TECDAX	24,3%	22,6%	9,2%	38.070
All	18,9%	17,4%	7,9%	230.960

Table 5.7: Price risk (VaR, 10 day, 99%)

This table contains distribution statistics on price risk calculated as 10-day, 99% VaR according to equation (4.4).

5.2 Empirical net-return model with weighted spread

5.2.1 Motivation

In this section I present results of the empirical net-return model with weighted spread. In section 5.2.2, the impact of order size on the overall risk level will be analyzed. This analysis will allow to accept or reject the hypothesis that order size is an important factor in liquidity risk determination. I also empirically look at the effect of tail correlation on the overall risk estimate in section 5.2.3. Section 5.2.5 presents several robustness tests of the results.

5.2.2 Magnitude of liquidity impact

As background, table 5.7 contains average 10-day, 99 % VaR price risk estimates for each index. Overall average price risk was 18.9 %. As could have been expected, price risk is lowest in the DAX with an average of 16.3 %, followed by MDAX, SDAX and TecDAX. However, there is still quite large variation within the indices.

Table 5.8 contains the total risk estimates. Overall average total risk is 21 %, i.e. significantly higher than the price risk estimate. The increase of total risk with order size is already apparent in all indices. While the price risk in the TecDAX was significantly larger than in the MDAX and

Total risk, VaR(10 day, 99%) abs., liquidity-adjusted in %		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	**Mean**	17%	n/a	17%	17%	n/a	17%	n/a	17%	17%	n/a	18%	20%	20%	21%	21%	18%	0.82 ***
	Median	15%	n/a	15%	15%	n/a	15%	n/a	15%	16%	n/a	17%	18%	18%	17%	18%	17%	0.65 ***
	Std. Dev.	4%	n/a	5%	5%	n/a	5%	n/a	5%	5%	n/a	5%	8%	7%	7%	8%	6%	0.56 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	**Mean**	18%	19%	20%	20%	20%	20%	20%	22%	23%	22%	24%	n/a	n/a	n/a	n/a	21%	0.98 ***
	Median	17%	17%	17%	17%	17%	17%	18%	19%	21%	20%	20%	n/a	n/a	n/a	n/a	18%	0.91 ***
	Std. Dev.	8%	9%	9%	9%	9%	9%	9%	9%	10%	10%	11%	n/a	n/a	n/a	n/a	9%	0.36 ***
	Obs.	73,234	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	**Mean**	21%	20%	21%	22%	24%	24%	25%	27%	28%	30%	35%	n/a	n/a	n/a	n/a	24%	2.87 ***
	Median	19%	18%	18%	20%	21%	21%	22%	23%	26%	29%	30%	n/a	n/a	n/a	n/a	21%	2.70 ***
	Std. Dev.	8%	7%	7%	8%	9%	9%	9%	10%	9%	10%	16%	n/a	n/a	n/a	n/a	9%	1.30 **
	Obs.	70,048	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	**Mean**	25%	21%	22%	22%	22%	22%	23%	24%	26%	26%	30%	n/a	n/a	n/a	n/a	23%	1.63 ***
	Median	23%	19%	20%	20%	20%	20%	21%	22%	24%	24%	28%	n/a	n/a	n/a	n/a	20%	1.63 ***
	Std. Dev.	9%	8%	8%	9%	8%	9%	9%	9%	10%	9%	10%	n/a	n/a	n/a	n/a	9%	0.41 ***
	Obs.	36,980	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	**Mean**	20%	n/a	20%	20%	n/a	21%	n/a	22%	23%	n/a	23%	n/a	n/a	n/a	n/a	21%	0.96 ***
	Median	18%	n/a	18%	18%	n/a	19%	n/a	20%	21%	n/a	21%	n/a	n/a	n/a	n/a	19%	0.96 ***
	Std. Dev.	8%	n/a	8%	8%	n/a	9%	n/a	9%	9%	n/a	11%	n/a	n/a	n/a	n/a	9%	0.68 **
	Obs.	222,391	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 5.8: Absolute liquidity-adjusted total risk (VaR, 10 day, 99%)

This tables shows cross-sectional statistics on empirical, absolute total risk including a liquidity adjustment according to equation (4.7); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

SDAX, when looking at total risk, SDAX is slightly more risky with 24 % than the TecDAX with 23 %. This must be explained by the impact of liquidity. I will now investigate into this liquidity impact in detail.

I look at the total impact of liquidity $\lambda(q)$ on risk in a standard 10-day, 99% confidence-level VaR-setting according to equation (4.8). These parameters are typically used in a Basel II framework.[11] Table 5.9 on page 103 presents statistics on the overall liquidity impact $\lambda(q)$ by order size and index. On average over all stocks and across all order sizes, total risk - including liquidity risk - is 10% higher than price risk alone. DAX is generally the index with the lowest liquidity risk, while MDAX and TecDAX are second. SDAX consistently shows the highest liquidity risk levels across all order sizes. This finding is consistent with trading volumes and market values discussed in the market background section 3.2.

There is strong variation in liquidity impact between indices and within indices as indicated by standard deviations. Variation is of the same order of magnitude than the level. Impact is practically zero ($\leq$1%) in small order sizes of the DAX ($<$€ 250 thsd.). Liquidity impact can easily rise above 20% in large stock positions of the DAX or medium stock positions in small stocks. In an average € 1 million SDAX-positions, liquidity impact on risk rises to 30% of price risk at a 10-day horizon.

Especially interesting is the liquidity impact calculated with spread as revealed in the min-column.[12] Impact remains rather small across all stocks and comparable to the liquidity impact measured with XLM(10) and XLM(25) respectively. In SDAX and TecDAX it is slightly higher than in the smallest XLM bracket. Since median risk levels are comparable, this effect is probably due to few outliers as XLM and spread data come from two different databases.

[11] Cp. Dowd (2001), p.51.
[12] This corresponds to the risk measurement approach suggested by Bangia et al. (1999) applied to stocks.

λ(q), VaR(10 day, 99%) in % of price risk		Order size (in thsd. Euro)																Size
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	Mean	1%	n/a	1%	1%	n/a	1%	n/a	2%	4%	n/a	9%	16%	21%	24%	26%	10%	0.78 ***
	Median	1%	n/a	1%	1%	n/a	1%	n/a	1%	3%	n/a	6%	11%	19%	20%	25%	3%	0.78 ***
	Std. Dev.	1%	n/a	0%	0%	n/a	1%	n/a	2%	4%	n/a	8%	14%	16%	16%	18%	14%	0.84 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	Mean	2%	2%	2%	3%	4%	5%	7%	11%	15%	19%	21%	n/a	n/a	n/a	n/a	8%	0.58 ***
	Median	2%	1%	2%	2%	3%	4%	5%	9%	11%	14%	17%	n/a	n/a	n/a	n/a	4%	0.62 ***
	Std. Dev.	2%	3%	3%	5%	4%	5%	6%	9%	18%	35%	63%	n/a	n/a	n/a	n/a	22%	0.62 ***
	Obs.	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	Mean	9%	6%	7%	10%	13%	16%	20%	23%	22%	22%	30%	n/a	n/a	n/a	n/a	14%	0.35 ***
	Median	3%	3%	4%	7%	8%	9%	11%	14%	19%	20%	23%	n/a	n/a	n/a	n/a	8%	0.44 ***
	Std. Dev.	52%	8%	11%	16%	20%	29%	44%	48%	17%	15%	34%	n/a	n/a	n/a	n/a	27%	0.23 *
	Obs.	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	Mean	3%	1%	2%	3%	4%	5%	8%	11%	16%	18%	18%	n/a	n/a	n/a	n/a	7%	0.64 ***
	Median	1%	1%	1%	2%	3%	4%	6%	8%	13%	18%	16%	n/a	n/a	n/a	n/a	3%	0.66 ***
	Std. Dev.	5%	1%	1%	2%	4%	8%	7%	10%	18%	12%	13%	n/a	n/a	n/a	n/a	10%	0.66 ***
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	Mean	4%	n/a	3%	5%	n/a	7%	n/a	12%	13%	n/a	17%	n/a	n/a	n/a	n/a	10%	0.44 ***
	Median	2%	n/a	2%	2%	n/a	3%	n/a	7%	10%	n/a	13%	n/a	n/a	n/a	n/a	4%	0.62 ***
	Std. Dev.	30%	n/a	7%	10%	n/a	16%	n/a	25%	16%	n/a	42%	n/a	n/a	n/a	n/a	20%	0.42 **
	Obs.	217,436	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 5.9: Liquidity impact on risk (VaR, 10 day, 99%)

Table shows cross-sectional statistics of lambda, which is the impact of liquidity in percent of price risk according to (4.8); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient of log-size regressed on the log distribution statistic including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

Liquidity impact generally increases with order size.[13] To more systematically analyze this size effect, I separately estimated the impact of doubling order size on $\lambda(q)$ in percent in the last column. To do so, I regress the log row statistics on log order size including a constant intercept.[14] Size impact is the coefficient on log-size and indicates the curvature of the price impact function. It specifically investigates into the importance of price impact data in contrast to spread data only and abstracts from the different levels in liquidity risk between indices. Generally, the estimated price impact statistic is positive but smaller than one, which shows, that the liquidity impact (risk) function is concave.[15] The price impact is larger in the DAX, than in the other indices. Here, the difference between small, liquid and larger, less-liquid positions is especially pronounced. With size impact of 0.78, liquidity impact almost doubles in the DAX when doubling order size. In the other indices, liquidity impact is already large at small positions - hence the lower curvature. All size impacts are statistically significant at the 1%-level. The economically large size-impact statistic underlines the importance of using order book information beyond the spread for risk estimation - even in the DAX.

These results have important consequences for risk estimation techniques. First, I find that liquidity is an important component in total risk, especially in larger order sizes, where the price impact estimation error relative to price risk rises up to 30% at 10-day horizons. Second, estimating liquidity risk with spread data is no valid alternative, as liquidity risk impact in this size class is very small and strongly increases with size. Third, large variations indicate that constant scaling of price

[13] The decrease in the average SDAX position between € 250 thsd. and € 500 thsd. results from a non-constant sample effect. Large SDAX positions were continuously tradable only in later years. Therefore, risk estimates for large SDAX positions are calculated on a more liquid period depressing liquidity impacts compared to more continuously traded small positions. Cp. discussion in section 3.3.3.

[14] Ordinary least-squared regression equation is $log(Stat(q)) = c + log(q) + \epsilon$, with stat being the row statistic and c a constant intercept.

[15] This is consistent as already the price impact cost function is empirically found to be concave; cp. Hasbrouck (1991); Hausman et al. (1992).

risk across all stocks, often dubbed “hair cuts”, are probably insufficient and liquidity has to be accounted for specifically for each stock.

5.2.3 Correlation effect

Next, I specifically look into the tail correlation between mid-price return and liquidity cost. A correlation factor $\kappa(q)$ of zero corresponds to perfect tail correlation between liquidity and mid-price return. It mirrors the case that liquidity costs are highest when prices are lowest. Table 5.10 on page 106 shows the results based on 10-day, 99% VaR according to (4.10). Mean correlation factors range between 40% and 60% of liquidity risk. On average, 60% of the liquidity cost risk is diversified away. The negative correlation factor reveals that large, illiquid positions are relatively more liquid in crises. Stock market crashes seem to attract liquidity, which allows to liquidate less-liquid positions more cost-efficiently, however at lower prices. Since over half of the liquidity risk is diversified away, liquidity risk would be overestimated by about 100% at larger sizes when neglecting correlation (cp. equation (4.11)).

Correlation factors are quite uniform across order sizes and indices at around a negative 55 to 65 %. Only in the DAX it is slightly lower at about -40 %. Correlation plays an even larger role at the spread level, where it is consistently higher than in larger order sizes. This underlines the different dynamics between the spread, quoted by market makers, and weighted spread, which emerges from free market competition. Cross-sectional standard deviation is also quite constant. The size-independent nature is underlined by the statistically and economically insignificant price impact statistic.[16]

The $\kappa(q)$-statistic should be treated with care. The effect of correlation on total risk is substantial only if the liquidity risk is also substantial (cp. equation (4.11)). As liquidity risk is quite low at small positions the overall error remains small and the violation is less critical.

[16]Estimated in a linear regression of the distribution statistic on size.

