Spatial Covariance Based Downlink Beamforming in an SDMA Mobile Radio System

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***Abstract*—On the downlink of an SDMA mobile radio system, different users who share the same frequency band and time slot have to be spatially separated by beamforming. In this paper we analyze an approach to jointly calculate weights in such a way that all users receive their signal with a given signal-to-noise-and-interference ratio. The computations are based on spatial covariance matrices which describe the user-specific channels averaged over the fast fading. The presented algorithm succeeds in minimizing the array transmit power while spatially separating the users, no matter which rank their spatial covariance matrices have. But since it needs to solve a complex nonlinear constrained optimization problem, several approximations of this algorithm will be presented and compared to each other on the basis of array power, complexity, and convergence.**

***Index Terms*— Smart antennas, SDMA, beamforming, constrained optimization.**

I. Introduction

LET us assume a mobile radio system which operates in a number of different time, frequency, and/or code slots. Adding an SDMA component to this system (as described in [1] and [2]) can produce a number of communication channels, which is significantly higher than the number of separate slots. A basic prerequisite for SDMA is an M element antenna array at the base station, capable of spatially separating $K < M$ mobile users transmitting and receiving in the same slot.

On the uplink, the spatial separation of $K > 1$ users transmitting in the same slot can be done without beamforming. As shown in [2]–[4], numerous linear and nonlinear joint detection algorithms can be employed to exploit the degrees of freedom provided by an $M > K$ antenna array to extract the K signals transmitted by the users.

On the down-link, the spatial separation of the users is more difficult, since the mobiles cannot necessarily be expected to do joint detection and adaptive interference cancellation.

- The mobile terminals must be cheap, small, and light. Preferably, the mobile handy will not be equipped with an antenna array and sophisticated joint detection signal processing hardware.
- In general, the propagation delay of the mobile radio channel varies from user to user. On the uplink, user-specific transmission timing can easily compensate for these delays. On the down-link, the K signals transmitted by the base station cannot be timed in such a way that each user receives his signal synchronized with the interference from the signals bound for the $K-1$ co-users receiving in the same channel. Therefore, the mobile has little chance to effectively execute interference cancellation.

For these reasons, the appropriate way to do spatial separation on the down-link is to limit the interference received by the K users by beamforming at the base station (see Fig. 1). Unfortunately, the literature on beamforming mainly concentrates on the uplink case, where one desired signal has to be produced by applying one weight (vector) to the M signals received at the base station antenna array. Well-known algorithms to compute a useful weight are the "linearly constrained minimum variance beamformer" (plain or with spatial smoothing) [5], [6], the "Duvall beamformer" [7], and the "Single-Snapshot beamformer" [8], [9].

The generalization of these algorithms to the case of filtering out K desired signals can be done straightforwardly, since the underlying mathematical problem can be easily decomposed into K *independent computations* of K beamforming weights.

The downlink case is different. For instance, changing the weight $\boldsymbol{w}_1$ applied to the baseband signal $s_1(t)$ in Fig. 1 does not only change the useful signal power received by the user 1; it also changes the interference received by the other user 2. Therefore, beamforming for downlink transmission will always have to be like *one joint computation* of K weights to control useful signal power and interference for all K users.

A useful attempt to employ downlink beamforming to reduce interference and thus allow higher frequency reuse in cellular systems was presented in [10]. There, the base station applied weights in a way that the ratio of desired signal power and undesired interference power was maximized. The beamforming algorithms presented in [11] can be considered an extension of the mentioned algorithm to the SDMA case with the same slot reused by K users in the same cell. Both approaches have in common the drawback of increased array power at the base station, since putting too much emphasis on the minimization of interference tends to result in an undesirable array gain in multipath scenarios.

This paper makes use of ideas presented in [10]–[12], but its core algorithm, the "Power Minimizer," concentrates on a more ambitious target. The minimization of array power is the primary goal, while interference is suppressed no more than necessary, without attempting to completely cancel it at the expense of an undesirable array gain. This way, frequency reuse within the cell (SDMA) will be combined with the chance to allow higher frequency reuse in the entire cellular system.

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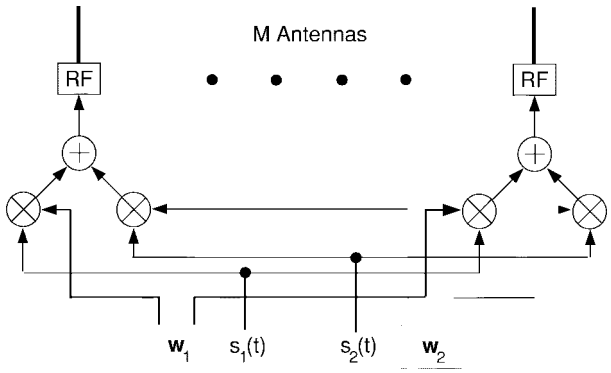

Fig. 1. Beamforming network for downlink transmission for the case of $K = 2$ users.

The paper is organized as follows. In Section 2 the channel model is explained. Furthermore, the user-specific spatial covariance matrices incorporating the channel parameters relevant for downlink beamforming are defined, and methods to estimate these matrices are mentioned.

The Power Minimizer itself is introduced in Section III. This algorithm states a nonlinear cost function with a number of nonlinear constraint equations, so that its straightforward solution turns out to be computationally expensive. Therefore, in the same section, the Linearized Power Minimizer is presented, which yields a feasible approximation of the exact solution of the Power Minimizer problem at greatly reduced computational costs. Based on the power emitted by the array and the convergence and the computational costs of the algorithm, a few variations of the Linearized Power Minimizer are then compared to each other, and to the exact solution.

With reasonable time-variance of the mobile radio channel, there is the chance to update the beamforming weights from weights calculated in the near past with reduced computational effort. In Section IV, two different ideas derived from the considerations made in Section III are presented which quickly update the Power Minimizer weights.

Finally, a short summary of the results concludes the paper in Section V.

II. The SDMA System Model

A. Notations

Throughout this paper, uplink and down-link variables are marked by the symbols $(\hat{\cdot})$ and $(\check{\cdot})$, respectively. The notation $E\{\cdot\}$ indicates the expectation of a random variable. The symbol $|\cdot|$ designates the magnitude of a complex number.

Vectors that are always considered to be column vectors are denoted by lower case boldfaced letters, and matrices by upper case boldfaced letters. The symbol $(\cdot)^*$ indicates the complex conjugate, the symbol $(\cdot)^H$ is the complex conjugate transposition of a matrix or vector.

The notation $||\cdot||$ designates the 2-norm of a vector. The sum of all entries of a matrix or vector is denoted by the symbol $||\cdot||_S$. It will only be applied to real-valued matrices and vectors, but it does not indicate a norm since the summation can result in negative values.

