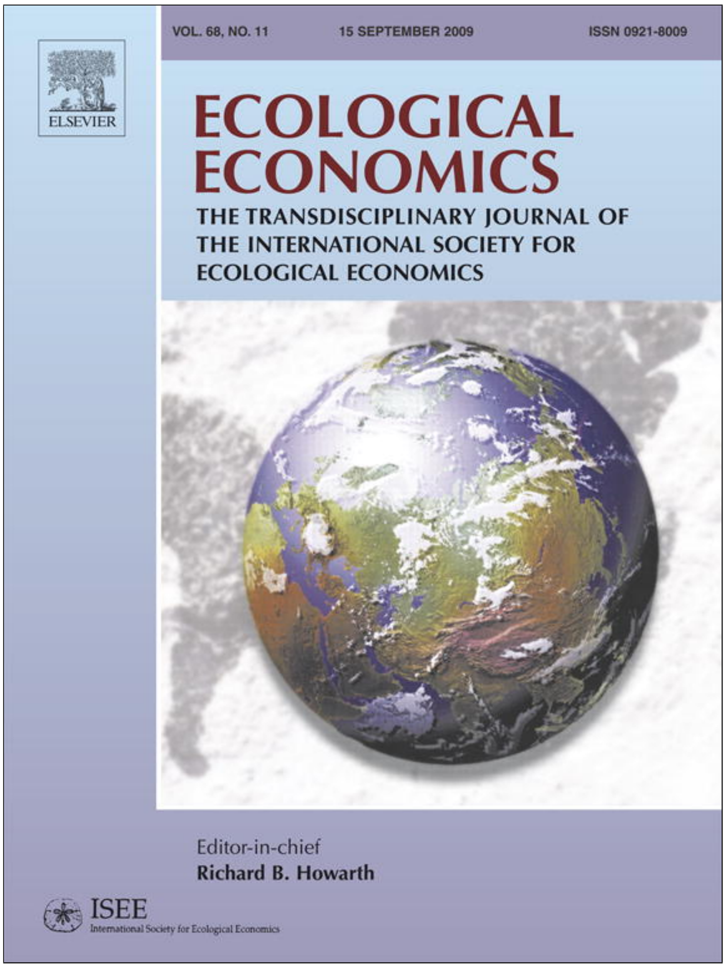
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ANALYSIS

Optimizing the shares of native tree species in forest plantations with biased financial parameters

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A B S T R A C T

Addressing uncertainty is a key requirement to follow the principle of precaution in sustainable ecosystem management. The maximization of worst-case outcomes according to the "maximin" decision rule, based on the two parameters mean and variance of a financial indicator, is a prominent approach to integrate uncertainty in decision-making. In forestry, the problem of selecting the optimum tree species combination for a forest plantation investment can be seen as a problem of optimal portfolio selection, to be solved according to the "maximin" decision rule. Yet, it is well known that portfolios computed from expected means and variances are highly sensitive to changes in the estimated parameters. The financial results may be poor if we rely too much on the historical data. This paper tests an extended worst-case model that considers a lower bound for the expected mean net present value (NPV) of a tree species portfolio and an upper bound for its variance. Biased expected mean NPVs, variances and correlations for the tree species *Picea abies* [L.] Karst. (Spruce) and *Fagus sylvatica* L. (Beech) were used to test the variability of the resulting tree species portfolios (27 scenarios). A comprehensive simulated data set, which was adopted from an existing study and defined as the independent reference, served to evaluate the financial performance of the tree species portfolios obtained from optimization with the biased data. Compared with the results of classical worst-case optimization instances, it was feasible to reduce the variability of tree species shares effectively when the optimization was carried out with the extended worst-case approach. Furthermore, the financial performance of this approach was better when tested with the independent data. The worst-case forest NPVs achieved with the extended approach were on average 10% (statistical confidence 0.95) or 147% (statistical confidence 0.99) greater in comparison to the results of the classical approach. The influence of the uncertainty parameter selection was tested and the results were discussed against the controversial viewpoints on the usefulness of the "information-gap decision theory". Finally, the significance of our results for sustainable ecosystem management is pointed out.

1. Introduction

Forestry approaches and management practices which may be identified as "close-to-nature", "nature-based", "near-natural" or "ecosystem management" have obtained increasing intellectual interest (Gamborg and Larsen, 2003; Bristow et al., 2006). Working with native tree species and mixed species approaches belong to this overall shift to ecologically oriented forest management. In Germany, the concerns on forest decline in the 1980s and the enormous damage caused by storms and insects to pure coniferous species forests (almost Spruce, which is not native to many sites where it is grown) have led to the abandonment of pure conifer forests in most official management rules (e.g., Baumgarten and von Teuffel, 2005; Fritz, 2006). Here, the trend towards broadleaves and native tree species is justified by the ecological benefits of these forest types, such as soil improvement, provisioning of ecosystem services, higher biodiversity and greater resilience, rather than by financial reasons (Knoke et al., 2008).

The shift towards a more natural forest composition is not necessarily supported by classical forest economic analysis, since the broadleaved species, native to most sites in Germany, are often the worse economic performers (Möhring, 2004; Knoke et al., 2005). Yet, native tree species are usually better adapted to site conditions and thus often suffer lower risks (von Lüpke and Spellmann, 1999). If they cover some parts of forest plantations, their greater resistance against biotic and abiotic hazards can also stabilize financial returns in the long run (Knoke et al., 2005). This fact would lead to sustainability not only from an ecological but also from an economical point of view.

Various studies in forest economics have already focused on the composition of tree species on a given forest land (e.g. Thomson, 1991; Deegen et al., 1997; Weber, 2002; Knoke et al., 2005, Knoke and Wurm, 2006; Knoke and Hahn, 2007; Knoke et al., 2008; Beinhofer, in press; Hildebrandt et al., in press). Other examples of financial optimization of

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species shares exist in the fields of fishery (Edwards et al., 2004) and grassland management (Koellner and Schmitz, 2006). The mentioned studies build on the fact that mixed ecosystems show effects of risk compensation similar to diversified portfolios of stocks (Knoke, 2008). However, as long-life forests differ from stock portfolios, their composition cannot be directly adjusted to current market developments, since a change of tree species composition is a long-term operation. Robust optimization methods, that secure acceptable financial results even under pessimistic assumptions, are thus needed.

