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Joint Network-Channel Coding for Wireless Relay Networks

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Contents

Zusammenfassung

Diese Arbeit behandelt den Entwurf von Kanalcodes für den Fehlerschutz in drahtlosen Relaisnetzen mit Rundsendungen und Zeitmultiplex. Als Netze werden der Relaiskanal, der Mehrfachzugriffsrelaiskanal und der bidirektionale Relaiskanal betrachtet. Die Codes für die letzteren beiden Kanäle beinhalten Netzcodierung am Relais. Netz- und Kanalcodierung wird gemeinsam entworfen, um Redundanz im Netzcode für einen verbesserten Fehlerschutz nutzen zu können. Die vorgeschlagenen Codeentwürfe basieren auf Turbo Codes. Eine mögliche Anwendung ist der zellulare Mobilfunk. Neben dem Entwurf von Codes wird untersucht, wie die Sendezeit zu den Quellen und dem Relais zugeteilt werden soll, und es wird ein System mit hierarchischer Modulation vorgeschlagen, dass eine Komplexitätsreduktion erlaubt.
Zusätzlich wird eine Methode vorgestellt, wie derzeitige Punkt-zu-Punkt Kommunikationssysteme mit paketübergreifender Kanalcodierung verbessert werden können.

Abstract

This work investigates the design of channel codes for error-correction in wireless relay networks with broadcast transmissions and time-division multiple access. In particular, the relay channel, the multiple-access relay channel (MARC) and the two-way relay channel (TWRC) are considered as networks. The coding schemes for the MARC and the TWRC include network coding at the relay. Network coding and channel coding is jointly designed with the aim to exploit redundancy in the network code for a better error-correction. The proposed coding schemes are based on turbo codes. A possible application are cellular mobile systems. Beside the design of new channel codes, it is considered how to allocate the transmission time to the source(s) and the relay. Moreover, a scheme with hierarchical modulation is proposed to lower the computational complexity of relaying systems.
It is also proposed to extend current H-ARQ schemes for point-to-point communication systems with cross-packet channel coding.

1

Introduction

Wireless telephony became an integral part of our everyday life during the last 20 years. Contrary to wireline telephony, the use of the wireless medium allows that the user is mobile and can communicate in all areas covered by the system. Moreover, the costs to establish the wireless infrastructure have the potential to be cheaper because it is not necessary to install wire to each user.
With the development of the internet, people became more and more interested to communicate beside voice also other data, for example video, music or other data files. Whereas mobile wireless communication systems of the second generation (e.g. GSM[1]) were mainly intended for the transmission of voice, wireless systems of the third (e.g. UMTS[2]) generation and future systems are intended also for other data traffic. Such data traffic requires much higher data rates than voice traffic.
The bandwidth suitable for mobile communication with electromagnetic waves is a scarce resource. Moreover, the transmission power is limited. Due to the limited bandwidth and transmission power, it is necessary to think about extensions of the point-to-point communication between base station and mobile equipment, which allow higher data rates by better exploiting the available bandwidth. One possible extension is to use relays. It is currently investigated by researchers in industry and academia how to include wireless relaying in future communication systems. For example, relaying is planned to be included in the IEEE standard 802.16j. An overview over relaying concepts for mobile communications is given in [PWS+04]. The purpose of relays is to support the communication between mobile and base station. We assume in this thesis that relays are not a source of own data. Relays are supposed to be simpler than base stations in terms of transmit power requirements, size and costs. Contrary to base stations, relays do not have

[1] GSM: Global System for Mobile Communications
[2] UMTS: Universal Mobile Telecommunications System

a wired connection to the backhaul network. Fixed relays can be installed for example on traffic lights on street crossings. Mobile relays can be installed for example on trains or taxis. Wireless relaying promises amongst others the following advantages compared to a point-to-point communication between base station and mobile station without relay[3]:

- If the relay is positioned in a clever way, it is very likely that the links from mobile and base station to the relay are of better quality than the direct link between mobile and base station. Then, relaying promises higher data rates than the point-to-point communication. Moreover, the mobile station can save transmission energy (battery lasts longer). It is also possible to extend the coverage of cells.
- The sink can receive both the signal from source and relay. Both the small-scale fading due to multipath propagation and the large-scale fading due to shadowing can be assumed independent for the source-sink and the relay-sink link. Therefore, relaying allows to gain diversity and to increase the robustness of the communication system.
- The use of relays increases the probability of a line-of-sight connection, especially on the link between a fixed relay and the base station. This allows to use parts of the spectrum which are not used currently due to their vulnerability against non-line-of-sight conditions.

We consider relay networks with broadcast transmissions and with time-division multiple-access (TDMA). That means that the nodes in the network are not allowed to transmit simultaneously during the same time in the same frequency band. The restriction to TDMA is suboptimal. However, TDMA schemes are well suited for a practical realization of relaying [PWS+04], because the nodes can operate in a half-duplex mode and a synchronization of the transmissions of different nodes is not required on a symbol or carrier phase level. We consider the following three relay networks which are depicted in Figure 1.1:

- The relay channel consists of one source, one relay and one sink. This setup can be either used for the uplink or for the downlink.
- The multiple-access relay channel (MARC) consists of two sources, one relay and one sink. The setup can be used for the cooperative uplink for two mobile stations.
- The two-way relay channel (TWRC) consists of two sources which want to exchange information with the help of one relay. This setup can be used for the cooperative uplink and downlink.

[3]For a fair comparison between systems with and without relay, we guarantee that all systems use the same total transmission time, the same total bandwidth and the same total transmission energy.

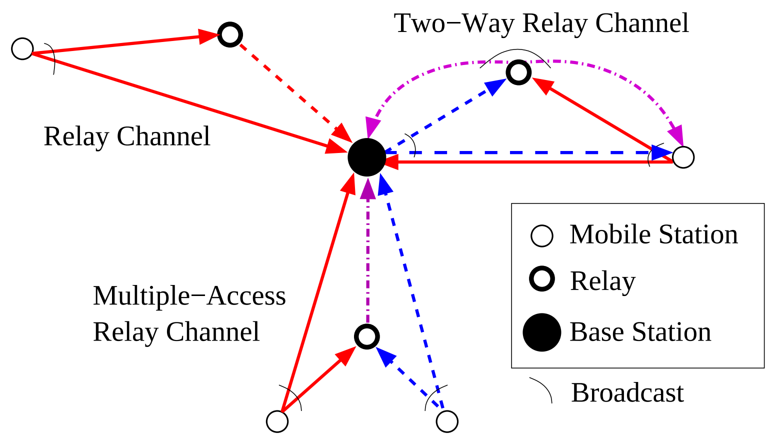


Figure 1.1: Relay channel, multiple-access relay channel and two-way relay channel.

In this thesis, we follow the approach that the adaption of channel coding to relaying allows to realize the promised gains. We consider the design of channel codes for these three relay networks and the allocation of the transmission time to the source(s) and the relay. The coding schemes for the MARC and the TWRC include network coding at the relay whereas the coding scheme for the relay channel includes routing. Network coding and channel coding is jointly designed with the aim to exploit redundancy in the network code for a better error protection. The proposed coding schemes are based on turbo codes.

This thesis is organized as follows:

Chapter 2 summarizes the required background knowledge about channel coding for a point-to-point communication without relay. Moreover, a new coding scheme, termed cross-packet channel coding, is proposed.

Chapter 3 summarizes background knowledge about network coding for error-free point-to-point networks.

Chapter 4 gives an overview over several communication strategies for wireless relaying. We show the advantage from exploiting broadcast transmission compared to simpler strategies and we explain why more complicated strategies with simultaneous multiple-access do not provide much advantage under the restrictions of the current technical possibilities. The overview allows to classify our work. It also allows to understand the relation between results from the literature which seem inconsistent on a first glance.

We explain in **Chapter 5** how joint routing and channel coding for the relay channel can be realized with distributed channel codes. We extend distributed channel coding in the following two directions: First, we propose to use hierarchical modulation at the source. This allows to reduce the computational complexity at the sink at the cost of a very small performance loss. Second, we consider how to optimally allocate the transmission time to source and relay.

In **Chapter 6**, we propose a joint network-channel coding scheme, termed turbo network code, for the multiple-access relay channel. The scheme allows to gain diversity for higher code rates than the system for the relay channel. We also consider how to optimally allocate the transmission time.

In **Chapter 7**, we propose a joint network-channel coding scheme for the two-way relay channel. The scheme allows to increase the data rate compared to the system for the relay channel. Again, we consider how to optimally allocate the transmission time.
Chapter 8 summarizes the results and states possible directions for future research.

Parts of the thesis have been published in [HSOB05], [HD06], [HH06], [Hau06], [HH07], [HC07].

2

Channel Coding for Point-to-Point Communication

The main focus of this thesis is on coding for small relay networks. In this chapter, we will explain the required basics about channel coding for a point-to-point communication with one data source and one data sink.
One contribution of this thesis is cross-packet channel coding. It is a coding scheme for a point-to-point communication. It is treated in Section 2.5.

2.1 Forward Error Correction (FEC) with Channel Coding

We consider the communication of data from a source to a sink. The data has to be transferred through a channel which outputs a disturbed version of its input. Examples for such point-to-point communications can be found in wireless and wireline communications and in storage systems (e.g. hard disks, CDs, DVDs). We will focus in this thesis on wireless communications, especially on cellular based mobile communication systems. In such systems either a mobile phone or a mobile computer transfers data, for example voice or pictures, to a base station (uplink) or the base station transfers data to the mobile equipment (downlink).
An aim of communications engineering is to enable a reliable communication, in the sense that the sink obtains the data with minimal disturbance. If we want to transfer digital data (bits), we can use forward error correction (FEC) with channel coding to protect the digital data against the channel disturbance. That means the data is channel encoded

before the channel. A binary channel encoder encodes a packet

$$\mathbf{u} = (u_1, u_2, \ldots, u_K)$$

of K data bits and outputs a block (codeword)

$$\mathbf{c} = (c_1, c_2, \ldots, c_N)$$

of $N \geq K$ code bits. The channel encoder includes $N - K$ redundant bits to allow the channel decoder at the sink to correct the channel disturbance. The code rate R_c of the channel code is defined as $R_\mathrm{c} = K/N$. Both the elements of $\mathbf{u}$ and $\mathbf{c}$ are defined to be in the binary Galois field GF(2) with the elements $\{0, 1\}$.

In this thesis, we will only consider linear channel codes. Then, the channel encoder is defined by the generator matrix $\mathbf{G}$ and the output of the channel encoder is given by

$$\mathbf{c} = \mathbf{u} \cdot \mathbf{G}. \tag{2.1}$$

Error correction has to be distinguished from error detection. Contrary to error correction, the aim of error detection is not to correct errors, but to detect at the sink that a packet contains an error. The dual problem to channel coding is source coding. The aim of source coding is to compress data and to remove all redundancy in the data. Even if the user is mostly not aware of it, channel coding is used in every day life. Channel codes are included in mobile communication systems, in television broadcast and in compact discs (CDs) and hard drives.

It is not possible to input bits into most physical channels. This concerns also the communication over the wireless medium with electromagnetic waves. In order to transmit the code bits over such physical channels (e.g. a wireless channel), a modulator has to map the digital bits to analog waveforms that match the characteristics of the channel. The mapping is performed by taking L code bits and selecting dependent on the L bits one of 2^L energy waveforms for transmission over the channel. Due to practical constraints the 2^L waveforms should have a similar shape and should only differ in the scale of the amplitude, in the shift of the phase or in the shift of the frequency. We will only consider the modulation of the amplitude and of the phase. Moreover, we consider the equivalent lowpass representation of the signals and use a time-discrete channel model. Then, a modulation can be described as a set $\mathcal{S}$ of 2^L complex numbers with the corresponding amplitude scalings and phase shifts and it is not necessary to consider the shape of the waveform. Examples for important alphabets are binary phase-shift-keying (BPSK) with $\mathcal{S}_2 = \{-1, +1\}$, quadrature phase-shift-keying (QPSK) with $\mathcal{S}_4 = \{-j, -1, j, 1\}$, 4-quadrature-amplitude-modulation (4-QAM) and 16-quadrature-amplitude-modulation (16-QAM). The definitions of 4-QAM and 16-QAM can be found in [Pro95]. Each BPSK symbol carries $L = 1$ bit, each QPSK and each 4-QAM symbol carries $L = 2$ bits and each 16-QAM symbol carries $L = 4$ bits. With increasing L the data rate can be increased. However, the Euclidean distance between the constellation points decreases and the probability that the receiver detects the wrong symbol increases as well. The modulator transforms the block $\mathbf{c}$ of N code bits to a block

$$\mathbf{x} = (x_1, x_2, \ldots, x_M)$$

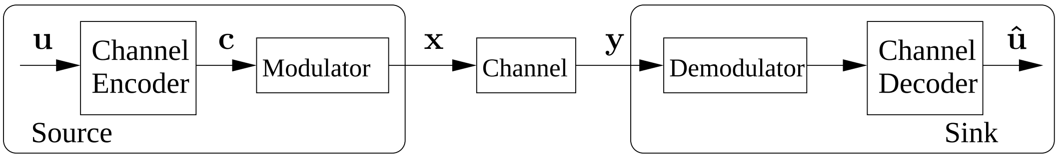


Figure 2.1: System model for point-to-point communication.

of $M = N/L$ symbols from the set $\mathcal{S}$. The rate R of channel code and modulator is defined as $R = K/M = R_c \cdot L$.
The channel disturbs the symbols $\mathbf{x}$ and outputs the disturbed symbols

$$\mathbf{y} = (y_1, y_2, \ldots, y_M).$$

The sink receives $\mathbf{y}$. The demodulator and the channel decoder try to recover the original data $\mathbf{u}$ from $\mathbf{y}$ and output the estimate $\hat{\mathbf{u}}$. The aim of the design of channel coding and modulation is to minimize the bit errors between $\mathbf{u}$ and $\hat{\mathbf{u}}$ given the allowed rate R and other constraints, for example the transmission power or energy for wireless systems. The transmission power corresponds to the mean square of the amplitude of the symbols $\mathbf{x}$. Figure 2.1 depicts a model of the considered system. More information about channel coding and modulation can be found in [LC04, BB99, Pro95, CHIW98, CF07].

2.2 Channel Model for Wireless Signal Propagation

We assume a wireless channel model with noise, propagation path-loss and fading.

2.2.1 Noise

Wireless communication is limited by electromagnetic noise. The main types of noise are thermal noise at the receiver, artificial man-made noise, atmospheric and galactic noise [Yac93]. The thermal noise is caused by the thermal chaotic motion of electric charge carriers (e.g. electrons) in the receiver. Thermal noise disappears at the temperature of zero Kelvin.
A main origin of man-made noise is emitted by cars, when the spark ignites the mixture of gasoline vapor and air [PP00, Yac93]. The man-made noise dominates in urban areas for the part of the spectrum below 4000 MHz and in suburban areas below 1000 MHz [Yac93]. The level of man-made noise increases with decreasing frequency. For example, the level at 1000 MHz is 25 dB lower than the one at 100 MHz. The thermal noise is dominant above 4000 MHz in urban areas and above 1000 MHz in suburban areas. Atmospheric and galactic noise is negligible for frequencies below 500 MHz.
The received signal at the sink has to pass through a bandpass filter with a bandwidth W large enough to not distort the transmitted waveform. We make the common assumption that the noise after the receiver filter is additive and superimposes on the transmitted signal, that the power spectral density is $N_0/2$ over the filtered bandwidth and that the probability density function has a Gaussian shape. Noise according to this model is called additive white Gaussian noise (AWGN).

2.2.2 Propagation Path-Loss

For wireless relay networks, often the topology of the network is given, for example that the relay is at half distance between source and sink. The free-space transmission formula of Friis [Rap99, Yac93] allows to calculate the received power $P_r(d)$ at the sink when the transmitter at the source sends with transmission power P and when source and sink have the distance d:

$$P_r(d) = P \cdot G_1 \cdot G_2 \cdot \left(\frac{\lambda}{4\pi d}\right)^n \tag{2.2}$$

The formula depends additionally on the gains G_1 and G_2 of the transmission and receive antenna, and the wavelength λ of the signal. The path-loss exponent is given by $n = 2$. The formula is only valid in the far-field of the transmission antenna, when the distance d is larger than the Fraunhofer distance $d_\mathrm{F} = 2D^2/\lambda$ where D is the size of the transmission antenna.

We are only interested how the received power at distance d relates to the received power at a reference distance d_0. This relation is given by

$$P_r(d) = P_r(d_0) \cdot \left(\frac{d_0}{d}\right)^n . \tag{2.3}$$

Only for the transmission in the free-space, the path loss exponent is given by $n = 2$. For the transmission in areas where obstacles (e.g. buildings, trees or mountains) or the ground reflect, absorb, diffract or scatter the electromagnetic waves, often other path-loss exponents between $n = 2$ and $n = 6$ are assumed, which were determined experimentally [Rap99, Page 104] [Yac93]. For UMTS, a path-loss exponent of $n = 3.52$ is assumed [HT01, Page 153].

The ratio of the received power at distance d to the received power at distance d_0 in dB is given by $10 \cdot n \cdot \log 10(d_0/d)$. Figure 2.2 depicts the ratio for several path-loss exponents n. If a relay is at half distance between source and sink ($d/d_0 = 0.5$) and the source broadcasts to the relay and the sink, the SNR advantage of the relay versus the sink varies between 6.02 dB ($n = 2$) and 18.06 dB ($n = 6$) dependent on the assumed n.

2.2.3 Fading

In Section 2.2.2, we described that the mean signal decreases with distance d as d^{-n} where the pathloss exponent is $n = 2$ for a transmission over the free-space. Absorption, reflection, scattering and diffraction at obstacles can increase the pathloss exponent n.

Obstacles cause also other effects which can be only modeled with statistical methods which are described in [TV05, Yac93]. First, the shadowing of the receiver behind large obstacles (e.g. buildings or hills) causes large-scale fading. Large-scale fading is modeled with a lognormal distribution and changes, if the mobile station leaves or enter the shadow of an obstacle. As obstacles can have shadows up to several 100 meters or even more (e.g. mountains), the fading changes relatively slow. How fast the change occurs, depends on the velocity of the mobile station.

Second, multipath propagation through reflections causes small-scale fading. The reflected signals have a longer distance to the receiver compared to the direct signal. The

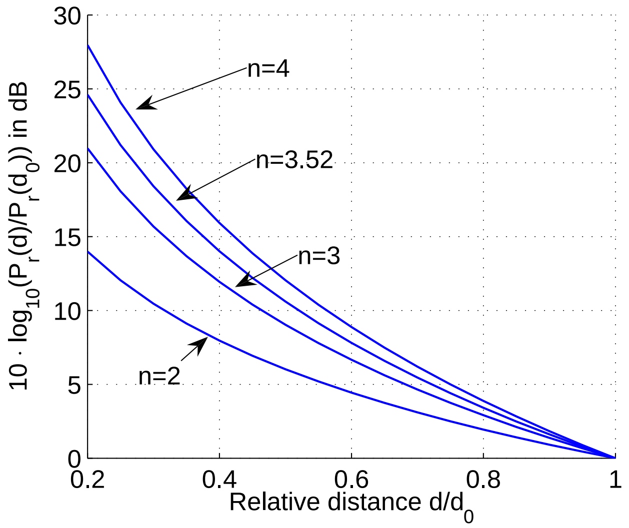


Figure 2.2: Ratio of received power at distance d to the received power at distance d_0 in dB.

reflected signals and the direct signal can superimpose constructively or nonconstructively depending on the phase difference of the signals. If the mobile station changes its position only by $\lambda/4$ (for example $\lambda/4 = 7.5$ cm for a frequency of 1 GHz), a constructive superposition can already change to a nonconstructive one. Therefore, fading caused by multipath propagation can change relatively fast. If the sink only receives reflected signals, small-scale fading is modeled with a Rayleigh distribution. If there is a line-of-sight connection between source and sink and the sink receives both reflected signals and the direct signal, small-scale fading is modeled with a Ricean distribution.
The rapidity of the fading is described by the coherence time T_c of the channel. Whereas most diversity schemes (e.g. delayed retransmission from source, multiple antennas at transmitter or receiver) can only gain diversity against small-scale fading, relays allow both to gain diversity against small-scale fading and shadowing.

2.2.4 Channel Model

For the design and evaluation of channel codes a simplified time-discrete channel model which takes into account additive white Gaussian noise and the propagation path-loss is sufficient. We assume that the source modulates the real and the imaginary part of the waveform from the lowpass spectrum to the bandpass spectrum around the carrier frequency f_c with $\cos(2\pi f_c t)$ and $-\sin(2\pi f_c t)$, respectively. The sink demodulates the bandpass signal with $2 \cdot \cos(2\pi f_c t)$ and $-2 \cdot \sin(2\pi f_c t)$. Then, the useful signal without noise in the lowpass spectrum is identical at source and sink. Whereas the noise in the bandpass bandwidth W has a power spectral density $N_0/2$, the equivalent lowpass noise has a power spectral density N_0 in the frequency range $|f| \leq W/2$ and zero otherwise. If diversity is relevant for the design and evaluation, we assume a simple model with small-scale fading according to a Rayleigh distribution. The coherence time of the fading

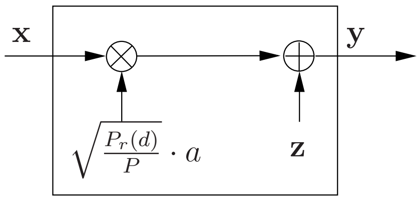


Figure 2.3: Channel model with fading a and noise samples $\mathbf{z}$. The receiver power at the sink $P_r(d)$ depends on the distance d between source and sink.

is assumed long enough such that the fading does not change for the transmission of one block with M transmitted symbols. Diversity is relevant, if the sink receives several transmissions with unequal fading.

The elements of transmitted symbols $\mathbf{x}$ are normalized such that the mean square of the amplitude is $\sqrt{P}$. The received block after the matched filter at the receiver of the sink with distance d to the source is given by

$$\mathbf{y} = \sqrt{\frac{P_r(d)}{P}} \cdot a \cdot \mathbf{x} + \mathbf{z} = h \cdot \mathbf{x} + \mathbf{z}, \tag{2.4}$$

where $P_r(d)$ is the received power at the sink, W is the bandpass bandwidth used by the system and the channel coefficient h is given by $h = \sqrt{P_r(d)/P} \cdot a$. The elements z_i of the noise vector $\mathbf{z}$ are Gaussian distributed[1] with variance $N_0 \cdot W$ (variance $N_0 \cdot W/2$ in each of the two dimensions of a complex number):

$$z_i = \hat{z} + j \cdot \hat{\hat{z}} \quad \text{with} \quad \hat{z}, \hat{\hat{z}} \sim \mathcal{N}\left(0, N_0 \cdot W/2\right)$$

The fading coefficient a is generated according to

$$a = \hat{a} + j \cdot \hat{\hat{a}} \quad \text{with} \quad \hat{a}, \hat{\hat{a}} \sim \mathcal{N}\left(0, 0.5\right).$$

The absolute value $|a|$ of the fading coefficient is Rayleigh distributed. Figure 2.3 depicts a block diagram of the channel model. We assume that the coherence time T_c is short enough such that the fading coefficient of one transmission with M symbols is not correlated with the one of the previous transmission. We only consider fading, when diversity is relevant for the coding method. Otherwise, we set a to $a = 1$.

The instantaneous signal-to-noise ratio (SNR)

$$\gamma = |a|^2 \cdot \frac{P_r(d)}{N_0 \cdot W} = \frac{|h|^2 \cdot P}{N_0 \cdot W} \tag{2.5}$$

is defined as the ratio of the received power and the noise power. The average SNR is given by

$$\rho = \frac{P_r(d)}{N_0 \cdot W} \tag{2.6}$$

[1] A variable z is Gaussian distributed with mean μ and variance σ^2, when its probability density function (PDF) is given by $p(z) = \frac{1}{\sqrt{2\pi \cdot \sigma^2}} \exp\left(\frac{-(z-\mu)^2}{2 \cdot \sigma^2}\right)$. Such a PDF is denoted as $\mathcal{N}(\mu, \sigma^2)$.

with $\gamma = |a|^2 \cdot \rho$. A bandpass bandwidth W (equivalent lowpass bandwidth $W/2$) allows a minimum symbol duration $T_s = 1/W$. That means that it is possible to send $T/T_s = T \cdot W$ complex symbols during time T. If we express the average received power $P_r(d) = E_s/T_s$ in terms of the received symbol energy E_s and the symbol duration $T_s = 1/W$, we find that the average SNR ρ can be formulated as $\rho = E_s/N_0$. A detailed explanation of wireless communication models can be found in [Pro95].

2.3 Information Theory - Limits for Channel Codes

The aim of channel coding is to provide a reliable communication with as little redundancy as possible. A small amount of redundancy means to use a high code rate $R = K/M$. Reliable communication means that an arbitrary small probability of decoding error can be achieved. Information theory provides a limit for the rate R and defines the capacity C for a channel. The channel capacity is defined as the maximum achievable rate. It is not possible for any coding scheme to enable a reliable communication with a rate R larger than C. The derivation of the capacity C is based on random codes with a very large block length.
For a complex valued AWGN channel with SNR γ as described in the last section, the capacity C in bits per symbol is given by [Pro95]

$$\boxed{C = \mathcal{C}(\gamma) = \log_2(1+\gamma)} \tag{2.7}$$

under the assumption of a Gaussian distributed channel input variable. The achievable data rate in bits per time unit depends on the number of symbols we can send per time unit. A bandpass bandwidth W allows to send $T/T_s = T \cdot W$ complex symbols during time T.
We know from (2.7) that the required SNR γ_0 for a reliable communication with rate R_0 is given by

$$\gamma_0 = 2^{R_0} - 1. \tag{2.8}$$

Although a Gaussian distributed channel input could achieve the largest capacity, in practical channel encoded and modulated systems, the channel input variable will normally not follow a Gaussian distribution, but will be a variable from a discrete alphabet, such as 4-QAM or 16-QAM. The channel capacity $C = \mathcal{C}_k$ for a discrete input alphabet $\mathcal{S}_k$ can be calculated numerically [Pro95] as

$$\mathcal{C}_k(\gamma) = \max_{P(x)} \sum_{x_i \in \mathcal{S}_k} \int_{-\infty}^{+\infty} p(y|x_i) P(x_i) \log_2 \frac{p(y|x_i)}{p(y)} \mathrm{d}y \tag{2.9}$$

where the SNR γ influences the probability density function $p(y|x_i)$. Figure 2.4 depicts the channel capacities $\mathcal{C}(\gamma)$ and $\mathcal{C}_k(\gamma)$ in bits per symbol for different channel input variables x_i (Gaussian distributed, BPSK, 4-QAM, 16-QAM or 64-QAM) dependent on the SNR γ.
Information theory and the concept of channel coding were introduced in 1948 [Sha48]. We refer to [CT91] for more information about information theory. Although it was shown

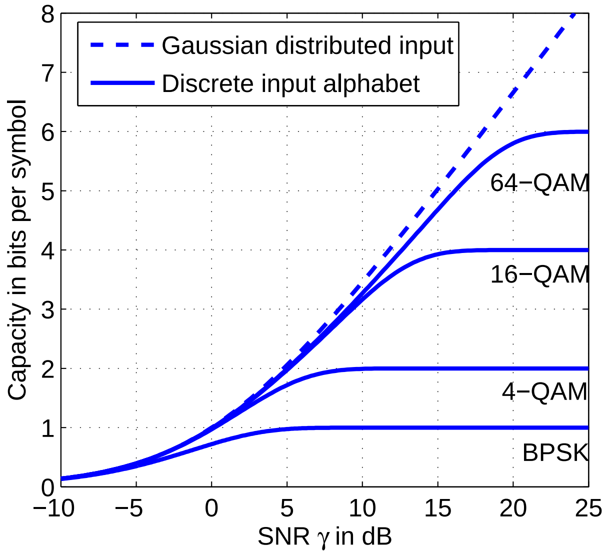


Figure 2.4: Channel capacities in bits per symbol for different channel input variables x_i (Gaussian distributed, BPSK, 4-QAM, 16-QAM or 64-QAM).

in [Sha48] that the capacity in (2.7) is achievable with very long, randomly chosen codes, it was not clear how to realize a capacity-achieving code with efficient encoding and decoding. A binary random code without structure with packet length K requires a list with 2^K entries specifying how to encode a packet to a codeword. Such lists require too much memory for large block lengths. For example, the storage of all codewords for $K = 100$ information bits and $N = 200$ code bits would require $2^{100} \cdot 200/(8 \cdot 10^9)$ Gigabytes $= 3.17 \cdot 10^{22}$ Gigabytes. Moreover, a maximum-likelihood decoding is not computationally feasible for such codes. In the next section, we will describe types of channel code that allow efficient encoding and decoding due to their specific structure.

2.4 Some Types of Channel Codes for Wireless Communication

In this section, we describe some types of channel codes for wireless communications which are relevant for this thesis. We only consider binary and linear codes. Then, the channel code design can be seen as the problem how to choose the generator matrix $\mathbf{G}$ in (2.1). Beside the minimization of the bit or packet error rate at the output of the channel decoder for a given rate R or the maximization of the rate R for a given tolerable error rate, the engineering problem is also to design a channel encoder and decoder such that the cost of implementing the encoder and decoder falls within acceptable limits. Moreover, it is often important that the block length of the packet is not too large to allow the receiver to decode with a tolerable delay.
We consider a system according to Figure 2.1. The received signal $\mathbf{y}$ at the data sink is first processed by the demodulator. The demodulator does not make a hard decision on

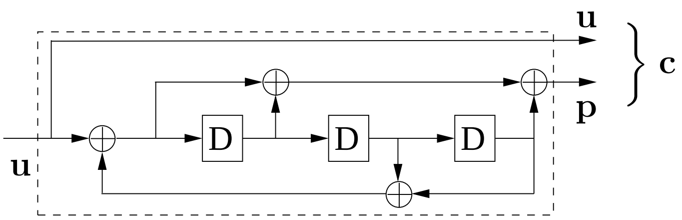


Figure 2.5: Example for a convolutional encoder with memory $m = 3$. This convolutional encoder is systematic and recursive.

a symbol, but delivers a soft decision

$$\mathbf{r} = (r_1, r_2, \ldots, r_N)$$

to the decoder. A soft decision includes information how reliable the decision is. We always assume that a soft demodulation is used. The soft decision contains a log-likelihood ratio (LLR) $r_i = L(c_i|y_j)$ for each of the code bits c_i in $\mathbf{c}$ conditioned on one channel output y_j. The channel output y_j is the disturbed version of the symbol x_j carrying the code bit c_i. The conditioned LLR $L(c|y)$ is defined as

$$L(c|y) = \ln \frac{P(c = 1|y)}{P(c = 0|y)}. \tag{2.10}$$

The framework for the LLR algebra is explained in [HOP96]. For example, a LLR of $r_i = 0$ corresponds to $P(c_i = 1|y_j) = 0.5$, a LLR of $r_i = 1$ to $P(c_i = 1|y_j) = 0.73$ and a LLR of $r_i = 5$ to $P(c_i = 1|y_j) = 0.99$. For efficient wireless communication it is very important to forward soft decisions instead of hard decisions from the demodulator to the channel decoder.