The notations $\tilde{\lambda}(\cdot)$ and $\tilde{\boldsymbol{u}}(\cdot)$ designate the largest positive eigenvalue of a Hermitian matrix and a corresponding normalized eigenvector. They will also be referred to as the "dominant eigenvalue" and "dominant eigenvector," respectively. Analogously, the "dominant generalized eigenvector" $\tilde{\boldsymbol{u}}(\boldsymbol{X}_1, \boldsymbol{X}_2)$ of the Hermitian matrix pair $(\boldsymbol{X}_1, \boldsymbol{X}_2)$ is a normalized eigenvector corresponding to the largest positive eigenvalue λ, which solves the generalized eigenproblem

$$\boldsymbol{X}_1 \boldsymbol{u} = \lambda \boldsymbol{X}_2 \boldsymbol{u}. \tag{1}$$

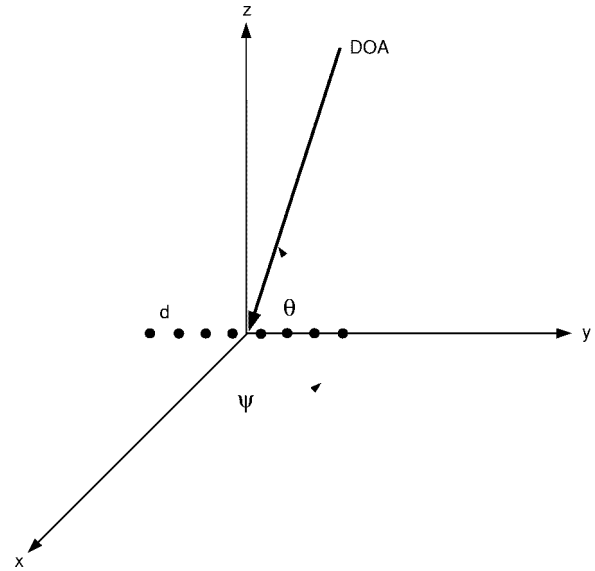

Fig. 2. Azimuth ψ and elevation θ of a wavefront impinging on a ULA.

B. The Uplink Channel

We assume that on the uplink, each user k produces a baseband signal $\hat{s}_k(t)$ which will be modulated onto a sinusoidal carrier with the wavelength $\hat{\lambda}$. Without loss of generality the baseband signal is subject to the constraint equation $E\{|\hat{s}_k(t)|^2\} = 1$. Assuming a WSSUS multipath channel, each signal reaches the base station through a high number Q_k of propagation paths produced by diffractions and scatterings [13], [14]. Each path q can be characterized by the time delay τ_{kq} and a time-varying complex amplitude

$$\hat{b}_{kq}(t) = A_{kq} e^{j2\pi f_{kq} t + j\phi_{kq}(\hat{\lambda})}. \tag{2}$$

The complex amplitude $\hat{b}_{kq}(t)$ incorporates the transmission factor A_{kq}, the Doppler frequency f_{kq} and the frequency dependent phase shift $\phi_{kq}(\hat{\lambda})$.

The distance from the array to all users, reflectors, and scatterers is assumed to be much larger than the array spacing, so that all wavefronts arriving at the array are planar (far-field approximation). Moreover, the bandwidth of all baseband signals received from (and transmitted to, respectively) the users is considered to be much smaller than the reciprocal of the maximum time that a planar radio wave needs to propagate along the array (narrow-band approximation).

With these assumptions, the propagation of a single wavefront from antenna element to antenna element within an array can be modeled by means of additive phase shifts [15]. Stacking these phase shifts yields the so-called array response vector or steering vector $\boldsymbol{a}_{kq}$. It depends on the configuration of

the antenna array, the carrier wavelength $\hat{\lambda}$ and the direction of arrival (DOA) of the wavefront which can be described by the azimuth–elevation pair (ψ, θ). For example, the array response vector of a uniform linear antenna array (ULA) consisting of omnidirectional elements equally spaced by the distance d (see Fig. 2) can be calculated by

$$\hat{\boldsymbol{a}} = (1, e^{\triangle\phi}, \cdots, e^{(M-1)\triangle\phi})^T, \triangle\phi = 2\pi j d \sin\psi \cos\theta / \hat{\lambda}. \tag{3}$$

The summation of all signals transmitted by the K users yields the uplink receive vector

$$\hat{\boldsymbol{x}}(t) = \sum_{k=1}^{K} \sqrt{\hat{P}_k} \sum_{q=1}^{Q_k} \hat{b}_{kq}(t) \hat{s}_k(t - \tau_{kq}) \hat{\boldsymbol{a}}_{kq} \tag{4}$$

with $\hat{P}_k$ denoting the RF power corresponding to the signal transmitted by the user k. The signal power $\|\hat{\boldsymbol{x}}(t)\|^2$ received at the array is subject to fading.

C. The Downlink Channel

The period $\triangle T$ between signal reception on the uplink and the reception of the corresponding answer at the mobile is called the dwell period [16]. It is composed of the signal processing time at the base station (including parameter estimation and beamforming) and the period of time the signal needs to cover the distance between the user and the base station. We assume that for any mobile radio system, $\triangle T$ will always be short enough to render the downlink channel for each user spatially reciprocal to his uplink channel. This means that the number Q_k of propagation paths and the corresponding DOA's (ψ_{kq}, θ_{kq}), the transmission factors A_{kq}, the time delays τ_{kq}, and the Doppler frequencies f_{kq} remain unchanged. The time-varying downlink amplitude can then be expressed as

$$\check{b}_{kq}(t) = A_{kq} e^{j2\pi f_{kq}(t+\triangle T) + j\phi_{kq}(\check{\lambda})}. \tag{5}$$

This spatial reciprocity is the premise to do the downlink separation of the K users by arrival-statistic based beamforming. Each user k is associated with a specific beamforming weight vector $\boldsymbol{w}_k$ and with a baseband signal $\check{s}_k(t)$ with $E\{|\check{s}_k(t)|^2\} = 1$. After joint transmission of the weighted signals, the user k receives the downlink baseband signal

$$\check{x}_k(t) = \sum_{q=1}^{Q_k} \check{b}_{kq}(t) \check{\boldsymbol{a}}_{kq}^H \sum_{l=1}^{K} \check{s}_l(t - \tau_{kq}) \boldsymbol{w}_l. \tag{6}$$

