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Risk Premia on Credit and Equity Markets

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Abstract

Risk premia influence asset prices on both equity and credit markets. Most research on risk premia has so far looked at either equity or credit markets separately. However, these two markets are not separated: Both markets offer claims on the same underlying (i.e. companies' assets) and most investors have access to both markets. Therefore, risk premia on equity markets can be compared to risk premia on debt markets and vice versa. We use this comparability idea to address certain questions concerning risk premia on both equity and credit markets.

We start by analyzing credit spreads on credit markets. Practitioners frequently price credit instruments using real-world quantities (PD, EL) and adding a (credit) risk premium. We analyze these credit risk premia within structural models of default based on calibrations from historical equity risk premia. We first analyze a Merton framework and find that i) credit risk premia constitute a significant part of model-implied spreads and ii) this part increases with increasing credit quality. In addition, credit risk premia are hardly affected by moving to more advanced structural models of default.

We use these observations to propose a new approach for estimating the equity premium using CDS spreads and structural models of default. Although the equity premium is – both from a conceptual and empirical perspective – a widely researched topic in finance, there is still no consensus in the academic literature on its magnitude. Based on a Merton model, a simple estimator for the market Sharpe ratio and the equity premium can be derived. This estimator has several advantages: First, it offers a new line of thought for estimating the equity premium which is not directly linked to current methods. Second, it is only based on observable parameters. We neither have to calibrate dividend or earnings growth, which is usually necessary in dividend/earnings discount models, nor do we have to calibrate asset values or default barriers, which is usually necessary in traditional applications of structural models. Third, our estimator is robust with respect to model changes.

We apply our estimator to more than 150,000 CDS spreads from the U.S., Europe, and Asia from 2003-2007. Our estimates yield equity premia of 6.50% for the U.S., 5.44% for Europe, and 6.21% for Asia based on 5-year CDS spreads. Due to some conservative assumptions these estimates are upper limits for the equity premium. Using 3-, 7-, and 10-year CDS spreads yields similar results and offers an opportunity to estimate the term structure of risk premia.

Besides the magnitude of the equity premium, the time series behavior of risk premia is another important issue in finance. We use the estimator described above to calibrate the term structure of risk premia before and during the 2007/2008 financial crisis. We find that the risk premium term structure was flat before the crisis and downward sloping during the crisis. The instantaneous risk premium increased significantly during the crisis, whereas the long-run mean of the risk premium process was of the same magnitude before and during the crisis.

These results convey the idea that (marginal) investors have become more risk averse during the crisis. Investors were, however, well aware that risk premia will revert to normal levels again. As a result, short-term risk premia increased more than long-term risk premia. The slope of the risk premium term structure (measured as 10-year Sharpe ratio minus 3-year Sharpe ratio) was approximately zero before the 2007/2008 financial crisis and became negative during the 2007/2008 financial crisis. Based on theoretical arguments one would also expect this slope to be a factor in asset pricing, although our short sample period does not allow for a direct validation.

Both applications – estimating equity premia and calibrating the risk premium term structure – benefit from the same underlying reason: Risk premia can be more easily measured on credit than on equity markets. It is easier to estimate the necessary input factors on credit markets than to estimate the necessary input factors on equity markets. In addition, distinct maturities are available. Therefore, we think that our approach is not limited to the applications developed in this thesis but also offers a basis for analyzing further research questions conerning risk premia on financial markets.

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List of Abbreviations

Abbreviation	*Definition*
APT	Arbitrage pricing theory
BIS	Bank for International Settlement
bp	Basis points
CAPM	Capital Asset Pricing Model
CBOE	Chicago Board Option Exchange
CDS	Credit default swap
CIR	Cox-Ingersoll-Ross
CMA	Credit Market Association
DCF	Discounted cash flow
DDM	Dividend discount model
EDF	Expected default frequency
EDM	Earnings discount model
e.g.	Exempli gratia
EIR	Equity-implied rating
EL	Expected loss
EP	Equity premium
et al.	Et alii
etc.	Et cetera
GMM	Generalized methods of moments
i.e.	Id est
ISDA	International Swaps and Derivatives Association
LGD	Loss given default
Log	Logarithm

Abbreviation	*Definition*
OTC	Over-the-counter
OUP	Ornstein Uhlenbeck process
PD	Probability of default
RIM	Residual income model
RoE	Return on equity
RR	Recovery rate
SR	Sharpe ratio

List of Symbols

Symbol	*Definition*
B_t	Brownian motion
CF_t	Cash flow
D_t	Dividends
δ	Payout ratio
E_t	Earnings
k	Discount factor
κ	Mean reversion speed
L	Default barrier
μ	Real-world asset value drift
$\bar{\mu}$	Long-run mean of equity premium
N	Nominal value of debt
PD^P	Actual/Real-world default probability
PD^Q	Risk-neutral default probability
Φ	Normal distribution
r, r_f	Risk-free rate
ρ	Correlation
RI_t	Residual income
s	CDS spread
σ	Volatility
τ, T	Maturity
θ	Sharpe ratio
$\bar{\theta}$	Long-run mean of Sharpe ratio
V_t	Asset value

1. Introduction

1.1. Motivation

Based on common sense and empirical observations, most agents are risk averse, i.e., if they have the choice between a riskless and a risky investment offering the same expected return they prefer the riskless investment.[1] Therefore risky assets – with risk that cannot be totally diversified – are expected to offer an (expected) excess return above the return on riskless assets.[2] Risk premia are a compensation for this non-diversifiable risk. Throughout this thesis, we will use the term "risk premium" as the expected excess return of a risky asset above the risk-free rate. The terms equity (risk) premium and credit risk premium are used for risk premia on equity and credit markets. The equity premium is certainly the single most cited risk premium. It is defined as the difference between the (expected) return on a market portfolio of equities (e.g. S&P 500, DAX) and the risk-free rate.

Risk premia are of central importance for several areas of modern finance. First, risk premia play a major role in asset allocation decisions. They determine the average return an investor can expect to earn on a risky portfolio. Risk premia – in addition to individual preferences – should therefore influence the allocation of funds to riskless and risky assets.[3] E.g., if equities earn an average return of 20% above the risk-free rate, investors should be more inclined to invest in equities than if they only earn an average excess return of 1%. Thus, risk premia should reflect the risk aversion of the marginal

[1] Cf. Sharpe (1965), Friend/Blume (1975), Pindick (1988), and Paun (2008).
[2] Cf. section 2.2.3 for a literature review of excess returns on equities and section 2.3.3 for a review of risk premia on credit instruments.
[3] Cf. Tobin (1958) and Markowitz (1952).

investor: Risky assets have to earn a return where demand and supply of risky assets are balanced.

Second, from a valuation perspective, risk premia determine the correct discount rate for the respective cash flows. Given a certain set of uncertain cash flows subject to systematic risk, a higher risk premium results in a higher discount rate and therefore in a lower asset or company value.[4] Unfortunately, these risk premia are not directly observable which poses some challenges in practical applications. E.g., during the 2007/2008 financial crisis, accountants were faced with a big problem when valuing companies: When using historical risk premia they were not able to come up with model-based DCF values close to the market values – even with very conservative cash flow projections. They concluded that risk aversion and risk premia must have increased during the turmoil – although a reliable estimate was hard to determine.[5]

Third, the magnitude and behavior of risk premia is of large academic interest because it helps to understand the way financial markets work and market participants behave. In particular, the behavior of the equity premium has been subject to intensive debate.[6] Research has shown that dividend/price ratios have some ability to predict future equity returns over longer horizons, giving rise to academic research on return predictability, time variation, and mean reversion in the equity premium.[7] Of course, the results of this fundamental research also have implications on the applications discussed above. The importance of risk premia is certainly not limited to the examples mentioned so far. It is, however, not our target to describe all areas where risk premia play an important role. Our concern is rather to point out that risk premia are at the heart of finance and indispensible for

[4] This is based on the net present value rule, cf. Brealey et al. (2008) for an overview. The most prominent concepts for the determination of discount rates are the Capital Asset Pricing Model (cf. Sharpe (1964), Mossin (1966) and Lintner (1965)), the arbitrage pricing theory (cf. Ross (1976)) and the Fama/French model (cf. Fama/French (1993, 1996)).

[5] These observations are based on informal discussions with accountants at major accounting firms.

[6] Cf. section 2.2.3.2 and Cochrane (2005) for on overview.

[7] Cf. Fama/French (1988), Cochrane (2005), and Campbell/Viceira (1999) for an application to asset allocation decisions.

explaining asset prices and asset returns, and for managing financial assets.

Some people might argue that risk premia are not necessary for some of the applications described above due to risk-neutral pricing rules.[8] Why should we care about risk premia when we are equipped with a rich theory that expresses prices as risk-neutral expected payoffs discounted by the risk-free rate? This risk-neutral valuation has become popular in many areas of modern finance, especially for the pricing of derivative claims. At first glance, the Black/Scholes (1974) option pricing formula seems to be independent from any risk premium. Bond and credit default swap prices can also be determined using risk-neutral default probabilities.[9] If markets are complete and in the absence of arbitrage, all claims can be priced using a unique risk-neutral probability measure.[10] However, there are fundamental reasons why risk premia are indeed important. First, the risk-neutral measure Q has to be calibrated. The prices of the assets used to calibrate Q have to be found by other models.[11] Risk-neutral pricing rules are therefore an appropriate tool to price derivative claims such as options on stocks, but it is usually hard to use it for pricing the underlyings of these derivative claims. In addition, it is interesting to learn something about the factors driving the value of assets and not simply to determine the value itself. Risk premia affect asset values and this effect can only be analyzed by looking at real-world quantities.

As mentioned earlier, the single most cited risk premium is certainly the equity premium. Although it is of major importance for many purposes, there is still no consensus on the correct magnitude of the equity

[8] Cf. Elton et al. (2001) for a similar discussion.

[9] Cf. Jarrow/Turnbull (1995) for a rigorous analysis.

[10] This is the essential result of Harrison/Pliska (1981) and Harrison/Kreps(1979). Technical conditions apply.

[11] Frequently, liquid instruments are used to calibrate Q (e.g. equities). Then other instruments – such as options – can be priced based on this calibration. Mathematically, the risk-neutral probability measure can also be derived from the real-world probability measure via Girsanov's Theorem. In this transformation, risk premia play a role.

premium. There are mainly four different approaches for measuring the equity premium (or any other risk premium):[12] First, it can be estimated from historical averages, thereby assuming that history is a good proxy for the present and future situation. Second, implied equity risk premia can be extracted from market prices by implicitly solving valuation equations under the real-world measure. Third, theoretical approaches based on utility functions and corresponding risk aversion parameters can be used. Fourth, expert estimates can be used – although this is not very satisfactory from an academic point of view.

Fortunately, markets are not totally separated from each other, so risk premia on different markets can be compared. This has two reasons: First, it can be reasonably assumed that investors on different markets are not too different from each other. As an extreme example, it seems implausible that investors in Germany demand an equity premium of 50%, while investors in France demand an equity premium of 0.1%. Second, within globally integrated financial markets there is competition between markets and investors. In the example above, French investors would simply invest in Germany, which would raise asset prices in Germany and decrease asset prices in France until risk premia were comparable. Academic research about equity premia in different countries – which on average resulted in equity premia below the U.S. estimates – is one of the reasons why historical equity premia in the U.S. are now seen as upward biased.[13]

A comparision of risk premia is possible not only between equity markets in different countries, but also between different asset classes. This is the basic idea of this thesis. We focus on the comparison of risk premia on credit and equity markets. In general, bonds are less risky than equities. As a result, the average risk premium on bonds should be lower than the

[12] Cf. section 2.2 for a detailed literature overview for all four approaches.

[13] Cf. Dimson et al. (2003, 2006) for historical equity premia of 17 countries. Of course, a one-on-one comparison of equity premia is not possible since these may differ due to different volatilities, correlations with the global market portfolio, and market frictions such as taxes and transaction costs.

equity risk premium. Although this is of course a very vague relation, it already helps to reject some of the results from classical utility theory: Mehra/Prescott (1985) determined an equity premium of less than 0.35% using standard risk aversion parameters and standard utility theory. Bond risk premia are above 0.35%, therefore equity premia of less than 0.35% do not appear reasonable.[14] Within this thesis, we will model the relationship between credit risk premia and equity premia in more detail, i.e., we will derive formulas such as

$$\text{Equity Premium} = \text{f}\,(\,\text{Credit Risk Premium, Other Parameters}\,) \tag{1.1}$$

This formula can be used in (at least) two ways: First, to derive credit risk premia from equity premia. Second, to derive equity premia from credit risk premia. We will use the second approach for our empirical application and derive the equity premium from credit default swap prices. In addition, we will try to answer some fundamental issues about the behavior of the equity premium and risk premia in general.

This choice of application is not arbitrary, but has a deeper reasoning: We think that credit markets are better suited to measure current, implied risk aversion than equity markets. Why is this the case? It is easier to estimate the necessary input factors on credit markets than to estimate the necessary input factors on equity markets. This is due to both a rich academic and practitioner's literature on the estimation of real-world cash flows on credit instruments[15] and a limited time horizon for which estimates have to be made. To measure implied risk premia, expected cash flows have to be equated to current market prices. To estimate cash flows for equities, earnings estimates are necessary. For bonds or credit default swaps, estimates of the default probability (PD) and recovery rate (RR) need to be available. For credit instruments, generally accepted estimates for PD and RR exist, these estimates are based on (partially) objective criteria and only need to

[14] Cf. Hull et al. (2005) for a good overview of the magnitude of bond risk premia. A detailed literature overview can be found in section 2.3.3.

[15] Cf. section 2.3.2 for a literature review.

be estimated for a limited time horizon, usually less than 10 years.[16] In contrast, earnings forecasts from analysts – which are most frequently used as proxies for expected earnings[17] – are subjective and have to be available for an infinite time horizon to determine implicit discount rates.[18]

However, estimating implied risk premia from credit markets also has some disadvantages. With illiquid bond markets, the determination of an accurate market price used to be a major problem until about a decade ago. This has changed with the rise of credit default swap (CDS) markets that offer a purer measure of credit risk than bonds and are standardized with respect to maturities, seniority, and other features. Although CDS markets are OTC markets, they have become the benchmark for credit risk – especially since the introduction of CDS indices in the U.S., Europe, and Asia in 2003/2004.[19]

The derivation of equity premia from credit risk premia via (1.1) induces additional model risk. The function f has to be specified and it may depend on the certain model setup and on other parameters. In this thesis, we will use structural models of default to link risk premia on equity markets to risk premia on credit markets.[20] The academic literature has developed a variety of structural models with different assumptions concerning the default mechanism and the underlying processes. It is a well-established fact from academic research that these different models yield very different results when applied in practice (cf. section 2.3.1.3 for details). At this point, one of the main contributions of this thesis will be to demonstrate that for our specific application the difference between the main structural models

[16] An overview of methodologies for estimating PD and RR can be found in section 2.3.2.

[17] Cf. Claus/Thomas (2001) and Gebhardt et al. (2001) for example.

[18] A short overview of analyst forecasts can be found in section 2.2.2.

[19] Cf. Jakola (2006) for an overview of the growth and importance of the CDS market and FitchRatings (2006) for a case study of bond vs. CDS liquidity.

[20] Structural models seem to be the first choice to link valuation from equity and credit markets since they model both equity and debt as a function of the same underlying process (asset value process). However, other – more ad-hoc – models may also be applied, cf. section 2.4 for an overview.

of default is quite small. While model risk is a major problem for standard applications, it is not – or only to a much smaller extent – a problem for our application.

Finally, besides offering an easier estimation of input factors, estimating equity risk premia from credit risk premia allows for further applications. There is a large advantage of CDS if risk premia are supposed to be estimated for a certain time horizon. E.g., assume we are faced with a five-year strategic asset allocation decision.[21] In this case, we are interested in expected risk premia over the next five years. CDS offer distinct maturities which can be used to determine five-year CDS-implied equity risk premia via (1.1). In contrast, the duration of equities is difficult to determine accurately and usually exceeds 20 years.[22] Estimating implied equity risk premia from CDS separately for each maturity allows constructing a term structure of risk premia. Tools from the interest rate literature can then be applied to estimate mean reversion, volatility, and long-run means.[23]

In summary, credit markets have two main advantages for estimating risk premia: First, real-world expectations of market participants can be measured more accurately than on equity markets. Second, credit instruments such as bonds and CDS have fixed maturities – which allows estimating risk premia for distinct time horizons. If one is interested in the equity risk premium, the main challenge is the transformation of risk premia from credit markets to equivalent risk premia on equity markets. This requires certain model assumptions which have to be thoroughly analyzed.

[21] In contrast to strategic asset allocation decisions, tactical asset allocation decisions also try to exploit differences in short-term risk premia on different markets. As we assume that markets are in equilibrium, we focus on strategic asset allocation in this setting.

[22] In a Gordon constant-growth model, the duration of equity is the reciprocal of the dividend yield. A 5% dividend yield – which is rather high for ordinary stocks – corresponds with a duration of 20 years in this model. Lower dividend yields result in larger durations.

[23] These are mainly the tools from affine term-structure models (pricing formulas, Kalman filter methodology), cf. Bolder (2001) for a good summary.

1.2. An introductory example

As discussed in the last section, the main idea of this thesis is the comparability of risk premia between equity and credit markets. We will formally derive the functional relationship between equity and credit risk premia in chapter 3 for several standard structural models of default. In this section, we want to demonstrate the idea with a simple binomial tree model. The focus of this introductory example is neither to derive a specific formula or relationship nor to get any reasonable results concerning the magnitude of these risk premia. Its aim is simply to demonstrate the mechanics and to give an intuition why credit markets may be more useful for deriving risk premia than equity markets are.

What do we need to estimate risk premia? Risk premia are the expected excess returns on financial assets. Therefore, we need i) an estimate of expected future cash flows, ii) the current price of the asset, and iii) the risk-free rate. i) and ii) allow for an estimation of expected returns and – substracting the risk-free rate – expected exess returns. Mathematically, we need the real and risk-neutral probability measures. The "difference" is the risk premium.[24]

Let us start by looking at a situation as depicted in figure 1.1. We operate in a single-period, binomial model with one single firm X for which both bonds (zero bonds with face value \$100) and equity are traded and we assume a risk-free rate of 0%.[25] The state "down" represents default of firm X, the state "up" represents survival of firm X. Equity of company X is traded at \$40. In case of default, equity holders receive nothing, therefore the payoff

[24] The formal link between the real-world probability measure, the risk-neutral probability measure, and risk premia is established by Girsanov's Theorem. In addition, no-arbitrage, completeness, and technical conditions have to be met to guarantee existence and uniqueness. This introductory example tries to motivate the link and will therefore contain a more informal discussion of risk premia.

[25] The example could easily be extended to a non-zero interest rate or to coupon-bearing bonds. The assumptions were made for reasons of simplicity for the following calculations.

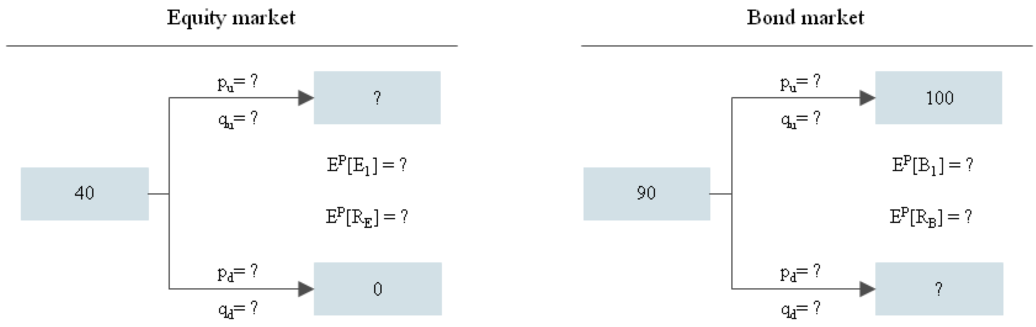

Figure 1.1.: Introductory example: Bond and equity market setup. p/q denote real-world/risk-neutral probabilities. Subscript u and d denote states "up" and "down". $E^P[E_1]/E^P[B_1]$ denote expected cash flows and $E^P[R_E]/E^P[R_B]$ expected returns of equities and bonds.

to equity holders in state "down" is \$0. The payoff to bond holders in state "up" is the face value of the bond – assumed to be \$100 – since there is no default in this state. The price of the bond is \$90 which can be observed from the bond market. All information depicted in figure 1.1 can be gained by i) observing market prices on bond and equity markets and ii) assuming a strict-priority rule in case of default. No forecasts, expert estimates or judgements for either bond or equity markets are necessary up to this point.

In the next step, we add two additional pieces of information to our binomial tree model, which require some kind of estimation procedure: The (real-world) default probability of the bond is assumed to be 1% and the (real-world) recovery rate of the bond is assumed to be 50% of the nominal value, i.e. \$50 (cf. figure 1.2). This information could be gathered from ratings of either the main rating agencies or from corresponding academic research.

With this information, we are now able to calibrate the risk-neutral probabilities for state "up" and "down" by equating the expected risk-neutral payoff to the current bond price.[26] The resulting risk-neutral probabilities

[26] As mentioned above, we have assumed a zero risk-free rate.

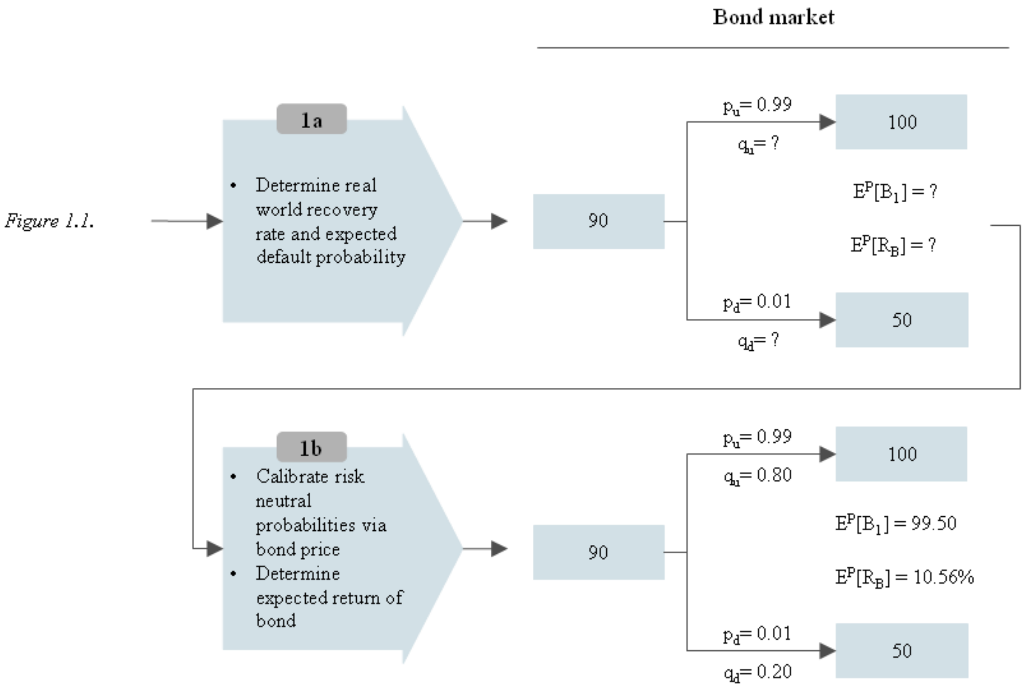

Figure 1.2.: Introductory example: Calibration of real-world probability measure (P) and risk-neutral probability measure (Q) via the bond market. For details cf. figure 1.1.

are $q_u = 0.80$ and $q_d = 0.20$.[27] The expected return of the bond is 10.56%.[28] This is also the excess return of the bond as the risk-free rate is 0%.

The risk-neutral probabilities enable us to derive the payoff to the equity holders in state "up" ($50), cf. figure 1.3.[29] Together with the real-world probabilities, we are able to determine the real-world expected payoff to the equity holders ($49.50) and the expected yield for the equity holders (23.75%).[30] Since the risk-free rate is 0%, the excess return above the risk-

[27] These are derived via $(1 - q_d) \cdot \$100 + q_d \cdot \$50 \stackrel{!}{=} 90$ and $q_u = 1 - q_d$.

[28] The expected real-world payoff is $(1-p_d) \cdot \$100 + p_d \cdot \$50 = 99\% \cdot \$100 + 1\% \cdot \$50 = \$99.50$. Therefore, the expected return is $\$99.50/\$90 - 1 = 10.56\%$. Please note that this is different from the yield of the bond, which is the *promised* return of the bond.

[29] $80\% \cdot x + 20\% \cdot 0 \stackrel{!}{=} 40$ yields $x = 50$.

[30] The expected payoff is calculated as $99\% \cdot \$50 + 1\% \cdot \$0 = \$49.50$, the expected return

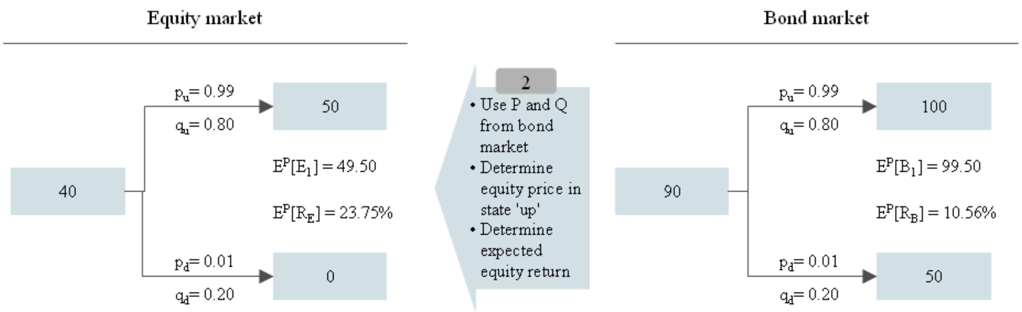

Figure 1.3.: Introductory example: Derivation of equity risk premium via bond market information. For details cf. figure 1.1.

free rate to equity holders is also 23.75%.

We are therefore able to determine excess expected equity returns via the bond market with the following assumptions:

1. No-arbitrage: Equity and bond markets are integrated, in particular there is no arbitrage.
2. Observable market prices: Market prices from credit and equity markets are observable.
3. Model setup: Binomial model and strict priority rule.
4. Calibration: Estimates for the default probability and recovery rate are available.

No-arbitrage: This is a standard assumption in financial modeling.[31] The same procedure as described above could of course also be used to find possible arbitrage opportunities.

Observable market prices: As discussed in the last section, reliability of prices for credit instruments is a much lesser concern today than it used

as \$49.50/\$40 = 23.75%.

[31] Cf. Duffie (1996), Cochrane (2005), and LeRoy/Werner (2006) for an in-depth discussion.

to be a few years ago, especially with the introduction of CDS indices in 2003/2004.

Model setup: The procedure outlined above has to make assumptions about the link between equity and debt cash flows. A binomial model combined with a strict priority of payments is assumed. In more advanced models, other assumptions will be necessary (e.g. first-passage-style default mechanism vs. zero-bond-style default mechanism). A key concern of the applications developed in this thesis will be the robustness with respect to model changes.

Calibration: We have assumed availability of real-world default probability and recovery rate estimates. This is a very crucial point. We could easily determine risk premia on equities and bonds in our example if we had assumed instead i) availability of estimates for the probabilities for the "up" or "down" state from equity markets and ii) the payoff in the "up-state" for equities. As discussed in the last section, estimating expected cash flows from credit markets is much easier than estimating cash flows on equity markets due to partially objective models and the limited time horizon for which estimates have to be made.

This introductory example has demonstrated the basic mechanism and assumptions for linking risk premia on credit markets to risk premia on equity markets. We will now formulate the explicit research questions for this thesis.

1.3. Research questions and contribution

This thesis aims to compare risk premia on equity and credit markets. Based on theoretical arguments, we argue that credit markets have some advantages in measuring risk premia and risk aversion of market participants.[32]

[32]Unfortunately, it is not possible to conduct an empirical test to compare the performance of our equity premium estimates with other equity premium estimates. This is due to our short sample period (2003-2008) – which is limited by the availability of reliable credit default swap data. In general, any backtesting of aggregate expected equity return estimates is problematical due to the large standard deviation of returns

We explore the theoretical link between equity premia and credit risk premia, use this link to estimate the equity premium from CDS spreads and answer an open issue from the asset pricing literature (time variation and mean reversion of risk premia). In particular, this thesis addresses the following research questions:

1. How can the link between risk premia on equity and credit markets be modeled within standard structural models of default? What are the properties of this relationship?
   a) Derivation and analysis within a standard Merton framework, in particular i) derivation of formulas to link credit risk premia to the equity premium, asset, and market Sharpe ratio, ii) analysis of the relative importance of asset/market Sharpe ratio assumptions for credit risk premia, and iii) analysis of the properties of the relationship (sensitivities, concavity/convexity, limit behavior).
   b) Derivation and analysis within more advanced structural models of default, in particular i) robustness of formulas derived in the Merton framework with respect to model changes and ii) analysis of influence of additional parameters and features of these advanced structural models.[33]
2. How can this link be used to estimate the equity premium and market Sharpe ratio from CDS spreads (CDS-implied equity premium, CDS-implied market Sharpe ratio)?
   a) Implementability: Derivation of an implementable formula based on observable market parameters. Discussion of data availability and implementation issues.

compared to the mean returns, cf. Poterba/Summers (1988).

[33] We will focus on the Duffie/Lando (2001) framework, which includes unobservable asset values and was the first structural model of default consistent with reduced-form pricing. This allows us to integrate the two main types of credit risk models (structural, reduced-form). A special case of the Duffie/Lando model is the Leland/Toft (1996) model (for zero asset value uncertainty). Due to our specific design, we will also capture all Brownian motion type structural models of default with a fixed default barrier, which is usually the case.

b) Estimation of equity premia from CDS spreads and proxies for real-world default probabilities for the U.S., Europe, and Asia before the 2007/2008 financial crisis (2003/2004 – 2007) and comparison with other equity premium estimates.

c) Analysis of the robustness of our estimator with respect to model changes and noise in the estimation of input parameters.

3. Can the link between equity premia/Sharpe ratios and credit risk premia be used to shed light on the existence of time variation and mean reversion of risk premia?

   a) Estimation of the term structure of market Sharpe ratios for the U.S. and Europe before and during the 2007/2008 financial crisis based on different CDS maturities.

   b) Analysis of time variation in CDS-implied market Sharpe ratios. This is done via an application of standard methodologies from the interest rate literature (in particular Kalman filtering) to determine the parameters of the instantaneous Sharpe ratio process from the time series of market Sharpe ratio term structures.

These topics are relevant for both academia and practical applications: The theoretical analysis of the link between equity and credit risk premia offers a new perspective on some recent research in the credit risk area. In particular, it offers a theoretical explanation for the large importance of risk premia for credit spreads which has been previously indicated by empirial studies.[34] In addition, it can help to explain the low explanatory power of fundamental variables for credit spread movements.

This thesis also adds to the equity premium literature. The equity premium is of major importance for many asset pricing and asset allocation decisions. There is still no consensus on the magnitude of the equity premium – estimates from major finance journals range from as low as approx.

[34]Cf. Elton et al. (2001) and Amato/Remolona (2003).

0.1% to more than 8%.[35] The methodology developed and proposed in this thesis offers a new line of thought for estimating the equity premium and is independent from the other main approaches discussed in the current literature.

There are many open issues concerning both the behavior of aggregate (expected) returns and the cross-section of returns. This thesis argues that the credit markets may be helpful in explaining some of these issues. In particular, our last research question analyzes the time variation in risk premia. Equity market research finds some evidence for time variation in equity risk premia – although return data of usually more than 50 years is necessary to yield any significant results.[36]

1.4. Structure of analysis

Chapter 2 provides a survey about the existing literature on general asset pricing, equity valuation and equity premia, credit valuation, and credit risk premia and the link between risk premia on equity and credit markets. Chapter 3 provides a theoretical analysis of the link between credit risk premia and equity premia within structural models of default. This theoretical link is implemented in chapter 4. Data on CDS spreads from 2003-2007 is used to estimate CDS-implied equity premia for the U.S., Europe, and Asia. Chapter 5 applies the estimation methodology from chapter 4 to estimate a term structure of risk premia. A time series of term structures before and during the 2007/2008 financial crisis is used to estimate the parameters of the instantaneous risk premium process (mean reversion, volatility, long-run mean). Chapter 6 concludes and provides an outlook for possible future areas of research.

[35] Cf. Mehra/Prescott (1985) for estimates of as low as 0.1% and Easton et al. (2002) for estimates of more than 8% for the 1990s. A general overview of equity premium estimates can be found in section 2.2.3.

[36] Cf. Campbell/Viceira (1999).

2. Existing literature and review of standard models

In section 2.1, we start with a brief review of the literature on general asset pricing. Section 2.2 follows with a discussion of equity valuation including different valuation models, cash flow estimation, and equity risk premia. Section 2.3 gives an overview of the main concepts of credit pricing including pricing models, expected loss estimation, and credit risk premia. Section 2.4 concludes this chapter with an overview of the existing literature concerning the link between risk premia on equity and credit markets.

2.1. General asset pricing theory

We will briefly discuss general asset pricing theory in this section. However, the focus of this section is more to present some of the main ideas that we will use later in chapters 3-5 rather than to provide a detailed discussion of all areas of modern asset pricing. An in-depth treatment of these topics can be found in Duffie (1996), Kruschwitz (2002), Copeland et al. (2005), Cochrane (2005), LeRoy/Werner (2006), and Brealey et al. (2008).

The value of an investment project generating a cash flow of CF_1 in year 1 can be determined according to Brealey et al. (2008) with the present value rule, i.e.,

$$V_0 = \frac{1}{1+k} CF_1, \tag{2.1}$$

where k is an appropriate discount factor. So far, this is not a theory but an identity since the discount factor k is not specified and does not even

have to be unique for investments generating the same cash flow CF_1. The crucial point in valuation theory is the determination of the discount rate k, also called the cost of capital or the hurdle rate. At this point, several questions are of interest. First, is this discount factor unique or does it depend on the preferences of an individual investor who has to deal with a certain investment problem? Second, what are the properties of k, i.e., in which cases is it low and in which cases is it high?
Financial theory builds mainly on three assumptions to answer these questions: First, it is assumed that investors want to be rich, i.e., they prefer more money to less. Second, investors are risk averse, i.e., they prefer a riskless investment to a risky investment if expected returns are the same. Third, there is a functioning capital market. The first assumption guarantees that non-monetary properties of the cash flow CF_1 do not have an effect on its valuation, e.g., as long as the payoff is exactly the same, people do not care whether the payoff CF_1 comes from a tobacco company or from a micro-finance institution. In addition, this first assumption usually ensures that arbitrage opportunities are exploited and therefore markets can reasonably be assumed to be arbitrage-free. The second assumption gives rise to risk premia for systematic (non-diversifiable) risk. The third assumption ensures that wealth can be transformed into different time patterns of consumption and thereby decouples investment and consumption decisions.

In the absence of uncertainty, Fisher (1930) was the first to develop a separation theorem on this basis. He showed that if capital markets are frictionless in the sense that borrowing and lending rates are the same, then there is a unique value for any riskless cash flow. A unique borrowing and lending rate ensures that agents can transform wealth and consumption in time for a market-wide unique discount factor. In this case, the present value can be calculated by setting k in (2.1) equal to this unique risk-free rate. However, this unique present value rule is already violated if different rates for (risk-free) borrowing and lending exist (cf. Hirshleifer (1958)).

If securities/investments are not riskless but their payoff depends on the state of the economy things become more complicated. The discount rate k depends on the riskiness of the cash flows, giving rise to so-called stochastic discount factors. In this case, there are two main theoretical approaches which provide guidance for valuation.

First, utility-based approaches. These approaches assume that agents' utility increases with increasing consumption and that utility functions are concave, i.e., "the last bite is never as satisfying as the first" (Cochrane (2005), p. 5). In addition, agents are assumed to maximize the discounted value of utility. In this setting it can be shown that the value of an investment equals the sum of the payoffs for each state of the economy multiplied with the marginal rates of substitution for each state (cf. Leroy/Werner (2006) and Cochrane (2005)), i.e.

$$V_0 = E^P\left[\frac{1}{1+k'}CF_1\right] = \frac{1}{1+k}E^P\left[CF_1\right] \text{ with } \frac{1}{1+k'} = \beta\frac{u'(c_1)}{u'(c_0)}, \quad (2.2)$$

where β denotes a subjective discount factor, u denotes the utility function of the investor, and c_t denotes the consumption in year t. However, the discount factor $\frac{1}{1+k}$ does not have to be unique, but may depend on the individual preferences of agents. Generally, cash flows that positively covary with consumption are subject to a higher average discount factor, cf. Cochrane (2005). It can further be shown that in the absence of arbitrage a positive valuation functional exists. If markets are complete, i.e., if every claim is attainable and can be generated by a certain set of assets, then this valuation functional is unique. In this case, the marginal rate of substitution is the same for each investor, cf. LeRoy/Werner (2006). Utility-based approaches provide deep insights and explanatory power of the mechanics of security markets. However, they rely on some restrictive assumptions, in particular the dependence of utility on consumption.[1]

A second approach simply assumes that prices on security markets are given and does not directly specify the underlying mechanics, cf. Duffie (1996) for

[1] E.g., some people might have a consumption target and decide to work longer until they have earned enough to satisfy this target. In this case, the utility function would also have to include the negative utility of longer working hours or a later retirement age.

example. As in the utility-based approaches, it can be shown that if markets are arbitrage free, claims in the asset span have a unique price and discount rate. If markets are complete, every claim has a unique price and discount rate.

If markets are arbitrage-free and complete, valuation can also be performed with a unique risk-neutral probability measure[2] and risk-neutral and real-world valuation lead to the same result, i.e.,

$$V_0 = \frac{1}{1+k} E^P \left[CF_1 \right] = \frac{1}{1+r} E^Q \left[CF_1 \right], \tag{2.3}$$

where r denotes the risk-free rate.[3] In this case, all cash flows are discounted with the risk-free rate but expectation is taken under the risk-neutral probability measure. A general approach to risk-neutral valuation was provided by Harrison/Kreps (1979) and Harrison/Pliska (1981).

Depending on the underlying assumptions, various models for the cross-section of returns have been proposed. The most famous is the Capital Asset Pricing Model (CAPM), which relies on the assumption of either normally distributed returns or on agents that optimize their portfolios based on mean/variance criteria, cf. Brealey et al. (2008) and LeRoy/Werner (2006). The CAPM is a one-factor model; multi-factor models such as the arbitrage pricing theory (APT) and the Fama/French three-factor model have also been proposed. However, it can be shown that a one-factor model is sufficient to capture the cross-section of expected returns if i) markets are arbitrage-free and complete and ii) agents have homogeneous expectations,

[2] If market are arbitrage-free, a risk-neutral probability measure exists. If markets are arbitrage-free and complete, the risk-neutral probability measure is also unique. Technical conditions apply. Cf. Duffie (1996).

[3] The equivalence of real-world and risk-neutral valuation applies to the value of a contingent claim. It should be noted that c.p. statements usually give different results depending on whether they are performed in the real or risk-neutral world. E.g., if risk aversion increases, the price of a call option will decrease if real-world expected payoffs are assumed to stay the same. However, it will remain unchanged if risk-neutral expected payoffs are assumed to stay the same. Cf. also Berg et al. (2009) for a similar discussion concerning the effect of volatility on option prices.

cf. Duffie (1996) and LeRoy/Werner (2006). No further assumptions concerning the distribution of returns[4] and the form of utility functions is required. However, the special design of the CAPM (the single factor is the return on the market portfolio) is only valid if the additional assumptions mentioned above are made.

The formulas developed above can be generalized to the multi-period case, i.e.,

$$V_0 = \sum_{t>0} E^P \left[\frac{1}{1+k_t} CF_t \right] \tag{2.4}$$

$$V_0 = \sum_{t>0} E^Q \left[\frac{1}{1+r_t} CF_t \right] \tag{2.5}$$

Throughout this thesis, we will abstract from any individual or aggregate utility functions but simply assume that unique real-world and risk-neutral probability measures exist. We will draw on formulas (2.4) and (2.5) to develop prices for equity and credit instruments. In addition, we will usually assume that we can work in a single-factor setting and – in the empirical part – that this single factor is the market portfolio.

2.2. Equity valuation

The models in this section will be specific cases of the net present value rule (2.4). The value of a company (or a share of a company) in t will be denoted by V_t. The models in the next paragraphs will differ in the specification of cash flows (dividends vs. residual income vs. earnings). The estimation of cash flows will be discussed in section 2.2.2. Section 2.2.3 provides a literature review on risk premia on equity markets.

[4] Apart from the condition that the first two moments must exist.