2.4.1 Convolutional Codes

Convolutional codes were introduced in 1955. They are treated in detail in [LC04] and references therein. Convolutional codes are highly structured and allow a simple implementation and a good performance for short block lengths. Nevertheless, they are still far away from reaching the capacity limit.
The encoder for a convolutional code contains memory for m bits. The encoder output bits at any given time unit depend linearly on the input bits at that time and on the m memory bits. Figure 2.5 depicts the block diagram for a convolutional encoder with memory $m = 3$, with one input and two outputs. The block with the D represent a memory element for one bit. The memory elements forward the bits with a delay of one time unit. The operator $\oplus$ represents a modulo-2 addition. The code bits $\mathbf{c}$ contain the information bits $\mathbf{u}$ and additional parity bits $\mathbf{p}$. The output $\mathbf{c}$ of the convolutional code can be interpreted as a discrete convolution of the input sequence $\mathbf{u}$ and the impulse responses of the convolutional code. The structure of the generator matrix $\mathbf{G}$ of a convolutional code can be found in [LC04]. It is helpful for a reliable decoding to input m tail bits after the sequence $\mathbf{u}$ into the encoder such that all memory elements are forced back to the

zero-state. The code rate R_c of a convolutional code with one input and two outputs and with m tail bits is given by $R_c = K/(2 \cdot K + 2 \cdot m)$. The channel code whose encoder is depicted in Figure 2.5 is systematic and recursive. The code is systematic because the information bits $\mathbf{u}$ are included in the code bits $\mathbf{c}$. The code is recursive because the current memory bits are fed back to calculate the new memory bits. As we will only use in this thesis the convolutional code in Figure 2.5 with feedforward generator 15 and feedback generator 13 (both in octal), we refer to [LC04] for a more general description of convolutional codes.

There are many possibilities how to decode a convolutional code. We will concentrate on soft-input soft-output (SISO) decoders. An important SISO decoder is the BCJR algorithm [BCJR74]. It outputs the a posteriori LLRs $L(u_k|\mathbf{r})$ about the information bits and takes into account the LLRs for all code bits $\mathbf{r}$, a priori information about the information bits and exploits the code structure and redundancy included by the channel encoder. The decoder can make a hard decision $\hat{u}_k$ for each information bit u_k dependent on the sign of $L(u_k|\mathbf{r})$. The BCJR algorithm maximizes the a posteriori probability that an information bit is correctly decoded (MAP decoder). The coding structure of convolutional codes allows that the MAP decoder is computationally feasible and that the computational complexity only grows linearly with the block length. That means the decoding complexity per code bit is independent of the code length. The complexity of the BCJR algorithm grows exponentially with the memory m of the convolutional code. There are suboptimal versions of the BCJR algorithm whose complexity is reduced and whose decoding performance is only slightly degraded. Important suboptimal versions are the soft-output Viterbi algorithm (SOVA) [HH89] and the LOG-MAP algorithm according to [RVH95]. We use the LOG-MAP algorithm according to [RVH95] with a correction term from a look-up table for simulation results in this thesis. An overview about several suboptimal MAP decoders is given in [LC04] and in [VS01].

Convolutional codes are used in many practical systems [CHIW98]. GSM uses a convolutional code with memory $m = 4$. DVB [Eur04] uses a concatenation of a Reed-Solomon code and a convolutional code with memory $m = 6$. Convolutional codes are also used for deep-space communications.

The output of a convolutional code can be punctured. Then, certain bits from the output of the channel encoder $\mathbf{c}$ are periodically deleted. The resulting code has a higher rate R_c than the unpunctured mother code. By varying the puncturing rule, a family of punctured codes can be obtained from the mother code whose codes rates can vary between the rate of the mother code and $R_c = 1$. A family of punctured codes fulfills the rate-compatibility restriction, if all the code bits of a high rate punctured code are used by the lower rate codes. Rate-compatible punctured convolutional codes were proposed in [Hag88]. Rate-compatible punctured codes are helpful to realize hybrid ARQ/FEC systems (see Section 2.5). It is required that the sink knows the position of the punctured bits. The demodulator at the sink has no information whether the punctured bits are '0' or '1' and includes values of zero in $\mathbf{r}$ for the punctured bits before the channel decoding.

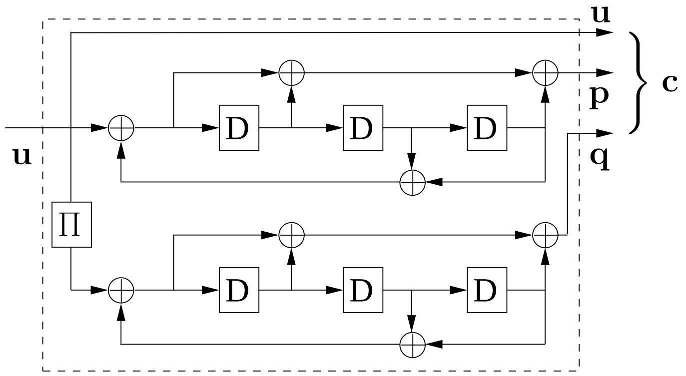


Figure 2.6: Example for a PCCC encoder.

2.4.2 Parallel Concatenated Convolutional Codes - Turbo Codes

Parallel concatenated convolutional codes (PCCC) were proposed in [BGT93] in 1993. These codes provided an astonishing performance of 0.5 dB from the capacity limit of an AWGN channel constrained to binary input at a code rate $R_{\mathrm{c}} = 0.5$ and with a bit error rate of 10^{-5}. This exceeded the performance of previously known codes by around two decibels.

Figure 2.6 depicts the encoder of an example for a PCCC. A PCCC consist of two systematic and recursive convolutional codes which are parallely concatenated. The first convolutional code outputs the information bits $\mathbf{u}$ and the parity bits $\mathbf{p}$. The second convolutional code processes an interleaved version $\Pi(\mathbf{u})$ of $\mathbf{u}$. The block with Π represents the interleaver. The second code only outputs parity bits $\mathbf{q}$ and no information bits. In the example in Figure 2.6, we used the convolutional code from Figure 2.5. The code bits $\mathbf{c}$ contain the information bits $\mathbf{u}$ and the parity bits $\mathbf{p}$ and $\mathbf{q}$. Again, it is helpful to input m tail bits after the sequence $\mathbf{u}$ into each of the two convolutional encoder such that all memory elements are forced back to the zero-state. The code rate R_{c} of a PCCC with memory-m convolutional codes is given by $R_{\mathrm{c}} = K/(3 \cdot K + 4 \cdot m)$. The interleaver is an important part of the code. Its aim is to reorder the bits in a pseudorandom manner. PCCCs are linear codes.

The PCCC was termed turbo code in [BGT93], because the proposed decoding method reuses soft information in analogy to the reuse of the exhaust gas in turbo engines. Figure 2.7 depicts a diagram of the turbo decoder. The vectors $\mathbf{r}_u$, $\mathbf{r}_p$ and $\mathbf{r}_q$ are subsets of the soft information $\mathbf{r}$ from the demodulation which correspond to $\mathbf{u}$, $\mathbf{p}$ and $\mathbf{q}$, respectively. The turbo decoder contains one SISO convolutional decoder for each of the two convolutional codes in the turbo encoder. Turbo decoding starts by processing the first SISO decoder. The first SISO decoder calculates a posteriori LLRs $L^{-}(\hat{\mathbf{u}}) = L(u_k|\mathbf{r}_u, \mathbf{r}_p)$ about all information bits and takes into account the LLRs for the code bits of the first convolutional encoder $\mathbf{r}_u$ and $\mathbf{r}_p$. Initially, no a priori information $L_{\mathrm{e}}^{|}(\hat{\mathbf{u}})$ is available Next, the second SISO decoder is processed. The second SISO decoder obtains LLRs for the parity bits of the second convolutional code $\mathbf{r}_q$ from the demodulation and a priori information about the information bits $L_{\mathrm{e}}^{-}(\hat{\mathbf{u}})$ in the interleaved version from the first SISO decoder. It is

(a)

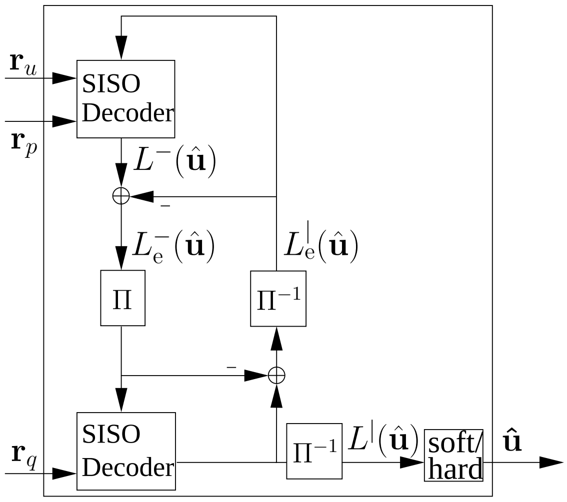


Figure 2.7: PCCC decoder.

important that the a priori information from the first SISO decoder contains only extrinsic information $L_{\mathrm{e}}^{-}(\hat{\mathbf{u}}) = L^{-}(\hat{\mathbf{u}}) - L_{\mathrm{e}}^{|}(\hat{\mathbf{u}})$. That means that only information is forwarded to the next decoding block which was newly obtained in the last decoding block. The second SISO decoder calculates a posteriori LLRs $L^{|}(\hat{\mathbf{u}})$ about the information bits and forwards extrinsic information $L_{\mathrm{e}}^{|}(\hat{\mathbf{u}}) = L^{|}(\hat{\mathbf{u}}) - L_{\mathrm{e}}^{-}(\hat{\mathbf{u}})$ to the first SISO decoder. Then, the first SISO decoder can be processed again whereas the new a priori information $L_{\mathrm{e}}^{|}(\hat{\mathbf{u}})$ has to be considered. Both SISO decoders can be processed several times. Each time the decoding result can improve due to the new a priori information from the other SISO decoder. After several iterations of the procedure, the first or the second SISO decoder can output a hard decision $\hat{\mathbf{u}}$ made from the a posteriori LLRs $L^{-}(\hat{\mathbf{u}})$ or $L^{|}(\hat{\mathbf{u}})$. The second SISO decoder has to deinterleave the LLRs. The deinterleaver is represented in Figure 2.7 by Π^{-1}. Turbo decoding is explained in more detail in [BGT93, HOP96, VS01, LC04]. Iterative decoding schemes were also proposed in other ways. A unified framework for iterative decoding and an overview of related proposals is presented in [HOP96].

Due to the impressive performance, turbo codes gained a lot of interest and are used in current communication systems. For example, UMTS uses the code depicted in Figure 2.6. The UMTS code uses a pseudorandom interleaver which can be constructed easily with an algebraic algorithm [Eur00]. A method to evaluate an upper bound to the bit error probability of turbo codes was proposed in [BM96b]. A comparison of the error rate between uncoded communication, convolutional codes and turbo codes can be found in [LC04]. The design of turbo codes was considered in [BM96a].

An intuitive explanation for the usefulness of turbo codes is that the code obtains a good trade-off between structure and randomness [LC04]. The derivation of the information-theoretic capacity is based on random codes. Therefore, it seems reasonable that randomness in a code helps to achieve a performance close to the information-theoretic limit. The turbo code includes randomness due to the pseudorandom interleaver. On the other side, a code needs structure to allow a feasible decoding also for very long blocklengths. Convolutional codes are very structured. As the turbo code contains convolutional codes

beside the interleaver, the close-to-optimal iterative decoding method is feasible.
PCCCs can be punctured in a similar way as convolutional codes. The rate-compatible puncturing of PCCCs was considered in [RM00].

2.5 Hybrid ARQ/FEC with Cross-Packet Channel Coding

2.5.1 FEC versus ARQ

It is also possible to use automatic repeat request (ARQ) [LC04] for error correction instead of forward error correction (FEC). An FEC system contains as main component an error-correcting code with the purpose to correct errors at the sink. Contrary, the main components of an ARQ system are an error-detection code and a feedback channel from the sink to the source. The error-detection decoder at the sink has to decide whether the packet with the information bits was received correctly or not. If the error-detection decoder decides that the packet was not received correctly, the sink instructs the source with one bit through the feedback channel to retransmit the same packet. The retransmissions can continue until the error-detection decoder decides that the packet was received correctly. Error-detection can be realized with very high reliability with a relatively small number of parity-check digits.
If a feedback channel is not available or if the delay requirements do not allow to wait for a retransmission, FEC is the only choice to correct communication errors. For example, FEC is suitable for the communication of real-time applications such as voice, which is sensitive to delay and can tolerate a certain amount of communication errors.
If a feedback channel is available and the delay requirements allow retransmissions, both FEC and ARQ can be used. The main disadvantage of an ARQ scheme is that many retransmissions are necessary for channels with low SNR and thus, the data throughput falls rapidly with decreasing SNR. The main disadvantage of an FEC scheme is that it is hard to achieve the same system reliability as with ARQ, because the probability that the decoder can correct errors is always lower than the probability that the decoder can detect errors. For example, data traffic requires very low error rates. For a wireless communication system with time-varying fading and SNR, it is very difficult to achieve efficiently such low error rates with an FEC scheme.

2.5.2 Conventional Hybrid ARQ/FEC

Hybrid ARQ/FEC (H-ARQ) allows to combine the advantages of both ARQ and FEC [LC04]. At the source, the error-detection encoder first attaches a small number of parity-check bits (error-detection bits) to the packet with the information bits. Then, the packet $\mathbf{u}$ (including the error-detection bits) is encoded with the FEC encoder. At the sink after FEC decoding, the parity-check bits allow the error-detection decoder to decide whether to request through the feedback channel a retransmission. Contrary to a pure ARQ system, the FEC code decreases the error probability after one transmission. ARQ strategies

can be combined with FEC to yield type I or type II H-ARQ systems.
If the data cannot be decoded correctly with H-ARQ schemes of type I, the receiver discards the first transmission and requests a new transmission.
For H-ARQ schemes of type II, when the first transmission cannot be decoded without error and a retransmission is required, the receiver combines the values of the first transmission with the received values of the second transmission, which can contain additional (incremental) redundancy. H-ARQ of type II can be easily realized with rate-compatible punctured channel codes (see Section 2.4). The error rate after the second transmission is lower than the one after the first transmission because of the following two reasons:

- Firstly, the transmission of incremental redundancy results in a lower code rate and in a better error protection.
- Secondly, if the coherence time is short enough such that each transmission experiences another fading coefficient, additional diversity can be gained with each retransmission.

Figure 2.8 depicts a diagram of the transmission system with H-ARQ (type II) for two packets $\mathbf{u}_1$ and $\mathbf{u}_2$, containing K_1 and K_2 bits, respectively. The channel encoder processes the first packet of information bits $\mathbf{u}_1$ and outputs a block of N code bits $\mathbf{c}_1$. The H-ARQ functionality selects a punctured version $\mathbf{c}_{1,1}$ with N_1 code bits from the codeword $\mathbf{c}_1$. The modulator (mod.) maps the punctured codeword $\mathbf{c}_{1,1}$ to the block of $M_1 = N_1/L$ symbols $\mathbf{x}_{11}$. The first transmission from the source to the sink contains $\mathbf{x}_{11}$. The rate after the first transmission is given by $R = K_1/M_1$. The demodulator (dem.) processes $\mathbf{y}_{11}$, which is a disturbed version of $\mathbf{x}_{11}$, and outputs LLRs $\mathbf{r}_{1,1}$. The parts in $\mathbf{r}_1$ which correspond to non-punctured bit of $\mathbf{c}_1$ are filled with the LLRs in $\mathbf{r}_{1,1}$. The parts in $\mathbf{r}_1$ which correspond to punctured bits of $\mathbf{c}_1$ are filled with the log-likelihood value of '0'. The channel decoder outputs an estimate $\hat{\mathbf{u}}_1$ based on the input $\mathbf{r}_1$. If the sink cannot successfully decode the codeword, a retransmission $\mathbf{x}_{12}$ from the source is necessary. The error detection can be done with a cyclic redundancy check (CRC), which has to be attached to the packet $\mathbf{u}_1$ before encoding. The retransmission $\mathbf{x}_{12}$ with $M_2 = N_2/L$ symbols carries another subset $\mathbf{c}_{1,2}$ with N_2 code bits from $\mathbf{c}_1$. The rate after the second transmission is given by $R = K_1/(M_1 + M_2)$. The sink stores the state of the LLRs $\mathbf{r}_1$ after the first transmission and updates $\mathbf{r}_1$ after the demodulation of $\mathbf{y}_{12}$ to $\mathbf{r}_{1,2}$, which is a disturbed version of $\mathbf{x}_{12}$. Using the updated $\mathbf{r}_1$, the channel decoder outputs again an estimate $\hat{\mathbf{u}}_1$. If the channel encoder is linear, the two outputs $\mathbf{c}_{1,1}$ and $\mathbf{c}_{1,2}$ of the conventional H-ARQ encoding system (channel encoder and H-ARQ functionality) can be expressed as

$$\mathbf{c}_{1,1} = \mathbf{u}_1 \cdot \mathbf{G}_{11} \quad \text{and} \quad \mathbf{c}_{1,2} = \mathbf{u}_1 \cdot \mathbf{G}_{12}. \tag{2.11}$$

Figure 2.10 depicts the largest coherence time T_c which still allows a diversity gain from the retransmission. It is possible to extend the described system and to allow more than one retransmission.
The next packet of information bits $\mathbf{u}_2$ is treated separately from the first packet $\mathbf{u}_1$.

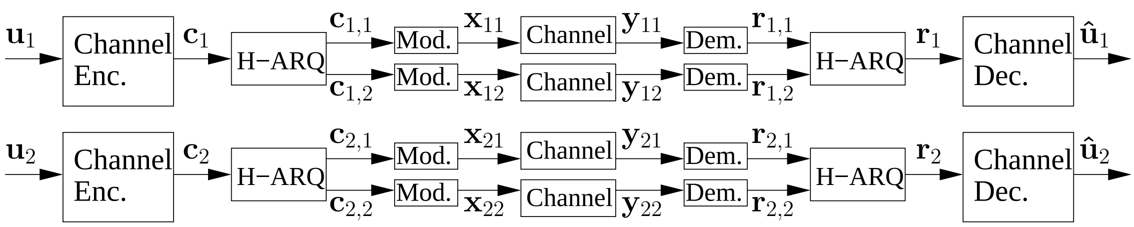


Figure 2.8: Transmission of two packets $\mathbf{u}_1$ and $\mathbf{u}_2$ with conventional H-ARQ.

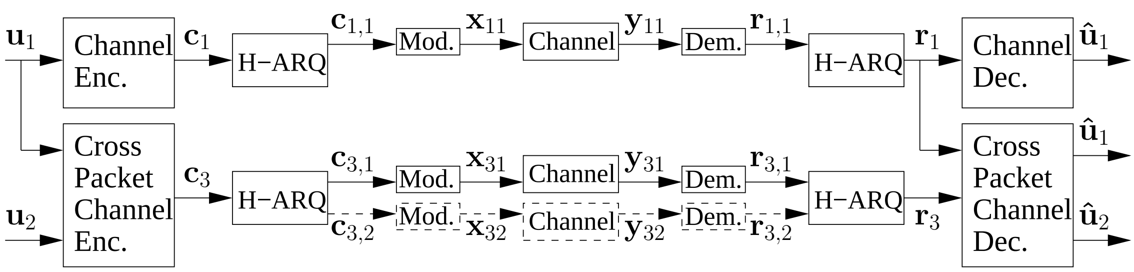


Figure 2.9: Transmission of two packets $\mathbf{u}_1$ and $\mathbf{u}_2$ with H-ARQ with cross-packet channel coding.

2.5.3 Extension with Cross-Packet Channel Coding

H-ARQ with cross-packet channel coding [HC07] extends conventional H-ARQ schemes of type II. The essential part of the idea is that if a retransmission is required for the first packet $\mathbf{u}_1$, the communication of the first and of the second packet $\mathbf{u}_1$ and $\mathbf{u}_2$ is considered jointly. The retransmission should allow the sink to decode both packets $\mathbf{u}_1$ and $\mathbf{u}_2$. Let us consider the case where the two packets $\mathbf{u}_1$ and $\mathbf{u}_2$ have the same number of information bits and where the first transmission and the retransmission have the same number of symbols to explain the motivation for cross-packet channel coding. Contrary to the conventional H-ARQ scheme, the total code rate does not decrease with cross-packet coding after the second transmission because both the total number of information bits and the total number of transmitted symbols is doubled. A constant total code rate means that the spectral efficiency does not decrease with the retransmission. However, the error rate of the first packet still decreases due to the diversity gain from the second transmission. More generally, cross-packet coding allows to gain diversity from retransmissions with a higher code rate than conventional H-ARQ systems.
In this section, we will design a cross-packet channel code in Section 2.5.4. Moreover, we will show the gain from cross-packet channel coding analytically in Section 2.5.5 and with numerical simulations results in Section 2.5.6. Our numerical results will show that the drawback for the second packet from cross-packet channel coding is negligible.

Figure 2.9 depicts a diagram of the transmission system with H-ARQ and cross-packet coding. The first transmission $\mathbf{x}_{11}$ is the same as in the system without cross-packet coding. If the sink requires a retransmission, we allow $\mathbf{u}_1$ to be retransmitted jointly with the initial transmission of $\mathbf{u}_2$ using cross-packet coding. The cross-packet channel encoder

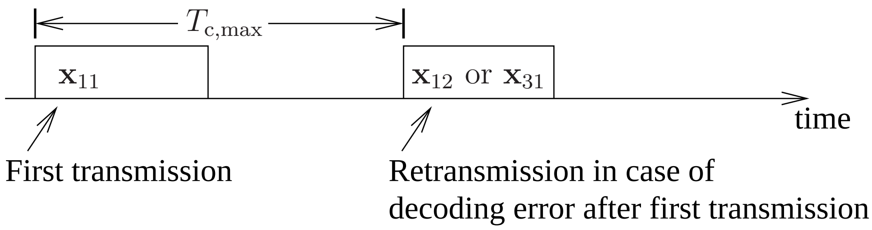


Figure 2.10: Illustration of first transmission $\mathbf{x}_{11}$ and retransmission $\mathbf{x}_{12}$ (no CPC) or $\mathbf{x}_{31}$ (CPC): We also depict the largest coherence time $T_{c,\max}$ which still allows a diversity gain from the retransmission.

outputs the codeword $\mathbf{c}_3$ which is based on the inputs $\mathbf{u}_1$ and $\mathbf{u}_2$. The output of a binary and linear cross-packet channel encoder is given by

$$\mathbf{c}_3 = \mathbf{u}_1 \cdot \mathbf{G}_1 \oplus \mathbf{u}_2 \cdot \mathbf{G}_2, \tag{2.12}$$

where $\oplus$ depicts a modulo-2 addition. The H-ARQ functionality punctures $\mathbf{c}_3$ and outputs a block of N_2 bits $\mathbf{c}_{3,1}$, which are modulated to $\mathbf{x}_{31}$. The block of $M_2 = N_2/L$ symbols $\mathbf{x}_{31}$ is sent in the second transmission to the sink. If the encoders are linear, the two outputs $\mathbf{c}_{1,1}$ and $\mathbf{c}_{3,1}$ of the H-ARQ encoding system (channel encoder and H-ARQ functionality) with cross-packet coding can be expressed as

$$\mathbf{c}_{1,1} = \mathbf{u}_1 \cdot \mathbf{G}_{11} \quad \text{and} \quad \mathbf{c}_{3,1} = \mathbf{u}_1 \cdot \mathbf{G}_{12} \oplus \mathbf{u}_2 \cdot \mathbf{G}_{21}. \tag{2.13}$$

Contrary to the outputs of the conventional H-ARQ system in (2.11), the second output includes also the second packet $\mathbf{u}_2$. The demodulator at the sink processes the channel output $\mathbf{y}_{31}$ to LLRs $\mathbf{r}_{3,1}$. The H-ARQ functionality transforms $\mathbf{r}_{3,1}$ to LLRs $\mathbf{r}_3$. The LLRs $\mathbf{r}_3$ correspond to the codeword $\mathbf{c}_3$. The cross-packet channel decoder delivers estimates $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ about both packets of information bits based on $\mathbf{r}_1$ and $\mathbf{r}_3$.
By varying K_2, we can scale the level of cross-packet channel coding. For $K_2 = 0$, the system falls back to the system without cross-packet channel coding. Then, the first packet $\mathbf{u}_1$ is transmitted most reliable and the code rate is $R = K_1/(M_1 + M_2)$. By increasing K_2, the code rate increases to $R = (K_1 + K_2)/(M_1 + M_2)$ and thus, the spectral efficiency increases as well. The level of cross-packet channel coding is described by the parameter $\sigma = K_2/K_1$.
It is also possible to provide more retransmissions, for example a third transmission $\mathbf{x}_{32}$ with M_3 symbols which carry N_3 additional code bits. The proposed system could be also extended by allowing to cross-packet code more than two packets. We do not consider the cross-packet coding of more than two packets in this thesis.

The proposed system could also be adapted for systems without ARQ where feedback with requests for retransmissions is not necessary. Then, cross-packet coding would be used for every transmission. Such a code is related to unit-memory convolutional codes [Lee76, TJ83].

2.5.4 Code Design: Iterative Cross-Packet Channel Decoding

In this section, we propose a specific code design for the cross-packet channel encoder and decoder. We only consider binary and linear codes. Then, the cross-packet channel code design can be seen as the problem how to choose the generator matrices $\mathbf{G}_1$ and $\mathbf{G}_2$ in (2.12). The cross-packet channel decoder for our code design is based on the iterative exchange of soft information. Although we will only consider turbo codes, the proposed cross-packet code design can be realized with any channel code which allows a soft-input soft-output decoding. Cross-packet coding based on LDPC codes is explained in [CC07]. Our design objective for the cross-packet channel encoder is that the bits of the first and the second packet are mixed properly such that the cross-packet channel decoder is able to decode the two packets jointly as one code.

Cross-Packet Channel Encoder

Figure 2.11 (a) depicts a block diagram of the proposed cross-packet channel encoder. If K_1 and K_2 are equal, the information bits of an interleaved version of packet $\mathbf{u}_1$ and the bits in the packet $\mathbf{u}_2$ appear alternately at the input of a channel encoder. If K_1 and K_2 are not equal, we can easily extend the method by writing the interleaved bits row-by-row into a matrix with $K_{\min} = \min\{K_1, K_2\}$ columns and reading the bits out column-by-column as input for the channel encoder. We first write all the bits from the smaller packet and then all the bits from the larger packet into the matrix. This permutation rule is called periodic block interleaver in [CC81]. It is not necessary that the last row is filled completely. If K_1 and K_2 are equal, the described periodic block interleaver falls back to the method of taking the interleaved bits alternately. The input of the channel encoder is termed $\mathbf{u}_3$. The packet $\mathbf{u}_3$ is channel encoded to obtain the output $\mathbf{c}_3$.
The interleaver Π in the cross-packet encoder shown in Figure 2.11 (a) is determined according to the UMTS standard [Eur00, VS01].
The H-ARQ functionality after both the channel encoder and the cross-packet channel encoder uses regular puncturing schemes, similar to the one described in [RM00], to choose for each transmission N_1 or N_2 out of all code bits. The only difference after cross-packet encoding is that we always puncture the systematic bits of $\mathbf{c}_3$ that correspond to $\mathbf{u}_1$ because they are already included in the first transmission $\mathbf{x}_{11}$.

Cross-Packet Channel Decoder

Figure 2.11 (b) depicts a block diagram of the cross-packet channel decoder. It delivers the estimates $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ based on the demodulator outputs $\mathbf{r}_1$ and $\mathbf{r}_3$.
The cross-packet decoder contains two SISO decoders. The upper SISO decoder corresponds to the channel encoder used for encoding of $\mathbf{u}_1$. The lower SISO decoder corresponds to the channel encoder within the cross-packet encoder. The two SISO decoder exchange iteratively soft information similar as in a conventional turbo decoder as explained in Section 2.4.2. First, the upper SISO decoder calculates a posteriori LLRs

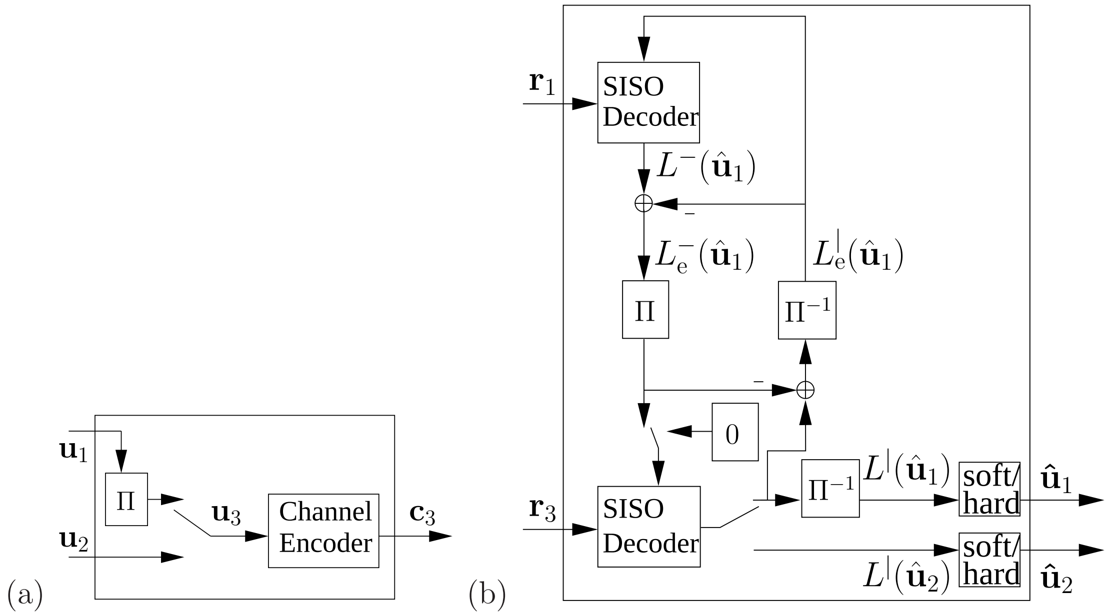


Figure 2.11: (**a**): Cross-packet channel encoder. **b**): Cross-packet channel decoder.