D. Parameter Estimation

The expectation of the signal power received by the user k and created by the weight vector $\boldsymbol{w}_l$ can be computed as

$$\begin{aligned} S_{kl} &= E\left\{ \left| \sum_{q=1}^{Q_k} \check{b}_{kq} \check{\boldsymbol{a}}_{kq}^H \check{s}_l(t - \tau_{kq}) \boldsymbol{w}_l \right|^2 \right\} \\ &= \boldsymbol{w}_l^H \left(\sum_{q_1=1}^{Q_k} \sum_{q_2=1}^{Q_k} \check{\boldsymbol{a}}_{kq_1} \check{\boldsymbol{a}}_{kq_2}^H E\{\check{b}_{kq_1}^*(t) \check{b}_{kq_2}(t)\} \right. \\ &\quad \left. \times E\{\check{s}_l^*(t - \tau_{kq_1}) \check{s}_l(t - \tau_{kq_2})\} \right) \boldsymbol{w}_l. \end{aligned} \tag{7}$$

If we then assume identical stationary statistical properties of all user baseband signals $\check{s}_1(t) \cdots \check{s}_K(t)$, the expectation S_{kl} can be expressed in matrix form as

$$\begin{aligned} S_{kl} &= \boldsymbol{w}_l^H \boldsymbol{C}_k \boldsymbol{w}_l \\ &= \boldsymbol{w}_l^H \left(\sum_{q_1=1}^{Q_k} \sum_{q_2=1}^{Q_k} \check{\boldsymbol{a}}_{kq_1} \check{\boldsymbol{a}}_{kq_2}^H x_{\mathrm{ss}}(\tau_{kq_1} \right. \\ &\quad \left. - \tau_{kq_2}) E\{\check{b}_{kq_1}^*(t) \check{b}_{kq_2}(t)\} \right) \boldsymbol{w}_l \end{aligned} \tag{8}$$

with the autocorrelation function of the signals $\check{s}_l(t)$ denoted by $x_{\mathrm{ss}}(\tau)$.

According to (8), all spatial and temporal parameters corresponding to the channel from the base station to the user k are incorporated in the so-called spatial covariance matrix $\boldsymbol{C}_k$ [11]. Since the optimization of the beamforming weights $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$ goes hand in hand with the optimization of the expectations S_{kl}, the beamforming procedure needs estimates of the matrices $\boldsymbol{C}_1 \cdots \boldsymbol{C}_K$. These estimates have to be obtained at the base station by means of the uplink receive vectors $\hat{\boldsymbol{x}}(t)$.

Depending on the channel situation, two parametrizations of the spatial covariance matrix $\boldsymbol{C}_k$ can be found in the literature.

1) The Spatial Signature Model: Let us make the following assumptions.

- The system uses the same carrier frequencies for uplink and downlink transmission which results in $\hat{\lambda} = \check{\lambda}$ and $\hat{\boldsymbol{a}}_{kq} = \check{\boldsymbol{a}}_{kq}$.
- The channel is not frequency selective so that the time delays τ_{kq} are identical for all paths $q = 1 \cdots Q_k$.
- The user velocity is low and the dwell time $\triangle T$ is very short so that the complex amplitudes $\hat{b}_{kq}(t)$ and $\check{b}_{kq}(t)$ are identical.

In this case, which is approximately given in the European DECT-1800 system, (8) yields

$$\boldsymbol{C}_k = \boldsymbol{r}_k \boldsymbol{r}_k^H, \qquad \boldsymbol{r}_k = \sum_{q=1}^{Q_k} \hat{b}_{kq}, \hat{\boldsymbol{a}}_{kq} \tag{9}$$

with $\boldsymbol{r}_k$ often referred to as the spatial signature of the user k.

Obviously the spatial covariance matrix $\boldsymbol{C}_k$ is identical to the rank one outer product of the spatial signature $\boldsymbol{r}_k$ measured on the uplink. This implies the following advantages:

- The uplink parameter estimation is not difficult, since the spatial covariance matrices can be computed by means of the spatial signatures. The spatial signatures can be easily estimated by means of training sequences [2] or even blindly [17].
- Since there is no need for DOA estimation, diffuse multipath $(Q_k \to \infty)$, beams with nonnegligible angular spread (as measured in [18]) or near-field phenomena do not impair the performance of the parameter estimation procedure. Moreover, the way of estimating $\boldsymbol{r}_k$ does not depend on the shape of the array, whereas well-known direction-finding algorithms like Root-MUSIC [19], ESPRIT [20], Unitary ESPRIT [21], or 2D Unitary ESPRIT [22] need to make assumptions on the array configuration.
- As the spatial covariance matrices will always have rank one, parts of the weight optimization procedure can be executed with low computational costs.

2) The Discrete DOA Model: For the European GSM system and the American IS-54 and IS-95 standards, at least one of the following assumptions is true:

- it is an FDD system so that $\hat{\lambda} \neq \check{\lambda}$ and $\check{\boldsymbol{a}}_{kq} \neq \hat{\boldsymbol{a}}_{kq}$ hold,
- the products of Doppler frequencies and dwell time are large enough to render the complex amplitudes $\check{b}_{kq}$ and $\hat{b}_{kq}$ uncorrelated.

In this case, the spatial signatures estimated on the uplink cannot be reused for downlink transmission. This has been impressively demonstrated in [23] and [24]. Note that reusing the spatial signatures is critical even in DECT systems, since the downlink user separation faces severe degradation if the dwell period is nonneglegible [16].

An alternative model is used for this case, assuming all paths $q = 1 \cdots Q_k$ are uncorrelated:

$$E\{\check{b}^*_{kq_1}(t)\check{b}_{kq_2}(t)\} = \begin{cases} A^2_{kq} & \text{if } q_1 = q_2 = q \\ 0 & \text{else.} \end{cases} \tag{10}$$

Moreover, $x_{\mathrm{ss}}(0) = 1$ holds, so that the spatial covariance matrix—averaged over the fast fading—can be approximated by

$$\boldsymbol{C}_k = \sum_{q=1}^{Q_k} A^2_{kq}\check{\boldsymbol{a}}_{kq}\check{\boldsymbol{a}}^H_{kq} \tag{11}$$

and does in general have a rank which is larger than one. An approximation of the spatial covariance matrix of each user can be computed by means of the DOA's and the transmission factors of the Q_k strongest wavefronts impinging on the array.

In nonline-of-sight environments, the number of wavefronts can be prohibitively high. Nevertheless, the model (11) can still be used, assuming there are $\tilde{Q}_k \ll Q_k$ "diffuse" wavefronts, each composed of a high number of "discrete" subwaves with similar DOA's. The signal power transmitted by these "diffuse" wavefronts is subject to fading. In this case, any term A_{kq} used in (11) has to be interpreted as an estimate of the medium-term expectation of the transmission factor $A_{kq}(t)$ corresponding to the "diffuse" wavefront q.

The following basic strategies of applying direction-finding algorithms to the multiuser spatial estimation problem were presented in [25].