2.2.1. Valuation models

2.2.1.1. Dividend discount model

The dividend discount model (DDM) is probably the most simple valuation approach. The dividends are the cash flows that an investor receives on an equity investment. Consequently,

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}, \tag{2.6}$$

where D_t denotes the (expected) dividend[5] in year t and k denotes the discount rate which is assumed to be constant for notational convenience at this stage. Assuming constant growth rates for the dividends together with a constant discount rate results in the Gordon dividend discount model

$$V_0 = \frac{D_1}{k - g_d}, \tag{2.7}$$

where g_d is the (constant, ever-lasting) growth rate of dividends. These formulas were first developed by Williams (1938). The latter formula is named after Gordon (1962) and is therefore called the Gordon dividend growth model.[6]

For practical applications, formula (2.6) requires too many estimates of future dividends to be applicable. In contrast, (2.7) uses too few dividend forecasts to yield reasonable results.[7] Therefore, several variations of these formulas have been developed to account for data availability and practicability considerations. The valuation formula (2.6) is usually split into two to three subperiods. In the first subperiod (3-5 years), explicit analyst es-

[5]For notational convenience we will denote expected dividends also with D_t instead of $E^P[D_t]$ as long as the meaning is clear from the context. This also applies to other quantities such as earnings or book values in the following sections.

[6]Here, g_d denotes the discrete growth rate. In continuous time $V_t = \int_{t=0}^{\infty} D_0 e^{\tilde{g}_d t} e^{-kt} dt = D_0/(k - \tilde{g}_d)$ with the continuous growth rate $\tilde{g}_d$.

[7]E.g., consider the current 2007/2008 financial turmoil where some companies are not expected to pay any dividend at all in the next fiscal year. In this case, (2.7) yields a company value of zero and/or g_d is undefined.

timates for the expected dividends are available. In the second subperiod, growth is assumed to converge to a long-run or economy-wide mean. In the third stage, this long-run growth rate is assumed to persist forever so that (2.7) can be applied. Some of the models omit the second stage and directly use a constant growth rate after the first stage.

Brigham et al. (1985) is one of the first examples of a two-stage DDM. In the first stage (3 years), analyst forecasts are used to estimate future dividends. In the second stage, a constant growth model is applied and the growth rate is estimated as the retention rate of earnings multiplied with the book return on equity.[8] Gordon/Gordon (1997) propose a two-stage DDM and assume that the profitability (RoE) in the second stage equals the cost of capital (k). Botosan/Plumlee (2002) use analyst forecasts for the stock price at the beginning of the second stage as a proxy for the company value at the beginning of the second stage. Three-stage models have been developed by Malkiel (1979) and Lee et al. (2007). Malkiel assumes a convergence of the growth rate of dividends to the long-run growth rate of the U.S. economy in the second stage. This growth rate is also the growth rate for the third stage. Lee et al. (2007) assume that the growth rate of earnings converges to the long-run growth rate of the economy. In addition, the retention rate is assumed to converge to a steady-state retention rate.

DDMs can be applied to determine company values (if discount rates and expected dividends are available) or to implicitly determine discount rates and equity premia (if today's market values and expected dividends are available). The results of empirical studies using DDM to estimate equity premia are discussed in section 2.2.3.

2.2.1.2. Residual income model

The residual income model (RIM) has been developed by Preinreich (1938) and Edwards/Bell (1961). It is based on a simple reformulation of the div-

[8]E.g., growth rate of dividends is assumed to be equal to the growth rate of book equity.

idend discount model in terms of book value of equity and earnings using the clean surplus relation and the transversality condition. In the following, we will briefly develop the RIM based on a very general idea of Ohlson (1998, 2000) and Ohlens/Jüttner-Nauroth (2005). The main idea is to use a telescoping series of *any* sequence (y_t) which satisfies the transversality condition $y_t/(1+k)^t \to 0$ as $t \to \infty$ so that

$$0 = y_0 - \frac{(1+k)y_0}{1+k} + \frac{y_1}{1+k} - \frac{(1+k)y_1}{(1+k)^2} +/- \ldots = y_0 + \sum_{t=1}^{\infty} \frac{y_t - (1+k)y_{t-1}}{(1+k)^t}.$$

Adding this telescoping series to the right-hand side of the dividend discount model (2.6) yields

$$V_0 = y_0 + \sum_{t=0}^{\infty} \frac{D_t + y_t - (1+k)y_{t-1}}{(1+k)^t}. \tag{2.8}$$

Now, any sequence (y_t) could be used as long as the transversality condition is fulfilled. To derive the residual income model, set

$$y_t = BV_t,$$

where BV_t denotes the expected book value of equity in t. This yields

$$V_0 = BV_0 + \sum_{t=0}^{\infty} \frac{D_t + BV_t - (1+k)BV_{t-1}}{(1+k)^t}.$$

So far, the derivation has been purely mathematical. The economic ingredient for the residual income model is the so-called clean surplus relation, which links dividends (D_t), earnings (E_t), and the book value of equity (BV_t) via

$$BV_t - BV_{t-1} = E_t - D_t. \tag{2.9}$$

This clean surplus relation states that the difference in book values between t and $t-1$ is equal to the earnings in period t less the dividend that is paid out to shareholders. It therefore requires that all changes that affect the book value of equity should be included in earnings. The extent to which the clean surplus relation holds is dependent on the specific accounting rules applied in determining book values and earnings. We will discuss the validity of this assumption in section 2.2.2. Plugging (2.9) into equation (2.8) yields

$$\begin{aligned} V_0 &= BV_0 + \sum_{t=1}^{\infty} \frac{E_t - kBV_{t-1}}{(1+k)^t} = BV_0 + \sum_{t=1}^{\infty} \frac{RI_t}{(1+k)^t} \\ & \text{with } RI_t = E_t - k \cdot BV_{t-1}. \end{aligned} \tag{2.10}$$

The term RI_t is called the residual income (or abnormal earnings, value added). It represents the earnings in t less the cost of capital on the book value of equity. Some authors have argued that in an equilibrium residual income should be approx. zero in the long-run as companies' expected returns should equal their cost of capital.[9] However, residual income is an accounting figure and therefore dependent on specific accounting rules. Due to accounting conservatism it will be on average larger than zero, cf. Zhang (2000) for a theoretical argumentation and Myers (1999) and Cheng (2005) for empirical studies.

For an implementation, (2.10) can be expressed as a one-/two-/three-stage model as discussed in section 2.2.1.1. Botosan (1997) uses a two-stage RIM together with analyst forecasts for the share price at the beginning of the second stage. Claus/Thomas (2001) apply the RIM to the aggregate market, i.e., they use aggregate book values, earnings, and dividends. They apply a two-stage model where growth in the second stage is assumed to be equal to the expected rate of inflation. Gebhardt et al. (2001) implement a three-stage RIM where RoE is assumed to revert to the historical median industry RoE in the second stage. Easton et al. (2002) apply a model with a simultaneous estimation of the long-run growth rate.

The key advantage of the residual income approach is its focus on earnings and book values instead of dividends. Earnings and book values are not as dependent on the retention rate as dividends. In addition, the ter-

[9] Cf. Gebhardt et al. (2001) and Claus/Thomas (2001) for a detailed discussion.

minal value makes up a much smaller proportion in residal income models compared to dividend discount models. Since terminal values are hard to determine due to the uncertainty in the corresponding growth rate assumptions, the RIM methodology seems to be better suited to determine company values. However, there are additional underlying assumptions (in particular clean surplus) and the accuracy may be spurious due to methodological issues related to accounting conservatism, cf. Zhang (2000).

As for the dividend discount model, residudal income models can either be applied to determine company values or to implicitly solve for the required discount rate and determine the implied equity premia. We will discuss the results of empirical studies using RIM for the estimation of equity premia in section 2.2.3.

2.2.1.3. Earnings discount model

A general earnings discount model was developed by Ohlson/Jüttner-Nauroth (2005) and implemented by Gode/Mohanram (2003).[10] It is based on the same formula (2.8) as the residual income model. Instead of the series of book values BV_t, the series of capitalized (expected) earnings E_{t+1}/k is used for y_t, which results in

$$V_0 = \frac{E_1}{k} + \sum_{t=1}^{\infty} \frac{1/k \cdot (E_{t+1} - (1+k)E_t + kD_t)}{(1+k)^t} =: \frac{E_1}{k} + \sum_{t=1}^{\infty} \frac{z_t}{(1+k)^t}.$$

Here, $k \cdot z_t = E_{t+1} - E_t - k(E_t - D_t)$ has the interpretation of expected (out)performance in $t+1$. It is the amount by which the expected earnings in $t+1$ exceed last year's earnings, including an adjustment for the retention made in the last year. If the payout ratio is assumed to be 100% (i.e., $D_t = E_t$), then $k \cdot z_t$ simply equals the earnings growth ($E_t - E_{t-1}$), cf. Ohlson/Jüttner-Nauroth (2005) for a detailed discussion. In the next step, Ohlson/Jüttner-Nauroth (2005) make an assumption about the behavior of

[10]The Gode/Mohanram study was indeed published earlier than the model it is based on. In addition, both papers were published in the same journal (Review of Accounting Studies).

z_t, i.e.,

$$z_t = (1+\gamma)z_{z+1}$$

for a constant $0 \leq \gamma < k$[11] to derive the valuation formula (cf. Ohlson/Jüttner-Nauroth (2005))

$$V_0 = \frac{E_1}{k} + \frac{z_1}{k-\gamma} = \frac{E_1}{k} \cdot \frac{g_2 - \gamma}{k-\gamma}, \tag{2.11}$$

where g_2 is the growth in earnings from $t = 1$ to $t = 2$ exceeding the cost of capital under the assumption of a 100% retention rate.[12] Ohlson/Jütttner-Nauroth (2005) also show that, under certain conditions, $\frac{E_t}{E_{t-1}} \rightarrow (1+\gamma)$ as $t \rightarrow \infty$ so that γ plays the role of long-term earnings growth.

Formula (2.11) can be simplified and related to the price-earnings growth ratio (PEG) by setting $\gamma = 0$:

$$\frac{V_0}{E_1} = \frac{g_2}{k^2} \tag{2.12}$$

and, by setting g_2 equal to the cost of capital k, to the price/earnings ratio (PE):[13]

$$\frac{V_0}{E_1} = \frac{1}{k}. \tag{2.13}$$

Again, formulas (2.11)-(2.13) can be applied to determine company values or discount rates and equity premia. Section 2.2.3 provides a review of the literature that uses earnings discount models to estimate the equity premium.

[11] $\gamma < k$ is mathematically necessary for convergence, $\gamma \geq 0$ is assumed for economic reasons, cf. Ohlson/Jüttner-Nauroth (2005).

[12] g_2 is formally defined as $g_2 = \frac{E_2 - E_1}{E_1} + \frac{kD_1}{E_1}$. The first term is the expected growth rate of earnings from $t = 1$ to $t = 2$, the second term adjusts these earnings for a 100% retention rate.

[13] Please note again that g_2 is the growth rate of earnings assuming a 100% retention rate. Therefore, the natural benchmark for g_2 is k.

2.2.2. Estimation of cash flows

For the application of either the dividend discount model, the residual income model, or the earnings discount model, estimates for future dividends and earnings are necessary.[14] These estimates can either be derived with statistical procedures or expert estimates (e.g., by analysts) can be used.

The overwhelming part of the literature uses analyst forecasts to estimate dividends and earnings. These forecasts are available either via Value Line or I/B/E/S. Value Line provides its own forecasts, whereas I/B/E/S collects forecasts from professional analysts all over the world.[15] I/B/E/S provides estimates on approx. 60,000 companies in 67 countries around the world. Estimates are made on a monthly basis and cover several key financial measures such as sales, earnings, dividends, etc. Estimates are usually available for the next 3-5 years.

The use of earnings estimates in academic studies poses several challenges. First, its availability is limited to the next 3-5 years. Since the largest part of a company value is usually determined by cash flows beyond the next 5 years, this poses a significant challenge and requires subjective estimates for long-run growth by the authors. Second, the scope is limited. Smaller companies are not covered and it can be observed that coverage expands in bull markets and declines in bear markets. Third, academic studies have consistently documented an upward bias in these estimates. However, the magnitude seems to have declined over time, so that the magnitude of today's bias is far from clear. Several explanations for this upward bias are provided: Analysts may issue optimistic forecasts to generate revenue with the respective clients (Francis/Philbrick (1993), Dugar/Nathan (1995), Lim (2001)), they may be irrational and overoptimistic (Abarbanell/Bernard (1992), Elgers/Lo (1994)) or there is some kind of selection bias (McNi-

[14] In addition, the residual income model requires an estimation of book values. If the clean surplus assumption holds, all future expected book values can be derived from today's book values and dividend and earnings forecasts.

[15] Cf. www.valueline.com and www.thomsonreuters.com/products_services/ibes.

chols/O'Brien (1997)). Gu/Wu (2003) have argued that analysts estimate median dividends and earnings instead of means. Since expected earnings are usually negatively skewed, medians are above means which may explain the bias.[16] Fourth, residual income models require clean surplus accounting. Clean surplus accounting is violated by US-GAAP and other accounting standards. The most prominent example are revaluation reserves which increase/decrease equity capital without increasing/decreasing earnings. If forecasts of earnings, dividends, and book values are not in line with clean surplus, this poses additional problems for empirical applications, cf. Ohlson (2000) for an overview of specific problems for residual income models.

Besides analyst forecasts, one could also draw on pure statistical estimates for dividends and earnings. E.g., if profitability and growth are mean-reverting, the corresponding parameters could be determined to estimate future earnings, cf. Kengelbach et al. (2007) for a comprehensive empirical study. In addition, different predictive power of cash flow and accrual components for the explanation of future earnings has been documented in the literature, cf. Sloan (1996) and Kaserer/Klinger (2008). These findings could also be used to estimate future earnings. Unfortunately, empirical studies of the performance of pure statistical estimates and comparisons with analyst forecasts are rare, cf. Kothari (2001) for an overview. Most studies rely on (subjective) analyst forecasts to estimate future expected earnings.

2.2.3. Risk premia on equity markets

Research on expected equity returns can be categorized broadly into i) research on aggregate equity returns and ii) research on the cross-section of returns. The standard model for the cross-section of returns is the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Mossin (1966), and Lintner (1965). Although it partially relies on restrictive assumptions, it clearly dominates practical applications.[17] Other models have been pro-

[16] A good overview of several possible explanations for bias in analyst forecasts can also be found in Brown (1993).

[17] Cf. Brealey et al. (2008).

posed, in particular the consumption CAPM (Breeden (1979)) and multi-factor models including the Arbitrage Pricing Theory (Ross (1976)), and the Fama/French (1993, 1996) three-factor model. This thesis focuses on aggregate equity returns; therefore, these cross-sectional models and their performance are not a core part. We will, however, make use of the CAPM in some cases in the empirical part (chapters 4 and 5). By doing so, we follow the main consensus among financial academics (cf. Welch (2008)).[18] In the following section (section 2.2.3.1), we will provide a more detailed overview of the main results in the literature on the magnitude of the equity premium. Then we will briefly review the literature on the behavior of the equity premium (section 2.2.3.2).

2.2.3.1. Magnitude

In the literature, four methods are discussed to estimate future expected equity returns. First, historical equity premia can be used as a proxy. Second, implied equity premium estimates can be used. These estimates are calculated by implicitly solving DCF-valuation formulas derived in section 2.2.1 for the discount rate. Third, approaches based on utility functions can be applied. Fourth, one may simply rely on expert estimates.

Historical equity premia: There are various studies on historical equity premia in the U.S. and other countries. The most comprehensive one was probably conducted by Dimson et al. (2006). This study includes 17 countries and each of the 17 countries is covered for the period from 1900-2005 (106 years). In addition, a comparable methodology for all countries has been applied. Historical equity premia from this study are depicted in table 2.1.[19] The equity premium is depicted using either short-term government bonds ("Bills", left-hand side of the table) or long-term government bonds

[18] We also do want to point out that – based on the considerations from section 2.1 – one-factor models are sufficient to explain the systematic part of the cross-section of expected returns. Therefore, the main simplification of the CAPM is not the focus on one single systematic factor but the specific choice of this factor (expected return of the market portfolio).

[19] The table is taken directly from Dimson et al. (2006), table 3.

Table 2.1.:
Historical equity premia for 17 countries from 1900-2006.
This table depicts historical equity premium estimates for 17 countries and is taken from Dimson et al. (2006). The left-hand side of the table shows historical equity premia as the difference between historical equity returns and returns on *short*-term government bonds ("Bills"), the right-hand side shows the difference between historical equity returns and returns on *long*-term government bonds ("Bonds"). Equity premia are in local currencies, the row "World" is value-weighted in US-$. All numbers are in percent.

	Historical Equity Premium Relative to Bills				Historical Equity Premium Relative to Bonds			
Country	Geometric Mean	Arithmetic Mean	Standard Error	Standard Deviation	Geometric Mean	Arithmetic Mean	Standard Error	Standard Deviation
Australia	7.08	8.49	1.65	17.00	6.22	7.81	1.83	18.80
Belgium	2.80	4.99	2.24	23.06	2.57	4.37	1.95	20.10
Canada	4.54	5.88	1.62	16.71	4.15	5.67	1.74	17.95
Denmark	2.87	4.51	1.93	19.85	2.07	3.27	1.57	16.18
France	6.79	9.27	2.35	24.19	3.86	6.03	2.16	22.29
Germany[a]	3.83	9.07	3.28	33.49	5.28	8.35	2.69	27.41
Ireland	4.09	5.98	1.97	20.33	3.62	5.18	1.78	18.37
Italy	6.55	10.46	3.12	32.09	4.30	7.68	2.89	29.73
Japan	6.67	9.84	2.70	27.82	5.91	9.98	3.21	33.06
Netherlands	4.55	6.61	2.17	22.36	3.86	5.95	2.10	21.63
Norway	3.07	5.70	2.52	25.90	2.55	5.26	2.66	27.43
South Africa	6.20	8.25	2.15	22.09	5.35	7.03	1.88	19.32
Spain	3.40	5.46	2.08	21.45	2.32	4.21	1.96	20.20
Sweden	5.73	7.98	2.15	22.09	5.21	7.51	2.17	22.34
Switzerland	3.63	5.29	1.82	18.79	1.80	3.28	1.70	17.52
U.K.	4.43	6.14	1.93	19.84	4.06	5.29	1.61	16.60
U.S.	5.51	7.41	1.91	19.64	4.52	6.49	1.96	20.16
Average	**4.81**	**7.14**	**2.21**	**22.75**	**3.98**	**6.08**	**2.11**	**21.71**
World-ex U.S.	4.23	5.93	1.88	19.33	4.10	5.18	1.48	15.19
World (US-$)	**4.74**	**6.07**	**1.62**	**16.65**	**4.04**	**5.15**	**1.45**	**14.96**

[a]Germany omits 1922-23.

("Bonds", right-hand side of the table) and either arithmetic or geometric averages.[20] For most applications, the arithmetic average is closer to the correct equity premium than the geometric average.[21] Relative to short-term government bonds, the average arithmetic mean of the equity premium of all 17 countries in local currencies is 7.14%. Value-weighted and in US-$ the arithmetic mean is 6.07%. The arithmetic mean ranges from 4.51% (Denmark) to 10.46% (Italy). Equity premia relative to long-term bond returns are approx. 1% lower (6.08% on average) and range from 3.27% (Denmark) to 9.98% (Japan). Geometric averages are approx. 2% lower ranging from from 2.80% (Belgium) to 7.08% (Australia) relative to short-term bonds and from 1.80% (Switzerland) to 6.22% (Australia) relative to long-term bonds. Other studies of historical equity premia report similar results for the overall level of equity premia, although the reported risk premia are dependent on the specific time horizon, cf. e.g. Ibbotson (2008) for equity premia from 1970-2007.[22]

Using historical equity premia for expected future equity premia has some

[20] Dimson et al. (2006) use bond *returns* – and not yields – to determine the equity premium. Although this seems natural, the resulting effects should be kept in mind. E.g. the equity premium for Germany is higher than the real rate of return on equities (0.74% higher for the geometric average, 6.98% higher for the arithmetic average, cf. Dimson et al. (2006)) because of the hyperinflation in the 20s in Germany and the corresponding poor bond returns.

[21] The geometric average is always smaller or equal to the arithmetic average, the difference is $0.5\sigma^2$ for normal distributions and infinite observations. There are mainly three reasons why the arithmetic average is better suited for most applications: First, the arithmetic average is an unbiased estimator for the one-year equity premium, whereas the geometric average is downward biased, cf. Blume (1974). Second, the N-th power of the expected equity returns (1 + risk-free rate + equity premium) using the arithmetic average is a better estimator for the N-period return than the geometric average if N is a lot smaller than the number T of historical returns used to estimate the equity premium, cf. Blume (1974). With 106 years of historical data (i.e., $T = 106$) in the Dimson et al. (2006) study, $T >> N$ will usually be the case for practical applications. Third, Cooper (1996) shows that even the arithmetic average leads to an upward biased estimator for the present value of cash flows (the upward bias in the geometric average is even larger). However, he shows that in realistic cases the correct discount rate is very close to the arithmetic average.

[22] The results for Germany in Dimson et al. (2006) differ significantly from the results in Stehle (2004), although the study of Dimson et al. (2006) is partly based on data from Stehle (2004). Dimson et al. use additional data from Ronge (2002) for the period from 1900 to 1953 for the German market. Due to the hyperinflation in the 20s, equity premia are larger for this subperiod.

disadvantages. First, it is based on the assumption that past observations are a good guide for future expectations. E.g., since 1900, transaction costs and liquidity relative to bonds are likely to have decreased in most of the markets, which may justify lower equity premia. On the other hand, volatility has generally increased over the last decades, which would result in higher equity premia if Sharpe ratios are assumed to remain constant. Second, historical equity premia may be subject to survivorship bias (cf. Brown et al. (1995) and Li/Xu (2002) for a discussion). However, Dimson et al. (2006) estimate that their countries covered at least 90% of the global market capitalization in the year 1900 and estimate the survivorship bias at less than 0.1% in their sample. Third, if equity premia are volatile, then an increase in stock prices may be due to a decrease in implied equity premia. In this case, historical data may yield spurious results. Fourth, standard errors of estimates are quite large even over a 106-year horizon. Dimson et al. (2006) report standard errors of approx. 2% on average and of 1.62% for the world portfolio (relative to bills). Therefore, a 95% confidence interval for the world equity premium would still cover a range of approx. +/- 3% around the mean estimates.

Implied equity premium estimates: Table 2.2 gives an overview of studies using either a dividend discount model (DDM), a residual income model (RIM), or an earnings discount model (EDM) to estimate implied equity premia for the U.S.

The resulting equity premia range from as low as 1% to slightly more than 7%. Implied equity premium estimates have largely contributed to the perception that historical equity premia for the U.S. market – which are usually close to or above 7% – are upward biased due to favorable conditions on the U.S. market over the last 50-100 years. However, these estimates also have their shortcomings. First, all approaches are heavily dependent on terminal value assumptions.[23] Even residual income models, which are supposed to be least dependent on these assumptions (cf. Claus/Thomas (2001) for a

[23] Earnings/dividends/residual income at the beginning of the terminal value period and the growth rate during the terminal value period.

Table 2.2.:
Implied equity premium estimates for the U.S.

This table gives an overview of the main studies on implied equity premium estimates for the U.S. *DDM* denotes dividend discount model, *RIM* denotes residual income model, *EDM* denotes earnings discount model. The column *Risk-free rate* denotes the risk-free rate that is used in the respective study to determine the equity premium.

Author (Year)	Country	Sample period	Type	Approach	Equity Premium	Risk-free rate
Malkiel (1979)	U.S.	1960-1976	DDM	Three-stage, g_d=growth of economy	4.5%[a]	10Y Gov
Brigham et al. (1985)	U.S.	1966-1984	DDM	Two-stage, $g_d = RoE \cdot retention$	5.6%	20Y Gov
Harrison/Marston (2001)	U.S.	1982-1998	DDM	One-stage	7.1%	30Y Gov
Claus/Thomas (2001)	U.S.	1985-1998	DDM	One-stage, g_d based on analyst forecasts	7.3%	10Y Gov
Fama/French (2002)	U.S.	1951-2000	DDM	Based on hist. div. yields and div. growth	2.6%	6M CP
Botosan/Plumlee (2005)	U.S.	1983-1993	DDM	Two-stage, $RoE = CoE$ in second stage	2.1%	5Y Gov
Claus/Thomas (2001)	U.S.	1985-1998	RIM	Two-stage, RI grows with expected inflation	3.4%	10Y Gov
Gebhardt et al. (2001)	U.S.	1979-1995	RIM	Three-stage, RoE → median industry RoE	2.7%	10Y Gov
Easton et al. (2002)	U.S.	1981-1998	RIM	Two-stage, simult. estimation of growth rate	6.0%	5Y Gov
Gode/Mohanram (2003)	U.S.	1984-1998	RIM	Three-stage, RoE → median industry RoE	3.2%	10Y Gov
Chen et al. (2004)	U.S.	1993-2001	RIM	Three-stage, RoE → median industry RoE	5.0%	10Y Gov
Botosan/Plumlee (2005)	U.S.	1983-1993	RIM	Three-stage, RoE → median industry RoE	1.0%	5Y Gov
Fama/French (2002)	U.S.	1951-2000	EDM	Based on hist. div. yields, earnings growth	4.3%	6M CP
Gode/Mohanram (2003)	U.S.	1984-1998	EDM	Based on OJN (2005)	5.6%	10Y Gov
Chen et al. (2004)	U.S.	1993-2001	EDM	Based on OJN (2005)	7.2%	10Y Gov
Easton (2004)	U.S.	1981-1999	EDM	Based on Price Earnings Ratio	1.2%[b]	10Y Gov
Easton (2004)	U.S.	1981-1999	EDM	PEG-Ratio, simult. estim. of growth rate	3.6%[c]	10Y Gov
Botosan/Plumlee (2005)	U.S.	1983-1993	EDM	Based on OJN (2005)	6.6%	5Y Gov
Botosan/Plumlee (2005)	U.S.	1983-1993	EDM	Based on PEG-ratio	5.0%	5Y Gov

Abbreviations: g_d: dividend growth, RI: Residual Income, RoE: Return on Equity, CoE: Cost of Equity
PEG: Price-Earnings-Growth, OJN: Ohlson/Jüttner-Nauroth, CP: Commercial Paper, Gov: Government
Y: years, M: months, simult: simultaneous, estim: estimate

[a] Own approximation based on figure given in the source.
[b] Own calculation based on table given in the source.
[c] Own calculation based on table given in the source.

detailed discussion) yield values for the equity premium ranging from 1-6%. In fact, these high sensitivities with respect to terminal value assumptions are one of the major reasons why implied equity premium estimates are not frequently used in practical applications.[24] In addition, the estimates rely on analyst forecasts for dividends and earnings. These forecasts are subjective and have been shown to be upward biased, cf. section 2.2.2. It should also be noted that conservative accounting generally causes a downward bias of most implied equity premium estimates (cf. Zhang (2000) and Myers (1999)) and that these estimates provide inconsistent values for the term structure of risk premia.[25]

Approaches based on utility functions: Equity premia can – at least in theory – also be derived from approaches based on utility functions. Mehra/Prescott (1985, 2003) point out the fact that classical approaches combined with reasonable parameters for risk aversion lead to equity premia that are below 1% ("Equity Premium Puzzle"). Thus, alternative approaches have been developed, including habit formation, idiosyncratic risks, disastrous rare events and market imperfections to partially resolve the puzzle.[26] However, utility-based approaches only play a minor role in practical applications, cf. Brealey et al. (2008). They are therefore more useful as a theoretical and qualitative tool rather than as a means to directly estimate risk premia for equities or on any other market. We will therefore not go into detail about these models.

Expert estimates: With inconclusive evidence from historical risk premia,

[24] Cf. Welch (2008) for an overview of financial economists views on different approaches for determining cost of capital.

[25] Implied equity premium estimates usually determine the implied cost of capital for each company separately and aggregate these estimates based on the *current* market capitalization of the companies. Expected market capitalizations in the following years will shift gradually towards companies with higher expected cost of capital – unless this is not compensated by higher dividend payouts. Therefore, the expected equity premium will gradually increase over the following years. However, all authors report the – lowest – first year estimate. Unfortunately, we have not seen any author who has pointed out this fact so far.

[26] Cf. Cochrane (2005) for an overview.

Author (Year)	Country	Period	Experts	Equity Premium arith.	Equity Premium geom.	r_f
Welch (2000)	U.S.	1997-1999	U.S. fin. prof.	7.1%	5.2%	1M Gov
Welch (2001)	U.S.	Aug. 2001	U.S. fin. prof.	5.5%	4.7%	1M Gov
Welch (2008)	U.S.	Dec. 2007	U.S. fin. prof.	5.7%	5.0%	1M Gov
Graham/Harvey (2008)	U.S.	2000-2008	U.S. CFOs	2.4%-4.7%		10Y Gov
Fernandez (2009)	U.S.	n/a	100 textbooks	6.6%		n/a
Audit Companies	Ger.	n/a	Valuation experts	4-5%	n/a	n/a

Table 2.3.:
Expert estimates for the equity premium
This table depicts expert estimates for the equity premium from several sources. *Period* is the survey period, r_f denotes the risk-free rate used to determine the equity premium. *U.S. fin. prof.* denotes U.S. finance professors, *CFO* denotes Chief Financial Officers. *Y* denotes years, *M* denotes months.

implied risk premia, and utility-based approaches, many practitioners draw back on expert estimates. Table 2.3 depicts frequently used and cited expert estimates. It is not our main target to give an overview of all estimates and surveys regarding the equity premium, cf. Welch (2000, 2008) and Graham/Harvey (2008) for an overview. However, we want to point out that most estimates currently lie in the area of 3-6% for the arithmetic average. The surveys by Welch (2000, 2001, 2008) among U.S. finance professors revealed equity premia between 5.5% and 7.1% for the arithmetic average. Geometric averages are approx. 1-2% lower. Graham/Harvey (2008) summarize quarterly surveys among U.S. CFOs from 2000-2008. These estimates range from 2.4% to 4.7%. However, they seem to be geometric rather than arithmetic averages. Fernandez (2009) summarizes the equity premia used in approx. 100 textbooks, the average is 6.6%. Finally, audit companies in Germany usually use 4-5% equity premium for the German market.

2.2.3.2. Time series behavior

Besides the (average) magnitude of the equity premium, the time series behavior of expected excess returns has been subject to debate in the academic literature. In the survey of Welch (2000), 20% of the market participants offered monotonically increasing and 50% monotonically decreasing equity premium term structures. This indicates that market participants seem to have the perception that expected returns are indeed not constant but vary over time. Based on arguments from utility theory, expected Sharpe ratios are dependent on risk aversion levels and the volatility of consumption, cf. Cochrane (2005). This also offers a theoretical motivation for time-varying expected returns.

Unfortunately, risk premium processes are inherently hard to measure. The discussion in the last subsection has shown that estimating the average magnitude of the equity premium is already subject to significant noise. Consequently, an estimation of the parameters of the equity premium process is difficult. To give an intuition for the problems concerned with estimating time variation in equity premia, assume that equity premia EP_t are mean-reverting and excess returns r_t are modeled as the sum of the equity premium and a normally distributed error term, i.e.,

$$
\begin{aligned}
EP_t &= EP_{t-1} + \kappa(\bar{\mu} - EP_{t-1}) + \sigma_\epsilon \epsilon_t,\ 0 < \kappa < 1 && (2.14)\\
r_{t+1} &= EP_t + \sigma_u u_{t+1}\\
&= EP_{t-1} + \kappa(\bar{\mu} - EP_{t-1}) + \sigma_\epsilon \epsilon_t + \sigma_u u_{t+1} && (2.15)\\
\text{with} \quad & \rho(u_t, \epsilon_t) = \rho. && (2.16)
\end{aligned}
$$

If we are only able to look at realized returns (i.e., (2.15)), we are faced with several problems: First, it is hard to disentangle noise in realized returns (σ_u) and volatility in expected returns (σ_ϵ). Second, our ability to measure mean reversion is limited by the amount of noise both in realized returns (σ_u) and in expected returns (σ_ϵ). Third, the correlation between u and ϵ is crucial. If ρ is small enough, then returns may be negatively autocorrelated, while expected returns are positively autocorrelated, cf. appendix B.1 for a derivation of exact formulas.

In a study of historical returns of 17 countries from 1900-2006 Dimson et al. (2006) find an average positive autocorrelation of 0.07 for realized aggregate real returns, although the autocorrelation is only positive in 2 of the 17 markets at the 95% significance level – which is only slightly more than one would expect by pure chance. Poterba/Summers (1988) find positive autocorrelation in returns over short horizons and negative correlation over longer horizons when looking at the U.S. and 17 other countries. Although these results are not statistically significant, Poterba/Summers (1988) point out that substantial movements in expected returns are necessary to economically account for these results. This discrepancy between statistical and economical evidence is a consequence of the large standard deviation of equity returns relative to their unconditional mean.

The return predictability literature[27] has tried to explain future excess returns by certain parameters such as dividend/price ratios or earnings/price ratios. Assume that the equity premium EP_t can be predicted by a parameter x_t (e.g., dividend/price ratios or earnings/price ratios), i.e.,[28]

$$EP_t = a + bx_t \tag{2.17}$$

and that excess returns r_t are the sum of the equity premium and a normally distributed error term

$$r_{t+1} = EP_t + \sigma u_{t+1} = a + bx_t + \sigma u_{t+1}. \tag{2.18}$$

Again, if we are only able to look at realized returns (i.e., (2.18)), our ability to get a precise estimate for b is limited by the amount of noise $\sigma \cdot u$. In practice, the high volatility of equity returns makes it hard to get any significant statistical results on the basis of realized returns even over very large sample periods (50-100 years), cf. Cochrane (2005) and Poterba/Summers (1988). However, besides being a pure econometrical exercise, there is a stronger economic motivation behind this kind of return predictability. For example, dividend/price ratios (d/p ratios) have to predict either dividend growth or future returns.[29] As Cochrane (2005) points out, this is not a theory but an identity. If prices move up, then, based on a simple DCF argumentation,

[27]Cf. Cochrance (2005) for an overview.
[28]One could easily add an error term to this equation; we omit it to get a better intuition.
[29]Or price/dividend-ratios must be allowed to grow explosively (bubble).

either dividends have to increase or the discount factor has to decrease. This identity can help to outwit pure econometric analysis. Roughly speaking, authors have found that the evidence of d/p ratios to predict future excess returns is stronger than for d/p ratios to predict future dividend growth (cf. Fama/French (1988), Cochrane (1992)). D/p ratios are time-varying and mean-reverting. Therefore, the same should be true for expected (excess) returns.[30]

Other authors have used term spreads, T-bill rates, earnings, and macro variables to predict excess returns (Fama/French (1989), Lamount (1998), Lettau/Ludvigson (2001)). However, all of these studies are only able to find indirect indications for time-varying expected returns and the econometric validity of the results is disputed. Comprehensive overview articles that reject the return predictability hypothesis based on statistical insignificance results are Goyal/Welch (2008) and Boudoukh et al. (2008). It should be noted, that these studies do not find any predictability of d/p ratios for dividend growth either. We know, however, that either i) d/p ratios predict dividend growth or ii) d/p ratios predict expected returns must be valid. These articles do not find enough statistical evidence to support i) on its own, however, they also do not reject ii). Ang/Bekaert (2008) also reject return predictability based on earnings/price ratios. Moreover, they even find that earnings yields significantly predict future earnings growth. Overview articles arguing in favor of return predictability are Campbell et al. (2008) and Cochrane (2008).

Using a specification based on d/p ratios similar to (2.14)-(2.18), Campbell/Viceira (1999) estimate the parameters of the equity premium process. They estimate a mean reversion parameter of 0.20, a volatility σ_ϵ of 3.1% and a long-run mean equity premium of 4.2%. In addition, they estimate a negative correlation between the error term ϵ and u of -0.70. Their results are, however, only significant when looking at a time series of more than 100

[30] This change in expected returns is also not due to changes in the risk-free rate, cf. Cochrane (2005).

years. Using shorter time horizons no longer yields significant coefficients. This is not surprising given the discussion above.

2.2.3.3. Synopsis: Risk premia on equity markets

Taken together, the literature on the magnitude of equity premia suggests that:

- Reasonable expected equity premia for the U.S. are between 2% and 7%.
- However, there is neither an agreement on the best methodology for estimating expected equity premia nor on any narrower range for the magnitude of the equity premium.
- An estimation of equity premia from historical returns has the major disadvantage that it does not include current information. This disadvantage is especially important if we keep in mind the major changes that financial markets have seen over the last 100 years and the fact that equity premia are likely to be time-varying.
- Implied equity premium estimates are largely dependent on terminal value assumptions. In addition, despite the fact that a variety of approaches exist, these approaches are all exposed to the same shortcomings[31] and therefore do not provide independent results.

The review of the literature on time variation in risk premia (equity premium and market Sharpe ratio) has revealed the following:

- There is some evidence, both empirical as well as based on theoretical models, that risk premia are not constant but are time-varying and mean-reverting.
- This evidence suggests that risk premia are lower after bull markets and higher after bear markets, i.e., marginal investors require a higher

[31] Bias in analyst forecasts, long-run growth assumptions, and problems associated with conservative accounting.

compensation for systematic risk in bad states of the economy than in good states of the economy.

- Time-varying risk premia are hard to measure from realized returns due to econometric problems (especially high standard deviations of returns relative to mean returns) and no consensus on the degree of time variation exists.

2.3. Credit pricing

2.3.1. Pricing models

In this section we will briefly discuss the main pricing models for bonds and credit default swaps (CDS) that have been explored in the literature. The price of a defaultable bond in t with maturity T, coupon c, and a recovery rate of RR will be denoted by $B^d(t, T, c, RR)$. The spread of a CDS is denoted by s and is defined as the regular payment by the protection buyer for which the fixed and the floating leg have the same value, i.e., for which the initial value of the CDS is zero. For notational convenience we will assume a constant risk-free rate r, a face value of 1 for all bonds, a constant recovery rate RR, and continuous coupon payments.

2.3.1.1. Yield-based pricing

Yield-based pricing is the equivalent to the DCF models for equity valuation.[32] In these models, the value of a bond or CDS is determined as the real-world expected cash flow discounted with an appropriate risk-adjusted discount rate. It is very important to differentiate between the promised yield and the expected yield in this case, cf. Brealey et al. (2008). The *promised yield* is the return on a bond if all payments are made, i.e. if no

[32] Unfortunately, most books on credit pricing do not include yield-based pricing models. A basic discussion can be found in Brealey et al. (2008). This section is based on similar considerations for risk-neutral reduced-form pricing models, cf. Duffie/Singleton (2003) and Schönbucher (2003) for example. A decomposition of the promised yield into its components which is similar to the one described in this section can be found in Elton et al. (2001).

default occurs. Therefore, by definition, the value of a bond is the discounted value of the promised cash flows discounted by the promised yield Y_p:

$$B^d(0,T,c,RR) \quad = \quad \underbrace{\int_0^T ce^{-Y_p t}dt}_{\text{coupon}} + \underbrace{e^{-Y_p T}}_{\text{principal}} = \frac{c}{Y_p}(1-e^{-Y_p T}) + e^{-Y_p T}$$

If a coupon bond is traded at par ($B^d = 1$), the coupon of the bond is equal to the promised yield of the bond ($c = Y_p$). The promised yield is also the maximum return that can be achieved if all promised payments are made and the bond is held to maturity. If we refer to the yield of a bond, we will usually refer to this promised yield if not stated otherwise.

In case of default, some payments will not be made and therefore the *expected* yield is lower. The expected yield (Y_e) can be calculated by setting the current value of a bond equal to the expected discounted cash flows. In order to determine the expected cash flows, we will have to specify the default probability for each $t \in [0,T]$. For ease of notation, we will assume that default can be modeled with a homogeneous Poisson process (cf. Schönbucher (2003)). In this case, default arrives with a constant (real-world) default intensity λ^P and the cumulative default probability between t and T ($PD^P(0,t)$) can be calculated as

$$PD^P(0,t) = 1 - e^{-\lambda^P t}.$$

The default intensity is the instantaneous loss in survival probability. We are now able to derive the price of a bond as

$$\begin{aligned} B^d(0,T,c,RR) &= \int_0^T \underbrace{ce^{-\lambda^P t}e^{-Y_e t}}_{\text{coupon}} + \underbrace{RR\lambda^P e^{-\lambda^P t}e^{-Y_e t}}_{\text{recovery}} dt + \underbrace{e^{-\lambda^P T}e^{-Y_e T}}_{\textit{principal}} \\ &= \frac{c+\lambda^P RR}{Y_e+\lambda^P}\left[1-e^{-(Y_e+\lambda^P)T}\right] + e^{-(\lambda^P+Y_e)T}. \end{aligned} \tag{2.19}$$

Therefore, the bond is traded at par ($B^d = 1$) if

$$c \quad = \quad Y_e + \lambda^P - \lambda^P RR = Y_e + \lambda^P LGD \tag{2.20}$$

where LGD denotes the loss given default ($LGD = 1 - RR$). Therefore, yields on corporate bonds are the sum of the expected return (Y_e) and the real-world expected loss ($\lambda^P LGD$). The real-world expected loss is the part of the yield which is lost due to default, the rest of the yield is a true excess return which compensates for the additional systematic risk of bonds. Based on models for i) the real-world expected loss and ii) the expected return on corporate bonds, we would now be able to price corporate bonds using formula (2.19). The spread of a corporate bond is the yield less the risk-free rate (r), i.e.,

$$s = c - r = (Y_e - r) + \lambda^P LGD. \tag{2.21}$$

The bond spread is therefore the sum of the expected *excess* return and the expected loss.
Based on arbitrage arguments, bond spreads are equal to CDS spreads in this simple setting (cf. Schönbucher (2003)). CDS spreads can therefore also be decomposed into an expected loss part and a risk premium part. However, CDS are initially worth zero, so the risk premium part cannot be directly interpreted as an excess return on the CDS.