$L^{-}(\hat{\mathbf{u}}_1)$ from its input $\mathbf{r}_1$. Initially, no a priori information $L_{\mathrm{e}}^{|}(\hat{\mathbf{u}}_1)$ about $\mathbf{u}_1$ is available ($L_{\mathrm{e}}^{|}(\hat{\mathbf{u}}_1) = \mathbf{0}$). Then, the lower SISO decoder calculates a posteriori LLRs $L^{|}(\hat{\mathbf{u}}_3)$ based on $\mathbf{r}_3$ and on a priori information about $\mathbf{u}_1$ from the upper SISO decoder. The extrinsic information $L_{\mathrm{e}}^{-}(\hat{\mathbf{u}}_1) = L^{-}(\hat{\mathbf{u}}_1) - L_{\mathrm{e}}^{|}(\hat{\mathbf{u}}_1)$ has to be interleaved and mixed with zeros in order to obtain a priori information about $\mathbf{u}_3$. The output of the lower SISO decoder $L^{|}(\hat{\mathbf{u}}_3)$ is split into LLRs $L^{|}(\hat{\mathbf{u}}_1)$ and $L^{|}(\hat{\mathbf{u}}_2)$ about $\mathbf{u}_1$ and $\mathbf{u}_2$. We can calculate extrinsic information $L_{\mathrm{e}}^{|}(\hat{\mathbf{u}}_1) = L^{|}(\hat{\mathbf{u}}_1) - L_{\mathrm{e}}^{-}(\hat{\mathbf{u}}_1)$ about $\mathbf{u}_1$ which can be exploited as a priori information by the upper SISO decoder for the next decoding round. It is possible to apply the turbo principle [Hag97] and iteratively exchange soft information about $\mathbf{u}_1$ between the two decoders several times. The lower decoder outputs hard estimates $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ after several iterations.

2.5.5 Outage Behavior

The capability of systems to gain diversity in a fading environment can be analyzed in terms of outage probabilities [OSW94]. We consider the outage behavior for both the system with and without cross-packet channel coding after the first and after the second transmissions. We assume the block fading channel model as described in Section 2.2. The fading coefficient of the first transmission is termed a_1. The fading coefficient of the second transmission is termed a_2. The average SNR is given by ρ for all transmissions. The instantaneous SNR is given by $\gamma_1 = |a_1|^2 \cdot \rho$ and by $\gamma_2 = |a_2|^2 \cdot \rho$ for the first and the second transmission, respectively.
We will define the outage event OUT for all situations separately. It depends on the fading coefficients, respectively the instantaneous SNRs, whether an outage occurs. The outage event reflects the situation that a reliable communication is not possible. The definitions

of the outage event are based on information-theoretic results and are not dependent on the code design. The event $\overline{\text{OUT}}$ is defined as the complement of the event OUT.

First Transmission

The first transmission of M_1 symbols to communicate the packet with K_1 information bits is identical for both systems. Given the instantaneous SNR $\gamma_1 = |a_1|^2 \cdot \rho$, the capacity C in bits per complex channel use is given by $C = \mathcal{C}(\gamma_1) = \log_2(1 + \gamma_1)$ under the assumption of a Gaussian distributed channel input variable (compare (2.7)).
The receiver can decode reliably the first packet with K_1 transmission bits, if the instantaneous SNR γ_1 has a value such that the following inequality holds:

$$K_1 \leq M_1 \cdot \mathcal{C}(\gamma_1) \tag{2.14}$$

Therefore, we define the event $\overline{\text{OUT}}$ after one transmission as

$$\boxed{\left[K_1 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right)\right].} \tag{2.15}$$

Second Transmission without Cross-Packet Coding

If we use no cross-packet channel coding, the transmitter sends M_2 more symbols to help the receiver to decode the K_1 information bits. The receiver can decode reliably the first packet with K_1 transmission bits, if the instantaneous SNRs γ_1 and γ_2 have values such that the following inequality holds [Che03] [TV05, (5.85)]:

$$K_1 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right) + M_2 \cdot \mathcal{C}\left(\gamma_2\right) \tag{2.16}$$

We define the event $\overline{\text{OUT}}$ after two transmissions without cross-packet coding as

$$\boxed{\left[K_1 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right) + M_2 \cdot \mathcal{C}\left(\gamma_2\right)\right].} \tag{2.17}$$

Second Transmission with Cross-Packet Coding

If we use cross-packet channel coding, the transmitter sends M_2 new symbols to help the receiver to decode the K_1 information bits and K_2 new information bits. The receiver can decode reliably both the first packet with K_1 transmission bits and the second packet with K_2 information bits, if the instantaneous SNRs γ_1 and γ_2 have values such that the following inequalities hold:

$$K_1 + K_2 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right) + M_2 \cdot \mathcal{C}\left(\gamma_2\right) \tag{2.18}$$
$$K_2 \leq M_2 \cdot \mathcal{C}\left(\gamma_2\right) \tag{2.19}$$

The derivation of (2.18) and (2.19) is given in the Appendix A.1. For $K_2 = 0$, the condition for reliable decoding with cross-packet coding falls back to the one without cross-packet coding in (2.16).

We want to consider the outage behavior of the first packet and the of the second packet separately. The first packet with K_1 information bits can be decoded reliably either if the first transmission was successful or if the second transmission with cross-packet coding was successful. That means either the Condition (2.14) or both the Conditions (2.18) and (2.19) have to be fulfilled. The event $\overline{\mathrm{OUT}}$ for the first packet after two transmissions with cross-packet coding is defined as

$$\boxed{\left[K_1 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right)\right] \vee \Big(\left[K_1 + K_2 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right) + M_2 \cdot \mathcal{C}\left(\gamma_2\right)\right] \wedge \left[K_2 \leq M_2 \cdot \mathcal{C}\left(\gamma_2\right)\right]\Big).} \tag{2.20}$$

The second packet with K_2 information bits can be decoded reliably if the second transmission with cross-packet coding was successful. That means both of the Conditions (2.18) and (2.19) have to be fulfilled. The event $\overline{\mathrm{OUT}}$ for the second packet after two transmissions with cross-packet coding is defined as

$$\boxed{\left[K_1 + K_2 \leq M_1 \cdot \mathcal{C}\left(\gamma_1\right) + M_2 \cdot \mathcal{C}\left(\gamma_2\right)\right] \wedge \left[K_2 \leq M_2 \cdot \mathcal{C}\left(\gamma_2\right)\right].} \tag{2.21}$$

The outage probabilities under the constraint that we use coded modulation with a discrete modulation alphabet $\mathcal{S}_k$ are obtained, when we replace $\mathcal{C}$ by $\mathcal{C}_k$ as defined in (2.9).

Comparison for Coded Modulation

We want to compare the conventional H-ARQ system and the H-ARQ system with cross-packet channel coding regarding the ability to gain a diversity order of two for the first packet after two transmissions. We assume a system with coded modulation where each symbol carries L code bits ($\mathcal{C}(\gamma \rightarrow \infty) = L$). A system has a diversity order of two, if it can tolerate that one of the two transmissions is in a very deep fade. That means that no outage occurs for either $\gamma_1 \rightarrow 0$ or $\gamma_2 \rightarrow 0$. We want to know the maximum code rate $R_\mathrm{c} = R/L$ which allows a diversity order of two.

Let us first consider the conventional H-ARQ system. If the first transmission is in a very deep fade ($\gamma_1 \rightarrow 0$), we can observe from the outage definition in (2.17) that it is necessary that

$$K_1 \leq M_2 \cdot \mathcal{C}(\gamma_2) \leq M_2 \cdot L \tag{2.22}$$

is fulfilled to allow a diversity order of two. Otherwise (2.17) can never be fulfilled for $\gamma_1 \rightarrow 0$. If the second transmission is in a very deep fade ($\gamma_2 \rightarrow 0$), we can observe from the outage definition in (2.17) that it is necessary that

$$K_1 \leq M_1 \cdot \mathcal{C}(\gamma_1) \leq M_1 \cdot L \tag{2.23}$$

is fulfilled to allow a diversity order of two. From (2.22) and (2.23) we conclude that $M = M_1 + M_2$ has to fulfill $M \geq 2 \cdot K_1/L$. Therefore, we know that the code rate

$R_\mathrm{c} = R/L = K_1/(M \cdot L)$ has to fulfill the following condition such that the conventional H-ARQ system allows a diversity order of two:

$$\boxed{R_\mathrm{c} \leq 1/2} \tag{2.24}$$

That means it is impossible for the conventional H-ARQ system to obtain a diversity order of two for $R_\mathrm{c} > 1/2$.

Let us consider the H-ARQ system with cross-packet coding. We only consider the first packet. If the first transmission is in a very deep fade ($\gamma_1 \to 0$), we can observe from the outage definition in (2.20) that it is necessary that

$$K_1 + K_2 \leq M_2 \cdot \mathcal{C}(\gamma_2) \leq M_2 \cdot L \tag{2.25}$$

and

$$K_2 \leq M_2 \cdot \mathcal{C}(\gamma_2) \leq M_2 \cdot L \tag{2.26}$$

is fulfilled to allow a diversity order of two. As $K_2 \leq K_1 + K_2$, the fulfillment of (2.25) involves the fulfillment of (2.26) and thus, we do not have to consider (2.26). If the second transmission is in a very deep fade ($\gamma_2 \to 0$), we can observe from the outage definition in (2.20) that it is necessary that

$$K_1 \leq M_1 \cdot \mathcal{C}(\gamma_1) \leq M_1 \cdot L \tag{2.27}$$

is fulfilled to allow a diversity order of two. From (2.25) and (2.27) we conclude that $M = M_1 + M_2$ has to fulfill $M \geq (2 \cdot K_1 + K_2)/L$ what leads to

$$1 \geq \frac{2 \cdot K_1 + K_2}{M \cdot L} = \frac{K_1 + K_2}{M \cdot L} \cdot \frac{2 \cdot K_1 + K_2}{K_1 + K_2} = R_\mathrm{c} \cdot \frac{2+\sigma}{1+\sigma}$$

with $R_\mathrm{c} = R/L = (K_1 + K_2)/(M \cdot L)$ and $\sigma = K_2/K_1$. Therefore, we know that the code rate R_c has to fulfill the following condition such that the H-ARQ system with cross-packet coding allows a diversity order of two for the first packet:

$$\boxed{R_\mathrm{c} \leq \frac{1+\sigma}{2+\sigma}} \tag{2.28}$$

By comparing the conditions in (2.28) and (2.24), we conclude that the system with cross-packet coding allows a diversity order of two for larger rates R_c than the conventional H-ARQ system whose rate is limited to $R_\mathrm{c} \leq 1/2$. For example, the rate of the system with cross-packet coding is limited to $R_\mathrm{c} \leq 2/3$, if the first and the second packet have the same length ($\sigma = 1$). If the second packet contains no information bits ($\sigma = 0$), (2.28) falls back to (2.24).

2.5.6 Simulation Results

Simulation Setup and Parameters

We compare the proposed system with cross-packet channel coding to the reference system with conventional H-ARQ. We assume the block fading channel model as described in Section 2.2. The fading coefficient of the first transmission is termed a_1. The fading coefficient of the second transmission is termed a_2. The average SNR is given by ρ for all transmissions. The instantaneous SNR is given by $\gamma_1 = |a_1|^2 \cdot \rho$ and by $\gamma_2 = |a_2|^2 \cdot \rho$ for the first and for the second transmission, respectively.
We simulate the code design described in Section 2.5.4 and measure the bit error rate (BER) and the packet error rate (PER) for BPSK ($L = 1$) after the first transmission and after one retransmission. We choose the channel encoder which encodes $\mathbf{u}_1$ and the channel encoder in the cross-packet channel encoder which encodes $\mathbf{u}_3$ to be the parallel concatenated convolutional code (PCCC) which is used in UMTS [Eur00]. This code including the required interleaver is described in [VS01] and its encoder is depicted in Figure 2.6. The PCCC channel decoders use four iterations and the cross-packet channel decoder uses also four iterations.
We also evaluate the outage probabilities according to the outage definitions in Section 2.5.5 for BPSK modulation by generating samples of the random fading coefficients and counting the number of outages. The packet error rate corresponds to the outage rate. The outage rate can be seen as information-theoretic benchmark for the coding system. The outage rate can be generated with much less computational effort than the bit and packet error rate (by a factor of more than 100000 with our computer system).
The information bits are grouped in packets of $K_1 = K_2 = 1500$ bits and the first transmission contains $M_1 = 1750$ BPSK symbols. That means that the rate after the first transmission is given by $R_\mathrm{c} = K_1/M_1 = 6/7$.

Simulation Results with Rate $R_\mathrm{c} = 4/7$ after Second Transmission

First, we consider the case that the code rate after the second transmission (first retransmission) is given by $R_\mathrm{c} = (K_1 + K_2)/(M_1 + M_2) = 4/7$. Then, the system with cross-packet coding with $K_1 = K_2$ is allowed to retransmit $M_2 = 3500$ BPSK symbols whereas the conventional system with $K_2 = 0$ is allowed to retransmit $M_2 = 875$ BPSK symbols. As pointed out in the comparison in Section 2.5.5, we expect the system with cross-packet coding to outperform the conventional system because the conditions for a diversity order of two in (2.25) and (2.27) are fulfilled for cross-packet coding whereas the corresponding conditions in (2.22) and (2.23) for the conventional system are not fulfilled.

Figure 2.12 depicts the BER, PER and outage rate after the first and the second transmission. The conventional H-ARQ system achieves a lower error rate after the second transmission compared to the performance after the first transmission due to the decreased code rate R_c. Therefore, the offset of the error rate curves for $R_\mathrm{c} = 4/7$

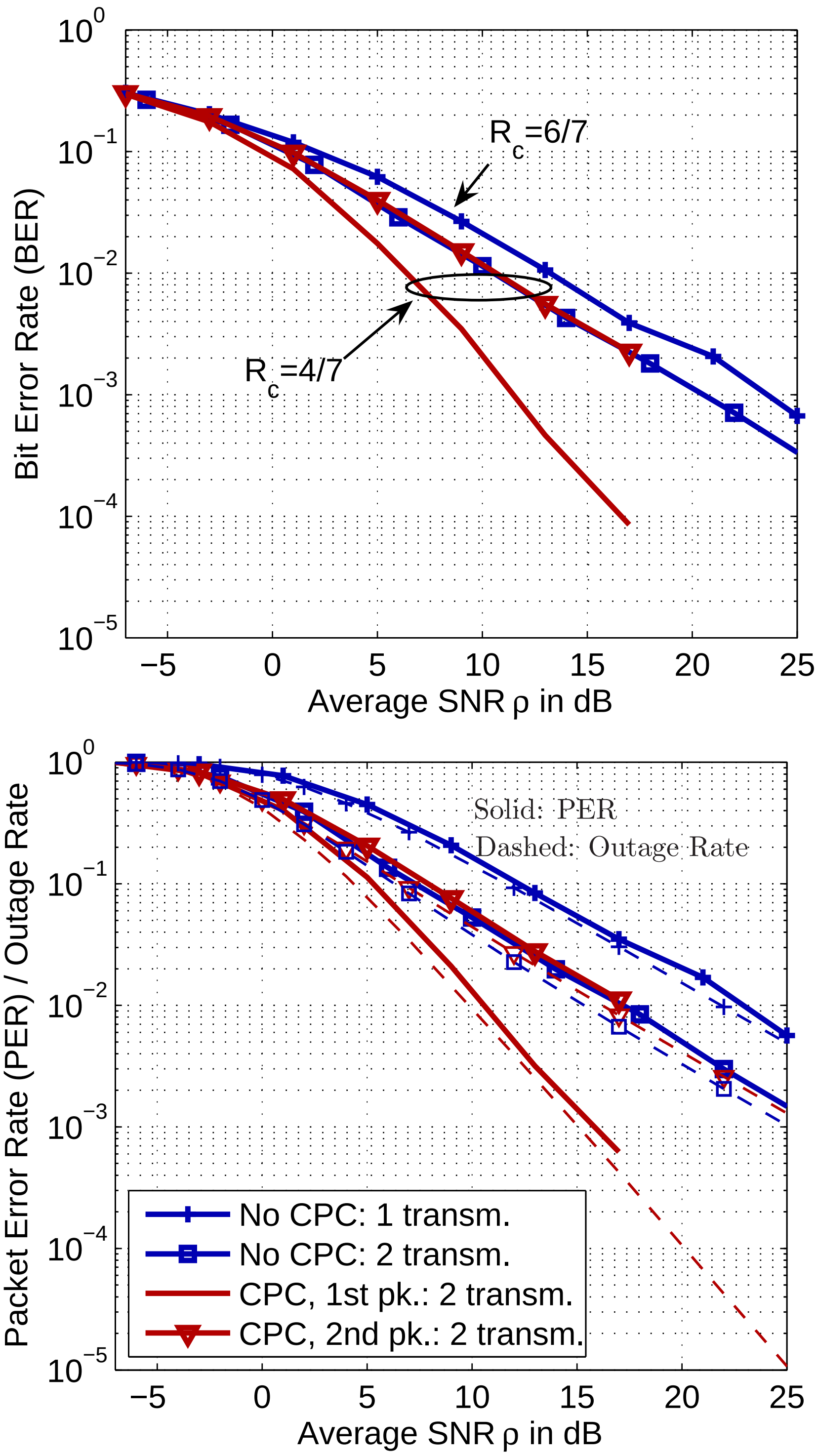


Figure 2.12: Bit error rate (BER) and packet error rate (PER) for the first and the second packet (pk.) $\mathbf{u}_1$ and $\mathbf{u}_2$ for H-ARQ with cross-packet channel coding (CPC) after the first and the second transmission (transm.) with $K_1 = K_2 = 1500$ bits, $M_1 = 1750$ and $M_2 = 3500$ BPSK symbols. The error rates of the reference system without CPC are also depicted for the same rate with $K_1 = 1500$ and $K_2 = 0$ bits, $M_1 = 1750$ and $M_2 = 875$ BPSK symbols.

is shifted to smaller SNRs compared to the curves for $R_c = 6/7$. As expected, the conventional system does not gain diversity from the second transmission. This can be observed from the constant slope of the curves after the first and the second transmission. It is not possible for the conventional system to compensate a very deep fade during the first transmission because the $M_2 = 875$ BPSK symbols of the second transmission are not enough symbols to transport the $K_1 = 1500$ information bits to the sink.
Contrary, the system with cross-packet coding allows to gain diversity for the first packet. This can be observed from the steeper slope of the error rate curves after the second transmission. The $M_2 = 3500$ BPSK symbols of the second transmission can transport the $K_1 + K_2 = 3000$ information bits to the sink even if a very deep fade occurred during the first transmission. As the second packet is carried as well, the code rate is not lower compared to the reference system. Beside the diversity gain, the decreased code rate can also be efficiently exploited with cross-packet coding. This can be observed from the shifted offset of the error rate curves. There is no benefit or drawback for the second packet from cross-packet coding. Its decoding performance is almost not affected by cross-packet coding.
The gap between the outage rates and the corresponding packet error rates is very small. That means the outage behavior can be useful as benchmark for the coding system and that the proposed code design performs close to the optimum.

As expected, the system with cross-packet coding can gain diversity for $R_c > 1/2$ compared to the conventional H-ARQ system.

Simulation Results with Rate $R_c = 3/7$ after Second Transmission

Next, we consider the case that the code rate after the second transmission (first retransmission) is given by $R_c = (K_1 + K_2)/(M_1 + M_2) = 3/7$. Then, the system with cross-packet coding with $K_1 = K_2$ is allowed to retransmit $M_2 = 5250$ BPSK symbols whereas the conventional system with $K_2 = 0$ is allowed to retransmit $M_2 = 1750$ BPSK symbols. As pointed out in the comparison in Section 2.5.5, we expect both the system with cross-packet coding and the conventional system to gain a diversity order of two because the conditions in (2.25) and (2.27) are fulfilled for cross-packet coding and the corresponding conditions in (2.22) and (2.23) for the conventional system are also fulfilled.

Figure 2.13 depicts the BER, PER and outage rate after the first and the second transmission. The conventional H-ARQ system achieves a lower error rate after the second transmission compared to the performance after the first transmission due to the decreased code rate R_c and due to an increased diversity order. Beside the shifted offset of the error rate curves due to the decrease of the code rate to $R_c = 3/7$, there is also a steeper slope of the curves after the second transmission. Contrary to $M_2 = 875$, it is possible for the conventional system to compensate a very deep fade during the first transmission because the $M_2 = 1750$ BPSK symbols of the second transmission can transport the $K_1 = 1500$ information bits to the sink.

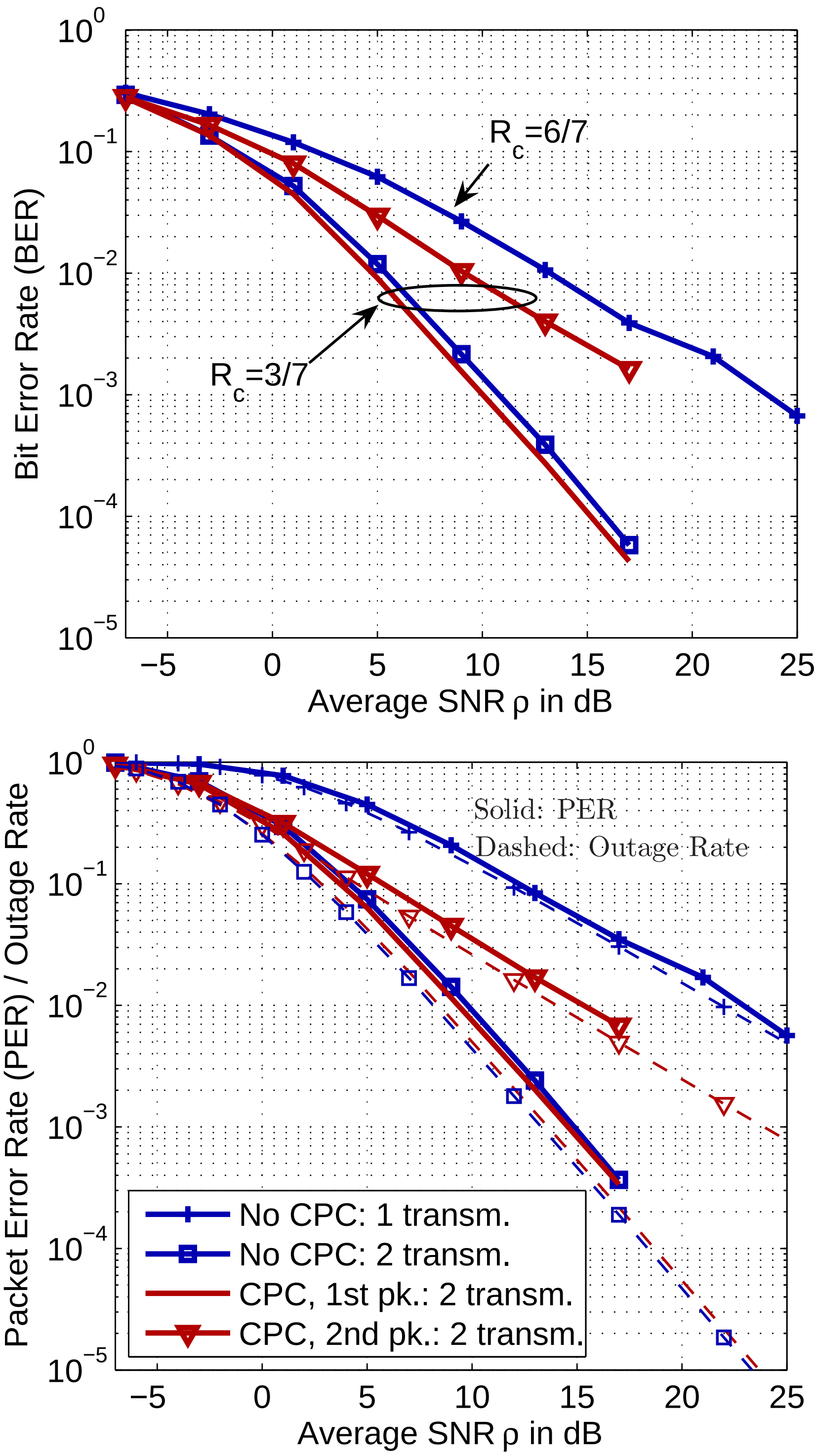


Figure 2.13: Bit error rate (BER) and packet error rate (PER) for the first and the second packet (pk.) $\mathbf{u}_1$ and $\mathbf{u}_2$ for H-ARQ with cross-packet channel coding (CPC) after the first and the second transmission (transm.) with $K_1 = K_2 = 1500$ bits, $M_1 = 1750$ and $M_2 = 5250$ BPSK symbols. The error rates of the reference system without CPC are also depicted for the same rate with $K_1 = 1500$ and $K_2 = 0$ bits, $M_1 = 1750$ and $M_2 = 1750$ BPSK symbols.

The system with cross-packet coding achieves almost the same performance for the first packet than the conventional system. As the decoder of the system with cross-packet coding works with a longer code length than the conventional system, the performance is slightly better. As the second packet is only contained in the second transmission, there is no diversity gain for the second packet. There is no benefit or drawback for the second packet from cross-packet coding compared to a conventional coding system after one transmission.
Again, the gap between the outage rates and the corresponding packet error rates is very small.

As expected, the system with cross-packet coding and the conventional H-ARQ system both can gain diversity and achieve the same performance for $R_\mathrm{c} \leq 1/2$.

2.6 Summary

We explained the assumed wireless channel model and background knowledge about information theory and about important types of channel codes for wireless communication, which is required for the next chapters.
Section 2.5 described H-ARQ with cross-packet channel coding, which extends current H-ARQ schemes for point-to-point communications. In contrast to current H-ARQ schemes, the transmission of two consecutive packets of information bits is considered jointly. If a retransmission for the first packet is necessary, we process both packets jointly. This should allow that both packets can be decoded without errors at the receiver after the retransmission. We considered the outage behavior and proposed a specific code design for cross-packet coding which can be decoded efficiently in an iterative way. Simulation results confirmed our analytical conclusions that the proposed cross-packet coding design allows to gain diversity from retransmissions with higher code rates than conventional H-ARQ systems. Contrary to conventional H-ARQ systems, it is possible to gain a diversity order of two for a code rate $R_\mathrm{c} > 1/2$ with cross-packet coding.

3

Network Coding for Error-Free Point-to-Point Networks

In the next chapters, we will design network coding jointly with channel coding to improve the error correction in wireless relay network. In this chapter, we will give a short introduction to network coding. Network coding generalizes routing and was initially proposed to increase the throughput in networks with error-free point-to-point links. Wireline networks are modeled normally as point-to-point networks. Contrary, models for wireless networks often include broadcast transmissions and multiple-access models.

3.1 From Routing to Network Coding

Network coding [ACLY00] is a concept that allows to increase the throughput in networks. The basic idea is that intermediate nodes in a network are allowed not only to route but also to combine incoming data from different nodes with coding operations. The purpose of network coding and routing is different than channel coding. According to the OSI model[1], channel coding is part of the physical layer whereas routing/network coding is located in the network layer. Channel coding is performed in the physical layer to protect the communication over point-to-point links against transmission errors. The physical layer is the lowest layer and has to provide an error-free network to the upper layer. Network coding and routing is performed in the network layer with the aim to transfer information efficiently through the (error-free) network. Layered architecture models as the OSI model allow to handle the complexity of large communication networks. Each layer fulfills its specific tasks and delivers a more abstract and simpler model of the

[1]OSI model: Open Systems Interconnection (OSI) basic reference model

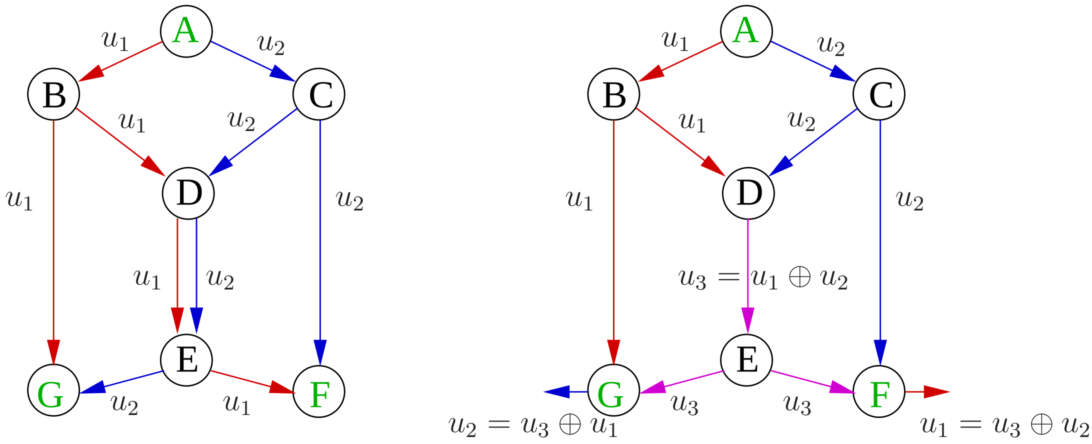


Figure 3.1: Example for the superiority of network coding over routing from [ACLY00].

network to its upper layer. This allows to split the communication network design into several easier design problems which are easier to solve. Nevertheless a layered design approach can be in general suboptimal to a cross-layer design approach, where several layers are designed jointly.

In error-free point-to-point networks, network coding can be advantageous to routing, if a communication scenario is considered, which exceeds the case with one source and one sink. The canonical example for the superiority of network coding over routing is the "butterfly network" [ACLY00]. Figure 3.1 depicts the example. Source A wants to multicast two bits u_1 and u_2 to sink G and sink F. Each of the error-free point-to-point links in the network can transmit one bit per time unit. Figure 3.1 (left) depicts the solution with routing to the problem. The link from node D to node E acts as bottleneck and has to be used twice. The other links have to be used once.
Figure 3.1 (right) depicts the solution with network coding. Node D applies network coding and performs a modulo-2 addition of the two incoming nodes ($u_3 = u_1 \oplus u_2$). The bit u_3 is forwarded to both sinks via node E. The sinks receive one of the bits u_1 and u_2 uncoded and can recover the other bit with the help of u_3 with another modulo-2 addition ($u_2 = u_1 \oplus u_3$, $u_1 = u_2 \oplus u_3$). Contrary to the routing solution, all links have to be used once and one channel use is saved through network coding.
The authors derived in [ACLY00] the maximum information flow which can be multicasted from one source to several sinks in an error-free point-to-point network. This result is summarized in the next section.