κ(q) VaR(10 day, 99%) in % of liquidity risk		Order size (in thsd. Euro)																Size
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	Mean	-64%	n/a	-40%	-41%	n/a	-43%	n/a	-45%	-49%	n/a	-34%	-41%	-40%	-40%	-41%	-41%	0.00
	Median	-68%	n/a	-41%	-41%	n/a	-41%	n/a	-44%	-48%	n/a	-49%	-47%	-42%	-41%	-41%	-44%	0.00
	Std. Dev.	21%	n/a	15%	15%	n/a	16%	n/a	17%	18%	n/a	87%	47%	30%	24%	19%	36%	0.00
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	Mean	-70%	-62%	-62%	-61%	-63%	-64%	-63%	-63%	-64%	-60%	-59%	n/a	n/a	n/a	n/a	-62%	0.00 **
	Median	-72%	-66%	-66%	-62%	-63%	-67%	-64%	-61%	-62%	-63%	-60%	n/a	n/a	n/a	n/a	-63%	0.00 **
	Std. Dev.	15%	17%	17%	18%	16%	16%	17%	16%	18%	24%	17%	n/a	n/a	n/a	n/a	18%	0.00
	Obs.	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	Mean	-67%	-61%	-65%	-65%	-66%	-66%	-64%	-60%	-57%	-59%	-54%	n/a	n/a	n/a	n/a	-63%	0.00 ***
	Median	-68%	-61%	-65%	-65%	-65%	-66%	-64%	-59%	-56%	-61%	-57%	n/a	n/a	n/a	n/a	-64%	0.00 **
	Std. Dev.	19%	19%	16%	18%	16%	16%	18%	15%	17%	18%	16%	n/a	n/a	n/a	n/a	17%	0.00
	Obs.	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	Mean	-72%	-66%	-66%	-64%	-66%	-66%	-67%	-66%	-63%	-65%	-62%	n/a	n/a	n/a	n/a	-65%	0.00 **
	Median	-73%	-66%	-66%	-66%	-70%	-66%	-67%	-69%	-65%	-63%	-65%	n/a	n/a	n/a	n/a	-66%	0.00 *
	Std. Dev.	17%	16%	17%	18%	16%	16%	14%	18%	16%	13%	15%	n/a	n/a	n/a	n/a	16%	0.00
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	Mean	-68%	n/a	-59%	-59%	n/a	-61%	n/a	-59%	-59%	n/a	-50%	n/a	n/a	n/a	n/a	-58%	0.00 **
	Median	-70%	n/a	-61%	-61%	n/a	-61%	n/a	-58%	-58%	n/a	-56%	n/a	n/a	n/a	n/a	-60%	0.00 **
	Std. Dev.	18%	n/a	19%	19%	n/a	18%	n/a	18%	19%	n/a	56%	n/a	n/a	n/a	n/a	25%	0.00 **
	Obs.	217,436	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 5.10: Correlation factor (VaR, 10 day, 99%)

Table shows cross-sectional statistics of the correlation factor, which measures correlation between liquidity cost and mid-price return according to (4.10); min-column measures the effect at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of size in a linear regression of the distribution statistic on size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

Overall, these empirical results refute the common assumption of perfect tail correlation, i.e. that it is reasonable to simple add up price and liquidity risk. Doing so would overestimate total risk, especially in large, more illiquid order sizes. These results resolve the discussion, whether the perfect tail correlation assumption is valid or not. The representative, empirical results are in line with the argument of Francois-Heude and Van Wynendaele (2001), who criticize the perfect correlation assumption of Bangia et al. (1999). However, the overall effect of this assumption remains small if the liquidity impact is small in total. It might also be different in other assets like currencies, which were analyzed by Bangia et al. (1999), but I see no a priori reason why this should be the case. I also hypothesize that correlation effects should be similar for other liquidity cost measures, because these proxy for the same phenomenon. Overall, my results indicate, that tail correlation is important and should be taken into account in illiquid stock positions.

5.2.4 Liquidity impact at shorter horizons

Risk on a 10-day horizon calculated above, provides a comparable reference to the standard statistics usually requested by financial regulators. However, as noted already in section 2.1.2.2, when correctly and directly accounting for liquidity risk, the 10-day horizon gets the notion of management reaction time instead of liquidation time. In order to stick to the original intention behind VaR, what a portfolio is worth in the worst case, I also calculate VaR at a 1-day horizon. This statistic is also more comparable to the intraday results available so far.

In table 5.11 the average daily price risk for each index is shown.

Index	Mean	Median	Std. Dev.	Obs.
DAX	5,6%	5,5%	1,1%	43.710
MDAX	6,1%	5,7%	2,2%	71.458
SDAX	7,2%	6,5%	3,0%	72.313
TECDAX	8,2%	7,9%	1,8%	36.801
All	6,7%	6,0%	2,5%	224.282

Table 5.11: Price risk (VaR, 1 day, 99%)
This table contains distribution statistics on price risk calculated as 1-day, 99% VaR according to equation (4.4).

Overall average price risk is significantly lower at 6.7 % than in the 10-day case at 18.9 %, which is slightly smaller than the common square-root of time rule would suggest. The index rank is the same as in the 10-day calculation.

Table 5.12 presents the daily total risk estimates. The structure of the 10-day case is preserved. SDAX overtakes the TecDAX in the risk level. Similarly to above, I proceed with a more detailed analysis of this liquidity impact.

Table 5.13 shows the liquidity impact $\lambda(q)$ for a 1-day, 99% VaR according to equation (4.8). As expected, the relative liquidity impact magnifies when shortening horizons, because price risk is reduced while absolute liquidity risk remains unchanged. The structure between indices remains unchanged. While still being negligible in small DAX positions, total risk including liquidity is almost double the price risk for large positions. Average € 1 million SDAX positions have a >90% liquidity risk impact. Even in some small positions, liquidity plays a substantial role with liquidity impact surpassing 10% in the SDAX for small position sizes. Overall, these results are comparable to the 2-30% range found in other studies.[17]

The size-impact statistic reveals a very similar curvature in magnitude in the daily compared with the 10-day case. All size impacts are statistically significant at the 1% level. Correlation effects are similar

[17] Cp. Francois-Heude and Van Wynendaele (2001); Giot and Grammig (2005); Angelidis and Benos (2006).

Total risk, VaR(1 day, 99%)		Order size (in thsd. Euro)																Size
abs., liquidity-adjusted in %		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	**Mean**	6%	n/a	6%	6%	n/a	6%	n/a	6%	6%	n/a	7%	9%	10%	10%	10%	7%	0.88 ***
	Median	6%	n/a	6%	6%	n/a	6%	n/a	6%	6%	n/a	6%	8%	8%	9%	9%	6%	0.58 ***
	Std. Dev.	1%	n/a	1%	1%	n/a	1%	n/a	1%	1%	n/a	2%	5%	4%	4%	4%	3%	0.73 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	**Mean**	6%	6%	6%	6%	7%	7%	7%	8%	10%	10%	11%	n/a	n/a	n/a	n/a	8%	1.16 ***
	Median	6%	6%	6%	6%	6%	7%	7%	8%	10%	9%	9%	n/a	n/a	n/a	n/a	7%	0.90 ***
	Std. Dev.	2%	2%	3%	2%	2%	2%	3%	3%	4%	4%	5%	n/a	n/a	n/a	n/a	3%	0.57 ***
	Obs.	73,234	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	**Mean**	9%	8%	8%	9%	11%	12%	12%	14%	15%	17%	22%	n/a	n/a	n/a	n/a	11%	2.73 ***
	Median	7%	7%	8%	9%	9%	10%	11%	11%	13%	15%	16%	n/a	n/a	n/a	n/a	9%	1.91 ***
	Std. Dev.	5%	4%	3%	5%	7%	7%	7%	10%	7%	9%	16%	n/a	n/a	n/a	n/a	7%	1.99 ***
	Obs.	70,048	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	**Mean**	9%	7%	7%	8%	8%	9%	9%	10%	12%	13%	15%	n/a	n/a	n/a	n/a	9%	1.69 ***
	Median	8%	7%	7%	7%	8%	8%	9%	10%	12%	11%	13%	n/a	n/a	n/a	n/a	8%	1.37 ***
	Std. Dev.	3%	2%	2%	2%	2%	3%	4%	4%	5%	5%	6%	n/a	n/a	n/a	n/a	4%	0.94 ***
	Obs.	36,980	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	**Mean**	7%	n/a	7%	7%	n/a	8%	n/a	9%	10%	n/a	11%	n/a	n/a	n/a	n/a	9%	1.14 ***
	Median	7%	n/a	7%	7%	n/a	7%	n/a	8%	9%	n/a	9%	n/a	n/a	n/a	n/a	7%	0.65 ***
	Std. Dev.	4%	n/a	3%	3%	n/a	5%	n/a	6%	5%	n/a	7%	n/a	n/a	n/a	n/a	5%	1.03 **
	Obs.	222,391	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 5.12: Absolute liquidity-adjusted total risk (VaR, 1 day, 99%)

This tables shows cross-sectional statistics on empirical, absolute total risk including a liquidity adjustment according to equation (4.7); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

λ(q), VaR(1 day, 99%) in % of price risk		Order size (in thsd. Euro)															Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	**Mean**	3%	n/a	2%	2%	n/a	3%	n/a	5%	11%	n/a	30%	56%	68%	76%	79%	32%	0.82 ***
	Median	3%	n/a	2%	2%	n/a	2%	n/a	4%	7%	n/a	20%	32%	42%	61%	69%	9%	0.79 ***
	Std. Dev.	2%	n/a	1%	1%	n/a	2%	n/a	4%	11%	n/a	29%	56%	57%	54%	56%	47%	0.90 ***
	Obs.	42,129	n/a	42,650	42,650	n/a	42,650	n/a	42,650	42,646	n/a	42,604	41,716	39,970	38,225	36,343	412,104	
MDAX	**Mean**	7%	6%	8%	10%	14%	18%	28%	44%	68%	76%	77%	n/a	n/a	n/a	n/a	31%	0.62 ***
	Median	6%	5%	6%	7%	11%	13%	22%	37%	56%	66%	61%	n/a	n/a	n/a	n/a	15%	0.66 ***
	Std. Dev.	6%	9%	16%	12%	17%	18%	24%	31%	44%	51%	51%	n/a	n/a	n/a	n/a	39%	0.40 ***
	Obs.	69,578	73,902	73,697	73,053	72,414	71,794	70,480	67,645	59,972	52,777	46,239	n/a	n/a	n/a	n/a	661,973	
SDAX	**Mean**	30%	17%	27%	41%	60%	68%	75%	80%	70%	70%	96%	n/a	n/a	n/a	n/a	52%	0.33 ***
	Median	12%	13%	18%	32%	43%	49%	58%	56%	62%	51%	73%	n/a	n/a	n/a	n/a	33%	0.34 ***
	Std. Dev.	102%	17%	35%	48%	83%	88%	102%	118%	46%	54%	96%	n/a	n/a	n/a	n/a	76%	0.25 *
	Obs.	69,988	68,497	64,068	60,824	57,733	54,871	49,291	39,714	23,114	13,985	8,363	n/a	n/a	n/a	n/a	440,460	
TECDAX	**Mean**	11%	6%	8%	11%	17%	23%	32%	47%	62%	67%	66%	n/a	n/a	n/a	n/a	29%	0.60 ***
	Median	7%	5%	7%	10%	13%	17%	28%	42%	56%	61%	67%	n/a	n/a	n/a	n/a	14%	0.62 ***
	Std. Dev.	28%	3%	5%	8%	15%	25%	26%	36%	64%	47%	34%	n/a	n/a	n/a	n/a	37%	0.62 ***
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	**Mean**	15%	n/a	13%	18%	n/a	29%	n/a	43%	52%	n/a	59%	n/a	n/a	n/a	n/a	36%	0.43 ***
	Median	7%	n/a	7%	9%	n/a	13%	n/a	33%	43%	n/a	53%	n/a	n/a	n/a	n/a	17%	0.61 ***
	Std. Dev.	60%	n/a	23%	31%	n/a	54%	n/a	65%	50%	n/a	53%	n/a	n/a	n/a	n/a	52%	0.22 *
	Obs.	217,436	n/a	217,548	213,653	n/a	206,264	n/a	183,967	152,727	n/a	113,237	n/a	n/a	n/a	n/a	1,833,877	

Table 5.13: Liquidity impact on risk (VaR, 1 day, 99%)

Table shows cross-sectional statistics of lambda, which is the liquidity impact on risk in percent of price risk according to equation (4.8); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the increase in risk in percentage points when doubling order size, measured as coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

in structure and magnitude when compared to the 10-day horizon as can be seen from table 5.14. The importance of the correlation effect is confirmed.

5.2.5 Robustness tests

5.2.5.1 Effects of change of risk measure

Recently, literature has discussed coherent risk measures as alternative to Value-at-Risk to overcome the shortfalls of VaR like non sub-additivity.[18] This raises the question, if results would change significantly when switching to a different risk measure. To test if results are robust or specific to the VaR, I calculate expected shortfall,[19] which is the expected loss in the worst α-percent of the cases. I continue to use the basic approach detailed in section 4.2.1 on page 85, but I replace VaR with expected shortfall (ES) defined as follows.

$$ES^{\alpha,\Delta t} = E(r|r < r^{\alpha}) \quad (5.3)$$

When I calculate risk based on expected shortfall instead of value-at-risk as displayed in table 5.15 effects of order size get accentuated. Generally speaking, results are structurally similar when measuring risk as ES compared to VaR. While total risk estimates increase, the impact of liquidity is comparable even in the tail of the distribution. Methodology and results are therefore quite robust to a change to the expected shortfall risk measure.