We could easily extend formula (2.19) to inhomogeneous Poisson processes or Cox processes. In this case, the equivalence between the coupon and the sum of expected yield and expected loss only holds as an approximation. This approximation is, however, a very good approximation for reasonable calibrations, cf. Schönbucher (2003) and Duffie/Singleton (2003) for a discussion and a more thorough treatment of Poisson and Cox processes.

Taken together, the yield on a bond is the maximum return on a bond if it is held to maturity. This yield can be decomposed into the expected loss and the expected return. The bond spread – defined as the yield less the risk-free rate – can be decomposed into the sum of the expected loss and the expected excess return.

2.3.1.2. Reduced-form credit pricing

Reduced-form models rely on an exogenous specification of the default time. In a strict sense, the yield-based pricing model described in the last subsection is also a reduced-form model. Most textbooks, however, usually refer to reduced-form pricing models in a risk-neutral setup. Reduced-form models have been developed by Litterman/Iben (1991), Hull/White (1995), Madan/Unal (1995), Jarrow/Turnbull (1995), and Duffie/Singleton (1997, 1999). In this section, we briefly discuss the main features which are important for this thesis. In a risk-neutral pricing framework, the value of a defaultable zero-coupon bond with zero recovery $B^d(0, T, \mathit{coupon} = 0, RR = 0)$ can be determined as

$$B^d(0, T, 0, 0) = E^Q[\, e^{-rt} 1_{\{\tau > T\}} \,] = e^{-rt}(1 - PD^Q(0, T)),$$

where τ denotes the default time and $PD^Q(0, T)$ denotes the cumulative risk-neutral default probability of the bond. The risk-neutral default probability can be determined via any suitable model. In a reduced-form approach, this default probability is assumed to be exogenously specified. If we further operate in a continuous-time setting, the instantaneous default probability is called "default intensity" and denoted with λ^Q in the risk-neutral world. The default probability in reduced-form models can be calculated as

$$\begin{aligned} PD^Q(0, T) &= 1 - E^Q[e^{-\int_0^T \lambda_s^Q ds}] \qquad (2.22) \\ &\underset{\text{constant}\,\lambda}{=} 1 - e^{-\lambda^Q T}. \end{aligned}$$

A mathematical rigorous treatment including the conditions for the existence and uniqueness of the corresponding risk-neutral probability measure can be found in Jarrow/Turnbull (1995) and Duffie/Singleton (1997, 1999). We could now go on by specifying a certain process for the default intensity and use standard theory to evaluate (2.22). Such a procedure can be found in Duffie/Singeton (2003).[33] For the following argumentation, we

[33] The expression (2.22) is very similar to standard interest rate theory, so that results from the interest rate theory can be applied to pricing defaultable bonds. If the default

will assume for simplicity reasons that the default intensity is constant and that coupons are paid continuously. In this case, the price of a defaultable coupon-bearing bond with constant recovery rate RR is

$$\begin{aligned} B^d(0,T,c,RR) &= \int_0^T \left[ce^{-\lambda^Q t} + RR\lambda^Q e^{-\lambda^Q t}\right] e^{-rt}dt + e^{-\lambda^Q T}e^{-rT} \\ &= \frac{c+\lambda^Q RR}{r+\lambda^Q}\left[1-e^{-(r+\lambda^Q)T}\right] + e^{-(r+\lambda^Q)T}. \end{aligned} \tag{2.23}$$

Therefore, the bond is traded at par if

$$c = r + \lambda^Q - \lambda^Q RR = r + \lambda^Q LGD, \tag{2.24}$$

This coupon of par bonds is also called the yield. Consequently, the yield of the bond is the sum of the (instantaneous) risk-free interest rate and the (instantaneous) risk-neutral expected loss ($\lambda^Q \cdot LGD$). The bond spread is defined as the yield of the bond less the risk-free rate, i.e., the risk-neutral expected loss.

If we value a CDS based on the same assumptions, we have to evaluate the premium leg

$$PV(\text{Premium leg}) = \int_0^T se^{-\lambda^Q t}e^{-rt}dt$$

and the default leg

$$PV(\text{Default leg}) = \int_0^T LGD\lambda^Q e^{-\lambda^Q t}e^{-rt}.$$

The spread of a CDS is the value for s for which both legs have the same value, i.e.,

$$s = \lambda^Q \cdot LGD. \tag{2.25}$$

The spread of a CDS is therefore equal to the bond spread, which is also

intensity belongs to the class of affine processes, tractable solutions can be found. The Vasicek model and the CIR model are popular choices in this case, cf. Schönbucher (2003) and Duffie/Singleton (2003).

the risk-neutral expected loss. The same result could have been achieved by arbitrage arguments, since in our setting a portfolio of a bond and a CDS is riskless and should therefore earn a return equal to the risk-free rate.

In practice, things turn out to be more complicated. There are several candidates for the risk-free rate, the risk-free rate and the default intensity are not constant, and payments are not made continuously. The resulting formulas are usually easy to derive but might look uncomfortably complex. All in all, these assumptions have only a minor effect for our purposes. Our main results from this section are still approximately valid:

- Credit spreads are equal to the the risk-neutral expected loss.
- Credit spreads are equal to the sum of the real-world expected loss and the expected excess return ("risk premium").

In addition, recovery rates may not be constant but also subject to systematic risk, and other factors like taxes and liquidity may influence spreads and prices. For CDS pricing, counterparty risk and the value of the delivery option are additional issues. These topics are discussed in more detail in subsection 2.3.3.

2.3.1.3. Structural models of default

Structural models model equity and debt as contingent claims on the company's assets. They can be applied to the pricing of both equity and debt. Structural models have, however, been applied to a much larger extent to the pricing of corporate debt. We therefore present the main literature on structural models of default in this section.

The most frequently cited structural model of default is the one developed by Black/Scholes (1973) and Merton (1974) ("Merton framework"). In this framework, the company's debt is assumed to consist of a single zero bond with face value N and maturity T. The company's assets are assumed to follow a geometric Brownian motion. At maturity, equity holders will pay back the debt if the asset value is above N and declare bankruptcy if the asset value has fallen below N. Therefore, the value of debt is equal to the value of a risk-free bond plus a short European put option on the company's assets, where the strike equals the nominal of the zero bond and the maturity equals the maturity of the zero bond. Formulas derived for European put options in Black/Scholes (1973) can be applied to determine the value of corporate debt.

The Merton framework has been extended by numerous authors to incorporate more realistic default mechanisms and different asset value processes. The Black/Cox (1976) model was the first first-passage time model. In this model, default is modeled as the first time that the asset value process crosses a certain default barrier. This allows for a default before the maturity of debt. Using a geometric Brownian motion and a constant default barrier, a closed-form solution exists. Geske (1977) introduced a model which captures interest-paying debt. Longstaff/Schwartz (1995) extended the model to allow for stochastic interest rates. They modeled interest rates with a Vasicek process and derived semi-closed-form solutions for defaultable bonds. Colin-Dufresne/Goldstein (2001) develop a model which allows for mean-reverting leverage ratios and explicitly analyze credit spreads in this context.

The models mentioned above treat the default barrier as exogenous. In contrast, Leland (1994) and Leland/Toft (1996) have developed a model with an endogenous default barrier. Equity owners will endogenously choose an optimal leverage and liquidation strategy to maximize the value of equity. Leland (1994) and Leland/Toft (1996) included taxes and bankruptcy costs in their model; therefore the theorem of Modigliani/Miller (1958) (irrelevancy of capital structure) no longer holds. In these settings, high bankruptcy costs make equity preferable to debt. The tax advantage of debt works in the opposite direction. Based on certain assumptions concerning the asset value process, tax advantage and bankruptcy costs, an optimal leverage and default barrier can be chosen. Strategic default models further develop this idea to allow for a strategic default, i.e., firms may default strategically and renegotiate debt contracts to extract concessions from bondholders. These models are based on the empirical observations that bankruptcy procedures include considerable opportunistic behavior and deviations from absolute priority (Franks/Torous (1989, 1994), Weiss (1990)). Strategic default models include the models of Anderson/Sundaresan (1996), Anderson/Sundaresan/Typhon (1996), and Mella-Barral/Perraudin (1997).

A main shortcoming of all models discussed above are the low short-term default probabilities implied by these models. Since default is modeled as a diffusion of a stochastic process, default is totally predictable and short-term (instantaneous) default probabilities converge to zero. Zhou (1997) introduced a model which allows for jumps in the asset value process, therefore resulting in more realistic short-term default probabilities. Duffie/Lando (2001) introduced the first structural model which yields a default intensity, bridging the gap between reduced-form models and structural models of default. In the Duffie/Lando model, asset values are assumed to be unobservable and investors only receive noisy information about the true asset value. They calculate explicit formulas for the distribution function of the asset value and for the default intensity. Other models with incomplete information were proposed by Gisecke (2004) and Coculescu et al. (2008). A commercial model (Credit Grades) has been implemented by Lardy et al. (2002). The models of Gisecke (2004) and Lardy et al. (2002) incorporate an uncertain default barrier, while the asset value is observable. Unfortunately, as long as the asset value is above its running minimum, the instantaneous default probability is also zero in this framework and a default intensity does not exist.

There are various studies which analyze the pricing performance of structural models of default. Jones et al. (1984) document that a simple Merton model generates spreads which are far below empirical observations for bond spreads of investment grade firms. However, one of the main problems when applying structural models of default is the calibration of input parameters. Subsequent work has shown that various structural models are indeed able to generate spreads which are in line with bond market observations, although the variations are quite large, cf. Eom et al. (2004) and Schaefer/Strebulaev (2008). There is also a growing literature on the pricing performance of structural models for CDS spreads. Predescu (2005) studies a Merton and Black/Cox setting, Hull et al. (2004a) analyze a Merton model, Chen et al. (2006) also include the Longstaff/Schwartz model. Other authors use simple regression analysis to link parameters that have an influence on credit spreads in structural models of default to CDS spread observations, cf. Cossin/Hricko (2001), Houweling/Vorst (2005), and Ericsson et al. (2006). Huang/Zhou (2008) determine the input parameters implicitly using time series of credit spreads for different maturities and a GMM estimation procedure. Most studies find that there are reasonable parameter combinations which could explain the credit spread, but it is very hard to determine the corresponding input parameters exogenously. Arora et al. (2005) test the ability of structural and reduced-form models to discriminate between defaulters and non-defaulters and find that these model classes yield similar accuracy ratios. They also analyze the pricing performance of an EDF-based structural model for pricing CDS and find that it even outperforms the Hull/White reduced-form model in most cases.

2.3.2. Estimation of expected loss

The expected loss[34] is the product of the probability of default (PD) and the recovery rate (RR).[35] The expected loss is crucial for several applications: First, it has been shown that prices react to new information on either default probabilities or recovery rates,[36] so any pricing model which ignores this piece of information is likely to be inferior to models that include PD and RR estimates. Second, PD and RR are needed to determine real-world quantities for a portfolio of bonds, loans, or CDS, e.g., the value-at-risk

[34]Within this subsection, all quantities are real-world quantities if not stated otherwise.
[35]The default probability can be either a cumulative quantity, a per-annum quantity, or an instantaneous quantity. Consequently, the expected loss can also be either cumulative, per annum or instantaneous.
[36]Cf. Hull et al. (2004b).

or expected loan loss provisions. Third, there is a growing literature which judges the performance of structural models on both actual and risk-neutral quantities, cf. Huang/Huang (2003), Chen et al. (2009). Fourth – and this is the main application within this thesis – the expected loss is necessary to determine risk premia from credit spreads and bond yields based on formulas (2.20) and (2.21).

2.3.2.1. Probability of default

Market participants mainly use three different sources for estimating default probabilities: Agencies' ratings, implementations of structural models of default and discriminant/hazard rate models.

Agencies' ratings: The three dominant rating agencies are Moody's, Fitch, and Standard & Poors (S&P). Agencies provide ratings for different issuer types (e.g. public sector, corporates, securitizations/SPVs), purposes (e.g., liquidity, credit risk) and seniorities (e.g., senior, subordinated). We will focus on the senior unsecured corporate rating in this section since our focus is on risk premia for corporate debt and senior unsecured is the dominant seniority for traded bonds and CDS. Senior unsecured ratings are denoted with Aaa, Aa, A, Baa, Ba, ..., C (Moody's) and AAA, AA, A, BBB, BB, ..., C (S&P, Fitch).[37] Subcategories (notches) have been established, which are denoted with 1, 2, 3 (Moody's) and +/- (S&P, Fitch) respectively, e.g., Aa1/Aa2/Aa3 and Aa+/Aa/Aa-. Ratings can be mapped to historical cumulative default probabilities for different maturities, i.e., $(Rating,\ Maturity) \rightarrow PD, \{Aaa, Aa, A, Baa, ..., C\} \times \{1, 2, ..., T\} \rightarrow [0, 1]$. These mappings are provided by the rating agencies and are publicly available.[38] A mapping of Moody's rating grades to historical default probabilities – which we will use in our empirical section – is depicted in appendix A.1.1.[39] There are two main aspects that have to be considered when deal-

[37]These are rating categories of non-defaulted issuers. The default rating category is usually denoted with D.

[38]Cf. Moody's (2008a), FitchRatings (2008), S&P (2008).

[39]The raw historical default probabilities provided by the rating agencies are usually smoothed either via a log/log-regression or via transition matrices and a Markov

ing with agencies' ratings: First, agencies' ratings are through-the-cycle ratings, i.e., agencies do not aim to assign ratings in such a way that default probabilities for a certain rating category are constant across time. Rather ratings are relative assessments of credit quality, cf. Löffler/Posch (2007), p. 73 ff. for an in-depth discussion. Therefore, a mapping of rating grades to *implied/current* default probabilities requires further information, e.g., the current point in the business cycle. However, agencies' ratings and the corresponding historical default probabilities may act as a good proxy for the average default probability over a whole economic cycle. Second, there is no coherent approach of the three main rating agencies whether their ratings are assessments of default probabilities or assessments of the expected loss.[40]

Implementation of structural models of default: Second, PD estimates can be gained from implementations of structural models of default. This is usually done pragmatically by practitioners. The most prominent implementation of a structural model of default is the KMV model (Moody's KMV (2007)). KMV calculates expected default frequencies (EDFs), which are point-in-time estimates for the default probability and available for maturities from 1 to 10 years. They are based on a calibration of a distance-to-default (DtD) measure[41] to default probabilities. For the application in this thesis, it is important to note that the calibration is done via a large historical database which is used to map KMVs distance-to-default measure to

assumption, cf. Bluhm et al. (2003) and appendix A.1.1 for details.

[40] Moody's current official document on its rating methodology states that "Moody's bond ratings are predictions of relative creditworthiness, which can be defined as a relative expected loss rate." (Moody's (2002)) while they state in the same document that "There is an expectation that ratings will, on average, relate to subsequent default frequency, although they typically are not defined as precise default rate estimates." Moody's (2008b) seems to indicate that ratings are assigned on an expected loss basis. Fitch states that "... issuers are typically assigned Issuer Default Ratings that are relative measures of default probability". (www.fitchratings.com, Rating Definitions, 13 Feb 2009). S&P indicates that issuer ratings are more PD-based than EL-based (cf. S&P (2009).

[41] The distance-to-default measures the number of standard deviations that the asset value of a company is away from the default barrier. The default barrier in the KMV model is defined as the short-term debt plus 50% of the long-term debt, cf. Moody's KMV (2007).

default probabilities. Therefore, no direct assumptions concerning the asset value process – especially the drift of the process – are necessary.

Discriminant analysis and hazard rate models: Third, models based on regression, discriminant analysis, and hazard rate models are used to estimate default probabilities. These models provide point-in-time estimates[42] for the default probability and use certain covariates to predict default probabilities. The first academic model was developed by Altman (1968), who used accounting variables[43] to predict one-year ahead default probabilities. It was later enhanced by Zmijewski (1984), who also discusses certain estimation biases which may arise in standard applications. Shumway (2001) developed the first academic multi-period hazard rate model and uses accounting variables as well as market-based measures (market capitalization, excess returns, idiosyncratic standard deviation).[44] Duffie et al. (2007) (continuous time) and Löffler/Maurer (2008) (discrete) are examples of hazard rate models which predict multi-year default probabilities including both accounting variables and market variables. Recently, FitchRatings (cf. FitchRatings (2007)) has also launched an equity-implied rating (Fitch-EIR) which is based on a hazard rate specification and also uses macro-variables to predict default.

The performance of rating models is usually measured via the accuracy ratio or area under curve (discrimination), the binomial test (calibration) and the Brier score (both discrimination and calibration), cf. Löffler/Posch

[42]Hazard rate models aim directly at estimating default probabilities, not only relative rankings of obligors. Therefore they usually provide point-in-time estimates for the default probabilities. However, these models may still fail to capture the impact of the economic cycle as long as covariates that may capture this economic cycle – such as equity returns, leverage and macro variables – are not included.

[43]Altman used the covariates working capital/total assets, retained earnings/total assets, EBIT/total assets, market value of equity/book value of total debt, and sales to total assets and determined default probabilities based on discriminant analysis. The resulting score is also called Z-score.

[44]These models have been used by regulators and practitioners to predict default probabilities before Shumway, cf. Shumway (2001). However, Shumway was the first to put this procedure on a sound statistical basis.

(2007) for details. Practitioners' models (KMV EDF, Fitch EIR) and modern hazard rate models which include market-based covariates usually have a better performance than agencies' ratings and pure accounting-based measures such as Altman's Z-score. The advantage of using market-based covariates is especially pronounced for short-term horizons and diminishes for longer horizons, cf. Löffler/Maurer (2008). For an overview of the performance of different approaches cf. Kealhofer (2000), Shumway (2001), Löffler (2004), Moody's KMV (2007), FitchRatings (2007), and Löffler/Maurer (2008).

2.3.2.2. Recovery rate

All rating agencies collect historical data on recovery rates. These differ by seniority (e.g., senior vs. subordinated) and type of loan (e.g., bank loan vs. bond). Moody's historical recovery rates are publicly available. The average senior unsecured recovery rate has been 35% (value-weighted) and 37% (issuer-weighted) from 1982-2007, cf. Moody's (2008). Altman/Kishore (1996) report an average senior unsecured recovery rate of 48% and a median of 41% for a sample from 1978-1995. Duffee (1999) and Driessen (2005) use a recovery rate of 44%.

Besides mean and median values, variations in recovery rates and explanatory factors for these variations are of major interest for empirical applications. All three major rating agencies have started to introduce recovery ratings which offer obligor-specific estimates for the recovery rate. However, these estimates are usually only provided for sub-investment grade entities, cf. Altman (2006) for an overview of recovery ratings. All studies find a significant dependence of the recovery rate on the seniority. E.g., Moody's (2008) reports average recovery rates as low as 17% for subordinated bonds and as high as 66% for senior secured bank loans. Similar, but slightly lower variations are reported in Altman/Kishore (1996). The industry sector seems to explain some variations in recovery rates, cf. Altman/Kishore (1996) and Moody's (2004), although most sectors have seen historical recovery rates between 30% and 50%. Recovery rate variations are significantly smaller than PD variations, cf. Moody's (2007a). Moody's (2007b) report only minor variations in recovery rates across industry sectors. The initial rating and the time between origination date and default date do not have any significant effect on the recovery rate (Altman/Kishore (2008)). The rating one year prior to default does seem to have some effect – with better ratings corresponding to higher recovery rates – but the effect diminishes if the rating three or more years prior to default is considered (Moody's (2007a)). Most studies report an impact of the economic cycle on average recovery rates – low recovery rates go hand in hand with economic downturns and with high default frequencies (Moody's (2007a), Altman et al. (2005), Hu/Perraudin (2002), and Frye (2000a,b)). In contrast, Carey/Gordy (2003) do not find a positive correlation between recovery rates and defaults. Chava et al. (2006) set up a model where the expected recovery rate can be explained by the coupon rate, the 3-month Treasury yield, the issue size, and the seniority. Other parameters do not add any significant improvement to the overall fit in their model. Moody's (2004) performs similar studies on a broader set of parameters. They find a significant effect for only a few firm-specific, industry-specific, and macroeconomic factors.

Markit, a provider of credit default swap data, also asks market participants to provide a (risk-neutral) recovery estimate together with each single CDS spread. These estimations are very close to 40%. Huang/Zhou (2009) report an average recovery rate of 40.30% based on Markit data. The estimates range from an average of 40.92% for Aa-rated obligors to 38.23% for B-rated obligors. Discussions with market participants also indicated that a value of 40% for the risk-neutral recovery rate is frequently used by practitioners for pricing purposes.

Taken together, reasonable estimates for the average senior unsecured recovery rate are between 35-45%. Recovery rates vary largely by seniority. In addition, recovery rates vary by industry and with the economic cycle. However, this variation is partly disputed and by far not as large as variations in the corresponding default probabilities. Other parameters only seem to have a minor effect on recovery rates.

2.3.3. Risk premia on credit markets

2.3.3.1. Bonds

Research on risk premia on corporate bonds has soared over the last decade, cf. Elton et al. (2001), Collin-Dufresne et al. (2001), Longstaff et al. (2005), Liu et al. (2007) and Bühler/Trapp (2008). Specifically, most researchers try to explain the spread on corporate bonds by certain parameters that have been proposed in various pricing models. Most studies define the bond spread as the difference between the yield on a corporate bond and the yield on government bonds. These government bonds act as a benchmark for a risk-free investment. There is still no consensus on the detailed decomposition of bond spreads into their components. Qualitatively, however, most researchers agree that the spread should be decomposed into four categories: The expected loss, a premium for bearing systematic risk, a liquidity premium, and a tax effect:[45]

$$\text{Bond spread} = \text{EL} + \text{RP}_{\text{Systematic risk}} + \text{RP}_{\text{Liquidity}} + \text{Tax effect}. \tag{2.26}$$

Studies do, however, vary in their assessment of the relative importance of these effects. Economically, the spread should equal the (real-world) expected loss if credit risk is totally diversifiable, all securities are totally liquid, and in the absence of taxes. If defaults (or recovery rates) are dependent on the business cyle – i.e., higher default rates coincide with poor equity returns – then a premium for systematic risk is justified. Liquidity premia are hard to capture in standard neoclassical models but are common knowledge among asset managers and can easily be included in reduced-form models,

[45] The sum of expected loss and the risk premium for systematic risk is usually called the credit risk premium. We will give an exact definition of the term credit risk premium for our applications in chapter 3.

cf. Duffie/Singleton (2003). Since bond spreads are usually subject to taxation, the respective tax rates or relative tax rates compared to risk-free instruments do of course matter when pricing corporate bonds.

Elton et al. (2001) explicitly determine the expected loss and the tax effect on corporate bonds. They derive the expected loss based on agencies' ratings and the corresponding historical loss rates. In their model, the tax effect arises due to the different tax treatment of corporate bonds and Treasury bonds in the U.S. After deducting the expected loss and the tax component, they analyze the remaining component and find that it strongly covaries with equity returns and is therefore likely to constitute a premium for systematic risk.[46]

Collin-Dufresne et al. (2001) analyze credit spread changes and find that only a part of these changes can be explained by factors that are usually assumed to play a role in standard structural models of default. However, they find that a common, unidentified systematic component drives credit spread changes. They conclude that bond spread changes are driven by local supply/demand shocks.

Liu et al. (2007) find, based on a reduced-form setup, that taxes account for 60%, 50%, and 37% of the corporate-Treasury yield spread for Aa, A, and Baa ratings. This proportion is higher for shorter maturities and lower for longer maturities.

Driessen (2005) analyzes the components of corporate bond spreads in a reduced-form setup and finds that for a Baa-rated, 10-year maturity bond, the tax effect explains approx. 30% of the total spread and liquidity accounts for 10-15% of the spread. These ratios increase for shorter maturities. He also finds that systematic risk is priced in bonds.

Bühler/Trapp (2008) find that the liquidity premium amounts to on average 35% of the total spread. The liquidity component is higher for higher rated bonds (AA: 48%, A: 44%, BBB: 34%, B: 15%). In addition, the liquidity

[46] However, a main shortcoming of this study is the use of constant, historical loss rates based on agencies' ratings. Since these are through-the-cycle ratings, it is not clear if the correlation with equity returns is partly also attributable to time-varying expected loss.

component is time-varying.[47]
A good overview of studies on bond risk premia and their magnitude can also be found in Hull et al. (2005). Hull et al. (2005) conclude that a reasonable estimate for a liquidity premium on corporate bonds is between 10 and 25 bp.
Finally, Longstaff et al. (2005) document that bond spreads heavily depend on the definition of the risk-free rate. They analyze Treasuries, the standard risk-free benchmark in academic studies, RefCorp bonds, which are guaranteed by the Treasury[48] and Swap rates, which are usually used as risk-free rates by practitioners. The difference between Treasuries and RefCorp bonds is approx. 20 bp on average, the difference between Treasuries and swap rates is approx. 60 bp.

Taken together, the literature suggests that i) systematic risk is priced in corporate bonds, ii) liquidity makes up a significant part of the bond spread, and iii) the bond spread and the part of the spread which is attributable to taxes is heavily dependent on the definition of the risk-free rate (risk-free rate problem).

2.3.3.2. Credit default swaps

CDS spreads are seen as a purer measure of credit risk in the academic literature, cf. Longstaff et al. (2005), Ericsson et al. (2006), Bühler/Trapp (2008), and Huang/Zhou (2009). CDS provide a direct spread measure, they therefore avoid the problem of defining an appropriate risk-free rate. In addition, in contrast to bonds, CDS are unfunded exposures and are not in

[47] It should be noted that Bühler/Trapp (2008) force their model to split the total spread into a credit risk component, a liquidity component, and a correlation component, i.e. these three parts always add up to 100%. The credit risk component captures both expected loss and a premium for systematic risk. Any potential tax effects will be included either in the credit risk or liquidity component.

[48] RefCorp bonds are bonds issued by the Resulution Funding Corporation, which is a government agency. They have the same credit risk as Treasury bonds since they are guaranteed by the Treasury. In addition, they receive the same tax treatment as U.S. Treasury bonds. However, they are less liquid, cf. also Longstaff (2004) and Hull et al. (2005).

fixed supply. Therefore, they are much less prone to liquidity distortions. For intuition, assume that an investor requires a riskless return of 5% for liquid government bonds, a liquidity premium of 50 bp for corporate bonds, and a credit risk premium (EL + systematic risk) of 100 bp. A portfolio of a corporate bond (long) and a CDS (as protection buyer) is (credit) risk-free but is not as liquid as a government bond. The CDS spread is therefore more likely to be 100 bp than 150 bp.

Based on these and similar arguments, Longstaff et al. (2005) argue that CDS are pure measures of credit risk. Bühler/Trapp (2008) formalized this idea in a reduced-form model and estimated the part of the spread which is due to a liquidity premium to be only 4% on average compared to 35% for bonds for a sample of Euro area CDS. Ericsson et al. (2006) analyze CDS residuals after adjusting the CDS spread for model-implied pure credit spreads and find that – in contrast to bond residuals – no relationship between these CDS residuals and liquidity proxies exists. They conclude that CDS spreads do not seem to be as prone to non-default factors as bond spreads.

Tang/Yan (2007) analyze liquidity costs and liquidity risk in a market microstructure setting based on adverse selection, search frictions, and inventory costs. They find that CDS sellers capture a liquidity risk premium in the CDS market; therefore, the fair CDS spread is likely to be closer to the bid than to the ask spread. The estimates for their total sample are 10% (liquidity costs) and 20% (liquidity costs + liquidity risk). However, the period considered by Tang/Yan (June 1997 to March 2006) covers an early period of the CDS market and their sample also includes very infrequently traded counterparties. Tang/Yan also point out the fact that CDS liquidity has strongly increased over their sample period and that actively traded names give rise to a much smaller liquidity premium. Therefore, these findings may well be consistent with the magnitudes reported in Bühler/Trapp (2008). CDS market liquidity has strongly increased over the last years, especially since the introduction of index trading in 2003/2004, cf. Jakola (2006) and FitchRatings (2006).

Besides less liquidity distortions, the CDS market also seems to be quite efficient in incorporating new information: Hull et al. (2004b) find that credit rating announcements are anticipatd in the CDS market, Norden/Weber (2004) find that the CDS market leads the stock market in case of reviews for downgrades. In addition, the market volume of CDS has become larger than the volume of corporate bonds.[49]

While using CDS spreads instead of bond spreads has advantages concerning tax and liquidity effects, there are also some disadvantages. First, CDS are subject to counterparty risk. Ceteri paribus, this should decrease CDS spreads. Longstaff et al. (2005) argue that this counterparty risk is likely to be very small. However, since the bankruptcy of Lehman Brothers the perception of counterparty risk may well have changed in the market. Unfortunately, there are no comprehensive post-Lehman studies of the effect of counterparty risk available yet. Second, CDS contracts involve a delivery option for the protection buyer. Ceteri paribus, this effect should decrease the recovery rate on CDS and therefore increase CDS spreads. However, the main impact of this delivery option is likely to come from restructuring events. The ISDA has developed certain restructuring clauses for CDS contracts which are mainly aimed at minimizing this delivery option. An overview of the potential effects can be found in Mithal (2002) and J.P. Morgan (2004).

2.3.3.3. Synopsis: Risk premia on credit markets

Taken together, the literature on credit risk premia suggests that:

- Systematic risk is priced in both bonds and CDS.
- Bond spreads are heavily influenced by the choice of the risk-free rate (risk-free rate problem). This is of special importance in the U.S.,

[49] The amount outstanding of worldwide corporate bonds as of December 2008 (floating and fixed rate) was approximately 2,200 bn US-$, the nominal value of single-name CDS contracts was 33,000 bn US-$ with a gross market value of 1,900 bn US-$ (as of June 2008), cf. BIS (2009). Although this CDS volume also includes sovereigns and financial institutions, a large part is likely to be corporate CDS.

where government bonds have a tax advantage compared to corporate bonds. CDS spreads are not subject to the risk-free rate problem.[50]

- Bond spreads incorporate a significant liquidity premium. CDS spreads may incorporate a liquidity premium due to market microstructure effects. However, this premium is likely to be small ($\approx 5\%$) for frequently traded CDS counterparts, e.g., constituents of one of the main CDS indices in the U.S., Europe, and Asia.

- Theory suggests that CDS spreads are affected by counterparty risk and a delivery option. However, the delivery option is likely to have only a minor effect on CDS spreads due to limitations with respect to potentially deliverable bonds. Most studies also assume that counterparty risk has only a minor effect on CDS spreads. However, the effect of counterparty risk may have to be scrutinized in more detail after the Lehman default.

Throughout this thesis, we will follow the standard argumentation in the literature and assume that CDS spreads are a pure measure of credit risk and that counterparty risk and delivery options can be neglected, i.e.,

$$\text{CDS spread} = \text{EL} + \text{RP}_{\text{Systematic risk}} \tag{2.27}$$

We also do not incorporate a tax premium. In this sense, the premium for systematic risk is a pre-tax risk premium. Of course, if risk premia on debt and equity markets are compared, the different tax treatment may result in different pre-tax risk premia. We will discuss this issue briefly in chapters 3-5.

[50] In simple models with continuous spread payments and constant default intensity, the valuation of a CDS is also not dependent on the risk-free rate, cf. (2.25). If spreads are not payed continuously and default intensities are not constant, a very small impact of the risk-free rate on the CDS spread arises, cf. Duffie/Singleton (2003) and Schönbucher (2003).

2.4. The link between equity and credit risk premia

Qualitative link: The idea that risk premia on credit markets and equity markets are interrelated is not new. Keim/Stambaugh (1986) find that the spread between yields on low-grade corporate bonds and the yield on Treasury bills predicts future stock market excess returns. They interpret this as a time-varying expected return, i.e., a higher spread between corporate bonds and Treasuries indicates higher risk aversion and therefore higher future expected returns. Fama/French (1989) find that "predictable variation in stock return is [...] tracked by variables commonly used to measure default and term premiums in bond returns". They measure the default premium as the difference in yields between a market portfolio of corporate bonds and Aaa-rated bonds. In these cases, credit spreads are used as an indicator for risk aversion. This establishes a qualitative link without making any predictions about the quantitative link. In addition, these approximations are rough since credit spreads of course reflect not only risk aversion but also expected loss, and – to a certain extent – liquidity and tax premia (cf. section 2.3.3). Other papers using similar approximations include Jagannathan/Wang (1996) and Chen et al. (1986).

Beta-based link: Cornell/Greene (1991) and Fama/French (1993) go one step further and calculate CAPM betas of bonds. Cornell/Greene (1991) derive betas of 0.25 for high-grade bond funds compared to 0.52 for low-grade bond funds.[51] Fama/French (1993) derive betas of approx. 0.20 for investment grade bonds and 0.30 for sub-investment grade bonds. These values could be used to determine equity premia if bond spreads and the expected loss are known. However, this is a very inaccurate procedure due to estimation problems for beta, liquidity, and tax distortions in bond spreads and the assumptions that the CAPM can be directly applied to bonds.

[51] Funds are defined as low-grade bond funds in Cornell/Greene (1991) if at least two thirds of the portfolio is invested in bonds rated Baa or lower. The remaining funds are defined as high-grade funds.

Link via structural models of default: The first papers that exploited simple relationships in the Merton framework to estimate Sharpe ratios implied from bond spreads and estimates for the real-world default probability include Bohn (2000), Kealhofer (2003b), and Berg/Willershausen (2005). However, Duffie/Singleton (2003) point out the fact that these formulas are formally justified only in a Merton framework.
Huang/Huang (2003) calibrate various structural models of default to historical default probabilities and historical equity premia including a model with jumps, stochastic leverage, and mean-reverting asset risk premia. They find that only a small portion of bond spreads can be explained by credit risk. Especially for investment grade bonds, they are only able to explain 20-30% of the spread with credit risk. This is in line with the findings presented in section 2.3.3 about liquidity and tax effects on corporate bonds.
Chen et al. (2009) use structural models of default with countercyclical default boundaries and time-varying asset premia based on the framework developed by Campbell/Cochrane (1999) to fit historical yield spreads to historical default probabilities and historical Sharpe ratio.
Cooper/Davydenko (2003) use a simple Merton model to extract estimates for the expected loss from bond spreads and equity premium estimates. They also briefly discuss and apply their procedure to estimate equity premia from bond spreads and historical loss rates and find an average equity premium of 4.8% with a range from 3.1% for Aa-rated companies to 8.5% for B-rated companies. However, their procedure requires estimates for the default barrier – which they assume to be equal to the face value of debt – and of the (average) equity volatility until maturity.
All in all, these approaches have gained limited acceptance so far for several reasons. First, all of the models above analyze bond spreads which are subject to liquidity and tax distortions as discussed in the last subsection.[52] Second, most of these models use aggregate or average data per rating

[52]An exception is Berg/Willershausen (2005), who also analyze CDS spreads. They do, however, analyze only a very small sample (one day, approx. 20 German companies in the DAX).

grade, ignoring convexity or concavity effects, cf. David (2007). Third, many of these models rely on estimates of parameters which are usually hard to determine, e.g. default barriers, asset volatilities, etc. Fourth, the derived relationships usually only hold for specific models, so model robustness is a crucial point.[53]

Note on risk premia in reduced-form models: Within structural models, risk premia arise due to different drifts of the asset value process in the real and risk-neutral world. In contrast, in reduced-form models, risk premia are either captured by a drift adjustment of the intensity process or by different $t = 0$ real-world and risk-neutral default intensities (jump-to-default premium), cf. Jarrow et al. (2005). Several studies analyze risk premia in a reduced-form setup, i.e., Duffee (1999), Berndt et al. (2005) and Driessen (2005) and the results are usually not directly comparable to bond and CDS premia of the form (2.26) and (2.27). However, we do want to point out that reduced-form and structural models can be linked. The first structural model that yields a default intensity was developed by Duffie/Lando (2001). Both the drift adjustment in the default intensity process and the jump-to-default premium are driven by the Sharpe ratio of the underlying asset value process. However, the jump-to-default premium is also influenced by the degree of asset value uncertainty. Coculescu et al. (2008) provide a generalization of the Duffie/Lando model with a continuous observation process and refer to this risk premium as "imperfect information" risk premium. Economically, the idea can best be explained by the delayed observation idea: If observations of asset values are noisy, investors also have to rely on older observations. E.g., as a simple case one might assume that an investor knows the asset value one year ago with certainty but does not have any information on today's asset value. In this case, the investor is also exposed to systematic changes in asset values in the *past* and not only to systematic future asset value changes. However, the underlying driver is

[53]Although Huang/Huang (2003) analyze several structural models of default, they also do not include a model which is consistent with reduced-form pricing such as the model developed by Duffie/Lando (2001).

the same in both cases, i.e., the systematic risk of the company's cash flows and the Sharpe ratio of the company's assets.

Synopsis: Taken together, it will be of major importance for our empirical application to i) develop a model which relies on observable parameters as far as possible, ii) test the robustness of our model, especially also with respect to advanced models such as the Duffie/Lando framework which are compatible with reduced-form pricing, iii) use company-level data instead of aggregate/mean data, and iv) use CDS spreads instead of bond spreads.

In addition, most academic research described above uses structural models and estimates for Sharpe ratios/equity premia to estimate credit spreads. However, there is a very large sensitivity of these calculations with respect to the Sharpe ratio/equity premium assumption (cf. Chen et al. (2009)). We will argue in chapter 3 that these models are much better suited to go the opposite direction, i.e., to derive Sharpe ratios and equity premia from credit spreads. Used this way around, the disadvantage of a high sensitivity with respect to the Sharpe ratio/equity premium turns out to be an advantage: The formulas are quite stable if used the other way around.

3. From actual to risk-neutral default probabilities

This chapter establishes the theoretical link between actual and risk-neutral default probabilities within structural models of default.[1] The difference between actual and risk-neutral default probabilities is called the credit risk premium. We will analyze the functional form and the drivers of credit risk premia in a simple Merton framework and in more advanced structural models of default.

Section 3.1 starts with a motivation. It gives some intuition why this difference is a key to understanding credit markets and why it is so important for pricing bonds and credit default swaps. We will argue that credit markets are different e.g. from option markets because market participants in the credit markets actually do use real-world quantities for pricing purposes. We will also give intuition for the main results in this chapter: First, risk premia constitute a significant part of the credit spread and second, risk premia implied by a simple Merton model are also a good proxy in more advanced structural models. Section 3.3 formalizes these ideas for the Merton framework, section 3.4 analyzes the Duffie/Lando (2001) model and – as special cases – the Black/Cox (1976) and Leland/Toft (1996) model.

3.1. Motivation and intuition

Let us look at an investor who wants to price the "first" bond/CDS/loan of a company X. With "first", we mean that there are no other *credit* instruments

[1]For this chapter cf. also Berg (2009a).

that an investor can use for pricing purposes. Of course, if spreads for a 3-year and a 4-year bond of company X are known, these are reasonable bounds for a 3.5-year maturity bond of company X. Or, if spreads of bonds are known, spreads of credit default swaps can be determined based on simple arbitrage arguments and vice verca.[2] In simple terms: If we know that one of the credit instruments has a spread of 50 bp (300 bp), then all similar credit instruments will also have a spread of approx. 50 bp (300 bp). But how do we know, if the "fair" spread of any of these instruments should be 50 bp or 300 bp? There are mainly three categories of pricing models that an investor can use:

1. Determine risk-neutral default intensities and use the respective reduced-form models to determine the correct spread and price (cf. section 2.3.1.2).

2. Choose a (structural) model of default, calibrate all parameters and determine spread and price (cf. section 2.3.1.3).

3. Determine the real-world default probability and real-world expected loss and transform this real-world expected loss to a spread / risk-neutral default probability by adding a risk premium (cf. section 2.3.1.1).

The first approach requires an estimate of the risk-neutral default intensity. This risk-neutral default intensity can possibly be derived from any debt security of company X. This approach is therefore a good approach to price the "second" debt instrument of company X, but it is not suited to find a spread for the "first" or for all debt instruments. In practice, a calibration of the risk-neutral default intensity without using any debt instruments of company X itself (or a similar company) is not straigthforward.