Optimization of tree species composition poses a challenge: Different to other assets, we have neither stock market data for the single forest stands, nor time series for timber prices or biophysical risks like storms, insect attacks or snow breakage, that comprise several rotations. We thus have to simulate the required financial data over long time periods that we need to address in forestry. The resulting estimates on financial parameters are highly uncertain on the one hand. On the other hand, the optimal tree species composition is extremely sensitive to these parameter estimates. Based on various scenarios on plantation management, different biophysical risks and risk attitudes of decision-makers, the derived optimal shares of the broadleaved tree species *Fagus sylvatica* L. (European Beech), which is native to most sites in Germany, may range between almost zero and more than 60% (Knoke et al., 2005; Knoke and Wurm, 2006). This considerable range should warn us not to rely too much on one simulated scenario. Moreover, highly variable results are not convincing for forest practitioners and thus, for the case of Germany, forest practitioners often base their tree species choice primarily on intuitive considerations.

In order to optimize the composition of tree species, scientific studies often apply optimization methods which are based on the mean net present value (NPV) and its variance. From studies of other fields than forestry it is well known that, despite the theoretical success of the mean-variance models (Bai et al., 1997), the practical relevance of the resulting portfolios is generally not very significant (Goldfarb and Iyengar, 2003). Solutions of mean-variance optimization are generally extremely sensitive to changes in the financial input parameters, with the latter usually being simulated exclusively on a historical data basis. However, it is not possible to reflect future changes from historical data and thus alternatives for a "robust" optimization have been proposed. The so called "information-gap decision theory" (IGT), promoted by Ben-Haim (2006), has recently gained considerable popularity especially in ecological studies. These studies optimize, for example, habitats or reserves under severe uncertainty (e.g. Regan et al., 2005; Halpern et al., 2006; Moilanen et al., 2006). But also regarding financial optimization IGT has been considered as a possible solution for decision making under severe uncertainty (e.g. Ben-Haim, 2005; McCarthy and Lindenmayer, 2007; Knoke, 2008). Nevertheless, both the originality and usefulness of IGT have already been sharply questioned. Sniedovich (2007) has shown that IGT is only a simple variant of the classical "maximin" decision-rule. He has stated that IGT carries out only a very local consideration of uncertainty. Although Knoke (2008) has pointed out the analytical appeal of IGT, he has also demonstrated that the results from this optimization model do not differ from those obtained with classical optimization methods, as for example the minimization of the probability of failure.

Forest ecosystem management needs to address future uncertainty more comprehensively than it has done up to now. When optimizing tree species shares it is very likely that the operation can be carried out only with biased input parameters, since we have to anticipate states that lie very far in future. We should thus not have too much confidence in our modeled financial input parameters. We rather have to address the severe uncertainty inherent in the estimated financial data. According to the principle of precaution we are to consider that our estimates for the expected mean and its variance may be too optimistic. Hence, we want to test whether tree species portfolios obtained from optimization with biased financial parameters can nevertheless provide comparatively high financial outcomes, even in the case when extremely wrong parameter estimates are used as an input. We therefore find it necessary to look for rather stable solutions under variable input parameters that deliver acceptable financial results (i.e. positive and relatively high NPV) even under pessimistic scenarios. Our objective is thus to test the stability of simple tree species portfolios and their financial performance for various biased estimates on means and variances of the tree species' NPVs. This is done under a classical and an extended modeling approach, which considers information-gaps. We thus do not change the considered decision rule in principle, but we a priori take into account that the simulated data may be biased.

2. Methods

Establishing young forest stands usually means providing products and services for future generations and thus decision-making has to consider sustainability requirements. When seeking sustainable ecosystem management, scientists often recommend a perspective of precaution (Figge and Hahn, 2004; Wunder, 2000; Knoke and Moog, 2005; Weber-Blaschke et al., 2005; Krysiak, 2006). The point of view of precaution holds not only for ecological but also for financial aspects. Sustainable management should secure future management options and stabilization of long-term financial returns. To carefully address the uncertainty situation under which we have to decide on tree species, we may analyze the financial consequences of various tree species portfolios under pessimistic assumptions in order to make sure that even then acceptable results are secured.

Dealing with uncertainty can principally be seen as a two-player game, as it has been explained in detail by, for example, Sniedovich (2007). Dependent on the attitude of the decision-maker (first player), whether pessimistic or optimistic, it is assumed that nature (second player) plays against or with him. So called "maximin" rules are the classical, but still popular decision-rules. "Maximin" decision-making reflects risk-aversion and is principally able to support sustainable management. It tells us that we should rank alternatives by their worst possible outcomes; the alternative whose worst outcome is superior to the worst outcome of others has to be selected:

$$\max_{d\in D}\ \min_{s\in S(d)}\ f(d,s)$$

This means, that the decision maker would first select a decision $d \in D$ from the decision space available to him to maximize the outcome $f\ (d,s)$. Nature would then, given d, select a state $\{S(d):d \in D\}$ to minimize the outcome $f\ (d,s)$, an assumption that reflects the risk-avoiding attitude of the decision-maker. The decision-maker has then to find the decision which performs best under this pessimistic assumption.

The application of "maximin" rule requires estimating possible worst-case outcomes (the minimized outcomes provided by nature to the pessimistic decision-maker) of the various possible choices. This would also enable optimizing decisions on portfolio selection, seen as a "maximin" game. Consider a forest plantation as a portfolio of various independent tree species and assume that the value of this portfolio, V_p, given by the sum of all discounted future net revenues (i.e. the net present value, NPV), should be maximized according to the "maximin" rule. Let the distribution function of possible portfolio NPVs, V_p, be known and normal: $V_p \sim N(E(V_p),VAR(V_p))$. Given an accepted statistical confidence level, $1-\alpha$, the expected worst-case result, V_{w-c}, can then be predicted and maximized by means of selecting the optimum shares of the tree species in the forest portfolio, $\mathbf{f}^{*}$, as the vector of tree species shares that maximizes the worst-case forest NPV, V_{w-c}:

$$\max_{(\mathbf{f}:\ 1^T\mathbf{f}=1,\ V_{w-c},V_p\in V,\ \alpha>0)} V_{w-c} = \Phi^{-1}\left(\alpha, E\left(V_p\right), \sqrt{VAR\left(V_p\right)}\right) \qquad (1)$$

with V as the set of possible forest portfolio NPVs and Φ^{-1} as the inverse of the normal distribution function.