5.2.5.2 Effects of time variation

As further robustness test, I calculate monthly, rolling estimates of lambda to counter concerns that results are due to the long estimation period. This test also addresses any concerns for non-constant-sample

[18] Cp. Artzner et al. (1997); Acerbi and Scandolo (2007).
[19] Also called 'conditional value-at-risk' or 'expected tail loss'.

κ(q) VaR(1 day, 99%) in % of liquidity risk		Order size (in thsd. Euro) Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
DAX	Mean	-62%	n/a	-41%	-43%	n/a	-46%	n/a	-50%	-53%	n/a	-43%	-42%	-42%	-39%	-39%	-44%	0.00 *
	Median	-66%	n/a	-42%	-43%	n/a	-45%	n/a	-52%	-56%	n/a	-47%	-44%	-44%	-41%	-39%	-44%	0.00 *
	Std. Dev.	24%	n/a	18%	17%	n/a	17%	n/a	16%	16%	n/a	34%	19%	15%	14%	11%	19%	0.00
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,716	39,970	38,225	36,343	412,463	
MDAX	Mean	-64%	-57%	-57%	-59%	-58%	-59%	-56%	-54%	-48%	-46%	-45%	n/a	n/a	n/a	n/a	-55%	0.00 ***
	Median	-67%	-57%	-58%	-59%	-59%	-59%	-57%	-55%	-48%	-47%	-46%	n/a	n/a	n/a	n/a	-55%	0.00 ***
	Std. Dev.	17%	17%	17%	16%	15%	14%	14%	12%	13%	18%	16%	n/a	n/a	n/a	n/a	16%	0.00
	Obs.	69,578	74,779	74,574	73,930	73,291	72,671	71,357	68,520	60,784	53,461	46,741	n/a	n/a	n/a	n/a	670,108	
SDAX	Mean	-59%	-51%	-52%	-52%	-50%	-50%	-47%	-48%	-47%	-50%	-41%	n/a	n/a	n/a	n/a	-50%	0.00 ***
	Median	-63%	-51%	-50%	-50%	-51%	-49%	-47%	-49%	-46%	-54%	-41%	n/a	n/a	n/a	n/a	-50%	0.00
	Std. Dev.	16%	19%	16%	16%	14%	15%	16%	16%	14%	15%	17%	n/a	n/a	n/a	n/a	16%	0.00
	Obs.	69,988	69,081	64,254	60,824	57,798	54,871	49,291	39,780	23,114	13,985	8,363	n/a	n/a	n/a	n/a	441,361	
TECDAX	Mean	-64%	-51%	-50%	-51%	-53%	-52%	-54%	-51%	-47%	-49%	-47%	n/a	n/a	n/a	n/a	-51%	0.00 **
	Median	-64%	-54%	-52%	-50%	-54%	-52%	-55%	-50%	-47%	-48%	-44%	n/a	n/a	n/a	n/a	-51%	0.00 ***
	Std. Dev.	13%	16%	17%	16%	13%	14%	12%	12%	15%	11%	13%	n/a	n/a	n/a	n/a	14%	0.00
	Obs.	35,741	37,133	37,133	37,126	37,075	36,949	36,299	33,958	26,995	20,641	16,031	n/a	n/a	n/a	n/a	319,340	
All	Mean	-62%	n/a	-51%	-52%	n/a	-53%	n/a	-51%	-49%	n/a	-44%	n/a	n/a	n/a	n/a	-50%	0.00 ***
	Median	-64%	n/a	-50%	-52%	n/a	-52%	n/a	-52%	-49%	n/a	-46%	n/a	n/a	n/a	n/a	-50%	0.00 **
	Std. Dev.	18%	n/a	18%	17%	n/a	16%	n/a	14%	15%	n/a	24%	n/a	n/a	n/a	n/a	17%	0.00
	Obs.	217,436	n/a	218,671	214,590	n/a	207,201	n/a	184,968	153,599	n/a	113,798	n/a	n/a	n/a	n/a	1,843,272	

Table 5.14: Correlation factor (VaR, 1 day, 99%)

Table shows cross-sectional statistics of the correlation factor, which measures correlation between liquidity cost and mid-price return according to (4.10); min-column measures the effect at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of size in a linear regression of the distribution statistic on size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

λ(q), ES(10 day, 99%) in % of price risk		Order size (in thsd. Euro)																Size impact
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	
DAX	Mean	1%	n/a	1%	1%	n/a	1%	n/a	2%	3%	n/a	12%	17%	22%	23%	26%	9%	0.82 ***
	Median	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	4%	8%	12%	13%	16%	2%	0.70 ***
	Std. Dev.	1%	n/a	0%	0%	n/a	1%	n/a	1%	3%	n/a	33%	35%	36%	37%	38%	26%	1.12 ***
	Obs.	423	n/a	431	431	n/a	430	n/a	427	412	n/a	388	354	335	314	289	3,811	
MDAX	Mean	2%	2%	2%	3%	4%	5%	7%	9%	13%	13%	15%	n/a	n/a	n/a	n/a	6%	0.51 ***
	Median	1%	1%	1%	2%	3%	3%	5%	7%	9%	9%	11%	n/a	n/a	n/a	n/a	3%	0.53 ***
	Std. Dev.	2%	4%	3%	4%	7%	7%	13%	9%	13%	11%	11%	n/a	n/a	n/a	n/a	9%	0.30 ***
	Obs.	780	826	812	803	791	774	724	653	550	425	376	n/a	n/a	n/a	n/a	6,734	
SDAX	Mean	5%	5%	6%	7%	9%	13%	14%	14%	21%	25%	25%	n/a	n/a	n/a	n/a	10%	0.40 ***
	Median	2%	2%	3%	4%	5%	7%	9%	12%	17%	22%	22%	n/a	n/a	n/a	n/a	6%	0.54 ***
	Std. Dev.	13%	6%	12%	8%	11%	26%	29%	13%	16%	22%	16%	n/a	n/a	n/a	n/a	17%	0.20 *
	Obs.	608	564	499	431	397	370	319	234	154	98	55	n/a	n/a	n/a	n/a	3,121	
TECDAX	Mean	2%	1%	2%	2%	3%	4%	6%	9%	12%	15%	19%	n/a	n/a	n/a	n/a	6%	0.63 ***
	Median	1%	1%	2%	2%	3%	3%	4%	6%	9%	14%	14%	n/a	n/a	n/a	n/a	3%	0.59 ***
	Std. Dev.	2%	1%	1%	2%	3%	4%	6%	8%	11%	16%	27%	n/a	n/a	n/a	n/a	9%	0.77 ***
	Obs.	169	347	333	329	334	313	300	261	196	153	107	n/a	n/a	n/a	n/a	2,673	
All	Mean	3%	n/a	3%	3%	n/a	5%	n/a	8%	11%	n/a	15%	n/a	n/a	n/a	n/a	8%	0.48 ***
	Median	1%	n/a	1%	2%	n/a	3%	n/a	5%	7%	n/a	10%	n/a	n/a	n/a	n/a	3%	0.52 ***
	Std. Dev.	7%	n/a	7%	5%	n/a	13%	n/a	9%	12%	n/a	25%	n/a	n/a	n/a	n/a	16%	0.34 **
	Obs.	1,980	n/a	2,075	1,994	n/a	1,887	n/a	1,575	1,312	n/a	926	n/a	n/a	n/a	n/a	16,339	

Table 5.15: Liquidity impact on risk (ES, 10 day, 99%)

Table shows cross-sectional statistics of lambda, which is liquidity impact in percent of price risk according to equations (4.8) and (5.3); Min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^2 of log-size in a regression of the log distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

bias, because I calculate risk estimates only on stocks included in the index due to data availability. Because empirical percentiles cannot be calculated on monthly samples of daily data, I chose a straight-forward mean-variance estimation procedure. For each date, I calculate the 20-day backward variance σ_r of continuous price return and assume that daily expected return is zero. Relative price risk on a 99% confidence level is then defined as

$$VaR^{1\%}_{price} = 1 - exp\left(-2.33 \times \sigma_r\right) \tag{5.4}$$

Similarly, I calculate liquidity-adjusted total risk with the mean μ_{rnet} and standard deviation σ_{rnet} of 20-day backward net-return distribution

$$VaR^{1\%}_{total}(q) = 1 - exp\left(\mu_{rnet}(q) - 2.33 \times \sigma_{rnet}(q)\right) \tag{5.5}$$

with net returns calculated according to equation (4.5). I then calculate the liquidity impact $\lambda(q)$ according to equation (4.8). Neglect of negative skewness and high kurtosis (fat tails) makes this procedure simple, but it might underestimate risk. Due to the underestimation, absolute values need to be treated with care, but are still - as lower bound - a suitable indicator for the time variation of the liquidity impact on risk, especially if skewness and kurtosis are fairly constant.

Rolling total risk estimates are presented in table 5.16.

L-adj. VaR(10d, 99%) in %		Order size (in thsd. Euro) Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	Size impact
DAX	**2002**	24%	n/a	24%	24%	n/a	24%	n/a	24%	25%	n/a	26%	28%	29%	30%	31%	26%	1.32 ***
	2003	16%	n/a	17%	17%	n/a	17%	n/a	17%	17%	n/a	18%	20%	21%	21%	22%	18%	0.94 ***
	2004	9%	n/a	9%	9%	n/a	9%	n/a	10%	10%	n/a	10%	11%	12%	12%	13%	10%	0.70 ***
	2005	8%	n/a	8%	8%	n/a	8%	n/a	8%	8%	n/a	8%	9%	9%	9%	10%	9%	0.29 ***
	2006	9%	n/a	9%	9%	n/a	9%	n/a	10%	10%	n/a	10%	10%	10%	10%	11%	10%	0.29 ***
	2007	10%	n/a	10%	10%	n/a	10%	n/a	10%	10%	n/a	10%	11%	11%	11%	11%	11%	0.22 ***
	2008	16%	n/a	15%	15%	n/a	15%	n/a	15%	15%	n/a	15%	15%	16%	16%	16%	15%	0.18 **
	All	12%	n/a	12%	12%	n/a	12%	n/a	12%	12%	n/a	13%	13%	13%	14%	14%	13%	0.37 ***
	Δ2002-2008 a.	-12%	n/a	-12%	-12%	n/a	-12%	n/a	-12%	-13%	n/a	-14%	-15%	-16%	-16%	-17%	-14%	-0.96
MDAX	**2002**	21%	21%	21%	22%	22%	23%	24%	26%	28%	27%	27%	n/a	n/a	n/a	n/a	23%	1.69 ***
	2003	16%	16%	16%	16%	16%	16%	17%	19%	21%	23%	24%	n/a	n/a	n/a	n/a	17%	1.87 ***
	2004	11%	12%	12%	12%	12%	12%	13%	13%	16%	17%	19%	n/a	n/a	n/a	n/a	13%	1.46 ***
	2005	11%	11%	11%	11%	11%	11%	11%	11%	13%	14%	15%	n/a	n/a	n/a	n/a	12%	0.80 ***
	2006	12%	12%	12%	12%	12%	12%	13%	13%	13%	14%	15%	n/a	n/a	n/a	n/a	13%	0.59 ***
	2007	13%	13%	13%	13%	13%	13%	14%	14%	14%	15%	15%	n/a	n/a	n/a	n/a	14%	0.49 ***
	2008	20%	19%	19%	20%	20%	20%	20%	20%	20%	21%	22%	n/a	n/a	n/a	n/a	20%	0.50 ***
	All	14%	14%	14%	14%	14%	14%	14%	15%	16%	16%	17%	n/a	n/a	n/a	n/a	15%	0.65 ***
	Δ2002-2008 a.	-7%	-7%	-7%	-8%	-8%	-8%	-9%	-11%	-13%	-11%	-10%	n/a	n/a	n/a	n/a	-8%	-1.12
SDAX	**2002**	24%	23%	27%	35%	41%	47%	47%	32%	4%	n/a	n/a	n/a	n/a	n/a	n/a	28%	-1.45
	2003	19%	19%	20%	21%	23%	25%	28%	31%	35%	28%	29%	n/a	n/a	n/a	n/a	22%	2.98 ***
	2004	14%	15%	15%	16%	18%	20%	21%	25%	31%	31%	33%	n/a	n/a	n/a	n/a	18%	4.49 ***
	2005	13%	13%	13%	13%	14%	15%	16%	18%	24%	27%	30%	n/a	n/a	n/a	n/a	15%	3.83 ***
	2006	14%	14%	14%	15%	15%	15%	16%	18%	23%	27%	31%	n/a	n/a	n/a	n/a	17%	3.53 ***
	2007	15%	15%	15%	15%	15%	16%	16%	17%	20%	23%	26%	n/a	n/a	n/a	n/a	17%	2.22 ***
	2008	20%	22%	22%	22%	21%	21%	22%	24%	28%	33%	35%	n/a	n/a	n/a	n/a	23%	2.80 ***
	All	16%	16%	16%	16%	17%	17%	18%	20%	23%	25%	28%	n/a	n/a	n/a	n/a	18%	2.59 ***
	Δ2002-2008 a.	-8%	-7%	-11%	-18%	-24%	-29%	-28%	-13%	19%	-3%	-1%	n/a	n/a	n/a	n/a	-11%	3.95
TecDAX	**2002**	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
	2003	21%	21%	21%	22%	22%	23%	24%	25%	27%	29%	28%	n/a	n/a	n/a	n/a	23%	1.87 ***
	2004	17%	17%	17%	17%	17%	18%	19%	20%	22%	24%	25%	n/a	n/a	n/a	n/a	18%	1.86 ***
	2005	13%	13%	13%	13%	13%	14%	14%	15%	17%	20%	21%	n/a	n/a	n/a	n/a	15%	1.76 ***
	2006	15%	15%	15%	15%	15%	15%	16%	16%	18%	20%	23%	n/a	n/a	n/a	n/a	16%	1.49 ***
	2007	16%	16%	16%	16%	16%	16%	17%	17%	18%	20%	21%	n/a	n/a	n/a	n/a	17%	1.03 ***
	2008	24%	23%	23%	23%	23%	23%	23%	24%	26%	26%	29%	n/a	n/a	n/a	n/a	24%	1.11 ***
	All	16%	16%	16%	16%	17%	17%	17%	18%	20%	21%	23%	n/a	n/a	n/a	n/a	18%	1.45 ***
	Δ2002-2008 a.	-5%	-5%	-5%	-5%	-5%	-6%	-6%	-7%	-8%	-7%	-6%	n/a	n/a	n/a	n/a	-5%	-0.54
All	**2002**	22%	n/a	23%	23%	n/a	24%	n/a	25%	26%	n/a	26%	n/a	n/a	n/a	n/a	24%	0.96 ***
	2003	18%	n/a	18%	18%	n/a	19%	n/a	20%	21%	n/a	21%	n/a	n/a	n/a	n/a	20%	0.96 ***
	2004	13%	n/a	13%	14%	n/a	15%	n/a	15%	16%	n/a	15%	n/a	n/a	n/a	n/a	15%	0.61 **
	2005	11%	n/a	11%	11%	n/a	12%	n/a	13%	14%	n/a	14%	n/a	n/a	n/a	n/a	12%	0.96 ***
	2006	13%	n/a	13%	13%	n/a	13%	n/a	14%	15%	n/a	16%	n/a	n/a	n/a	n/a	14%	0.84 ***
	2007	14%	n/a	14%	14%	n/a	14%	n/a	15%	16%	n/a	17%	n/a	n/a	n/a	n/a	15%	0.84 ***
	2008	20%	n/a	20%	20%	n/a	20%	n/a	21%	22%	n/a	23%	n/a	n/a	n/a	n/a	21%	0.84 ***
	All	14%	n/a	14%	15%	n/a	15%	n/a	16%	16%	n/a	17%	n/a	n/a	n/a	n/a	15%	0.72 ***
	Δ2002-2008 a.	-8%	n/a	-8%	-9%	n/a	-9%	n/a	-9%	-10%	n/a	-9%	n/a	n/a	n/a	n/a	-9%	-0.30