3.2 Information-Theoretic Capacity of Networks

It has been known for more than 50 years that the maximum information flow in a network with one source and one sink is limited by the weakest set of links which cut the source from the sink completely. This cut can be interpreted as the bottleneck for the information flow. The max-flow min-cut theorem provides a simple but powerful

rule, which can be applied to any network. Determining maximum flows is a linear programming problem and can be solved in a computationally efficient way also for large networks [AMO93]. Routing the information through the networks allows to achieve the maximum flow.
Contrary to unicast (one source transfers information to one sink), it was not clear how to determine the maximum information flow for multicast (one source transfers identical information to several sinks) for a long time. It was shown in [ACLY00], that the max-flow min-cut interpretation is also valid for the maximum information flow for multicast. In general, it is necessary to use network coding to achieve the maximum multicast flow. Let us explain this in more detail.

A unicast denotes the communication from one source to one sink in a network. It was shown in [FF56, DF56, EFS56] that the unicast capacity in a point-to-point network has a max-flow min-cut interpretation. The unicast capacity can be achieved with routing [EFS56]. Let us shortly explain how to calculate the unicast capacity for a given network. We consider a network with V nodes and E links. Each link connects two nodes. One node is a source and one node is a sink. We are interested at what rate we can send information from the source to the sink through the network. The network is represented as the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. $\mathcal{V} = \{1, 2, \cdots, V\}$ is the set of the V vertices in the graph which correspond to the V nodes in the network. $\mathcal{E} = \{(u_1, v_1), (u_2, v_2), \cdots, (u_E, v_E)\}$ is the set of all directed edges which corresponds to the E links in the network. The edge (u, v) connects the vertices u and v where u is the tail and v is the head of the directed edge. The source node is denoted as vertex 1, the sink node is denoted as vertex V. Each edge (u, v) has a certain capacity C_{uv} which gives the rate at which information can be communicated over the link from vertex u to v.
The capacity between the source and the sink can be calculated according to a max-flow min-cut interpretation. That means that the maximum possible flow of information through the network (max-flow) is constrained by the edges with the smallest sum of capacities which cut the source from the sink (min-cut). The min-cut between the source and the sink can be interpreted as a bottleneck for the flow of information. Formally, a cut between node 1 and node V is a partition of the vertex set $\mathcal{V}$ into two subsets $\mathcal{S}$ and $\bar{\mathcal{S}} = \mathcal{V} - \mathcal{S}$ such that $\mathcal{S}$ contains v and $\bar{\mathcal{S}}$ contains u. The value $F(\mathcal{S})$ of the cut describes the information flow which is possible over the cut. It is defined as the sum of the capacities of all edges which cross the cut and have their tail in $\mathcal{S}$ and their head in $\bar{\mathcal{S}}$:

$$F(\mathcal{S}) = \sum_{(u,v)\in\mathcal{E}, u\in\mathcal{S}, v\in\bar{\mathcal{S}}} C_{uv} \tag{3.1}$$

We denote the set of all possible cuts between the source node 1 and the sink node V as Γ. The capacity C of the network is given by the cut with the minimal flow value:

$$C = \min_{\mathcal{S}\in\Gamma} F(\mathcal{S}) \tag{3.2}$$

Whereas a unicast communication consists of one source and one sink, a multicast communication consists of several sinks which all want to obtain the same information. It was shown in [ACLY00] that the multicast capacity in a point-to-point network has also a max-flow min-cut interpretation. The multicast capacity C is achievable, if the min-cut between the source and each sink is larger or equal than C. In general, it is necessary to use network coding to achieve the multicast capacity.

3.3 Design of Network Codes

It was shown in [ACLY00] that network coding has the potential to multicast more efficiently than routing. Motivated by the results in [ACLY00], research groups considered the problem how to design network codes.
The design approaches can be classified into the following two directions:

- Deterministic design of network codes
- Randomized network codes

3.3.1 Deterministic Design of Network Codes

The first approach assumes knowledge about the complete network. Based on this knowledge a deterministic network code is designed and each node is assigned a specific coding operation.
It was shown in [LYC03] that linear network coding with finite alphabet size is sufficient to achieve the multicast capacity. In [KM03], an elegant algebraic characterization of linear network coding schemes and a polynomial time algorithm to verify a given linear coding scheme was given. In [JSC$^+$05], a polynomial time algorithm for multicast network code construction was given based on [LYC03] and [KM03].

3.3.2 Randomized Network Codes

The second approach is motivated by the fact that very large networks (e.g. the internet) change permanently and that an exact knowledge of the complete current network is not available. In [HKM$^+$03, HMK$^+$06] and references therein, a randomized network coding strategy was presented. The idea is that the information at the source is split into several blocks which are transferred into the network. All nodes other than the receiver nodes perform random linear coding operations of the available blocks. The operations are chosen independently at each node. The transmission from a node has to include as overhead the information which blocks were coded at the node. The advantage of the randomized coding strategy is that no central knowledge or planning of the network and the network code is necessary. Moreover, the randomized approach is robust to changing network conditions.
It was shown in [HMK$^+$06], that random linear codes achieve the multicast capacity asymptotically for large code lengths.

The authors of [GR05] proposed a scheme for content distribution over peer-to-peer networks which is based on random network coding. The system was called Avalanche in [GR05]. Simulation results in [GR05] for network models with several hundred nodes showed that network coding outperforms content distribution systems with coding only at the source (e.g. a modified fountain coding approach [BCMR04]) or systems without any coding (e.g. the currently used BitTorrent system [Coh03]). According to the results in [GR05], network coding can improve the throughput by $20-30\%$ compared to coding only at the source and by 2-3 times compared to the system without coding. Moreover, it was observed that the system is more robust to server and node departures with network coding.

3.4 Network Coding and Channel Coding

In the previous part of this chapter, we described the capacity gain through network coding for multicast in error-free point-to-point networks. In this section, we consider point-to-point networks with erroneous channels. In order to protect the information against errors, channel coding has to be considered. We describe the following two possibilities how to combine network coding and channel coding: Separated or joint network coding and channel coding. These two approaches resemble the concepts of joint or separated source coding and channel coding (see [Hag95] and references therein).

3.4.1 Separated Network Coding and Channel Coding

Separated network coding and channel coding means that channel coding is used in the physical layer to protect the point-to-point links from errors and network coding is used in the network layer to transfer the information efficiently through the error-free network. This layered approach simplifies the complexity of the system, because the system design is split into the following two tasks: channel coding for a point-to-point communication and network coding for the error-free point-to-point network. Researchers and engineers can concentrate on one of these two problems. In order to illustrate separated network-channel coding we consider a node with two incoming links which performs network encoding such as node D in the network in Figure 3.1. We assume that the links are erroneous. Figure 3.2 depicts channel coding and network coding at node D for this case. First, the two incoming links are channel decoded in the physical layer to obtain the information packets $\mathbf{u}_1$ and $\mathbf{u}_2$. The physical layer delivers $\mathbf{u}_1$ and $\mathbf{u}_2$ to the network layer. Then, the network encoder in the network layer outputs $\mathbf{u}_3 = \mathbf{u}_1 \oplus \mathbf{u}_2$. Then, the packet $\mathbf{u}_3$ is delivered to the physical layer and channel encoded to the packet of code bits $\mathbf{c}_3$. Assuming a linear channel encoder with generator matrix $\mathbf{G}$, the output of the network encoder can be expressed as

$$\mathbf{c}_3 = (\mathbf{u}_1 \oplus \mathbf{u}_2) \cdot \mathbf{G}. \tag{3.3}$$

A node which performs network decoding such as node G in the network in Figure 3.1 works according to the same principle. First, the incoming links are channel decoded in the physical layer to obtain $\mathbf{u}_1$ and $\mathbf{u}_3$. Then, $\mathbf{u}_2 = \mathbf{u}_1 \oplus \mathbf{u}_3$ is obtained by the network decoder in the network layer.

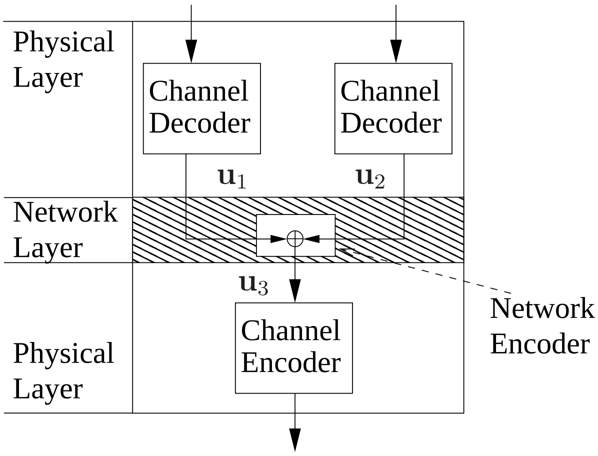


Figure 3.2: Network encoding at a node for a system with separated network coding and channel coding.

It was shown in [SYC06] that the capacity of a point-to-point network with independent point-to-point links (wireline networks normally fulfill these conditions) can be achieved with separated network-channel coding.

3.4.2 Joint Network Coding and Channel Coding

Joint network coding and channel coding is a more general approach than separated network-channel coding. Instead of splitting the overall problem into two separated tasks, error protection and network information transfer are considered jointly. Instead of guaranteeing the error-free transmission for each point-to-point link, we are only interested to guarantee error-free decoding at the nodes[2]. A node has to decode the data using the input from all incoming links. If a node has more than one incoming link, error-free decoding at the sink can be possible even if error-free decoding of the point-to-point links is not possible. Joint network-channel coding is useful, if the network code contains redundancy. Analogous to joint source-channel coding where the remaining redundancy after the source encoding helps the channel code to combat noise, joint network-channel coding allows to exploit the redundancy in the network code to support the channel code for a better error protection. Joint network-channel coding requires that one channel code is distributed in the network to include several links and is not anymore performed locally for one point-to-point link. The usefulness and the design of joint network-channel codes for specific wireless relay networks is discussed in the next chapters. We will realize joint network-channel coding by passing soft information between channel decoders and the network decoder.
Whereas a separated network-channel coding approach seems sufficient for wireline networks, a joint network-channel coding approach is required for efficient information trans-

[2]We will only consider decode-and-forward strategies in this thesis where the intermediate nodes decode the data. This approach could be extended to other approaches where it is not required that the intermediate nodes decode the data without error.

fer in wireless networks with broadcast transmissions [EMH+03, RK06].
A new concept for error correction in random network coding is introduced in [KK08].

3.4.3 Network Coding with Erasure Correction

Network coding with erasure correction is a method that can be categorized between separated and joint network-channel coding. Conventional channel codes including error detection capabilities are performed locally for one point-to-point link in the physical layer. Network coding is performed in the network layer. If the error detection recognizes errors in a packet after channel decoding, the physical layer delivers to the network layer that the packet was erased. The network layer works on a network of erasure channels. It was shown in [KM03] and references therein that network coding increases the robustness in such networks compared to routing.
We will compare two methods using either joint network-channel coding or network coding with erasure correction in Chapter 6 for a small relay network with noisy fading channels. We will show that the method using joint network-channel coding benefits from delivering soft decisions from the channel decoders to the network decoder. The method using network coding with erasure correction loses performance because hard decisions are delivered from the channel decoders to the network decoder.
In this thesis, we will classify network coding with erasure correction as a separated network-channel coding method, because channel coding is still performed locally for each point-to-point link.

3.5 Summary

Network coding generalizes routing and allows to increase the throughput and the robustness in communication networks. Whereas the capacity of a point-to-point network with independent point-to-point links (wireline networks normally fulfill these conditions) can be achieved with separated network and channel coding, the separation fails optimality for wireless networks with broadcast transmissions. In the next chapters, we will show how joint network-channel codes can be designed for specific small wireless networks.

4

Wireless Networks compared to Point-to-Point Networks

In the previous chapter, we explained how network coding can improve routing in wireline point-to-point networks. Contrary to the wireline communication, the wireless medium allows to use broadcast transmissions, to optimally allocate the resources (e.g. transmission time, bandwidth or transmission power) to the transmitting nodes and to simultaneously access the wireless medium.

In this chapter, we want to give an overview over communication strategies in wireless networks. Due to the complexity of wireless network communication, we will focus in this chapter on the communication from one source to one sink with the help of one relay. The main focus of this thesis is on coding schemes for wireless networks with broadcast transmissions. We will show the advantage from exploiting the broadcast transmissions compared to simpler strategies. Moreover, we will explain why more complicated strategies do not provide much advantage under the restriction of the current technical possibilities. This overview allows to classify our work and simplifies the comparison with related work. It also allows to understand the relation between results from the literature, which seem inconsistent at a first glance. We derive how to optimally share the total available transmission time between source and relay. This will help us to design practical coding schemes for wireless relaying with broadcast transmission in the following chapters.

4.1 Properties of the Wireless Medium

In Chapter 3, we described networks with point-to-point links. Some pairs of the nodes in such networks are linked and a certain information rate can be reliably communicated over

this link. A link connects two nodes and the different links do not influence each other. If a network is wireline connected, it is from its nature a point-to-point network. Contrary, the wireless medium is shared between all nodes and it shows a broadcast nature. That means that a transmitted signal is received by all nodes which are close enough to the transmitting node[1]. The wireless broadcast nature was exploited from the beginning of electrical communication for broadcast applications, such as television and radio. For individual unicast applications, such as telephony, wireline networks were mostly used, because this is the natural medium when each user needs a separate link for its own communication. Despite its natural inappropriateness for individual communication, the wireless medium became popular for individual communication, because it allows the user to be mobile and provides more flexibility and has the potential to provide lower costs for establishing the infrastructure.
In this section, we give an overview how individual communication can be established over the wireless medium. The easiest way to allow multiple nodes to access the common wireless medium is to use time- or frequency-division multiplexing and to divide the total available time or frequency equally between all transmitting nodes. We describe the following three possibilities how the performance of a wireless network can be improved:

- First, we can optimally allocate the available resources (time, frequency, power/energy) to the transmission of the nodes. **Optimal resource allocation** can improve the performance compared to an equal division of the resources.
- Second, we can reuse resources and allow **simultaneous multiple access** (SMA) to the wireless medium for several transmitters at the same time in the same frequency band. Simultaneous multiple access[2] can be either done non-cooperatively or cooperatively.
  If there is no cooperation, the simultaneous multiple-access will decrease the signal quality because the transmitters interfere with each other. If the interference is small enough, this strategy can still improve the performance because the transmitters do not have to share the wireless medium and can use the available frequency for the total available time.
  If there is cooperation, several transmitters can synchronize their carriers in such a way that their signals superimpose constructively at the final receiver (beamforming). Cooperative simultaneous multiple-access is related to systems with multiple transmit antennas.
- Third, transmitting nodes can use **broadcast transmissions** which can be listened to by several receiving nodes.

These three principles can be applied separately or can be also combined yielding certain transmission strategies. We will take a look at the achievable rate of specific transmission

[1] In principle, all nodes receive the transmitted signal. However, nodes in a certain distance will only receive a signal far below the noise level.

[2] We use the term 'simultaneous multiple-access' in contrast to multiple-access schemes where different transmissions are divided orthogonally, for example time-division multiple access (TDMA) or frequency-division multiple access (FDMA).

strategies and give an overview over the necessary technical requirements (e.g. synchronization, half-duplex or full-duplex relay) for the different strategies.
Note that the achievable rate for a transmission strategy can be different dependent whether the time T or the bandwidth W is shared between the transmitting nodes, because contrary to the used time, the used bandwidth influences the SNR (compare (2.5)).
If we compare relaying strategies with the point-to-point communication without relay, we have to agree on constraints for the transmission power and energy for a fair comparison.
For wireless networks, several possibilities how to constrain the resources are used in the literature. Beside constraining the used bandpass bandwidth W and the used time T, we can use additionally one or several of the following constraints:

- Individual constraint on the transmitted energy E_i during the time T for each node $i \in \{1, 2, \ldots, S\}$ in the network: The individual energy constraint is reasonable for systems where the battery power at the nodes is the limiting factor. The constraint on the transmitted energy E_i during time T is equivalent to a constraint on the average power E_i/T of node i.

- Individual constraint on the transmitted maximum power P_i for each node i in the network: The individual maximum power constraint is reasonable for systems where the hardware at the transmitter limits the transmission power or where the nodes are only allowed to transmit with a certain power to minimize the risk to cause damage on the environment or on the user's health.
  Note that there is a difference between the constraints on the individual maximum power P_i and on the individual energy E_i, because the maximum transmission power P_i and the transmission time T_i of the node i influence the transmitted energy $E_i = P_i \cdot T_i$ of the node. The transmission time T_i of a node can change dependent on the used strategy. For example, if we allow simultaneous multiple-access, each node will transmit for a larger percentage of the total time T compared to a strategy with time-division.

- Constraint on the total transmitted energy $\sum_{i=1}^{S}(P_i \cdot T_i)$ of all nodes in the network during the time T: This constraint could be used for interference-limited wireless systems, because the total transmitted energy indicates the interference to systems which reuse the same frequency at the same time. Moreover, this constraint is reasonable for networks where all nodes in the network are sources.
  Contrary to the individual energy constraints, the constraint on the total transmitted energy allows to optimize the allocation of the energy to the different nodes.

The comparison of communication strategies will have different results dependent on the used constraint. The choice between the total and the individual energy constraint corresponds to the question whether the nodes in a network have to share their transmission energies with a new node or whether they are still allowed to use the same energies. For a point-to-point channel as treated in Chapter 2, the three constraints have the same meaning. We will concentrate on relaying strategies which provide a fair comparison in every aspect. That means that the individual transmission power P_i of each node i in the

network is not larger than the power P at the source of the point-to-point communication and that the total transmitted energy is not larger compared to the point-to-point communication.
The choice of the transmission strategy, the choice between time- and frequency-separation and the choice of the resource constraint result in many combinations. In the following, we will consider the achievable rate of some important strategies. We will consider the uplink from the mobile station (MS) to the base station (BS) with the help of a relay (R) as example for a small wireless network.

Although the following overview is limited to a very small network, we will not consider the following aspects:

- We only consider decode-and-forward strategies, where the relay decodes the information from the mobile station completely. We do not consider amplify-and-forward and compress-and-forward strategies [KGG05].
- If the strategies contain a relay-receive and a relay-transmit phase, the described strategies could be further improved, if the transmit-phase of the relay is split in many phases. Then, the timing of these transmit-phases can be exploited to transfer information to the sink [Kra04, Kra07, LHK08]. We only consider strategies with a fixed timing of the transmit phase where the timing is independent of the information.
- We do not consider fading channels and diversity in the overview in this chapter.

4.2 Wireless Networks as Point-to-Point Networks

In order to establish a point-to-point network for individual communication over the wireless medium, the available bandpass bandwidth W or the available time T can be shared to establish independent point-to-point links. The uplink of K_{M} information bits from the mobile station to the base station over the relay requires two orthogonal point-to-point links, one link from the mobile station to the relay and one link from the relay to the base station. The mobile station transmits the block of $M_{\mathrm{M}} = W_{\mathrm{M}} \cdot T_{\mathrm{M}}$ symbols $\mathbf{x}_{\mathrm{M}}$ with power P_{M} during the relay-receive phase and uses as resources the bandwidth W_{M}, the time T_{M} and the energy $E_{\mathrm{M}} = P_{\mathrm{M}} \cdot T_{\mathrm{M}}$. The relay transmits the block of $M_{\mathrm{R}} = W_{\mathrm{R}} \cdot T_{\mathrm{R}}$ symbols $\mathbf{x}_{\mathrm{R}}$ with power P_{R} during the relay-transmit phase and uses as resources the bandwidth W_{R}, the time T_{R} and the energy $E_{\mathrm{R}} = P_{\mathrm{R}} \cdot T_{\mathrm{R}}$. Dependent on the choice to use time or frequency division, we set T_{M}, T_{R}, W_{M} and W_{R} to

- $T_{\mathrm{M}} = \theta \cdot T$, $T_{\mathrm{R}} = (1-\theta) \cdot T$ and $W_{\mathrm{M}} = W_{\mathrm{R}} = W$ for time division.
- $W_{\mathrm{M}} = \theta \cdot W$, $W_{\mathrm{R}} = (1-\theta) \cdot W$ and $T_{\mathrm{M}} = T_{\mathrm{R}} = T$ for frequency division.

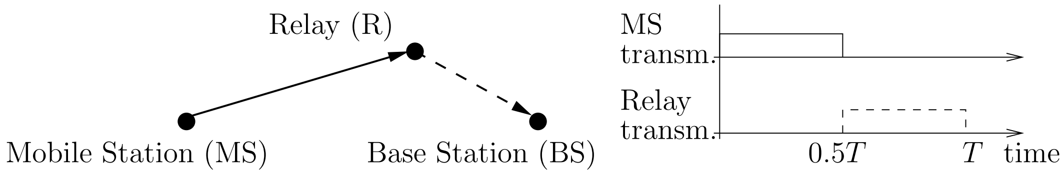


Figure 4.1: Wireless relay communication as point-to-point network with equal time-sharing: Mobile station and relay use the channel for the half of the total time T for their transmissions. For the example with the SNRs $\tilde{\gamma}_{\mathrm{MR}} = 7$ and $\tilde{\gamma}_{\mathrm{RB}} = 15$, a transmission rate of $R = 1.50$ bits per channel use can be achieved from mobile station to base station, if the individual powers are constrained.

The parameter θ determines what percentage of the total time/frequency is allocated to the mobile station ($0 \leq \theta \leq 1$). The channel model for the two point-to-point communications is given analog to the model in Section 2.2 by

$$\mathbf{y}_{\mathrm{MR}} = h_{\mathrm{MR}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MR}} \qquad \text{(Relay-receive phase)} \qquad (4.1)$$

$$\mathbf{y}_{\mathrm{RB}} = h_{\mathrm{RB}} \cdot \mathbf{x}_{\mathrm{R}} + \mathbf{z}_{\mathrm{RB}}, \qquad \text{(Relay-transmit phase)} \qquad (4.2)$$

where $\mathbf{y}_{\mathrm{MR}}$ and $\mathbf{y}_{\mathrm{RB}}$ denote the received signal at the relay and the base station, respectively. The noise and the channel coefficients are denoted as $\mathbf{z}_{\mathrm{MR}}$ and h_{MR} for the MS-R link and as $\mathbf{z}_{\mathrm{RB}}$ and h_{RB} for the R-BS link. The SNR on the MS-R link γ_{MR} is given by $\gamma_{\mathrm{MR}} = |h_{\mathrm{MR}}|^2 \cdot P_{\mathrm{M}}/(N_0 \cdot W_{\mathrm{M}})$, the SNR on the R-BS link γ_{RB} is given by $\gamma_{\mathrm{RB}} = |h_{\mathrm{RB}}|^2 \cdot P_{\mathrm{R}}/(N_0 \cdot W_{\mathrm{R}})$. Figure 4.1 illustrates the relay communication for equal time division ($\theta = 0.5$).

We want to consider the achievable rate with the relay communication. For comparison, we first consider the achievable rate R (equal to the capacity C_{MB}) for the point-to-point communication without relay as described in Chapter 2. The mobile station transmits with power $P_{\mathrm{M}} = P$ and uses the energy $E_{\mathrm{M}} = P \cdot T$. The SNR on the MS-BS link is given by $\tilde{\gamma}_{\mathrm{MB}} = |h_{\mathrm{MB}}|^2 \cdot P/(N_0 \cdot W)$ whereas h_{MB} denotes the channel coefficient on the MS-BS link. The achievable rate R in bits per channel use is given according to (2.7) by

$$R = C_{\mathrm{MB}} = \mathcal{C}(\tilde{\gamma}_{\mathrm{MB}}) = \log_2(1 + \tilde{\gamma}_{\mathrm{MB}}). \qquad (4.3)$$

To simplify the notation for the consideration of the relay communication, we define that the SNR on the MS-R link γ_{MR} is given by $\gamma_{\mathrm{MR}} = \tilde{\gamma}_{\mathrm{MR}} = |h_{\mathrm{MR}}|^2 \cdot P/(N_0 \cdot W)$, if the mobile station transmits with power $P_{\mathrm{M}} = P$ and uses the full bandwidth $W_{\mathrm{M}} = W$ and that the SNR on the R-BS link γ_{RB} is given by $\gamma_{\mathrm{RB}} = \tilde{\gamma}_{\mathrm{RB}} = |h_{\mathrm{RB}}|^2 \cdot P/(N_0 \cdot W)$, if the relay transmits with power $P_{\mathrm{R}} = P$ and uses the full bandwidth $W_{\mathrm{R}} = W$. Then, the SNRs γ_{MR} and γ_{RB} are related to the transmission powers P_{M} and P_{R} and to the bandwidths W_{M} and W_{R} as

$$\gamma_{\mathrm{MR}} = \frac{P_{\mathrm{M}} \cdot W}{P \cdot W_{\mathrm{M}}} \cdot \tilde{\gamma}_{\mathrm{MR}} \quad \text{and} \quad \gamma_{\mathrm{RB}} = \frac{P_{\mathrm{R}} \cdot W}{P \cdot W_{\mathrm{R}}} \cdot \tilde{\gamma}_{\mathrm{RB}}. \qquad (4.4)$$

As the considered relay communication represents a point-to-point network, separate network coding and channel coding is optimal and thus, conventional channel coding can be used on the physical layer to provide a reliable point-to-point communication with a rate below or equal to the capacity of the point-to-point link. The capacities of the MS-R and the R-BS link are termed C_{MR} and C_{RB}, respectively. The achievable rate R from MS to BS can be calculated according to the max-flow min-cut theorem in (3.2) described in Chapter 3.2. For our example, the mobile station can transmit $M_{\mathrm{M}} = T_{\mathrm{M}} \cdot W_{\mathrm{M}} = \theta \cdot T \cdot W$ and the relay can transmit $M_{\mathrm{R}} = T_{\mathrm{R}} \cdot W_{\mathrm{R}} = (1-\theta) \cdot T \cdot W$ complex symbols per time unit T and thus, we obtain the following achievable rate $R = K_{\mathrm{M}}/(M_{\mathrm{M}} + M_{\mathrm{R}})$ in bits per channel use:

$$\boxed{R = \min\{C_{\mathrm{MR}}, C_{\mathrm{RB}}\} = \min\left\{\theta \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right), (1-\theta) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right)\right\}}. \tag{4.5}$$

Dependent on the choice how to constrain the resources, we set P_{M} and P_{R} to

- $P_{\mathrm{M}} = P$ and $P_{\mathrm{R}} = P$ for the individual power constraint ($P_{\mathrm{M}} = P_{\mathrm{R}} = P$). The SNRs are given by $\gamma_{\mathrm{MR}} = \tilde{\gamma}_{\mathrm{MR}}$ and by $\gamma_{\mathrm{RB}} = \tilde{\gamma}_{\mathrm{RB}}$ for time division. For frequency division, the SNRs are given by $\gamma_{\mathrm{MR}} = \tilde{\gamma}_{\mathrm{MR}}/\theta$ and by $\gamma_{\mathrm{RB}} = \tilde{\gamma}_{\mathrm{RB}}/(1-\theta)$.

- $P_{\mathrm{M}} = P \cdot T/T_{\mathrm{M}}$ and $P_{\mathrm{R}} = P \cdot T/T_{\mathrm{R}}$ for the individual energy constraint ($P_{\mathrm{M}} \cdot T_{\mathrm{M}} = P_{\mathrm{R}} \cdot T_{\mathrm{R}} = P \cdot T$). The SNRs are given by $\gamma_{\mathrm{MR}} = \tilde{\gamma}_{\mathrm{MR}}/\theta$ and by $\gamma_{\mathrm{RB}} = \tilde{\gamma}_{\mathrm{RB}}/(1-\theta)$ both for time and frequency division.

- $P_{\mathrm{M}} = \beta \cdot P \cdot T/T_{\mathrm{M}}$ and $P_{\mathrm{R}} = (1-\beta) \cdot P \cdot T/T_{\mathrm{R}}$ for the constraint on the total energy ($P_{\mathrm{M}} \cdot T_{\mathrm{M}} + P_{\mathrm{R}} \cdot T_{\mathrm{R}} = P \cdot T$). The parameter β determines what percentage of the total energy is allocated to the source ($0 \le \beta \le 1$). The SNRs are given by $\gamma_{\mathrm{MR}} = \beta \cdot \tilde{\gamma}_{\mathrm{MR}}/\theta$ and by $\gamma_{\mathrm{RB}} = (1-\beta) \cdot \tilde{\gamma}_{\mathrm{RB}}/(1-\theta)$ both for time and frequency division.
  We denote it as fair comparison in every aspect, if the total energy constraint is satisfied and if the constraints $P_{\mathrm{M}} \le P$ and $P_{\mathrm{R}} \le P$ are also fulfilled.