However, as reported above it is well known that the optimum portfolio composition, $\mathbf{f^*}$, is generally highly sensitive to the estimated portfolio parameters, mean and variance $(\tilde{E}(V_p),\tilde{V}AR(V_p))$ (e.g., Goldfarb and Iyengar, 2003). A biased estimation of future timber prices, their volatility, logging costs, timber growth, tree species survival probabilities or other important factors can lead to a very poor approximation of the true future values. We should therefore define uncertainty sets that possibly include the true values for our parameters $(E(V_p),VAR(V_p))$ to consider the severe uncertainty involved with their estimation.

For the expected NPV of the forest portfolio, V_p, we can formulate the following uncertainty set:

$$U\left(\tilde{E}\left(V_p\right),h\right) = \left\{E\left(V_p\right) : \left|E\left(V_p\right) - \tilde{E}\left(V_p\right)\right| \le h\right\}, h \ge 0$$

with

$$\tilde{E}\left(V_p\right) = \tilde{\mathbf{v}}^T\mathbf{f} \tag{2}$$

$$1^T\mathbf{f} = 1$$

$$h = \omega\,\bar{\tilde{v}}$$

Here $\tilde{\mathbf{v}}$ is a vector of estimated NPVs of the considered single tree species and $\mathbf{f}$ is a vector of the possible shares of the tree species. The uncertainty level, h, has to be selected by the decision-maker. It describes the information-gap that results from the difference between what we know and what is needed to draw a correct decision. We propose to estimate this uncertainty level proportional to a naive approximation of the forest NPV, which does not consider the shares of the tree species. The approximation is formed by the arithmetic mean of the single per unit NPVs of the considered tree species. The number ω is a decimal number that controls the level of uncertainty, h. Note that the uncertainty set described with Eq. (2) is a simple information-gap model (Ben-Haim, 2006). It comprises all possible true mean forest portfolio NPVs $(E(V_p))$, whose deviation from the estimated mean NPV $(\tilde{E}(V_p))$, quantified by the absolute value of the difference between the estimated NPV and a member of U, is not greater than h. The uncertainty set in Eq. (2) thus contains all possible true but unknown mean NPVs consistent with the estimated mean NPV.

We may now use Eq. (1) to estimate a possible worst-case forest NPV with the following pessimistic lower bound of the true but unknown expected forest NPV:

$$E\left(V_p\right) = \tilde{\mathbf{v}}^T\mathbf{f} - \omega\,\bar{\tilde{v}} \tag{3}$$

Not only the true mean NPV, but also its variance can only be approximated with a given, probably great, uncertainty. The variability derived from historical data may not be representative for future variability. The variance of the forest portfolio is subject to the uncertainty reflected in the estimated covariance matrix, $\tilde{\Sigma}$. The true covariance matrix can be seen as a member of the following uncertainty set:

$$U\left(\tilde{\mathbf{\Sigma}},a\right) = \left\{\mathbf{\Sigma} : \left\|\mathbf{\Sigma} - \tilde{\mathbf{\Sigma}}\right\|_F \le a\right\}, a > 0$$

with

$$\tilde{V}AR\left(V_p\right) = \mathbf{f}^T\tilde{\mathbf{\Sigma}}\,\mathbf{f} \tag{4}$$

$$a = \sqrt{\sum_i \left|\Delta VAR(V_i)\right|^2}$$

$$\hat{a} = \varepsilon\;\overline{\tilde{V}AR(V_i)}$$

Here, $\mathbf{\Sigma}$ is the true covariance matrix, $\overline{\tilde{V}AR(V_i)}$ is the arithmetic mean of the estimated variances of the NPVs of single tree species and $||\cdot||_F$ is the Frobenius norm of the matrix which contains the differences between $\mathbf{\Sigma}$ and $\tilde{\mathbf{\Sigma}}$, $_\Delta VAR(V_i)$. For the sake of simplicity we assume the covariances to be exactly known in $\tilde{\mathbf{\Sigma}}$ so that there are no differences between $\mathbf{\Sigma}$ and $\tilde{\mathbf{\Sigma}}$ in this regard (we later test whether this simplification can be justified). We can thus see a as the norm obtained from the unknown differences between the estimated and the true variances of the individual tree species' NPVs. We propose to estimate the upper bound of this norm, $\hat{a}$, proportional to the arithmetic mean of the NPV variances for the single tree species. The number ε is again a decimal that defines the level of uncertainty in the variance estimation, which has to be decided on by the decision-maker.

We may now write for the true but unknown forest portfolio variance as a pessimistic upper bound to be used in Eq. (1):

$$VAR\left(V_p\right) = \mathbf{f}^T\left(\tilde{\mathbf{\Sigma}} + \hat{a}\ \mathrm{I}\right)\mathbf{f} \tag{5}$$

The assumption will be discussed later that all the unknown variance components to be considered for every tree species are the same, independent from the specific variance estimated for a single tree species. For a robust optimization it is important that the unknown variance components are really independent from the biased parameters of the single variances of the tree species' NPVs. If we would estimate them proportionally to the biased estimates of the variances of tree species' NPVs, we would only increase the bias and the composition of the portfolios would even become more unbalanced and variable. The shares of tree species whose estimated NPV variances are great would be further reduced and the shares of those whose NPV variances are small would further increase. We inevitably considered the unknown part of variance as a black box, as de facto no information is available about additional systematic and unsystematic risks that the future may bring. To assure independence from the estimated values the upper bounds of the unknown variance components were thus computed naively proportional to the arithmetic mean of the variances of the single tree species (see Eq. (4)).

We assume that the naive estimation of the uncertainty levels, which is independent from the biased estimates, will stabilize the results of the optimal tree species shares, even if we vary the biased estimates for means and variances in Eq. (1). Below we compare the optimal portfolio composition for a two-species forest plantation, derived by using the biased estimates for means and variances to maximize Eq. (1) (classical worst-case modeling), with an optimization based on means and variances and adjusted to uncertainty according to Eq. (3) and (5) (extended worst-case modeling). Simulated data sets, published by Knoke and Wurm (2006), were defined as reference data and utilized to evaluate the financial performance of the optimizations carried out with biased parameters.