The second approach is also not suited for practical applications due to the problems of calibrating the input parameters of the structural model,

[2]In practice, even the derivation of CDS spreads from bond spreads is not straightforward due to liquidity premia, counterparty risk and market microstructure issues, cf. Hull et al. (2005) and Longstaff et al. (2005).

especially the default barrier and the correct type of structural model (e.g. first-passage- vs. zero-bond-style). Estimation errors and model errors are usually too large to derive any meaningful price. Academic studies therefore come to conflicting evidence concerning the pricing performance of structural models (cf. section 2.3.1.3).

The third approach is the only one which is practically applicable. Accepted estimates for real-world default probabilities are widely available, e.g. from agencies' ratings, models such as KMV EDFs or Fitch EIRs and bank's internal models. Indeed, ratings are a predominant factor in the market for bonds and CDS and most communication in the market is done via PDs and recovery rates (RR).[3] Availability of recovery rate estimates is also increasing, e.g., all major rating agencies have introduced recovery rate ratings, commercial tools such as CreditEdge+ provide recovery rate estimates and academic research has also soared over the last years, cf. Altman (2006) for an overview.

This third – hybrid – approach requires two steps: First, an estimation of the real-world default probability and real-world expected loss. Second, a methodology for adding a risk premium. This chapter deals with the second step. We derive two main results which are briefly explained based on intuitive arguments in the next paragraphs.

The first result is related to the relative importance of the risk premium: The risk premium is a significant part of the model-implied credit spread.[4] For all investment grade obligors it is larger than the real-world expected loss for reasonable parameter combinations. E.g., for a Baa-rated obligor the one-year expected loss is not more than 20 bp.[5] The risk premium for equi-

[3] E.g., the market is clustered into investment grade and sub-investment grade using agencies' ratings, reports on the credit market frequently make statements on the (expected) development of foreclosures/defaults and migration probabilities. In contrast, we have never seen a report about exercise probabilities e.g. in the S&P 500 option market.

[4] Without liquidity and tax premia.

[5] Based on Moody's (2007), cf. appendix A.1.1.

ties is usually supposed to be in the area of 5%, i.e. 500 bp. If a Baa-rated bond is only 1/20 as "risky" as equities then the risk premium will exceed the expected loss. Intuition tells us that this is the case. E.g., in a CAPM setting, a beta of only 0.05 is necessary to induce such a risk premium.[6] We will more formally discuss this issue in section 3.3.

The second result concerns the difference between the Merton framework and other structural models of default: The risk premium, i.e., the difference between actual and risk-neutral default probability, will stay (almost) the same independent of the model choice and independent of parameters such as the asset volatility or the asset value uncertainty. The only parameter that does influence the risk premium is of course the Sharpe ratio of the asset value process.[7] This may seem amazing at first, but there is also a good intuition for it: All "features" that have been added to the Merton framework usually have an effect on both actual and risk-neutral default probabilities in the same direction. To mention just a few examples:

- If we move from a zero-bond style model to a first-passage time model, both actual and risk-neutral default probability will increase.
- If we increase the default barrier, both actual and risk-neutral default probabilities will increase.
- If we introduce asset value uncertainty, both actual and risk-neutral default probabilities will increase.
- If we increase the asset volatility, both actual and risk-neutral default probabilities will increase.

Again, this result is more formally analyzed in section 3.4.

[6] Cornell/Green (1991) find betas between 0.25 and 0.50 for bonds, Fama/French (1993) find betas between 0.19 and 0.30, cf. section 2.4.

[7] One might criticise this approach for rejecting structural models of default due to calibration problems, but at the same time using structural models to transform real-world quantities into risk-neutral quantities. In fact, we do not state that structural models are per se wrong, rather it takes a lot of effort to calibrate them correctly so that more convenient ways have been found to predict real-world PDs.

3.2. Definition of absolute and relative credit risk premia

The derivation of a credit spread from an expected loss corresponds to the derivation of risk-neutral quantities from real-world quantities. If recovery rates are assumed to be constant, the remaining task is to derive risk-neutral default probabilities from actual default probabilities. The difference between risk-neutral and actual default proabibilities will be denoted as "credit risk premium". Similar to Amato/Remolona (2005) and Hull et al. (2005) we will define the absolute credit risk premium ($AbsCRP$) as the difference and the relative credit risk premium ($RelCRP$) as the quotient of risk-neutral and actual PDs:[8]

Definition 3.2.1 *(Absolute and relative credit risk premium)*

- *The absolute credit risk premium (AbsCRP) is defined as the difference between the cumulative risk neutral (PD^Q) and the cumulative actual default probability (PD^P), i.e.*

$$AbsCRP := PD^Q - PD^P$$

- *The relative credit risk premium (RelCRP) is defined as the absolute credit risk premium divided by the actual default probability, i.e.*

$$RelCRP := \frac{PD^Q - PD^P}{PD^P} = \frac{PD^Q}{PD^P} - 1$$

Please note that all quantities are cumulative quantities. Economically, a constant absolute credit risk premium means that investors require the same excess return independent of the credit quality. A constant relative credit risk premium would imply that investors require the same excess return per unit of default probability independent of the credit quality.

[8] Amato/Remolona (2005) use the term "risk premium" for the absolute credit risk premium, "price of default risk" for the relative credit risk premium and "risk adjustment" for the relative credit risk premium minus 1.

We will analyze the absolute and relative credit risk premium within structural models of default. Unlike classical applications of structural models, we are therefore not interested in the actual and risk neutral default probabilities itself but are simply interested in the *difference* (or quotient) of actual and risk-neutral default probabilities. As we will see, this makes most of the calibration process usually needed redundant.

3.3. Merton framework

3.3.1. Model setup

In the Merton framework, asset values are assumed to follow a geometric Brownian motion. Debt is modeled via a single zero bond with maturity T. A default can therefore only occur at maturity. More formally, we will assume

$$\begin{array}{rcl} \text{Asset value process}: & dV_t = \mu V_t dt + \sigma V_t dW_t & (\text{real world}) \\ & dV_t = r V_t dt + \sigma V_t dW_t & (\text{risk}-\text{neutral world}) \\ \text{Default mechanism}: & PD^P = P[V_T < N] & (\text{real world}) \\ & PD^Q = Q[V_T < N] & (\text{risk}-\text{neutral world}) \end{array}$$

where V_t is the asset value process, μ and r are the actual and risk-neutral drift, σ denotes the asset volatility, W_t is a Brownian motion and N is the face value of the zero bond.

3.3.2. Credit risk premia in the Merton framework

In the Merton framework, the real-world default probability ($PD^P(T)$) between $t = 0$ and $t = T$ can be calculated as:[9]

$$\begin{aligned} PD^P(T) &= P[\ V_T < N\ ] = P[\ V_0 \cdot e^{(\mu - \frac{1}{2}\sigma^2)\cdot T + \sigma \cdot B_T} < N\ ] \\ &= P\left[\ \sigma \cdot B_T < ln\left(\frac{N}{V_0}\right) - (\mu - \frac{1}{2}\sigma^2)\cdot T\ \right] \end{aligned}$$

[9] See Duffie/Singleton (2003) or Berg/Kaserer (2008) for details.

$$= \Phi \left[\frac{ln\frac{N}{V_0} - (\mu - \frac{1}{2}\sigma^2) \cdot T}{\sigma \cdot \sqrt{T}} \right] \tag{3.1}$$

The risk-neutral default probability can be calculated accordingly as

$$PD^Q(T) = Q[\, V_T < N \,] = \Phi \left[\frac{ln\frac{N}{V_0} - (r - \frac{1}{2}\sigma^2) \cdot T}{\sigma \cdot \sqrt{T}} \right]. \tag{3.2}$$

Combining (3.1) and (3.2) yields:

$$PD^Q(PD^P) = \Phi \left[\Phi^{-1}(PD^P(T)) + \frac{\mu - r}{\sigma} \cdot \sqrt{T} \right]. \tag{3.3}$$

The relationship between the risk-neutral and the actual default probability is therefore independent of the asset value V_t, the nominal value of the zero bond N and the asset volatility. Only the actual default probability, the asset Sharpe ratio ($SR_V := \frac{\mu - r}{\sigma}$) and the maturity enter the formula. A graphical illustration of the relationship between risk-neutral and actual default probabilities, the Sharpe ratio, and maturity is given in Figure 3.1.

3.3.3. Implications

3.3.3.1. The relative importance of risk premia

The relative importance of (model-implied) risk premia for credit spreads is often underestimated. An overview of the literature on credit risk premia can be found in Hull et al. (2005) and Amato (2005). To demonstrate the relative importance of risk premia, we first transform cumulative PDs taken

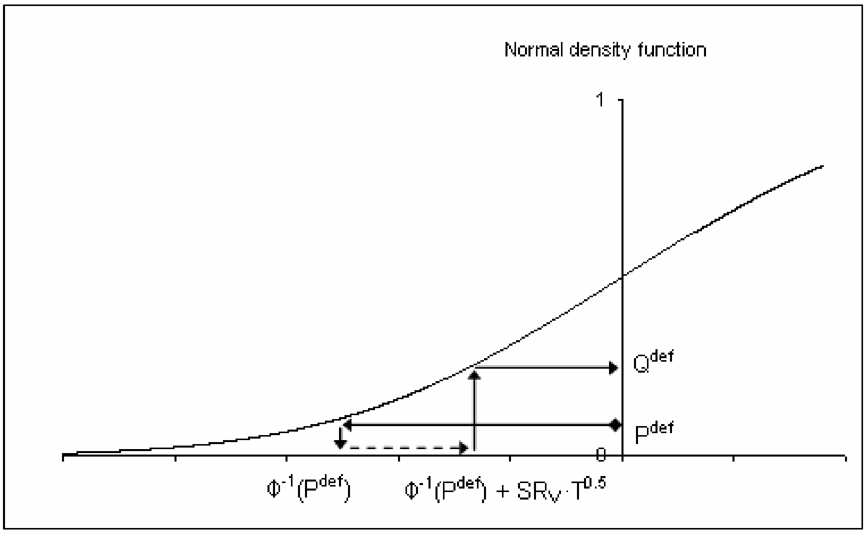

Figure 3.1.: Illustration of the relationship between actual and risk-neutral default probabilities in the Merton framework. PD^{def}: actual cumulative default probability, PD^Q: risk-neutral cumulative default probability, SR_V: Sharpe ratio of the assets, T: maturity.

from Moody's (2007) to per annum real-world PDs and ELs via[10]

$$PD^P_{pa} = 1 - (1 - PD^P_{cum})^{1/T}, \quad EL^P_{pa} = PD^P_{pa} \cdot LGD,$$

where subscripts "pa" and "cum" denote per annum and cumlative quantities, T denotes the maturity, superindex "P" denotes a real-world quantity and LGD is the loss given default, assumed to be constant and therefore equal in the real and risk-neutral world. We then calculate model-implied

[10]This transformation assumes constant conditional default probabilities. Our key results are not significantly influenced by this assumption. This assumption was taken for simplicity reasons to make the formulas easily readable. Alternatively, a constant hazard rate model could be applied, i.e. $\lambda^P = 1/T \cdot ln(1 - PD^P_{cum})$. where λ^P denotes the real-world hazard rate. Again, the differences for the following analysis are minimal.

spreads s via[11]

$$PD^Q_{cum} = f(PD^P_{cum})\,\text{via}\,(3.3),\;\; PD^Q_{pa} = 1-(1-PD_{cum})^{1/T},\;\; s = PD^Q_{pa}\cdot LGD,$$

i.e. in the first step, we transform the actual cumulative default probability to a risk-neutral cumulative default probability via (3.3), in the second step we annualize this cumulative risk-neutral PD, and in the third step we transform it into a risk-neutral EL / credit spread by multiplying the risk-neutral PD with the loss given default.

In order to calculate the risk-neutral default probabilities via (3.3) the asset Sharpe ratio has to be specified. Historical market Sharpe ratios for equity markets have been approximately 40%. Using an asset/market-correlation of 0.5 this can be transformed into an asset Sharpe ratio of 20% which we use as a base case. In addition we set $LGD = 60\%$ based on Moody's (2007). The resulting parameters are depicted in table 3.1 for the main CDS maturities of 3, 5, 7 and 10 years. In addition, the table shows the part of the spread which is due to a risk premium, defined as $1-EL/spread$. For investment grade obligors, the risk premium accounts for approx. 2/3 of the spread, for sub-investment grade obligors for approx. 1/2 of the spread. The part of the spread due to the risk premium increases with increasing maturity.

The asset Sharpe ratio itself is hard to determine accurately. First, historical Sharpe ratios on equity markets might not be equal to current market-implied Sharpe ratios. Second, correlations between assets and the market portfolio are not easy to determine – at least not on a single-obligor basis. Third, credit instruments are usually subject to higher tax rates in most countries. This would also affect before-tax Sharpe ratios.

[11] For underlying assumptions see footnote 10. In addition we assume that default can only happen at the end of each period so that the default-leg=payment-leg condition becomes $LGD \cdot \sum(1-PD^Q_{pa})^{t-1}\cdot PD^Q_{pa}\cdot(1+r)^{-t} = s\cdot\sum(1-PD^Q_{pa})^{t-1}(1+r)^{-t}$ so that $s = PD^Q_{pa}\cdot LGD$.

	T = 3				T = 5			
	PD_{cum} (%)	EL p.a. (bp)	s (bp)	Risk Prem (% of s)	PD_{cum} (%)	EL p.a. (bp)	s (bp)	Risk Prem (% of s)
Aa	0.05%	1	3	69%	0.17%	2	8	74%
A	0.23%	5	13	64%	0.60%	7	24	69%
Baa	1.00%	20	48	58%	2.17%	26	71	63%
Ba	4.41%	90	180	50%	7.86%	97	215	55%
B	19.42%	417	680	39%	28.41%	388	678	43%

	T = 7				T = 10			
	PD_{cum} (%)	EL p.a. (bp)	s (bp)	Risk Prem (% of s)	PD_{cum} (%)	EL p.a. (bp)	s (bp)	Risk Prem (% of s)
Aa	0.26%	2	10	78%	0.34%	2	11	82%
A	0.90%	8	29	73%	1.17%	7	31	78%
Baa	3.11%	27	81	67%	4.06%	25	85	71%
Ba	10.72%	96	229	58%	14.05%	90	234	61%
B	37.01%	383	696	45%	48.62%	386	727	47%

Table 3.1.:
Relative importance of credit risk premia in the Merton framework for an asset Sharpe ratio of 20%.

This table depicts the part of the credit spread which is due to a risk premium based on a Merton framework for maturities of T = 3, 5, 7 amd 10 years and different rating grades. PD_{cum} denotes cumulative real-world default probability, *EL p.a.* denotes annualized real-world expected loss in basis points, s denotes the credit spread and *Risk Prem* denotes the percentage of the spread which is due to a risk premium ($:= 1 - EL/s$). LGD was assumed to be 60%.

Asset Sharpe ratio (%)	PD_{cum} (%)	EL p.a. (bp)	s (bp)	Risk Prem (% of s)
10%	2.17%	26	44	40%
20%	2.17%	26	71	63%
30%	2.17%	26	111	76%
40%	2.17%	26	165	84%
50%	2.17%	26	239	89%

Table 3.2.:
Sensitivity of credit spread with respect to asset Sharpe ratio assumption (Baa, 5-years).
Percentage of credit spread due to a risk premium based on a Merton framework for a maturity of T = 5 and a Baa-rating for different asset Sharpe ratio assumptions. Column labels as in table 3.1.

We calculate the resulting spread for different assumptions for the asset Sharpe ratio for the Baa, 5-year maturity case (table 3.2). The spread is highly dependent on the assumption about the asset Sharpe ratio, indeed the spread can be almost 10 times the expected loss for an asset Sharpe ratio of 50%. This high sensitivity of the (model-implied) spread with respect to the Sharpe ratio assumption has several implications. First, if changes in the asset Sharpe ratio are hard to explain, then spread movements will also be hard to explain. Second, standard proxies for the real-world default probability will not be as important in explaining spread changes as some researchers might assume. This theoretical analysis confirms findings e.g. of Elton et al. (2001). Third, credit spreads provide a good way of extracting risk aversion. If two thirds of the spread are due to a risk premium, then even with a noisy estimate for the real-world expected loss, risk premia can be extracted with a reasonable accuracy. We will exploit this finding in our empirical applications in chapter 4 and 5.

3.3.3.2. Functional form of credit risk premia

Based on (3.3) we now want to formulate the main characteristics of credit risk premia in the Merton framework. For ease of notation we will omit the variable T and use the notations $PD^Q := PD^Q(T)$ and $PD^P := PD^P(T)$.

Proposition 3.3.1 *The following statements about the risk neutral default probability, the absolute and the relative credit risk premium hold true as long as the asset Sharpe ratio SR and the maturity T are larger than zero:*

1. *The risk-neutral default probability as a function of the actual default probability is increasing and concave with $PD^Q(PD^P) \geq PD^P$, $PD^Q(0) = 0$ and $PD^Q(1) = 1$*

2. *The absolute credit risk premium $AbsCRP$ as a function of the actual default probability is increasing and concave for $PD^P < \Phi[-0.5 \cdot SR\sqrt{T}]$ with $AbsCRP(0) = 0$*

3. *The relative credit risk premium $RelCRP$ as a function of the actual default probability is decreasing and convex for $PD^P < 50\%$ with $\lim_{PD^P \to 0} RelCRP = \infty$ and $RelCRP(1) = 0$*

The proof can be found in appendix B.2.[12] Proposition 3.3.1 is illustrated in figure 3.2.

(1.) and (2.) state that risk-neutral default probabilities are larger than actual default probabilities. This simply means, that – if agents are risk averse – they will demand a higher return on risky bonds than on riskless assets. The difference is zero for $PD^P = 0$ and it is higher for larger actual default probabilities. This is in accordance with the general idea that investors require a higher return for higher risk and that risk increases with increasing default probability. The upper limit is not relevant for practical purposes.[13]

(3.) states that the ratio of risk-neutral to actual default probabilities will increase with increasing credit quality, e.g. the ratio will be higher for Aa-rated obligors than for Ba-rated obligors. In addition, the increase will be

[12] Please note that the restrictions concerning PD^P are sufficient but not necessary, see appendix B.2 for details.

[13] Please note, that the upper limit $\Phi[-0.5 \cdot SR_V \cdot \sqrt{T}]$ decreases with increasing Sharpe ratio and increasing maturity. Even for a maturity of 10 years and a Sharpe ratio of 40% this upper bound is approx. 26% which is somewhere between a Ba and B rating (cf. appendix A.1.1). For higher default probabilities the difference decreases, but this is purely a mathematical exercise. For very high default probabilities the recovery rate risk is more important than the default risk. Therefore, the difference between risk neutral and actual expected loss may still rise further.

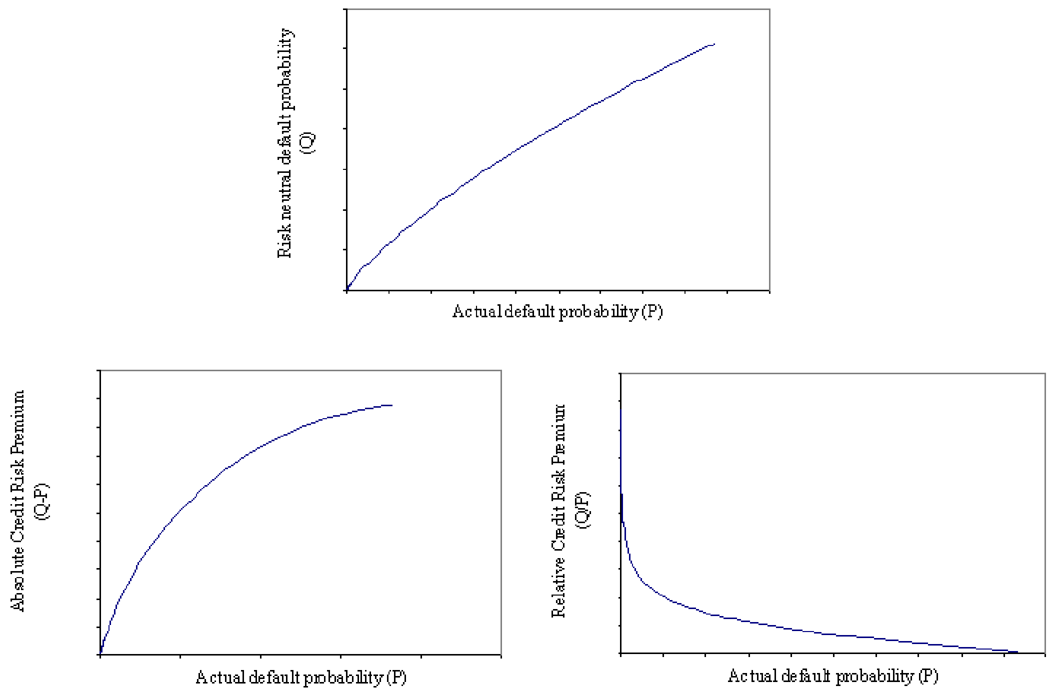

Figure 3.2.: Illustration of the relationship between actual cumulative default probability and the risk-neutral cumulative default probability, the absolute and the relative credit risk premium in the Merton framework. $P = PD^P$ and $Q = PD^Q$ denote the cumulative actual and cumulative risk-neutral default probability.

steeper than linear and the ratio will converge to infinity if the actual default probability approaches zero. Some authors (e.g. Amato (2005)) directly use the relative credit risk premium as a measure of risk aversion. Based on the Merton model, the relative credit risk premium is, however, not only dependent on the risk aversion (i.e. Sharpe ratio), but also (heavily) dependent on the credit quality. This is in line with the results from Hull et al. (2005).

3.4. Other structural models of default

The results from the previous section are formally only justified in a Merton framework. We will analyze other models in this section. In particular, we will analyze the relationship between actual and risk-neutral default probabilities in the Duffie/Lando (2001) framework ("D/L framework"). The D/L framework introduces asset value uncertainty, therefore resulting in more realistic short-term default probabilities and credit spreads. It was the first structural model which yields a default intensity and is therefore consistent with reduced-form credit pricing. In addition, the D/L model captures other frequently used models as special cases. Setting the asset value uncertainty to zero yields the Leland/Toft (1996) model. Due to our special calibration approach, we (implicitly) cover all strategic default models with a geometric Brownian motion as asset value process and with a constant default barrier.

3.4.1. Model setup

The main feature in the D/L framework is the introduction of uncertainty about the current asset value. Default probabilities can be calculated as a weighted average of the respective first-passage time default probabilities, where the weight is given by the probability distribution of the asset value:

$$PD^P_{DL}(t,T) = \int_L^\infty \underbrace{PD^P_{FP}(t,T,x)}_{\text{PD(first passage time) if } \mathrm{V_t = x}} \underbrace{g^P(x|\hat{V}_t, v_0, t)}_{\text{Prob., that } \mathrm{V_t = x}} dx \quad (3.4)$$

$$PD^Q_{DL}(t,T) = \int_L^\infty PD^Q_{FP}(t,T,x) g^Q(x|\hat{V}_t, v_0, t) dx. \quad (3.5)$$

Here, $PD^P_{DL}(t,T)$ and $P^Q_{DL}(t,T)$ denote the cumulative actual and risk-neutral default probability between t and T, $\hat{V}_t$ denotes the noisy observation of the asset value in t and v_0 denotes the non-noisy asset value in $t = 0$. $PD^P_{FP}(t,T,x)$, $PD^Q_{FP}(t,T,x)$ is the default probability in a Black/Cox first-passage time framework, i.e. the actual/risk neutral probability that an asset value process starting in t at $V_t = x$ will fall below the default barrier up to time T:

$$PD_{FP}^{P}(t,T) = \Phi\left(\frac{b - m^P(T-t)}{\sigma\sqrt{T-t}}\right) - e^{\frac{2m^P b}{\sigma^2}}\Phi\left(\frac{b + m^P(T-t)}{\sigma\sqrt{T-t}}\right) \quad (3.6)$$

$$PD_{FP}^{Q}(t,T) = \Phi\left(\frac{b - m^Q(T-t)}{\sigma\sqrt{T-t}}\right) - e^{\frac{2m^Q b}{\sigma^2}}\Phi\left(\frac{b + m^Q(T-t)}{\sigma\sqrt{T-t}}\right) \quad (3.7)$$

with $b = ln(\frac{L}{V_t})$, $m^P = \mu - \frac{1}{2}\sigma^2$, $m^Q = r - \frac{1}{2}\sigma^2$ and $\sigma = \sigma_V$. $g^Q(x|V_t, v_0, t)$, $g^P(x|V_t, v_0, t)$ are the density functions of the asset value V_t under the actual and risk-neutral probability measure. In their model setup, D/L derive an explicit closed-form solution for these probability density functions (cf. Duffie/Lando (2001)). Among other parameters, it is dependent on the degree of asset value uncertainty (α) and the time that has passed since the last non-noisy observation (t). This density function is different in the real and risk-neutral world. Intuitively, the investor processes two pieces of information: First, the noisy observation of the asset value, which is assumed to be unbiased. Second, the fact that the company has not defaulted in $[0, t]$. The second piece of information gives rise to a risk premium, i.e. actual and risk-neutral densities differ.

3.4.2. Implementation

There is no closed-form solution for the relationship between actual and risk-neutral default probabilities in the D/L framework. We therefore proceed numerically in the following way:

1. Choose Parameters: Choose a specific rating grade (RAT), maturity (T), asset volatility (σ), risk-neutral asset value drift (m)[14], asset Sharpe ratio (SR), asset value uncertainty (α), time passed since last non-noisy observation of the asset value (t), and difference between asset value in $t = 0$ and the noisy information in t ($R := ln(\hat{V}_t/v_0)$). Without loss of generality we set the default barrier (L) to 100.

2. Determine the risk-neutral PD in the Duffie/Lando framework

[14] The risk-neutral drift may deviate from the risk-free rate due to payouts (e.g. dividends, share buybacks).

a) Based on (3.4), determine the asset value $\hat{V}_t$ that yields the cumulative actual default probability PD^P_{RAT} for the respective rating grade. Cumulative default probabilities per rating grade (PD^P_{RAT}) are taken from Moody's (2007) and are depicted in appendix A.1.1.[15] Based on the specifications in step 1, $\hat{V}_t$ is the only free parameter on the right-hand side of (3.4).

b) Determine $PD^Q_{D/L}$ based on (3.5).

3. Determine the risk-neutral PD in the Merton framework via (3.3).

4. Determine the error defined as the quotient between Duffie/Lando risk-neutral PD and Merton risk-neutral PD, i.e.

$$Err := \frac{PD^Q_{D/L}}{PD^Q_{Merton}}. \tag{3.8}$$

Repeat steps 1 to 4 for all reasonable parameter combinations. Reasonable parameter combinations choosen for this study were:

- Rating grades: Aa, A, Baa, Ba, B (incl. respective cumulative actual default probabilities based on appendix A.1.1)
- $T = 1, 3, 5, 7, 10$ (standard CDS maturities)
- σ: 3%, 5%, 7.5%, 10%, 15%, 20%, 30%
- m: -2.5%, 0%, 2.5%, 5%
- SR: 10%, 20%, 30%, 40%
- α: 0%, 10%, 30%
- t: 1, 3 years[16]
- R: R=0 and R=+/- 2 standard deviations of the asset value process.

[15] Please note that default probabilities from other sources, e.g. Fitch or S&P could also be used. Rating grades simply act as a natural way of identifying distinct default probabilities.

[16] The case $t = 0$ is already captured by the case $\alpha = 0\%$.

This yields a total of 50,400 combinations (2,016 for each combination of rating grade and maturity). For each combination actual and risk-neutral default probabilities were determined with the 3-step approach given above.[17]

3.4.3. Implications

3.4.3.1. Illustrative example

To demonstrate the methodology, we start with a short numerical example by setting

$$RAT = Baa, T = 5, SR = 20\%,$$

$$m = 0\%, \sigma = 15\%, L = 100, \alpha = 10\%, t = 1, R = 0.$$

Based on historical default probabilities, the 5-year Baa cumulative default probability is approx. 2.17% (see appendix A.1.1). The asset value which yields 2.17% real-world default probability for this specific parameter combination is 203.02 (determined via (3.4)). Based on (3.5), the risk-neutral default probability for this parameter combination (including $V_t = 203.02$) is 5.70%. In the Merton framework, a real-world default probability of 2.17%, an asset Sharpe ratio of 20%, and a maturity of 5 years yield a risk-neutral default probability of 5.80% using formula (3.3). The resulting difference between the Merton framework and the D/L framework is therefore rather small (5.80% vs. 5.70%) for this demonstrative example.

3.4.3.2. The default timing effect

We first analyze a Baa-rated obligor, 5-year maturity with a Sharpe ratio of 20%. For corporates, the Baa rating grade is the single largest rating category (by number of issuers and volume). 5-years is the standard maturity in CDS markets and an asset Sharpe ratio of 20% is in line with historical data for the equity markets, cf. the discussion in section 3.3. Restricting the analysis to this case allows to study some effects in more detail. Still, the results are generally valid for other rating categories, maturities and Sharpe

[17] The numerical evaluations of the integrals (3.4) and (3.5) were performed using adaptive quadrature based on the quadgk function in MATLAB.

ratios, too. General results will be presented in subsection 3.4.3.4.

The results of all Baa/5-year/SR=20% combinations are shown in figure 3.3 in column “Total”. Please note that all of these combinations have the

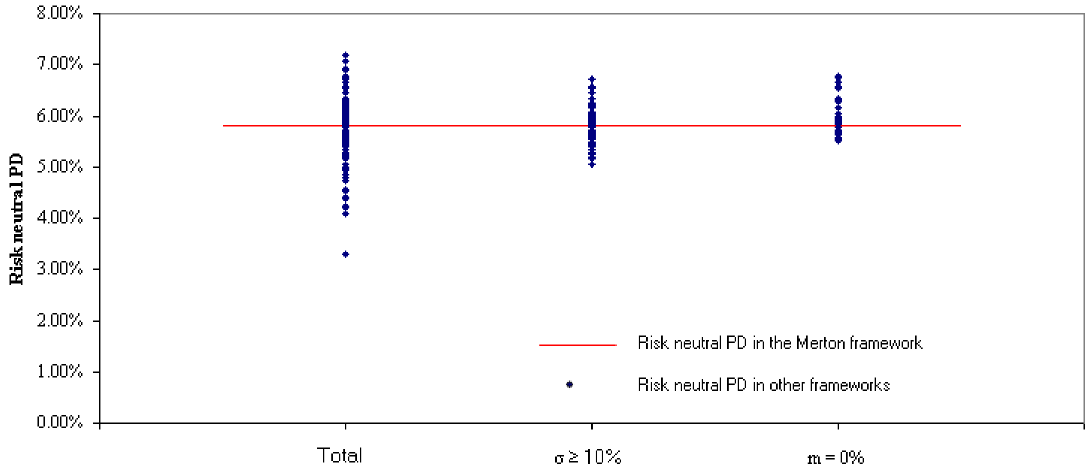

Figure 3.3.: Risk-neutral default probabilities in the D/L framework for the Baa/5-year/20% Sharpe ratio case for different parameter combinations. *Total* contains all parameter combinations for this case as described in subsection 3.4.2. $\sigma \geq 10\%$ restricts the asset volatility to values larger or equal to 10% (“non-financials”), $m = 0\%$ restricts the risk-neutral asset value drift relative to the default barrier to 0% (“constant leverage”).

same real-world PD (2.17%, since both rating and maturity are fixed) and the same Merton-implied risk-neutral PD (5.80%, risk-neutral PDs in the Merton framework are only dependent on the real-world PD, maturity and the asset Sharpe ratio). The risk-neutral PDs in the D/L framework range from 3.29% to 7.18%.

If we look closer at the parameter combinations which lead to the lowest risk-neutral PDs we find that they are characterized by low asset volatilities and a large drift. Once the analysis is restricted to asset volatilities of larger than 10% or constant leverage ($m = 0\%$), the resulting PDs are off by not more than 15% from the Merton PD (figure 3.3, column 2 and 3). Asset volatilities smaller than 10% can rarely be observed for non-financials. E.g. in the KMV database, less than 10% of the non-financials have asset volatilities smaller than 10%. Constant leverage is also a frequently stated assumption.

Of course, a further restriction on the asset volatility and drift might be economically justified in certain cases, but it does not help to gain insight into the economics that are driving these results. The economics behind these results are as follows: Combinations of high drift and low asset volatility lead to situations where a company either defaults "very early" or "never at all". Economically, if default occurs soon then the investor is only exposed to systematic risk for a short period. E.g., assume as an extreme case that there is an obligor which either defaults in 1 month with probability 2.17% based on the state of the economy in 1 month or never defaults at all. Clearly, the 5-year cumulative default probability is 2.17%. The investor is, however, only exposed to systematic risk over the first month, afterwards the investment is risk-free. Therefore, the risk premium would be smaller than for an obligor which might also default in 3-, 4- or 5-years from today.

To analyze this default timing argument closer, we define the expected conditional default time as

$$DT := E^P[\tau|\tau < T], \tag{3.9}$$

where τ is the time of default. DT describes the expected time of default under the condition that default occurs until T. For the Merton framework DT is always equal to T since default can only happen at maturity. Based on the observation in the Merton framework (difference between actual and risk-neutral PD increases with increasing maturity) and the indications from above it is natural to assume that DT plays a role in the difference between actual and risk-neutral PDs in the D/L framework as well. Figure 3.4 plots the risk-neutral PDs from the Baa/5-years/SR=20% example as a function of the expected conditional default time. Indeed, a clear relationship can

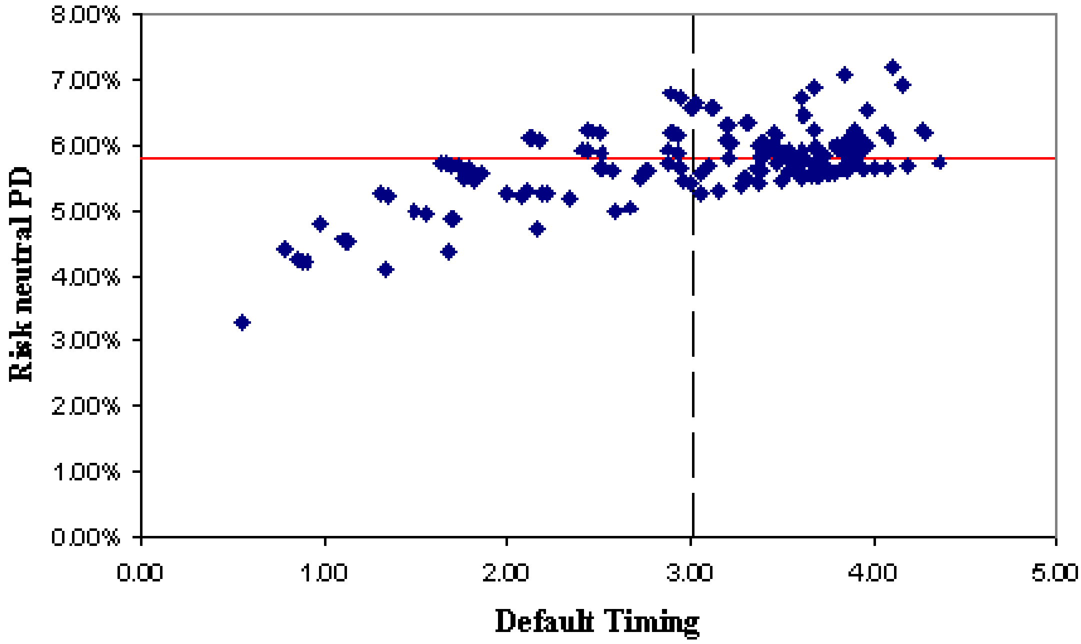

Figure 3.4.: Risk-neutral default probabilities in the D/L framework for the Baa/5-year/20% Sharpe ratio case for different parameter combinations as a function of the conditional default time as defined in (3.9).

be seen: The lower DT, the lower the risk-neutral PD. Based on empirical observations from Moody's (2007), if a Baa-rated obligor defaults in the next 5 years the default will happen on average after 3.01 years (cf. appendix A.1.2) as indicated by the dotted line in figure 3.4. Restricting the range of reasonable expected default times again results in risk-neutral PDs close to the Merton framework.

3.4.3.3. The asset value uncertainty effect

We now expand our analysis to different maturities for a Baa-rated obligor but still use an asset Sharpe ratio of 20%. All the effects mentioned in the last subsection are still valid and will be discussed in a broader analysis in section 3.4.3.4. Here we want to focus on the effect of asset value uncertainty. It is well known from Duffie/Lando (2001) that asset value uncertainty increases short-term default probabilities. In this section, we will demonstrate that it also increases the *difference* between actual and risk-neutral default probabilities.
Figure 3.5 shows the effect of asset value uncertainty on credit risk premia for an asset volatility of 15% and a risk-neutral drift of 0%.[18] Instead of plotting risk-neutral PDs, it shows the quotient between the risk-neutral PD in the D/L framework and the risk-neutral PD in the Merton framework. This allows us to plot all maturities on the same scale. Please also note that real-world PDs for one maturity are the same for all asset value uncertainties but that the asset value $\hat{V}_t$ differs. Figure 3.5 indicates two main findings: First, a higher asset value uncertainty leads to a higher credit risk premium. A higher asset value uncertainty effectively means that an investor has to rely more heavily on the $t = 0$ asset value and the information contained in the survival up to t. This exposes the investor to additional systematic risk. An investor is not only exposed to systematic risk for the maturity T, but effectively for a timeframe of $t + T$, cf. Schönbucher (2003), p. 280 for an intuitive description of this "delayed observation" idea. Second, the effect is more pronounced for shorter maturities. The relative difference between $t + T$ and T simply increases with a shorter maturity T. Although not depicted in figure 3.5, the effect is also larger for higher rating grades.

This asset value uncertainty effect is surely an interesting effect when thinking about the high CDS spreads during the 2007/2008 financial crisis. However, this effect is rather small for reasonable asset value uncertainties[19]

[18] The main findings do not change for other parameter combinations which are available on request.

[19] The maximum asset value uncertainty in figure 3.5 is 30%. Based on a normal distri-

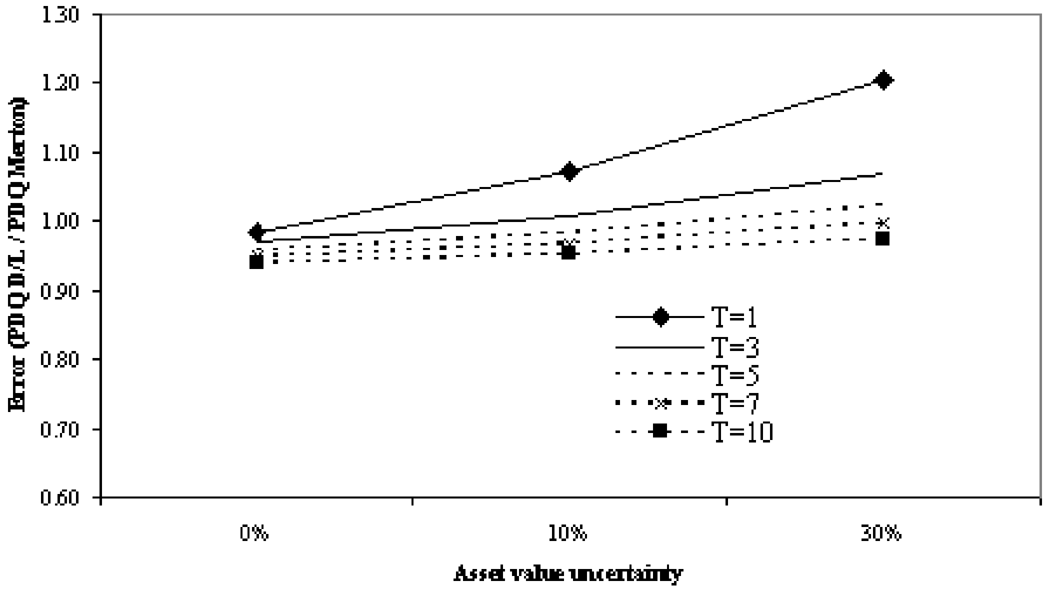

Figure 3.5.: Ratio of risk-neutral PD in the D/L framework and risk-neutral PD in the Merton framework for different asset value uncertainties and maturities. Models are calibrated to the same real-world default probability. The y-axis plots the quotient of D/L risk-neutral PD and Merton risk-neutral PD. Parameter choices are: $RAT = Baa$, $SR = 20\%$, $m = 0\%$, $\sigma = 15\%$, $t = 1$, $R = 0$.

and maturities of 3 years or longer.