Time and frequency division only yield different rates, if we choose the individual power constraint. Time-division under the individual power constraint also fulfills the total energy constraint and provides a fair comparison in every aspect. For frequency division, there is no difference between the individual power and the individual energy constraint. The parameter θ can be either chosen to a fixed value, for example $\theta = 0.5$ for equal time or frequency allocation, or the parameter can be chosen to $\theta = \theta^*$ such that the rate R is maximized:

$$\theta^* = \arg\max_{\theta} R \tag{4.6}$$

If the total energy is constrained, the energy allocation parameter β can be also optimized:

$$[\theta^*, \beta^*] = \arg\max_{[\theta,\beta]} R \tag{4.7}$$

The SNRs are only independent of θ for time division with the individual power constraint. Then, we can easily find the optimal time allocation parameter θ^* by solving the equation

$\theta^* \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) = (1-\theta^*) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$ and obtain[3]

$$\boxed{\theta^* = \frac{\mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}})}} \tag{4.8}$$

and

$$\boxed{R = \frac{\mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}})}.} \tag{4.9}$$

Example 4.1
Let us consider an example with the SNRs $\tilde{\gamma}_{MB} = 1$, $\tilde{\gamma}_{MR} = 7$, and $\tilde{\gamma}_{RB} = 15$. These SNRs are obtained, if the relay is on a line between mobile and base station with the relative MS-R distance $d_{MR}/d_{MB} = 0.557$ for a path-loss exponent $n = 3.33$.
The achievable rate R in bits per channel use for the point-to-point communication without relay is given by

$$R = \mathcal{C}(\gamma_{MB} = 1) = 1.00.$$

The achievable rate in bits per use for the relay communication with equal time-sharing ($\theta = 0.5$) and a constraint on the individual transmission powers P_M and P_R to the one of the point-to-point communication ($P_M = P_R = P$) is given by

$$R = \min\{0.5 \cdot \mathcal{C}(\gamma_{MR} = 7), 0.5 \cdot \mathcal{C}(\gamma_{RB} = 15)\} = \min\{1.50, 2.00\} = 1.50.$$

Compared to the point-to-point communication, the relay communication with equal time-sharing allows to increase the rate from 1.00 to 1.50 bits per channel use. The comparison is fair, because in both strategies the same total energy and the same transmission power is used.
Next, we consider the example, if the common medium is divided such that the rate R from source to sink is maximized. The R-BS link in the example in Figure 4.1 has a higher SNR than the MS-R link and thus, the R-BS capacity is larger than the MS-R capacity. The high R-BS capacity is waisted because the smaller MS-R capacity acts as bottleneck. The rate R can be enlarged, if the MS-R link gets a larger fraction of the total available time than the R-BS link. According to (4.8), it is optimal for the example with $\mathcal{C}(\gamma_{MR} = 7) = 3.00$ and $\mathcal{C}(\gamma_{RB} = 15) = 4.00$ to allocate $\theta^* = 57\%$ of the available time to the mobile station. Then, the rate R in bits per channel use is given by

$$R = \frac{3.00 \cdot 4.00}{3.00 + 4.00} = 1.71$$

according to (4.9). Compared to the equal time allocation ($\theta = 0.5$), the optimal time allocation increases the rate from $R = 1.50$ to $R = 1.71$ bits per channel use. The optimal resource allocation for the example is illustrated in Figure 4.2.
Table 4.1 summarizes all cases for the example. The cases which do not provide a fair

[3]The derivation of (4.8) and (4.9) follows from the derivation of (4.21) and (4.22) in the next section.

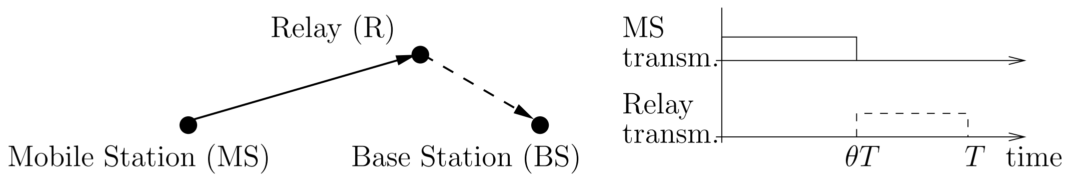


Figure 4.2: Wireless relay communication as point-to-point network with optimal time-sharing: The mobile station uses the channel for $\theta \cdot T$ and the relay for $(1-\theta) \cdot T$. For the example with the SNRs $\tilde{\gamma}_{\mathrm{MR}} = 7$ and $\tilde{\gamma}_{\mathrm{RB}} = 15$, a transmission rate of $R = 1.71$ bits per channel use can be achieved for $\theta = 57\%$, if the individual powers are constrained.

comparison in every aspect are highlighted with a gray background. The energy-allocation parameter β is always optimized numerically with an intensive computer search. In general, the achievable rate R depends strongly on the chosen constraint. The optimization of the time/frequency-allocation always increases the rate.

The example demonstrates that even a simple form of relaying without broadcast and simultaneous multiple-access can outperform clearly a communication without relay. Relaying allows to increase the data rate significantly (from 1.00 to 1.71 bits per channel use in our example) and allows at the same time to save energy (battery lasts longer) at the mobile station (reduced from $P \cdot T$ to $P \cdot T_M = 0.57 \cdot P \cdot T$ in our example). The comparison is fair because the total transmission time, the total transmission energy and the used bandwidth do not increase.

The technical requirement for the described relaying strategy is easy to fulfill. If the wireless network is established by time-division multiplexing, the nodes have to be synchronized such that the transmit phases do not overlap. Time-division multiplexing has already been included in current wireless systems, for example in GSM, and will cause no problems for future systems. A synchronization at the symbol or carrier-phase level or a full-duplex relay are not required.

We considered wireless point-to-point networks with time- or frequency-division and explained that optimal allocation of the wireless medium can improve the achievable rate. Beside the optimal allocation of the wireless medium, it is also possible to reuse the wireless medium. The mobile station and the relay can both access simultaneously the channel for the full time ($T_{\mathrm{M}} = T_{\mathrm{R}} = T$, $W_{\mathrm{M}} = W_{\mathrm{R}} = W$) and can both transmit $M = T \cdot W$ symbols. This requires the relay to be a full-duplex relay and to transmit and receive simultaneously. The simultaneous multiple-access occurs non-cooperatively. Figure 4.3 illustrates the relay communication as point-to-point network with simultaneous multiple-

Constraint	θ	β	P_{M}	P_{R}	E_{M}	E_{R}	R
Point-to-Point Communication							
$P_{\mathrm{M}} = P$, $E_{\mathrm{M}} = P \cdot T$	/	/	P	-	$P \cdot T$	-	1.00
Relay Communication with equal time-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	0.50	/	P	P	$0.50 \cdot P \cdot T$	$0.50 \cdot P \cdot T$	1.50
$E_{\mathrm{M}} = E_{\mathrm{R}} = P \cdot T$	0.50	/	$2.00 \cdot P$	$2.00 \cdot P$	$P \cdot T$	$P \cdot T$	1.95
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.50	0.68	$1.36 \cdot P$	$0.64 \cdot P$	$0.68 \cdot P \cdot T$	$0.32 \cdot P \cdot T$	1.70
Relay Communication with optimal time-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	0.57	/	P	P	$0.57 \cdot P \cdot T$	$0.43 \cdot P \cdot T$	1.71
$E_{\mathrm{M}} = E_{\mathrm{R}} = P \cdot T$	0.585	/	$1.71 \cdot P$	$2.41 \cdot P$	$P \cdot T$	$P \cdot T$	2.16
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.55	0.605	$1.10 \cdot P$	$0.88 \cdot P$	$0.61 \cdot P \cdot T$	$0.39 \cdot P \cdot T$	1.72
Relay Communication with equal frequency-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	0.50	/	P	P	$P \cdot T$	$P \cdot T$	1.95
$E_{\mathrm{M}} = E_{\mathrm{R}} = P \cdot T$	0.50	/	P	P	$P \cdot T$	$P \cdot T$	1.95
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.50	0.68	$0.68 \cdot P$	$0.32 \cdot P$	$0.68 \cdot P \cdot T$	$0.32 \cdot P \cdot T$	1.70
Relay Communication with optimal frequency-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	0.585	/	P	P	$P \cdot T$	$P \cdot T$	2.16
$P_{\mathrm{M}} = P_{\mathrm{R}} = P/2$	0.61	/	$0.50 \cdot P$	$0.50 \cdot P$	$0.50 \cdot P \cdot T$	$0.50 \cdot P \cdot T$	1.68
$E_{\mathrm{M}} = E_{\mathrm{R}} = P \cdot T$	0.585	/	P	P	$P \cdot T$	$P \cdot T$	2.16
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.55	0.605	$0.61 \cdot P$	$0.39 \cdot P$	$0.61 \cdot P \cdot T$	$0.39 \cdot P \cdot T$	1.72

Table 4.1: Relay communication as point-to-point network: Achievable rate R in bits per channel use for the example with $\tilde{\gamma}_{\mathrm{MB}} = 1$, $\tilde{\gamma}_{\mathrm{MR}} = 7$, and $\tilde{\gamma}_{\mathrm{RB}} = 15$ under different power/energy constraints. The cases which provide a fair comparison in every aspect are on white background. The parameter θ determines what percentage of the total time/frequency is allocated to the mobile station. The parameter β determines what percentage of the total energy is allocated to the mobile station.

access. The channel model is given by

$$\mathbf{y}_{\mathrm{MR}} = h_{\mathrm{MR}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MR}} \tag{4.10}$$

$$\mathbf{y}_{\mathrm{RB}} = h_{\mathrm{RB}} \cdot \mathbf{x}_{\mathrm{R}} + h_{\mathrm{MB}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{RB}} \tag{4.11}$$

whereas the channel coefficient for the MS-BS link is denoted as h_{MB}. The interference from the mobile station can be seen as additional noise which lowers the SNR at the base station from γ_{RB} to the signal-to-interference-and-noise ratio (SINR)

$$\hat{\gamma}_{\mathrm{RB}} = \frac{\gamma_{\mathrm{RB}}}{1 + \gamma_{\mathrm{MB}}}. \tag{4.12}$$

The achievable rate $R = K_{\mathrm{M}}/M$ for this strategy is given by

$$R = \min\{C_{\mathrm{MR}}, C_{\mathrm{RB}}\} = \min\{\mathcal{C}(\gamma_{\mathrm{MR}}), \mathcal{C}(\hat{\gamma}_{\mathrm{RB}})\}. \tag{4.13}$$

with the SNRs

$$\gamma_{\mathrm{MR}} = \frac{P_{\mathrm{M}}}{P} \cdot \tilde{\gamma}_{\mathrm{MR}} \quad , \quad \gamma_{\mathrm{RB}} = \frac{P_{\mathrm{R}}}{P} \cdot \tilde{\gamma}_{\mathrm{RB}} \quad \text{and} \quad \gamma_{\mathrm{MB}} = \frac{P_{\mathrm{M}}}{P} \cdot \tilde{\gamma}_{\mathrm{MB}}. \tag{4.14}$$

The SINR $\hat{\gamma}_{\mathrm{RB}}$ is given by (4.12).
Dependent on the choice how to constrain the resources, we set P_{M} and P_{R} to

- $P_{\mathrm{M}} = P$ and $P_{\mathrm{R}} = P$ for the individual power and the individual energy constraint. Then, the system requires the double amount of total energy compared to the point-to-point system.
- $P_{\mathrm{M}} = \beta \cdot P$ and $P_{\mathrm{R}} = (1-\beta) \cdot P$ for the constraint on the total energy. The parameter β determines again what percentage of the total energy is allocated to the mobile station.

Example 4.2 (continued)
Table 4.2 contains the rate R under the two constraints for the example with $\tilde{\gamma}_{MB} = 1$, $\tilde{\gamma}_{MR} = 7$ and $\tilde{\gamma}_{RB} = 15$. For the constraint on the total energy, the parameter β is chosen numerically such that the rate R is maximized. Then, a data rate of $R = 2.33$ bits per channel use can be achieved. With growing interference from MS to BS, the rate R will decrease and can also become smaller than for the case without reusing the time slots. Again, the cases which provide a fair comparison to the point-to-point communication in every aspect are highlighted.

Reusing is difficult to realize for the described small network with one relay, because the intensity of the electro-magnetic near field of the transmitted signal is much higher

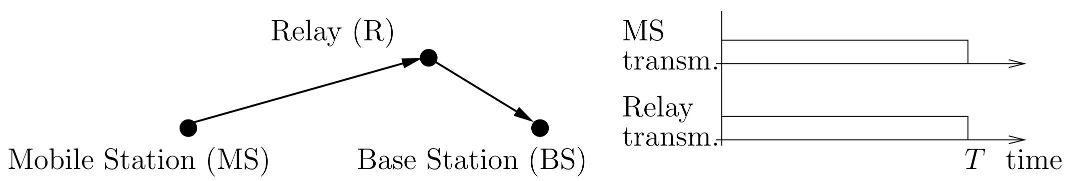


Figure 4.3: Wireless relay communication as point-to-point network with simultaneous multiple-access: Both the mobile station and the relay use the channel for the total time T for their transmissions. For the example with the SNRs $\tilde{\gamma}_{\mathrm{MR}} = 7$, $\tilde{\gamma}_{\mathrm{RB}} = 15$ and $\tilde{\gamma}_{\mathrm{MB}} = 1$, a transmission rate of $R = 2.33$ bits per channel use can be achieved for a fair comparison in every aspect. The strategy requires a full-duplex relay.

Constraint	θ	β	P_{M}	P_{R}	E_{M}	E_{R}	R
Full-Duplex Relay Communication with simultaneous multiple-access							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	/	/	P	P	$P \cdot T$	$P \cdot T$	3.00
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	/	0.58	$0.58 \cdot P$	$0.42 \cdot P$	$0.58 \cdot P \cdot T$	$0.42 \cdot P \cdot T$	2.32

Table 4.2: Wireless relay communication as point-to-point network with simultaneous multiple-access: Achievable rate R in bits per channel use for the example with $\tilde{\gamma}_{\mathrm{MB}} = 1$, $\tilde{\gamma}_{\mathrm{MR}} = 7$, and $\tilde{\gamma}_{\mathrm{RB}} = 15$ under different power/energy constraints. This strategy requires a full-duplex relay.

than that of the far field of the received signal at the relay. Reusing can be helpful for half-duplex relays, if the information is transmitted over several relays. Figure 4.4 depicts an example with two half-duplex relays. The second relay can reuse the time slot of the mobile station and the two transmissions interfere with each other. If the distance is large enough or if there are buildings between MS and BS, the received power at the base station of the signal transmitted by the mobile station will be small enough that this interference can be neglected compared to the received power from the second relay.

Reuse and sharing is a competence of the medium access control (MAC) layer. The decision how to reuse and share the medium is not relevant for wireline point-to-point links because each wire is a separate medium and cannot be shared or reused by an other link. As we have still a point-to-point network without broadcasts, the capacity can be still achieved with a system with separated routing/network coding (in the network layer) and conventional channel coding (in the physical layer). It is not necessary to design new channel codes for point-to-point networks. More details about specific transmission methods (like sharing and reusing) for large wireless point-to-point networks can be found for example in [TG03] and references therein. In [Zan05], the author considers the influence of channel coding to a wireless network communication which is established as point-to-point network with an uncoordinated, random, contention-based medium access protocol with collision detection (ALOHA protocol). In [Daw04, Chapter 2.2.3], the author considers resource allocation (power, time) for wireless point-to-point networks with several relays. Both the cases with and without simultaneous multiple access (interference) are

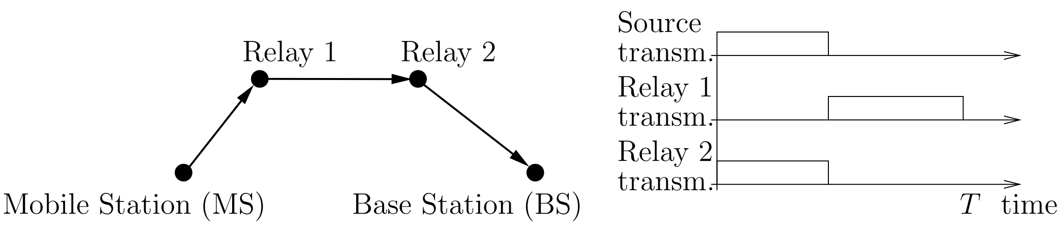


Figure 4.4: Wireless communication with two relays as point-to-point network with partial simultaneous multiple-access. Mobile station and relay 1 share the total time for their transmission. Relay 2 uses the channel simultaneous with the mobile station.

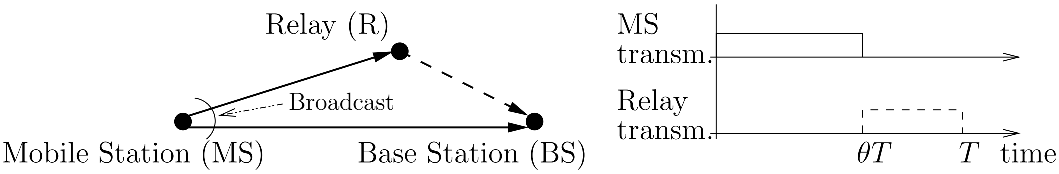


Figure 4.5: Wireless relay communication with broadcast and optimal time-sharing: For the example with the SNRs $\tilde{\gamma}_{\mathrm{MR}} = 9$, $\tilde{\gamma}_{\mathrm{RB}} = 4$ and $\tilde{\gamma}_{\mathrm{MB}} = 1$, a transmission rate of $R = 2.00$ bits per channel use can be achieved for $\theta = 2/3$, if the individual powers are constrained.

considered. In [Daw04, Chapter 5], various system level aspects regarding the extension of CDMA-based cellular systems with relaying are considered.

4.3 Wireless Networks with Broadcast

In this section we consider wireless relaying with broadcast transmissions. The wireless broadcast nature allows in principle the nodes to listen to every transmission in the network. Of course, it is only useful to listen to a signal, if enough power can be received of it. We consider again the uplink of K_{M} information bits with the help of a relay as described at the beginning of Section 4.2. The mobile station transmits M_{M} symbols during the relay-receive phase and the relay transmits M_{R} symbols during the relay-transmit phase. Contrary to Section 4.2, we exploit the wireless broadcast nature and include that the base station can already listen to the transmission of the mobile station during the relay-receive phase. The relay communication with broadcast is illustrated in Figure 4.5 and the channel model is given by

$$\mathbf{y}_{\mathrm{MR}} = h_{\mathrm{MR}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MR}} \qquad \text{(Relay-receive phase)} \tag{4.15}$$

$$\mathbf{y}_{\mathrm{MB}} = h_{\mathrm{MB}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MB}} \qquad \text{(Relay-receive phase)} \tag{4.16}$$

$$\mathbf{y}_{\mathrm{RB}} = h_{\mathrm{RB}} \cdot \mathbf{x}_{\mathrm{R}} + \mathbf{z}_{\mathrm{RB}}, \qquad \text{(Relay-transmit phase)} \tag{4.17}$$

where $\mathbf{y}_{\mathrm{MB}}$ denotes the received signal at the base station during the relay-receive phase. The other variables are defined analogously to the denotation in the previous section. The SNR on the MS-BS link is given by $\gamma_{\mathrm{MB}} = |h_{\mathrm{MB}}|^2 \cdot P_{\mathrm{M}}/(N_0 \cdot W_{\mathrm{M}}) = P_{\mathrm{M}} \cdot W \cdot \tilde{\gamma}_{\mathrm{MB}}/(P \cdot W_{\mathrm{M}})$.

The base station receives signals both from the mobile station and the relay. The two transmissions can be interpreted as redundant transmissions and can be used to help the channel coding for error protection. This requires to break with the separation of network layer and physical layer [EMH$^+$03, RK06] and requires a more complicated system design. It is necessary to design network coding, respectively routing, in the network layer and channel coding in the physical layer jointly. We will describe in the Chapters 5, 6 and 7 how to design joint network-channel codes, respectively joint routing-channel codes, for some small wireless relay networks.

We assume Gaussian channel and that the MS-R link is not worse than the MS-BS link ($\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$). Then, the achievable decode-and-forward rate $R = K_{\mathrm{M}}/(M_{\mathrm{M}} + M_{\mathrm{R}})$

for the relay communication with broadcast is given by [LV05b, Theorem 2]

$$\boxed{R = \min\{\theta \cdot \mathcal{C}(\gamma_{\mathrm{MR}}), \theta \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1-\theta) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\}.} \tag{4.18}$$

The SNRs and the parameter θ are defined as in the last section. Again, we can choose between time-and frequency division and there are the same options how to choose the power/energy constraint. If the base station is not able to receive the signal from the mobile station ($\gamma_{\mathrm{MB}} = 0$), the system with broadcast (Equation (4.18)) falls back to the system without broadcast (Equation (4.5)). Contrary to relaying without broadcast transmission, it is not possible that the rate of the relay communication is smaller than the one of the point-to-point communication, because the relay communication falls back to the point-to-point communication for $\theta = 1$. Therefore, relaying with broadcast provides a robust system, which also works efficiently, when we chose to use the relay, even though the direct MS-BS link has a high SNR. This is important for mobile communications,where the SNRs can change rapidly.
As in the previous section, the parameter θ can be chosen to $\theta = \theta^*$ such that the rate R is maximized:

$$\theta^* = \arg\max_{\theta} R \tag{4.19}$$

We only consider the case with time division and the constraint $P_{\mathrm{M}} = P_{\mathrm{R}} = P$ in more detail. Then, mobile station and relay use the same transmission power and the same total energy as the point-to-point communication and the comparison is fair in every aspect. The SNR values $\gamma_{\mathrm{MB}} = \tilde{\gamma}_{\mathrm{MB}}$, $\gamma_{\mathrm{MR}} = \tilde{\gamma}_{\mathrm{MR}}$ and $\gamma_{\mathrm{RB}} = \tilde{\gamma}_{\mathrm{RB}}$ are independent of the resource allocation parameter θ. Note that the authors in [LV05b] considered frequency-division with individual power constraint, respectively the time- or frequency-division case with individual energy constraint and thus, the SNR is scaled by the parameter θ in [LV05b] and the optimization problem is much more complicated than in our case. We showed in [HH06] that for time-division a closed expression for the optimization in (4.19) can be found under the assumption $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{RB}})$. The maximization of R in (4.19) can be easily solved, because $\theta \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1-\theta) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) = \mathcal{C}(\gamma_{\mathrm{RB}}) + \theta \cdot (\mathcal{C}(\gamma_{\mathrm{MB}}) - \mathcal{C}(\gamma_{\mathrm{RB}}))$ is monotone decreasing with θ for $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{RB}})$ and $\theta \cdot \mathcal{C}(\gamma_{\mathrm{MR}})$ is monotone increasing with θ. Therefore, the solution of the maximization can be always found at the cross-over point of $\theta \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1-\theta) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$ and $\theta \cdot \mathcal{C}(\gamma_{\mathrm{MR}})$ and we can find the optimal parameter θ^* solving the following equation:

$$\theta^* \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) = \theta^* \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1-\theta^*) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) \tag{4.20}$$

Solving this equation leads to the optimal value for the time sharing parameter

$$\boxed{\theta^* = \frac{\mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})}} \tag{4.21}$$

and to the following closed expression for R:

$$\boxed{R = \frac{\mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})}} \tag{4.22}$$

Constraint	θ	β	P_{M}	P_{R}	E_{M}	E_{R}	R
Relay with equal time-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	0.50	/	P	P	$0.50 \cdot P \cdot T$	$0.50 \cdot P \cdot T$	1.50
$E_{\mathrm{M}} = E_{\mathrm{R}} = P \cdot T$	0.50	/	$2.00 \cdot P$	$2.00 \cdot P$	$P \cdot T$	$P \cdot T$	1.95
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.50	0.87	$1.74 \cdot P$	$0.26 \cdot P$	$0.87 \cdot P \cdot T$	$0.13 \cdot P \cdot T$	1.86
Relay with optimal time-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P$	2/3	/	P	P	$2/3 \cdot P \cdot T$	$1/3 \cdot P \cdot T$	2.00
$E_{\mathrm{M}} = E_{\mathrm{R}} = P \cdot T$	0.73	/	$1.37 \cdot P$	$3.70 \cdot P$	$P \cdot T$	$P \cdot T$	2.48
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.64	0.72	$1.13 \cdot P$	$0.78 \cdot P$	$0.72 \cdot P \cdot T$	$0.28 \cdot P \cdot T$	2.01
Relay with equal frequency-division							
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.50	0.87	$0.87 \cdot P$	$0.13 \cdot P$	$0.87 \cdot P \cdot T$	$0.13 \cdot P \cdot T$	1.86
Relay with optimal frequency-division							
$P_{\mathrm{M}} = P_{\mathrm{R}} = P/2$	0.73	/	$0.50 \cdot P$	$0.50 \cdot P$	$0.50 \cdot P \cdot T$	$0.50 \cdot P \cdot T$	1.85
$E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$	0.64	0.72	$0.72 \cdot P$	$0.28 \cdot P$	$0.72 \cdot P \cdot T$	$0.28 \cdot P \cdot T$	2.01

Table 4.3: Relay communication with broadcast: Achievable rate R in bits per channel use for the example with $\tilde{\gamma}_{\mathrm{MB}} = 1$, $\tilde{\gamma}_{\mathrm{MR}} = 7$, and $\tilde{\gamma}_{\mathrm{RB}} = 15$ under different power/energy constraints.

Example 4.3 (continued)
For the previous example with $\mathcal{C}(\gamma_{MR} = 7) = 3.00$, $\mathcal{C}(\gamma_{RB} = 15) = 4.00$ and $\mathcal{C}(\gamma_{MB} = 1) = 1.00$, the rate R in bits per channel use according to (4.22) is given by

$$R = \frac{3.00 \cdot 4.00}{3.00 + 4.00 - 1.00} = 2.00$$

and the optimal time-allocation parameter is given by $\theta^* = 2/3$ according to (4.21). From the comparison with the achievable relaying rate without broadcast in (4.9), we know that the broadcast transmission at the mobile station increases the achievable data rate from $R = 1.71$ to $R = 2.00$ bits per channel use.
Table 4.3 summarizes several cases for the example. The energy-allocation parameter β is always optimized numerically with an intensive computer search. We see from the comparison of the Tables 4.1 and 4.3 that the rate can be improved, if the base station listens to the transmission of the mobile station. Again, the optimization of the time/frequency allocation allows to increase the rate.

We have seen in the last section that the achievable rate with the strategy with simultaneous access of mobile station and relay to the wireless medium decreases with growing interference from mobile station to base station. Contrary, the benefit of the broadcast transmission increases with growing SNR γ_{MB} on the MS-BS link.

Wireless networks with broadcast transmissions do not require a full-duplex relay or a synchronization at the symbol or carrier level and thus, there are no hardware constraints which do not allow the implementation in practical systems. As the efficient application of broadcast transmissions requires to break with the separation of network and physical layer, this strategy may not be reasonable for large networks with complicated routing or network coding tasks. In large networks the separation of layers is a very important concept to handle the complexity of the system. However, for small networks as our example with one relay, a partly joint design of network layer and physical layer seems reasonable, because no complicated mechanisms in the network layer are required. Therefore, the use of broadcast transmissions is considered as practical for cellular based mobile communication systems [PWS+04], if only a very small number of hops have to be supported.
Practical distributed channel coding schemes for relay communication with broadcast were proposed on the basis of convolutional codes in [SE04, HN06] and on the basis of turbo codes in [ZV03, SV04]. The distributed channel coding schemes on the basis of LDPC codes in [CdBSA07] can be easily simplified to realize a relay communication with broadcast. We will consider these coding schemes in more detail in Chapter 5.
The outage behavior of relay communication strategies with broadcast and fading channels is considered in [LTW04].

For general networks with broadcast, it is more difficult to determine the achievable rate than for the example in Figure 4.5 with one relay. As networks with broadcast are not point-to-point networks, we cannot use the max-flow min-cut theorem in (3.2) anymore. The determination of the capacity for networks with broadcast has not been solved completely. A first step to solve this problem was done in [RK06]. In [RK06], the authors established a max-flow min-cut theorem for deterministic broadcast networks.

4.4 Wireless Networks with Broadcast and Simultaneous Multiple-Access

The wireless relay networks with broadcast can be further extended by allowing several nodes to access the wireless medium simultaneously. For example, mobile station and relay in the previous example can transmit simultaneously to the base station. We consider two strategies for the example, one with partial simultaneous multiple-access and one with full simultaneous multiple-access. Whereas the strategy with full simultaneous multiple-access requires a full-duplex relay, the strategy with partial simultaneous multiple-access only requires a half-duplex relay. Both strategies include the cooperative simultaneous multiple-access.

