ON THE MOBILE RADIO CAPACITY INCREASE THROUGH SDMA

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ABSTRACT

In a cellular mobile radio system an SDMA component can be implemented for the reuse of radio channels physically incorporated by time, frequency or code slots. In this paper we will present simulation results on an SDMA downlink channel allocation scheme operating in a typical urban mobile radio cell. Based on the resulting blocking probabilties we will give estimations on the capacity increase which can be expected after adding an SDMA component to a conventional mobile radio system.

1. INTRODUCTION

The basic idea of SDMA is the RWC (reuse within cell) of a radio channel, incorporated by an FDMA, TDMA or CDMA slot, by $K > 1$ different users.

On the uplink the spatial separation of K signals can be done by exploiting the information supplied through an $M \geq K$ element antenna array at the base. The data sampled at the array can be used to estimate the fast fading channel impulse responses relevant for the K user-specific uplink channels. These channel estimates can then be fed to a linear [1] [2] or non-linear [3] [4] data detector yielding estimates for the symbols transmitted by each user.

Since the mobiles are supposed to be equipped with a single antenna only, there is no way of executing downlink joint detection at the mobiles. Therefore, on the downlink the spatial separation of the K users operating in the same channel has to be done by beamforming.

Moreover, in many mobile radio systems the channel impulse responses estimated on the uplink cannot be directly reused as beamformer inputs due to the frequency and/or time gap between uplink and downlink channel. Therefore, the adaptive control of the beamforming weights can only be based on the uplink channel estimates averaged over the fast fading.

The medium term downlink channel of each user $k = 1 \cdots K$ can be efficiently described by means of the $M \times M$ spatial covariance matrix

$$\boldsymbol{C}_k = \sum_{q=1}^{Q_k} A_{kq}^2 \boldsymbol{a}_{kq} \boldsymbol{a}_{kq}^H. \qquad (1)$$

The number of propagation paths between the locations of the base station and the user k is denoted by Q_k. Each path $q = 1 \cdots Q_k$ is described by its average transmission factor A_{kq} and the array steering vector $\boldsymbol{a}_{kq}$ incorporating its direction of arrival (DOA). An efficient algorithm named Unitary ESPRIT to estimate these parameters in real time was presented in [5].

The spatial covariance matrices $\boldsymbol{C}_1 \cdots \boldsymbol{C}_K$ do not contain any information about the fast fading relevant for downlink transmission. Therefore, the average SNIR necessary for each downlink receiver will be much higher than the average SNIR the base station antenna array can cope with during uplink reception.

This implies that in contrast to most conventional mobile radio systems the SDMA downlink, not the SDMA uplink, will be the critical link. Hence, the SDMA capacity increase through the reuse of resources within a cell has to be evaluated according to the average number of users which can spatially separated by beamforming on the SDMA downlink.

2. SPATIAL SEPARABILITY

On the downlink the spatial separation of K different users can only be managed in a robust way, if the DOAs of all users in one channel are not too close to each other. Otherwise, downlink transmission will face severe problems, which can be illustrated by the example depicted in fig. 1:

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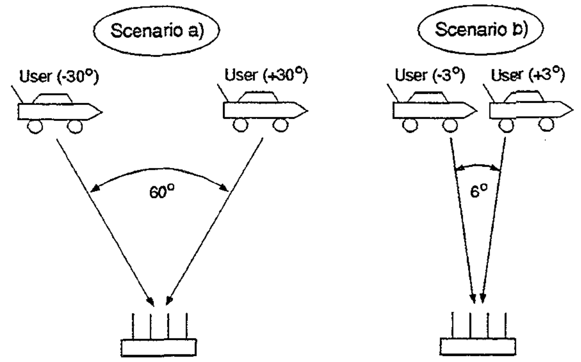

Figure 1: a) A spatially well separable scenario and b) a spatially badly separable scenario

$K = 2$ users, each characterized by a single propagation path with identical attenuations, have to be separated by means of an $M = 4$ element uniform linear antenna array. In case a) the azimuths of the two corresponding DOAs are quite different ($\psi_1 = +30^0$, $\psi_2 = -30^0$), whereas in case b) they are nearly identical ($\psi_1 = +3^0$, $\psi_2 = -3^0$).