- The uplink receive vectors $\hat{\boldsymbol{x}}(t)$ are used to estimate the DOA's of all dominant wavefronts impinging on the array [19]–[21]. Then the baseband signals transmitted by each wavefront are separated by means of a DOA-based spatial filter which also yields the transmission factors of all wavefronts. Finally, the users are separated by means of the information carried by the demodulated baseband signals [26].
- The $M \cdot K$ channel impulse responses of all user–antenna pairs are estimated by means of orthogonal training sequences. These estimates can be used to separately calculate the spatial parameters of each user [2].

As an alternative, the spatial covariance matrices of all users may be directly estimated from the uplink receive vectors. Algorithms doing this can be found in [10] and [27].

III. Downlink Beamforming With the Power Minimizer

A. Motivation and Problem Formulation

In an interference-limited cellular mobile radio system, the noise power N_k relevant for each user is determined by intercell interference, but not by receiver-inherent thermal or quantization noise. If there are many interfering sources, the BER of the mobile data detector is, in general, only dependent on the level of interference but not on its origin (intercell or intracell interference, respectively). Hence, it does not make any sense to force the intracell interference caused by SDMA down to zero at the expense of the unnecessary increase in the power emitted at the base and, therefore, of the interference emitted into neighboring cells.

The power minimizing beamformer does not necessarily cancel all intracell interference but still minimizes the expectation of the array power while maintaining a given SNIR for all users. These minimum mean square type demands can be put mathematically as the optimization problem

$$\underset{\boldsymbol{w}_1 \cdots \boldsymbol{w}_K}{\text{minimize}} \left\{ \check{P} = \sum_{k=1}^{K} \|\boldsymbol{w}_k\|^2 \right\} \tag{12}$$

with the K constraint equations

$$\frac{S_{k,k}}{\mathrm{SNIR}} = N_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} S_{kl}, \qquad k = 1 \cdots K. \tag{13}$$

The mathematical problem stated in (12) and (13) is not trivial to solve. In contrast to the beamforming problem on the uplink, there is no way here to create K subproblems which separately optimize single weight vectors $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$, as done in [26]. This leaves us with a nonlinear optimization problem with nonlinear constraint equations, which is critical to solve under real time conditions.

In the following subsections, we will present various ways to solve the Power Minimizer beamforming problem.

B. The Nonlinear Approach

After stacking the weight vectors in a complex $(M \cdot K) \times 1$ variable vector $\boldsymbol{w}$, the problem (12), (13) can be reformulated as the minimization of the real-valued target function

$$f(\boldsymbol{w}) = \boldsymbol{w}^H \boldsymbol{w} \tag{14}$$

subject to the K real-valued constraint equations

$$h_k(\boldsymbol{w}) = \boldsymbol{w}^H \boldsymbol{D}_k \boldsymbol{w} - 1 = 0, \qquad k = 1 \cdots K. \tag{15}$$

The $(M \cdot K) \times (M \cdot K)$ constraint matrix $\boldsymbol{D}_k$ corresponding to the user k is defined as

$$\boldsymbol{D}_k = \begin{pmatrix} -\frac{\boldsymbol{C}_k}{N_k} & & & & \boldsymbol{0} \\ & \ddots & & & \\ & & \frac{\boldsymbol{C}_k}{N_k\,\mathrm{SNIR}} & & \\ & & & \ddots & \\ \boldsymbol{0} & & & & -\frac{\boldsymbol{C}_k}{N_k} \end{pmatrix}. \tag{16}$$

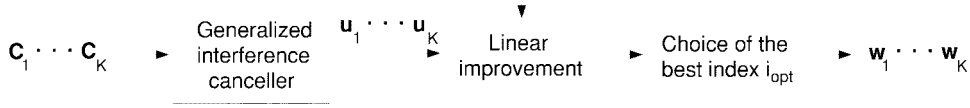

Fig. 3. The linearized power minimizer.

The global optimization of the vector $\boldsymbol{w}$ can be done by well-known nonlinear algorithms based on penalty functions, Lagragian multipliers or combinations thereof. Some of them have been analyzed and compared in [28]–[30]. One example is the "Augmented Lagrangian algorithm for nonlinear EQualities" (ALEQ) which minimizes the unconstrained augmented Lagrangian function

$$L_a(\boldsymbol{w}, \lambda_1 \cdots \lambda_K) = f(\boldsymbol{w}) - \sum_{k=1}^{K} \lambda_k h_k(\boldsymbol{w}) - \frac{\mu_k}{2} h_k^2(\boldsymbol{w}). \quad (17)$$

Infinitesimally incrementing the real-valued weights $\mu_1 \cdots \mu_K$ from 0 to ∞ will always lead to the global optimum of the constrained problem.

C. The Linearized Power Minimizer

All well-known nonlinear algorithms applied to our power minimizing problem have one decisive drawback in common: if the starting point of the iteration is far away from the global optimum, the convergence is very slow, so that an intelligent initialization procedure is necessary. On the other hand, if a fairly good starting point is known, locally convergent algorithms exploiting the special structure of the constraint matrices [see (16)] can be utilized with a dramatically reduced computational burden.

Therefore, we will present an algorithm especially adapted to the power minimizing problem formulated in (12) and (13). In the following, we will refer to this algorithm as the "Linearized Power Minimizer." Its basic idea is to break up the optimization of the variable vector $\boldsymbol{w}$ defined in (14) into the separate optimization of the unit vectors $\boldsymbol{u}_1 \cdots \boldsymbol{u}_K$ along with the adaptation of the K squared lengths $v_1 \cdots v_K$ defined as

$$v_k = \|\boldsymbol{w}_k\|^2, \qquad \boldsymbol{u}_k = \boldsymbol{w}_k / \sqrt{v_k}. \quad (18)$$

The Linearized Power Minimizer is illustrated by the flow chart in Fig. 3.

1) The Generalized Interference Canceller: There are three variations to calculate preliminary weights $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$:

- For $k = 1 \cdots K$, maximize the desired signal power caused by $\boldsymbol{u}_k$

$$\underset{\boldsymbol{u}_k}{\text{maximize}} \left\{ S_{k,k} = \boldsymbol{u}_k^H \boldsymbol{C}_k \boldsymbol{u}_k \right\} \qquad \forall \ \|\boldsymbol{u}_k\| = 1. \quad (19)$$

  The solution of (19) is given by the dominant eigenvector $\tilde{\boldsymbol{u}}(\boldsymbol{C}_k)$ of $\boldsymbol{C}_k$.
  The squared lengths $v_1 \cdots v_K$ can be calculated by means of the SNIR constraint equations. Plugging (8) and (18) into (13) yields the linear equation

$$\boldsymbol{\Psi} \begin{pmatrix} v_1 \\ \vdots \\ v_K \end{pmatrix} = \boldsymbol{\Psi} \boldsymbol{v} = \boldsymbol{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \quad (20)$$

  with the real-valued constraint matrix $\boldsymbol{\Psi}$ given by

$$\boldsymbol{\Psi} = \begin{pmatrix} \frac{\boldsymbol{u}_1^H \boldsymbol{C}_1 \boldsymbol{u}_1}{N_1 \, \text{SNIR}} & \cdots & -\frac{\boldsymbol{u}_K^H \boldsymbol{C}_1 \boldsymbol{u}_K}{N_1} \\ \vdots & \ddots & \vdots \\ -\frac{\boldsymbol{u}_1^H \boldsymbol{C}_K \boldsymbol{u}_1}{N_K} & \cdots & \frac{\boldsymbol{u}_K^H \boldsymbol{C}_K \boldsymbol{u}_K}{N_K \, \text{SNIR}} \end{pmatrix}. \quad (21)$$

  Solving (20) and applying (18) yields the weights $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$.