4.4.1 Partial Simultaneous Multiple-Access

As in the previous section, the strategy with partial multiple-access requires a division of the total time or frequency into one relay-receive phase and one relay-transmit phase. Again, the parameter θ defines the allocation of the total time/frequency to the two phases. During the relay-receive phase, the mobile station transmits the block of M_{M} symbols $\mathbf{x}_{\mathrm{M}}$ with power P_{M} and the relay and the base station receive $\mathbf{y}_{\mathrm{MR}}$ and $\mathbf{y}_{\mathrm{MB}}$, respectively. During the relay-transmit phase, the mobile station transmits the block of M_{R} symbols $\mathbf{x}_{\mathrm{M}}^{(2)}$ with power $P_{\mathrm{M}}^{(2)}$, the relay transmits the block of M_{R} symbols $\mathbf{x}_{\mathrm{R}}$ with power P_{R} and the base station receives $\mathbf{y}_{\mathrm{RB}}$. The mobile station uses the energy $E_{\mathrm{M}} = P_{\mathrm{M}} \cdot T_{\mathrm{M}} + P_{\mathrm{M}}^{(2)} \cdot T_{\mathrm{R}}$ during time T. Figure 4.6 illustrates the strategy for time-division. The channel mode is given by

$$\mathbf{y}_{\mathrm{MR}} = h_{\mathrm{MR}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MR}} \qquad \text{(Relay-receive phase)} \qquad (4.23)$$
$$\mathbf{y}_{\mathrm{MB}} = h_{\mathrm{MB}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MB}} \qquad \text{(Relay-receive phase)} \qquad (4.24)$$
$$\mathbf{y}_{\mathrm{RB}} = h_{\mathrm{RB}} \cdot \mathbf{x}_{\mathrm{R}} + h_{\mathrm{MB}}^{(2)} \cdot \mathbf{x}_{\mathrm{M}}^{(2)} + \mathbf{z}_{\mathrm{RB}} \qquad \text{(Relay-transmit phase)} \qquad (4.25)$$

whereas $h_{\mathrm{MB}}^{(2)}$ denotes the channel coefficient for the MS-BS link during the relay-transmit phase. Mobile station and relay can synchronize their carriers in such a way that their signals superimpose constructively at the base station (beamforming). Then, the same effects occur as with antenna arrays at the transmitter. However, such a carrier synchronization is regarded as very difficult to realize with current hardware [BL06], especially if two or three of the nodes are mobile. The achievable rate of this strategy was considered in [HMZ05] for decode-and-forward and compress-and-forward. Again, we assume Gaussian channel and that the MS-R link is not worse than the MS-BS link ($\gamma_{\mathrm{MB}} \leq \gamma_{\mathrm{MR}}$). Then, the achievable decode-and-forward rate $R = K_{\mathrm{M}}/(M_{\mathrm{M}} + M_{\mathrm{R}})$ in bits per channel use for the strategy with partial multiple-access is given in [HMZ05, Proposition 2] and

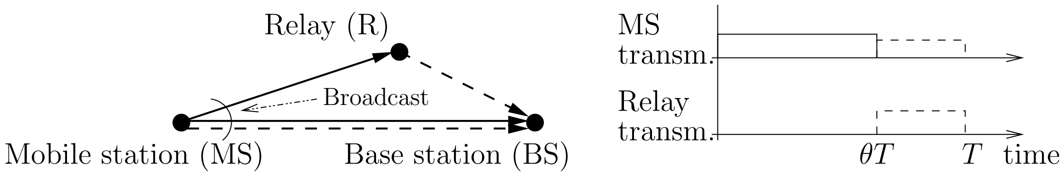


Figure 4.6: Wireless relay communication with broadcast and partial simultaneous multiple-access for time-division: Then, the mobile station transmits for the total time T and the relay for $(1-\theta)\cdot T$. For the example with the SNRs $\tilde{\gamma}_{SR}=7$, $\tilde{\gamma}_{RD}=15$ and $\tilde{\gamma}_{SD}=1$, a transmission rate of $R=2.01$ bits per channel use can be achieved for $\theta=0.64$ for a fair comparison in every aspect. This strategy requires a carrier phase synchronization of the signals from mobile station and relay such that the signals superimpose constructively at the base station.

in [KSA03, Theorem 2] by

$$\boxed{\begin{aligned} R = \min\Big\{&\theta\cdot\mathcal{C}(\gamma_{\mathrm{MR}})+(1-\theta)\cdot\mathcal{C}\left((1-\varphi)\cdot\gamma_{\mathrm{MB}}^{(2)}\right), \\ &\theta\cdot\mathcal{C}(\gamma_{\mathrm{MB}})+(1-\theta)\cdot\mathcal{C}\left(\gamma_{\mathrm{RB}}+\gamma_{\mathrm{MB}}^{(2)}+2\sqrt{\varphi\cdot\gamma_{\mathrm{MB}}^{(2)}\cdot\gamma_{\mathrm{RB}}}\right)\Big\}\end{aligned}} \tag{4.26}$$

whereas the SNR $\gamma_{\mathrm{MB}}^{(2)}$ is defined as $\gamma_{\mathrm{MB}}^{(2)}=\frac{|h_{\mathrm{MB}}^{(2)}|^2\cdot P_{\mathrm{M}}^{(2)}}{N_0\cdot W_{\mathrm{R}}}=\frac{P_{\mathrm{M}}^{(2)}\cdot W}{P\cdot W_{\mathrm{R}}}\cdot\tilde{\gamma}_{\mathrm{MB}}$. The other SNRs and the parameter θ are defined as in the last section. If $P_{\mathrm{M}}^{(2)}$ is set to $P_{\mathrm{M}}^{(2)}=0$, the system with broadcast and partial multiple-access (Equation (4.26)) falls back to the system with broadcast and without multiple-access (Equation (4.18))[4]. The parameter φ determines the correlation between the signal of mobile station and relay during the relay-transmit phase ($0\le\varphi\le 1$). If there is no correlation ($\varphi=0$), no beamforming gain can be achieved. Dependent on the choice how to constrain the resources, we set P_{M}, $P_{\mathrm{M}}^{(2)}$ and P_{R} to

- $P_{\mathrm{M}}=P_{\mathrm{M}}^{(2)}=P$ and $P_{\mathrm{R}}=P$ for time-division with individual power constraint ($P_{\mathrm{M}}=P_{\mathrm{M}}^{(2)}=P_{\mathrm{R}}=P$) or to $P_{\mathrm{M}}=\kappa\cdot P$, $P_{\mathrm{M}}^{(2)}=(1-\kappa)\cdot P$ and $P_{\mathrm{R}}=P$ for frequency-division with individual power constraint ($P_{\mathrm{M}}+P_{\mathrm{M}}^{(2)}=P_{\mathrm{R}}=P$). The

[4]Note that it is not possible to recognize directly in the original papers that [LV05b, Theorem 2] is a special case of [HMZ05, Proposition 2] because the SNRs are scaled by $1/\theta$ or by $1/(1-\theta)$ in [LV05b, Theorem 2] whereas the SNRs are not scaled in [HMZ05, Proposition 2], although it is mentioned in both papers that the results are valid for both time- and frequency-division. The reason for the difference is that different constraints are used. Whereas the result in [HMZ05] can be obtained for time-division with the individual power constraint, the result in [LV05b] can be obtained either for the constraint on the individual energy or for frequency-division for the constraint on the individual power. The detailed treatment of the constraints in this thesis helps to understand the relation between the two papers.
Note that the SNRs in the achievable rate for the system without broadcast and without simultaneous multiple-access given in [KGG05, Equation (59)] are also scaled by $1/\theta$ or by $1/(1-\theta)$. The relation to [HMZ05, Proposition 2] can only be recognized, if the constraints are changed.

parameter κ determines what percentage of the mobile station power, respectively energy, is used in the relay-receive phase.

- $P_{\mathrm{M}} = \kappa \cdot P \cdot T/T_{\mathrm{M}}$, $P_{\mathrm{M}}^{(2)} = (1-\kappa) \cdot P \cdot T/T_{\mathrm{R}}$ and $P_{\mathrm{R}} = P \cdot T/T_{\mathrm{R}}$ for the individual energy constraint ($P_{\mathrm{M}} \cdot T_{\mathrm{M}} + P_{\mathrm{M}}^{(2)} \cdot T_{\mathrm{R}} = P_{\mathrm{R}} \cdot T_{\mathrm{R}} = P \cdot T$). The parameter κ determines what percentage of the energy of the mobile station is used in the relay-receive phase.
- $P_{\mathrm{M}} = \beta \cdot \kappa \cdot P \cdot T/T_{\mathrm{M}}$, $P_{\mathrm{M}}^{(2)} = \beta \cdot (1-\kappa) \cdot P \cdot T/T_{\mathrm{R}}$ and $P_{\mathrm{R}} = (1-\beta) \cdot P \cdot T/T_{\mathrm{R}}$ for the constraint on the total energy ($P_{\mathrm{M}} \cdot T_{\mathrm{M}} + P_{\mathrm{M}}^{(2)} \cdot T_{\mathrm{R}} + P_{\mathrm{R}} \cdot T_{\mathrm{R}} = P \cdot T$).
  We denote it as fair comparison in every aspect, if the total energy constraint is satisfied and if for time-division the constraints $P_{\mathrm{M}} \leq P$, $P_{\mathrm{M}}^{(2)} \leq P$ and $P_{\mathrm{R}} \leq P$ or for frequency-divsion the constraints $P_{\mathrm{M}} + P_{\mathrm{M}}^{(2)} \leq P$ and $P_{\mathrm{R}} \leq P$ are also fulfilled.

If the carrier synchronization is not established, the achievable rate can be found by setting φ to $\varphi = 0$. Compared to the other strategies without simultaneous multiple-access, there are two more parameters (φ and κ) which can be optimized. For the asynchronous case ($\varphi = 0$) with individual power constraint, a closed expression for the optimized θ is given in [NHSP07]. The closed expression in (4.21) from [HH06] for the relay communication with broadcast can be seen as a special case of the expression from [NHSP07].

Example 4.4 (continued)
Table 4.4 summarizes several cases for the example with $\tilde{\gamma}_{MB} = 1$, $\tilde{\gamma}_{MR} = 7$ and $\tilde{\gamma}_{RB} = 15$. The parameters are optimized with an intensive computer search. If the total energy is constrained, the gain from the partial simultaneous multiple-access (PSMA) is small. For example, the PSMA allows to increase the rate R from $R = 2.01$ to $R = 2.03$ bits per channel use for optimal frequency-division. The beamforming gain is not obvious for this example. Beamforming helps, if the relay is close to the mobile station. We will consider this in the comparison in Section 4.5. If the individual transmission power is constrained, PSMA increases the rate at the cost of a larger total energy.

Practical distributed channel coding schemes for relay communication with broadcast and partial simultaneous multiple access were proposed on the basis of LDPC codes in [CdBSA07] and on the basis of turbo codes in [ZD05b]. The outage behavior of relay communication strategies with broadcast, PSMA and fading channels is considered in [NBK04].

Constraint	θ	φ	κ	β	$\frac{P_\mathrm{M}}{P}$	$\frac{P_\mathrm{M}^{(2)}}{P}$	$\frac{P_\mathrm{R}}{P}$	$\frac{E_\mathrm{M}}{PT}$	$\frac{E_\mathrm{R}}{PT}$	R
Relay Communication with equal time-division										
$P_\mathrm{M} = P_\mathrm{M}^{(2)} = P_\mathrm{R} = P$	0.50	0.00	/	/	1.00	1.00	1.00	1.00	0.50	2.00
$P_\mathrm{M} = P_\mathrm{M}^{(2)} = P/2$, $P_\mathrm{R} = P$	0.50	0.00	/	/	0.50	0.50	1.00	0.50	0.50	1.38
$E_\mathrm{M} = E_\mathrm{R} = P \cdot T$	0.50	0.00	0.71	/	1.42	0.58	2.00	1.00	1.00	2.06
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$	0.50	0.04	0.77	0.86	1.32	0.40	0.28	0.86	0.14	1.91
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$, $\max\{P_\mathrm{M}, P_\mathrm{M}^{(2)}, P_\mathrm{R}\} \leq P$	0.50	0.06	0.58	0.86	1.00	0.72	0.28	0.86	0.14	1.87
Relay Communication with optimal time-division										
$P_\mathrm{M} = P_\mathrm{M}^{(2)} = P_\mathrm{R} = P$	0.62	0.07	/	/	1.00	1.00	1.00	1.00	0.38	2.22
$P_\mathrm{M} = P_\mathrm{M}^{(2)} = P/2$, $P_\mathrm{R} = P$	0.69	0.04	/	/	0.50	0.50	1.00	0.50	0.31	1.67
$E_\mathrm{M} = E_\mathrm{R} = P \cdot T$	0.73	0.31	0.99	/	1.36	0.04	3.70	1.00	1.00	2.49
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$	0.64	0.81	0.97	0.75	1.14	0.06	0.69	0.75	0.25	2.03
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$, $\max\{P_\mathrm{M}, P_\mathrm{M}^{(2)}, P_\mathrm{R}\} \leq P$	0.66	0.36	0.94	0.70	1.00	0.12	0.88	0.70	0.30	2.01
Relay Communication with equal frequency-division										
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$	0.50	0.04	0.77	0.86	0.66	0.20	0.14	0.86	0.14	1.91
Relay Communication with optimal frequency-division										
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$	0.64	0.81	0.97	0.75	0.73	0.02	0.25	0.75	0.25	2.03

Table 4.4: Relay communication with broadcast and partial simultaneous multiple access: Achievable rate R in bits per channel use for the example with $\tilde{\gamma}_\mathrm{MB} = 1$, $\tilde{\gamma}_\mathrm{MR} = 7$, and $\tilde{\gamma}_\mathrm{RB} = 15$ under different power/energy constraints. The cases which provide a fair comparison in every aspect are on white background. The parameter φ determines the correlation between the signals of mobile station and relay during the relay-transmit phase. The parameter κ determines what percentage of the source energy is used during the relay-receive phase.

4.4.2 Full Simultaneous Multiple-Access

The strategy with full multiple-access requires a full-duplex relay. The mobile station transmits the block of $M = T \cdot W$ symbols $\mathbf{x}_\mathrm{M}$ with power P_M and the relay transmits the block of M symbols $\mathbf{x}_\mathrm{R}$ with power P_R for the full time ($T_\mathrm{M} = T_\mathrm{R} = T$) in the full bandwidth ($W_\mathrm{M} = W_\mathrm{R} = W$). Figure 4.7 illustrates this strategy. The channel mode is given by

$$\mathbf{y}_\mathrm{MR} = h_\mathrm{MR} \cdot \mathbf{x}_\mathrm{M} + \mathbf{z}_\mathrm{MR} \tag{4.27}$$

$$\mathbf{y}_\mathrm{RB} = h_\mathrm{RB} \cdot \mathbf{x}_\mathrm{R} + h_\mathrm{MB} \cdot \mathbf{x}_\mathrm{M} + \mathbf{z}_\mathrm{RB}. \tag{4.28}$$

The variables and the SNRs are defined as in the last sections. Again, beamforming can be used, if the carrier phases from mobile station and relay are synchronized such that the signals superimpose constructively at the base station. As mentioned previously,

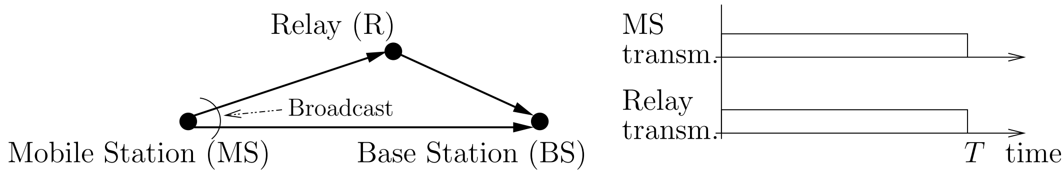


Figure 4.7: Wireless relay communication with broadcast and simultaneous multiple-access: For the example with the SNRs $\tilde{\gamma}_{SR} = 7$, $\tilde{\gamma}_{RD} = 15$ and $\tilde{\gamma}_{SD} = 1$, a transmission rate of $R = 2.60$ bits per channel use can be achieved from source to sink for a fair comparison in every aspect. The strategy requires a full-duplex relay.

a carrier synchronization may not be easily realizable, especially if two or three of the nodes are mobile. The achievable decode-and-forward rate for this strategy was considered in [CG79]. The achievable rate $R = K_{\mathrm{M}}/M$ in bits per channel use under the assumption of Gaussian channels is given by [CG79, Theorem 5]

$$\boxed{R = \min\left\{\mathcal{C}\left((1-\varphi)\cdot\gamma_{\mathrm{MR}}\right), \mathcal{C}\left(\gamma_{\mathrm{RB}} + \gamma_{\mathrm{MB}} + 2\sqrt{\varphi\cdot\gamma_{\mathrm{MB}}\cdot\gamma_{\mathrm{RB}}}\right)\right\}.} \tag{4.29}$$

The SNRs are defined as in the last section. Again, the parameter φ determines the correlation between the signal of mobile station and relay ($0 \leq \varphi \leq 1$). Dependent on the choice how to constrain the resources, we set P_{M} and P_{R} to

- $P_{\mathrm{M}} = P$ and $P_{\mathrm{R}} = P$ for the individual power and the individual energy constraint. Then, the system requires the double amount of total energy compared to the point-to-point system.
- $P_{\mathrm{M}} = \beta\cdot P$ and $P_{\mathrm{R}} = (1-\beta)\cdot P$ for the constraint on the total energy. The parameter β determines what percentage of the total energy is allocated to the mobile station.

The rate R can be maximized over the parameter φ. If the total energy is constrained, the rate can be also maximized over β.

Example 4.5 (continued)

Table 4.5 summarizes two cases for the example with $\tilde{\gamma}_{MB} = 1$, $\tilde{\gamma}_{MR} = 7$ and $\tilde{\gamma}_{RB} = 15$. The full-duplex relay communication with full simultaneous multiple access increases the rate from $R = 2.03$ to $R = 2.60$ bits per channel use.

However, full-duplex relays are regarded as difficult to realize with current technology and most of the current research is focused on half-duplex relays. Distributed channel

Constraint	φ	β	P_M	P_R	E_M	E_R	R/W
Full-Duplex Relay Communication with simultaneous multiple-access							
$P_\mathrm{M} = P_\mathrm{R} = P$	0.00	/	P	P	$P \cdot T$	$P \cdot T$	3.00
$E_\mathrm{M} + E_\mathrm{R} = P \cdot T$	0.03	0.75	$0.75 \cdot P$	$0.25 \cdot P$	$0.75 \cdot P \cdot T$	$0.25 \cdot P \cdot T$	2.60

Table 4.5: Wireless relay communication with broadcast and simultaneous multiple-access: Achievable rate R in bits per time unit for the example with $\tilde{\gamma}_\mathrm{MB} = 1$, $\tilde{\gamma}_\mathrm{MR} = 7$, and $\tilde{\gamma}_\mathrm{RB} = 15$ under different power/energy constraints. This strategy requires a full-duplex relay.

coding schemes for full-duplex relaying were proposed on the basis of turbo codes in [ZD05a] and on the basis of LDPC codes (respectively factor graphs) in [KAA04]. In [Daw04, Chapter 2.2.4]. the author considers the allocation of the transmission power in a system with several relays with broadcast and full simultaneous multiple-access.

For general networks with broadcast and simultaneous multiple-access, an upper bound (termed cut-set bound) on the achievable rate is given in [CT91, Page 445].

4.5 Comparison

In this section, we want to compare the achievable rate for several strategies. We consider the uplink with a variable position of the relay as depicted in Figure 4.8. The SNR on the MS-BS link is given by $\tilde{\gamma}_\mathrm{MB} = 1$ and the MS-BS distance is given by d_MB. The relay has a variable position between mobile and base station and the MS-R and the R-BS distances are given by d_MR and d_RB with $0 \leq d_\mathrm{MR} \leq d_\mathrm{MB}$ and $d_\mathrm{RB} = d_\mathrm{MB} - d_\mathrm{MR}$. The path-loss exponent is set to $n = 3.33$ and the SNRs on the MS-R and the R-BS link are given according to Section 2.2 by $\tilde{\gamma}_\mathrm{MR} = \tilde{\gamma}_\mathrm{MB} \cdot (d_\mathrm{MB}/d_\mathrm{MR})^n$ and by $\tilde{\gamma}_\mathrm{RB} = \tilde{\gamma}_\mathrm{MB} \cdot (d_\mathrm{MB}/d_\mathrm{RB})^n$, respectively. For the relative MS-R distance $d_\mathrm{MR}/d_\mathrm{MB} = 0.557$, we obtain the example which we considered in detail in the last sections. Note that the SNRs are only given by $\tilde{\gamma}_\mathrm{MB}$, $\tilde{\gamma}_\mathrm{MR}$ and $\tilde{\gamma}_\mathrm{RB}$, if the transmission power is given by P and if the full bandwidth W is used. Otherwise the SNRs scale with the power and bandwidth according to (4.4).

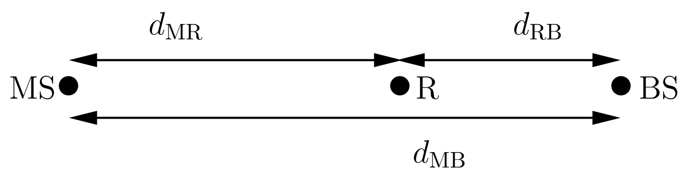


Figure 4.8: We compare relaying strategies dependent on the relative MS-R distance $d_\mathrm{MR}/d_\mathrm{MB}$. The SNR on the MS-BS link is fixed to $\tilde{\gamma}_\mathrm{MB} = 1$ and the path-loss exponent is set to $n = 3.33$.

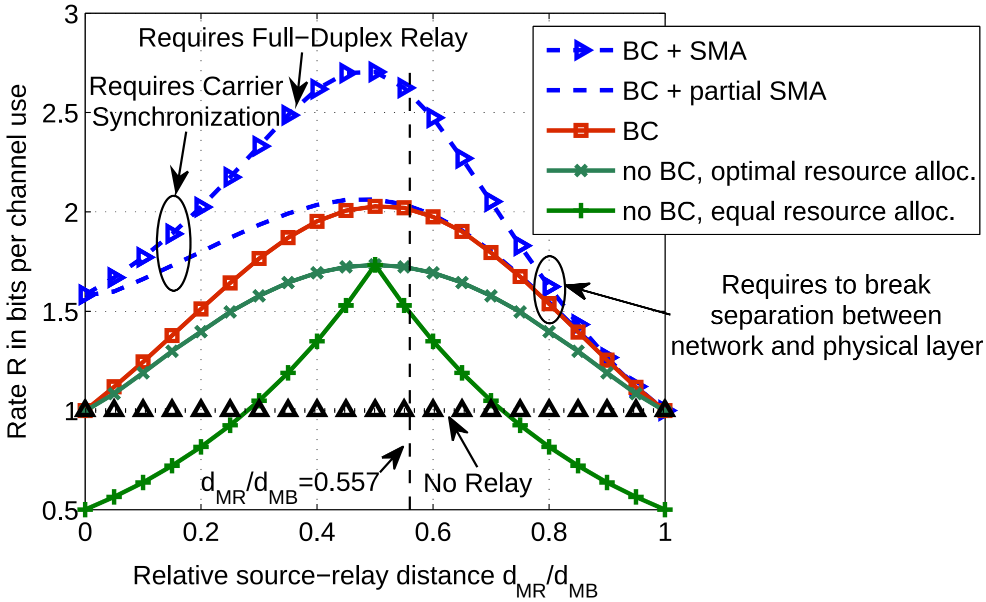


Figure 4.9: Comparison of rate R over the relative MS-R distance for the following decode-and-forward strategies: Full-duplex relaying with broadcast (BC) and simultaneous multiple-access (SMA), relaying with BC and partial SMA (with carrier synchronization), relaying with BC, relaying with optimal resource allocation, relaying with equal resource allocation.

Figure 4.9 depicts the achievable rate R in bits per channel use for several strategies dependent on the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$. The achievable rates are determined under the total energy constraint $E_{\mathrm{M}} + E_{\mathrm{R}} = P \cdot T$ and it is assumed that the channel inputs are Gaussian distributed. The resource allocation parameters are numerically optimized, except for the relaying with equal resource allocation. Then, we assume equal time-division with equal transmission powers at source and relay ($P_{\mathrm{M}} = P_{\mathrm{R}} = P$). The point-to-point communication without relay achieves a rate of $R = 1$ bits per channel use independent of the relay position. First, we consider the relay communications which represent point-to-point networks. That means that the wireless broadcast nature is not exploited. The relay communication with equal resource allocation only outperforms the point-to-point communication, if the relay is not close to mobile or base station. Contrary, the relay communication with optimal resource allocation outperforms the point-to-point communication also for relay positions close to mobile or base station. These strategies are regarded as easy to implement in practical systems.

Next, we consider the relay communication with broadcast transmission (BC). Then, it is exploited that the base station can listen to the transmission of the mobile station and the rate can be increased. The broadcast transmission requires to break with the separation between network layer and physical layer. Nevertheless, the implementation in practical systems is regarded as practical for small networks such as our example with one relay.

The strategy with BC and partial simultaneous multiple access (SMA) allows beamforming. This allows to increase the rate for relay positions close to the mobile station.

Even if the relay is very close to the mobile station[5], the rate can be increased by more than 50% in our example. As beamforming requires to synchronize the carrier phases of mobile station and relay such that the signal superimpose constructively at the base station, the implementation is not regarded as practical with current technology [BL06]. The rate of the strategy with BC and partial SMA without beamforming is given by (4.26) for $\varphi = 0$. We calculated the rate over the relative source-relay distance. For the sake of clarity, the rate is not depicted in Figure 4.9 because it is not possible to distinguish the curve from the rate with broadcast and without partial SMA. Therefore, we conclude that the partial SMA does not help for this example, if the carrier phases are not synchronized and if the total energy is constrained. If the individual transmission power is constrained, the partial SMA allows to increase that rate at the cost of an increased total energy.
The strategy with full-duplex relaying with broadcast (BC) and simultaneous multiple access (SMA) clearly outperforms the other strategies. However, the full-duplex requirement is a crucial handicap for the application in practical systems.
We only considered decode-and-forward strategies in this chapter. If the relay is close to the sink, the relay can also be helpful, if it has not decoded the information without error. Such strategies are termed compress-and-forward or estimate-and-forward. These strategies allow to increase the rate for relays close to the sink compared to decode-and-forward (see [KGG05, Figure 16]). Practical coding schemes according to the compress-and-forward idea were considered in [SV05, YVT+05] for a network with one source and one relay and in [YK07] for a network with two sources and one relay. The coding schemes are based on the idea that the relay does not make a hard decision, but processes and forwards soft information.

The results in Figure 4.9 are based on the assumption that the channel inputs are Gaussian distributed. In practical systems the channel inputs will be from a discrete alphabet. In order to obtain the achievable rate R under the assumption that a 4-QAM constellation is used, we replace $\mathcal{C}(\cdot)$ by $\mathcal{C}_4(\cdot)$ in (4.5), (4.18), (4.26) and (4.29). The function $\mathcal{C}_4(\cdot)$ is defined in (2.9). It can be precalculated and stored in a look-up table for the calculations of the achievable rate of the relay communications. Figure 4.10 depicts the achievable rate R in bits per channel use for several strategies under the assumption of 4-QAM. Contrary to the Gaussian distributed channel input, the function $\mathcal{C}_4(\cdot)$ is limited by $L = 2$, even for very high SNRs. Therefore, the relay communications without broadcasts (point-to-point networks) are not able to achieve a larger rate than one. The benefit compared to the point-to-point communication without relay is very small. If the relay is close to mobile or base station, the relay communication loses rate compared to the case without relay.
The relay communication with broadcast clearly outperforms the relay communications without broadcast. As the point-to-point communication is a special case of the relay communication with broadcast, the rate is never lower than the one of the point-to-point communication.

[5]Beamforming does not work, if the mobile station and relay antenna are at exactly the same position. Beamforming requires a small distance between two antennas, approximately half of the wavelength.

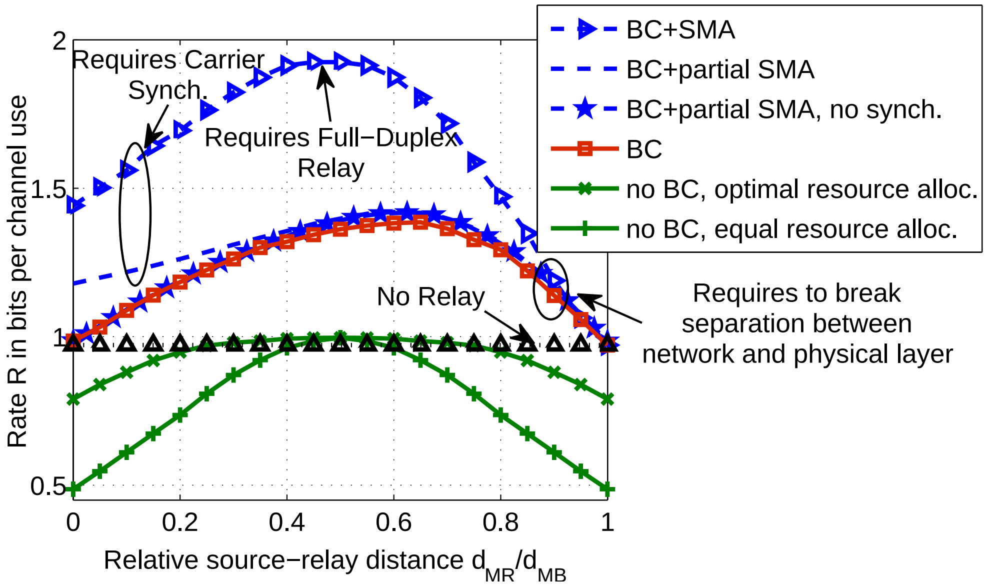


Figure 4.10: Comparison of rate R under the assumption of 4-QAM for the following decode-and-forward strategies: Full-duplex relaying with broadcast (BC) and simultaneous multiple-access (SMA), relaying with BC and partial SMA (with and without carrier synchronization), relaying with BC, relaying with optimal time/frequency allocation, relaying with equal time/frequency allocation.

The relay communication with broadcast and partial SMA achieves a beamforming gain for relay positions close to the mobile station. We also depict the rate, if no carrier synchronization is available ($\varphi = 0$). Then, a very small rate gain can be achieved compared to the system without partial SMA for relative source-relay distances around $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.6$. However, we do not think that this small gain justifies to make the system more complicated by including partial SMA.
Again, the relay communication with broadcast and full SMA achieves the highest rate at the cost of a full-duplex relay.

From the comparison of the relaying strategies unter the total energy constraint, we conclude that the relay communication with broadcast provides a good trade-off between performance and technical practicality. We want to consider this system in more detail. Time-division relaying is regarded as advantageous compared to frequency-division, because the allocation of the transmission time can be adapted easier than the less flexible frequency allocation [HMZ05]. Therefore, we only consider time-division. Under the total energy constraint, we can optimize the time-allocation parameter θ and the energy-allocation parameter β. The variable parameters require that the transmission powers of mobile station and relay are controlled dependent on the channel SNRs. We want to consider a simplified system without power control. Then, the transmission powers P_{M} and P_{R} at mobile station and relay are constrained to $P_{\mathrm{M}} = P_{\mathrm{R}} = P$ and it is only necessary to optimize the time-allocation parameter θ. The achievable rate R is

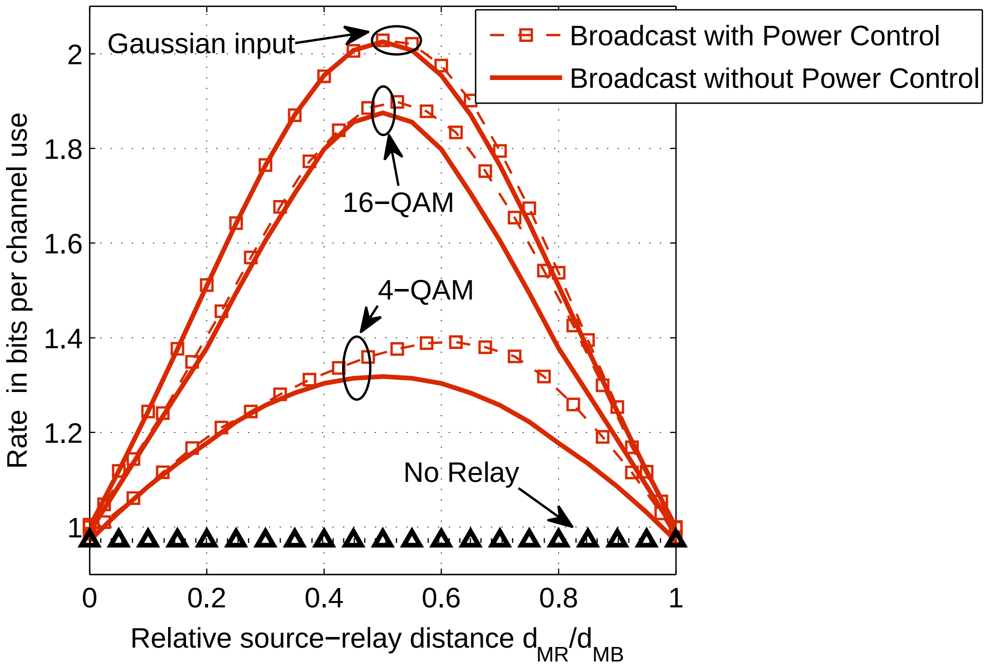


Figure 4.11: Influence of power control for time-division relaying with broadcast.

given by the closed expression in (4.22) and it is not necessary to optimize numerically. We want to know the rate penalty, if no power control is used. Figure 4.11 depicts the rate R for the relay communication with broadcast both with and without power control. For the Gaussian channel input, the rate penalty is very small, if no power control is used. For 16-QAM and 4-QAM, there is a rate penalty around the relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.7$. For example, if we use power control for 4-QAM at $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.6$, the mobile station transmits with power $P_{\mathrm{M}} = (\beta/\theta) \cdot P = 1.29 \cdot P$ for $\theta = 71\%$ of the time, the relay transmits with power $P_{\mathrm{R}} = ((1-\beta)/(1-\theta)) \cdot P = 0.30 \cdot P$ for $(1-\theta) = 29\%$ of the time and a rate of $R = 1.39$ bits per channel use is achieved. If no power control is used, the mobile station transmits with power $P_{\mathrm{M}} = P$ for $\theta = 68\%$ of the time and the relay transmits with power $P_{\mathrm{R}} = P$ for $(1-\theta) = 32\%$ of the time and a rate of $R = 1.30$ bits per channel use is achieved.
Although power control can be useful for time-division relaying in certain situations, we will use the simpler system without power control in the following chapters of this thesis.