3.4.3.4. Extended results

So far, the results have been restricted to a rating of Baa and a Sharpe ratio of 20%. Table 3.3 depicts the extended results for other ratings and a Sharpe ratio of 20%, table 3.4 for a Sharpe ratio of 30%.[20] As in figure 3.5, error terms are shown in the tables, i.e., the quotient between the risk-

bution and without any other information this basically means that a 95% confidence interval for the asset value is approx. +/- 60% – which seems to be a quite wide range.

[20]Results for Sharpe ratios of 10% and 40% available upon request.

neutral PDs in the D/L framework and in the Merton framework once both frameworks have been calibrated to the same real-world PD.

For a Sharpe ratio of 20% (table 3.3), the quotient between the risk-neutral D/L PDs and the risk-neutral Merton PDs can range from as far as 0.33 (Aa, 10-years) to 2.12 (Aa, 1-year). Once mild conditions on either the asset volatility ($\sigma \geq 10\%$), the risk-neutral drift relative to the default barrier ($m = 0\%$) or the default timing (within +/- 20% of empirical averages based on Moody's (2007), cf. A.1.2) are imposed, the differences are a lot smaller. E.g., for the 5-year maturity, the quotient is always between 0.83 and 1.30 for a reasonable default timing. Relative differences are smaller for longer maturities and lower ratings. As expected, higher Sharpe ratios slightly increase the difference between the Merton framework and the D/L framework. Risk-neutral default probabilities can be interpreted as a sum of real-world PDs and a risk premium. A higher Sharpe ratio leads to a higher risk premium (while PDs stay the same by definition), therefore increasing the difference between different model approaches.

All in all, one should take care with using the simple Merton transformation (3.3) from actual to risk-neutral default probabilities for very short maturities. For longer maturities, the Merton transformation is still a good approximation for the relationship between actual and risk-neutral PDs.

		Total		$\sigma \geq 10\%$		$m = 0\%$		DT reasonable	
Maturity	Rating	Min	Max	Min	Max	Min	Max	Min	Max
1	Aa	0.94	2.12	0.98	1.84	0.99	2.07	1.20	1.96
	A	0.93	1.94	0.98	1.71	0.99	1.88	1.19	1.78
	Baa	0.92	1.75	0.98	1.57	0.98	1.68	1.14	1.68
	Ba	0.91	1.55	0.97	1.41	0.98	1.45	0.91	1.50
	B	0.90	1.31	0.97	1.21	0.98	1.19	0.94	1.31
	All	0.90	2.12	0.97	1.84	0.98	2.07	0.91	1.96
3	Aa	0.69	1.54	0.94	1.39	0.97	1.49	0.87	1.50
	A	0.68	1.45	0.94	1.33	0.97	1.40	0.85	1.42
	Baa	0.69	1.36	0.93	1.26	0.97	1.30	0.87	1.33
	B	0.71	1.26	0.91	1.18	0.96	1.18	0.89	1.24
	B	0.76	1.14	0.91	1.07	0.96	1.03	0.93	1.11
	All	0.68	1.54	0.91	1.39	0.96	1.49	0.85	1.50
5	Aa	0.52	1.36	0.90	1.25	0.96	1.30	0.85	1.30
	A	0.54	1.30	0.88	1.21	0.96	1.24	0.83	1.24
	Baa	0.57	1.24	0.87	1.16	0.95	1.17	0.86	1.17
	Ba	0.62	1.17	0.86	1.11	0.95	1.08	0.89	1.11
	B	0.72	1.09	0.86	1.03	0.94	0.99	0.92	1.05
	All	0.52	1.36	0.86	1.25	0.94	1.30	0.83	1.30
7	Aa	0.42	1.28	0.85	1.19	0.95	1.22	0.77	1.22
	A	0.45	1.23	0.83	1.15	0.95	1.16	0.82	1.16
	Baa	0.50	1.18	0.81	1.11	0.94	1.11	0.87	1.11
	Ba	0.57	1.12	0.81	1.07	0.94	1.03	0.87	1.03
	B	0.71	1.06	0.84	1.01	0.94	0.97	0.94	1.02
	All	0.42	1.28	0.81	1.19	0.94	1.22	0.77	1.22
10	Aa	0.33	1.22	0.76	1.14	0.94	1.14	0.75	1.01
	A	0.37	1.18	0.74	1.11	0.93	1.10	0.73	0.97
	Baa	0.43	1.14	0.73	1.08	0.93	1.05	0.81	1.05
	Ba	0.53	1.09	0.75	1.04	0.92	0.99	0.87	0.99
	B	0.72	1.03	0.83	1.00	0.93	0.96	0.95	1.01
	All	0.33	1.22	0.73	1.14	0.92	1.14	0.73	1.05

Table 3.3.:
Credit risk premia in the Duffie/Lando framework: Extended results, asset Sharpe ratio = 20%

Minimum and maximum quotient of risk-neutral PDs in the D/L framework and risk-neutral PDs in the Merton framework (for various parameter combinations) once actual PDs have been calibrated to the same level. Asset Sharpe ratio is 20%. For actual PDs cf. appendix A.1.1, for parameter combinations cf. subsection 3.4.2. *Total* contains all parameter combinations as described in subsection 3.4.2. $\sigma \geq 10\%$ restricts the asset volatility to values larger or equal to 10% ("non-financials"), $m = 0\%$ restricts the risk-neutral asset value drift relative to the default barrier to 0% ("constant leverage"), *DT reasonable* restricts the average default time to values +/-20% compared to values based on Moody's (2007), cf. appendix A.1.2.

		Total		$\sigma \geq 10\%$		$m = 0\%$		DT reasonable	
Maturity	Rating	Min	Max	Min	Max	Min	Max	Min	Max
1	Aa	0.91	2.95	0.97	2.41	0.98	2.84	1.45	2.62
	A	0.90	2.58	0.97	2.15	0.98	2.46	1.28	2.46
	Baa	0.88	2.22	0.96	1.89	0.97	2.08	0.88	2.08
	Ba	0.86	1.84	0.96	1.61	0.97	1.67	0.86	1.76
	B	0.85	1.44	0.95	1.29	0.97	1.25	0.91	1.44
	All	0.85	2.95	0.95	2.41	0.97	2.84	0.86	2.62
3	Aa	0.57	1.83	0.91	1.59	0.96	1.74	0.79	1.77
	A	0.56	1.68	0.90	1.48	0.95	1.58	0.82	1.63
	Baa	0.57	1.53	0.88	1.37	0.95	1.41	0.84	1.48
	Ba	0.60	1.37	0.87	1.24	0.94	1.21	0.86	1.32
	B	0.68	1.18	0.86	1.08	0.93	1.02	0.90	1.18
	All	0.56	1.83	0.86	1.59	0.93	1.74	0.79	1.77
5	Aa	0.38	1.52	0.83	1.36	0.94	1.42	0.76	1.48
	A	0.40	1.43	0.82	1.29	0.93	1.32	0.81	1.34
	Baa	0.44	1.33	0.80	1.21	0.92	1.20	0.85	1.25
	Ba	0.51	1.22	0.79	1.13	0.92	1.07	0.85	1.16
	B	0.64	1.11	0.81	1.02	0.92	0.97	0.91	1.07
	All	0.38	1.52	0.79	1.36	0.92	1.42	0.76	1.48
7	Aa	0.29	1.39	0.75	1.26	0.92	1.28	0.74	1.28
	A	0.32	1.32	0.73	1.20	0.91	1.19	0.81	1.19
	Baa	0.37	1.24	0.72	1.14	0.90	1.10	0.79	1.10
	Ba	0.46	1.16	0.72	1.07	0.90	1.01	0.85	1.10
	B	0.64	1.07	0.79	1.00	0.91	0.95	0.92	1.04
	All	0.29	1.39	0.72	1.26	0.90	1.28	0.74	1.28
10	Aa	0.21	1.29	0.64	1.18	0.90	1.16	0.64	0.99
	A	0.25	1.23	0.62	1.13	0.89	1.09	0.73	1.09
	Baa	0.31	1.17	0.62	1.08	0.88	1.02	0.78	1.02
	Ba	0.42	1.10	0.66	1.03	0.88	0.95	0.83	1.00
	B	0.66	1.03	0.78	0.98	0.90	0.94	0.95	1.02
	All	0.21	1.29	0.62	1.18	0.88	1.16	0.64	1.09

Table 3.4.:
Credit risk premia in the Duffie/Lando framework: Extended results, asset Sharpe ratio = 30%

Minimum and maximum quotient of risk-neutral PDs in the D/L framework and risk-neutral PDs in the Merton framework (for various parameter combinations) once actual PDs have been calibrated to the same level. Asset Sharpe ratio is 30%. For a detailed description see table 3.3.

4. Estimating equity premia from CDS spreads

4.1. Motivation

This chapter draws upon the theoretical results from chapter 3. We have documented two main results in chapter 3 which are useful for this chapter: First, model-implied credit spreads are very sensitive with respect to the asset Sharpe ratio. Second, the relation between the risk-neutral and actual default probability from the Merton framework is still approximately valid in more advanced structural models of default. These two observations suggest that it is promising to estimate Sharpe ratios and equity premia from credit spreads. The first observation is an indication that this procedure might yield quite robust results with respect to noise in the input parameters when applied in practice. The second observation indicates that the results will also be robust with respect to model changes.

Generally, as discussed in section 2.4, there are four major things that we have to consider: First, our model should be based on observable parameters. Second, it should be robust with respect to model changes. Third, we should use individual company data instead of aggregates due to convexity effects. Fourth, we should use CDS spreads instead of bond spreads due to the risk-free rate problem and liquidity distortions.

In section 4.2 we will develop a model which relies on observable parameters and show that is is robust with respect to model changes.[1] These results are mainly based on the findings from chapter 3 although the results from chapter 3 cannot always be transformed one-to-one to the application in this chapter. In section 4.3 we will describe the data and discuss implementation issues. We will use CDS spreads from the U.S., Europe, and Asia on an individual company level. Section 4.4 discusses the results for the U.S. and 5-year CDS spreads, section 4.2.2 shows the results for further maturities and other markets. Robustness tests are shown in section 4.6.

4.2. Model setup

Subsection 4.2.1 starts with the classical Merton model. We derive a simple Merton estimator for the market Sharpe ratio and the equity premium. This estimator is only based on observable parameters, i.e., the risk-neutral and actual default probability, the maturity, and equity correlations. Subsection 4.2.2 and appendix C.1 expand this framework to first-passage-time models, models with endogenous default barrier, and a model with unobservable asset values based on Duffie/Lando (2001).

4.2.1. Estimating equity premia in the Merton framework

Our model is based on the relationship between actual and risk-neutral default probabilities in the Merton framework which we derived in section 3.3.2 (cf. formula (3.3)):[2]

$$PD^Q(t,T) = \Phi\left[\,\Phi^{-1}(PD^P(t,T)) + \frac{\mu - r}{\sigma} \cdot \sqrt{T-t}\,\right], \qquad (4.1)$$

Here, PD^Q and PD^P denote cumulative risk-neutral and actual default probabilities, μ and σ are the real-world drift and the volatility of the asset value process, r denotes the risk-free rate, $T-t$ denotes the maturity of the bond, and Φ denotes the cumulative standard normal distribution function.

[1] For this chapter cf. also Berg/Kaserer (2008).

[2] In section 3.3.2 we always worked with $t = 0$. In this section we will explicitly express the formulas as functions of t and T to allow for variations in both time and maturity.

Formula (4.1) can be easily transformed to calculate the Sharpe ratio of the company's assets (SR_V):

$$SR_V := \frac{\mu - r}{\sigma} = \frac{\Phi^{-1}(PD^Q(t,T)) - \Phi^{-1}(PD^P(t,T))}{\sqrt{T-t}}. \tag{4.2}$$

Therefore we define our estimator for the asset Sharpe ratio in the Merton framework as

$$\widehat{\gamma}_{\mathrm{SR_V,Merton}} := \frac{\Phi^{-1}(PD^Q(t,T)) - \Phi^{-1}(PD^P(t,T))}{\sqrt{T-t}}. \tag{4.3}$$

It should be noted, that formula (4.2) is still correct if a non-stochastic, constant payout ratio δ is introduced. Relationship (4.2) is a central formula in this chapter. It has two main advantages that make it convenient for our purpose: First, it directly yields the Sharpe ratio of the assets, i.e., neither μ_V and σ_V nor V_t, N, or r have to be estimated separately. In contrast to other applications of structural models, we do not have to calibrate any parameter of the asset value process. The company Sharpe ratio can simply be estimated based on actual and risk-neutral default probabilities and the maturity. Second, it is robust with respect to model changes. This will be discussed in the next subsection and in the appendix.

If we try to estimate the market Sharpe ratio, we are faced with an additional problem: The Sharpe ratio of the assets $\frac{\mu_V - r}{\sigma_V}$ will usually differ from the market Sharpe ratio, since the assets V_t will not necessarily be on the efficient frontier. The Sharpe ratio of the assets does not only capture the risk preference of investors, but also depends on the correlation of the assets with the market portfolio. The market Sharpe ratio can be calculated via a straight forward application of the continuous time CAPM:[3]

$$\mu_V = r + \frac{\mu_M - r}{\sigma_M} \cdot \rho_{V,M} \cdot \sigma_V \quad \Leftrightarrow \quad \frac{\mu_M - r}{\sigma_M} = \frac{\mu_V - r}{\sigma_V} \cdot \frac{1}{\rho_{V,M}}, \tag{4.4}$$

where $\rho_{V,M}$ denotes the correlation coefficient between the asset returns and

[3] We assume $\rho_{V,M} \neq 0$.

the market returns.

Therefore we will need an estimate of the correlation between the asset value and the market portfolio. This correlation $\rho_{V,M}$ can be approximated by the correlation between the corresponding equity return and the market return (denotet by $\rho_{E,M}$), i.e. by

$$\rho_{V,M} \approx \rho_{E,M}. \tag{4.5}$$

The error of this approximation is negligible, since – within the Merton framework – the equity value of a company equals a deep-in-the-money call option on the assets. The option is deep-in-the-money, since annual default probabilities are less than 0.4% for investment grade companies and less than 10% for all obligors rated B and above. For deep-in-the-money options, gamma is approx. zero, i.e., we have an almost affine linear relationship between asset and equity value, cf. Hull (2005) for example. For reasonable parameter choices, the approximation error is less than 3% (for rating grades above B) and 1% (for investment grade ratings) respectively (cf. Appendix C.3 for details). Hence, the following approximation holds:

$$\frac{\mu_M - r}{\sigma_M} \approx \frac{\Phi^{-1}(PD^Q(t,T)) - \Phi^{-1}(PD^P(t,T))}{\sqrt{T-t}} \cdot \frac{1}{\rho_{E,M}}.$$

Therefore, we define the Merton estimator of the market Sharpe ratio as:

$$\widehat{\gamma}_{\mathrm{SR_M,Merton}} := \frac{\Phi^{-1}(PD^Q(t,T)) - \Phi^{-1}(PD^P(t,T))}{\sqrt{T-t}} \frac{1}{\rho_{E,M}}. \tag{4.6}$$

Including the (expected) volatility of the market portfolio σ_M yields an estimator for the equity premium:

$$\widehat{\gamma}_{\mathrm{EP,Merton}} := \frac{\Phi^{-1}(PD^Q(t,T)) - \Phi^{-1}(PD^P(t,T))}{\sqrt{T-t}} \frac{\sigma_M}{\rho_{E,M}}. \tag{4.7}$$

4.2.2. Estimating equity premia in other frameworks

Of course, our estimator $\widehat{\gamma}_{\text{EP,Merton}}$ for the equity premium is formally only justified in a Merton framework. Moving to more elaborated structural models of default usually has a significant impact on actual and risk-neutral default probabilities. E.g., in a first-passage-time framework with zero drift in the real world, actual default probabilities are twice as high as actual default probabilities in the Merton framework for the same parameterization ("reflection principle").

Fortunately, our estimator does not only include the actual default probability but the *difference* between (the inverse of the cumulative normal distribution function of) the risk-neutral and (the inverse of the cumulative normal distribution function of) the actual default probability. This difference can be shown to be very robust with respect to model changes.

We have already given some intuition for this in section 3.1 and formally shown this robustness in section 3.4 for the derivation of risk-neutral PDs from actual PDs for the Duffie/Lando (2001) model. The results from section 3.4 are, however, not one-to-one transferable for the application in this section. In section 3.4, we used a relationship of the form $PD^Q = f(PD^P, SR_V)$ whereas in this section we apply a relationship of the form $SR_V = f(PD^P, PD^Q)$. This may of course also affect the model robustness. The robustness with respect to model changes for the purpose of this section (i.e., formula (4.2)) is analyzed in more detail in appendix C.1 based on a first-passage-time framework, endogenous default frameworks and the Duffie/Lando (2001) framework with unobservable asset values. The difference between the asset/market and the equity/market correlation in these frameworks – which might affect the approximation (4.5) – is analyzed in appendix C.3. The findings mainly confirm the robustness results from section 3.4 for this application.

4.3. Data and implementation

In each week our sample consists of the intersection of a) on-the-run companies in the CDX.NA.IG index[4], b) the credit default swap (CDS) database of CMA (credit markets association), and c) the KMV EDF database. The Dow Jones CDX.NA.IG-index is the main CDS index in North America. It covers the 125 most liquid North American investment grade CDS.[5] We used 5-year CDS spreads to derive risk-neutral default probabilities because the 5-year maturity is the most liquid one. EDFs (expected default probabilities) from Moody's KMV data base were used as a proxy for the actual default probabilities and correlations with the S&P500-index as a proxy for the correlations with the market portfolio. For all parameters, we used weekly data from the period from April 2003 until June 2007.[6]

Credit Default Swaps are OTC credit derivatives that have become widely popular over the last years with growth rates of over 100% (nominal value) in 2005 and 2006 and total outstanding market volume of approx. $26 trillion at the end of 2006 (ISDA (2006)). Their main mechanism is quite simple: The protection buyer periodically pays a predefined premium to the protection seller (usually quarterly). In case of a credit event, the protection seller has to cover the losses incurred on a predefined reference obligation, i.e., he has to pay an amount equal to the difference between the nominal and the current market value of the predefined reference obligation to the protection buyer. As usual, put into practice, things turn out to be more complicated: The credit event has to be precisely defined, a basket of reference obligations has to be specified[7] and the term "market value" at the

[4]Our data sample starts in 04/2003 whereas the first CDX.NA.IG index starts in 10/2003. For the dates before 10/2003 we used the constituents of the CDX.NA.IG 1 index. The results do not materially differ if we start our sample period in 10/2003.

[5]Although the CDX.NA.IG index is a North American index we will frequently refer to "U.S." since the vast majority of constituents is based in the U.S. Only two non-U.S.-based companies are in our sample (Bombardier and Alcan (both from Canada)).

[6]EDFs from KMV are only available on a monthly basis. We assumed EDFs to be constant within each month.

[7]Defining only a single reference obligation is not possible for practical reasons, which usually leads to a "cheapest-to-deliver" option for the protection buyer, who can

time of default has to be clearly specified. As in most academic research (e.g. Berndt et.al (2005)), we will assume that the extent of these specification does not have a significant value and therefore CDS can be prized as if these implicit options were not part of the game.

The 5-year CDS spreads (bid/ask/mid) used in our analysis were taken from Datastream. These data is compiled and provided by Credit Market Analysis (CMA) who collects CDS data from a range of market contributors from both buy- and sell-side institutions. Only dates with at least one trade or firm bid for the respective CDS are used to avoid potential errors from pure market maker data. We used CDS mid spreads for our analysis. Bid/ask-spreads served for consistency checks and sensitivity analysis. Since data quality is always an issue in over-the-counter markets, we also used Bloomberg data sources as a quality check. The differences were minimal, probably also due to the fact that we have choosen the 125 most liquid counterparts in the market which should enhance data quality as well. The risk-neutral default probability PD^Q was derived by the approximation $PD^Q = 1 - exp\left(-\frac{s}{LGD} \cdot T\right)$ out of the CDS spread s with maturity T and the risk-neutral loss given default LGD (cf. Duffie/Singleton (2003) and the similar discrete time calculations in section 3.3.3). A recovery rate $(1 - LGD)$ of 45% was used and robustness tests were conducted.

As discussed in section 2.3.2.1, there are three main sources for real-world default probabilities that market participants use: Agencies' ratings, ratings based on a Merton-type distance-to-default measure and hazard rate model. We use expected default frequencies (EDFs) from Moody's KMV data base as our primary proxy for the actual default probabilities. Robustness tests based on agencies' ratings and hazard rate models are provided in the robustness section. EDFs are default probabilities, which are based on a Merton-style structural framework, cf. Moody's KMV (2007). The calibration is, however, done more pragmatically based on a large set of historical

normally choose which reference obligation to sell to the protection seller in case of a default.

data and on discriminant analysis. EDFs are widely used in the banking industry and also constitute a part of some of the internal rating systems of large banks. They have also been used in academic studies such as in Berndt et al. (2005). We used 1-year EDFs (and the respective equivalent rating grades from Aaa to B3) and derived multi-year EDFs by Moody's cumulative default probabilities per rating grade.[8] The cumulative default probabilities were determined via a logarithmic approach based on raw data from Moody's (2007). The resulting table of cumulative default probabilities can be found in Appendix A.1.1. The main advantage of EDFs compared to other ratings for our purpose is its link to market data: The current asset volatility and equity value are direct input parameters, therefore EDFs constitute a "point-in-time" estimation of the current default probability. In contrast to EDFs, the ratings of the large rating agencies are defined as "through-the-cycle"-ratings, which – in effect – results in different default probabilities for a specific rating grade dependent on the current overall economic outlook, cf. section 2.3.2.1.

We used 3-year weekly[9] correlations between the reference entities share price returns and the S&P-500 index. The share prices were taken from Datastream. We used median industry correlations since industry wide estimations of correlations have lower standard errors than a company by company estimation. This procedure also allowed to include companies without a 3-year equity price history. The industry sector classification was based on the sub-indices of the CDX.NA.IG index.

Expected volatilities for the market portfolio were approximated by implied volatilities from the VIX term structure. Data was collected directly

[8] Cf. Appendix A.1.1 for details. Elton et al. (2001) use a similar approach based on transition matrices. We have opted for a direct cumulative estimation because of indications that rating migrations are non-Markovian and cannot be explained by constant transition probabilities (Farnsworth/Li (2007)). The differences are, however, minimial.

[9] The calibration of correlations has a minor effect on the overall result, using 2-year or 1-year correlations did not alter results significantly.

Variable	N	Mean	Median	Std. dev.	25th Perc	75th Perc
CDS mid	24,785	54.53	39.80	58.41	25.50	61.00
CDS offer	24,785	56.76	42.00	59.31	27.20	63.50
CDS bid	24,785	52.37	37.70	57.69	23.70	59.00
Δ(bid, offer)	24,785	4.40	4.00	3.02	3.00	5.00
EDF1	24,785	0.17%	0.07%	0.50%	0.04%	0.15%
EDF5	24,785	1.90%	1.26%	2.58%	0.85%	2.14%
ρ	24,785	0.52	0.53	0.08	0.46	0.59
Implied vol	24,785	17.14%	16.31%	2.36%	15.43%	18.80%

Table 4.1.:
Descriptive statistics
Descriptive statistics for input parameters. The sample consists of the intersection of the KMV database, the CDX.NA.IG on-the-run companies and the CMA CDS database (via datastream) from April 2003 to June 2007. *EDF1/EDF5* denote 1- and 5-year cumulative default probabilities based on KMV EDFs. ρ denotes the correlation between equity returns and S&P 500 returns. *Implied vol* denotes the implied market volatility taken from the VIX term structure published by the CBOE based on mid option prices for maturities from 18-23 months.

from the CBOE webpage[10]. We used implied volatilities based on mid options prices for maturities from 18-23 months which was the longest maturity bucket that was consistently available.

Our final data set consists of 24,785 date/company-combinations for which 5-year CDS spreads and EDFs were available. Table 4.1 gives an overview of the main input parameters.[11]

[10]Chicago Board Option Exchange, www.cboe.com/publish/vixtermstructure/vixtermstructure.xls.

[11]Based on 222 weeks in our sample period and 125 on-the-run constituents in the CDX.NA.IG index the theoretical maximum is 27,750 date/company-combinations. Therefore, we have data available for approx. 90% of the theoretical maximum. This is probably also due to the fact that we used constituents of the most liquid CDS index and our sample period starts approximately at the same time when index trading – and therefore also liquidity – took off in the CDS markets.

4.4. Results for 5-year CDS in the U.S.

Based on the data described in section 4.3 and the Merton estimator for the equity premium (4.7), the company Sharpe ratio (4.3) and the market Sharpe ratio (4.6) derived in section 4.2, we estimate the implicit equity premium and company and market Sharpe ratios for each of the 24,785 observations. Table 4.2 provides the results on a yearly basis in column (3) to (8).

Our estimation yields an average equity premium of 6.50% for the U.S. market. The average company Sharpe ratio is 19.33% and the average market Sharpe ratio is 38.77%. The median values are even lower with 5.95% for the equity premium and 18.17% and 35.30% for the company and market Sharpe ratio. We would already like to mention here that all these values are upper limits for the equity premium. This is due to some implicit conservative assumptions, especially concerning the part of the CDS spread which is due to credit risk (we assume 100%) and the recovery rate (our assumption of 45% seems to be an upper limit); cf. section 4.6 for details.

Looking at each year of our sample period separately shows a quite homogenous result: The implicit equity premium estimates range from 5.16% in 2003 to 7.18% in 2005. The year 2005 also exhibits the largest one-year increase in the equity premium up 23% from 5.84% in 2004. CDS premia were still as high in 2005 as in 2004 – especially due to an increase in spreads in the second quarter around the downgrades of Ford and General Motors – while EDFs were decreasing (2.37% vs. 1.54%) due to bullish equity markets and lower volatilities . Correlations were decreasing from 0.55 to 0.50 while implied volatilities decreased from 18.28% to 16.04% resulting in an almost unchanged term $\frac{\sigma_M}{\rho_{E,M}}$. It seems plausible to assume that these downgrades have led to an increase in risk aversion among market participants. We would like to point out that the implied equity premia also increased in the second quarter of 2007 at the beginning of the subprime crisis. Due to low CDS spreads in the first quarter of 2007, average estimates for the first

Table 4.2.:

Company Sharpe Ratio, Market Sharpe Ratio and Equity Premium Estimates based on U.S. 5-year CDS spreads from April 2003 to June 2007

The sample consists of the intersection of the KMV database, the CDX.NA.IG on-the-run companies and the CMA CDS database (via datastream) from April 2003 to June 2007. For correlations, median industry correlations have been used based on 3-year weekly returns. Implied volatilities are based on maturities from 18-23 months from the VIX term structure (mid prices) of the CBOE. *N* denotes the number of weekly company/date combinations. *s* denotes the CDS spread, *EDF5* denotes the 5-year cumulative default probability based on KMV EDFs, ρ denotes the Equity/Market-correlation and σ_M denotes the implied S&P 500 volatility. Averages are calculated as unweighted averages over all observations.

		Estimates						Mean of input parameters			
Parameter	Year	N	Mean	Median	Std. dev.	25th Perc	75th Perc	s	EDF5	ρ	σ_M
Sharpe ratio company	2003	4,235	13.44%	12.47%	14.18%	5.08%	20.68%	75.10	3.72%		
	2004	5,916	16.77%	16.39%	11.52%	9.19%	23.57%	58.91	2.37%		
	2005	5,860	21.62%	20.18%	13.13%	13.33%	28.15%	57.34	1.54%		
	2006	5,835	22.52%	21.25%	12.99%	13.36%	30.29%	41.12	1.00%		
	2007	2,939	22.07%	20.21%	14.16%	12.48%	30.07%	37.10	0.82%		
	2003-2007	24,785	19.33%	18.17%	13.50%	10.63%	26.59%	54.53	1.90%		
Sharpe ratio market	2003	4,235	24.51%	21.80%	25.85%	8.36%	38.36%	75.10	3.72%	0.57	
	2004	5,916	31.77%	30.06%	22.34%	16.44%	45.81%	58.91	2.37%	0.55	
	2005	5,860	44.49%	40.83%	27.62%	25.45%	59.78%	57.34	1.54%	0.50	
	2006	5,835	46.88%	41.87%	29.49%	25.99%	63.11%	41.12	1.00%	0.50	
	2007	2,939	45.92%	40.82%	31.62%	24.72%	62.04%	37.10	0.82%	0.50	
	2003-2007	24,785	38.77%	35.30%	28.51%	19.31%	54.23%	54.53	1.90%	0.52	
Equity Premium	2003	4,235	5.16%	4.56%	5.49%	1.76%	8.02%	75.10	3.72%	0.57	20.93%
	2004	5,916	5.84%	5.53%	4.15%	2.97%	8.35%	58.91	2.37%	0.55	18.28%
	2005	5,860	7.18%	6.55%	4.54%	4.03%	9.69%	57.34	1.54%	0.50	16.04%
	2006	5,835	7.17%	6.46%	4.55%	3.91%	9.67%	41.12	1.00%	0.50	15.23%
	2007	2,939	7.08%	6.25%	4.89%	3.77%	9.59%	37.10	0.82%	0.50	15.39%
	2003-2007	24,785	6.50%	5.95%	4.74%	3.32%	9.10%	54.53	1.90%	0.52	17.14%

half of 2007 are, however, almost the same than in 2006 (7.08% vs. 7.17%).

We would also like to emphasize the fact that our results stem from very different conditions on the credit markets. Looking at CDS spreads, they averaged 75.10 bp in the year 2003. This was accompanied by large EDFs (3.72%), large correlations (0.57) and a high implied volatility (20.93%). In the first half of 2007, spreads were less than half the spreads of 2003 (37.10 bp), EDFs were less than a fourth of their 2003 levels (0.82% vs. 3.71%), correlations were down to 0.50 and implied volatility was also significantly lower than in 2003 (15.39% vs. 20.93%). The fact that equity premium estimates were very similar throughout this time period indicates, that our estimates are not simply a result of a specific set of parameters but exhibit a certain robustness to changing market conditions. If at all, there seems to be a small tendency for equity premia to rise when credit markets are bullish (e.g. default rates and spreads decrease), although this is not or only partially true for the 2006 and 2007 period (from 2005 to 2006 Sharpe ratios increased but the implied equity premium decreased due to lower estimates for the implied volatility).

To get a first indication why these results are so robust we can decompose the spread into a part which is due to expected loss and into a risk premium. In 2007, for example, the average cumulative actual default probability was 82 bp, i.e., the cumulative expected loss was 45 bp (using LGD=55%) and the per annum expected loss was approx. 9 bp. If we compare this expected loss to the average spread of 37 bp, we see that approx. 75% (28bp/37bp) of the spread is due to a risk premium. Doubling the risk premium while keeping the expected loss constant would require a 75% increase in the CDS spread. Doubling the risk premium by keeping the CDS spread constant is only possible with negative – and therefore unreasonable – EDFs. This intuitively indicates why our results are so robust. Small uncertainty or noise in the spread or the actual default probability simply does not significantly effect the results. We will discuss several robustness tests in more detail in section 4.6.

4.5. Results for further maturities and from other markets

We have expanded our analysis to maturities of 3, 7, and 10 years and to European and Asian reference entities, too. Maturities of 3, 5, 7, and 10 year are the standard maturities for which CDS indices are provided by "markit".

Using other maturities than 5 year serves several purposes: First, they offer a robustness check of our results from the previous section. On average, risk premia estimates based on 3, 7, and 10 year maturities should not largely deviate from the results of the respective 5-year maturities. Second, these results could be used to identify a term structure of risk premia. We are not aware of any empirical analysis so far which has captured risk premia term structures. Therefore, comparison to other studies is of course limited but our results could offer a starting point for the discussion.

An application of the methodology to Europe and Asia also offers several perspectives. First, the results itself are of course interesting for an estimation of equity premia on these markets. Second, the results offer a good possibility to validate the robustness of the U.S.-results. If equity markets are globally integrated, investors should demand a similar risk premium across different countries/regions. We would therefore expect equity premium estimates in a similar magnitude as based on U.S. data. Third, U.S., Europe, and Asia offer a certain diversity concerning the loss experience and credit quality over our sample period. While the U.S. market was still in the aftermath of the Enron and Worldcom defaults at the beginning of our sample period and suffered the downgrades of Ford and GM in 2005, Europe did not suffer any comparable big-scale losses and had, on average, a better credit quality than the U.S. market. The Asian market did not suffer any unexpected large losses, too, but had on average a significantly lower credit quality than the U.S. market. These markets therefore offer a good opportunity to check if our estimator is robust with respect to these different credit market conditions.

Again, our data sample consists of the intersection of the KMV database, the main CDS index for the respective markets and the CMA CDS database. We used the iTraxx Europe index for Europe and the iTraxx Asia ex Japan index for the Asian market.[12] Only on-the-run companies were considered. The iTraxx Europe IG index consists of 125 investment grade constituents and is rolled over every 6 months. Index trading started later than in the U.S. (June 2004 vs. October 2003). The iTraxx Asia index started with 30 constituents in July 2004, it was later enlargerd to 50 constituents (effecive date 9/20/2005).[13] Due to the later start of index trading compared with the U.S. and data availability our sample period starts at the beginning of 2004, so our sample includes the time period from January 2004 until June 2007. Again, we used the first series of the respective index to define on-the-run companies before the effective date of the first series. CDS spreads were based on the CMA database. For comparability, we included only weeks where spreads for all maturities (3, 5, 7 and 10 years) were available. Actual default probabilities were determined via EDFs from KMV. We used the same methodology as for the U.S. to transfer 1-year EDFs to cumulative default probabilities.[14] The DJStoxx 600 (Europe) and the S&P Asia 50 (Asia) were used for an estimation of correlations. Median correlations per industry sector were again used for reasons of robustness. Implicit volatilities were calculated based on the VSTOXX Volatility sub-index 24 months.[15] For the Asian market, implicit volatility indices are not available,

[12] There is also an iTraxx index covering Japan. We have choosen the iTraxx Asia ex Japan index to cover countries which seem to offer the best independent view compared with the U.S. and Europe. The biggest countries in the iTraxx Asia ex Japan are Korea, Hong Kong, Singapore, Malaysia, China, and Taiwan. Together these countries offer a good perspective on a region where experience with corporate finance, derivative products and governance structures seem to be significantly different from the U.S. and Europe.

[13] After our sample period, effective 9/20/2007, it was again enlargerd to 70 constituents.

[14] Migration probababilities and cumulative loss rates are very similar for the U.S. and Europe, cf. Moody's (2008). Historical default data for the Asian market is rare. We are though not aware of any arguments why migration behavior should be different in Asia and think that this approach gives estimates which are as close as possible to what market participants would assume.

[15] Implied volatiities for longer maturities are not available due to the lack of liquid option

therefore we used rolling 1-year historical volatilities of the S&P 50 Asia index.

Table 4.3 provides the results for the 3-, 5-, 7- and 10-year maturities for the U.S., Europe, and Asia from 2004-2007. Please note that the 5-year results for the U.S. differ slightly from the previous section since only weeks where spreads for all maturities were available have been included in this data sample.

For the U.S., results based on 3-, 7-, and 10-year maturities are similar – but slightly smaller – than for the 5-year maturities. For the 2004-2007 period the equity premium estimation based on 5-year maturities is 7.00% while the estimates for the 3-, 7- and 10-year maturities were 6.43%, 6.62% and 6.26% respectively. The market Sharpe ratio estimates range from 39.32% (T=10) to 43.85% (T=5). All maturities show quite similar results for each year with equity premium estimates ranging from 4.80% (T=10, 2004) to 7.33% (T=5, 2005). All maturities exhibit an increase in the implied equity premium from 2004 to 2005 while the effect for other years is quite small. These results confirm our analysis for the equity premium from the last section.

For Europe, implied equity premium estimates are lower than for the U.S. They range from 5.03% (T=3) to 5.44% (T=5). Estimates for the 7-year maturity (5.24%) and the 10-year maturity (5.06%) yield similar results. Lower equity premia for Europe compared to the U.S. are consistent with both historical experience as well as evidence from other implied equity premium estimates.[16] Market Sharpe ratios for Europe are also lower than for the U.S., ranging from 26.33% (T=3) to 28.39% (T=10) compared to a range of 39.32% to 43.85% for the U.S. This is also consistent with the the-

markets for longer maturities. Implied volatilities do though have the characteristic that they are less volatile for longer maturities. If at all, our results would therefore be even smoother if volatilities for longer maturities were available.

[16] For example, Claus/Thomas (2001) estimates an equity premium of 3.40% for the U.S. while estimates for the UK, France and Germany are 2.81%, 2.60% and 2.02%.

oretical argument that – from a global perspective – the U.S. market should be closer to the global market portfolio and therefore closer to the capital market line. The difference between estimates for the U.S. and Europe is especially pronounced in 2004 where implied equity premia estimates for Europa are as low as 1.87% (T=3). Excluding 2004 from the analysis does, however, still result in lower estimates for Europe compared to the U.S.

Average equity premia estimates for Asia are between the estimates from the U.S. and Europe. The lowest average estimates comes from the 10-year maturity (5.60%) and the highest from the 3-year maturity (6.50%) with estimates for the 5- and 7-year maturities in between (6.21% and 5.84%). The market Sharpe ratio estimates range from 35.54% to 41.40%. Again, all yearly estimates are quite similar with the lowest estimate of 4.98% (T=10, 2005) and the highest estimate of 7.63% (T=3, 2006). Interestingly, the increase in risk premia from 2004 to 2005 which occured both for the U.S. and for Europe was much less pronounced for Asia. Market Sharpe ratios in Asia were increasing from 2004 to 2005 – but significantly less than in the U.S. and Europe – while equity premia estimates were even decreasing due to decreasing volatilities.

All in all, the results based on 3-, 7- and 10-year maturities as well as the estimates for Europe and Asia confirm the results of the previous section and even lead to smaller equity premium estimates. Again, the resulting implicit equity premia are lower than based on historical estimates.

4.6. Robustness

4.6.1. Sensitivity with respect to noise in input parameters

In subsection 4.2.2 and appendix C.1 we have shown that the results are quite robust with respect to model changes. Besides misspecifying the model, a wrong measurement of the input parameters poses another possible source

Table 4.3.:

Company Sharpe Ratio, Market Sharpe Ratio and Equity Premium Estimates based on International 3-, 5-, 7-, 10-year CDS spreads from January 2004 to June 2007

The sample consists of the intersection of the KMV database and the major CDS indices for the U.S. (CDX.NA.IG), Europe (iTraxxEurope) and Asia (iTraxxAsia ex Japan) from January 2004 to June 2007. Only on-the-run companies are considered. CDS spreads are obtained from CMA through datastream where we included only company/date-combinations where spreads for all maturities (3-, 5-, 7- and 10-years) were available. For correlations, median industry correlations have been used based on 3-year weekly returns. Implied volatilities are based on maturities from 18-23 months from the VIX term structure (mid prices) of the CBOE for the U.S., the VSTOXX sub-index 24 months for Europe and 1-year historical volatilities for Asia. N denotes the number of CDS spreads for each maturity, T denotes the CDS maturity. Averages are calculated as unweighted averages over all observations.

		U.S. N=16,761				Europe N=17,786				Asia N=3,926			
Parameter	Year	T=3	T=5	T=7	T=10	T=3	T=5	T=7	T=10	T=3	T=5	T=7	T=10
Sharpe	2004	15.62%	16.70%	14.73%	13.79%	4.14%	6.70%	5.86%	5.54%	15.12%	15.09%	13.67%	13.13%
ratio	2005	19.86%	22.11%	20.79%	19.68%	12.36%	13.91%	13.43%	12.98%	17.59%	16.55%	15.64%	15.21%
company	2006	20.75%	22.77%	21.78%	20.54%	15.61%	16.78%	16.41%	15.97%	20.65%	19.27%	17.72%	16.66%
	2007	20.67%	22.29%	22.09%	21.20%	17.00%	16.47%	16.34%	16.26%	15.38%	15.16%	15.39%	15.30%
	04-07	19.57%	21.44%	20.31%	19.22%	12.34%	13.65%	13.19%	12.83%	18.02%	17.16%	16.13%	15.50%
Sharpe	2004	29.91%	31.79%	28.08%	26.29%	8.20%	12.04%	10.57%	9.97%	31.49%	32.49%	29.69%	28.78%
ratio	2005	40.82%	45.36%	42.63%	40.35%	26.54%	29.09%	27.96%	26.83%	40.43%	38.05%	35.94%	34.93%
market	2006	43.37%	47.30%	45.19%	42.57%	33.59%	35.64%	34.80%	33.68%	49.23%	45.79%	42.04%	39.45%
	2007	43.06%	46.32%	45.91%	44.02%	36.26%	35.08%	34.65%	34.10%	34.02%	33.40%	33.71%	33.45%
	04-07	40.20%	43.85%	41.56%	39.32%	26.33%	28.39%	27.40%	26.48%	41.40%	39.43%	37.03%	35.54%
Equity	2004	5.51%	5.82%	5.13%	4.80%	1.87%	2.73%	2.38%	2.24%	5.65%	5.81%	5.31%	5.15%
Premium	2005	6.60%	7.33%	6.88%	6.51%	4.91%	5.37%	5.16%	4.94%	5.73%	5.42%	5.13%	4.98%
	2006	6.64%	7.23%	6.90%	6.50%	6.36%	6.75%	6.59%	6.37%	7.63%	7.10%	6.51%	6.10%
	2007	6.64%	7.14%	7.07%	6.78%	6.89%	6.66%	6.58%	6.48%	5.84%	5.73%	5.79%	5.74%
	04-07	6.43%	7.00%	6.62%	6.26%	5.03%	5.44%	5.24%	5.06%	6.50%	6.21%	5.84%	5.60%

of inaccuracy. First, parameters might have been estimated with noise. Our estimator (4.7) is convex in PD^P and $\rho_{E,M}$ and concave in the CDS spread s. Therefore, noise in the measurement of the CDS spread causes our estimator to be downward biased, noise in PD^P and $\rho_{E,M}$ to be upward biased. The net effect is likely to be small and lead to an upward bias. Second, we may have systematically under-/overestimated any of the input parameters. We therefore tested the sensitivity of our results with respect to all input parameters. The results for the U.S. are shown in Table 4.4.[17] We would especially like to point out two facts: First, parameter changes of 10% relative to its original value result in an equity premium of approx. 10%/0.6 percentage points higher/lower for all parameters in our model. Second, the sensitivity is decreasing with increasing maturity. I.e., if input parameters have the same noise for all maturities then estimates based on 10-year maturities will be more accurate than estimates based on 3-year maturities. Of course, these sensitivities must be analyzed in combination with the accuracy of the respective input parameters. I.e., a high sensitivity is worse if the respective input parameter cannot be accurately determined, it is less harmful if the respective input parameter can be determined with very little noise. We will perform various robustness tests in the following subsections.