  Even though $\boldsymbol{\Psi}$ is invertible with probability 1, the solution $\boldsymbol{v}$ of (20) does not necessarily yield feasible weights. Since the elements of $\boldsymbol{v}$ represent "squared lengths", the set $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$ can only be considered valid, if $\boldsymbol{v}$ passes the validity test

$$\boldsymbol{\Psi}^{-1} \boldsymbol{1} = \boldsymbol{v} \in \mathbb{R}_+^K. \quad (22)$$

- For $k = 1 \cdots K$, maximize the ratio of the desired signal power to the undesired signal power caused by $\boldsymbol{u}_k$

$$\underset{\boldsymbol{u}_k}{\text{maximize}} \left\{ \frac{S_{k,k}}{\sum_{\substack{l=1 \\ l \neq k}}^{K} S_{l,k}} = \frac{\boldsymbol{u}_k^H \boldsymbol{C}_k \boldsymbol{u}_k}{\boldsymbol{u}_k^H \bar{\boldsymbol{C}}_k \boldsymbol{u}_k} \right\} \qquad \forall \|\boldsymbol{u}_k\| = 1. \quad (23)$$

  The solution of (23) is given by the dominant generalized eigenvector $\tilde{\boldsymbol{u}}(\boldsymbol{C}_k, \bar{\boldsymbol{C}}_k)$ of the matrix pair $(\boldsymbol{C}_k, \bar{\boldsymbol{C}}_k)$. The squared lengths $v_1 \cdots v_K$ and the weights $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$ can then be computed analogously to the first variation.

  This variation is based on ideas presented in [10]–[11].

- Execute the first two procedures and continue with the better of the two sets $\boldsymbol{u}_1 \cdots \boldsymbol{u}_K$. It makes sense to define the "better" set as the one resulting in the lower array power $\check{P} = \|\boldsymbol{v}\|_S$ [provided that $\boldsymbol{v}$ is valid according to (22)].

It is possible to stop the power minimizing procedure here ending up in what we will call the "Generalized Interference Canceller." It can be regarded as a preliminary approximation of the solution to the original power minimization problem stated in (12) and (13).

2) Linear Improvement: Even if the weight vectors yielded by the Generalized Interference Canceller have passed the validity test (22), they may be far away from the optimal solution of the Power Minimizer. Therefore, in the following we will present a method to drastically improve the preliminary solution $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$ yielded by the Generalized Interference Canceller. As the necessary computations only include linear operations and the calculation of a dominant eigenvector, we named this procedure "Linear Improvement" of the weights. Note that this procedure can even be applied if the unit vectors $\boldsymbol{u}_1 \cdots \boldsymbol{u}_K$ yielded by the Generalized Interference Canceller failed the validity test.

Setting out from

$$\check{P} = \sum_{k=1}^{K} ||\boldsymbol{w}_k||^2 = ||\boldsymbol{v}||_S = ||\boldsymbol{\Psi}^{-1}\mathbf{1}||_S = ||\boldsymbol{\Psi}^{-1}||_S \tag{24}$$

the power minimizing problem (12), (13) can be reformulated as the minimization problem

$$\underset{\boldsymbol{u}_1 \cdots \boldsymbol{u}_K}{\text{minimize}}\{||\boldsymbol{\Psi}^{-1}||_S\} \tag{25}$$

with the $K+1$ constraint equations

$$||\boldsymbol{u}_1|| = \cdots = ||\boldsymbol{u}_K|| = 1, \qquad \boldsymbol{\Psi}^{-1}\mathbf{1} \in \mathbb{R}_+^K. \tag{26}$$

The basic idea of the Linear Improvement procedure is to solve the problem stated in (25) and (26) with restricted degrees of freedom. Only one specific unit vector $\boldsymbol{u}_i$ will be optimized with the $K-1$ others left constant. The dramatic simplification of the problem results from the fact that for any $i = 1 \cdots K$, the cost function defined in (25) can be transformed into

$$||\boldsymbol{\Psi}^{-1}(\boldsymbol{u}_i)||_S = \frac{\boldsymbol{u}_i^H \boldsymbol{B}_i \boldsymbol{u}_i}{\boldsymbol{u}_i^H \boldsymbol{A}_i \boldsymbol{u}_i} \tag{27}$$

with the matrices $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ being functions of the vectors $\boldsymbol{u}_{j \neq i}$, but not of the vector $\boldsymbol{u}_i$. Moreover, it can be proved that the matrix $\boldsymbol{B}_i$ is invertible. Therefore, the dominant eigenvector $\tilde{\boldsymbol{u}}(\boldsymbol{\Upsilon}_i)$ of the solution $\boldsymbol{\Upsilon}_i$ of the linear Hermitian system $\boldsymbol{B}_i\boldsymbol{\Upsilon}_i = \boldsymbol{A}_i$ solves the restricted power minimizing problem, unless the resulting matrix $\boldsymbol{\Psi}(\boldsymbol{u}_i = \tilde{\boldsymbol{u}}(\boldsymbol{\Upsilon}_i))$ fails the validity test stated in (22).

To clarify the following description, we will introduce the symbols $|\boldsymbol{\Psi}|$, $|\boldsymbol{\Psi}(r;c)|$ and $|\boldsymbol{\Psi}(r_1, r_2; c_1, c_2)|$. They denote the determinant of $\boldsymbol{\Psi}$, the determinant of $\boldsymbol{\Psi}$ deprived of its row r, and its column c and the determinant of $\boldsymbol{\Psi}$ deprived of its rows r_1 and r_2 $(r_1 \neq r_2)$ and its columns c_1 and c_2 $(c_1 \neq c_2)$. For the special case of $K = 2$ users $|\boldsymbol{\Psi}(r_1, r_2; c_1, c_2)|$ is defined to be equal to 1.