Table 5.16: Liquidity-adjusted total risk (rolling VaR, 10-day, 99%)
Table shows liquidity-adjusted total risk by sub-sample according to equation (4.8) calculated with a rolling mean-variance estimation; a. statistic shows absolute change between 2003 and 2008 when 2002 number not available; min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

Average total risk, measured as 10-day, 99 % VaR, declined from 2002 to 2008 by 9 percentage points from 24 % to 15 %. This decline is visible in all indices. In all sub-periods there is an increase in risk with order size in all indices and sub-periods. I will now turn to the analysis of the liquidity component itself.

Results for $\lambda(q)$ on the basis of a 10-day, 99% VaR according to (4.8) and (5.5) are displayed in table 5.17.

Avg. λ(q), VaR(10d, 99%)		Order size (in thsd. Euro)																Size
in % of price risk		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	**2002**	1%	n/a	1%	1%	n/a	1%	n/a	2%	4%	n/a	10%	18%	17%	17%	18%	8%	0.77 ***
	2003	1%	n/a	1%	1%	n/a	1%	n/a	2%	4%	n/a	9%	19%	23%	25%	25%	10%	0.82 ***
	2004	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	8%	15%	21%	25%	29%	10%	0.83 ***
	2005	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	3%	7%	11%	14%	19%	6%	0.70 ***
	2006	0%	n/a	0%	1%	n/a	1%	n/a	1%	2%	n/a	3%	5%	7%	10%	14%	4%	0.65 ***
	2007	0%	n/a	0%	0%	n/a	1%	n/a	1%	1%	n/a	2%	4%	5%	7%	9%	3%	0.60 ***
	2008	0%	n/a	0%	0%	n/a	0%	n/a	1%	1%	n/a	2%	3%	5%	7%	8%	3%	0.63 ***
	All	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	5%	10%	13%	16%	18%	6%	0.74 ***
	Δ2002-2008 [a.]	-1%	n/a	0%	0%	n/a	0%	n/a	-1%	-1%	n/a	-4%	-8%	-4%	-2%	0%	-1%	
MDAX	**2002**	4%	6%	7%	8%	10%	12%	16%	19%	28%	31%	30%	n/a	n/a	n/a	n/a	12%	0.42 ***
	2003	3%	3%	4%	5%	7%	10%	16%	24%	35%	40%	37%	n/a	n/a	n/a	n/a	14%	0.63 ***
	2004	2%	2%	3%	4%	5%	6%	11%	18%	32%	35%	37%	n/a	n/a	n/a	n/a	13%	0.73 ***
	2005	2%	2%	2%	3%	3%	4%	6%	10%	21%	29%	34%	n/a	n/a	n/a	n/a	10%	0.74 ***
	2006	1%	1%	1%	2%	2%	3%	4%	6%	12%	17%	22%	n/a	n/a	n/a	n/a	7%	0.71 ***
	2007	1%	1%	1%	2%	2%	2%	3%	4%	8%	12%	17%	n/a	n/a	n/a	n/a	5%	0.65 ***
	2008	1%	1%	1%	1%	1%	2%	2%	3%	6%	10%	13%	n/a	n/a	n/a	n/a	4%	0.67 ***
	All	2%	2%	3%	3%	4%	6%	8%	12%	20%	24%	26%	n/a	n/a	n/a	n/a	10%	0.61 ***
	Δ2002-2008 [a.]	-2%	-3%	-4%	-4%	-5%	-6%	-7%	-7%	-8%	-6%	-4%	n/a	n/a	n/a	n/a	-2%	
SDAX	**2002**	13%	12%	27%	61%	91%	120%	153%	115%	79%	n/a	n/a	n/a	n/a	n/a	n/a	36%	0.56 **
	2003	10%	12%	17%	26%	36%	42%	40%	41%	40%	51%	64%	n/a	n/a	n/a	n/a	27%	0.32 ***
	2004	6%	7%	11%	20%	27%	35%	40%	52%	46%	38%	120%	n/a	n/a	n/a	n/a	24%	0.49 ***
	2005	6%	5%	7%	11%	15%	19%	24%	29%	32%	41%	37%	n/a	n/a	n/a	n/a	17%	0.46 ***
	2006	3%	4%	5%	7%	10%	13%	19%	28%	37%	36%	37%	n/a	n/a	n/a	n/a	15%	0.56 ***
	2007	2%	2%	3%	4%	5%	7%	10%	17%	31%	38%	43%	n/a	n/a	n/a	n/a	13%	0.71 ***
	2008	2%	2%	2%	3%	5%	6%	10%	16%	26%	32%	39%	n/a	n/a	n/a	n/a	10%	0.75 ***
	All	6%	6%	9%	14%	17%	21%	23%	28%	34%	38%	43%	n/a	n/a	n/a	n/a	18%	0.42 ***
	Δ2002-2008 [a.]	-7%	-6%	-18%	-47%	-74%	-99%	-130%	-86%	-45%	-13%	-21%	n/a	n/a	n/a	n/a	-18%	
TecDAX	**2002**	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	
	2003	2%	3%	4%	5%	8%	11%	15%	21%	25%	28%	31%	n/a	n/a	n/a	n/a	11%	0.57 ***
	2004	2%	3%	4%	5%	8%	12%	18%	26%	26%	29%	31%	n/a	n/a	n/a	n/a	13%	0.60 ***
	2005	2%	2%	3%	5%	6%	8%	11%	17%	27%	38%	41%	n/a	n/a	n/a	n/a	12%	0.67 ***
	2006	2%	2%	2%	3%	4%	5%	7%	13%	22%	27%	31%	n/a	n/a	n/a	n/a	10%	0.68 ***
	2007	1%	1%	2%	2%	3%	4%	5%	8%	15%	19%	23%	n/a	n/a	n/a	n/a	8%	0.66 ***
	2008	1%	1%	1%	2%	2%	3%	4%	6%	11%	15%	16%	n/a	n/a	n/a	n/a	5%	0.65 ***
	All	2%	2%	3%	4%	6%	8%	11%	16%	22%	27%	29%	n/a	n/a	n/a	n/a	11%	0.62 ***
	Δ2002-2008 [a.]	0%	-1%	-1%	-1%	-2%	-4%	-4%	-5%	-3%	-1%	-1%	n/a	n/a	n/a	n/a	-1%	
All	**2002**	6%	n/a	8%	10%	n/a	13%	n/a	12%	14%	n/a	15%	n/a	n/a	n/a	n/a	13%	0.14 ***
	2003	5%	n/a	7%	9%	n/a	14%	n/a	19%	21%	n/a	21%	n/a	n/a	n/a	n/a	15%	0.31 ***
	2004	3%	n/a	5%	8%	n/a	14%	n/a	20%	22%	n/a	23%	n/a	n/a	n/a	n/a	15%	0.42 ***
	2005	3%	n/a	4%	5%	n/a	9%	n/a	14%	19%	n/a	24%	n/a	n/a	n/a	n/a	11%	0.53 ***
	2006	2%	n/a	2%	3%	n/a	6%	n/a	12%	16%	n/a	19%	n/a	n/a	n/a	n/a	9%	0.59 ***
	2007	1%	n/a	2%	2%	n/a	4%	n/a	8%	14%	n/a	19%	n/a	n/a	n/a	n/a	7%	0.70 ***
	2008	1%	n/a	1%	2%	n/a	3%	n/a	7%	11%	n/a	13%	n/a	n/a	n/a	n/a	6%	0.69 ***
	All	3%	n/a	4%	6%	n/a	9%	n/a	14%	17%	n/a	20%	n/a	n/a	n/a	n/a	11%	0.45 ***
	Δ2002-2008 [a.]	-3%	n/a	-4%	-4%	n/a	-4%	n/a	2%	3%	n/a	5%	n/a	n/a	n/a	n/a	-2%	

Table 5.17: Liquidity impact on risk (rolling VaR, 10-day, 99%)

Table shows mean lambda, which is liquidity impact in percent of price risk by subsample calculated with a rolling mean-variance estimation of Value-at-Risk (10-day, 99%) according to (4.8) based on (5.5); a. Statistic shows absolute change between 2003 and 2008 when 2002 number not available; min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient in 10^{-2} of log-size in a regression of the log distribution statistic on log-size including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

The impact of liquidity on risk has generally declined over time across all indices. In all years, the liquidity impact strongly increased with order size as the size-impact statistic reveals. The prior finding of the index rank (DAX, MDAX / TecDAX, SDAX) is confirmed and stable over time. TecDAX, however, was shortly more liquid after its initiation in 2003 until 2004. Although to be interpreted with care, the liquidity impact probably remained non-negligible during the low-risk period from 2006-2007. The impact of liquidity on total risk was certainly economically significant in the crises periods of 2002-2003 and in 2008.

Results for the whole panel ('all') have to be treated with care, because they are distorted by the non-constant sample effect. Over the years, the liquidity of less-liquid stocks strongly improved, which made their liquidity cost data increasingly available. As consequence, less-liquid, high-cost stocks are increasingly included in the sample, which increases the average risk estimate. However, individual year estimates have almost no sample bias and underline, that liquidity impact is economically significant.

If skewness and kurtosis would be included, these findings are also likely to get confirmed, as the one-time liquidity cost deduction will probably introduce additional skewness, which keeps the relation between price and liquidity risk valid. Overall, this confirms that liquidity price impact is economically significant enough to encourage integration into risk measurement systems.

5.2.5.3 Effects of portfolio diversification

I showed, that liquidity risk is economically significant when looking at individual stocks in the different indices. But does this result persist when looking at portfolios of stocks? If diversification between mid-prices of different stocks is larger than between liquidity of different stocks, liquidity impact might be substantially reduced.