4.6.2. Robustness: CDS spread

General remarks and bid/ask spreads: We have used several measures to ensure that our CDS data is not significantly biased in any direction. First, our data source (CMA) is not based on a single market participant but based on data from several buy and sell side contributors. Second, we have compared our spreads to data from Bloomberg with no significant differences. Third, we have used constituents of the most liquid indices, which should enhance liquidity and data quality for the respective constituents. In addition our data sample should be easily comparable and reproducable and is not biased towards more recent dates.

[17] Our robustness tests concentrate on the U.S. since the U.S. market provides the widest choice of studies and historical data for robustness tests. Sensitivities are similar for Europe and Asia and are available on request.

	T=3 2004-2007		T=5 2003-2007		T=7 2004-2007		T=10 2004-2007	
	EP new	Δ (rel.)	EP new	Δ (rel.)	EP new	Δ (rel.)	EP new	Δ (rel.)
Base Model	6.43%		6.50%		6.62%		6.26%	
s +10%	7.12%	10.62%	7.16%	10.13%	7.20%	8.79%	6.80%	8.70%
s -10%	5.69%	-11.59%	5.78%	-11.02%	5.99%	-9.54%	5.67%	-9.42%
RR + 10%	7.05%	9.51%	7.09%	9.07%	7.14%	7.86%	6.74%	7.78%
RR -10%	5.88%	-8.67%	5.96%	-8.24%	6.15%	-7.13%	5.82%	-7.05%
EDF +10%	5.82%	-9.53%	5.93%	-8.82%	6.13%	-7.33%	5.81%	-7.18%
EDF -10%	7.10%	10.42%	7.12%	9.61%	7.15%	7.97%	6.74%	7.74%
Corr +10%	5.85%	-9.09%	5.91%	-9.09%	6.02%	-9.09%	5.69%	-9.09%
Corr -10%	7.15%	11.11%	7.22%	11.11%	7.36%	11.11%	6.95%	11.11%
Vola +10%	7.08%	10.00%	7.15%	10.00%	7.28%	10.00%	6.88%	10.00%
Vola -10%	5.79%	-10.00%	5.85%	-10.00%	5.96%	-10.00%	5.63%	-10.00%

Table 4.4.:
Sensitivities of equity premium estimates for the U.S. market
Sensitivities of equity premium estimates for U.S. market. *Base model* denotes the model with parameter choice as in section 4.4 and 4.5. $T = 3$, $T = 5$, $T = 7$ and $T = 10$ denote estimations based on different CDS maturities. *EP new* denotes equity premium estimate after change in input parameter as described in the first column. Δ *(rel.)* denotes the change of the equity premium estimate relative to the base model. *s* denotes the CDS spread for the respective maturity, *RR* denotes the recovery rate, *EDF* denotes the Expected default frequency based on KMV EDFs, *Corr* denotes the Asset/Market-Correlation and *Vola* denotes the implied market volatility.

Besides specific shortcomings of OTC markets, bid/ask spreads pose a natural noise in our data. We have used mean CDS spreads in our analysis. The average bid/ask-spread is only 4 bp in our sample which is probably also due to the fact that – in each week – we have only used the 125 most liquid CDS in the market. Using bid or ask quotes changes our average equity premium by less than 5%/0.3 percentage points.

Portion of CDS spread attributable to credit risk: There is no consensus on the part of the CDS spread which is due to credit risk in the academic literature. In general, however, CDS are seen to be a rather pure measure of credit risk, at least in comparison with bond spreads and reasonable estimates for non-credit-risk components are signficantly below 10% of the CDS spread, cf. section 2.3.3. We do not aim to quantitatively account for a possible part of the spread which is not due to credit risk. We do only want to stress that all these effects will lead to a decrease in the implicit equity premium since they result in a lower portion of the CDS spread attributable to credit risk. Therefore the derived equity premium of 6.50% should be regareded as an upper limit for the equity premium.

4.6.3. Robustness: Recovery rate

Our recovery rate assumption of 45% may lead to biased results due to several reasons.

First, our estimator is convex in the recovery rate. If recoveries are not constant but vary e.g. across industries then our estimator is downward biased. However, the convexity is not strong and recovery rate variations are not likely to be very pronounced, cf. section 2.3.2.2. Therefore, this downward bias should be rather small.
Second, the real-world recovery rate may be higher than 45% on average. Based on the literature presented in section 2.3.2.2 this seems unlikely. Indeed, we choose a 45% recovery rate because this seemed to be an upper limit for the average real-world recovery rate. Chava et al. (2006) set up a model where the expected recovery rate can be explained by the coupon rate, the 3-month Treasury yield, the issue size, and the seniority. Other covariates analyzed by Chava et al. do not improve out-of-sample performance. Using their regression results for the expected actual recovery rate indicates again, that our recovery rate of 45% is an upper limit for the expected recovery rate.[18] In addition, our sample consists of CDS with maturities up to 10 years from 2003-2007 which effectively means that recovery rates from 2003-2017 – e.g. a 15-year-horizon – are relevant for our averages. On the aggregate level, this should also help to mitigate some of the effects

[18]Based on Chava et al. (2006), the expected recovery rate for senior unsecured bonds can be estimated as $0.5183 + 0.0182 \cdot couponrate - 0.0319 \cdot 3 - month - Treasury - yield - 0.0332 \cdot log(issuesize)$, where coupon rate and Treasury yield are measured in percentage and the issue size is measured in \$'000. Even a very conservative calibration for our purpose of ($couponrate = 6,\ 3 - month - Treasury = 1,\ issuesize = \$10m$) results in an expected recovery rate of approx. 45% (i.e. 46%). Certainly, average coupon rates have been lower than 6%, the average 3-month Treasury yield has been higher than 1% and the average issue size for our sample has been higher than 10m, therefore the abovementioned calibration is conservative for our purpose.

induced by time-varying recovery rates.
Third, a countercyclical time-varying recovery rate results in risk-neutral recovery rates which are lower than actual recovery rates. Again, our recovery rate of 45% is an upper limit for the recovery rate.

All in all, a recovery rate of 45% used in our calculations seems to be an upper limit. Lower recovery rates would result in even lower Sharpe ratio estimates. Therefore our estimations poses an upper limit for the market Sharpe ratio.

4.6.4. Robustness: Actual default probabilities

First, it is very important to note that our main target is to determine the PD estimates that are used by market participants. E.g., if there was a better estimate for the real-world default probability than that used by market participants, it could be used to exploit arbitrage opportunities but it could not be used to gauge market participants risk aversion. Market participants rely almost entirely on three types of PD estimates: agencies' ratings, distance-to-default-based measures such as KMV EDFs and hazard rate models. So far we have used EDFs as our primary source for the actual default probability. Here, we will perform robustness tests based on agencies' ratings and a hazard rate model.

First, we have used agencies' ratings with the corresponding cumulative default probabilities as a robustness check. Unfortunately, these ratings are through-the-cycle estimates of the default probability. Using agencies' ratings therefore requires the assumption that we cover a whole economic cycle.[19] This assumption is probably most realistic for 5-year CDS – where we have covered the longest period from 2003-2007 including the high-expected default year 2003 – and for the longest maturity in our sample (10-year). We have averaged the ratings of Moody's, S&P and Fitch in our calculation

[19] More exactly, it requires the assumption, that investors average expectations over our sample period correctly mirror an economic cycle.

and determined multi-period default probabilities based on appendix A.1.1. Only observations where at least one of the agencies' ratings was available could be included which slightly decreased our data sample. Results are reported in Table 4.5. For the 5-year CDS sample from 2003-2007 we estimate very similar default probabilities (1.77% vs. 1.65%) and equity premia (6.70% vs. 6.66%). The estimated equity premium based on 10-year CDS spread are even lower (5.08% (Agencies) vs. 6.35% (EDF)) but this may be due to a good credit environment from 2004–2007 – an effect which is certainly even more pronounced for shorter maturities.

Hazard rate models provide another robustness check for both 1-year and mulit-year default probabilities. There is a large literature on hazard rate models for the U.S. market, e.g. Shumway (2001) and Chava/Jarrow (2004). Unfortunately, most of these models only estimate one-year ahead default probabilities and can therefore only be used as a robustness check for one-year default probabilities. Löffler/Maurer (2008) estimate a discrete duration model for conditional default probabilities up to 5 years ahead. They use accounting covariates (e.g. EBIT, total assets) as well as market variables (e.g. return, volatility) similar to Shumway (2001) for default prediction. Details about their model can be found in appendix A.2. For the estimates based on Löffler/Maurer (2008), we excluded financial services companies – which are excluded in their methodology – and all companies where Compustat data was not available. The results are reported in Table 4.5. The estimates for the cumulative default probability are slightly higher resulting in slightly lower equity premia estimates, but both are very similar to the EDF model. Five (three) year cumulative default probabilities are 1.89% (0.72%) for the Löffler/Maurer model compared to 1.79% (0.59%) for the EDF model. The resulting equity premia are 6.74% (5-year) and 6.11% (3-year) compared to 6.96% (5-year) and 6.61% (3-year) for the EDF model.

Of course, accuracies for estimating equity premia in our model should always be interpreted compared to alternative techniques for estimating equity premia. An inaccuracy of 10% in the average default probability results in an increase/decrease of our equity premium estimate by approx. 10%/0.6 percentage points. This sensitivity is comparable to the sensitivity of the long-run growth rate in dividend discount models.[20] In contrast to long-run growth rates we do though have (at least) partially objective criteria for default prediction. The sensitivity is also lower for higher rated obligors – where realized default rates are usually very noisy – and higher for lower rated obligors – where defaults can be observed more frequently and default probabilities are thereofore more stable. In addition – in contrast to dividend/earnings forecasts – default predictions are not systematically biased.

4.6.5. Robustness: Asset correlations

We have used median industry equity correlations as a proxy for asset correlations. Theoretical evidence for our framework suggests, that equity correlations are very good proxies for asset correlations (cf. appendix C.3) but we have also performed robustness checks based on other measures of asset correlations.

Correlations enter our formula in the denominator (cf. (4.6)). If the estimation of the correlations is unbiased but exhibits noise, our estimator will therefore be upward biased. If, for example, the "true" correlation would be 0.50 but due to noise we estimate (with equal probability) either 0.4 or 0.6, then the estimator is unbiased but the inverse of the correlation is upward biased (2.08 vs. 2.00). Therefore our estimations are upper limits for the true equity premium.

As a robustness check, we used Basel-II-correlations instead of asset correlations. Asset correlations in the Basel-II-framework have been determined as a result of intensive debate between regulators, academics and practi-

[20] Based on a Gordon model the equity premium estimate in a dividend discount model is $EP = g + d - r_f$ where EP is the equity premium estimate, g the growth rate of dividends, d the dividend yield and r_f the risk-free rate. If $d \approx r_f$ then the sensitivity with respect to g is approx. 1.

	T=3 2004-2007		T=5 2003-2007		T=7 2004-2007		T=10 2004-2007	
Parameter	EDF Model	New Model	EDF Model	New Model	EDF Model	New Model	EDF Model	New Model
Agencies' Ratings								
N	16,032	16,032	23,100	23,100	16,032	16,032	16,032	16,032
PD	0.58%	0.78%	1.77%	1.65%	2.25%	2.89%	3.74%	4.78%
EP	6.52%	4.29%	6.70%	6.66%	6.71%	5.26%	6.35%	5.08%
Discrete Duration Model								
N	10,078	10,078	14,700	14,700				
PD	0.59%	0.72%	1.79%	1.89%				
EP	6.61%	6.11%	6.96%	6.74%				

Table 4.5.:
Equity premium estimates based on different proxies for the real-world default probability
This table shows equity premium estimates where agencies' ratings and default probabilities based on a discrete duration model have been used as proxies for the real-world default probability. Agencies' ratings are based on average ratings of Moody's, S&P and Fitch with corresponding cumulative default probabilities based on appendix A.1.1. The discrete duration model is based on Löffler/Maurer (2008) and excludes all financial services companies and all companies where the respective balance sheet and P&L data was not available on Compustat. *EDF Model* denotes the model with EDFs as proxies for the real-world default probability. *New Model* denotes the model with either PDs based on agencies' ratings or based on Löffler/Maurer (2008). *N* denotes the number of observations, *PD* the average cumulative default probability and *EP* the equity premium estimation. Equity premia estimations for the EDF model deviate from the previous sections due to a different sample size.

tioner, cf. BIS (2005), and are therefore a combined results of many studies on asset correlations. For large corporates, asset correlations in the Basel-II-framework are a function of the one-year actual default probability. It is important to notice that Basel-II uses asset/asset-correlations whereas we are interested in asset/market-correlations. In a single-factor framework like Basel-II, asset/market-correlations can be inferred from asset/asset-correlations by simply taking the square root of the asset/asset-correlations.

Average Basel-II-inferred correlations are 0.48, slightly smaller than the average median industry correlation of 0.52. Basel-II-correlations are though less dispersed. For investment grade ratings, Basel-II asset/asset-correlations are always between 0.20 and 0.24 (for large corporates) which is equivalent to asset/market-correlations between 0.45 and 0.49. A lower dispersion leads to smaller averages since the asset/market-correlation enters our formula in the denominator, see (4.6). The resulting equity premium estimate is 6.73% (instead of 6.50%). All in all, the results are very similar to the results based on equity correlations.

5. The term structure of risk premia

5.1. Motivation

Practitioners and academics usually claim that risk premia must have increased significantly during the 2007/2008 financial crisis to justify the returns and valuations seen in the market. This raises a set of questions: How can this "gut feeling" by market participants be scrutinized in a solid methodological framework? And: Is the change in risk premia expected to be a permanent shift or do market participants expect risk premia to revert back to normal levels once the crisis is over? If risk premia are indeed volatile and mean reverting a risk premium term structure emerges. E.g., during times when (marginal) investors demand above average risk premia – such as gut feeling suggests for the 2007/2008 financial crisis – short duration assets would be expected to have a larger risk premium than longer duration assets.

This chapter analyzes the risk premium term structure before and during the 2007/2008 financial crisis and estimates the corresponding parameters of the instantaneous risk premium process (long-run mean, mean reversion speed, volatility).[1] Throughout this chapter the risk premium is measured as the market Sharpe ratio, i.e., the excess return of the market portfolio per unit of standard deviation. Of course, a measure such as the equity premium could also be used. The equity premium does, however, have the drawback

[1]For this chapter cf. also Berg (2009b).

that it combines risk aversion ("Sharpe ratio") and the quantity of risk ("volatility"). The target of this chapter is not to state that the quantity of risk has increased during the 2007/2008 financial crisis. Instead, the focus is on the excess return per unit of risk measured via the market Sharpe ratio.

A review of the literature on time-varying risk premia was given in section 2.2.3.2. In this chapter, we take a new approach and use the estimator for the market Sharpe ratio derived in section 4.2 to identify time-varying risk premia. This approach offers two distinctive advantages: First, implicit risk premia can be used instead of realized returns. Second, in contrast to equities, credit instruments are available for a variety of distinct maturities. In particular, for credit default swaps (CDS), standard maturities (usually 3, 5, 7, 10 years) have been established. Therefore, Sharpe ratios can be extracted from the credit markets for each maturity separately. This in turn yields a term structure of Sharpe ratios. Based on a time series of Sharpe ratio term structures the parameters of the instantaneous Sharpe ratio process can be estimated with a very high accuracy.[2] Indeed, the methodology proposed in this chapter results in reliable estimations for the parameters of the Sharpe ratio process with sample periods as small as 12 months.

Section 5.2 presents the theoretical framework. Section 5.3 describes our data sources. Section 5.4 provides the empirical results for the estimation of the term structure of Sharpe ratios and the parameters of the instantaneous Sharpe ratio process. Robustness tests are shown in section 5.5.

[2] It is well known from the interest rate literature that adding cross sections (i.e. different maturities) greatly enhances the estimation accuracy for the underlying process. Especially in situations where short-term interest rates are far above or below the long-run mean, adding information from further maturities decreases the resulting standard errors for the estimation of the long-run mean and mean reversion speed by a factor of up to 100.

5.2. Model setup

This section first links asset valuation to debt valuation (subsection 5.2.1) and then introduces a process for the instantaneous Sharpe ratio (subsection 5.2.2) in order to derive an estimator for the Sharpe ratio term structure (subsection 5.2.3, based on the results of section 4.2) and for the parameters of the Sharpe ratio process (subsection 5.2.4, based on a Kalman Filter approach).

5.2.1. Asset value process and default mechanism

The model setup in this chapter is mainly based on the framework presented in chapter 4.2. to estimate Sharpe ratios from CDS spreads. However, we will introduce time variation in Sharpe ratios in this section. Generally, the following two input factors have to be specified:

- The dynamics of the asset value process (including the dynamics of the asset Sharpe ratio).
- The default mechanism which – in addition to the asset value process – determines the default time.

Asset value process: The real-world asset value process V_t is modeled as a diffusion with a risk-free rate r, a payout ratio δ, a mean-reverting market Sharpe ratio θ_t^V and an asset volatility σ^V.

$$\text{Asset value process}: \quad dV_t = (\theta_t^V \cdot \sigma^V + r - \delta - \sigma_V^2/2)V_t dt + \sigma_V V_t dW_t^V \tag{5.1}$$

$$\text{CAPM condition}: \quad \theta_t^V = \rho\theta_t, \quad \text{with } \rho = Corr(r_M, r_V) \tag{5.2}$$

$$\text{Sharpe ratio process}: \quad \theta_t = \mu^\theta(\theta_t)dt + \sigma^\theta(\theta_t)dW_t^\theta. \tag{5.3}$$

Equation (5.2) assumes that the continuous time CAPM holds, i.e., that the company Sharpe ratio (θ^V) is the product of the asset/market-correlation (ρ) and the market Sharpe ratio (θ). Here, r_M and r_V denote the return on the market portfolio and the asset return. The last equation formulates the process for the instantaneous Sharpe ratio. This process will be specified in more detail in section 5.2.2.

The formulas above can be easily reformulated in terms of the equity premium π_t instead of the market Sharpe ratio θ_t. The Sharpe ratio was choosen because it is a purer measure of risk aversion (whereas the equity premium measures both risk aversion as well as the quantity of risk).

Default mechanism: As in section 4.2, a simple Merton model is assumed, i.e., default can only happen at the end of maturity if the asset value is lower than the default barrier. Therefore, actual (PD^P) and risk-neutral (PD^Q) default probability can be calculated as

$$PD^P = P[V_T^P < L] \tag{5.4}$$

$$PD^Q = Q[V_T^Q < L] \tag{5.5}$$

The choice of such a specific default mechanism may seem to be a rather hard restriction. Fortunately, our results are robust with respect to the use of other models. This issue will be discussed in more detail in subsection 5.2.3.

5.2.2. A process for the instantaneous Sharpe ratio

The instantaneous Sharpe ratio θ_t is modeled as a mean-reverting Ornstein-Uhlenbeck process (OUP), i.e.,

$$\text{Process}: \quad d\theta_t = \kappa(\bar{\theta} - \theta_t)dt + \sigma dW_t^\theta \tag{5.6}$$

$$\text{Solution}: \quad \theta(s) = \theta(t)e^{-\kappa(s-t)} + \bar{\theta}\left(1 - e^{-\kappa(s-t)}\right) + \int_t^s \sigma e^{-\kappa(s-\nu)} dW_\nu^\theta \tag{5.7}$$

$$\text{Average}: \quad \Theta(t,\tau) := \frac{1}{\tau}E^P\left[\int_t^{t+\tau} \theta(s)ds\right] = \bar{\theta} + \frac{1}{\tau}(\theta_t - \bar{\theta})\frac{1 - e^{-\kappa\tau}}{\kappa}. \tag{5.8}$$

Please note that the average Sharpe ratio $\Theta(t,\tau)$ is defined as a real-world arithmetic average rather than a risk-neutral average of the respective discount rates.[3] The reason behind that definition will become clearer in the next subsections.

The choice of a specific process for the instantaneous Sharpe ratio process is by no means trivial. Several other candidates (e.g. CIR-process) would certainly also qualify. The OUP was choosen for several reasons: First, it is able to capture mean reversion and volatility. Second, its parameters can be easily interpreted ($\bar{\theta}$: long-run mean, κ: mean reversion speed, σ: volatility). Third, it is analytically tractable. Fourth, it has also been used by other authors for a similar purpose (e.g. Huang/Huang (2003) and Campbell/Viceira (1999)).

There are several possible underlying reasons why Sharpe ratios may be time-varying. One reason may be a time-varying risk aversion of the marginal investor. But even under the assumption of constant relative risk aversion, time-varying expected returns may emerge if the volatility of consumption is time-varying. We do not aim to explain the drivers of time-varying expected returns here, rather we take the indications of several academic studies (see Fama/French (1988) and Cochrance (1992) for an overview) as a motivation to analyze whether expected time-varying Sharpe ratios can be validated based on current asset prices.

5.2.3. Estimating Sharpe ratios from CDS spreads

This subsection derives a formula to estimate both the Sharpe ratio of the underlying firm's asset value process and the market Sharpe ratio directly from CDS spreads and estimates for the real-world default probability. We have shown in section 4.2, formula (4.2) and (4.4) that the following formula for the market Sharpe ratio SR_M can be derived in a Merton framework

[3] I.e. $E^P\left[\int_t^{t+\tau}\theta(s)ds\right]$ instead of the usual "spot-rate" definition $lnE^Q\left[e^{-\int_t^{t+\tau}\theta(s)ds}\right]$.

with constant Sharpe ratios:[4]

$$SR_M = \frac{\Phi^{-1}(PD^Q(t,\tau)) - \Phi^{-1}(PD^P(t,\tau))}{\sqrt{\tau}} \frac{1}{\rho_{V,M}}, \tag{5.9}$$

where PD^Q and PD^P denote the cumulative risk-neutral and actual default probability, T denotes the maturity and and $\rho_{V,M}$ denotes the asset/market correlation.

Although this estimator is derived in a simple Merton framework, we have shown in section 4.2.2 and appendix C.1 that the estimator is still robust in first-passage-time frameworks, frameworks with endogenous default barrier and frameworks with unobservable asset values (Duffie/Lando (2001) model). Huang/Huang (2003) also show that – given a certain actual default probability – the risk-neutral default probability is almost the same for the main structural models of default in the literature. They analyze the Longstaff/Schwartz (1995) model with stochastic interest rates, the Leland/Toft (1996) model with endogeneous default, strategic default models of Anderson/Sundaresan/Tychon (1996) and Mella-Barral/Perraudin (1997), and a model with mean-reverting leverage ratios. Their analysis also includes a model with time-varying asset risk premium.

Appendix C.2 explicitly discusses the case of time variation in Sharpe ratios and shows that the Sharpe ratio estimator above is approximately true for a model with time-varying Sharpe ratios if the constant Sharpe ratio SR_M is substituted by its real-world arithmetic average $\Theta(t,\tau)$ as defined in (5.8):

$$\Theta(t,\tau) \approx \frac{\Phi^{-1}(PD^Q(t,\tau) - \Phi^{-1}(PD^P(t,\tau))}{\sqrt{\tau}} \frac{1}{\rho_{V,M}} \tag{5.10}$$

The cumulative risk-neutral default probability PD^Q can be dervied from CDS spreads s for a maturity T via the relationship (cf. Duffie/Singleton

[4]The Sharpe ratio of the underlying firm's asset value process can be estimated by omitting the $\rho_{V,M}$-term.

(2003))

$$PD^Q = 1 - e^{-s/LGD \cdot T}, \tag{5.11}$$

where LGD denotes the loss given default. The respective calibration issues are discussed in the empirical part.

5.2.4. Estimating the parameters of the instantaneous Sharpe ratio process

Unfortunately, the Sharpe ratio process θ_t cannot be observed directly. Instead, only the average expected Sharpe ratios $\Theta(t_i, \tau_j)$ can be measured via (5.10) and this measurement may be subject to noise. Therefore, the Kalman filter methodology is needed in order to estimate the parameters of the Sharpe ratio process. The application of this methodology is similar – but not equal – to the literature on interest rate processes.[5] In our case, the transition and measurement equations can be derived based on equation (5.7) and (5.8), i.e.,

$$\begin{aligned} \theta_{t_i} &= F\theta_{t_{i-1}} + C + \epsilon_{t_i} && (5.12) \\ \Theta(t_i, \tau) &= H\theta_{t_i} + A + \nu_{t_i} && (5.13) \end{aligned}$$

with

$$\begin{aligned} F &= e^{-\kappa \Delta t} \\ C &= \left(1 - e^{-\kappa \Delta t}\right) \cdot \bar{\theta} \\ \epsilon_{t_i} &\sim \mathrm{N}\left(0, \frac{\sigma^2}{2\kappa}\left(1 - e^{2\kappa \Delta t}\right)\right) \\ (H)_j &= \frac{1}{\tau_j} \frac{1 - e^{-\kappa \Delta t}}{\kappa} \\ (A)_j &= \left(1 - \frac{1}{\tau_j} \frac{1 - e^{-\kappa \Delta t}}{\kappa}\right) \cdot \bar{\theta} \\ \nu_{t_i} &\sim \mathrm{N}\left(0, R^2 \cdot I\right), \end{aligned}$$

[5] Cf. Bolder (2001) for a good overview of the Kalman filter approach and interest rate modeling.

where I denotes the identity matrix and τ denotes a vector of all available maturities. R is unknown and estimated within the Kalman filter methodology. The corresponding log-likelihood function is given by

$$l(\kappa, \bar{\theta}, \sigma, R) = -\frac{nNln(2\pi)}{2} - \frac{1}{2}\sum_{i=1}^{N} ln\left(det(var_i)\right) + err_i^t var_i^{-1} err_i$$
$$err_i = \Theta(t_i) - E\left[\Theta(t_i)|\mathcal{F}_{t_{i-1}}\right], \quad var_i = Var\left[\Theta(t_i)|\mathcal{F}_{t_{i-1}}\right]. \qquad (5.14)$$

Here, $\Theta(t_i)$ denotes a row vector where each row represents one maturity. The conditional expectations and variances are determined based on (5.13).

The parameter estimation proceeds in two steps:

- First, the Sharpe ratios $\Theta(t_i, \tau_j)$ are estimated based on (5.10) for all available maturities τ_j and for all available dates t_i.
- Second, the parameters of the Sharpe ratio process are estimated based on (5.12)-(5.14).

5.3. Data and implementation

The sample consists of weekly observations of 3-, 5-, 7-, and 10-year CDS spreads from on-the-run companies of the main CDS indices in the U.S. and Europe from April 2004 until September 2008. The CDX.NA.IG index was used for the U.S. and the iTraxx Europe index for Europe. Both indices consist of 125 members and are rolled over every six months (end of March and end of September). The first series of the iTraxx was launched on Mar, 20th, 2004, the first series of the CDX.NA.IG was launched on Nov, 20th, 2003. In order to obtain the same sample period for both Europe and the U.S. our sample starts for both regions in April 2004.

For each observation, the average market Sharpe ratio was estimated based on (5.10). The following input parameters were needed: a) CDS spreads and b) loss given default (to estimate the risk-neutral default probability based on (5.11)), c) real-world cumulative default probabilities, and d) correlation between asset returns and market returns.

The CDS spreads for 3-, 5-, 7-, and 10-year maturity were taken from CMA (Credit Markets Association) via Datastream. Mid spreads were used for the analysis. Together with each CDS spread CMA provides a veracity score which indicates if the spread is based on an actual trade, a firm bid or other sources (e.g. indicative bid, bond spread derived). A date/company-combination was only included in our sample if either trades or firm bids have been reported for all maturities (3,-5-,7-, 10-year) in that respective week. The loss given default was set to 60% based on Moody's (2007) and robustness tests were performed.

The actual cumulative default probability was determined based on expected default frequencies (EDFs) from Moody's KMV. Moody's KMV provides EDFs from 1- to 10-year maturities based on the distance-to-default measure. To calibrate the distance-to-default to default probabilities, KMV uses its proprietary database of historical default events. Therefore, there is no problem of any circular arguments, since the level of default probabilities does not rely on any Sharpe ratio or drift assumptions taken by Moody's KMV. Robustness tests based on a hybrid hazard-rate model have been conducted.

Correlations between asset returns and the market return were proxied by the median industry correlation between the corresponing equity returns and the market return (2-year weekly correlations between the performance index of the respective stocks and the major index in each region).[6] The data was taken from Datastream. Using equity instead of asset correlations is justified as equity is a deep-in-the-money call on the company's assets. Therefore, delta is approximately one and gamma approximately zero and correlations are (almost) the same. Median industry correlations were used

[6]The S&P 500 was used as the index for the U.S. and the Stoxx600 as the index for Europe.

for robustness reasons.

Finally, the sample period was split into two sub-periods: "Before Crisis" (April 2004-June 2007) and "During Crisis" (July 2007-September 2008). Of course there is no single starting date of the 2007/2008 financial crisis, therefore our division of the sample period is – to a certain extent – arbitrary.[7] Already in Feb. 2007, HSBC announced losses of \$ 10bn related to subprime mortgages. In April 2007, New Century Financial, one of the biggest mortgage lenders in the U.S., declared bankruptcy. The crisis accelerated in June and July 2007 when Bear Stearns had to inject \$ 3.2 bn to bail out two of its hedge fonds and when Moody's and Standard & Poor's downgraded more than 250 subprime RMBS. The Dow Jones index peaked as late as in October 2007. However, our main conclusions do also hold when choosing Q2 2007 or Q4 2007 as a starting point for the 2007/2008 financial crisis. It is not the target to show that certain risk premia changes happened *exactly* at the beginning of the crisis. Rather it should be demonstrated that the implied risk premia have gradually changed throughout the turmoil.

All in all, the study uses 20,215 observations for each maturity for the U.S. (14,416 before the crisis, 5,799 observations during the crisis) and 20,809 observations for each maturity for Europe (15,091 before, 5,718 during the crisis). The descriptive statistics for the input parameters are shown in table 5.1.

[7] The following information on the history of the financial crisis was taken from CNNMoney.com/Special report: Subprime crisis: A timeline (http://money.cnn.com/2008/09/15/news/economy/subprime_timeline/index.htm).

Table 5.1.:

Descriptive statistics for input parameters

The sample consists of the intersection of the KMV database, the CDX.NA.IG on-the-run companies (U.S.)/iTraxx on-the-run companies (Europe) and the CMA CDS database (via Datastream) from April 2004 to September 2008. $CDS3/CDS5/CDS7/CDS10$ denote 3-/5-/7-/10-year CDS spreads in bp. $EDF3/EDF5/EDF/7/EDF10$ denote 3-/5-/7-/10-year cumulative expected default frequencies from Moodys KMV. ρ denotes the equity/market correlation. Median industry correlations have been used based on 2-year weekly returns. The corresponding market returns are based on the return index of the S&P 500 (U.S.) and Stoxx600 (Europe). σ_M denotes the implied market volatility based on maturities from 18-23 months from the VIX term structure (mid prices) of the CBOE and the VStoxx sub-index. Averages are calculated as unweighted averages over all observations.

	Before Crisis (04/2004 – 06/2007)				During Crisis (07/2006 – 09/2008)			
Param.	N	Mean	Median	Stddev	N	Mean	Median	Stddev
				Panel A: U.S.				
CDS3	14,416	32.38	20.00	52.08	5,799	126.14	52.70	244.85
CDS5	14,416	53.74	37.70	62.69	5,799	133.84	70.00	204.20
CDS7	14,416	66.86	49.20	66.53	5,799	134.37	77.20	179.73
CDS10	14,416	79.74	60.70	69.61	5,799	136.95	85.40	161.53
EDF3	14,416	0.54%	0.30%	1.23%	5,799	1.11%	0.24%	4.67%
EDF5	14,416	1.21%	0.70%	1.98%	5,799	1.99%	0.60%	6.22%
EDF7	14,416	1.94%	1.18%	2.70%	5,799	2.87%	0.98%	7.48%
EDF10	14,416	3.01%	1.98%	3.75%	5,799	4.12%	1.59%	9.14%
ρ	14,416	0.51	0.52	0.08	5,799	0.52	0.52	0.06
σ_M	14,416	16.08%	15.92%	1.43%	5,799	23.37%	24.34%	2.36%
				Panel B: Europe				
CDS3	15,091	20.82	17.30	15.83	5,718	63.94	50.10	51.65
CDS5	15,091	34.28	29.20	23.80	5,718	82.29	64.95	60.85
CDS7	15,091	44.11	39.00	28.06	5,718	89.89	72.20	61.82
CDS10	15,091	54.36	49.30	31.92	5,718	96.33	79.50	61.73
EDF3	15,091	0.57%	0.36%	0.61%	5,718	0.38%	0.21%	0.51%
EDF5	15,091	1.35%	0.90%	1.32%	5,718	0.95%	0.55%	1.06%
EDF7	15,091	2.22%	1.53%	2.04%	5,718	1.60%	0.98%	1.64%
EDF10	15,091	3.50%	2.47%	3.09%	5,718	2.56%	1.59%	2.50%
ρ	15,091	0.57	0.58	0.11	5,718	0.59	0.55	0.10
σ_M	15,091	19.58%	18.92%	2.09%	5,718	24.36%	24.20%	1.68%

5.4. Results

5.4.1. Risk premium term structure

The Sharpe ratio estimates based on 3-, 5-, 7-, and 10-year CDS spreads are presented in figure 5.1 (U.S.) and 5.2 (Europe).[8] Both the U.S. and Europe show a very similar pattern of CDS-implied Sharpe ratios over the whole sample period. In addition, for both the U.S. as well as Europe CDS-implied Sharpe ratios before the 2007/2008 financial crisis are very similar for all maturities. In fact, the implied Sharpe ratio estimates for different maturities rarely deviate by more than five percentage points before the 2007/2008 financial crisis. The overall level of the Sharpe ratio estimates is slightly higher for the U.S. (predominantly between 20-50% before the crisis) than for Europe (predominantly between 10-40% before the crisis) as also mentioned in Berg/Kaserer (2008). This is consistent with both historical experience as well as with standard portfolio theory.[9]

During the financial crisis, beginning in July 2007, the CDS-implied Sharpe ratios show an interesting patters: For both the U.S. as well as for Europe implied Sharpe ratios generally increase during the 2007/2008 financial crisis. The increase is, however, much more pronounced for short-term maturities (3-year, 5-year) than for longer maturities (7-year, 10-year). As a result, the term structure of risk premia changes from a flat term structure before the 2007/2008 financial crisis to an inverse term structure during the 2007/2008 financial crisis. This term structure of Sharpe ratios is depicted in figure 5.3. In addition, the behavior of the risk premium term structure (flat before the crisis and inverse during the crisis) is very persistent as can be seen from figure 5.1 and 5.2. Indeed, the inverse nature of the risk premium term structure was prevalent in any week without exception from mid 2007 until the end of our sample period.

[8] Median Sharpe ratios for each date are depicted to decrease the influence of outliers. Mean estimates are, however, very similar but sligtly higher.

[9] Under the assumption that the U.S. market has a higher correlation with the global market portfolio than the European market, the European Sharpe ratios are smaller than U.S. Sharpe ratios based on CAPM considerations.

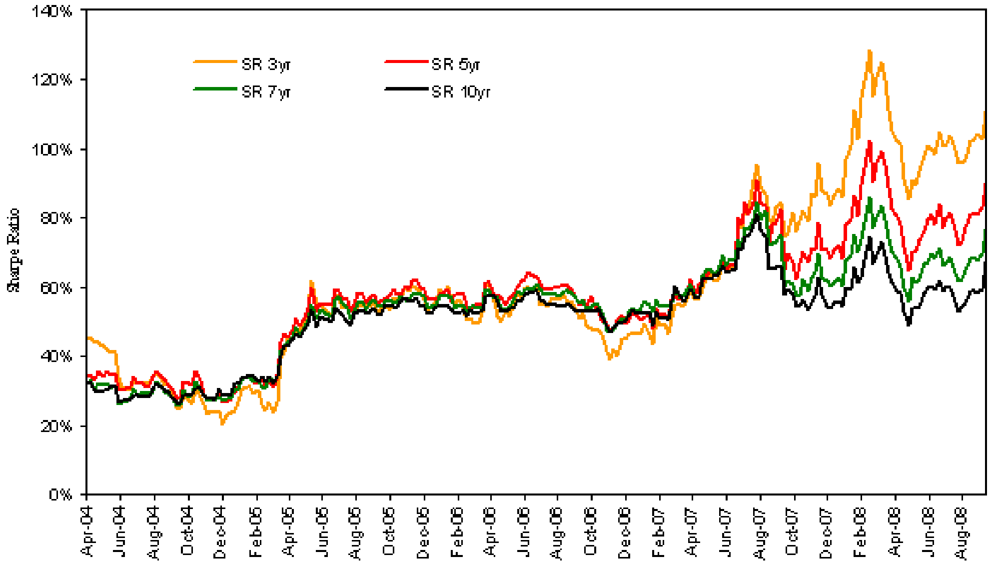

Figure 5.1.: CDS-implied Sharpe ratios for the U.S. (index: CDX.NA.IG) based on 3-/5-/7- and 10-year CDS spreads and EDFs form 04/2004-09/2008. *SR Xyr* denotes CDS-implied Sharpe ratio for X years.

The economic interpretation is straightforward: Assume that the risk aversion of the marginal investors is mean-reverting. Then the instantaneous Sharpe ratio will also be mean-reverting. If the instantaneous Sharpe ratio equals the long-run mean, a flat risk premium term structure will be observed. In contrast, if the current instantaneous Sharpe ratio is high, the expected Sharpe ratio will be a decreasing function of the maturity. Therefore, an inverse risk premium term structure emerges.

5.4.2. Slope of the risk premium term structure

Our methodology also allows for an estimation of the slope of the risk premium term structure. Similar to the interest rate literature, this slope deter-

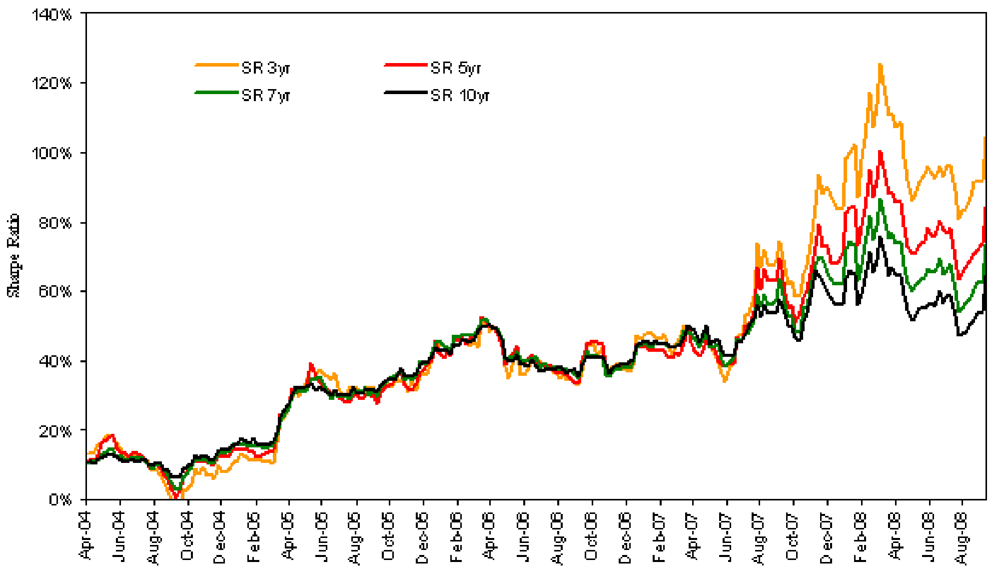

Figure 5.2.: CDS-implied Sharpe ratios for Europe (index: iTraxx Europe) based on 3-/5-/7- and 10-year CDS spreads and EDFs form 04/2004-09/2008. *SR Xyr* denotes CDS-implied Sharpe ratio for X years.

mines the difference between long-run and short run risk premia. Figure 5.4 depicts this slope based on the Sharpe ratio estimates from subsection 5.4.1.