The calculation of $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ is based on the equation

$$||\boldsymbol{\Psi}^{-1}||_S = |\boldsymbol{\Psi}|^{-1} \sum_{m=1}^{K}\sum_{n=1}^{K} (-1)^{m+n} |\boldsymbol{\Psi}(m;n)|. \tag{28}$$

Plugging (21) into (28) yields

$$||\boldsymbol{\Psi}^{-1}(\boldsymbol{u}_i)||_S = \frac{\boldsymbol{u}_i^H \left(b_{0,i}\mathbf{I} + \sum_{k=1}^{K} b_{k,i}\boldsymbol{C}_k\right)\boldsymbol{u}_i}{\boldsymbol{u}_i^H \left(\sum_{k=1}^{K} a_{k,i}\boldsymbol{C}_k\right)\boldsymbol{u}_i}. \tag{29}$$

Introducing the abbreviations

$$\alpha(k,i) = \begin{cases} (N_k \,\text{SNIR})^{-1} & \text{if } k = i \\ (-1)^{k+i+1} N_k{}^{-1} & \text{else} \end{cases}$$

and

$$\text{sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ \pm 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \tag{30}$$

the coefficients $a_{1,i} \cdots a_{K,i}$ and $b_{0,i} \cdots b_{K,i}$ used in (29) can be calculated as follows:

$$\begin{aligned} a_{k,i} &= \alpha(k,i)|\boldsymbol{\Psi}(k;i)| \qquad \forall k = 1 \cdots K, \\ b_{0,i} &= \sum_{m=1}^{K} (-1)^{m+i} |\boldsymbol{\Psi}(m;i)| \\ b_{k,i} &= \alpha(k,i) \sum_{m=1}^{K}\sum_{n=1}^{K} (-1)^{m+n} \text{sgn}((m-k)(n-i)) \\ &\quad \times |\boldsymbol{\Psi}(m,k;n,i)| \qquad \forall k = 1 \cdots K. \end{aligned} \tag{31}$$

After the execution of the Linear Improvement procedure for all indices $i = 1 \cdots K$, the optimal index i_{opt} has to be chosen. Again, it makes sense to choose the index i corresponding to that matrix $\boldsymbol{\Psi}_i$, which yields the smallest target function $||\boldsymbol{\Psi}_i^{-1}||_S$ [provided that $\boldsymbol{v}$ passes the validity check (22)].

After replacing the former unit vector $\boldsymbol{u}_{i_{opt}}$ by the new one, the whole Linear Improvement procedure can be repeated for all indices $i \neq i_{opt}$. This way the unit vectors $\boldsymbol{u}_1 \cdots \boldsymbol{u}_K$ can be improved step by step. After N Linear Improvements, the computation of the squared lengths $v_1 \cdots v_K$ and the weights $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$ according to (22) and (18) concludes the Linear Improvement of the Power Minimizer.

D. Convergence

The convergence of any beamforming algorithm heavily depends on the scenario incorporated by the spatial covariance matrices of the users to be spatially separated. For instance, in a scenario with two users having identical spatial covariance matrices $\boldsymbol{C}_1 = \boldsymbol{C}_2$, no beamforming scheme could ever realize an *SNIR* larger than 0 dB. Therefore, independently of the beamforming algorithm applied, an SDMA system has to employ an appropiate channel allocation scheme to avoid "spatially unseparable" scenarios.

In fact, the only safe way to prove the convergence of the Generalized Interference Canceller, the Linearized Power Minimizer or the ALEQ Power Minimizer in a specific scenario, is to execute the algorithm successfully. Nevertheless, reliable schemes to estimate the "spatial separability" of a scenario with comparably low computational costs have been presented and analyzed in [31] and [32].

As an example, let us assume K users with known spatial covariance matrices $\boldsymbol{C}_1 \cdots \boldsymbol{C}_K$. A fast and reliable scheme to evaluate the spatial separability of a scenario is as follows:[1]

- execute the third variation of the Generalized Interference Canceller,
- compute the constraint matrix $\boldsymbol{\Psi}$ and its inverse $\boldsymbol{\Psi}^{-1}$,
- if $\boldsymbol{\Psi}^{-1}$ exists and passes the validity test (22), the corresponding scenario will be assumed spatially separable and acceptable for beamforming.

E. Simulation Results

To compare the Generalized Interference Canceller (GIC) and the Linearized Power Minimizer (LPM) to the ALEQ

[1]This scheme is a variation of an SDMA channel allocation algorithm presented in [31].

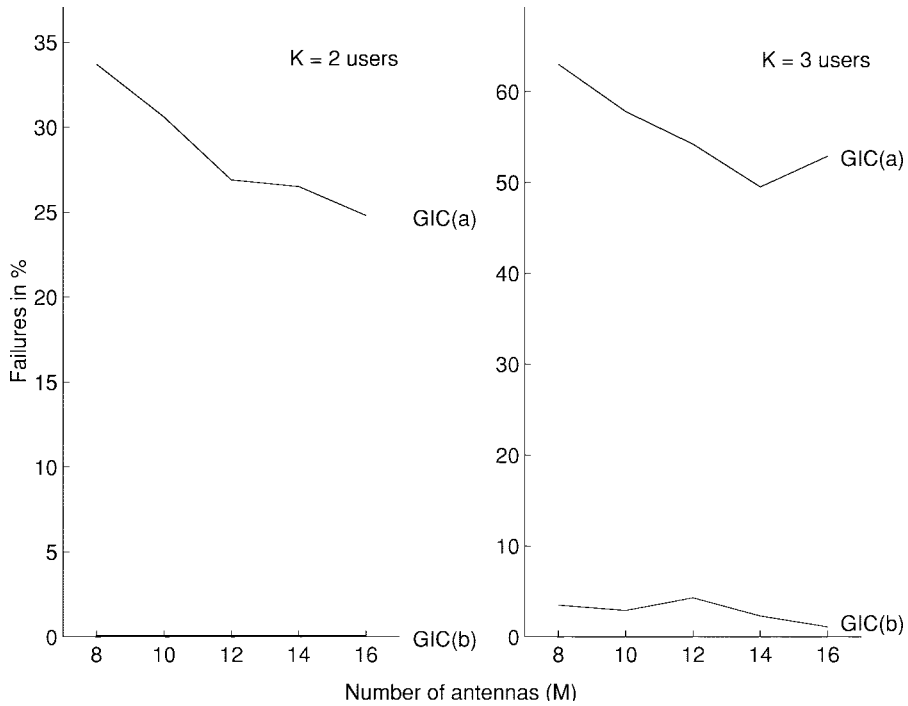

Fig. 4. Failure rate of the GIC (only scenarios with convergence of GIC (c) selected).

Power Minimizer (APM), these three schemes were implemented in C++ code. The comparison was based on the failure rate, the quality of the results incorporated by the array power $\check{P}$ defined in (12), and the computational costs measured by the number of executed complex floating point operations (c-flops).