2.1. Reference data sets

To evaluate the possible impact of integrating severe uncertainty into the optimization process, we used reference data sets from an existing study. Knoke and Wurm (2006) simulated long-term forest management data for the European tree species *Picea abies* [L.] Karst. (called Spruce from here onwards) and *Fagus sylvatica* L. (called Beech from here onwards) by means of the Monte-Carlo simulation technique. The plantation area of Spruce was extended far beyond its natural limits (Spiecker, 2003) in Germany, while Beech would dominate the natural vegetation cover in Central Europe. To assure comparability of the simulated NPVs with those derived in other forest science studies Knoke and Wurm (2006) selected an extremely long time horizon of 500 years. Therefore, the results are comparable

Table 1
Simulated financial data on NPV distributions (reference data sets) for various tree species shares (fraction of Spruce = 1-fraction of Beech), adopted from Knoke and Wurm (2006), with alterations.

Fraction of Beech	Moments of the reference data distributions of NPVs (Euro ha)										
	Mean	SD	Quantiles								
			0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
0	5028	2599	8827	8259	7928	7317	5447	2958	1545	444	− 1,083
0.05	4971	2468	8592	7994	7717	7157	5424	3029	1649	628	− 788
0.1	4914	2340	8355	7762	7535	6998	5321	3093	1768	817	− 540
0.15	4858	2214	8125	7617	7363	6823	5204	3139	1852	900	− 338
0.2	4801	2092	7881	7442	7188	6652	5122	3172	1903	1070	− 144
0.25	4744	1973	7697	7225	7008	6495	5038	3219	2028	1241	54
0.3	4688	1859	7497	7089	6832	6351	4938	3268	2158	1401	236
0.35	4631	1750	7295	6917	6672	6189	4771	3350	2137	1512	415
0.4	4574	1648	7087	6742	6506	6046	4671	3374	2245	1693	610
0.45	4518	1554	6892	6595	6347	5894	4588	3457	2344	1830	546
0.5	4461	1468	6731	6420	6184	5744	4512	3467	2460	1926	574
0.55	4404	1394	6592	6265	6037	5595	4499	3499	2563	1942	585
0.6	4348	1332	6448	6125	5887	5440	4478	3507	2637	1934	513
0.65	4291	1286	6336	5983	5727	5296	4440	3540	2638	1858	414
0.7	4234	1255	6242	5825	5607	5198	4407	3582	2630	1689	276
0.75	4178	1242	6160	5701	5467	5066	4360	3645	2503	1567	118
0.8	4121	1247	6078	5573	5352	4938	4329	3703	2495	1424	− 282
0.85	4064	1269	6001	5481	5269	4886	4308	3751	2357	1323	− 534
0.9	4008	1309	5934	5473	5210	4809	4286	3722	2300	1061	− 724
0.95	3951	1364	5858	5458	5159	4762	4276	3662	2055	858	− 1,072
1	3894	1433	5879	5427	5199	4735	4237	3595	1850	576	− 1,425

to those obtained under deterministic optimization for an unlimited time horizon, which forest economics studies often assume when applying the so called "Faustmann" approach (e.g., Chang, 1998). The rotation for both tree species was 100 to 110 years, depending on the stochastically simulated timber price, and they integrated natural hazard risks as well as timber price volatility. Distributions of NPVs (sums of discounted net revenue flows) were generated by means of 1000 repetitions for various portfolios of both tree species (Table 1).

The net revenue flows were discounted with an interest rate of 0.02. This interest rate is very low for a private investor. It was seen as the risk-free internal rate of return for an extremely long-term investment. Heal et al. (1996) pointed out, that individuals use interest rates in the order of 2% if the time horizon extends to one hundred years. The rotation periods in Germany (100 to 110 years in our study) are actually often extremely long (Moog and Borchert, 2001) and cannot be explained with high interest rates. The simulated NPV data, however, do not consider the uncertainty of tree plantation investments by means of a risk-adjusted interest rate (e.g., Kruschwitz, 2005). Knoke and Wurm (2006) used a 2% risk-free interest to compute NPVs, which are positive if the tree plantation investments' internal rates of return are greater than 2%. Instead of increasing the interest rate, the uncertainty was addressed explicitly by analyzing the range of NPVs under uncertainty and by maximizing their worst-cases.

Knoke and Wurm (2006) carried out their investigation considering the average growth conditions in Southern Germany. Based on reservation prices they applied flexible harvest policies, where the timing of timber harvests depended on the simulated timber price. The biophysical data resulted from projections of pure stands by means of growth models. Thereby the data excluded effects accruing by interactions of tree species in mixed forests stands, which consist of groups of different tree species or single tree mixtures (see Knoke and Seifert, 2008 for a first approach to consider this aspect).

A greater biophysical yield, as compared to Beech, resulted for the conifer Spruce. Given the absence of damages (meaning that the biophysical risks such as wind damage, snow breakage or insect attacks are ignored) an average volume increment of 10.30 m^3 ha^{-1} yr^{-1} was simulated for Spruce, while Beech showed only 6.92 m^3 ha^{-1} yr^{-1}. The integration of biophysical (wind damage, snow breakage, insect attacks) and timber market risks led to an extreme dispersion of the simulated NPVs (Table 1). A forest comprising 100% Beech showed a NPV of 3,894 Euro ha^{-1} and a standard deviation of ±1,433 Euro ha^{-1}. Pure Spruce was more profitable (NPV of 5,028 Euro ha^{-1}), but also more risky (standard deviation of 2,599 Euro ha^{-1}). While a species composition of 75 % Beech and 25% Spruce had a NPV proportional to the shares of both species, its standard deviation was only 1,242 Euro ha^{-1}, which was even smaller than that of the pure Beech forest. Obviously the tree species mixture resulted in risk compensation, due to slightly correlated

Table 2
Combinations of biased data on means and standard deviations as well as correlations generated to lie around the parameters of the reference data sets.