The resulting slope is close to zero before the 2007/2008 financial crisis, drops significantly at the end of the second quarter of 2007 and stays clearly negative until the end of our sample period (September 2008). Based on theoretical arguments, the change in the slope of the risk premium term structure should be a factor in asset pricing, too. Assume that we start with a flat risk premium term structure. Since we operate in a stationary setting, expected risk premia are then equal to today's risk premia. If the slope becomes negative (e.g. 3-yr Sharpe ratios are larger than 10-yr Sharpe ratios) assets with a longer duration[10] should perform better than assets

[10]Duration is defined as in the bond literature, i.e. how long on average an investor has

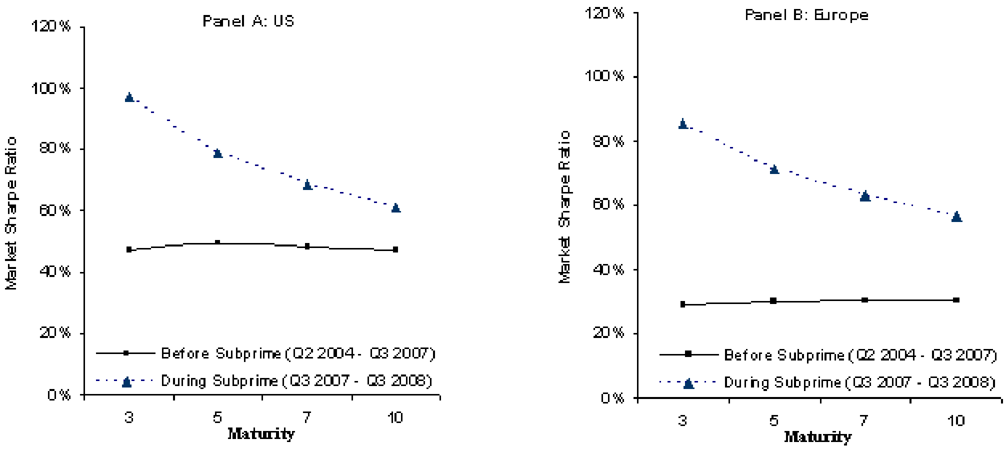

Figure 5.3.: Term structure of risk premia before (04/2004-06/2007) and during (07/2007-09/2008) the 2007/2008 financial crisis for the U.S. (index: CDX.NA.IG) and Europe (index: iTraxx Europe). x-axis: maturity, y-axis: CDS-implied market Sharpe ratio based on (5.10).

with a shorter duration (after controlling for other factors such as beta, size and market/book). On the other hand, if the slope becomes positive then short duration assets should perform better than long duration assets.[11] Factors such as the market-to-book ratio partially capture the duration of an asset. Therefore, the market-to-book ratio may indeed be justified as an asset pricing factor by changes in the underlying term structure of risk premia.[12] We think that this may be an area for future research.

to wait before receiving cash payments. The duration of a stock in a Gordon growth model is the reciprocal of the dividend yield. I.e. a stock with a 4% dividend yield would have a 25 year duration, a stock with a 2% dividend yield a duration of 50 years.

[11] Please note that it is not important if the slope incrases or decreases, rather it is important if it is higher/lower relative to its expected value.

[12] In this case one should, however, see both positive and negative returns on this factor depending on the change in the risk premium term structure relative to the expected values for that respective year.

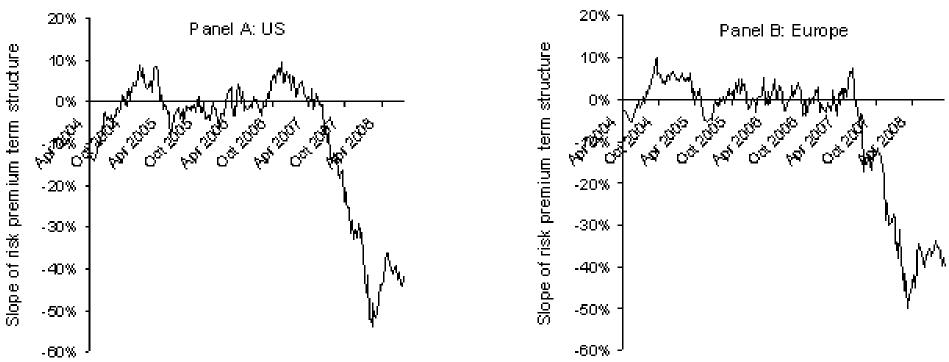

Figure 5.4.: Slope of the risk premium term structure measured as 10-year CDS-implied Sharpe ratio minus 3-year CDS-implied Sharpe ratio. Picture on left-hand side: U.S., picture on right-hand side: Europe.

5.4.3. Instantaneous Sharpe ratio process

Based on the estimates of the Sharpe ratio term structure from subsection 5.4.1 the parameters for the instantaneous Sharpe ratio process have been estimated as described in subsection 5.2.4. The resulting parameter estimates for the long-run mean Sharpe ratio $\bar{\theta}$, the mean reversion speed κ and the Sharpe ratio volatility σ_θ are shown for both the U.S. and Europe in table 5.2. The Kalman methodology has been applied for the total sample period (April 2004-September 2008) as well as for the "Before Crisis" (April 2004-June 2007) and "During Crisis" (July 2007-September 2008) subperiods. In addition, the average of the (filtered) instantaneous Sharpe ratio ($\oslash\theta_t$) is depicted in the second column.

Most importantly, table 5.2 reveals that the long-run mean estimate ($\bar{\theta}$) based on the "During Crisis" subperiod is of a similar magnitude than for the total sample period (41.5% vs. 43.8% for the U.S., 41.0% vs. 30.1% for Europe). In addition, the long-run mean estimate for the "During Crisis" subperiod is also of a similar magnitude than the average instantaneous

Sharpe ratio ($\oslash\theta_t$) estimate in the "Before Crisis" subperiod.[13]

In contrast to the long-run mean estimates, the estimates for the instantaneous Sharpe ratio (θ_t) are an order of magnitude higher for the "During Crisis" subperiod than for the "Before Crisis" subperiod (156.7% vs. 49.5% for the U.S., 153.5% vs. 32.1% for Europe). In addition, during the 2007/2008 financial crisis, the mean reversion parameter is significantly larger than zero with a similar mean reversion speed for the U.S. and Europe (0.59 and 0.63).

A special attention should be devoted to the standard errors. All standard errors are rather small. These small standard errors seem to be especially surprising for the "During Crisis" period – where parameter estimates are only based on 64 weekly observation. There are two (interlinked) reasons: First, this study uses cross-sectional information, e.g. maturities of 3-, 5-, 7- and 10-years. This cross-sectional information is very stable during the 2007/2008 financial crisis: The risk premium curve is inverse for every single week from mid 2007 on and the slope of the risk premium term structure is similar during the whole "During Crisis" subperiod. Second, it is a well known fact from interest rate modeling that, if information for several maturities is available, long-run means and mean reversion parameters can be estimated with higher accuracy when instantaneous rates are far above or below their long-run mean parameters. Indeed, if the parameters are estimated only based on one of the maturities, standard errors are more than 10 times higher.[14]

In particular the fact that the long-run mean is of a similar magnitude

[13]Please note that the mean reversion parameter estimate (κ) for the "Before Crisis" subperiod is not significantly above zero for both the U.S. and for Europe. Therefore, the long-run mean estimate for this subperiod becomes meaningless. In addition, the process becomes a martingale, i.e., expected conditional values for the instantaneous Sharpe ratio equal today's instantaneous Sharpe ratio.

[14]A potential bias in the estimates and standard errors could arise from the assumption of uncorrelated error terms. The Kalman procedure was therefore repeated assuming a homogeneous cross-sectional correlation of the error terms between 0 and 0.9. The resulting estimates for the long-run mean and the mean reversion speed did not change by more than 15% relative to the original estimates.

before and during the crisis seems to be an intuitive, but interesting finding. This finding probably does not come as a surprise to most researchers. However, this methodology is – to our best knowledge – the first one that is able to extract these risk premium term structures together with estimates for the Sharpe ratio process out of current asset prices with a satisfying accuracy. The main advantage of this approach is the feature, that it is able to estimate a whole term structure of risk premia for each date in our sample. Adding this cross-sectional information in addition to the pure time series evolvement of the implied or realized risk premia renders the estimates much more precise.

5.5. Robustness tests

5.5.1. General remarks on robustness

The question of robustness arises for almost every empirical study. In the context of the 2007/2008 financial crisis it does, however, gain special importance. Many economic parameters have seen extraordinary levels during the 2007/2008 financial crisis and may possibly distort our results. This may include the influence of certain subsamples, subperiods or a bias in the measurement of any of the input parameters.

Looking at certain subsamples – especially exclusion of financials – or certain subperiods during the 2007/2008 financial crisis does not change the main result of a downward sloping risk premium term structure during the 2007/2008 financial crisis. The sensitivity with respect to the input parameters is analyzed in the next step. First, a simple target value search is conducted. This target value search looks at each paramater separately and determines the value of this parameter as to yield a flat risk premium term structure during the crisis. Of course, this procedure is not able to gauge erros in several input parameters at the same time. In addition, it is not able to explain why any of the input parameters may have been biased

Table 5.2.:

Parameter estimates for instantaneous Sharpe ratio process

This table depicts the parameter estimates for the instantaneous Sharpe ratio process based on the 3-, 5-, 7- and 10-year Sharpe ratio estimates from subsection 5.4.1 and the Kalman filter methodology as described in subsection 5.2.4. *Total* denotes the total period form 04/2004-09/2008, *Before Crisis* denotes the period from 04/2004-06/2007, *During Crisis* denotes the period from 07/2007-09/2008. $\oslash\theta_t$ denotes the average of the instantaneous Sharpe ratio, $\bar{\theta}$ the long-run mean Sharpe ratio, κ the mean reversion, σ_θ the Sharpe ratio volatility and R the standard deviation of the error term. *SE* denotes the standard errrors for the respective parameter estimates. */** denotes significance at the 5%/1% level.

		Estimate				SE			
Period	$\oslash\theta_t$	$\bar{\theta}$	κ	σ_θ	R	$\bar{\theta}$	κ	σ_θ	R
Panel A: U.S.									
Total (04/2004-09/2008)	74.0%	43.8%**	0.37**	37.8%**	5.0%**	0.7%	0.02	3.2%	0.1%
Before Crisis (04/2004-06/2007)	49.5%	32.6%**	0.01	12.8%**	2.5%**	10.6%	0.01	1.1%	0.1%
During Crisis (07/2007-09/2008)	156.7%	41.5%**	0.59**	84.7%**	4.8%**	2.1%	0.09	14.1%	0.2%
Panel B: Europe									
Total (04/2004-09/2008)	54.6%	30.1%**	0.25**	34.4%**	3.7%**	0.6%	0.01	2.4%	0.1%
Before Crisis (04/2004-06/2007)	32.1%	5.0%	0.00	4.8%**	4.2%**	10.4%	0.00	0.3%	0.3%
During Crisis (07/2007-09/2008)	153.3%	41.0%**	0.63**	110.9%**	2.6%**	0.9%	0.05	13.0%	0.1%

during the crisis. Therefore, in a second step, a different proxy for the most crucial input parameter – the real-world default probability – is used. In addition, this study controls for parameters which have seen extraordinary levels during the 2007/2008 financial crisis via a regression approach. As a final check, market microstructure effects on CDS liquidity are analyzed.

5.5.2. Target search procedure

The target search proceeds in two steps. First, any of the input parameters (e.g., the default probability) is choosen. Second, the value for this parameter which would yield a flat risk premium term structure for the "During Crisis" subperiod (i.e. which yields the same 3-year CDS-implied Sharpe ratio than the 10-year CDS-implied Sharpe ratio) is implicitly determined. This study takes a very conservative position: It is assumed, that the parameter for the 10-year calculations remain unchanged while only the input parameters for the 3-year calculations are changed. E.g., the 3-year PD-estimates are increased until the same Sharpe ratio than with the *original* 10-year PD-estimates is obtained. Of course, if 3-year PD-estimates are downward biased one would expect the same for 10-year PD estimates. Therefore, the results provide a lower level of the change that is necessary to yield a flat term structure.

Table 5.3 depicts the value of our target search for the years 2007 and 2008. In the following, we focus on the values for the year 2008. The resulting values for the LGD are larger than 100% and therefore economically impossible. A correlation of 91% (U.S.) also seems unrealistic, a correlation of 102% (Europe) is economically impossible. A 60% (U.S.) / 61% (Europe) lower CDS spread is more than 10 times the bid/ask spread and cannot be explained by market microstructure issues alone. The 3-year PD has to increase by at least 182% (U.S.) and 211% (Europe) in 2008 to yield a flat term structure of risk premia. The resulting PDs are 4.26 % for the U.S. and 1.41% for Europe. Please note that our sample consists only of investment grade obligors since the CDS indices used include only investment grade obligors. The maximum 3-year cumulative PD for investment grade obligors from 1970-2006 based on Moody's (2007) was 1.22%.[15] The maximum 1-year default probability the investment grade obligors in the U.S. from 1920-2007 was 1.557% (in 1938). It would need three years in a row as bad as the worst year from 1920-2007 to yield cumulative PDs as high as 4.26 %. If it is assumed in addition that 10-year PDs would increase by the same absolute amount than the 3-year PDs, the target 3-year PDs increase to more than 5% for the U.S. and more than 1.7% for Europe. These values seem to be unreasonable given historical experience.

However, the PD estimate is probably the single most crucial input parameter of our procedure. Therefore we will perform two other robustness tests: First, an alternative measures for the PD estimate is applied, then a robustness test based on a linear regression of CDS spreads on certain parameters with very high/low levels during the 2007/2008 financial crisis is performed.

5.5.3. Different PD estimates

PD estimates can be categorized in three approaches:[16] Agencies' ratings (Moody's, Fitch, S&P), estimates derived from structural models of default (such as KMV EDFs) and hazard rate models. Agencies' ratings are (partly) expert-based and are "through-the-cycle" ratings, therefore they are not suited for this study. Hazard rate models similar to Shumway (2001) are especially popular for internal rating models of major banks. Recently, Fitch also launched its new equity-implied ratings (EIR). They use a hazard rate approach but also include the distance-to-default-measure as one of the covariates. In addition, financial ratios, market performance and macro variables are used for default prediction.[17]

[15] Cf. Moody's (2007), Exhibit 33. The study is a worldwide study but is dominated by U.S. and European data. The average default rate for Europe is usually lower than for the U.S., although Moody's does not provide a disaggregation on this level.
[16] Cf. also section 2.3.2.1.
[17] For details see FitchRatings (2007).

Table 5.3.:
Robustness: Target search procedure

This table shows the parameters that have to be used for the 3-year Sharpe ratio estimate in order to yield the same Sharpe ratio than for the 10-year Sharpe ratio estimate. The parameters for the 10-year Sharpe ratio estimate are left unchanged. Column "Q1/Q2 2007" shows the results for the second half of 2007, "Q1/Q2/Q3 2008" shows the results for the first three quarters of 2008. PD denotes the average real-world default probability, LGD denotes the loss given default, s the average CDS spread and ρ the average asset/market correlation. $Base\,Case$ is the average of the respective parameters without adjustments, $\%\,Change$ is the change in percentage relative to the base case and $Target$ the value of the respective parameter that is needed for a flat risk premium term structure. For the sample decomposition please see table 5.1. Averages are calculated as unweighted averages over all observations.

Parameter		Panel A: U.S.		Panel B: Europe	
		Q1/Q2 2007	Q1-Q3 2008	Q1/Q2 2007	Q1-Q3 2008
PD	Base Case	0.45%	1.53%	0.26%	0.46%
	% Change	76%	182%	75%	211%
	Target	0.84%	4.26%	0.46%	1.41%
LGD	Base Case	60%	60%	60%	60%
	% Change	60%	150%	62%	158%
	Target	96%	150%	97%	155%
s	Base Case	70.4	160.7	29.4	85.5
	% Change	-36%	-60%	-37%	-61%
	Target	43.4	64.4	18.2	33.2
ρ	Base Case	51%	53%	56%	60%
	% Change	32%	72%	36%	70%
	Target	68%	91%	76%	102%

The resulting term structure of risk premia using Fitch EIR as a proxy for the real-world default probability is shown in figure 5.5 together with the estimates based on Moody's KMV for the same sample. Financials had to be excluded as they are not covered by Fitch EIR. In addition, estimates for the cumulative default probability are only available for up to 5 years maturity. Therefore the term structure of risk premia was calculated as (5-year CDS-implied Sharpe ratio) minus (3-year CDS-implied Sharpe ratio). Figure 5.5 shows a very similar pattern for estimates based on either Moody's KMV or Fitch EIR. Indeed, the correlation coefficient between the two slopes are between 0.8 and 0.9. In both cases, the slope of the term structure is positive directly before the finanical crisis (first half of 2007) and becomes negative during the financial crisis. Our results are therefore not due to the specific distance-to-default-based specification of Moody's KMV but also holds if a larger amount of covariates is included for default prediction.

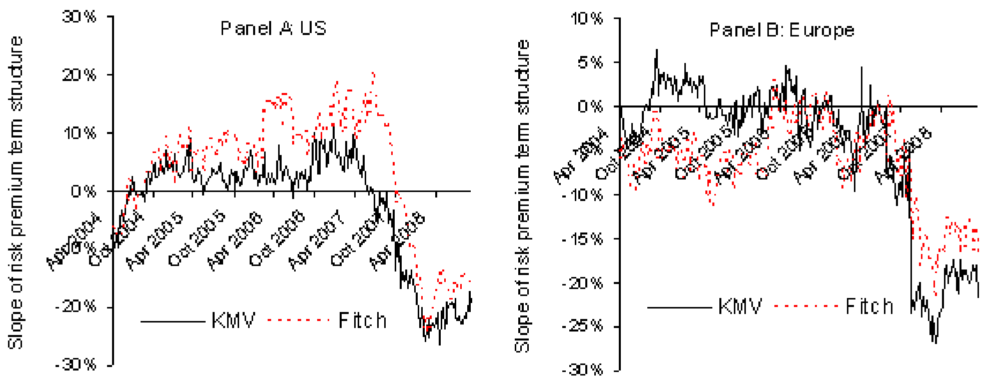

Figure 5.5.: Slope of the risk premium term structure measured as 5-year CDS-implied Sharpe ratio minus 3-year CDS-implied Sharpe ratio. Picture on left-hand side: U.S., picture on right-hand side: Europe. Solid line: Estimate based on Moody's KMV EDFs as a proxy for the real-world default probability, dotted line: Fitch EIR as a proxy for the real-world default probability.

5.5.4. Regression analysis

The following regression controls for the influence of several financial parameters that have seen extraordinary levels during the 2007/2008 financial crisis. These parameters may effect any of our input parameters, especially the estimate for the real-world default probability. We control for the effect of the following parameters:

- Implied Volatility (IV): Volatility may not be captured correctly in the KMV EDF estimates since these are based on historical volatilities. Implied volatility is measured based on short-term at-the-money option prices from Bloomberg.
- Skewness ($SKEW$): KMV uses a volatility-based measure in their distance to default. A low skewness might therefore underestimate the default probability.[18] Skewness is measured as the 180-days historical return skewness based on Datastream return data.[19]
- Information uncertainty ($UNCERT$): Based on Duffie/Lando (2001), higher information uncertainty leads to higher (short-term) default probabilities. Information uncertainty may have been larger during the 2007/2008 financial crisis, therefore leading to an underestimation of the true PDs. Information uncertainty is measured as the coefficient of variation of the 12-months ahead I/B/E/S earnings forecasts.[20]

Unfortunately, a direct adjustment of the PD estimates is not possible. Instead an indirect approch is necessary. In the first step, the impact of these parameters (together with the EDF) on CDS spreads is measured for

[18] Cf. Colin-Dufresne et al. (2001) for similar considerations. Colin-Dufresne (2001) use changes in the slope of the smirk of implied volatilities to control for jump magnitudes.
[19] We also applied two other measures of skewness: option-implid skewness and skewness implied from I/B/E/S forecast. The results did not significantly change.
[20] The coefficient of variation for I/B/E/S forecasts for the next fiscal year end and the year after the next fiscal year end is interpolated in order to receive a constant maturity 12-month ahead coefficient of variation.

each maturity τ via a regression, i.e.[21]

$$\begin{aligned} ln(\text{CDS spread}_\tau) \; &= \beta_0 \;\; +\beta_1 ln(EDF_\tau) + \beta_2 ln(IV) \\ &\qquad\quad +\beta_3 SKEW + \beta_4 ln(UNCERT) + \epsilon. \end{aligned}$$

In a second step the CDS-spread for the "During Crisis" subperiod are adjusted based on the assumption, that IV, $SKEW$ and $UNCERT$ would take average values of the "Before Crisis" subperiod. Based on these adjusted (lower) CDS spreads, the adjusted 3-, 5-, 7-, and 10-year Sharpe ratio estimates were determined. It should be noted that this is a very conservative robustness test: Some of these parameters may (and are likely to) be positively correlated with risk aversion. Subtracting the part of the CDS spread which is due to the high level of these parameters during the 2007/2008 financial crisis then also leads to a substraction of a "risk premium part" and not only to a substraction of the "PD part".

Indeed, the results from the regression are as expected: Especially IV and $UNCERT$ have an economically significant impact on CDS spreads, cf. table 5.4. In addition, the influence is larger for shorter CDS maturities. The effects are qualitatively the same for the U.S. and Europe although they are more pronounced for the U.S. Adjusting the CDS spreads based on these regression results yields lower Sharpe ratio estimates with a slightly smaller downward sloping trend. However, the general statement of an inverse term structure of risk premia during the 2007/2008 financial crisis does not change based on the adjusted CDS spreads (cf. figure 5.6).

5.5.5. Liquidity, market microstructure effects

In contrast to bonds, CDS are unfunded exposures without fixed supply and without large upfront payments. Both theoretical considerations as well as empirical studies indicate that liquidity has only a minor effect on CDS spreads (Longstaff et al. (2005), Ericsson et al. (2006) and Bühler/Trapp

[21] Berndt et al. (2005) use a similar regression of CDS spreads on EDFs. They also show that a log/log formulation performs better than a linear regression.

Table 5.4.:
Robustness: Regression analysis

This table depicts the results of the regression $ln(CDS - spread_{\tau}) = \beta_0 + \beta_1 ln(EDF_{\tau}) + \beta_2 ln(IV) + \beta_3 SKEW + \beta_4 ln(UNCERT) + \epsilon$. $ln(EDF)$ denotes the logarithm of the expected default frequency for the respective maturity, $ln(IV)$ denotes the logarithm of the short-term option implied volatility, $SKEW$ denotes the 180-days historical skewness in equity returns, $ln(UNCERT)$ denotes the 12-months ahead coefficient of variation of the I/B/E/S earnings forecasts. T-statistics are in paranthesis.

	Panel A: U.S.				Panel B: Europe			
	Maturity				Maturity			
	3	5	7	10	3	5	7	10
ln(EDF)	0.15	0.17	0.17	0.17	0.15	0.17	0.18	0.18
	(12.06)	(13.89)	(14.48)	(14.22)	(12.25)	(14.82)	(15.52)	(15.91)
ln(IV)	0.95	0.83	0.75	0.66	0.39	0.37	0.33	0.31
	(26.23)	(25.98)	(25.24)	(23.83)	(10.93)	(11.29)	(10.67)	(10.54)
SKEW	-0.02	-0.01	-0.01	-0.01	0.02	0.02	0.02	0.02
	(-2.94)	(-2.43)	(-2.28)	(-2.37)	(4.09)	(4.44)	(4.66)	(5.01)
ln(UNCERT)	0.25	0.21	0.19	0.18	0.06	0.04	0.04	0.03
	(22.00)	(20.44)	(20.11)	(19.92)	(4.60)	(3.69)	(3.60)	(3.17)

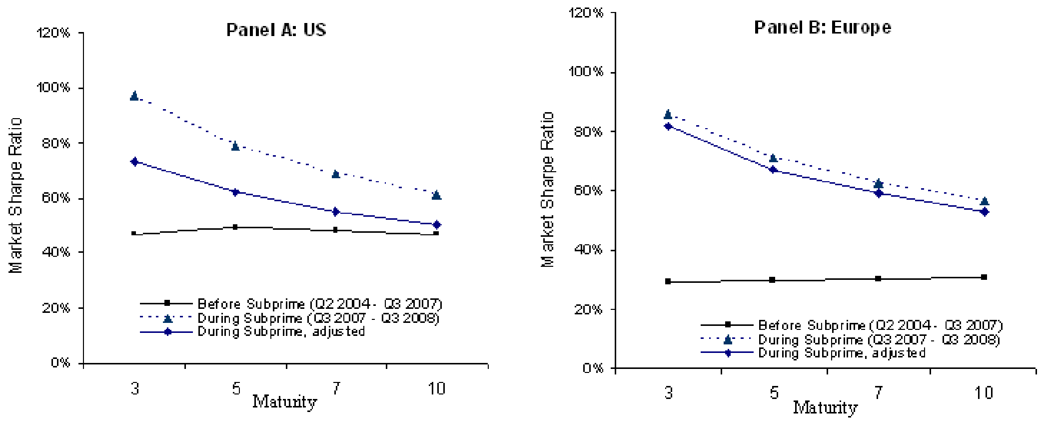

Figure 5.6.: Term structure of risk premia before (04/2004-06/2007) and during (07/2007-09/2008) the 2007/2008 financial crisis for the U.S. (index: CDX.NA.IG) and Europe (index: iTraxx Europe) before and after adjustments as described in table 5.4. x-axis: maturity, y-axis: CDS-implied market Sharpe ratio based on (5.10).

(2008) and the discussion in section 2.3.3). However, in an OTC market like the CDS market, market microstructure liquidity effects may drive valuations either below or beyond the "fair" market values due to supply/demand effects. These effects may be especially preeminant in turmoil periods such as the 2007/2008 financial crisis.

This study uses two measures to capture the potential effect of market microstructure liquidity effects: First, the bid/ask spread. Second, the veracity score as provided by CMA. The veracity score is a trade indicator. A veracity score of "1" denotes that a trade has taken place, a veracity score of "2" denotes a firm quote, lower veracity scores indicate even lower liquidity, e.g., indicative quotes or bond-derived CDS spreads. Figure 5.7 plots these measures for all maturities for both the "Before crisis" and "During crisis" subperiod. As expected, both liquidity measures indicate that the 5-year maturity is the most liquid one (lowest bid/ask spread, lowest average veracity score). *If* market microstructure effects dominate our results, we would expect U-shaped CDS-implied risk premia which are lowest for the most liquid – 5-year – maturity. In addition, there are even signs that trades during turmoil periods tend to concentrate in the most liquid maturity, i.e., 5-year CDS. Again, this result indicates that market microstructure effects are not the cause for the downward sloping risk premium curve.

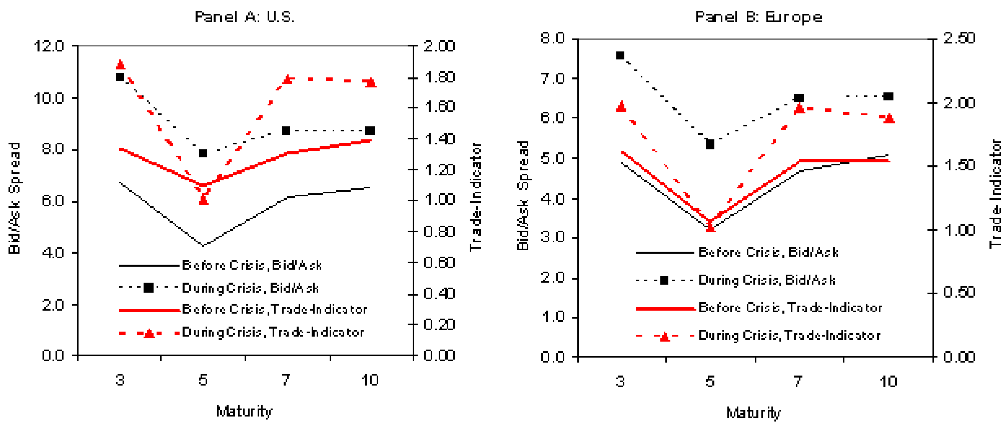

Figure 5.7.: Proxies for CDS liquidity for the U.S. (index: CDX.NA.IG) and Europe (index: iTraxx Europe) based on 3-/5-/7- and 10-year CDS spreads from 04/2004-09/2008. Solid lines: "Before crisis"-period, Dotted lines: "During crisis"-period. Black lines: Bid/Ask spread, red lines: Trade indicator based on CMA's veracity score.

6. Conclusion

6.1. Summary and implications

In this thesis, we postulated the idea that risk premia on different markets can – and should – be compared. Using structural models of default, we derived formulas to link risk premia on credit markets to the asset and market Sharpe ratio and to the equity premium.

Chapter 2 provided an overview of the current literature on equity valuation and credit pricing. In particular, models to measure risk premia on both equity and credit markets were discussed and existing studies linking these risk premia on debt and equity markets were summarized.

Chapter 3 provided the theoretical basis for chapters 4 and 5. We analyzed a Merton framework and demonstrated that risk premia constitute a significant part of model-implied credit spreads ($> 50\%$ for reasonable parameter combinations). In addition, risk premia are hardly affected by moving from a Merton framework to other structural models of default such as Black/Cox (1976), Leland/Toft (1994/1996), and Duffie/Lando (2001) once these models are calibrated to a common real-world default probability. Although actual and risk-neutral default probabilities are largely affected by model changes, the *difference* between actual and risk-neutral default probabilities is only marginally affected.

These results have several implications: First, simple rules derived in a Merton framework to transform real-world PDs and recovery rates into credit spreads are also a very good approximation in more advanced structural

models of default.[1] Second, the high importance of risk premia for credit pricing reveals a large hurdle for accurately pricing credit instruments, which requires a good estimation of the current risk aversion/risk premium. In practice, risk premia are very hard to determine accurately.
The large importance of risk premia combined with the robustness with respect to model changes is, however, also good news: It allows measuring risk premia from CDS spreads quite accurately. Since risk premia constitute the largest part of CDS spreads, the theoretical framework suggests that such an approach should be quite robust for estimating Sharpe ratios and equity premia.

Chapter 4 applied this idea to estimating equity premia and market Sharpe ratios from CDS spreads. We have developed implementable formulas to measure the risk attitude of investors based on credit valuations and transform it to an equity premium via structural models of default. This estimator only uses actual and risk-neutral default probabilities, the maturity, equity correlations, and – for the estimation of the equity premium – implied equity volatility. We neither have to calibrate a structural model, nor do we have to estimate earnings or dividend growth. In addition, this approach offers a totally new line of thought for estimating the equity premium that is not directly linked to current methods.
Compared to traditional DCF models used for estimating the equity premium, CDS spreads in our model correspond to the market value of equity, and actual default probabilities correspond to dividend/earnings forecasts and long-run growth assumptions. While CDS spreads and the market value of equity can both be determined with little noise, the crucial inputs are earnings- and long-run growth forecasts in the dividend-/earnings-discount models and the actual default probability in our model. There are two decisive advantages of estimating default probabilities compared to earnings forecasts and long-run growth rates: First, they only have to be estimated

[1] This may be an explanation for the good pricing performance of these models. Unfortunately, these models have not been empirically compared to other structural models in a larger-scale empirical study, cf. Arora et al. (2005) for a short study. We see this as an area for future research.

up to the maturity of the respective CDS. Second, there are (at least partially) objective criteria for estimating default probabilities. For that reason, different sources for the estimation of real-world default probabilities have led to quite robust results.
An empirical analysis of 5-year CDS spreads from Q2/2003-Q2/2007 (U.S.) and Q1/2004-Q2/2007 (Europe and Asia) respectively – i.e., before the 2007/2008 financial crisis – of the constituents of the main CDS indices in U.S., Europa, and Asia yielded upper limits for the equity premium of 6.50% (U.S.), 5.44% (Europe), and 6.21% (Asia). Various robustness tests have been performed.

In chapter 5, we analyzed the term structure of CDS-implied risk premia. In addition, we estimated the parameters of the instantaneous risk premium process based on a time series of risk premium term structures. We first developed the necessary methodologies and then applied our estimators to the constituents of the main CDS indices in the U.S. and Europe both before and during the 2007/2008 financial crisis.
We showed that the term structure of risk premia was flat before the 2007/2008 financial crisis and inverse during the crisis. These results are probably not surprising given the sentiment of market participants during the financial turmoil. However, the approach is to our best knowledge the first approach which is suited to monitor such a behavior of risk premia for different time horizons. Indeed, the standard errors of our estimates were small enough to see a significant difference in risk premium term structures before and during the 2007/2008 financial crisis.
Certainly, the 2007/2008 financial crisis is an unprecedented event in history. Therefore, quantitative research in such a period will always have its limits. It is possible that quantitative measures alone are not able to capture the dynamics of certain parameters. E.g., market participants may have a gut feeling about future default probabilities that are neither captured by agencies' ratings nor by any quantitative procedure. We analyzed alternative sources for the main input parameters and tried to control for the parameters that can most reasonably be linked to the abnormal situation

during the crisis (implied volatility, skewness, information uncertainty). Of course, in these turbulent times, room for misinterpretation remains. However, based on all evidence available, the main statements are quite robust. It would take extraordinary adjustments for any of the input parameters to come up with a risk premium term structure which is not inverse during the 2007/2008 financial crisis. Therefore, we are confident that the main results correctly reflect the situation during the financial turmoil.

Taken together, this thesis advocates the use of credit spreads to estimate risk premia. This link – which has long been established and applied qualitatively – seems to give interesting insights on a quantitative basis as well. The current risk aversion of investors seems to be better measurable based on credit markets than on equity markets. In addition, standard structural models yield almost the same functional relationship for converting risk premia from debt to equity markets, so model risk is smaller than would be suggested by traditional approaches of structural models of default.

6.2. Outlook

As discussed, we have proposed a new framework for estimating the equity premium based on CDS spread observations. Although we have performed several robustness checks, our approach will have to be challenged in several ways by future research. These include both model robustness as well as the empirical implementation.

Empirical implementation: Our approach suffers from three main limitations which might be an interesting area for further research: First, because of data limitations for the CDS market, we have been able to analyze only a rather limited period of time and a restricted number of companies. It will be interesting to see whether the approach remains robust as presumed if a whole economic cycle as well as a larger cross-section of firms can be integrated in the analysis. Second, the literature on the estimation of actual default probabilities is still evolving, especially concerning multi-year default prediction. Hence, it has to be seen whether our approach will remain robust with respect to future calibrations. Third, our approach still suffers from limitations, that it estimates an upper limit of the equity premium. By using methods that can split up the CDS spread in a part due to credit risk and in a part due to other factors (e.g., liquidity, taxes) the equity premium estimates could be rendered more precise. Besides liquidity impacts – which are likely to be small for CDS based on existing studies – we see the impact of a different tax treatment on equity and credit markets as an interesting area for further research which has so far been mostly ignored in this area of research.

Models: The literature on structural models is still evolving and new models are proposed on a frequent basis. We see three main areas for further research concerning the applications proposed in this literature. First, the analysis of the effect of time-varying asset Sharpe ratios, default boundaries, and recovery values along the model proposed by Chen et al. (2009). Second, a deeper analysis of the impact of information uncertainty. Third, a robustness test concerning the distributional assumptions taken in standard models.

Time-varying Sharpe ratios, default boundaries, and recovery rates: Chen et al. (2009) have argued that time-varying, countercylical Sharpe ratios, countercyclical default boundaries, and countercyclical loss given defaults may increase the difference between actual and risk-neutral default probabilities compared to the Merton framework. They observe that the price of a defaultable bond zero bond $B^d(0, T, 0, LGD)$ can be determined in a stochastic discount factor setting as[2]

$$\begin{aligned} B^d &= E^P\left[\frac{1}{1+k}(1 - 1_{\{\tau<T\}}LGD_\tau)\right] \\ &= \frac{1}{1+r_f}(1 - E^P\left[1_{\{\tau<T\}}LGD_\tau\right]) - Cov\left(\frac{1}{1+k}, 1_{\{\tau<T\}}LGD_\tau\right), \end{aligned}$$

[2]Cf. formula (2.2), Cochrane (2005), and LeRoy/Werner (2006).

where r_f denotes the risk-free rate, PD^P denotes the real-world default probability, $1/(1+k)$ is the stochastic discount factor, V_t the asset value, and LGD_τ the loss given default. This offers two channels that drive the difference between actual and risk-neutral quantities: First, a positive covariance between the pricing kernel and the default time. In structural models of default, this can be further broken down into 1a) a negative covariance between the pricing kernel and asset returns and 1b) a positive covariance between the pricing kernel and the default boundary. Third, a positive covariance between the pricing kernel and the loss given default (LGD_τ).
Our model considers the first channel, i.e., the negative covariance between asset prices and the pricing kernel. Chen et al. argue that the contribution of this channel can even be increased by introducing time-varying, countercyclical asset Sharpe ratios. They also conclude that countercyclical default boundaries might significantly increase credit risk premia. Finally, they note that the contribution by the third channel – positive covariance between the pricing kernel and the loss given default – is likely to be very small. Their analysis for the time-varying Sharpe ratio is based on levels that have been helpful in explaining the equity premium puzzle, the analysis of the time-varying default barrier is based on a goal-seek procedure. They do not make any statements about the empirical validity of the resulting time variation in either Sharpe ratios or default boundaries. Huang/Huang (2003) document that if time variation and countercyclicality in Sharpe ratios is fitted to historically observed parameters, the difference to a simple model without time variation is again very small, cf. also appendix C.2 for an intuitive explanation.
Taken together, there is theoretical evidence that countercylical Sharpe ratios and default boundaries might increase the difference between actual and risk-neutral default probabilities. In this case, our estimates for the equity premium would be further upward biased. However, there is small empirical evidence that the time variation in Sharpe ratios and default boundaries is large enough to lead to a significant deviation from the classical Merton model. A deeper analysis of these effects might be an interesting area for further research.

Information uncertainty: As documented in section 3.4, information uncertainty does not only influence default probabilities itself but also increases the difference between actual and risk-neutral default probabilities. This effect is especially pronounced for shorter maturities. However, research on the magnitude of information uncertainty is rare, cf. Duffie/Lando (2001) and Andrade et al. (2009), for example, so that an accurate calibration of this parameter is hardly possible. Further research in this area could be especially useful to explain differences between actual default probabilities and credit spreads for very short horizons.

Distributional assumptions: So far, we have focussed on a normal distribution driving log asset value changes. One could easily rewrite formula (4.2) to account for other distributional assumptions, i.e.,

$$SR_V := \frac{\mu - r}{\sigma} = \frac{F^{-1}(PD^Q(t,T)) - F^{-1}(PD^P(t,T))}{\sqrt{T-t}}, \tag{6.1}$$

where F denotes any distribution function. However, there are some problems with this approach. First, one might be inclined to replace the normal distribution by a fat-tailed distribution. However, if the logarithm of the asset returns is fat-tailed, neither the first nor the second moment of the asset value distribution exists.[3] Depending on the type of fat-tailed distribution, the Sharpe ratio of the log asset value process might exist while the Sharpe ratio of the asset value process itself does not exist.[4] This poses further challenges for the interpretation of historical equity premia and for asset valuation purposes. Second, if asset value processes are modeled in continuous time, the distributional assumption from a Brownian motion (i.e., normal distribution) cannot simply be substituted for a fat-tailed distribution, because this will usually require discontinuous trajectories, cf. Embrechts et al. (1997). Third, the derivation of formula (4.2) was based on the assumption that volatility in the risk-neutral world equals the volatility in

[3]Cf. Goldie/Klüppelberg (1998), Bamberg/Dorfleitner (2002), and Bamberg/Neuhierl (2008).