The Schwabing district in the city of Munich, Germany, was chosen as the coverage area of an (imaginary) SDMA mobile radio cell. With an average building height of 5 stories, this area measuring approximately 20 km^2 can be assumed a typical urban area in Europe. In this area, 57 different locations representing 57 different (imaginary) SDMA users were selected. For each user k, the spatial covariance matrix $\boldsymbol{C}_k$ describing the radio channel between him and the base station were calculated by means of all propagation paths within a attenuation range of 40 dB. The transmission factors, time delays, azimuths, and elevations of these paths were produced by a 3-D ray tracing tool described in [33], operating on a digitized map of the city of Munich.

The two cases $K = 2$ and $K = 3$ were simulated for a ULA with a variable number M of antennas. Each simulation run comprised 1000 out of $57!/(K!(57-K)!)$ different user combinations. For each run, one user combination was chosen in a random manner, but the run was repeated if the corresponding scenario had not been found spatially separable according to the procedure described in Section III-D.

The three variations of the GIC were compared in Figs. 4, 5, and 6. The first variation (a) suffers from a significant failure rate even though its performance is good in the event it yields a solution. On the other hand, the third variation (c) combines good performance with reasonable computational costs so that it was chosen as the initialization procedure for the LPM in Figs. 7 and 8.

The performance of the GIC, the LPM, and the APM algorithms is depicted in the Figs. 7 and 8. There is no need to show the corresponding failure rates since all three algorithms will always converge in combination with the channel allocation scheme described in Section III-D. The plots suggest the LPM with $N = 1$ or $N = 2$ Linear Improvements as an attractive beamforming scheme combining reasonable computational costs with good performance in terms of array power.

IV. Updating the Weights

In mobile radio channels, the spatial covariance matrices of all users are time-varying. Even though we assume that the dominant DOA's and the expectations of the corresponding transmission factors remain unchanged over a medium-term period, adapative beamforming schemes have to be considered to compensate for long-term lognormal fading effects.

Let us assume we have a new set of spatial covariance matrices $\boldsymbol{C}_1(t_2)\cdots\boldsymbol{C}_K(t_2)$ relevant for the time t_2. We also have an old set $\boldsymbol{w}_1(t_1)\cdots\boldsymbol{w}_K(t_1)$ of weights optimal for the time t_1. If the channel has not changed a great deal during the period from t_1 to t_2, the old weights should be close to the optimum of the Power Minimizer applied to the new matrices $\boldsymbol{C}_1(t_2)\cdots\boldsymbol{C}_K(t_2)$. This fact converts the formally global minimization problem into a local one.

We suggest two different approaches to compute updated weights $\boldsymbol{w}_1(t_2)\cdots\boldsymbol{w}_K(t_2)$.

- Apply the Linear Improvement procedure to the unit vectors $\boldsymbol{u}_1(t_1)\cdots\boldsymbol{u}_K(t_1)$ corresponding to the old weights. This algorithm can be interpreted as the Linearized Power Minimizer skipping the Generalized Interference Canceller procedure.
- Execute the ALEQ Power Minimizer with the start vector $\boldsymbol{w}$ yielded by stacking the old weights. Start with very

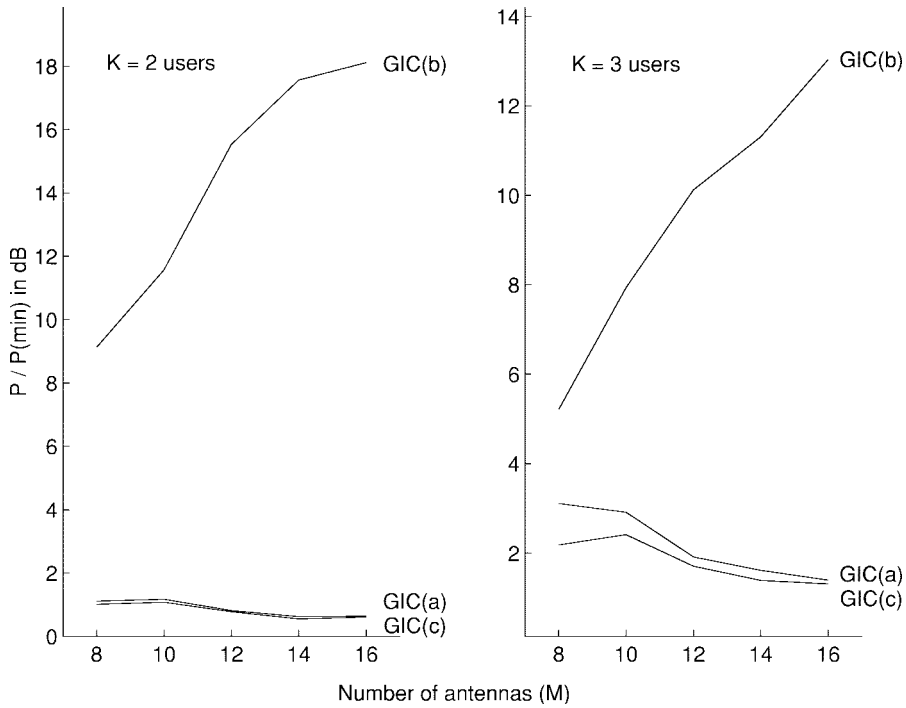

Fig. 5. Array power of the GIC.

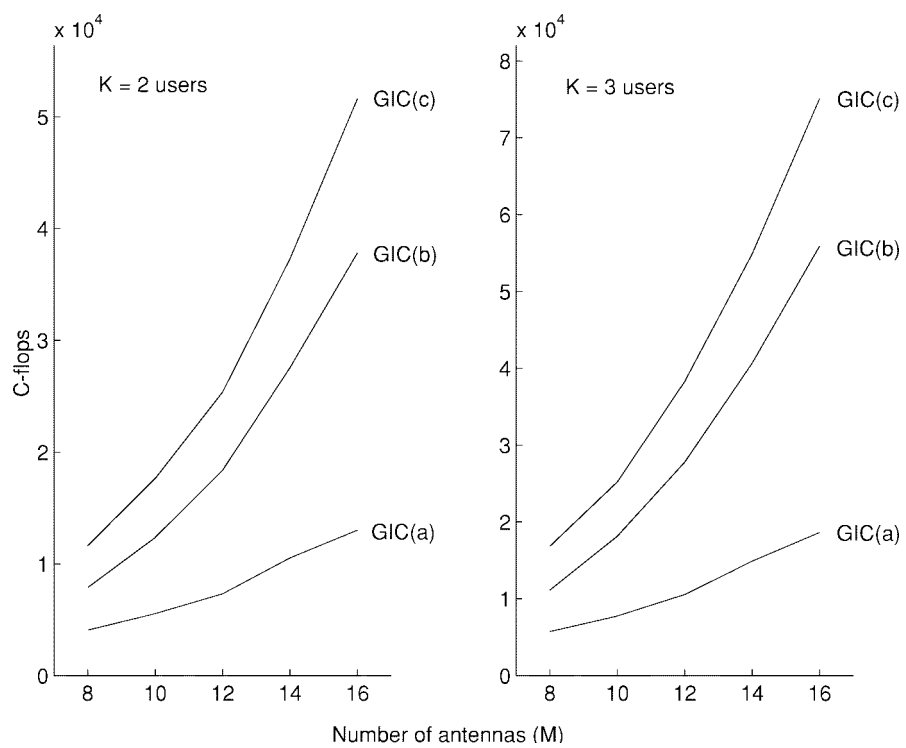

Fig. 6. Complex floating-point operations of the GIC.

large weights $\mu_1 \cdots \mu_K$ to force the algorithm to stay close to the starting point.