Scenario no.	Spruce variant	NPV (Euro ha)		Beech variant	NPV (Euro ha)		Coefficient of correlation
		Mean	Standard deviation		Mean	Standard deviation	
1	Optimistic	6536	4150	Optimistic	5062	2634	− 0.3
2		6536	4150		5062	2634	0.0
3		6536	4150		5062	2634	0.3
4	Optimistic	6536	4150	Realistic	***3894***	***1433***	− 0.3
5		6536	4150		***3894***	***1433***	***0.0***
6		6536	4150		***3894***	***1433***	0.3
7	Optimistic	6536	4150	Pessimistic	2726	1000	− 0.3
8		6536	4150		2726	1000	0.0
9		6536	4150		2726	1000	0.3
10	Realistic	***5028***	***2599***	Optimistic	5062	2634	− 0.3
11		***5028***	***2599***		5062	2634	0.0
12		***5028***	***2599***		5062	2634	0.3
13	Realistic	***5028***	***2599***	Realistic	***3894***	***1433***	− 0.3
14		***5028***	***2599***		***3894***	***1433***	***0.0***
15		***5028***	***2599***		***3894***	***1433***	0.3
16	Realsistic	***5028***	***2599***	Pessimistic	2726	1000	− 0.3
17		***5028***	***2599***		2726	1000	0.0
18		***5028***	***2599***		2726	1000	0.3
19	Pessimistic	3520	1500	Optimistic	5062	2634	− 0.3
20		3520	1500		5062	2634	0.0
21		3520	1500		5062	2634	0.3
22	Pessimistic	3520	1500	Realistic	***3894***	***1433***	− 0.3
23		3520	1500		***3894***	***1433***	***0.0***
24		3520	1500		***3894***	***1433***	0.3
25	Pessimistic	3520	1500	Pessimistic	2726	1000	− 0.3
26		3520	1500		2726	1000	0.0
27		3520	1500		2726	1000	0.3

Financial parameters of the reference data sets are given in bold and italics.

biophysical risks and weak, negatively correlated timber prices. The coefficient of correlation between the NPV of Spruce and Beech was almost zero.

2.2. Scenarios with biased means and variances of NPV

The above described, simulated data sets were used as a reference to evaluate the performance of the classical and the extended optimization approach. To test the sensitivity of the shares of tree species to changes in the input parameter estimates we used 27 combinations of biased mean NPVs, standard deviations and correlations (Table 2).

To generate this data we assumed a bias of ±30% for the expected mean NPVs and estimated the appropriate standard deviations in relation to the expected mean NPVs. For this purpose the difference of the reference data standard deviations of NPVs of both tree species was divided by the difference of the average NPVs of both species. The resulting standard deviation per unit of difference in NPV was then used to derive an adequate standard deviation depending on the chosen biased NPV (see Table 2). With minimum standard deviations of ±1500 (Spruce) and ±1000 (Beech) we defined the lowest limits for both tree species, since the estimation of the standard deviations proportionally to the differences in NPVs resulted in excessively small coefficients of variation (lower than 10%) for small NPVs.

2.3. Variability of tree species shares and financial performance of portfolios

First we compared the sensitivity of the tree species shares to changes in the input parameters. Based on the biased data, tree species shares were computed for every scenario (see Table 2) and separately for both maximization models by maximizing worst-case NPVs with the classical approach and with the one extended by information-gaps, as the extended worst-case approach. We required a 0.95 level of statistical confidence ($1-\alpha$, with $\alpha=0.05$) and set the relative level of uncertainty, ω, for the mean NPV to 0.3 (to be used according to Eq. (2)) and the relative level of uncertainty, ε, for the variance was defined 0.6 (to be used according to Eq. (4)). The sensitivity of the optimization to changes in these uncertainty parameters will be tested later.

As expected from the results of other studies, the resulting share of Beech varied greatly among the 27 scenarios under classical worst-case maximization, which was based on biased NPV means and variances. Shares between 0.35 and 0.85 were computed when based on the (biased) data for the 27 scenarios (Fig. 1).

The average share of Beech was 0.62 with ±0.13 as the standard deviation. With the extended worst-case optimization the variability of the forest portfolios could significantly be reduced. The minimum share of Beech was 0.45 and the maximum 0.60. The average share of

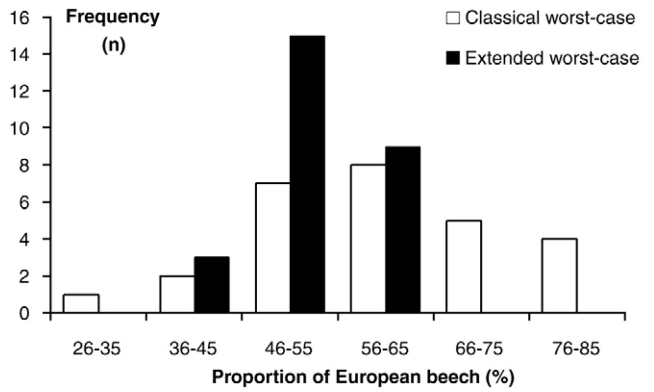

Fig. 1. Frequency of estimated optimum shares of Beech for 27 scenarios.

Table 3
Optimum shares of Beech and various forest NPVs obtained from optimization with biased data for a level of statistical confidence of $1-\alpha=0.95$.

Scenario no.	Classical worst-case			Extended worst-case		
	Proportion Beech	Worst-case read from reference data	Mean read from reference data	Proportion Beech	Worst-case read from reference data	Mean read from reference data
1	0.60	1934	4348	0.55	1942	4404
2	0.65	1858	4291	0.55	1942	4404
3	0.65	1858	4291	0.55	1942	4404
4	0.75	1567	4178	0.60	1934	4348
5	0.75	1567	4178	0.60	1934	4348
6	0.80	1424	4121	0.60	1934	4348
7	0.80	1424	4121	0.60	1934	4348
8	0.80	1424	4121	0.60	1934	4348
9	0.85	1323	4064	0.60	1934	4348
10	0.50	1926	4461	0.50	1926	4461
11	0.50	1926	4461	0.50	1926	4461
12	0.50	1926	4461	0.50	1926	4461
13	0.65	1858	4291	0.55	1942	4404
14	0.65	1858	4291	0.60	1934	4348
15	0.70	1689	4234	0.60	1934	4348
16	0.65	1858	4291	0.55	1942	4404
17	0.70	1689	4234	0.55	1942	4404
18	0.70	1689	4234	0.55	1942	4404
19	0.40	1693	4574	0.45	1830	4518
20	0.40	1693	4574	0.45	1830	4518
21	0.35	1512	4631	0.45	1830	4518
22	0.55	1942	4404	0.55	1942	4404
23	0.60	1934	4348	0.55	1942	4404
24	0.60	1934	4348	0.60	1934	4348
25	0.55	1942	4404	0.50	1926	4461
26	0.55	1942	4404	0.50	1926	4461
27	0.55	1942	4404	0.50	1926	4461
Mean	**0.62**	**1753**	**4325**	**0.54**	**1923**	**4411**
Standard-deviation	**0.13**	**200**	**148**	**0.05**	**34**	**57**

Beech resulted in 0.54 with a standard deviation of only ±0.05. Even under optimistic (Spruce) versus pessimistic (Beech) combinations (scenarios 7–9) and under pessimistic (Spruce) versus optimistic (Beech) combinations (scenarios 19–21) the share of Beech did not fluctuate greatly when optimized according to the extended worst-case method: the minimum value was 0.45, the maximum 0.60. For the case of classical worst-case optimization this was different. Here the share of Beech ranged between 0.35 and 0.85 for scenarios 7–9 and 19–21.