The beampatterns created by two optimized weight vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ yielding an SNIR of 10 dB for both users are depicted in fig. 2. In case b) the steep slopes of the patterns at the DOAs of the users ($\pm 3^0$) make two problems obvious:

1. The performance of the downlink beamformer is extremely sensitive to DOA estimation errors.
2. The DOAs of the users are far away from the maxima of the corresponding beampatterns. This waste of electromagnetic power generally results in unnecessary CCI in neighboring cells.

In this context it makes sense to call the case a) a "spatially well separable" scenario and the case b) a "spatially badly separable" scenario.

The problems occuring in scenarios like b) can be avoided by combining SDMA with at least one different multiple access scheme like FDMA, TDMA or CDMA which can supply the system with a number L of separate channels. A DOA sensitive channel allocation algorithm can then assign spatially badly separable users to different channels.

3. THE *EIGENVECTOR METHOD* - A DOA SENSITIVE DOWNLINK CHANNEL ALLOCATION SCHEME

Let us assume that prior to a new user's channel request there are $K^{(l)}$ users operating in each channel $l = 1 \cdots L$. The numbers $K^{(l)}$ are not necessarily equal. In order to avoid trivial solutions

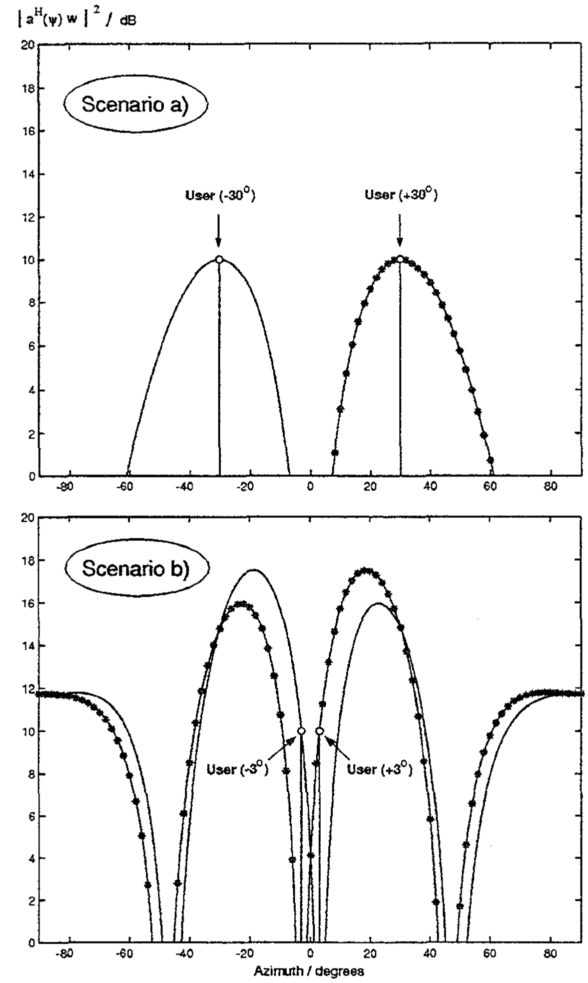

Figure 2: Beampatterns for the scenarios a) and b) created by a 4-element uniform linear antenna array

there are no vacant channels assumed ($K^{(l)} \neq 0$). Altogether $J - 1 = K^{(1)} + \cdots + K^{(L)}$ users are on air.

After adding a new user to the system, L^J new user-channel combinations will be possible, provided there are no restrictions concerning reallocations of any of the $J-1$ active users. In this case the number of combinations to be checked will be prohibitively high even in small systems (e.g. $7 \cdot 10^{11}$ for $L = 7$ and $K = 14$). Another reason, why we will not consider any user reallocations during the allocation procedure, is the additional signalling traffic caused by intracell handovers.

Therefore, we will solve the SDMA channel allocation problem by a two step procedure: First, finding the best channel $l_{opt} \in [1 \cdots L]$ for the new user; second, evaluating the spatial separabilty in that channel and then decide whether the new user will be allocated to the chosen channel or rejected.

The quality of any SDMA channel allocation scheme has to be evaluated according to how far the following goals can be put achieved:

1. Maximizing system capacity by maximizing the average number of users which can be accommodated in L channels.

2. Guaranteeing robust downlink communication by maximizing robustness of the beamformer against parameter estimation errors.