We want to consider the achievable rate for a variable position of both relay and base station. The relay is always on half-way between mobile and base station ($d_{\mathrm{MR}} = d_{\mathrm{RB}} = d_{\mathrm{MB}}/2$). The MS-BS distance is variable and the SNR on the MS-BS link γ_{MB} is in the range between -10 dB and 20 dB. Again, we set the path-loss exponent to $n = 3.33$. Then, the SNRs on the MS-R and the R-BS link are increased by $33.3 \cdot \log_{10}(2) = 10.0$ dB compared to the MS-BS link. Figure 4.12 depicts the rate R over the SNR on the MS-BS link for the time-division relay communication with broadcast (BC) on the left side and without broadcast on the right side. Power control is not used. For comparison, we also

show the rate for the point-to-point communication without relay, which has already been shown in Figure 2.4, and the rate for the full-duplex relay communication with broadcast and simultaneous multiple access (SMA). The relay communication with broadcast can gain up to 5 dB compared to the point-to-point communication and achieves clearly higher rates than the relay communication without broadcast. Contrary to the relaying scheme without broadcast, the scheme with broadcast guarantees that the rate is not smaller than the rate without relay. The rate for the relaying scheme without broadcast is obtained graphically by first shifting the rate of the point-to-point communication by $33.3 \cdot \log_{10}(2) = 10.0$ dB to the left and then dividing the rate by a factor of two.
Figure 4.13 depicts the rate R over the SNR on the MS-BS link for the time-division relay communication with broadcast (BC) for the path-loss exponent $n = 3.52$ (used in the UMTS model [HT01, Equation (8.1)]). The gain from relaying increases slightly compared to the comparison for the path-loss exponent $n = 3.33$ in Figure 4.12.

We conclude from our comparison that the use of broadcast transmission in wireless networks promises significant gains compared to the simpler strategy to realize wireless communication as a point-to-point network without broadcast transmissions. As the utilization of broadcast transmissions requires to break the separation between network and physical layer, it is mainly interesting for small networks, for example communication with the help of one relay. In the next chapters, we will consider practical coding schemes which should allow to exploit the promised gain from broadcast transmissions.

4.6 Summary

We compared several wireless relaying strategies and concluded that wireless relaying with broadcast transmissions provides a good trade-off between performance and technical feasibility. Then, the source (e.g. mobile station) broadcasts to relay and sink (e.g. base station) during the relay-receive phase and the relay transmits to the sink during the relay-transmit phase. Contrary to relaying without broadcast transmission, the broadcast strategy contains the point-to-point communication as special case. We showed that relaying with broadcast promises to gain up to 5 dB compared to the point-to-point communication for a realistic path-loss exponent of $n = 3.52$. The strategy requires to break with the separation of network and physical layer and to find new error-correction schemes. We will describe in the next chapters how such schemes can be designed based on practical channel codes (e.g. turbo codes or LDPC codes).
We also included more complicated wireless relaying strategies with simultaneous multiple access (e.g. source transmits also during relay-transmit phase) in our comparison and concluded that these strategies are regarded as difficult to realize with current technology, because they are based on a carrier phase synchronization at source and relay such that the signals superimpose constructively at the sink or require a full-duplex relay. If the synchronization cannot be established and if we constrain the total transmission energy, there is no significant performance gain from a second transmission of the source during the relay-transmit phase in the considered examples.

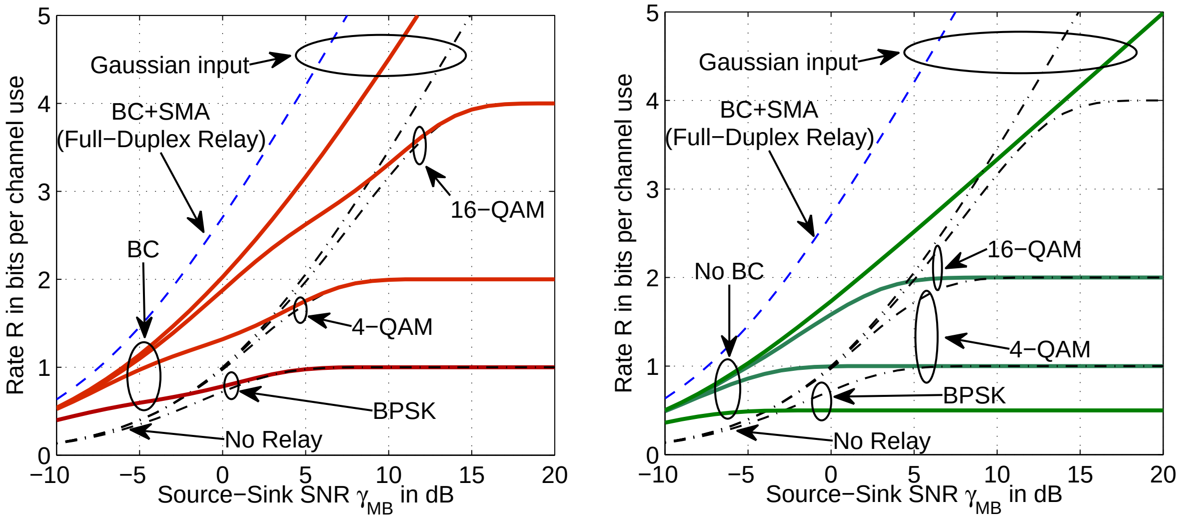


Figure 4.12: Achievable rate R for time division relaying with broadcast (left side) and without broadcast (right side) for a path-loss exponent of $n = 3.33$. The dashed-dot lines depict the rate for the point-to-point communication without relay.

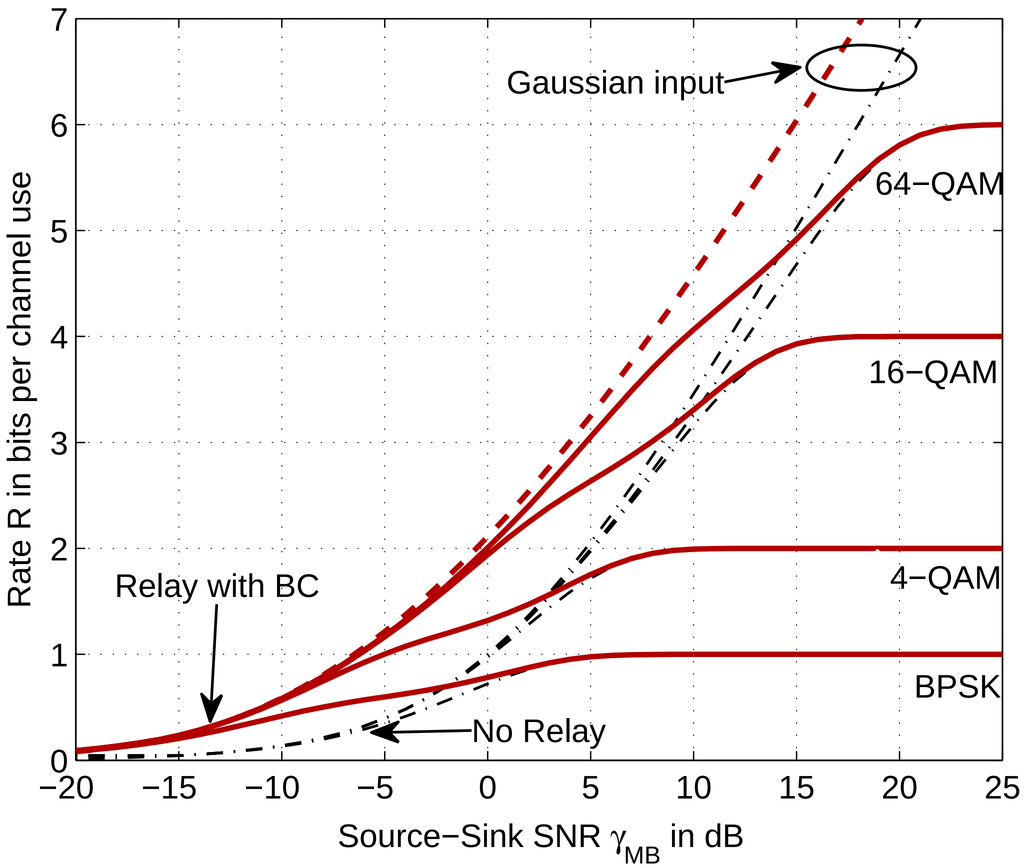


Figure 4.13: Achievable rate R for time division relaying with broadcast for a path-loss exponent of $n = 3.52$. The dashed-dot lines depict the rate for the point-to-point communication without relay.

5

Joint Routing-Channel Coding for the Relay Channel

We concluded in the last chapter that relaying with a broadcast transmission as described in Section 4.3 provides a good trade-off between performance and complexity. The broadcast transmission allows a significant increase of the data rate. The concept requires to break with the separation of routing in the network layer and channel coding in the physical layer and affords new coding schemes. In this chapter, we will explain how joint routing and channel coding schemes can be easily realized with distributed channel codes on the basis of rate-compatible punctured channel codes. Distributed channel coding was investigated based on convolutional codes [SE04,HN06], on parallel concatenated convolutional codes (turbo codes) [ZV03,SV04] and on serial concatenated convolutional codes [CV05]. The distributed channel coding schemes on the basis of LDPC codes in [CdBSA07] can be easily simplified to realize a relay communication with broadcast.
The diversity gain of relays was first considered in [SEA03] and in [LTW04] for Gaussian distributed channel input. Distributed channel coding was termed cooperative coding in [SEA03] and in [HN06] because a system with two sources was considered which co-operate and are relays for each other. As we only have one source, we prefer the term distributed channel coding. Cooperation with distributed channel coding is explained with illustrations in [LTL+06] and in [NHH04].
We will extend distributed channel coding in the following two directions:

- We will show how to extend distributed channel coding by using **hierarchical modulation** for the broadcast from source to relay and sink. This allows to reduce the computational complexity at the sink. As efficient relaying requires modulation constellations of high order, it can be useful to reduce the complexity for demodulation at the sink. In certain situations, the performance can be improved by using a non-equidistant constellation.

- In the last chapter, we derived how to optimally share the total transmission time between source and relay. We will apply this result to the practical coding system. Simulation results will confirm the usefulness of the **optimal time allocation**.

Beside the increase of the data rate, relaying allows to increase the robustness by providing spatial diversity in a fading environment. We will analyze the outage behavior of the considered relay system for coded modulation. Moreover, we will examine with Monte-Carlo simulations whether the coding scheme can realize the rate gain promised in the last chapter and the promised diversity gain for the considered examples.

5.1 From Hybrid ARQ/FEC to Distributed Channel Coding

We consider relaying with broadcast transmission for the uplink[1] in a cellular based mobile communication system as depicted in Figure 4.5. The mobile station wants to transmit a packet $\mathbf{u}_{\mathrm{M}}$ of K_{M} information bits to the base station with the help of a relay. We consider distributed channel coding for the time-division relay channel where the mobile station broadcasts to the relay and the base station in the first time slot (channel 1 or relay-receive phase) and the relay transmits to the base station in the second time slot (channel 2 or relay-transmit phase). If we use a conventional channel code for the point-to-point channel, the packet $\mathbf{u}_{\mathrm{M}}$ of K_{M} information bits is encoded to a block of N code bits which are sent via the modulated channel to the base station. Distributed channel coding means that the block of N is divided into two segments of lengths N_{M} and N_{R} with $N = N_{\mathrm{M}} + N_{\mathrm{R}}$. The first segment with N_{M} code bits is sent by the mobile station via the modulated channel 1 and and the other segment with N_{R} code bits is sent by the relay via the modulated channel 2. That means that one channel code is distributed to mobile station and relay. The relay listens to the transmission in channel 1 and decodes the information bits from the N_{M} code bits, reencodes the data to obtain the N_{R} code bits in the second segment. As the relay is assumed between the mobile and the base station, N_{M} code bits are sufficient to protect the data against transmission errors at the relay. If the relay only transmits in case of a request from the base station, distributed channel coding can be seen as a generalization of hybrid ARQ/FEC (H-ARQ) of type II [ZV05]. Instead of retransmitting the second segment with N_{R} code bits again from the source, the N_{R} code bits are transmitted from the relay. Of course distributed channel coding can be also implemented such that the transmission of the relay is done in any case. Then, feedback for request is not required. There are the following two advantages we can expect from using distributed channel coding compared to a H-ARQ system without relay:

- If the relay is closer to the sink than the source, the additional redundancy of the N_{R} code bits in the second segment has a better quality than transmissions from the

[1]The described system could be used for the downlink in the same way.

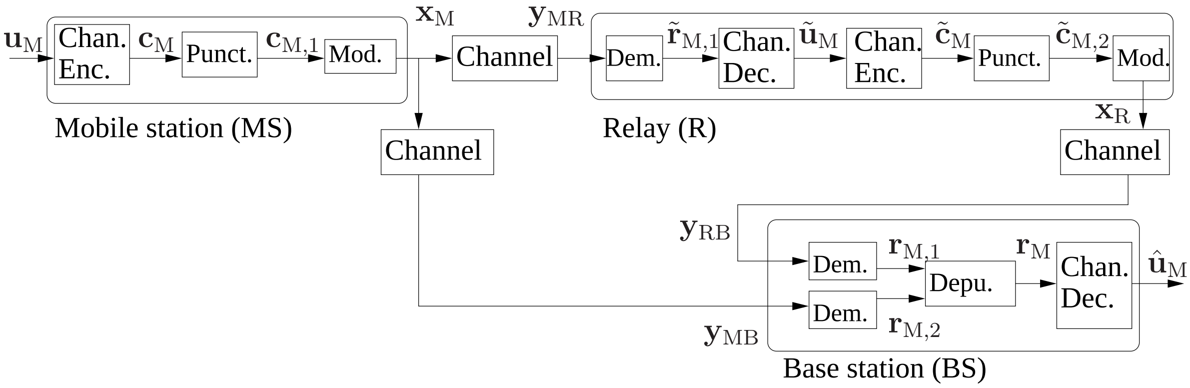


Figure 5.1: Block diagram of a distributed channel coding system.

source. Therefore, a better error protection can be achieved compared to a point-to-point communication. To provide the same error protection, we need a lower number of total code bits N for the relay channel compared to the point-to-point channel and thus, a higher rate and spectral efficiency can be achieved.

- The second transmission from the relay is transmitted from another position than the first transmission from the source. Both the fast fading due to multipath propagation and the slow fading due to shadowing can be assumed independent for the first and the second transmission. This allows the sink to gain diversity from the reception of the two transmissions. Contrary to H-ARQ, distributed channel coding provides also diversity against shadowing. Moreover, diversity can be gained even if the transmission from the relay is within the coherence time of the MS-BS channel. If the relay is positioned in a clever way such that there is always a line-oft-sight connection to the base station, the characteristic of the R-BS channel will be advantageous compared to the MS-BS channel.

We will assume in the following that the relay does not wait for a request and that it transmits in any case, if it decodes successfully.

5.2 System Description

Let us describe the system model in detail. For our simulation, we will use a distributed turbo code similar to the one proposed in [SV04]. Fig. 5.1 depicts the block diagram of the distributed channel coding system. The packet $\mathbf{u}_{\mathrm{M}}$ of K_{M} information bits is turbo encoded at the mobile station. The channel encoder outputs a block $\mathbf{c}_{\mathrm{M}}$ of N code bits. The K_{M} information bits include cyclic redundancy check (CRC) bits as error detection code. The puncturing block (punct.) chooses from $\mathbf{c}_{\mathrm{M}}$ a subset $\mathbf{c}_{\mathrm{M},1}$ which contains N_{M} code bits. The puncturing is done in a regular way, which is similar to the rate matching strategy described in [Eur00]. We do not puncture systematic and tail bits. The code bits $\mathbf{c}_{\mathrm{M},1}$ are modulated to the block $\mathbf{x}_{\mathrm{M}}$ containing $M_{\mathrm{M}} = N_{\mathrm{M}}/L$ symbols, whereas L is

the number of code bits carried by one symbol. The mobile station uses the transmission power P_M to broadcast $\mathbf{x}_\mathrm{M}$ to the relay and the base station.
The demodulation at the relay receives in the first time slot the disturbed symbols from the mobile station $\mathbf{y}_\mathrm{MR}$ and outputs LLRs $\tilde{\mathbf{r}}_{\mathrm{M},1}$ about the code bits $\mathbf{c}_{\mathrm{M},1}$. Based on $\tilde{\mathbf{r}}_{\mathrm{M},1}$, the channel decoder outputs an hard estimate $\tilde{\mathbf{u}}_\mathrm{M}$ about $\mathbf{u}_\mathrm{M}$. If the CRC indicates that the relay decoded successfully, the estimate $\tilde{\mathbf{u}}_\mathrm{M}$ is encoded with the same turbo code as at the mobile station to the block of code bits $\tilde{\mathbf{c}}_\mathrm{M}$. The puncturing block chooses from $\tilde{\mathbf{c}}_\mathrm{M}$ the subset $\tilde{\mathbf{c}}_{\mathrm{M},2}$ which contains N_R code bits. The subsets $\mathbf{c}_{\mathrm{M},1}$ and $\tilde{\mathbf{c}}_{\mathrm{M},2}$ are chosen to be disjoint. The code bits $\tilde{\mathbf{c}}_{\mathrm{M},2}$ are modulated to the block $\mathbf{x}_\mathrm{R}$ containing $M_\mathrm{R} = N_\mathrm{R}/L$ symbols. The relay uses the transmission power P_R to transmit $\mathbf{x}_\mathrm{R}$ to the base station. If the CRC indicates non-successful decoding, there are the following option what to do:

- The relay remains silent and does not transmit any symbols to circumvent error propagation to the sink. That means that only M_M symbols are transmitted in total.

- The relay ignores the CRC indication and reencodes the decoded information bits anyway. There is the danger that the errors at the relay propagate to the base station.

- If the relay is able to notify the mobile or base station that it decoded wrongly, using one bit in a feedback control channel, then the system can be extended in the following way.
  In case the relay decoded with error, the mobile station is notified that it has to transmit additional M_R symbols to the base station. Then, the system falls back to a point-to-point channel. The system transmits in total $M_\mathrm{M} + M_\mathrm{R}$ symbols, regardless whether the relay decodes with error or not.

- The system can be extended with soft relaying [SV05, YVT+05]. Then, the relay does not make a hard decision on the information bits in case of non-successful decoding, but forwards reencoded soft information to the base station.

The comparison of these schemes is beyond the scope of the thesis. We assume in the rest of the thesis that the relay remains silent in case of non-successful decoding.
The base station receives in the first time slot the disturbed symbols from the mobile station $\mathbf{y}_\mathrm{MB}$ and in the second time slot the disturbed symbols from the relay $\mathbf{y}_\mathrm{RB}$. The channel output $\mathbf{y}_\mathrm{MB}$ is demodulated to obtain LLRs $\mathbf{r}_{\mathrm{M},1}$ about $\mathbf{c}_{\mathrm{M},1}$ and $\mathbf{y}_\mathrm{RB}$ is demodulated to obtain LLRs $\mathbf{r}_{\mathrm{M},2}$ about $\tilde{\mathbf{c}}_{\mathrm{M},2}$. The depuncturing (depu.) combines LLRs $\mathbf{r}_{\mathrm{M},1}$ about $\mathbf{c}_{\mathrm{M},1}$ and the LLRs $\mathbf{r}_{\mathrm{M},2}$ about $\tilde{\mathbf{c}}_{\mathrm{M},2}$ to LLRs $\mathbf{r}_\mathrm{M}$ about $\mathbf{c}_\mathrm{M}$. The LLRs $\mathbf{r}_\mathrm{M}$ are used as input for a conventional turbo decoder as described in Section 2.4.2. The turbo decoder outputs the estimate $\hat{\mathbf{u}}_\mathrm{M}$ after several iterations.
In total, $M = M_\mathrm{M} + M_\mathrm{R}$ symbols are transmitted and the rate R in bits per channel use is given by $R = K_\mathrm{M}/M$. The time allocation parameter θ, which we introduced in Section 4.3, defines $M_\mathrm{M} = \theta \cdot M$ and $M_\mathrm{R} = (1-\theta) \cdot M$. We derived the value for θ which maximizes the achievable rate in (4.21).

The three channels are described in (4.15)-(4.17). The SNRs on the MS-BS link, MS-R link and the R-BS link are given by

$$\gamma_{\mathrm{MB}} = \frac{|h_{\mathrm{MB}}|^2 \cdot P_{\mathrm{M}}}{N_0 \cdot W} \tag{5.1}$$

$$\gamma_{\mathrm{MR}} = \frac{|h_{\mathrm{MR}}|^2 \cdot P_{\mathrm{M}}}{N_0 \cdot W} \tag{5.2}$$

$$\gamma_{\mathrm{RB}} = \frac{|h_{\mathrm{RB}}|^2 \cdot P_{\mathrm{R}}}{N_0 \cdot W}. \tag{5.3}$$

If the mobile station and the relay use the same transmission power $P_{\mathrm{M}} = P_{\mathrm{R}} = P$, the relation between the three SNRs γ_{MB}, $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{MR}})^n$ and $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{RB}})^n$ only depends on the distances d_{MB}, d_{MR} and d_{RB} and on the path-loss exponent n.
We consider the case with fading and without fading.

5.3 Extension with Hierarchical Modulation

In this section, we want to show how distributed channel coding can be extended with hierarchical modulation [HH07]. Before we describe the extended system, we motivate why hierarchical modulation can be useful.

5.3.1 Motivation to use Hierarchical Modulation

It is important to use a modulation scheme of a high order to exploit the high SNRs due to the relay. This can be recognized by taking a look at the achievable rates for relaying shown in Figure 4.13. As the SNR of the MS-BS link is smaller than the SNR of the MS-R link, a modulation scheme with lower order would be more suitable for the MS-BS link than for the link to the relay. The broadcast from the mobile station leads to the dilemma whether to adjust the order of the modulation scheme according to the SNR to the relay or according to the SNR to the base station.
A solution for this dilemma is the use of hierarchical modulation which is for example included in the digital video broadcast (DVB) standard [Eur04]. Hierarchical modulation means to use a modulation scheme which can be interpreted as a scheme with higher order by the receivers close to the transmitter and as a scheme with lower order by the receivers far from the transmitter. Such constellations can be interpreted as a superposition of several modulated signals. For example, a 4-QAM in 16-QAM constellation can be interpreted as a superposition of two 4-QAM constellations, where one of the 4-QAM signals uses a higher percentage of the total power than the other 4-QAM signal. It can be advantageous to use non-equidistant constellations for hierarchical modulation. The advantage of broadcast strategies, such as hierarchical modulation, was first shown in information theory in [Cov72]. The design for a real system applying such broadcast strategies was first described in [ROUV93]. The design of new hierarchical constellations was considered in [See05]. Broadcast strategies which are based on superposition are considered in many information-theoretic investigations about the relay

channel (e.g. [CG79, HMZ05, YE04, PdC07]). Beside in [HH07], other practical methods based on broadcast strategies for relay communication were proposed for example in [LV05a] and [LC07].
We describe in the following how a relay communication system with distributed turbo codes can be extended with the application of a hierarchical 4-QAM in 16-QAM constellation. This allows to reduce the computational complexity for the demodulation at the base station compared to the use of conventional 16-QAM at the price of a performance loss. We will evaluate in Section 5.7 the performance loss for several examples by simulation. Moreover, we will show in Section 5.7 for AWGN channels that the application of non-equidistant constellations can achieve a gain in term of bit and packet error rates in certain situations.
The proposed method can be easily extended to modulation constellations with higher order (e.g. 64-QAM) and can also be used for other distributed channel codes than distributed turbo codes.

5.3.2 System Description

We extend the system which we described in Section 5.2 with a hierarchical 4-QAM in 16-QAM constellation. For simplicity, we assume that the number of symbols M_{M} transmitted from the mobile station is half of the number of information bits K_{M} and that the number of symbols M_{R} transmitted from the relay is one quarter of the number of information bits K_{M} ($M_{\mathrm{M}} = K_{\mathrm{M}}/2$, $M_{\mathrm{R}} = K_{\mathrm{M}}/4$). The system can be easily adapted to other choices for M_{M} and M_{R} by changing the puncturing rule. A block diagram of the system is depicted in Fig. 5.2. The system is extended in the following aspects. The puncturing block chooses from $\mathbf{c}_{\mathrm{M}}$ two subsets $\mathbf{c}_{\mathrm{M,C}}$ and $\mathbf{c}_{\mathrm{M,E}}$, each with $2 \cdot M_{\mathrm{M}}$ bits. The block $\mathbf{c}_{\mathrm{M,C}}$ contains the common bits which are supposed to be detected in the first time slot at both the relay and the base station. The block $\mathbf{c}_{\mathrm{M,E}}$ contains the enhancement bits which are only supposed to be detected in the first time slot at the relay. We choose the systematic bits as common bits $\mathbf{c}_{\mathrm{M,C}} = \mathbf{u}_{\mathrm{M}}$. The enhancement bits in $\mathbf{c}_{\mathrm{M,E}}$ include every second redundancy bits[2] from each stream of the two parallel concatenated encoders within the turbo encoder. We want to use a non-equidistant 4-QAM in 16-QAM constellation. Such a constellation is constructed as follows. The common bits $\mathbf{c}_{\mathrm{M,C}}$ are mapped to M_{M} 4-QAM symbols $\mathbf{x}_{\mathrm{C}}$ and the enhancement bits $\mathbf{c}_{\mathrm{M,E}}$ are mapped to M_{M} 4-QAM symbols $\mathbf{x}_{\mathrm{E}}$. The symbols $\mathbf{x}_{\mathrm{M}}$ of the 4-QAM in 16-QAM constellation, which are transmitted by the mobile station, are constructed by superpositioning the two 4-QAM symbols $\mathbf{x}_{\mathrm{C}}$ and $\mathbf{x}_{\mathrm{E}}$:

$$\mathbf{x}_{\mathrm{M}} = \sqrt{1-\alpha} \cdot \mathbf{x}_{\mathrm{C}} + \sqrt{\alpha} \cdot \mathbf{x}_{\mathrm{E}} \tag{5.4}$$

The parameter α determines what percentage of the transmission power P is allocated to $\mathbf{x}_{\mathrm{E}}$. Fig. 5.3 depicts the 4-QAM in 16-QAM constellation for $\alpha = 0.02$. That means that 2% of the power is used to transmit the enhancement bits and 98% of the power is used to transmit the common bits. This assures that the common bits are much better protected than in a conventional, equidistant constellation. The equidistant 16-QAM constellation is obtained, if we choose the parameter α to be $\alpha = 0.2$. For $\alpha = 0$, we obtain an

[2] As we want to include also every second of the twelve tail bits, we omit six other redundancy bits.

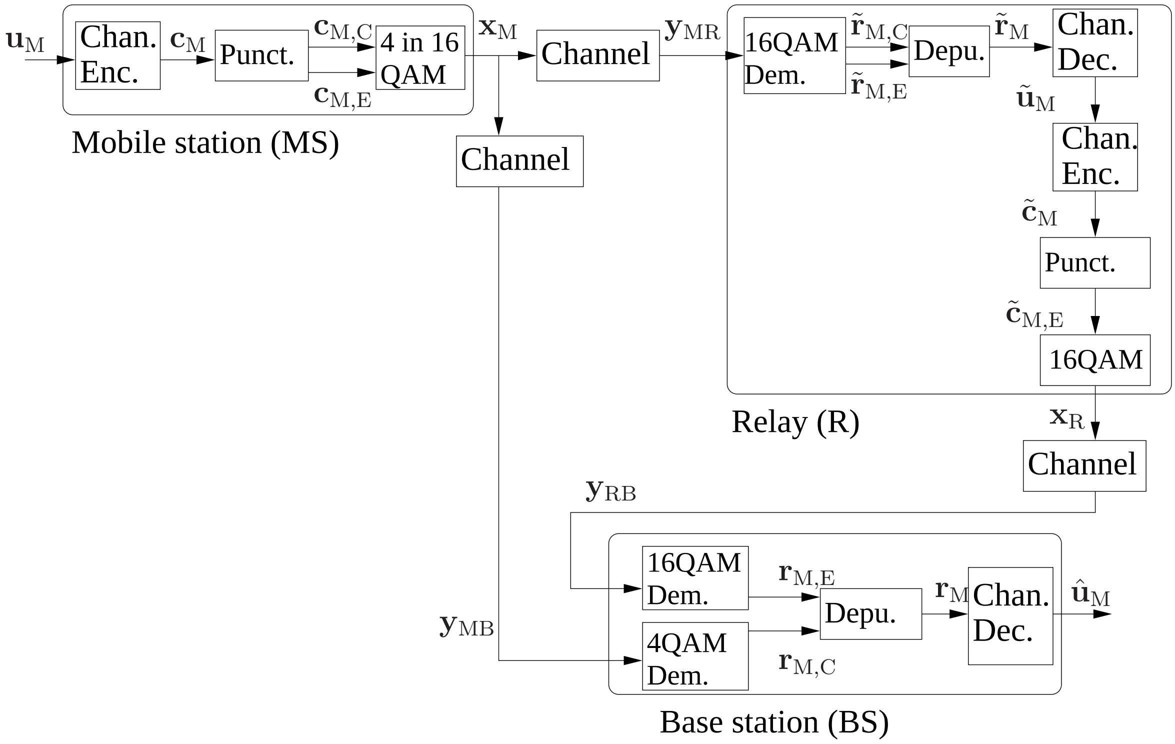


Figure 5.2: Block diagram of the communication from the mobile station to the base station with the help of a relay. The mobile station uses hierarchical 4-QAM in 16-QAM for the broadcast to the relay and to the base station.

equidistant 4-QAM constellation, which carries only the common bits. The hierarchical modulation (hier. mod.) maps two bits from $\mathbf{c}_\mathrm{M,C}$ and two bits from $\mathbf{c}_\mathrm{M,E}$ to one symbol. As symbols at minimum Euclidean distance differ in one bit, the constellation in Figure 5.3 is a Gray mapping. The first two bits are from $\mathbf{c}_\mathrm{M,C}$, the other two bits are from $\mathbf{c}_\mathrm{M,E}$. The mobile station transmits the block $\mathbf{x}_\mathrm{M}$ with M_M symbols in the first time slot.