To test the robustness of results against effects of portfolio diversification, I calculated daily value-weighted index returns and determined liquidity impact $\lambda(q)$ based on a 10-day, 99% VaR according to (4.8). While this methodology does not use optimized position weights, a value-weighted portfolio should show effects of diversification if there are any. Results are displayed in table (5.18). Estimates demonstrate, that liquidity impact on the portfolio level is of similar magnitude than on the average individual stock level (cp. table 5.9 on page 103). Especially in larger sizes, liquidity impact is increased at the portfolio level, e.g. it rises to 54% for the € 1 million position in the SDAX portfolio compared to 30% for the average individual stock position. This must be driven by larger liquidity commonality in larger sizes, i.e. diversification in liquidity between stocks decreases with larger sizes. Even for the all-stock portfolio liquidity impact levels are higher than for the average stock. Overall, my results are robust to diversification effects in stock portfolios.

5.2.6 Synopsis

In this section, I empirically tested the empirical net-return model based on weighted spread and its structure. Empirically, I find that impact of liquidity relative to price risk is small at small order sizes, especially at the spread level (<10% for 10-day, 99% VaR). However, it increases to 20-30% of price risk in larger sizes in illiquid indices as well as in the DAX. Results aggravate if I switch to daily VaR-horizons.

I also took a detailed look at tail correlation between liquidity and mid-price returns and showed that it is non-negligible. Liquidity risk would be overestimated by 100% if correlations are ignored. In the cases I identified above, where liquidity risk is an economically significant component of total risk, total risk will be severely overestimated if liquidity cost risk is simply added to existing risk measures. Therefore, several common approaches should be adapted to avoid this distortion.

I find that results are structurally similar when using expected shortfall instead of VaR risk measures. Results are therefore transferable. To check the time robustness of these findings, I employ a monthly, rolling mean-variance estimation method. Results are confirmed. Results are

λ(q) VaR(10 day, 99%) in % of liquidity risk		Order size (in thsd. Euro)																Size
		Min	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All	impact
DAX	**Estimate**	1%	n/a	1%	1%	n/a	1%	n/a	1%	2%	n/a	12%	22%	18%	25%	27%	10%	0.84 ***
	Obs.	42,129	n/a	42,710	42,710	n/a	42,710	n/a	42,710	42,706	n/a	42,663	41,759	40,002	38,294	36,388	412,652	
MDAX	**Estimate**	2%	2%	2%	2%	3%	5%	9%	5%	13%	21%	20%	n/a	n/a	n/a	n/a	7%	0.60 ***
	Obs.	73,279	74,858	74,679	74,040	73,365	72,737	71,409	68,543	60,788	53,501	46,793	n/a	n/a	n/a	n/a	670,713	
SDAX	**Estimate**	5%	32%	15%	27%	32%	52%	56%	26%	39%	45%	54%	n/a	n/a	n/a	n/a	35%	0.16 *
	Obs.	70,048	69,197	64,614	61,119	57,938	55,000	49,410	39,920	23,442	14,435	9,112	n/a	n/a	n/a	n/a	444,187	
TECDAX	**Estimate**	1%	1%	2%	2%	3%	4%	5%	8%	13%	15%	17%	n/a	n/a	n/a	n/a	6%	0.61 ***
	Obs.	36,980	37,157	37,157	37,150	37,099	36,973	36,323	34,028	27,077	20,868	16,291	n/a	n/a	n/a	n/a	320,123	
All stocks	**Estimate**	3%	n/a	2%	2%	n/a	3%	n/a	4%	5%	n/a	20%	n/a	n/a	n/a	n/a	6%	0.55 **
	Obs.	181,212	n/a	219,160	215,019	n/a	207,420	n/a	185,201	154,013	n/a	114,859	n/a	n/a	n/a	n/a	1,847,675	

Table 5.18: Liquidity impact (VaR, 10 day, 99%) by index portfolio

Table shows portfolio statistics of lambda, which is the impact of liquidity in percent of price risk according to (4.8); min-column measures risk at minimum spread level; all-column is average over all standardized order sizes, i.e. without minimum; size impact is the coefficient of log-size regressed on the log distribution statistic including an intercept; * indicates 10%, ** 5% and *** 1% confidence level of being different from zero based on a two-tailed test.

also similar for portfolios of stocks, when portfolio diversification is accounted for.

Overall, I strongly advocate the use of weighted spread data like XLM to improve risk estimates. Liquidity constitutes a large part of total risk, especially in larger positions and at short horizons - even in more liquid market segments.

5.3 Comparison of liquidity risk models

5.3.1 Motivation

As final analysis, I will run a horse race of liquidity risk models that are implementable in daily data. Based on the large data set described in chapter 3, I benchmark a large selection of models taken from section 2.2 as well as the propositions from chapter 4, an exercise that has not been conducted in the scientific literature so far. It will help to understand strengths and weaknesses and allows to devise concrete recommendations for practical liquidity risk measurement.

5.3.2 Selection of models

I sort liquidity risk models into two broad categories: Traceable and theoretical. A large stream of literature has developed theoretical modeling approaches, where implementation procedures are still missing and not obvious. These include Lawrence and Robinson (1995), Almgren and Chriss (2000) and Almgren (2003), Subramanian and Jarrow (2001), Hisata and Yamai (2000), Dubil (2003) and Engle and Ferstenberg (2007).[20] These models generally use optimal trading strategies to minimize the Value-at-Risk of a position including liquidity. However, empirical estimation techniques for the large range of parameters of these models still need to be developed.

[20] A more detailed discussion has been provided in section 2.2.2.4.

Among those liquidity risk models that are empirically traceable, several work on intraday or transaction data only. Berkowitz (2000a), Jarrow and Protter (2005b) and Angelidis and Benos (2006) belong to this class. In order not to completely neglect these, I choose Berkowitz (2000a), which seemed most promising to adapt for daily data. I include all traceable models available for daily data: Bangia et al. (1999), Cosandey (2001), Francois-Heude and Van Wynendaele (2001), Giot and Grammig (2005) as well as the modified add-on model (section 4.1) and the empirical net return model based on weighted spread (section 4.2). For all models I choose a straight forward implementation for daily stock data. I group the chosen models by the type of data required for their estimation: bid-ask-spread models, transaction or volume data models, and models requiring limit order book data.

5.3.3 Implementation specification

5.3.3.1 General approach

For all models, I calculate a standard, daily, relative Value-at-Risk (VaR) at a 99 % confidence level. In general, I tried to keep the implementation procedure as straight-forward as possible to allow for best comparisons.

Means, including those of liquidity costs, are generally calculated with a 20-day rolling procedure. If mid-price return is separately estimated in a normal-distribution framework, I set its mean to zero, as is common practice. I account for volatility clustering with the standard exponential weighted average (EWMA) model over 20 days by JP Morgan (1996) using a weight δ of 0.94 defined as

$$\sigma_t^2 = (1-\delta)\sum_{i=1}^{20}\delta^{i-1}r_{t-i}^2 + \delta^{20}r_{t-20}^2 \tag{5.6}$$

Where applicable I estimate skewness and excess-kurtosis with a simple non-weighted rolling procedure. The skewness of y is computed from historical data rolling over the last 500 days as $\gamma = \frac{1}{500}\sum_{t=1}^{500}(y_t - \mu_y)^3/\sigma_y^3$

with μ_y and σ_y as mean and volatility of y. The excess kurtosis for y is $\kappa = \frac{1}{500}\sum_{t=1}^{500}(y_t - \mu_y)^4/\sigma_y^4 - 3$.[21]

To allow for best comparison, I use the ten standardized order size classes to calculate the the liquidity risk for a stock position of a specific size from section 3.1.1.3.

For some models, I had to choose specific implementation approaches, that are not covered by above general description. These will be described in the following subsections. Liquidity risk models that can be implemented with the general implementation comments given in the previous section, will not be included.

5.3.3.2 Berkowitz (2000)

For Berkowitz (2000a) (section 2.2.2.2), future price is driven by risk factor changes and the liquidity impact of trading N_t number of shares as follows

$$P_{mid,t+1} - P_{mid,t} = C + \theta N_t + x_{t+1} + \epsilon_t \tag{5.7}$$

where θ is the regression coefficient, x_{t+1} is the effect of risk factor changes on the mid-price, C is a constant and ϵ_t the error term of the regression. θ can be understood as absolute liquidity cost per share traded. Although the original model is constructed on the basis of transaction data, I have tried to tune it as best as possible for the use in daily risk forecasts. Therefore, I approximated the transaction price with $P_{mid,t+1}$.

As the author does not go into implementation detail, I choose to estimate market risk effects as

$$x_{t+1} = \beta \times r_{M,t} \times P_{mid,t} \tag{5.8}$$

[21] To keep the sample as large as possible, I reduced the rolling window up to 20 days at the beginning of the sample, in order to also include the first two years into the results period. This discriminates models using skewness and kurtosis, but they nevertheless show superior performance as will be shown in section 5.3.5.

Liquidity Coefficient θ in Euro per million shares	Index				
	DAX	MDAX	SDAX	TECDAX	All
Mean	0.03	0.30	5.24	0.37	1.84
Median	0.01	0.05	0.17	0.04	0.03
Max.	0.23	12.50	1,777.00	24.40	1,777.00
Min.	-0.12	-14.30	-53.10	-3.95	-53.10
Std. Dev.	0.05	1.37	91.90	2.94	52.20
Signif. fraction at 95 % confidence	53%	36%	45%	54%	44%
Signif. fraction at 99 % confidence	44%	27%	37%	46%	36%

Table 5.19: Estimates of the liquidity measure θ

Table shows cross-sectional statistics of the estimated liquidity coefficient θ. The All-column contains the average over all indices. Significant fraction shows percentage of stocks with statistically significant theta at confidence level of 95 % and 99 % respectively.

where $\beta = Cov(r, r_M)/\sigma_{r_{market}}$ is the beta factor for each individual stock return on the 160-stock, value-weighted market portfolio return r_M over the sample period.[22]

Table 5.19 presents the regression estimates of the liquidity measure $\hat{\theta}$ for the sample period. The regression produces positive and negative estimates, which is slightly counter-intuitive as the liquidation of a position should always induce a price discount. $\hat{\theta}$ also varies strongly as indicated by standard deviation, minimum and maximum. In general, average liquidity costs per share are very small, in the order of one Euro per million shares. Only about half of the stocks have $\hat{\theta}$-values that are statistically significant different from zero. Therefore, I already doubt at this stage, that the liquidity measure implemented in daily data will produce accurate results.

I now calculate continuous, liquidity-adjusted net return as

$$rnet_t(q) = ln\left(1 + \left[\beta \times r_{M,t} - \hat{\theta} \times \frac{N_t + n}{P_{mid,t}}\right]\right) \qquad (5.9)$$

[22] Although this proceeding leads to a conceptually doubtful overlap between estimation and forecast period, this overlap generates a bias in favor of the model. Nevertheless, even positively biased estimates for this model provide poor results as is shown later in section 5.3.5. Risk-free rate is neglected due to the short time period.

for each standard-volume number of shares $n = q/P_{mid,t}$ to allow for later comparison with other liquidity risk models. The optimal trading strategy of the original model requires $1/h$'th of the position to be liquidated each day of the h-day horizon. For the daily horizon, the full position will be liquidated at once. I then define relative, liquidity-adjusted total risk as

$$L - VaR(q) = 1 - exp\left(\mu_{rnet(q)} + \hat{z}_\alpha \sigma_{rnet(q)}\right) \tag{5.10}$$

where $\mu_{rnet(q)}$ is the 20-day rolling net return mean and $\sigma_{rnet(q)}$ is the EWMA-estimated net return volatility. $\hat{z}_\alpha$ is the empirical percentile of the net return distribution.

5.3.3.3 Cosandey (2001)

The relative, liquidity-adjusted VaR of Cosandey (2001) (section 2.2.2.2) is implemented similar to the Berkowitz (2000a) approach described above but with net return defined as

$$rnet_t(q) = ln\left(r_{t+1} \times \frac{N_t}{N_t + n}\right) \tag{5.11}$$

where r is the mid-price return, N is the number of shares traded in the market and $n = q/P_{mmid}$ is the position size in number of shares. Risk is then defined as

$$L - VaR(q) = 1 - exp\left(\mu_{rnet(q)} + \hat{z}_\alpha \sigma_{rnet(q)}\right) \tag{5.12}$$

where $\mu_{rnet(q)}$ is the 20-day rolling net return mean and $\sigma_{rnet(q)}$ is the EWMA-estimated net return volatility. $\hat{\alpha}$ is the empirical percentile of the net return distribution.[23]

[23] I deviate from the original simulation approach, because, in my view, the key feature of this approach is new liquidity measure. Using a parametrization keeps approaches as comparable as possible. I also work with smoother continuous rather than discrete returns.

5.3.3.4 Francois-Heude and Wynendaele (2001)

In the original paper, Francois-Heude and Van Wynendaele (section 2.2.2.3) interpolate the liquidity cost function only from the best five limit-order-quotes made available by the Paris Stock Exchange. In favor of their approach, I use the liquidity cost function estimated as weighted spread from the whole limit order book as described at the beginning of this section.