3. Minimizing CCI in neighbouring cells by minimizing the average RF power emitted by the base.

A computationally efficient algorithm doing this job is given by the *Eigenvector Method* (named "Quick SB algorithm" in [6]). It is based on the following considerations:

Let us assume we want to supply one user, characterized by the receiver noise N (composed of thermal receiver noise and CCI from neighboring cells) and the spatial covariance matrix $\boldsymbol{C}$, with a given signal-to-noise-and-interference ratio $SNIR$ by applying the complex weight vector $\boldsymbol{w}$ at the M element base station antenna array. At the same time we want to minimize the average RF power P that has to be emitted at the base.

The RF transmit power P is proportional to the squared length $\boldsymbol{w}^H\boldsymbol{w}$ of the weight vector applied at the array, whereas the RF receive power S at the mobile antenna is proportional to the term $\boldsymbol{w}^H\boldsymbol{C}\boldsymbol{w}$. Therefore, the beamforming problem to calculate the optimal weight vector $\boldsymbol{w}$ can be mathematically put as the following constraint optimization problem:

$$\underset{\boldsymbol{w}}{\text{minimize}} \left\{P = \boldsymbol{w}^H\boldsymbol{w}\right\} \tag{2}$$

with

$$S = \boldsymbol{w}^H\boldsymbol{C}\boldsymbol{w} = N \cdot SNIR. \tag{3}$$

The solution $\boldsymbol{w}$ of the above problem is proportional to the dominant eigenvector $\tilde{\boldsymbol{u}}(\boldsymbol{C})$ of the spatial covariance matrix $\boldsymbol{C}$, i.e. the eigenvector corresponding to the largest eigenvalue $\tilde{\lambda}(\boldsymbol{C})$ of $\boldsymbol{C}$. Hence, the minimum RF transmit power P is given by the product $P = \tilde{\lambda}^{-1}(\boldsymbol{C}) \cdot N \cdot SNIR$.

Generalizing this result to the case of K users (indexed by $(\cdot)_1 \cdots (\cdot)_K$) being accommodated in K separate channels leads to the minimum RF transmit power

$$P_{min} = \sum_{k=1}^{K} \frac{N_k \cdot SNIR}{\tilde{\lambda}(\boldsymbol{C})}. \tag{4}$$

Let us now consider the SDMA case with K users operating in the same downlink channel. The beampatterns to separate the users $k = 1 \cdots K$ from each other on the downlink will be produced by the weights $\boldsymbol{w}_1 \cdots \boldsymbol{w}_K$. The corresponding beamforming problem can be put as follows:

$$\underset{\boldsymbol{w}_1 \ldots \boldsymbol{w}_K}{\text{minimize}} \left\{P = \sum_{k=1}^{K} \boldsymbol{w}_k^H\boldsymbol{w}_k\right\} \tag{5}$$

with the constraints for $k = 1 \ldots K$:

$$\frac{\boldsymbol{w}_k^H\boldsymbol{C}_k\boldsymbol{w}_k}{SNIR} = N_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \boldsymbol{w}_l^H\boldsymbol{C}_k\boldsymbol{w}_l. \tag{6}$$

No matter in which way this constraint optimization problem (5) & (6) will be solved mathematically, the downlink transmit power P will never be lower than the minimum power P_{min} defined in (4). Moreover, simulations show that the transmit power P will move further away from the minimum P_{min}, if the DOAs of the users move closer to each other. Finally, if the DOAs are just too close (or even identical), the problem (5) & (6) does not yield any solution at all, i.e. the users are spatially no longer separable by beamforming.

Therefore, it makes sense to use the ratio P/P_{min} as a measure for the spatial separabilty of a scenario. Doing this, $P/P_{min} = 1$ means optimal spatial separability, whereas $P/P_{min} \to \infty$ refers to the case of no spatial separability at all.

Based on this criterion, the *Eigenvector Method* is given by the following procedure:

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1. Estimate the spatial covariance matrix $\boldsymbol{C}$ of the new user requesting for a communication channel.

2. Compute a dominant unit eigenvector $\tilde{\boldsymbol{u}}$ of his spatial covariance matrix $\boldsymbol{C}$.

3. For all channels $l = 1 \cdots L$ do:

Estimate the receiver noise $N^{(l)}$ at the location of the new user's mobile in the specific channel l. In general this value $N^{(l)}$ predominantly results from CCI from neighboring cells and has to be measured by the mobile and communicated to the base station.