We can now use worst-case forest NPVs (for $\alpha=0.05$) from the reference data set (Table 1) for the regarding shares, which have resulted from optimizing with the biased data. Our simulation assumes that these values would have been the real outcome of a decision on species choice according to the biased data. A share of Beech of 0.35 would, for example, result in a worst-case NPV of 1512 Euro ha^{-1}, a Beech share of 0.85 gives 1323 Euro ha^{-1} (see Table 1). If we allocate reference data outcomes to all 27 choices of tree species guided by the biased data, the average worst-case forest value is 1753 (±200) Euro ha^{-1}, when classical worst-case maximization was applied, whereas the extended worst-case method achieves 1923 (±34) Euro ha^{-1} (Table 3).

Thus the average worst-case result increased by 10 % when the extended worst-case method was applied. The average forest NPV differed also in favor of the extended worst-case method and the standard deviation of the worst-case results was reduced by the factor of 6 when compared with that obtained from the classical worst-case method.

In contrast to the parameters mean and variance of the tree species' NPVs, the estimate for the correlation of NPVs seems to be rather less important, at least if we consider a range between −0.3 and +0.3. A variation within these limits led to a change in the species shares of maximally 10 percent points (for $1-\alpha=0.99$, see next

Table 4
Optimum shares of Beech and various forest NPVs obtained from optimization with biased data for a level of statistical confidence of $1-\alpha=0.99$.

Scenario no.	Classical worst-case			Extended worst-case		
	Proportion Beech	Worst-case read from reference data	Mean read from reference data	Proportion Beech	Worst-case read from reference data	Mean read from reference data
1	0.65	414	4291	0.60	513	4348
2	0.65	414	4291	0.60	513	4348
3	0.70	276	4234	0.60	513	4348
4	0.75	118	4178	0.65	414	4291
5	0.80	−282	4121	0.65	414	4291
6	0.85	−534	4064	0.65	414	4291
7	0.80	−282	4121	0.65	414	4291
8	0.85	−534	4064	0.65	414	4291
9	0.90	−724	4008	0.65	414	4291
10	0.50	574	4461	0.50	574	4461
11	0.50	574	4461	0.50	574	4461
12	0.50	574	4461	0.50	574	4461
13	0.65	414	4291	0.60	513	4348
14	0.70	276	4234	0.60	513	4348
15	0.75	118	4178	0.60	513	4348
16	0.70	276	4234	0.60	513	4348
17	0.75	118	4178	0.60	513	4348
18	0.80	−282	4121	0.60	513	4348
19	0.35	415	4631	0.45	546	4518
20	0.35	415	4631	0.40	610	4574
21	0.30	236	4688	0.40	610	4574
22	0.55	585	4404	0.55	585	4404
23	0.55	585	4404	0.55	585	4404
24	0.60	513	4348	0.55	585	4404
25	0.60	513	4348	0.55	585	4404
26	0.60	513	4348	0.55	585	4404
27	0.65	414	4291	0.55	585	4404
Mean	**0.64**	**211**	**4299**	**0.57**	**522**	**4383**
Standard-deviation	**0.16**	**390**	**178**	**0.07**	**68**	**82**

section and Table 4). For the case of the extended worst-case method the maximum change was only 5% points.

2.4. Demanding an increased statistical confidence for worst-case simulations

Here we modify the level of statistical confidence, which was originally set to $1-\alpha=0.95$. We can increase the cautiousness of the assumptions for the optimization when demanding a statistical confidence of $1-\alpha=0.99$, for which we carried out the following consideration. Under this assumption, the variability of optimum Beech shares slightly increased for the extended worst-case method. The optimum shares of Beech now ranged between 0.40 and 0.65 (Table 4), while previously they were 0.45 to 0.60 for $1-\alpha=0.95$.

However, the variation of optimum shares of Beech also increased for the case of classical worst-case modeling, where we obtained Beech shares ranging between 0.30 and 0.90 (0.35 to 0.85 for $1-\alpha=0.95$). Moreover, compared with the classical worst-case method, the worst-case NPV achieved under the extended method was at least as great or greater for every scenario (Table 4). On average the worst-case NPV from extended worst-case modeling was even 147% greater than that provided by classical worst-case modeling, while the achieved average NPV did not differ very much between both optimization methods.

2.5. The choice of the uncertainty parameters

The greater the selected uncertainty parameters ω and ε become, the more inclusive the uncertainty sets considered with Eq. (2) and (4) will be. To test the influence of the selected uncertainty horizon on the tree species composition, we selected the scenarios no. 9 and no. 21. Scenario no. 9 produced the greatest difference between the classical and the extended optimization. In terms of the estimated mean NPV it is optimistic for Spruce and pessimistic for Beech, while the standard deviation of Spruce is very high compared with that of Beech. The very small standard deviation of the Beech-NPVs caused a substantial proportion of this species (0.90 for $\alpha=0.01$) when the classical worst-case model was applied. In contrast, the share of Beech (0.65 for $\alpha=0.01$) remained comparatively stable under the extended worst-case model. Scenario no. 21 presented the opposite combination, which means that it was pessimistic for Spruce and optimistic for Beech in terms of the estimated mean NPV.

Variations of the uncertainty parameter ω, the classical robustness parameter maximized by means of the info-gap models proposed by Ben-Haim (2006), had no influence at all on the results of the optimization. If we only consider the possible uncertainty for the estimated mean NPV via an upper uncertainty boundary, we would thus obtain the same results as under the classical worst-case optimization. However, the variation of the uncertainty parameter ε, the parameter that accounted for a possible bias in the estimation of the variance of the NPVs, led to a considerable change of the optimum share of Beech (Fig. 2).