I specify risk in this model as

$$L - VaR(q) = 1 - exp(-z\sigma_r)\left(1 - \frac{\mu(q)_{WS}}{2}\right) + \frac{1}{2}\left(WS_t(q) - \mu(q)_{WS}\right) \tag{5.13}$$

where z is the normal percentile and σ_r the standard deviation of the mid-price return distribution. $\mu(q)_{WS}$ is the average spread for a security for order quantity q, and $WS_t(q)$ is the spread at time t.

5.3.3.5 Giot and Gramming (2005)

The relative, liquidity-adjusted total risk of Giot and Grammig (2005) (section 2.2.2.3) is calculated as

$$L - VaR(q) = 1 - exp\left(\mu_{rnet(q)} + z_{t,\alpha}\sigma_{rnet(q)}\right) \tag{5.14}$$

where $z_{t,\alpha}$ is the chosen percentile of the student distribution.[24] In order to ensure comparability, I stay with the EWMA-modeling of volatility and do not replicate their approach accounting for conditional heteroskedasticity. Because I implement their approach to daily instead of intraday data, I ignore their adjustment for diurnal variation in weighted spread.

[24] I take percentiles from the student distribution with 19 degrees of freedom due to the 20-day rolling window.

5.3.3.6 Modified liquidity risk model

As the modified add-on model can be used with bid-ask-spread as well as with weighted spread data, I include both versions. Similar to equation 4.3 in the *modified add-on model with bid-ask-spread*, I define the *modified add-on model with weighted spread* as

$$L - VaR = 1 - exp(\mu_r + \tilde{z}_\alpha(r) \times \sigma_r) \times \left(1 - \frac{1}{2}\left(\mu_{WS} + \tilde{z}_\alpha(WS) \times \sigma_{WS}\right)\right) \tag{5.15}$$

where $\tilde{z}_\alpha(r)$ is the percentile of the return distribution accounting for its skewness and kurtosis and $\tilde{z}_\alpha(WS)$ and the corresponding weighted spread distribution percentile. Accounting for four moments is performed with the Cornish-Fisher approximation 4.2.

In analogous application of the net-return model outlined in section 4.2, I also test another variant, the *modified net-return model with weighted spread*. It is also possible to use Cornish-Fisher approximated percentiles of the net return distribution, i.e. return net of weighted spread, and calculate risk as

$$L - VaR(q) = 1 - exp\left(\mu_{rnet(q)} + \tilde{z}_\alpha(q) \times \sigma_{rnet(q)}\right) \tag{5.16}$$

where $\tilde{z}_\alpha$ is the percentile estimated with the Cornish-Fisher approximation (4.2). This alternative parametrization does not rely on the assumption of t-distributed net returns or perfect return-liquidity correlation. This modified net-return model will be also included in the model selection.

5.3.4 Backtesting framework

I test the validity of risk forecasts for each model by comparing predicted risk with actual returns out-of-sample, similar to the backtesting approach described in 5.1.4.1. Actual realized losses are, however, calculated specific for each position q with the weighted spread under the assumption that the position has to be immediately liquidated against the limit order book

$$rnet_t(q) = r_t + ln\left(1 - \frac{1}{2}WS_t(q)\right) \quad (5.17)$$

That this assumption is valid in a large range of risk-related situations has been discussed in 3.1.1.2.

I calculate a L-VaR for all models at $1 - \alpha = 99\%$ confidence. If the L-VaR model correctly predicts risk, actual return should exceed VaR in only 1 % of all cases. Statistical significant deviations between predicted and actual risk are determined with the Kupiec (1995)-statistic according to equation 5.1.

Similar to section 5.1.4.1, I calculate the percentage of stocks, where the Kupiec statistic cannot reject precise risk estimation. This percentage will be called acceptance rate. If acceptance rates are averaged over all order sizes, I excluded bid-ask-spread rates to avoid double counting.[25] For stocks, where the deviation between predicted and real loss rates was significant, I determined if the violation occurred because risk was overestimated (fewer actual losses than predicted) or underestimated (more actual losses than predicted). These respective stock fractions are also determined.

When comparing models, I used a common sample. The large period of 5.5 years, i.e. 1.423 days, allows for very robust results of the Kupiec-statistic.

5.3.5 Backtesting results

5.3.5.1 Overall model ranking

Figure 5.3 shows the overall ranking of the tested liquidity risk models. Liquidity risk models are ranked by the overall average percentage of

[25] As bid-ask-spreads are reported for non-standardized order sizes only, there is potential overlap with weighted spread of small sizes.

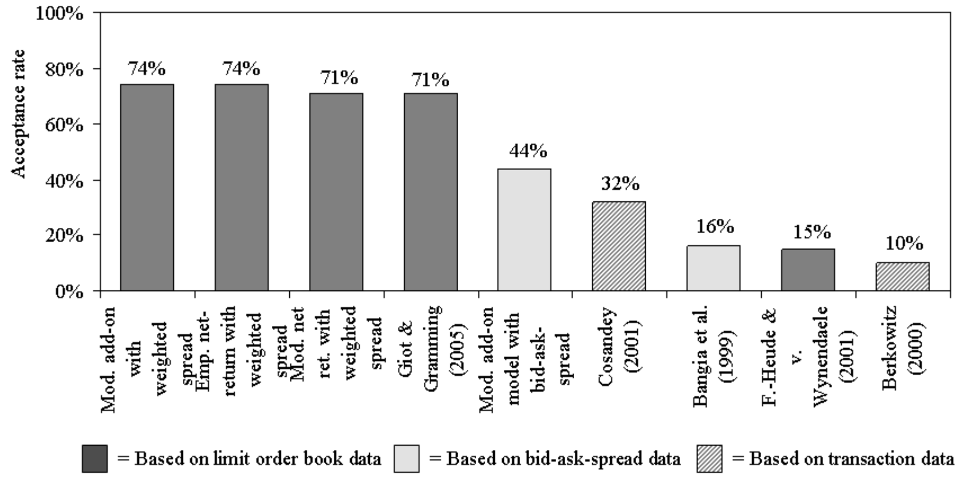

Figure 5.3: Ranking of liquidity risk models by overall acceptance rate
Figure shows overall acceptance rate averaged over all stocks and order sizes for each model. Acceptance rate is the percentage of stocks with statistically significant precise risk estimation according to Kupiec (1995).

stocks, for which risk was correctly estimated according to the Kupiec-statistic. In general, models based on the larger data set, limit order data, show superior performance with an acceptance rate of above 70 %. Best performing with 74 % is the Cornish-Fisher modified add-on approach with weighted spread and the empirical net return model based on weighted spread . This is closely followed by modified weighted spread net return and the t-distributed net-return approach by Giot and Grammig (2005) with a 71 % acceptance rate.

I would have expected the net-return Cornish-Fisher approach to be higher ranked than if return and liquidity percentiles are separately estimated, because correlation between return and liquidity are correctly accounted for. I hypothesize that forecasting of return and liquidity costs are more precise because the dynamics of both components are modeled separately. This compensates for the neglect of correlation. The t-distribution approach by Giot and Grammig (2005) seems to only partially account for the non-normality. The limit-order-book approach by Francois-Heude and Van Wynendaele (2001) is far behind on the second last place. I believe this is caused by the conceptual weakness of this model as described in section 2.2.2.3.

Although the modified add-on model based on bid-ask-spread, section 4.1, does not account for the price impact via weighted spread data, it surpasses with 44 % acceptance rate the Cosandey (2001) with 32 % acceptance. Bangia et al. (1999) is with 16 % overall acceptance better than Francois-Heude and Van Wynendaele (2001) and Berkowitz (2000a). The implementation attempt of the latter in daily data does not provide satisfactory results.

5.3.5.2 The impact of order size

The overall rank calculated as order-size average is influenced by the selection of size classes included. I therefore also calculated averages by individual order sizes. Table 5.20 shows the acceptance rate of the tested liquidity risk models by order size. The modified add-on model with weighted spread performs best in small to medium order sizes, while the best performing model in larger order sizes is Giot and Grammig (2005). The t-distribution seems to capture liquidity risk in larger order sizes very efficiently. The relatively low performance of the modified risk models with weighted spread in larger sizes is probably due to rising skewness and kurtosis for weighted spread in larger sizes caused by single outliers, which leads to imprecise Cornish-Fisher estimates.[26] Also, the assumption of perfect correlation leads to an overestimation of risk which has a significant impact in larger order sizes.[27] The hypothesis that the lower performance of the modified net return model with weighted spread is driven by the low forecastability of net return dynamics is substantiated. In lower order sizes, performance is more acceptable, because dynamics are mainly driven by mid-price return and liquidity is neglectable. In larger order sizes performance drops as liquidity dynamics are lost in the compounding of the net return.

[26] Cp. Jaeger (2004), p.16. and Zangari (1996), p.10.
[27] Cp. Stange and Kaserer (2008a).

Acceptance rate	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
Mod. add-on with weighted spread	81%	80%	76%	77%	82%	79%	79%	75%	66%	63%	60%	78%	63%	58%	56%	74%
Emp. net-return with weighted spread	78%	73%	76%	79%	77%	77%	82%	77%	71%	71%	75%	50%	42%	41%	34%	74%
Mod. net ret. with weighted spread	77%	73%	72%	72%	75%	72%	77%	71%	67%	62%	65%	83%	67%	53%	52%	71%
Giot & Gramming (2005)	62%	59%	58%	57%	63%	65%	76%	81%	85%	88%	84%	92%	86%	82%	88%	71%
Mod. add-on with bid-ask-spread	79%	82%	77%	65%	55%	51%	35%	29%	19%	11%	13%	15%	10%	7%	4%	44%
Cosandey (2001)	n/a	47%	39%	35%	22%	25%	21%	28%	33%	43%	44%	9%	6%	6%	6%	32%
Bangia et al. (1999)	39%	44%	31%	18%	16%	13%	11%	8%	7%	6%	5%	0%	0%	0%	0%	16%
F.-Heude & v. Wynendaele (2001)	3%	15%	15%	13%	16%	12%	14%	14%	18%	21%	20%	8%	9%	6%	12%	15%
Berkowitz (2000)	n/a	7%	9%	10%	8%	10%	8%	12%	10%	5%	14%	17%	14%	17%	19%	10%

Table 5.20: Acceptance rate of liquidity risk models by order size

Table shows acceptance rate by order size averaged over all stocks for each model. Acceptance rate is the percentage of stocks with statistically significant precise risk estimation according to Kupiec (1995).

Models based on bid-ask-spreads - not accounting for order size - show expectedly declining performance with rising order size, while the modified add-on model (4.1) consistently dominates. Cosandey (2001) shows a quite good performance for medium sizes, but very low at large order sizes. The assumption of linear price impact probably distorts results at order size extremes.

The discussion shows that overall ranking results remain valid with one exception. The rank of the top-performing limit order models is not fixed and - depending on the order size in question, the modified add-on model, the empirical net-return model and the Giot and Grammig (2005) model are probably all good choices in practice.

5.3.5.3 Type of misestimation

To allow a more detailed analysis of the reasons behind the individual model performance, table 5.21 shows the over- and underestimation rate of the tested liquidity risk models by order size. The first four limit order models are quite balanced and show underestimation as well as overestimation. The empirical net return model, for example, overestimates the risk of 14 % of the stocks and underestimates the risk for 13 % of the stocks. As mentioned earlier, the severe underestimation of bid-ask-spread models in large order sizes is expected due to their design. The strong general underestimation of the Francois-Heude and Van Wynendaele (2001) can probably be traced to the neglect of time variation, because the crises increase of liquidity cost has not been incorporated. The failure of the implementation of the Berkowitz (2000a)-model in daily data, probably lies in the fact that I had to use mid-price instead of transaction prices, which seems to smooth and therefore underestimate any liquidity effects. Based on these arguments, these two models should probably be ruled out for practical implementation.