Assume the new user has been allocated to the channel l, so that the channel now has $K = K^{(l)}+1$ users who will be indexed by $(\cdot)_1 \cdots (\cdot)_K$. With all spatial covariance matrices $\boldsymbol{C}_1 \cdots \boldsymbol{C}_K$, the corresponding dominant unit eigenvectors $\tilde{\boldsymbol{u}}_1 \cdots \tilde{\boldsymbol{u}}_K$ and the noise powers $N_1^{(l)} \cdots N_K^{(l)}$ assumed to be known, calculate the constraint matrix

$$\boldsymbol{\Psi} = \begin{pmatrix} \frac{\tilde{\boldsymbol{u}}_1^H\boldsymbol{C}_1\tilde{\boldsymbol{u}}_1}{N_1^{(l)}\,SNIR} & \cdots & -\frac{\tilde{\boldsymbol{u}}_K^H\boldsymbol{C}_1\tilde{\boldsymbol{u}}_K}{N_1^{(l)}} \\ \vdots & \ddots & \vdots \\ -\frac{\tilde{\boldsymbol{u}}_1^H\boldsymbol{C}_K\tilde{\boldsymbol{u}}_1}{N_K^{(l)}} & \cdots & \frac{\tilde{\boldsymbol{u}}_K^H\boldsymbol{C}_K\tilde{\boldsymbol{u}}_K}{N_K^{(l)}\,SNIR} \end{pmatrix}. \tag{7}$$

Solve the real-valued $K \times K$ system

$$\Psi \begin{pmatrix} P_1^{(l)} \\ \vdots \\ P_K^{(l)} \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}. \quad (8)$$

If the system does not have a (unique) solution or any entry $P_k^{(l)}$ is non-positive, the K users in the channel l will be considered spatially unseparable. Otherwise, an estimate of the downlink transmit power $P^{(l)}$ is given by

$$P^{(l)} = \sum_{k=1}^{K} P_k^{(l)}. \quad (9)$$

Calculate the ratio $P^{(l)}/P_{min}^{(l)}$ according to (4) and (9).

4. Select the optimal channel l_{opt} according to the lowest ratio $P^{(l)}/P_{min}^{(l)}$.

5. If the ratio $P^{(l_{opt})}/P_{min}^{(l_{opt})}$ is larger than 3 dB, then

    reject the new user,

otherwise

    allocate him to the channel l_{opt}.

We assume that an $SNIR$ of 10 dB guarantees robust downlink communication for every user in every channel.

4. CAPACITY

We will define the *capacity* of a mobile radio system in a traffic theory like manner:

> Capacity is the traffic $A = \mu T$ an SDMA system can support without exceeeding a maximum blocking probability B during the channel allocation procedure.

In this context the calling rate (in calls per second) is denoted by μ, whereas T designates the average duration (in seconds) of a call.

The corresponding traffic model is depicted in fig. 3: Each state in the Markov chain is characterized by the number J of users in the system. The transition from a state $J = j$ to a lower state $J = j - 1$ results from a (voluntarily) terminated call. Hence, the corresponding transition rate is given by the ratio j/T of the number of active users to the average call duration. The transition from a state $J = j$ to a higher state $J = j + 1$ is triggered by a user request resulting in a successful channel allocation. As long as there are still free channels available ($j < L$), the corresponding rate is identical to the calling rate μ. If all channels are occupied with at least one user ($j \geq L$), there is a chance the channel allocator rejects the new user, represented by the rejection rate $r(j)$.

There is no way of describing the rejection rates $r(j)$ and the resulting overall blocking probabilty B by means of analytic formulae. Therefore, we had to resort to simulations to estimate B as a function of the traffic $A = \mu T$ in realistic SDMA scenarios.

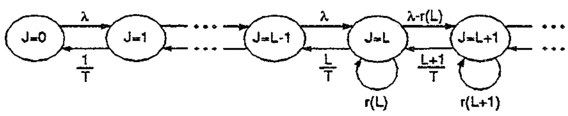

Figure 3: Traffic model for an SDMA system with L channels

5. SIMULATION RESULTS

Our SDMA capacity predictions were based on a Monte Carlo simulation of a single mobile radio cell. The statistics of the parameters defining the radio channels between the users and the base station were chosen in compliance with the results of the 3D channel measurements carried out in the city of Munich in 1995 [7].