The nature of worst-case modeling implies that the tree species with the smallest standard deviation always covers greater shares in the resulting portfolio, given a specific NPV. Setting the uncertainty parameter ε equal to zero means that the classical-worst case results are obtained, which reflect the effect of the level of the tree species' standard deviations very strongly. For the case of scenario no. 9 Beech has by far the lowest standard deviation ($\pm$1000 Euro ha^{-1}) and obtains shares between 0.85 ($1-\alpha=0.95$) and 0.90 ($1-\alpha=0.99$). Analyzing scenario no. 21 gives the opposite tendency; here Spruce shows the lower standard deviation of both species ($\pm$1500 Euro ha^{-1}) and achieves shares between 0.65 ($1-\alpha=0.95$) and 0.70 ($1-\alpha=0.99$).

Fig. 2 tells us that the optimum share of Beech tends towards 0.50 with growing uncertainty in terms of ε. Extended worst case optimization basically tends to equal weighted compositions for an increasing uncertainty horizon. That is intuitively clear: if the uncertainty set for the estimated variances is very inclusive and there is almost no information available, we have no good reasons to prefer one tree species. However, with an uncertainty value of $\varepsilon=0.6$ we still get optimal shares for Beech somewhat different to 50%.

Nevertheless, we intend to make clear that great robustness is not necessarily realized through models that prefer more conservative solutions (in terms of equal proportions) for increasing uncertainty. Deciding on tree species will always be a subjective decision. Our modeling allows for considering differences in the degree of confidence to the estimated financial data. If we place trust in the estimated data for one tree species but not for the other, we can model

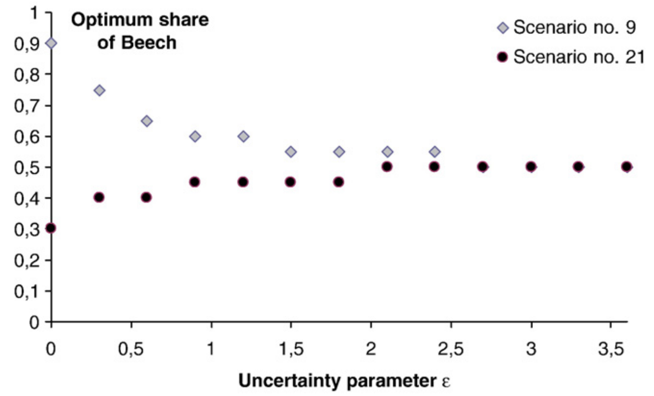

Fig. 2. Optimum shares of Beech for various levels of the uncertainty parameter ε derived from scenario no. 9 (mean NPV Spruce 6536 Euro ha$^{-1}\pm$4150; mean NPV Beech 2726 Euro ha$^{-1}\pm$1000; correlation coefficient +0.3, $\alpha=0.01$) and from scenario no. 21 (mean NPV Spruce 3520 Euro ha$^{-1}\pm$1500; mean NPV Beech 5062 Euro ha^{-1} $\pm$2634; correlation coefficient +0.3, $\alpha=0.01$).

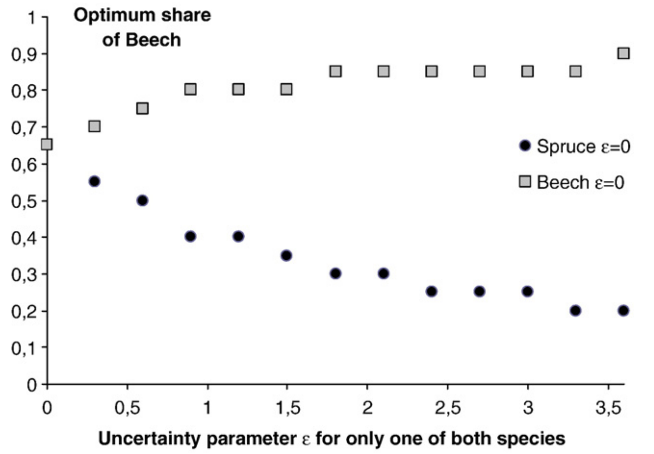

Fig. 3. Optimum shares of Beech for various levels of the uncertainty parameter ε, when the parameter ε is set to zero either for Spruce or for Beech (we applied data from scenario no. 14 mean NPV Spruce 5028 Euro $ha^{-1} \pm 2599$; mean NPV Beech 3894 Euro $ha^{-1} \pm 1433$; correlation coefficient 0, $\alpha = 0.05$).

the resulting info-gaps individually (Fig. 3). When accepting the variance of Spruce as it stands ($\varepsilon = 0$) but simultaneously being skeptical about the variance of Beech, we can increase the uncertainty parameter, ε, only for Beech. For this case, the share of Beech decreases from 0.65 ($\varepsilon = 0$ for Beech) to 0.20 ($\varepsilon = 3.6$ for Beech, scenario no. 14 was used for this example). If we do the same for Spruce, while setting ε equal to zero for Beech, the share of Beech increases from 0.65 ($\varepsilon = 0$ for Spruce) to 0.90 ($\varepsilon = 3.6$ for Spruce). It is thus clear that using info-gaps does not necessarily tend towards equal proportions.

3. Discussion

Our paper shows the possibility of stabilizing the results from optimizing tree species portfolios by means of integrating an upper bound for the variance estimation, which can be seen as the upper bound of an uncertainty set that possibly contains the true but unknown value of the portfolio variance. Depending on the level of the required statistical confidence (we tested for $1 - \alpha = 0.95$ and $1 - \alpha = 0.99$), an extended worst-case model achieved a 10 % or even a 147 % better worst-case performance. The average performance portfolios optimized with the extended model did not suffer when compared to the results of the classical worst-case model.