Overestimated L-VaR	Order size (in thsd. Euro)														
in % of total stocks	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
Mod. add-on with weighted spread	11%	2%	5%	6%	8%	8%	11%	16%	19%	23%	21%	14%	29%	39%	35%	12%
Emp. net-return with weighted spread	3%	1%	3%	5%	8%	8%	8%	16%	23%	23%	23%	50%	55%	56%	63%	14%
Mod. net ret. with weighted spread	3%	3%	5%	6%	9%	9%	12%	16%	19%	23%	17%	6%	24%	41%	48%	12%
Giot & Gramming (2005)	0%	0%	0%	2%	1%	1%	2%	2%	3%	5%	5%	3%	6%	15%	12%	2%
Mod. add-on with bid-ask-spread	8%	4%	3%	2%	1%	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%	1%
Cosandey (2001)	n/a	50%	59%	65%	77%	73%	78%	72%	67%	57%	56%	91%	91%	94%	94%	67%
Bangia et al. (1999)	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
F.-Heude & v. Wynendaele (2001)	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Berkowitz (2000)	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Underestimated L-VaR	Order size (in thsd. Euro)														
in % of total stocks	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
Mod. add-on with weighted spread	9%	18%	19%	17%	11%	13%	9%	9%	14%	14%	20%	8%	9%	3%	9%	14%
Emp. net-return with weighted spread	20%	26%	21%	16%	16%	15%	10%	7%	6%	6%	2%	0%	3%	3%	3%	13%
Mod. net ret. with weighted spread	19%	23%	23%	22%	16%	19%	10%	12%	14%	15%	17%	11%	9%	6%	0%	17%
Giot & Gramming (2005)	38%	41%	41%	41%	35%	33%	23%	17%	12%	7%	11%	6%	9%	3%	0%	26%
Mod. add-on with bid-ask-spread	13%	14%	21%	33%	44%	48%	65%	70%	81%	89%	87%	85%	90%	93%	96%	55%
Cosandey (2001)	n/a	3%	2%	0%	1%	1%	1%	0%	0%	0%	0%	0%	3%	0%	0%	1%
Bangia et al. (1999)	61%	56%	69%	82%	84%	88%	89%	92%	93%	94%	95%	100%	100%	100%	100%	84%
F.-Heude & v. Wynendaele (2001)	96%	85%	85%	87%	84%	88%	86%	86%	82%	79%	80%	92%	91%	94%	88%	85%
Berkowitz (2000)	n/a	93%	91%	90%	92%	90%	92%	88%	90%	95%	86%	83%	86%	83%	81%	90%

Table 5.21: Over- and underestimation rate by order size

Tables show over- and underestimation rate by order size averaged over all stocks for each model. Over- and underestimation rate is percentage of stocks with statistically significant deviating risk estimation according to Kupiec (1995).

Acceptance rate	Index				
	DAX	MDAX	SDAX	TECDAX	All
Mod. add-on with weighted spread	70%	72%	76%	74%	74%
Emp. net-return with weighted spread	61%	69%	79%	83%	74%
Mod. net ret. with weighted spread	64%	70%	74%	68%	71%
Giot & Gramming (2005)	70%	62%	75%	77%	71%
Mod. add-on with bid-ask-spread	46%	49%	38%	44%	44%
Cosandey (2001)	29%	23%	36%	31%	32%
Bangia et al. (1999)	10%	16%	16%	16%	16%
F.-Heude & v. Wynendaele (2001)	5%	9%	18%	19%	15%
Berkowitz (2000)	22%	8%	6%	4%	10%

Table 5.22: Acceptance rate of liquidity risk models by index

Table shows acceptance rate by index sub-sample averaged over all order sizes and all stocks for each model. Acceptance rate is the percentage of stocks with statistically significant precise risk estimation according to Kupiec (1995).

5.3.5.4 Robustness of model rank

As natural sub-samples, I used the four indices in my sample to check for the robustness of the model rank. Table 5.22 shows the acceptance rate of the tested liquidity risk models by index. The performance in the least liquid SDAX where liquidity effects are largest, is of particular importance. The first four models based on limit order data keep their superior performance, but switch ranks in some sub-samples. The modified add-on models delivers high acceptance rates more consistently than other models with acceptance rates never below 70 %. Therefore, the modified add-on model is recommendable when limit order book data are available.

The modified add-on model based on bid-ask-spread data consistently outperforms all other non-limit order data models as well as Francois-Heude and Van Wynendaele (2001). My adaptation of Berkowitz (2000a) has particular low acceptance rates in the less liquid indices. Its performance is best in the DAX, where liquidity is of minor importance. Hence, it cannot be recommended for daily risk forecasts. Above results are therefore generally confirmed.

Although a shortening of the period length reduces the reliability of the Kupiec-statistic, I split the period into two sub-periods and calculate

Acceptance rate	Sub-period	
	II/2002 - I/2005	II/2005 - II/2007
Mod. add-on with weighted spread	72%	79%
Emp. net-return with weighted spread	79%	79%
Mod. net ret. with weighted spread	69%	73%
Giot & Gramming (2005)	79%	74%
Mod. add-on with bid-ask-spread	43%	60%
Cosandey (2001)	33%	28%
Bangia et al. (1999)	24%	23%
F.-Heude & v. Wynendaele (2001)	26%	22%
Berkowitz (2000)	6%	11%

Table 5.23: Acceptance rate of liquidity risk models by sub-period
Table shows acceptance rate by sub-period averaged over all order sizes and all stocks for each model. Acceptance rate is the percentage of stocks with statistically significant precise risk estimation according to Kupiec (1995).

separate results for each sub-period as another robustness test. The acceptance rate by sub-period is presented in table 5.23. As the sub-period is significantly shorter than the full period, results of the Kupiec statistic are not directly comparable to the full period statistic. Levels can therefore not be compared across tables. I will look at the relative model rank only. In the first sub-period (II/2002 to I/2005), the model ranking is slightly different. The empirical net-return model with weighted spread (section 4.2) and Giot and Grammig (2005) now dominate the modified risk models. I hypothesize that this effect is driven by inefficient skewness and kurtosis estimates which are themselves caused by outliers during the turbulent first sub-period. An improved estimation technique for higher moments might help improve results in this particular situation, a point which is, however, left to future research. The ranking of the second sub-period (II/2005 to II/2007) is preserved.

The first four limit-order models therefore switch ranks in some sub-periods but keep their superiority as a group. The remaining ranking is preserved.

5.3.5.5 Detailed model performance

In the following I discuss the performance of each liquidity risk model in a more differentiated and detailed manner.

Models based on bid-ask-spread The model of Bangia et al. (1999) (section 2.2.2.1) has mediocre performance as displayed in table 5.24.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	23%	n/a	26%	23%	n/a	20%	n/a	11%	6%	n/a	0%	0%	0%	0%	0%	10%
MDAX	41%	39%	28%	18%	18%	11%	11%	7%	4%	4%	2%	n/a	n/a	n/a	n/a	16%
SDAX	44%	42%	31%	13%	10%	8%	7%	5%	6%	7%	8%	n/a	n/a	n/a	n/a	16%
TECDAX	34%	44%	32%	14%	12%	10%	10%	6%	8%	6%	6%	n/a	n/a	n/a	n/a	16%
All	39%	44%	31%	18%	16%	13%	11%	8%	7%	6%	5%	0%	0%	0%	0%	16%

Table 5.24: Acceptance rate of Bangia et al. (1999)-approach by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Because the Baniga model does not account for order size, it has particularly poor performance in large order sizes where liquidity costs are heavily underestimated by the bid-ask-spread. This is consistent in all indices. The acceptance rate in the DAX is particularly low, but comparatively high in larger sizes. Therefore the spread estimate seems to underestimate the real, crises liquidity cost even in smaller sizes.

Loebnitz (2006) suggests a correction of the Bangia et al. (1999) model. He argues that worst spreads should be conceptionally deduced from worst, not from current mid-prices. I also tested this variation. Results are displayed in table 5.25.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	**10**	**25**	**50**	**75**	**100**	**150**	**250**	**500**	**750**	**1000**	**2000**	**3000**	**4000**	**5000**	**All**
DAX	23%	n/a	20%	20%	n/a	20%	n/a	11%	3%	n/a	0%	0%	0%	0%	0%	8%
MDAX	38%	39%	28%	18%	18%	10%	10%	7%	4%	4%	2%	n/a	n/a	n/a	n/a	16%
SDAX	40%	40%	28%	13%	10%	8%	6%	5%	6%	7%	8%	n/a	n/a	n/a	n/a	15%
TECDAX	26%	38%	32%	14%	12%	10%	8%	6%	8%	6%	6%	n/a	n/a	n/a	n/a	15%
All	35%	42%	29%	18%	16%	12%	10%	8%	6%	6%	5%	0%	0%	0%	0%	16%

Table 5.25: Acceptance rate of Bangia et al. (1999) with Loebnitz correction by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. L-VaR is calculated as $L - VaR = 1 - exp(\alpha\sigma_r) \times (1 - 1/2(\mu_S + \hat{\alpha}_S\sigma_S))$. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Acceptance rates are slightly lower than in the original specification across all order sizes and indices. The overestimation of the original model caused by the conceptual imprecision seems to balance the general underestimation tendency, which yields overall more satisfactory results. Therefore, the suggested correction cannot be recommended.

In table 5.26, acceptance rate for the modified add-on approach with bid-ask-spread (section 4.1) is detailed.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	**10**	**25**	**50**	**75**	**100**	**150**	**250**	**500**	**750**	**1000**	**2000**	**3000**	**4000**	**5000**	**All**
DAX	76%	n/a	86%	81%	n/a	78%	n/a	72%	50%	n/a	26%	15%	10%	7%	4%	46%
MDAX	85%	85%	80%	72%	61%	52%	41%	32%	16%	10%	7%	n/a	n/a	n/a	n/a	49%
SDAX	77%	79%	68%	52%	39%	34%	21%	15%	12%	12%	13%	n/a	n/a	n/a	n/a	38%
TECDAX	76%	84%	82%	67%	55%	49%	33%	20%	12%	10%	10%	n/a	n/a	n/a	n/a	44%
All	79%	82%	77%	65%	55%	51%	35%	29%	19%	11%	13%	15%	10%	7%	4%	44%

Table 5.26: Acceptance rate of modified add-on approach with bid-ask-spread by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Compared with the models discussed in this paragraph so far, the modified add-on approach with bid-ask-spread offers substantial improvements with acceptance rates more than doubling. The detailed results at the spread level are as high as the overall results of the limit order models.

Therefore, risk seems to be adequately forecasted with this model when looking at positions tradable at the bid-ask-spread. The performance decline with order size is natural because this model does not account for the liquidity cost increase with size.

Models based on volume or transaction data My adaptation of the Berkowitz (2000a), as described in sections 2.2.2.2 and 5.3.3.2, does not perform very well across order sizes and indices as shown in table 5.27.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	n/a	n/a	24%	24%	n/a	24%	n/a	25%	27%	n/a	25%	17%	14%	17%	19%	22%
MDAX	n/a	10%	4%	5%	12%	6%	16%	6%	4%	9%	4%	n/a	n/a	n/a	n/a	8%
SDAX	n/a	7%	4%	7%	5%	5%	5%	7%	4%	4%	8%	n/a	n/a	n/a	n/a	6%
TECDAX	n/a	3%	3%	3%	3%	0%	0%	8%	8%	4%	16%	n/a	n/a	n/a	n/a	4%
All	n/a	7%	9%	10%	8%	10%	8%	12%	10%	5%	14%	17%	14%	17%	19%	10%

Table 5.27: Acceptance rate of Berkowitz (2000a) by index and order size
Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Results for the min-column (spread level) cannot be calculated because order size of spread is non-standardized and therefore missing in the adjustment procedure of Berkowitz (2000a). The original model is designed for transaction data. As described in section 5.3.3, the calculated acceptance rates are tuned to daily data. Unfortunately the results are not very promising. For daily data, at least in this implementation, the model of Berkowitz (2000a) cannot be recommended.

The Cosandey (2001) model (section 2.2.2.2 and 5.3.3.3) performs much better as can be seen from table 5.28.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	n/a	n/a	77%	77%	n/a	60%	n/a	34%	14%	n/a	0%	9%	6%	6%	6%	29%
MDAX	n/a	46%	30%	28%	19%	16%	16%	14%	13%	18%	20%	n/a	n/a	n/a	n/a	23%
SDAX	n/a	39%	30%	25%	21%	19%	21%	31%	47%	63%	75%	n/a	n/a	n/a	n/a	36%
TECDAX	n/a	47%	33%	22%	20%	18%	22%	27%	35%	40%	50%	n/a	n/a	n/a	n/a	31%
All	n/a	47%	39%	35%	22%	25%	21%	28%	33%	43%	44%	9%	6%	6%	6%	32%

Table 5.28: Acceptance rate of Cosandey (2001) by index and order size
Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Acceptance rates partially reach levels above 70 %, even in larger order sizes. However, performance is not consistently high. It varies between indices and order sizes with no apparent structural driver. Nevertheless, if only transaction data are available, it is the best model in the test.

Models based on limit order book data I now turn to the detailed performance results of the weighted spread models. Table 5.29 shows the Kupiec-accepted fraction of stocks for Francois-Heude and Van Wynendaele (2001) (section 2.2.2.3 and 5.3.3.4).

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	3%	n/a	3%	3%	n/a	3%	n/a	3%	3%	n/a	3%	8%	9%	6%	12%	5%
MDAX	4%	11%	6%	6%	9%	7%	10%	12%	9%	12%	12%	n/a	n/a	n/a	n/a	9%
SDAX	2%	13%	19%	15%	14%	13%	15%	19%	21%	24%	32%	n/a	n/a	n/a	n/a	18%
TECDAX	6%	16%	16%	16%	20%	18%	12%	15%	29%	23%	24%	n/a	n/a	n/a	n/a	19%
All	3%	15%	15%	13%	16%	12%	14%	14%	18%	21%	20%	8%	9%	6%	12%	15%

Table 5.29: Acceptance rate of Francois-Heude and Van Wynendaele (2001) by index and order size
Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

The detailed results are not very satisfactory. The model practically fails to predict risk well in the DAX, but performance is also quite low in the other indices. Especially, if compared to the other limit order models, the model of Francois-Heude and Van Wynendaele (2001) cannot be recommended.