We are assuming a ring-shaped cell which has an SDMA base station in its center equipped with a uniform linear M element antenna array. The maximum distance from any user to the base station is given by the outer ring radius 5 km, the minimum distance by the inner ring radius 0.1 km. The user locations are uniformly distributed in the ring and independent from each other. Assuming a typical urban area, the average attenuation corresponding to the user distance r_k can be approximated by $\overline{\rho}_k = 40\, lg(r_k/5m)$ dB.

The numbers Q_k of DOAs are assumed to be either 1 or 2 (with probability 0.5 each). The attenuation factors ρ_{kq} are log-normally distributed with the Suzuki parameter $S = 6$ dB and the average $\overline{\rho}_k$. Analogously, the noise and interference loads $N_k^{(l)}$ were assumed to be made up by CCI from log-normally shadowed users $10 - 30$ km away from the base (corresponding to a cellular frequency reuse factor $r = 3$).

Each DOA is assumed to consist of $R_{kq} \in [1 \cdots 50]$ propagation paths. The mean azimuths $\overline{\psi}_{kq}$ of all paths are uniformly distributed in the range $[-180^0; +180^0]$, whereas the mean elevations are constant ($\overline{\theta}_{kq} = 0^0$). The azimuths $\psi_{kq1} \cdots \psi_{kqR_{kq}}$ and elevations $\theta_{kq1} \cdots \theta_{kqR_{kq}}$ of all paths are both Gauss-distributed with the means $\overline{\psi}_{kq}$ and $\overline{\theta}_{kq}$ and the standard deviations $\delta\psi_{kq} = \delta\theta_{kq} = 5^0$.

Fig. 4 shows the number L of separate channels an SDMA base station needs to support a given traffic $A = \mu T$ in the cell. The number of antennas was varied from $M = 1$ to $M = 16$. The number of calls simulated for each point in the plot is 10000. The tolerable blocking probability was $B = 1\%$.

The result is, not surprisingly, the higher the number M of antennas, the lower the number L of channels necessary to support a given traffic A. As an example, consider the traffic $A = 30$ erl: A conventional system ($M = 1$) like GSM needs $L_1 = 42$ channels in order not to exceed the blocking probability $B = 1\%$. An SDMA system with $M = 8$ antennas needs $L_8 = 26$ channels and one with $M = 16$ antennas needs $L_{16} = 14$ channels. The corresponding results for $A = 60$ erl are: $L_1 = 73$, $L_8 = 32$ and $L_{16} = 24$.

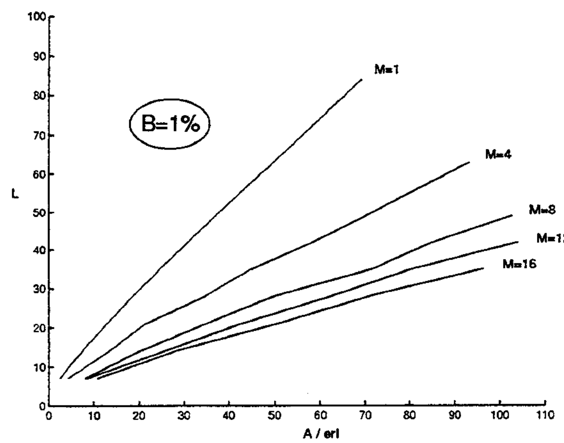

Figure 4: Supportable traffic A versus the number L of channels with a maximum blocking probability B = 1%

6. CONCLUSIONS

In this paper SDMA capacity is defined as the traffic A a system can support without exceeding a maximum blocking probability B during the channel allocation procedure. The capacity was estimated by simulating the performance of a specific SDMA channel allocation scheme, the *Eigenvector Method* in a realistic urban mobile radio cell. We assumed that channel allocation and, hence, system capacity is limited by the maximum number of users which can be accommodated on the downlink in a robust way.

As a result, the simulations yielded the number L of channels (i.e. FDMA, TDMA or CDMA slots) an operator must provide to be capable of managing a given traffic A (see fig. 4). The capacity increase L_1/L_M over a conventional system with a single antenna is rather dependent on the number M of antennas than on the traffic A, as shown in the table 1 which has been extracted from the plot shown in fig. 4.

L_1/L_M	A=20	A=40	A=60
$M = 1$	1.00	1.00	1.00
$M = 4$	1.49	1.66	1.73
$M = 8$	2.12	2.24	2.38
$M = 12$	2.53	2.63	2.74
$M = 16$	2.75	2.95	3.06

Table 1: The SDMA capacity increase L_1/L_M over a conventional system depending on the traffic A and the number M of antennas

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