[4]Formally, the Sharpe ratios and equity premia we estimated are log Sharpe ratios and equity premia. For a geometric Brownian motion ($dV_t = \mu V_t dt + \sigma V_t dB_t$), $E[V_t/V_{t-1}] = e^{\mu} \approx 1 + \mu$, so differences are minimal in this case.

the real-world. Volatility does not generally remain the same for changes of measure, e.g., in binomial models, risk-neutral and real-world volatilities are usually not the same.[5] Fourth, it can in general not be guaranteed that real-world and risk-neutral probability distributions belong to the same class F. Taken together, distributional assumptions deviating from the normal distribution might seem to be a natural robustness test for credit risk related topics and therefore an interesting area for further research – although there are substantial mathematical problems to overcome. However, we do want to point out that our approach is not subject to the same sensitivities with respect to the distributional assumption as classical applications of structural models of default. The reason is that we only look at the difference between actual and risk-neutral default probabilities. This difference is driven by the (average) slope of the distribution function, cf. figure 3.1 and formula (6.1).

Further applications: In chapter 5 we determined the slope of the risk premium term structure (defined as 10-year Sharpe ratio minus 3-year Sharpe ratio). This slope may be useful for several applications. First, practitioners might use it as a simple turmoil indicator. E.g., if the slope is a lot smaller than zero, there are two possible interpretations: Either standard methodology to estimate default probabilities or correlations does not work or short-term risk premia are indeed far above long-run levels. Both interpretations are likely to indicate a turmoil situation. This turmoil indicator may also be useful to assess if and when a turmoil has ended. Second, it can be used in asset allocation decisions as well as for asset pricing applications (e.g., company valuation). Finally, there are strong theoretic arguments that the slope of the risk premium term structure is also a factor in asset pricing. We see a deeper analysis of the relation of this risk premium term structure slope with other parameters such as equity returns, interest rates, and business sentiments as an interesting area for further research.

[5] However, Gisanov's theorem tells us that this is true in continuous time, i.e., the difference between the volatility in the real and risk-neutral world is only a second-order effect.

The general comparability idea can also be applied to other areas. So far, we have used this idea to estimate equity premia from CDS spreads and analyze the time-series behavior of CDS-implied Sharpe ratios. There are other open questions in finance which might be analyzed in this context. Recently, Philippon (2008) published a working paper which relies on estimates from the bond market to implement the q-theory of investment. He finds that the bond market's q fits the investment equation six times better than the usual measure of q derived from equity markets. The idea presented in this thesis could also be used to analyze the cross-section of returns. E.g., CDS-implied equity premia could be analyzed with respect to the relevance of certain factors proposed in the literature such as the Fama/French (1993, 1996) three-factor model. This might help to gain further insight into the economic drivers of some of these factors.

Synopsis: Taken together, we hope that we have provided some insight into the relationship between risk premia on debt and equity markets and that this work might be used as a basis for further research on this topic. We think that this area of research can be especially fruitful in times of turbulence such as the current financial crisis. While many investors may not like the current situation on the financial markets, it might indeed provide an excellent research basis for several issues in finance.

A. Default probabilities

A.1. Historical default probabilities from Moody's (2007)

A.1.1. Per rating grade and notch

For the mapping of Moody's rating grades to default probabilities we smoothed historically observed default frequencies from Moody's (2007) via a log-linear relationship, i.e., we performed the regression

$$ln(PD) = \beta_1 + \beta_2 \cdot NRG,$$

where NRG denotes the numerical rating grade ranging from 1 (Aaa) to 16 (B3) and PD denotes the historical default probabilities per rating grade.[1] The resulting cumulative default probabilities are shown in table A.1 (per rating grade and notch) and table A.2 (per rating grade).

A.1.2. Average default time

We calculated the average conditional default time based on a discrete approximation of (3.9):

$$DT = \frac{1}{PD^P_{cum}(T)} \sum_{t=1}^{T} (t - 0.5) \cdot \left[PD^P_{cum}(t) - PD^P_{cum}(t-1)\right] \qquad \text{(A.1)}$$

[1] The log-approach is a common approach for the calibration of default probabilities (cf. for example Bluhm et.al (2003)).

where $PD^P_{cum}(x)$ was determined based on table A.2. The resulting average conditional default times are depicted in table A.3.

A.2. Discrete duration model based on Löffler/Maurer (2008)

The model of Löffler/Maurer (2008) estimates cumulative default probabilities via a discrete duration model. The hazard function $h(t)$ is defined via

$$h(t+k) = P(Y_{t+k,t+k+1} = 1 | Y_{t+k-1,t+k} = 0, X_t) = \frac{1}{1 + exp(-\alpha_k - \beta_k X_t)}$$

where $Y_{t+k,t+k+1} \in \{0,1\}$ is the default indicator for the period $(t+k, t+k+1]$ and X_t denotes the vector of covariates. (α_k, β_k) is the vector of coefficient estimates, see table A.4 below. Multi-period default probabilities can be derived via

$$P(Y_{t,t+k} = 1) = 1 - \prod_{j=0}^{k-1} (1 - h(t+j)).$$

	Maturity									
RAT	1	2	3	4	5	6	7	8	9	10
Aaa	0.00	0.01	0.02	0.04	0.07	0.09	0.11	0.13	0.14	0.15
Aa1	0.00	0.01	0.03	0.07	0.11	0.14	0.17	0.19	0.20	0.22
Aa2	0.00	0.02	0.05	0.10	0.17	0.22	0.26	0.29	0.31	0.34
Aa3	0.01	0.03	0.08	0.16	0.26	0.33	0.39	0.43	0.47	0.51
A1	0.01	0.05	0.14	0.26	0.39	0.50	0.60	0.66	0.71	0.78
A2	0.02	0.09	0.23	0.40	0.60	0.76	0.90	0.99	1.08	1.17
A3	0.04	0.16	0.37	0.64	0.92	1.16	1.36	1.51	1.63	1.77
Baa1	0.08	0.28	0.61	1.00	1.42	1.76	2.06	2.28	2.47	2.68
Baa2	0.15	0.49	1.00	1.58	2.17	2.67	3.11	3.45	3.75	4.06
Baa3	0.28	0.86	1.64	2.50	3.34	4.06	4.70	5.22	5.68	6.14
Ba1	0.51	1.48	2.69	3.93	5.12	6.17	7.10	7.89	8.61	9.29
Ba2	0.95	2.57	4.41	6.20	7.86	9.38	10.72	11.95	13.04	14.05
Ba3	1.74	4.45	7.23	9.78	12.06	14.24	16.20	18.08	19.76	21.25
B1	3.20	7.72	11.85	15.41	18.51	21.64	24.49	27.35	29.94	32.14
B2	5.90	13.37	19.42	24.30	28.41	32.87	37.01	41.39	45.36	48.62
B3	10.85	23.18	31.82	38.31	43.61	49.94	55.93	62.64	68.72	73.54

Table A.1.:

Historical cumulative default probabilities for Moody's ratings per rating grade and notch in percent.

Cumulative default probabilities based on Moody's (2007) and based on a log-approach $ln(PD) = \beta_1 + \beta_2 \cdot NRG$, where NRG denotes the numerical rating grade ranging from 1 (Aaa) to 16 (B3). *RAT* denotes rating grade.

	Maturity									
RAT	1	2	3	4	5	6	7	8	9	10
Aa	0.00	0.02	0.05	0.10	0.17	0.22	0.26	0.29	0.31	0.34
A	0.02	0.09	0.23	0.40	0.60	0.76	0.90	0.99	1.08	1.17
Baa	0.15	0.49	1.00	1.58	2.17	2.67	3.11	3.45	3.75	4.06
Ba	0.95	2.57	4.41	6.20	7.86	9.38	10.72	11.95	13.04	14.05
B	5.90	13.37	19.42	24.30	28.41	32.87	37.01	41.39	45.36	48.62

Table A.2.:

Historical cumulative default probabilities for Moody's ratings per rating grade in percent.

Cumulative default probabilities based on Moody's (2007) and based on a log-approach $ln(PD) = \beta_1 + \beta_2 \cdot NRG$, where NRG denotes the numerical rating grade ranging from 1 (Aaa) to 16 (B3). *RAT* denotes rating grade. PDs for rating grades Aa, A, ... taken from respective PDs of Aa2, A2, ...

	Maturity									
RAT	1	2	3	4	5	6	7	8	9	10
Aa	0.50	1.29	2.07	2.78	3.44	3.92	4.35	4.64	4.92	5.31
A	0.50	1.24	1.98	2.64	3.25	3.73	4.15	4.47	4.78	5.16
Baa	0.50	1.19	1.86	2.46	3.01	3.48	3.90	4.26	4.60	4.97
Ba	0.50	1.13	1.70	2.22	2.70	3.16	3.58	3.98	4.36	4.73
B	0.50	1.06	1.51	1.91	2.28	2.72	3.14	3.60	4.03	4.40

Table A.3.:

Average conditional default time in years

Average conditional default time based on Moody's (2007) and formula (A.1) and table A.2.

	Prediction Horizon in Years				
	1	2	3	4	5
L	4.89***	2.97***	2.41***	1.49**	0.99*
	(0.53)	(0.41)	(0.39)	(0.46)	(0.48)
EBIT/TA	-1.85	0.95	-0.24	1.86	2.23
	(1.06)	(1.16)	(1.29)	(1.48)	(1.68)
EBIT/XINT	-0.3**	-0.33***	-0.19**	-0.29**	-0.31***
	(0.1)	(0.08)	(0.07)	(0.09)	(0.09)
SIZE	-0.18***	-0.2***	-0.19***	-0.22***	-0.21***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)
dTA	1.09**	1.13***	1.12**	1.17*	1.37**
	(0.39)	(0.34)	(0.4)	(0.5)	(0.45)
RET	-1.67***	-0.65***	-0.29	-0.24	-0.11
	(0.31)	(0.18)	(0.16)	(0.19)	(0.2)
VOLA	4.7***	4.39***	3.42**	0.66	0.33
	(1.01)	(1.11)	(1.07)	(1.36)	(1.39)
Constant	-9.05***	-7.53***	-7.08***	-6.47***	-6.12***
	(0.54)	(0.49)	(0.44)	(0.5)	(0.53)

Table A.4.:

Coefficients for discrete duration model based on Löffler/Maurer (2008)

This table contains the coefficient estimates from Löffler/Maurer (2008). *L*: leverage (= Total debt / (Total debt + Marketcap)), *XINT*: interest expenses, *SIZE*: log of the market cap divided by the S&P-500 market capitalization, *dTA*: one-year asset growth, *RET*: 12-month cumulative equity return, *VOLA*: 12-month monthly equity volatility.

B. Proofs

B.1. Autocorrelation of expected returns vs. realized returns

This section briefly discusses the pitfalls when using realized returns to make statements about expected returns. We will demonstrate that if expected excess returns are positively autocorrelated then realized excess returns may well be either positively autocorrelated, negatively autocorrelated, or not exhibit any autocorrelation at all.

To illustrate this point consider a simple model with mean-reverting equity premium EP_t and realized excess returns r_t (= returns above the risk-free rate) which are modeled as the sum of the equity premium and a noise term:[1]

$$EP_t = EP_{t-1} + \kappa(\mu - EP_{t-1}) + \epsilon_t \tag{B.1}$$

$$r_{t+1} = EP_t + u_{t+1} \tag{B.2}$$

$$\text{with} \quad \rho(u_t, \epsilon_t) = \rho \tag{B.3}$$

where $\kappa > 0$ is the mean reversion speed for the equity premium, μ is the long-run mean of the equity premium and u and ϵ are each serially uncorrelated standard normally distributed error terms with standard deviations σ_ϵ and σ_u. The two error terms are correlated with each other with a correlation of ρ.

[1] This is similar to Campbell/Viceira (1999).

In this setting, expected excess returns (=equity premium) have a positive serial autocorrelation if $\kappa < 1$:

$$\begin{aligned} \rho_{EP_t,EP_{t-1}} &= \frac{Cov(EP_t, EP_{t-1})}{\sigma_{EP_t}\sigma_{EP_{t-1}}} \\ &= \frac{Cov(EP_{t-1} + \kappa(\mu - EP_{t-1}) + \epsilon_t, EP_{t-1})}{\sigma_{EP_t}\sigma_{EP_{t-1}}} \\ &= (1-\kappa). \end{aligned} \tag{B.4}$$

In the last step we have used the fact that the process is stationary, therefore unconditional standard deviations of EP_t and EP_{t-1} are equal. The economic reason for this positive autocorrelation is that today's equity premium (EP_t) is the sum of yesterday's equity premium (EP_{t-1}) plus a mean-reverting part plus an error term. As long as $\kappa < 1$ today's equity premium will always depend on yesterday's equity premium in that sense that it will be also on average high if yesterday's equity premium was high.

The autocorrelation of realized returns can be calculated as follows:

$$\begin{aligned} Cov(r_{t+1}, r_t) &= Cov(EP_t + u_{t+1}, EP_{t-1} + u_t) \\ &= Cov(EP_t, EP_{t-1}) + Cov(EP_t, u_t) \\ &= (1-\kappa)Var(EP_t) + \rho\sigma_u\sigma_\epsilon, \end{aligned}$$

where we have again used the fact that the process EP_t is stationary and therefore unconditional variances of EP_t and EP_{t-1} are equal. If we insert the equation $Var(EP_t) = Var(EP_{t-1})$ into

$$Var(EP_t) = (1-\kappa)^2 Var(EP_{t-1}) + Var(\epsilon_{t+1})$$

we can calculate the variance of EP_t explicitly as

$$Var(EP_t) = \frac{Var(\epsilon)}{\kappa(2-\kappa)}$$

and therefore

$$Cov(r_{t+1}, r_t) = (1-\kappa) \cdot \frac{Var(\epsilon)}{\kappa(2-\kappa)} + \rho\sigma_u\sigma_\epsilon. \tag{B.5}$$

Since

$$Var(r_t) = Var(EP_t) + Var(u_t) = \frac{Var(\epsilon)}{\kappa(2-\kappa)} + Var(u_t) \tag{B.6}$$

it follows that the autocorrelation of realized returns is

$$\begin{aligned}
\rho_{r_t, r_{t-1}} &= (1-\kappa) \cdot \frac{\frac{Var(\epsilon)}{\kappa(2-\kappa)}}{\frac{Var(\epsilon)}{\kappa(2-\kappa)} + Var(u_t)} + \frac{\rho\sigma_u\sigma_\epsilon}{\frac{Var(\epsilon)}{\kappa(2-\kappa)} + Var(u_t)} && \text{(B.7)} \\
&= \underbrace{(1-\kappa)\frac{1}{1+\kappa(2-\kappa)\frac{Var(u)}{Var(\epsilon)}}}_{\text{Part 1}} + \underbrace{\rho\frac{\kappa(2-\kappa)\frac{\sigma_u}{\sigma_\epsilon}}{1+\kappa(2-\kappa)\frac{Var(u)}{Var(\epsilon)}}}_{\text{Part 2}} && \text{(B.8)}
\end{aligned}$$

- Part 1 is the part which is caused by the positive autocorrelation in the expected returns. If $\kappa = 1$ then the autocorrelation of expected returns is zero, therefore this part is also zero. In addition, it is a scaled version of (B.4): If the residual volatility σ_u of the realized returns is very large compared to the residual volatility σ_ϵ of the expected returns, part 1 will be small.

- Part 2 is the part which is caused by the negative correlation between the two error terms. If ρ is negative then positive returns (induced by the noise) result on average in a decrease of the equity premium. Or – interpreted the other way round – a decrease in the equity premium results in a decrease in the discount factor and therefore in an increase in prices and positive realized returns. I.e., equity premia are on average lower after a bull market (such as the end of 2006) and higher after a bear market (such as the beginning of 2009).

Taken together, if ρ is negative and small enough and σ_u is a lot larger than σ_ϵ then this will result in a negative autocorrelation for realized returns while expected returns are positively autocorrelated.

B.2. Proof of proposition 3.3.1

For ease of notation we will use the notation P for PD^P and Q for PD^Q. From (3.3), the first three derivatives of PD^Q with respect to PD^P are:

$$Q'(P) = e^{-0.5(SR^2 \cdot T + 2\cdot SR\sqrt{T}\Phi^{-1}(PD^P))} > 0 \tag{B.9}$$

$$Q''(P) = -Q'(P) \cdot SR_V \sqrt{T}\sqrt{2\pi} e^{0.5(\Phi^{-1}(PD^P))^2} < 0 \tag{B.10}$$

$$\begin{aligned} Q'''(P) &= -SR\sqrt{2\pi}\sqrt{T}\left[Q''(P)e^{0.5\left(\Phi^{-1}(P)\right)^2} + Q'(P)e^{\left(\Phi^{-1}(P)\right)^2}\sqrt{2\pi}\Phi^{-1}(P)\right] \\ &> 0 \text{ for } P < 50\% \end{aligned} \tag{B.11}$$

- (3.3.1.1.) follows directly from (B.9), (B.10) and (3.3)
- (3.3.1.2.) follows from (3.3.1.1.) and $(SR^2 \cdot T + 2SR\sqrt{T}\Phi^{-1}(P) < 0) \Leftrightarrow P < \Phi[-0.5 \cdot SR\sqrt{T}]$
- (3.3.1.3.): $RelCRP(1) = 1$ follows directly from (3.3).
  $\lim_{PD^P \to 0} RelCRP(PD^P) = \infty$ follows from an application of the rule of de l'Hospital and (B.9).
  To see that $f(P) := RelCRP(P)$ is decreasing, please note that the first derivative is $f'(P) = \frac{Q'(P) \cdot P - Q(P)}{P^2}$. Based on a Taylor expansion of $Q(P)$ in $P = 0$ at P it follows that $Q(0) = Q(P) - P \cdot Q'(P) + 0.5 \cdot \int_0^P \widetilde{p} \cdot Q''(\widetilde{p})d\widetilde{p}$.[2] It follows from (3.3) that $Q(0) = 0$ and from (B.10) that $Q''(P) < 0$. Therefore $f'(P) < 0$ and the relative credit risk premium is decreasing in P.
  To see that $f(P) := RelCRP(P)$ is concave, we first determine the second derivative of f with respect to P: $f''(P) = \frac{0.5 \cdot P^2 \cdot Q''(P) - P \cdot Q'(P) + Q(P)}{0.5 \cdot P^3}$.
  Again, a Tylor expansion leads to $Q(0) = Q(P) - P \cdot Q'(P) + 0.5 \cdot P^2 \cdot Q''(P) - 0.5 \cdot \int_0^P \widetilde{p}^2 \cdot Q'''(\widetilde{p})d\widetilde{p}$. Based on $Q(0) = 0$ and $Q'''(P) > 0$ for $P < 50\%$ (from (B.11)), it follows that $f''(P) > 0$, i.e. the relative credit risk premium is a convex function in P.

[2] A Taylor expansion would require $Q(P)$ to be differentiable in $P = 0$ which is not the case here. We can, however, repeat the same argument by choosing $\epsilon > 0$. Letting $\epsilon \to 0$ and using continuity of Q and Q'' for $P \in (0, \epsilon)$ yields the result.

C. Robustness of equity premium estimator for other frameworks

This chapter provides the results for the robustness of the equity premium estimator (4.7) derived in section 4.2 for other frameworks. Section C.1 discusses the robustness of the asset Sharpe ratio estimator for the Duffie/Lando (2001) model with unobservable asset value including the Black/Cox (1976) model and the Leland/Toft (1996) model as special cases. Section C.2 gives a rough approximation for a model with time-varying risk premia. Section C.3 discusses the robustness of asset/equity correlations which are needed to derive the market Sharpe ratio and the equity premium from the asset Sharpe ratio.

C.1. Framework with unobservable asset values

In this section we will show that the Merton estimator for the asset Sharpe ratio is robust with respect to model changes. We will analyze a framework with unobservable asset values based on Duffie/Lando (2001) and – implicitly – all endogenous default frameworks that yield a constant default barrier and assume a geometric Brownian motion for the asset value process (e.g. Leland (1994), Leland/Toft (1996)).

We will show the robustness of our asset Sharpe ratio estimator (4.3) derived in section 4.2 based on the implementation described in section 3.4.2. There, we determined the risk-neutral probability in the Duffie/Lando framework for 50,400 parameter combinations and defined an error term (cf.

(3.8)) as

$$Err := \frac{PD^Q_{D/L}}{PD^Q_{Merton}}.$$

In this section, we will also look at the difference between the Duffie/Lando framework and the Merton framework but instead of differences between risk-neutral PDs we are interested in differences between the Sharpe ratio estimator $\widehat{\gamma}_{SR_V,Merton}$ (cf. section 4.2, formula (4.3)) when applied to either the D/L risk-neutral PDs or the Merton risk-neutral PDs. This estimator applied to the D/L PDs is formally defined as

$$\widehat{\gamma}_{SR_V,Merton}(PD^Q_{D/L}) := \frac{\Phi^{-1}(PD^Q_{DL}(t,T)) - \Phi^{-1}(PD^P(t,T))}{\sqrt{T-t}}, \tag{C.1}$$

where PD^Q_{DL} denotes the risk-neutral PD derived in the Duffie/Lando framework (cf. step 3 in section 3.4.2) and PD^P is the cumulative actual default probability for the rating and maturity for the corresponding parameter combination. We will further define an adjustment factor AF via

$$AF := \frac{\widehat{\gamma}_{SR_V,Merton}(PD^Q_{D/L})}{\widehat{\gamma}_{SR_V,Merton}(PD^Q_{Merton})} = \frac{\widehat{\gamma}_{SR_V,Merton}(PD^Q_{D/L})}{SR_V}. \tag{C.2}$$

The last equation is valid because in the Merton framework the Sharpe ratio estimator exactly yields the asset Sharpe ratio SR_V, cf. section 4.2, formula (4.2). Therefore if the Duffie/Lando framework is the true framework then the Sharpe ratio can be estimated as

$$SR_V = \frac{\widehat{\gamma}_{SR_V,Merton}(PD^Q_{D/L})}{AF}.$$

We will now briefly give some insight into the different behavior of Err and AF. We will demonstrate that the slope of the cumulative distribution function between PD^Q_{Merton} and $PD^Q_{D/L}$ is decisive for determining Err from AF and vice versa. To see that, please note that

$$PD^Q_{D/L} \overset{(C.1)}{=} \Phi\left[\Phi^{-1}(PD^P) + \widehat{\gamma}_{SR_V,Merton}(PD^Q_{D/L})\sqrt{T-t}\right]$$

$$
\begin{aligned}
&\overset{C.2}{=} && \Phi\left[\Phi^{-1}(PD^P) + SR_V \cdot \sqrt{T-t} \cdot AF\right] \\
&= && \Phi\left[\Phi^{-1}(PD^P) + SR_V \cdot \sqrt{T-t} + SR_V \cdot \sqrt{T-t} \cdot (AF-1)\right] \\
&\overset{(3.3),\, linear\ approx.}{\approx} && PD^Q_{Merton} + m(\Phi^{-1}(PD^Q_{Merton})) \cdot SR_V \cdot \sqrt{T-t} \cdot (AF-1),
\end{aligned}
$$

where $m(\Phi^{-1}(PD^Q_{Merton}))$ denotes the slope of the cumulative normal distribution at $x = \Phi^{-1}(PD^Q_{Merton})$.[1] Therefore,

$$
\begin{aligned}
AF &\approx 1 + \frac{1}{SR_V\sqrt{T-t}} \frac{PD^Q_{D/L} - PD^Q_{Merton}}{m(\Phi^{-1}(PD^Q_{Merton}))} \\
\Leftrightarrow AF - 1 &\approx (Err-1) \cdot \frac{1}{SR_V\sqrt{T-t}} \cdot \frac{PD^Q_{Merton}}{m(\Phi^{-1}(PD^Q_{Merton}))},
\end{aligned}
$$

i.e., the adjustment factor exceeds one by the amount that Err exceeds one multiplied with a factor that depends on the asset Sharpe ratio, the maturity, and the risk-neutral Merton PD. The last term $\frac{PD^Q_{Merton}}{m(\Phi^{-1}(PD^Q_{Merton}))}$ can be explicitly evaluated.[2] This term is increasing with increasing risk-neutral Merton PD and is smaller than one for reasonable risk-neutral PDs.[3] Taken together

- The adjustment factor is close to one if SR_V is large or $\sqrt{T-t}$ is large.
- The adjustment factor is close to one if Err is close to one. We know from section 3.4 that this is on average the case for larger real-world PDs (longer maturities and lower ratings). In addition, we have demonstrated in section 3.4 that Err is larger for larger asset Sharpe ratios SR_V.
- The adjustment factor is close to one if $\frac{PD^Q_{Merton}}{m(\Phi^{-1}(PD^Q_{Merton}))}$ is small. This term is the smaller the smaller the risk-neutral Merton PD.

[1] Another choice for the slope would be the slope at $\Phi^{-1}PD^P_{Merton}$ or any other point between these two values.

[2] This term is equal to $\frac{PD^Q_{Merton}}{1/\sqrt{2\pi}e^{-0.5(\Phi^{-1}(PD^Q_{Merton}))^2}}$.

[3] It is smaller than one for all risk-neutral PDs smaller than 38.14%.

The net effect of these three items is not directly clear and we will have to draw back on our numerical results for an evaluation.

The results are reported in table C.1 (for a Sharpe ratio of 20%) and C.2 (for all Sharpe ratios between 10% and 40%) based on three scenarios as defined in section 3.4. Scenario 1 restricts the asset volatility to be larger or equal to 10%. Asset volatilities below 10% are usually only observed for financial services companies. Scenario 2 sets the risk-neutral drift of the asset value relative to the default barrier to zero. This captures the assumption of constant expected leverage. Scenario 3 captures all parameter combinations where the average default time is within +/- 20% of the historical averages for that respective rating grade based on appendix A.1.2.

In the following interpretation we will focus on table C.2.[4] If one does not restrict the parameter combination to one of these three scenarios, very large variations of the adjustment factor may occur. E.g., for a 5-year maturity, Baa-rating adjustment factors vary between 0.37 and 1.25. However, once any additional restrictions on either the volatility, the risk-neutral drift, or the default timing are applied, adjustment factors are close to one for all maturities between 3 and 10 years. E.g., for maturities between 3 and 10 years (these are the CDS maturities used in chapter 4 and 5) and for reasonable default timing, the adjustment factors for the Baa-rating are always between 0.76 and 1.36. Adjustment factors on average decrease with increasing maturity and are on average slightly above one for shorter maturities and slightly below one for longer maturities. Consequently, if the Duffie/Lando framework is the true framework, then our estimates for larger maturities will be slightly too low and our estimates for shorter maturities will be slightly too high. This may be one explanation why our results from chapter 4 usually show slightly lower 10-year CDS-implied equity premia than 3-year CDS-implied equity premia.
Finally, for 1-year maturities – which were not used in our empirical applications – adjustment factors can be very large (almost 2 even with restrictions

[4]Table C.1 is shown to allow for a direct comparison with table 3.3.

Table C.1.:

		Total		$\sigma \geq 10\%$		$m = 0\%$		DT reasonable	
Maturity	Rating	Min	Max	Min	Max	Min	Max	Min	Max
1	Aa	0.93	1.96	0.98	1.78	0.98	1.93	1.22	1.86
	A	0.90	1.96	0.97	1.77	0.98	1.91	1.25	1.83
	Baa	0.87	1.94	0.96	1.75	0.97	1.87	1.22	1.87
	Ba	0.81	1.90	0.94	1.70	0.96	1.77	0.81	1.84
	B	0.70	1.77	0.90	1.54	0.94	1.49	0.83	1.77
	All	0.70	1.96	0.90	1.78	0.94	1.93	0.81	1.87
3	Aa	0.67	1.39	0.95	1.30	0.97	1.36	0.87	1.37
	A	0.62	1.39	0.93	1.30	0.97	1.35	0.83	1.37
	Baa	0.55	1.39	0.91	1.29	0.96	1.33	0.82	1.36
	Ba	0.47	1.38	0.86	1.27	0.94	1.27	0.81	1.35
	B	0.37	1.35	0.76	1.18	0.89	1.08	0.81	1.27
	All	0.37	1.39	0.76	1.30	0.89	1.36	0.81	1.37
5	Aa	0.49	1.25	0.91	1.18	0.97	1.21	0.87	1.21
	A	0.45	1.25	0.89	1.18	0.96	1.20	0.83	1.20
	Baa	0.40	1.24	0.85	1.17	0.95	1.18	0.84	1.18
	Ba	0.34	1.24	0.78	1.15	0.92	1.12	0.84	1.15
	B	0.26	1.23	0.65	1.08	0.86	0.97	0.79	1.12
	All	0.26	1.25	0.65	1.18	0.86	1.21	0.79	1.21
7	Aa	0.41	1.18	0.88	1.13	0.96	1.14	0.81	1.14
	A	0.37	1.18	0.84	1.12	0.95	1.13	0.83	1.13
	Baa	0.33	1.18	0.79	1.11	0.94	1.11	0.85	1.11
	Ba	0.27	1.18	0.70	1.10	0.90	1.05	0.80	1.05
	B	0.21	1.17	0.56	1.03	0.82	0.92	0.82	1.07
	All	0.21	1.18	0.56	1.13	0.82	1.14	0.80	1.14
10	Aa	0.33	1.13	0.83	1.09	0.96	1.09	0.81	1.01
	A	0.30	1.13	0.78	1.08	0.95	1.07	0.76	0.98
	Baa	0.26	1.13	0.71	1.07	0.93	1.05	0.80	1.05
	Ba	0.22	1.12	0.61	1.06	0.88	0.99	0.81	0.99
	B	0.16	1.12	0.45	0.99	0.77	0.87	0.84	1.02
	All	0.16	1.13	0.45	1.09	0.77	1.09	0.76	1.05

Adjustment factors in the Duffie/Lando framework, SR=20%.
Minimum and maximum adjustment factors in the D/L framework. Asset Sharpe ratio is 20%. For actual PDs for each rating grade and maturity cf. appendix A.1.1, for parameter combinations cf. subsection 3.4.2. *Total* contains all parameter combinations as described in subsection 3.4.2. $\sigma \geq 10\%$ restricts the asset volatility to values larger or equal to 10% ("non-financials"), $m = 0\%$ restricts the risk-neutral asset value drift relative to the default barrier to 0% ("constant leverage"), *DT reasonable* restricts the average default time to values +/-20% compared to values based on Moody's (2007), cf. appendix A.1.2.

Table C.2.:

		Total		$\sigma \geq 10\%$		$m = 0\%$		DT reasonable	
Maturity	Rating	Min	Max	Min	Max	Min AF	Max AF	Min	Max
1	Aa	0.92	1.96	0.97	1.78	0.98	1.94	1.22	1.87
	A	0.89	1.96	0.97	1.77	0.98	1.92	1.20	1.90
	Baa	0.85	1.95	0.95	1.76	0.97	1.89	0.85	1.89
	Ba	0.78	1.91	0.93	1.71	0.96	1.80	0.78	1.84
	B	0.67	1.80	0.89	1.58	0.93	1.55	0.81	1.80
	All	0.67	1.96	0.89	1.78	0.93	1.94	0.78	1.90
3	Aa	0.63	1.39	0.93	1.30	0.97	1.37	0.86	1.37
	A	0.58	1.39	0.91	1.30	0.96	1.36	0.83	1.37
	Baa	0.51	1.39	0.88	1.29	0.95	1.34	0.82	1.36
	Ba	0.44	1.39	0.82	1.28	0.92	1.29	0.81	1.35
	B	0.35	1.36	0.71	1.21	0.86	1.13	0.81	1.33
	All	0.35	1.39	0.71	1.30	0.86	1.37	0.81	1.37
5	Aa	0.46	1.25	0.88	1.18	0.96	1.22	0.84	1.23
	A	0.42	1.25	0.84	1.18	0.94	1.21	0.83	1.22
	Baa	0.37	1.25	0.79	1.17	0.92	1.20	0.81	1.21
	Ba	0.32	1.24	0.71	1.16	0.88	1.15	0.80	1.20
	B	0.25	1.24	0.58	1.11	0.80	1.02	0.79	1.15
	All	0.25	1.25	0.58	1.18	0.80	1.22	0.79	1.23
7	Aa	0.38	1.18	0.82	1.13	0.95	1.15	0.81	1.14
	A	0.34	1.18	0.77	1.13	0.93	1.14	0.80	1.14
	Baa	0.30	1.18	0.71	1.12	0.90	1.13	0.81	1.13
	Ba	0.26	1.18	0.62	1.11	0.85	1.08	0.80	1.11
	B	0.19	1.17	0.49	1.07	0.74	0.96	0.81	1.10
	All	0.19	1.18	0.49	1.13	0.74	1.15	0.80	1.14
10	Aa	0.31	1.13	0.75	1.09	0.93	1.10	0.77	1.05
	A	0.28	1.13	0.69	1.09	0.91	1.09	0.76	1.05
	Baa	0.25	1.13	0.62	1.08	0.88	1.07	0.76	1.05
	Ba	0.20	1.13	0.53	1.07	0.81	1.03	0.79	1.05
	B	0.15	1.12	0.39	1.02	0.67	0.91	0.82	1.05
	All	0.15	1.13	0.39	1.09	0.67	1.10	0.76	1.05

Adjustment factors in the Duffie/Lando framework, all asset Sharpe ratios. Minimum and maximum adjustment factors in the D/L framework. For actual PDs for each rating grade and maturity cf. appendix A.1.1, for parameter combinations cf. subsection 3.4.2. This table includes all asset Sharpe ratios between 10% and 40%. *Total* contains all parameter combinations as described in subsection 3.4.2. $\sigma \geq 10\%$ restricts the asset volatility to values larger or equal to 10% ("non-financials"), $m = 0\%$ restricts the risk-neutral asset value drift relative to the default barrier to 0% ("constant leverage"), *DT reasonable* restricts the average default time to values +/-20% compared to values based on Moody's (2007), cf. appendix A.1.2.

on volatility, risk-neutral drift or default timing). This can be explained by the asset value uncertainty effect (cf. section 3.4.3.3). The asset value uncertainty therefore does not only seem to be an interesting feature for increasing default probabilities but also for increasing the difference between actual and risk-neutral default probabilities for very short maturities. For these short maturities, our Sharpe ratio estimator will usually be upward biased if the Duffie/Lando framework is the true framework.

C.2. Approximation for time-varying risk premia

If Sharpe ratios are time-varying then the derivation of the Merton framework does not hold anymore, even if we keep the assumptions concerning default timing and complete information. However, it can be shown that the formula from the Merton framework still approximates the average expected Sharpe ratio if Sharpe ratios follow an Ornstein-Uhlenbeck process. For ease of notation $\sigma_V = \sigma$, $\rho_{V,M} = \rho$ and $W_t^V = W_t$ is used. The real-world default probability can be derived as

$$\begin{aligned} P[V_{t+\tau} < L] &= P\left[V_t \cdot e^{\int_t^{t+\tau} \sigma\rho\theta_s ds + r - \frac{1}{2}\sigma^2 + \sigma W_\tau} < L\right] \\ &= P\left[\sigma W_\tau + \rho\sigma \int_t^{t+\tau} \theta_s ds < ln\left(\frac{L}{V_t}\right) - (r - 0.5\sigma^2)\tau\right] \\ &\approx \Phi\left[\frac{ln\left(\frac{L}{V_t}\right) - \sigma\rho E^P\left[\int_t^{t+\tau} \theta_s ds\right] - (r - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right] \end{aligned}$$

In the last row, the approximation

$$\begin{aligned} &Var\left(\sigma W_\tau + \rho\sigma \int_t^{t+\tau} \theta_s ds\right) \\ &= Var(\sigma W_\tau) + \rho^2\sigma^2 Var\left(\int_t^{t+\tau} \theta_s ds\right) + 2Cov\left(\sigma W_\tau, \int_t^{t+\tau} \theta_s ds\right) \\ &\approx Var(\sigma W_\tau) \end{aligned} \tag{C.3}$$

was used to substitute the integral by its expected value $E^P\left[\int_t^{t+\tau}\theta_s ds\right]$. The approximation is justified since θ_s is mean-reverting the volatility of $\rho\sigma\int_t^{t+\tau}\theta_s ds$ is a lot smaller than the volatility of σW_τ. In addition, the co-variance term will usually be negative if it is assumed that negative equity returns go hand in hand with an increase in risk aversion.

Accordingly, the risk-neutral default probability can be calculated as

$$Q[V_{t+\tau} < L] \quad = \quad \Phi\left[\frac{ln(\frac{L}{V_t}) - (r - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right]$$

so that

$$\begin{aligned}\frac{\Phi^{-1}(PD^Q(t,\tau) - \Phi^{-1}(PD^P(t,\tau))}{\sqrt{\tau}}\frac{1}{\rho} &\approx \frac{\sigma\rho E^P\left[\int_t^{t+\tau}\theta_s ds\right]}{\tau\sigma\rho} \\ &= \Theta(t,\tau).\end{aligned}$$

C.3. Asset/market vs. equity/market correlation

In this appendix, we will show that the correlation between asset returns and market returns in the Merton framework as well as in the Duffie/Lando framework is approximately equal to the correlation between equity returns and market returns. Since covariance matrices are always positiv semidefinit, it is sufficient to show that the correlation between asset returns and equity returns is approximately one.[5] The economic reason for a correlation of one between asset and equity values is quite simple: As equity is modeled in both frameworks as a deep-in-the-the-money[6] call option on the companies

[5]Since the covariance matrix is positv semidefinit, it has a non-negative determinant. The determinant of the covariance matrix equals $\sigma_1^2\sigma_2^2\sigma_3^2 \cdot (1 + 2\rho_{1,2}\rho_{1,3}\rho_{2,3} - \rho_{1,2}^2 - \rho_{1,3}^2 - \rho_{2,3}^2)$. If $\rho_{1,2} = 1$, it follows that $2\rho_{1,3}\rho_{2,3} - \rho_{1,3}^2 - \rho_{2,3}^2 = -(\rho_{1,3} - \rho_{2,3})^2 \geq 0$ and therefore $\rho_{1,3} = \rho_{2,3}$. We have also directly simulated the difference between the correlation between equity and market values and asset and market values. The results confirm the analysis in this appendix.

[6]Of course, this option is not by definition deep-in-the-money but rather depends on the "closeness" of the asset value to the default barrier. Looking at investment grade

assets[7], the sensitivity of the option with respect to the asset value is almost linear (i.e. the delta of the option is almost one and therefore gamma is close to zero). A linear relationship between two random variables in turn implies a correlation of one. The correlation will be the smaller, the less linear the relationship is, i.e., the higher the default probability (and therefore the less in-the-money the option) and/or the higher the effect of other input parameters that lead to non-linearities with respect to the asset value (e.g. taxes, insolvency costs in the Duffie/Lando framework). The determination of correlations is straightforward: First, a rating grade from Aa to B is choosen. Second, all reasonable combinations of input parameters – excluding the asset value – were depicted. In the third step the asset value was choosen as to yield the cumulative default probability of the respective rating grade choosen in the first step (cf. appendix A.1.1 for the cumulative default probabilities). In the last step, the correlations for these parameter combinations were numerically evaluated with a Monte Carlo simulation. The minimum correlation for each rating grade was plotted in figure C.1. The minimum correlation is always larger than 0.99 for investment grade obligors and larger than 0.96 for non-investment grade obligors.

ratings, the maximum one-year default probability is approximately 0.4% and the maximum 10-year cumulative default probability is approximately 8%. This motivates the use of the term "deep-in-the-money".

[7]The sort of option is of course different in both frameworks: a plain vanilla european call option in the Merton framework and a knock-out option in the Duffie/Lando framework.

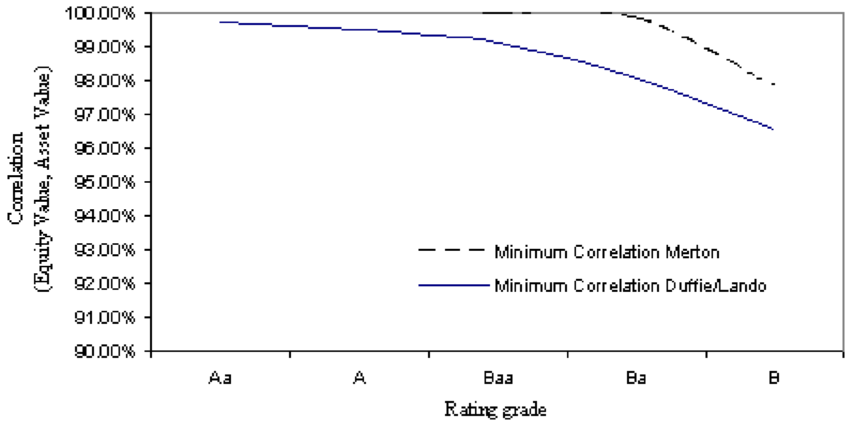

Figure C.1.: Correlation between equity and asset value in the Merton and Duffie/Lando (2001) framework. Parameter combinations for calculating the minimum: T(maturity):$3 - 10y$, $\sigma : 3\% - 30\%$, company Sharpe ratio: $10\% - 40\%$, risk-neutral asset growth rate after payouts: $0\% - 5\%$, α (asset value uncertainty): $0\% - 30\%$, T_1 (time since last certain asset value information): $0y - 3y$, other parameters: $r = 6\%$, default barrier=100, V_t choosen to fit target actual default probability for respective rating grade.

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