In table 5.30, detailed acceptance rates for Giot and Grammig (2005) (section 2.2.2.3 and 5.3.3.5) are shown.

Acceptance rate	Order size (in thsd. Euro)															
in % of stocks	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	86%	n/a	53%	53%	n/a	58%	n/a	61%	61%	n/a	72%	92%	86%	82%	88%	70%
MDAX	57%	41%	43%	45%	51%	55%	65%	74%	86%	88%	87%	n/a	n/a	n/a	n/a	62%
SDAX	57%	65%	65%	63%	67%	76%	81%	90%	89%	85%	87%	n/a	n/a	n/a	n/a	75%
TECDAX	64%	61%	67%	67%	76%	73%	80%	88%	91%	93%	85%	n/a	n/a	n/a	n/a	77%
All	62%	59%	58%	57%	63%	65%	76%	81%	85%	88%	84%	92%	86%	82%	88%	71%

Table 5.30: Acceptance rate of Giot and Grammig (2005) by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Acceptance rates are very high, often above 70 %. Performance is especially high in large order sizes in illiquid indices, i.e. where the liquidity adjustment is most needed. However, risk forecasts work less well in smaller order sizes.

The net-return model proposed in section 4.2 performs also quite well as can be seen the details in table 5.31.

Acceptance rate	Order size (in thsd. Euro)															
in % of stocks	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	86%	n/a	78%	78%	n/a	83%	n/a	78%	64%	n/a	57%	50%	42%	41%	34%	61%
MDAX	79%	69%	74%	77%	72%	68%	73%	63%	58%	64%	70%	n/a	n/a	n/a	n/a	69%
SDAX	76%	78%	78%	84%	77%	73%	82%	82%	80%	78%	83%	n/a	n/a	n/a	n/a	79%
TECDAX	74%	71%	80%	82%	90%	92%	90%	85%	77%	75%	90%	n/a	n/a	n/a	n/a	83%
All	78%	73%	76%	79%	77%	77%	82%	77%	71%	71%	75%	50%	42%	41%	34%	74%

Table 5.31: Acceptance rate of empirical net-return approach with weighted spread by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

As discussed above in the overview, performance is higher than in Giot and Grammig (2005) at smaller sizes but lower in larger order sizes. The detailed results show that this is mainly driven by DAX stocks. In the more important illiquid indices, the net-return model is slightly superior.

Similarly well are acceptance rates of the proposed modified add-on model with weighted spread (section 4.1 and 5.3.3.6) as is apparent from table 5.32.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	88%	n/a	64%	69%	n/a	69%	n/a	81%	78%	n/a	81%	78%	63%	58%	56%	70%
MDAX	84%	86%	80%	78%	84%	81%	78%	68%	54%	51%	49%	n/a	n/a	n/a	n/a	72%
SDAX	77%	84%	81%	78%	81%	79%	73%	75%	70%	67%	55%	n/a	n/a	n/a	n/a	76%
TECDAX	78%	63%	67%	73%	78%	80%	82%	77%	76%	73%	71%	n/a	n/a	n/a	n/a	74%
All	81%	80%	76%	77%	82%	79%	79%	75%	66%	63%	60%	78%	63%	58%	56%	74%

Table 5.32: Acceptance rate of modified add-on approach with weighted spread by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

Performance is slightly better in larger indices than in the empirical net-return model, but has some outliers in large MDAX sizes and small TecDAX sizes.

Finally, table 5.33 presents detailed results for the modified net-return model with weighted spread.

Acceptance rate in % of stocks	Order size (in thsd. Euro)															
	Min.	10	25	50	75	100	150	250	500	750	1000	2000	3000	4000	5000	All
DAX	85%	n/a	61%	61%	n/a	58%	n/a	67%	75%	n/a	74%	83%	67%	53%	42%	64%
MDAX	76%	78%	76%	73%	76%	70%	74%	68%	60%	59%	63%	n/a	n/a	n/a	n/a	70%
SDAX	77%	80%	79%	76%	75%	71%	76%	69%	74%	66%	58%	n/a	n/a	n/a	n/a	74%
TECDAX	72%	55%	61%	71%	73%	76%	82%	71%	62%	60%	71%	n/a	n/a	n/a	n/a	68%
All	77%	73%	72%	72%	75%	72%	77%	71%	67%	62%	65%	83%	67%	53%	52%	71%

Table 5.33: Acceptance rate of modified net return approach with weighted spread by index and order size

Table shows the fraction of stocks, where the L-VaR-model has been accepted by Kupiec-statistics. The min-column measures the acceptance rate for the minimum spread level, i.e. the bid-ask-spread. The All-column measures the average over all standardized order sizes, i.e. without the min-column.

As with the empirical net-return model, acceptance rates are lower in larger order sizes, quite consistent in all indices. Both models imprecisions seems to be driven by the compounding of mid-price return and liquidity cost processes in the net-return. This compounding limits forecastability if processes are very different, as is the case in larger order sizes. Overall, detailed results can be said to confirm earlier findings.

5.3.6 Synopsis

In this section, I have put a large selection of traceable liquidity risk models to the test in order to find out which is most suitable for daily risk estimation. I implemented Bangia et al. (1999), Berkowitz (2000a), Cosandey (2001), Francois-Heude and Van Wynendaele (2001), Giot and Grammig (2005) as well as the two model propositions from chapter 4 in a large sample of daily stock data over 5.5 years. I used a standard Kupiec (1995)-statistic to determine if models provide precise risk forecasts on a statistically significant basis.

I find, that available data is the main driver of the preciseness of risk forecasts. Models based on limit order data generally outperform models based on bid-ask-spread or transaction data. The latter (Cosandey (2001) and Berkowitz (2000a)) are highly approximate and should only be used if no other data are available. If limit order book data is available, an approach based on empirical or t-distributed net returns (section 4.2 or Giot and Grammig (2005)) as well as the modified add-on model (section 4.1) all show satisfactory results. The modified add-on model with limit order data shows the most consistent outperformance. If only bid-ask-spread data can be obtained, the modified add-on model with bid-ask-spreads (section 4.1) is recommendable. On the basis of transaction data, Cosandey (2001) provides suitable results for daily forecasts.

6 Conclusion

6.1 Summary and implications

In this thesis, I provided an up-to-date overview of the current state of market liquidity risk measurement. Chapter 2 summarized aspects of liquidity that are relevant to liquidity risk management. It clearly defined market liquidity from a cost perspective and showed that this perspective is consistent but advantageous to other existing liquidity definitions. I also provided an overview of liquidity risk models and described their less explicit assumptions and made similarities and discrepancies transparent.

Chapter 3 described the data set I used in the empirical analysis. Besides standard price, spread and volume data, I used a large data set of the weighted spread measure by Deutsche Börse, called Xetra Liquidity Measure (XLM). The weighted spread measure extracts liquidity costs by order size from the limit order book. It is a data-type made available by electronic exchanges only in recent years. With daily data of 160 stocks of the four major German stock indices over 5.5 years, it comprises the largest weighted-spread sample so far analyzed in academia. I outline under which assumptions weighted spread is a valid liquidity cost measure. I also provide a detailed empirical analysis of this weighted spread sample, which allows representative liquidity cost estimates by order size for the first time in literature. I find that liquidity cost significantly increase with order size. Time variation is significantly different for larger than for smaller order sizes. Both findings make it highly likely that weighted spread proves to be a superior liquidity measure in risk management as well. In general, assuming no effect of order size on liquidity cost will lead to significant distortions.

In chapter 4 I suggested two new liquidity risk models. The modified add-on model concerns a new way to account for non-normality in liquidity risk. With the help of the Cornish-Fisher approximation, the first four moments (including skewness and kurtosis) of a distribution can be taken into account when estimating percentiles of a distribution. The net-return model with weighted spread extends the approach of Giot and Grammig (2005). It provides a framework to analyze the use and effect of weighted spread in liquidity risk measurement. I developed a risk decomposition which allows to distill the effect of size and liquidity on risk as well as to extract the effect of liquidity-return correlation, an issue that has been disputed in the literature.

Chapter 5 contains the empirical analysis of this thesis. In a first step, I benchmarked the newly suggested modified add-on model against the original specification of Bangia et al. (1999). My new suggestion proves to be highly superior in terms of preciseness. The superiority is robust when looking at different stock or time sub-samples.

In a second step, I calculated liquidity risk with the net return model and weighted spread in daily horizons. Liquidity is found to be non-neglectable even at ten-day horizon. Liquidity risk is shown to significantly increase with order size. I also find that liquidity-return correlations are large and incur an overestimation of liquidity risk by 100 % when neglected. These results are robust when looking at the expected shortfall risk measure, risk over time and risk in a diversified stock portfolio.

In a final step, I ran a benchmark between all liquidity risk models available in daily data, an exercise that has not been done before. I find that models based on weighted spread, i.e., limit order data, provide risk forecasts with the highest preciseness. The newly suggested modified add-on model with weighted spread provides precise risk forecasts in subsamples most consistently. When having only transaction data available, the model by Cosandey (2001) is recommendable. When working with bid-ask-spread data, the modified add-on model with bid-ask-spread has the highest preciseness.

Overall, this thesis summarized the current state of research and extends it into several new directions. By using a unique, representative data-set and by suggesting new liquidity risk models, the thesis provides new and superior ways to account for liquidity in risk management.

6.2 Outlook

Market liquidity risk still provides a large realm of topics that require future research.[1] I believe, that answers to the following questions, which are - to the best of my knowledge - still open, would be especially interesting. A better understanding of certain aspects of market liquidity would be helpful and liquidity risk management also shows some loose ends.

First, although I hypothesized that optimal trading strategies do not provide any significant benefits from a risk perspective, they are certainly valid in normal market conditions and for block sales. The pressing question is how to estimate the parameters required for the optimal trading algorithms. What is the empirical benefit of different optimal trading strategies? In which situations are they (most) valid?

This issue can get tackled from a different perspective as well: When are liquidity prices efficient? If they are, then any optimal trading strategy will have to fail. It also only makes sense to add liquidity cost risk to price risk if prices not yet suitably reflect liquidity. If mid-prices already reflect overall liquidity, must any further adjustment be restricted to the individual trader's situation, must common liquidity effects be neglected?

Second, asset pricing questions based on more precisely estimated price impact curves would clarify the importance of liquidity costs to investors. Combining the weighted spread measure of the price impact curve with

[1] For this section, cp. Stange and Kaserer (2008b).

the distribution of trading volume yields the total cost paid by investors per stock. Is this total cost reflected in prices? It might also be possible to describe the whole price impact curve with theoretical, calibrated liquidity processes - similar to theoretical descriptions of the interest rate curve. This might help in situations, where the price impact curve is non-observable or where forecasting is very difficult.

Third, the most important issue for liquidity risk measurement is, in my view, the under-researched treatment of delay risk. The dynamics of delay (in crises) and its relation to the price dynamics is still unclear. When and for which assets does trading break down in crises? Further insight into empirical delay properties would help to choose an appropriate approach to integrate delay risk into liquidity risk measurement. This research topic would also have to tackle the question of how to measure and forecast delay, especially in markets where delay is important and market data is quite perforated. A subsequent empirical comparison of methods and magnitudes of liquidity risk in different asset classes would be interesting.

Fourth, the specification of size has been handled differently by different authors. When analyzing liquidity cost and risk, which specification is most suitable? Size can be defined as number of shares, volume in currency units or volume relative to the traded volume in the market. From the theoretical as well as the empirical perspective an analysis could be fruitful, which determines liquidity in a more precise and stable way. Section 3.3.5.1 can provide impetus here.

Fifth, the literature on market liquidity has been enriched by approaches that have not yet been used in liquidity risk management. Chacko et al. (2008) calculate liquidity cost in an option pricing framework, which is possible because liquidity can be interpreted as marketability option as discussed in section 2.2.1. Because it is implementable based on transaction data, it provides a traceable approach that is theoretically rigorous at the same time. It might be an interesting venue to explore from a liquidity risk perspective.

Sixth, liquidity risk management could still need some refinement. Duffie and Ziegler (2003) describe optimal liquidation strategies of portfolios in crises. I believe, that liquidity risk treatment of portfolios still has neglected potential for further insight. It might also be interesting to understand if it is possible to construct specific liquidity options, that could be used to hedge away the liquidity cost risk. Not long ago, volatility options became a traded contract in financial markets. Is there similar potential for liquidity options?

Market liquidity risk is still a relatively young research topic where further insight is possible beyond the refinement of existing ideas. With this thesis I hope to have contributed with new approaches and new empirical findings. Further insight is also very necessary as demonstrated by high interest from practitioners and regulators during the recent subprime events. Market liquidity risk can break financial institutions and reoccurs as topic in almost every modern financial crises. Understanding its structure and making true risks transparent is essential for steering financial institutions through turbulent times.

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