V. Conclusion

In this paper, we have presented three beamforming algorithms for the downlink of an SDMA mobile radio system: the Generalized Interference Canceller (GIC), the Linearized Power Minimizer (LPM), and the ALEQ Power Minimizer (APM). They have been described and compared to each other by extensive simulations in a realistic SDMA environment.

The APM yields weight vectors optimal in terms of array power emitted at the base but suffers from its high complexity (see Fig. 8). On the other hand, the GIC can be executed with very low computational costs at the expense of an increase

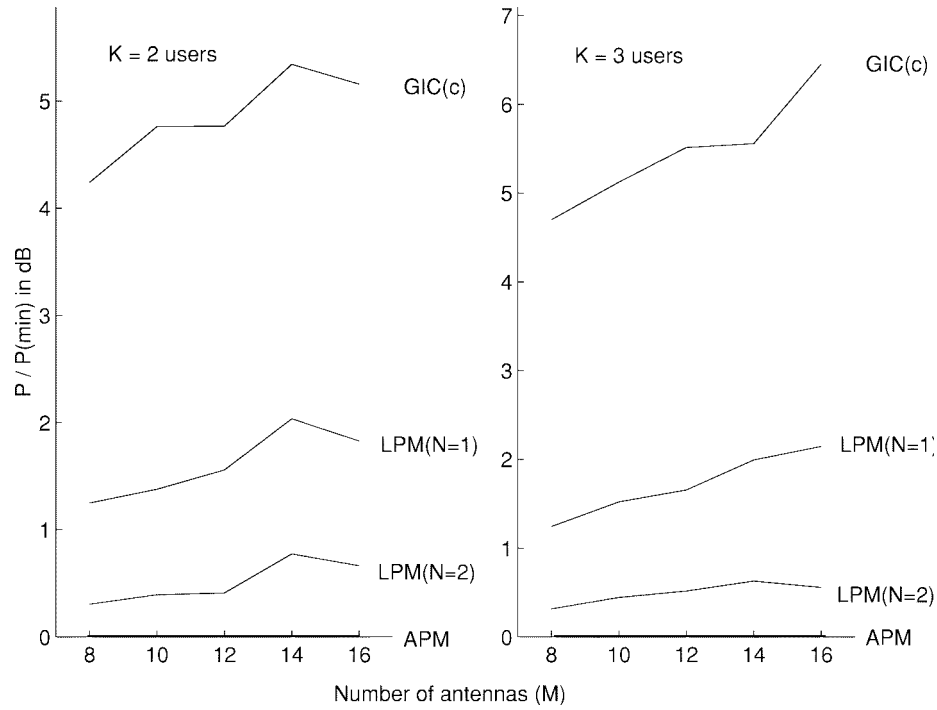

Fig. 7. Array power of the GIC, the LPM, and the APM.

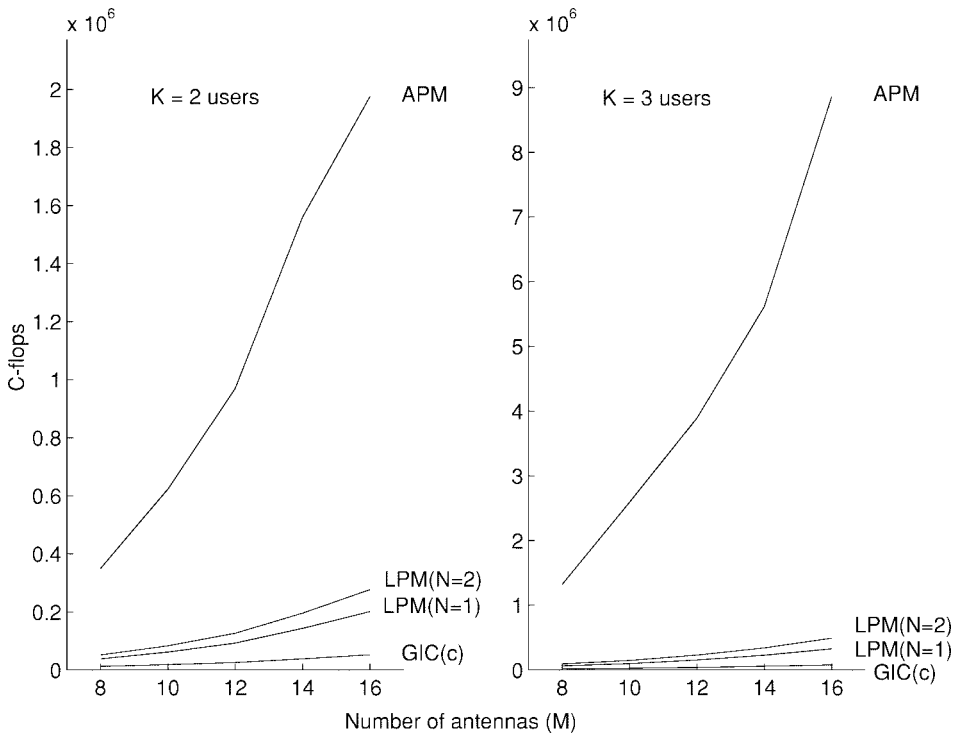

Fig. 8. Complex floating point operations of the GIC, the LPM, and the APM.

in array power between 4 and 6 dB (see Fig. 7). The LPM offers an attractive compromise of reasonable computational effort with good array power (less than 2 dB below the global optimum).

The comparison of beamforming algorithms is of course dependent on the channel allocation scheme as well. In our simulations we have implemented a method (see Section III-D) aimed at high SDMA system capacity while providing convergence for the GIC, LPM, and APM. Executing the proposed allocation method with the GIC variation (a) instead of the GIC variation (c) will result in a higher rejection rate due to the higher failure rate of the GIC(a) (see Fig. 4). As a consequence, the capacity of the SDMA system will decrease. On the other hand, the results of the GIC(a) will be less than 1 dB below the global optimum (see Fig. 5) with a minimum of computational complexity (see Fig. 6).

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