Considering an uncertainty set for the NPV variances is not common in financial modeling, but it helps to reduce the negative consequences of optimization which may result from possibly biased financial input data. From the field of statistics we know that we may be severely wrong when fitting regression curves too detailed to statistical observations. The random noise inherent in statistical data, which has no explanatory meaning, may then be generalized and we will be disappointed when applying the obtained regression curves to independent data. In financial modeling this is similar: we don't know whether we extrapolate relevant or irrelevant information from our simulated data into the future and we should thus consider that our data quality is probably very poor. To be on the safe side: it is rather likely that our data sets will not cover the possible future range of outcomes. If we are conscious of that fact, we can try to improve the data quality and our insight into their transferability. Nevertheless, there are limits of e.g. financial burden connected with data acquisition or accuracy of measurements and even in the case of perfect statistical efforts it remains a description of the past. Future changes remain undetected by this means and complications can arise in particular for long term optimization problems. According to the principle of precaution we can assume negative developments and use a robustness criterion as a quality parameter, which evaluates the resilience of possible solutions. The required uncertainty set, which can be interpreted as an information gap or a dilution of data, can be estimated by different ways.

Our study was partly inspired by the paper of Goldfarb and Iyengar (2003). In contrast to our approach, these authors based their consideration on a factor model for asset returns. With parameters assumed to be estimated by classical linear regression from historical market data, the factorial model predicts the deviation of asset returns from a vector of mean returns. The predicted deviation depends on the covariance matrix of the factor returns and the factor loading matrix. In addition to the mentioned components, Goldfarb and Iyengar (2003) included a residual error vector and defined uncertainty sets for the mean return vector, the factor loading matrix, the covariance matrix of factor returns and the residual error vector. A factorial model of the capital market as used in the Goldfarb and Iyengar (2003) paper finds — at least in part — its theoretical background in the "Capital Asset Pricing Model", CAPM (see Sharpe, 1964). The factorial approach was developed by Ross (1976). To derive the parameters for every factor of the factorial model the definition of a market portfolio is required, which should represent all possible risky investments. This seems impossible for a very long investment period, as it would be required for forest plantation investments. This market factorial model approach thus can hardly be used to evaluate forest plantation investments. We therefore decided to use a simpler model, which considered only the mean forest plantation NPVs derived from the vector of expected NPVs for the considered tree species and its covariance matrix. These were then corrected by means of additional variance components to consider a possible upper bound of the unknown true variance. We can compare our additional variance components with the residual error vector in the factorial model of Goldfarb and Iyengar (2003), which the authors did not estimate from their data. Analogous to the study of the mentioned authors we estimated the additional variance components by means of a diagonal matrix. However, the elements of our diagonal matrix were all the same, represented by a Frobenius norm, while Goldfarb and Iyengar (2003) estimated them proportionally to the covariance matrix of their linear factorial model. Tests of our model, with the additional variance components also being estimated proportionally to the expected covariance matrix of NPVs, have led however to rather poor results in our case: the variability of the tree species shares rather increased, when compared among the various tested scenarios, and the worst-case performance decreased. For a robust estimation of optimal tree species portfolios it thus appears essential to estimate additional variance components independent from the expected, likely biased NPV-variances of the tree species. If we use information from the expected values, we will indeed not make the optimization more stable, rather we would magnify the sensitivity of the optimization to changes in parameter estimates.

Our paper can also make a contribution to the controversial discussion on the value and usefulness of info-gap modeling (see e.g. Sniedovich, 2007). It became evident that considering a classical info-gap model, formulated to integrate a lower bound for the mean NPV did not change the results, when compared to the classical worst-case optimization. Rather the possible uncertainty in the variance estimation appears to be crucial. We thus conclude that considering info-gaps on variance estimation could certainly improve robust decision-making under severe uncertainty.

Robust optimization was already investigated by other authors (e.g., Bai et al., 1997; Ben-Tal and Nemirovski, 1998; Goldfarb and Iyengar, 2003; Sniedovich, 2007). These studies were based mainly on theoretical considerations (Ben-Tal and Nemirovski, 1998) or on theoretical considerations combined with artificial data (Goldfarb and Iyengar, 2003). However, in the case of ecosystem management, we hardly found any corresponding studies. In the field of forest science Knoke (2008) tested the IGT to increase the robustness of a tree species portfolio against adverse uncertainty. The tested approach was adopted from Ben-Haim (2005). In contrast to our present paper, a value-at-risk concept was used which aimed at maximizing the tolerance of an estimated failure probability to errors in the data set. Knoke (2008), who did not address the problem of errors in variance estimation, confirmed that the IGT approach achieved the same results as a minimization of the failure probability via a lower partial moment approach (LPM_0, see e.g. Lee and Rao, 1988).

Our paper offers a test of robust worst-case optimization and performance evaluation by means of simulated data. The simulated reference data set considered historical survival probabilities, timber price fluctuations and the resulting correlations of NPVs for the investigated tree species. This is a differentiation from the study of Goldfarb and Iyengar (2003), who, as already mentioned, tested their optimization algorithms by means of a completely artificial data set. A test with independent reference data sets and biased data sets was not carried out by Goldfarb and Iyengar (2003). For the field of ecosystem management our approach seems thus quite new.

It is clear that the disciplines of forest science and ecological ecosystem management in general, are just at the beginning of robust optimization under severe future uncertainty. However, if we rely on the classical IGT approach and ignore information gaps for the variances, we should be aware that we will probably not obtain new results from this type of optimization. In principle, the similarity between the results from IGT and those from classical optimization approaches was also already mentioned by Halpern et al. (2006).

We limited our investigation to a situation of mixing pure stands of two tree species and thus excluded the consequences of interactions between tree species. Future studies must combine bioeconomic modeling of more intimately mixed forest stands that contain interacting tree species with robust optimization techniques, to obtain recommendations for ecosystem management. First results show that stands of Spruce and Beech greatly benefit from more resistant Spruces, when the tree species are mixed in small groups at the stand level (Knoke and Seifert, 2008). Despite adverse effects from a reduced timber quality, the financial parameters NPV and standard deviation greatly benefit from gains in resistance of the conifer Spruce. This type of mixture even allows for comparatively great shares of Spruce without inflating risks too much.

The process of global warming will further intensify the problems in long-term ecosystem management planning. From this perspective the extended worst-case optimization seems promising. Ecosystem managers traditionally have to fulfill the claim for a careful management (precautionary approach) in order to secure sustainability, a concept as old as about 300 years in forestry (see von Carlowitz, 1713 and later Hartig, 1800 for a scientific consideration). As far as the financial core of sustainability is concerned, we may well address this sustainability concept by uncertainty adjusted, robust worst-case optimization.

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