The 16-QAM demodulation at the relay receives in the first time slot the disturbed symbols from the mobile station $\mathbf{y}_\mathrm{MR}$ and outputs LLRs $\tilde{\mathbf{r}}_\mathrm{M,C}$ about the common bits $\mathbf{c}_\mathrm{M,C}$ and LLRs $\tilde{\mathbf{r}}_\mathrm{M,E}$ about the enhancement bits $\mathbf{c}_\mathrm{M,E}$. The depuncturing combines the LLRs $\tilde{\mathbf{r}}_\mathrm{M,C}$ and $\mathbf{c}_\mathrm{M,E}$ to LLRs $\tilde{\mathbf{r}}_\mathrm{M}$ about the code bits $\mathbf{c}_\mathrm{M}$. Based on $\tilde{\mathbf{r}}_\mathrm{M}$, the turbo decoder outputs an hard estimate $\tilde{\mathbf{u}}_\mathrm{M}$ about $\mathbf{u}_\mathrm{M}$ after several iterations. If the CRC indicates that the relay decoded successfully, the estimate $\tilde{\mathbf{u}}_\mathrm{M}$ is encoded with the same channel code as at the mobile station to the block $\tilde{\mathbf{c}}_\mathrm{M}$. The puncturing block chooses from $\tilde{\mathbf{c}}_\mathrm{M}$ the subset $\tilde{\mathbf{c}}_\mathrm{M,E}$ of $N_\mathrm{R} = 4 \cdot M_\mathrm{R}$ code bits, which are mapped to M_R equidistant 16-QAM symbols. The block $\mathbf{x}_\mathrm{R}$ of M_R symbols is sent to the base station in the second time slot. If the CRC indicates non-successful decoding, there are the same options as in the system without hierarchical modulation.

The base station receives in the first time slot the disturbed symbols from the mobile station $\mathbf{y}_\mathrm{MB}$. The demodulation treats $\mathbf{y}_\mathrm{MB}$ as a block of 4-QAM symbols and only outputs LLRs $\mathbf{r}_\mathrm{M,C}$ about the common bits $\mathbf{c}_\mathrm{M,C}$. As the base station treats $\sqrt{\alpha}\cdot\mathbf{x}_\mathrm{E}$ as interference,

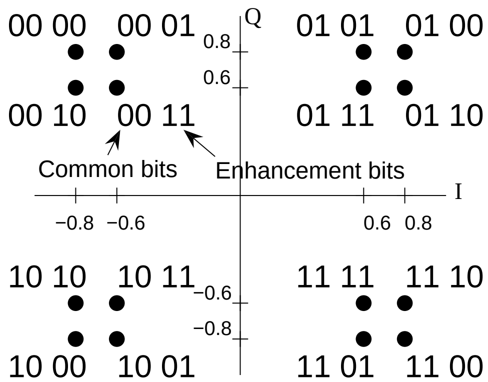


Figure 5.3: Mapping for the hierarchical modulation at the mobile station for a non-equidistant constellation with $\alpha = 0.02$. The common bits in $\mathbf{c}_{\mathrm{M,C}}$ are better protected than the enhancement bits in $\mathbf{c}_{\mathrm{M,E}}$.

we have to use the signal-to-interference-and-noise ratio

$$\hat{\gamma}_{\mathrm{MB}} = \frac{(1-\alpha)\cdot\gamma_{\mathrm{MB}}}{1+\alpha\cdot\gamma_{\mathrm{MB}}} \tag{5.5}$$

for the demodulation. If the relay decoded successfully, the base station receives in the second time slot the disturbed symbols from the relay $\mathbf{y}_{\mathrm{RB}}$ which are demodulated to the LLRs $\mathbf{r}_{\mathrm{M,E}}$ about the enhancement bits $\mathbf{c}_{\mathrm{M,E}}$. The turbo decoder at the base station uses both the LLRs $\mathbf{r}_{\mathrm{M,C}}$ and $\mathbf{r}_{\mathrm{M,E}}$ about the common and about the enhancement bits to output the estimate $\hat{\mathbf{u}}_{\mathrm{M}}$ after several iterations.
We will show simulation results for the proposed system in Section 5.7.1.

In the information-theoretic literature it is often proposed to split the information into two packets which are separately encoded. The two codewords are then superimposed. One of the packets is transferred over the direct MS-BS link, the other packet is transferred over the relay link. For a practical wireless system, our proposed approach with one distributed channel code has the following advantages. First, it is possible to gain diversity with a distributed code because the codebits of one channel code are transferred both over the MS-BS link and the R-BS link. Second, the codelength gets shorter, if we split the information into two packets.

5.4 Information-Theoretic Limits

The achievable decode-and-forward rate for the relay communication with a broadcast transmission as considered in this chapter is discussed in Section 4.3. Let us shortly summarize the important conclusions.
For $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$, the achievable decode-and-forward rate $R = K_{\mathrm{M}}/M$ for the relay

communication with broadcast is given in (4.18) by

$$\boxed{R = \min\{\theta \cdot \mathcal{C}(\gamma_{\mathrm{MR}}), \theta \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1-\theta) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\}}$$

with $\theta = M_{\mathrm{M}}/M$.
For $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$ and $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{RB}})$, the achievable rate R is given in (4.22) by

$$\boxed{R = \frac{\mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})},}$$

if θ is chosen to the optimal value (cf. (4.21))

$$\boxed{\theta^* = \frac{\mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})}.}$$

5.5 Outage Behavior

We consider the outage behavior of our assumed relay channel model. We assume the probability of erroneous error-detection through the CRC code to be much smaller than the probability of erroneous error-correction and neglect error-detection errors for the definition of the outage events. The definition of the outage event is based on information-theoretic results in Section 4.3 and does not depend on the specific code design and provides a benchmark for the considered codes. This can help to evaluate the performance of distributed channel coding faster than with time-consuming Monte-Carlo simulations. This is especially important because the relay channel has three SNR values as parameters compared to one SNR value for the point-to-point channel.
It depends on all three fading coefficients, whether the communication system is in outage or not. The definition of the outage event OUT correspond to the case that no reliable communication from the mobile station to the base station is possible. The event $\overline{\mathrm{OUT}}$ is defined as the complement of the event OUT.

The data of the mobile station can be decoded reliably at the base station according to (4.18), if the instantaneous SNRs γ_{MB}, γ_{MR} and γ_{RB} have values such that the following unequalities hold:

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right) \tag{5.6}$$
$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + M_{\mathrm{R}} \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) \tag{5.7}$$

Contrary to [LTW04, Equation (15)], it is assumed that relay and source transmit different code bits and that it is allowed that the total time is shared unequally. As an error detection is included in our system, the fulfillment of the condition

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) \tag{5.8}$$

allows also a reliable communication alternatively to the fulfillment of the Conditions (5.6) and (5.7). Then, the base station can decode reliably directly after the transmission

of the mobile station. The base station can decode the data reliably, either if (5.6) and (5.7) hold or if (5.8) holds.
We define the event $\overline{\mathrm{OUT}}$ for the relay channel as

$$\boxed{\begin{aligned}&\Big(\big[K_\mathrm{M} \le M_\mathrm{M} \cdot \mathcal{C}(\gamma_\mathrm{MR})\big] \wedge \big[K_\mathrm{M} \le M_\mathrm{M} \cdot \mathcal{C}(\gamma_\mathrm{MB}) + M_\mathrm{R} \cdot \mathcal{C}(\gamma_\mathrm{RB})\big]\Big)\\ \vee&\Big(\big[K_\mathrm{M} \le M_\mathrm{M} \cdot \mathcal{C}(\gamma_\mathrm{MB})\big]\Big).\end{aligned}} \tag{5.9}$$

Note that the event should be defined differently, if another option were chosen than that the relay remains silent in case of a decoding error at the relay.
The relation of distributed channel coding with hybrid ARQ/FEC for a point-to-point communication can be recognized, if we position the relay very close to the mobile station ($\gamma_\mathrm{MR} \to \infty$, $\mathrm{E}\{\gamma_\mathrm{RB}\} = \mathrm{E}\{\gamma_\mathrm{MB}\}$). Then, there is no difference whether the mobile station or the relay sends the retransmission and the definition of the event $\overline{\mathrm{OUT}}$ in (5.9) falls back to the definition for hybrid ARQ/FEC with one retransmission in (2.17) whereas the notation has changed ($\gamma_1 = \gamma_\mathrm{MB}$, $\gamma_2 = \gamma_\mathrm{RB}$, $M_1 = M_\mathrm{M}$, $M_2 = M_\mathrm{R}$).
The outage probabilities under the constraint that we use a discrete modulation alphabet $\mathcal{S}_k$ are obtained, when we replace $\mathcal{C}$ by $\mathcal{C}_k$ as defined in (2.9).

5.6 Allocation of Transmission Time to Source and Relay

We will see in the next section from the simulation results, that the performance of a distributed channel code depends strongly on the choice how to share the total transmission time T between the transmission time T_M of the mobile station (source) and the transmission time T_R of the relay. This is equivalent to the choice how to share the total number of symbols M between the number of symbols M_M transmitted from the mobile station and the number of symbols M_R transmitted from the relay. If we do not transmit enough symbols from the mobile station, the relay will not be able to decode successfully. If we transmit too many symbols from the mobile station, we will not exploit the relay efficiently.

For given channel SNRs γ_MB, γ_MR and γ_RB, we derived the optimal value θ^* for θ, which maximizes the achievable rate in (4.21). Figure 5.4 (left) depicts θ^* dependent on the MS-BS γ_MB. It is assumed that the relay is half-way between mobile and base station ($d_\mathrm{MR} = d_\mathrm{RB} = d_\mathrm{MB}/2$). Therefore, the SNRs γ_MR and γ_RB are given by $\gamma_\mathrm{MR} = \gamma_\mathrm{RB} = \gamma_\mathrm{MB} \cdot 2^n$ with path-loss exponent $n = 3.52$. Figure 5.4 (right) depicts θ^* dependent on the relative MS-R distance $d_\mathrm{MR}/d_\mathrm{MB}$ with $d_\mathrm{RB} = d_\mathrm{MB} - d_\mathrm{MR}$. The MS-BS SNR γ_MB is assumed to be $\gamma_\mathrm{MB} = 1$.

We also want to know a good choice for θ for a given rate $R = R_0$ and a given relation of the channel SNRs. For example, the SNR on the link between mobile station and base station γ_MB can be variable and the SNRs γ_MR and γ_RB can be given by $\gamma_\mathrm{MR} =$

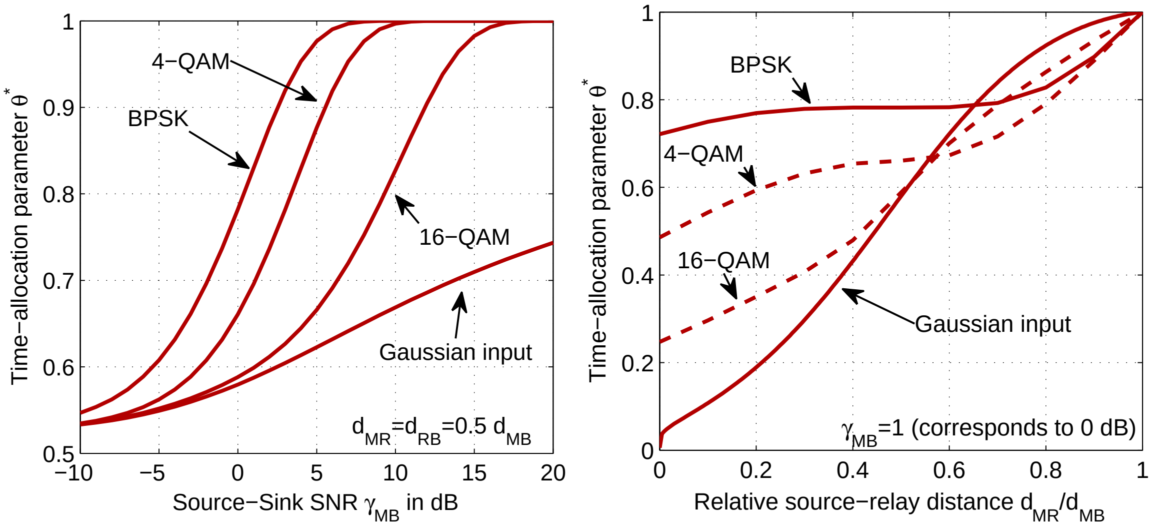


Figure 5.4: Optimal time-allocation parameter θ^* dependent on the MS-BS SNR (left) and dependent on the relative MS-R distance (right).

$\gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{MR}})^n$ and by $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{RB}})^n$. To find a good choice for θ, we first determine (numerically) the lowest SNR $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{MB},0}$ which allows to achieve the rate R_0 according to (4.22). If the relay is half-way between mobile and base station with $n = 3.52$, $\gamma_{\mathrm{MB},0}$ can be obtained graphically from Figure 4.13. The corresponding SNRs $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{MR},0}$ and $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{RB},0}$ can be calculated from $\gamma_{\mathrm{MB},0}$. Then, we obtain the optimal value θ^* according to (4.21) by setting the SNRs to $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{MB},0}$, $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{MR},0}$ and $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{RB},0}$.
Figure 5.5 (left) depicts θ^* dependent on the rate $R = R_0$ under the assumption that the relay is half-way between mobile and base station with path-loss exponent $n = 3.52$. Figure 5.5 (right) depicts θ^* dependent on the rate $R = R_0$ under the assumption that the MS-R distance is one fourth and the R-BS distance is three fourth of the MS-BS distance. That means that the SNR on the MS-R link is better by 21.19 dB and the SNR on the R-BS link is better by 4.40 dB than the SNR on the MS-BS link.
It is advantageous to optimize θ according to the given rate instead to the given SNRs, because the performance of the practical coding system will have a certain SNR gap to the theoretical limit.

The previous methods to choose θ are suitable for noisy channels without fading. If a noisy channel with fading is assumed, it is important to choose θ such that diversity can be gained. A system has a diversity order of two, if it can tolerate that one of the three links is in a very deep fade. That means that no outage occurs for either $\gamma_{\mathrm{MB}} \to 0$, $\gamma_{\mathrm{MR}} \to 0$ or $\gamma_{\mathrm{RB}} \to 0$. We assume the use of discrete modulation alphabets where each symbol carries L code bits. If either the MS-R link or the R-BS link is in a very deep fade ($\gamma_{\mathrm{MR}} \to 0$ or $\gamma_{\mathrm{RB}} \to 0$), we can observe of the outage definition in (5.9) that it is necessary that the information is transmitted over the direct MS-BS link. As $\mathcal{C}(\gamma_{\mathrm{MB}})$ is

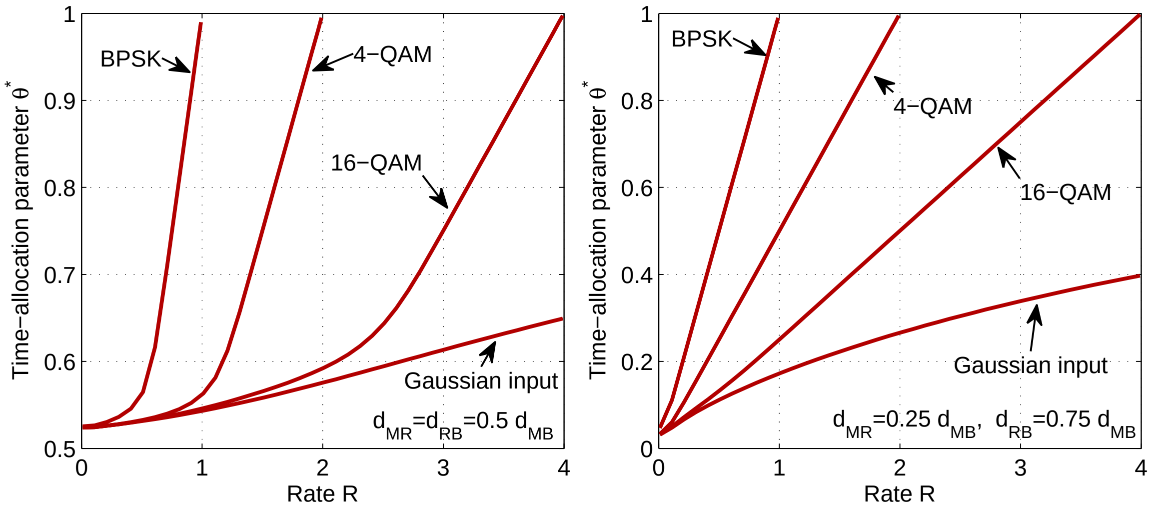


Figure 5.5: Optimal time-allocation parameter θ^* dependent on the rate R for two positions of the relay: Left: Relay positioned at half distance between mobile and base station. Right: MS-R distance is one fourth and the R-BS distance is three fourth of the MS-BS distance.

limited by $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq L$, it is necessary that

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot L = \theta \cdot M \cdot L \tag{5.10}$$

is fulfilled to allow a diversity order of two. Otherwise the Condition (5.9) can never be fulfilled for $\gamma_{\mathrm{MR}} \rightarrow 0$ or $\gamma_{\mathrm{RB}} \rightarrow 0$, respectively. If the MS-BS link is in a very deep fade ($\gamma_{\mathrm{MB}} \rightarrow 0$), we can observe of the outage definition in (5.9) that it is necessary that the information is transmitted over the relay route. As $\mathcal{C}(\gamma_{\mathrm{RB}})$ is limited by $\mathcal{C}(\gamma_{\mathrm{RB}}) \leq L$, it is necessary that beside the condition in (5.10)

$$K_{\mathrm{M}} \leq M_{\mathrm{R}} \cdot L = (1-\theta) \cdot M \cdot L \tag{5.11}$$

is also fulfilled to allow a diversity order of two. Otherwise (5.9) can never be fulfilled for $\gamma_{\mathrm{MB}} \rightarrow 0$. From (5.10) and (5.11), we obtain that the following condition for θ is necessary to allow a diversity order of two dependent on the code rate $R_{\mathrm{c}} = K_{\mathrm{M}}/(M \cdot L)$:

$$\boxed{R_{\mathrm{c}} \leq \theta \leq 1 - R_{\mathrm{c}}} \tag{5.12}$$

Figure 5.6 depicts the region for values of θ which allow a diversity order of two dependent on the code rate $R_{\mathrm{c}} = R/L$. Whereas a broad range of values for θ allow a diversity order of two for small rates R_{c}, θ has to be close to $\theta = 0.5$ to allow a diversity order of two for rates close to $R_{\mathrm{c}} = 0.5$. For $R_{\mathrm{c}} > 0.5$, it is not possible to gain a diversity order of two. The time allocation for the relay channel has also been optimized in [OMT06]. Contrary to our approach, the authors consider a protocol where the destination uses either the direct link or the relay link. Moreover, the authors consider Gaussian distributed channel inputs instead of coded modulation with a discrete constellation.

We conclude that the parameter θ should not only be chosen such that the rate is maximized for the noisy channel without fading. If possible, the choice for θ should be adapted such that diversity can be gained through relaying.

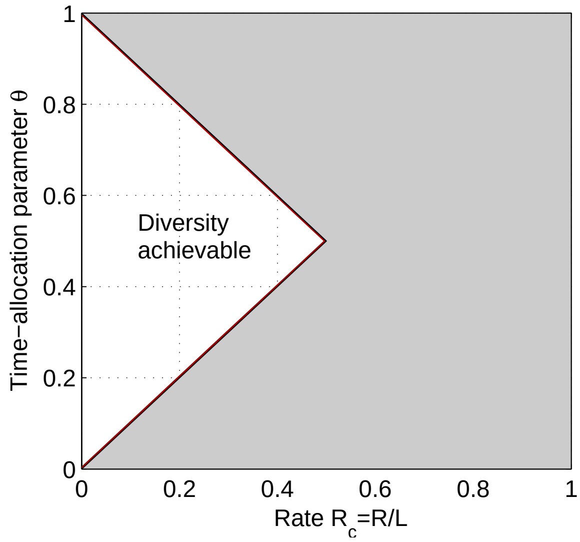


Figure 5.6: Values of θ which allow a diversity order of two dependent on the code rate $R_\mathrm{c} = R/L$.

5.7 Simulation Results

We compare the relay system with a point-to-point communication for AWGN channels with and without fading. In the relay system, the mobile station transmits M_M symbols with power $P_\mathrm{M} = P$ in the first time slot and the relay transmits M_R symbols with power $P_\mathrm{R} = P$ in the second time slots. In the system without relay, the mobile station transmits $\hat{M}_\mathrm{M} = M_\mathrm{M} + M_\mathrm{R}$ symbols with power P. All systems transmit in total $M = M_\mathrm{M} + M_\mathrm{R}$ symbols and thus, have the same rate $R = K_\mathrm{M}/M$. Moreover, the total transmission time, total transmission energy and the used bandwidth are identical for the relay system and the point-to-point system. The mobile station spends less transmission energy in the relay system ($\theta \cdot P \cdot T$ instead of $P \cdot T$).
We also include simple relaying without broadcast in the comparison. That means that the network consists of two point-to-point links where each is protected with a local channel code. Contrary to the distributed channel coding system, channel coding and routing are separated in different layers.
We choose the UMTS turbo code as described in Section 2.4.2 for the simulations. We choose the packet length K_M to $K_\mathrm{M} = 1500$ and run the decoder with 4 iterations.

5.7.1 Results without Fading

First, we compare the bit error rate (BER) and packet error rate (PER) at the base station for the AWGN channel without fading for a given rate R. The BER and PER is evaluated by a Monte Carlo simulation. We assume a pathloss exponent $n = 3.52$ as it is also assumed in the UMTS model [HT01, Equation (8.1)]. Initially, the relay is on halfway between mobile and base station and thus, the SNRs on the MS-R link and the R-BS link are increased by a factor of $2^{3.52}$ compared to the SNR on the MS-BS link

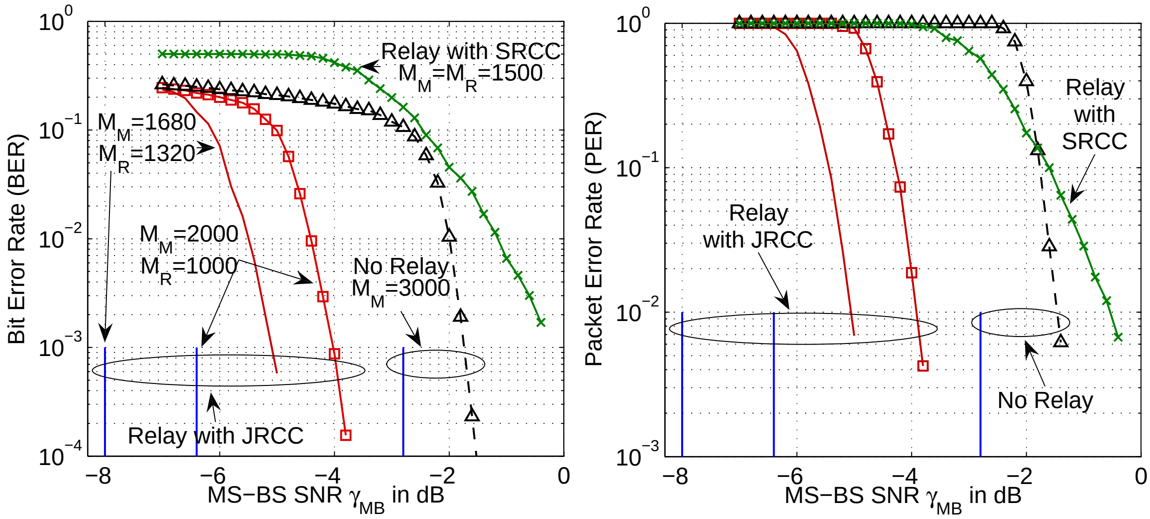


Figure 5.7: Bit and packet error rate for rate $R = 1/2$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

(corresponds to $35.2 \cdot \log_{10}(2)$ dB $= 10.6$ dB).
For the relay communication, we have to decide how to allocate the total number of symbols M to mobile station and relay. The choice is defined by the parameter $\theta = M_{\mathrm{M}}/M$. We will choose θ for the given rate R as described in Section 5.6 and depicted in Figure 5.5.
We want to compare the simulation results with the information-theoretic limits. That means we want to know the minimal SNR $\gamma_{\mathrm{MB},0}$ which theoretically allows a reliable communication with a given rate R. For the point-to-point communication with Gaussian input, this SNR can be obtained by finding the SNR γ_{MB} such that the rate R is obtained in (2.7). For the Gaussian input, the closed expression is given in (2.8). For discrete inputs, the SNR has to be found numerically with (2.9). It is also possible to find it graphically from Figure 2.4. For the relay communication with broadcast, the SNR $\gamma_{\mathrm{MB},0}$ has to be found numerically with (4.18). If the optimal time allocation is used the SNR $\gamma_{\mathrm{MB},0}$ can be found from (4.22) or graphically from Figure 4.13. For the relay communication without broadcast, the SNR $\gamma_{\mathrm{MB},0}$ has to be found numerically with (4.5) or with (4.9) for the optimal time allocation.

BPSK Modulation

First, we consider the system with BPSK modulation. Figure 5.7 depicts the BER and PER for the rate $R = 1/2$ information bits per symbol. As one packet contains $K_{\mathrm{M}} = 1500$ information bits, we are allowed to transmit $M = 3000$ symbols. The theoretical limits under the assumption of BPSK modulation are also shown. The communication without relay achieves a PER of 10^{-2} at an SNR of -1.5 dB. The gap to the theoretical limit is approximately 1.3 dB.
Next, we consider the relay communication with broadcast and the distributed turbo code as joint routing-channel code (JRCC). The optimal time-sharing parameter θ^*

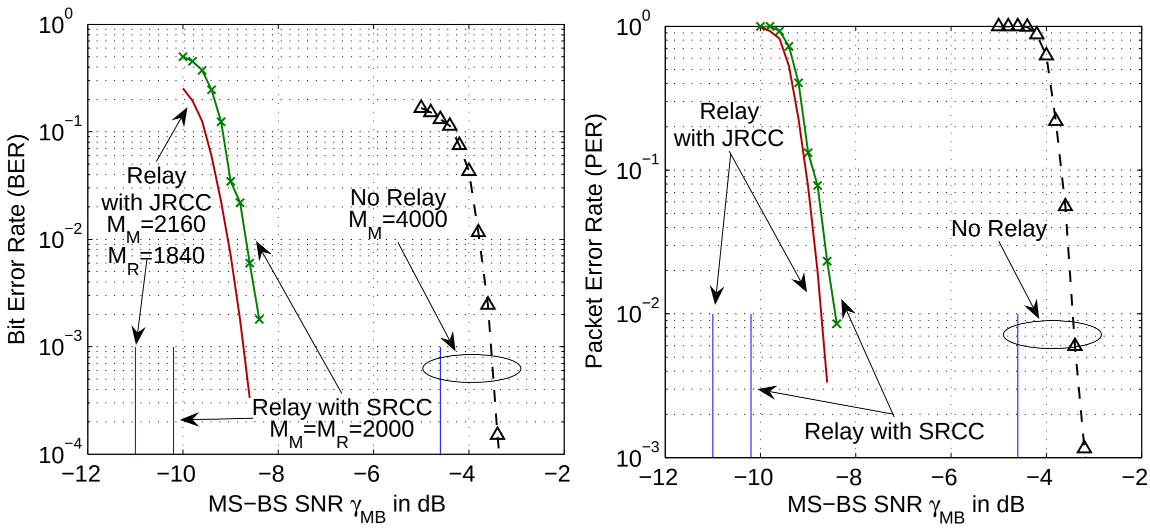


Figure 5.8: Bit and packet error rate for rate $R = 3/8$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

for $R = 1/2$ is given by $\theta^* = 0.56$. That means that the mobile station transmits $M_{\mathrm{M}} = \theta^* \cdot M = 1680$ symbols and the relay $M_{\mathrm{R}} = (1 - \theta^*) \cdot M = 1320$ symbols. The distributed turbo code outperforms the point-to-point communication by more than 3.4 dB. The gap to the theoretical limit is 2.9 dB. We also measured the error rates for the relay system with $M_{\mathrm{M}} = 2000$ and $M_{\mathrm{R}} = 1000$ ($\theta = 2/3$). This choice for θ degrades the performance by more than 1 dB compared to the optimal value.
For the simple relay communication without broadcast and separate routing and channel coding (SRCC), it is only possible to transmit uncoded information from the mobile station to the relay and from the relay to the base station ($M_{\mathrm{M}} = M_{\mathrm{R}} = 1500$). The performance is worse than without relay.

Next, we consider a lower rate of $R = 3/8$ information bits per symbol. We are allowed to transmit $M = 4000$ symbols. Figure 5.8 depicts the error rates and the theoretical limits. The communication without relay achieves a PER of 10^{-2} at an SNR of -3.4 dB. The gap to the theoretical limit is approximately 1.2 dB.
The optimal time-sharing parameter θ^* for the relay communication with broadcast and joint routing and channel coding and $R = 3/8$ is given by $\theta^* = 0.54$. That means that the mobile station transmits $M_{\mathrm{M}} = \theta^* \cdot M = 2160$ symbols and the relay $M_{\mathrm{R}} = (1 - \theta^*) \cdot M = 1840$ symbols. The distributed turbo code outperforms the point-to-point communication by more than 5.3 dB. The gap to the theoretical limit is 2.3 dB.
Contrary to $R = 1/2$, the simpler relay communication without broadcast and SRCC with $M_{\mathrm{M}} = M_{\mathrm{R}} = 2000$ achieves a performance close to the system with broadcast.

Next, we consider a higher rate of $R = 3/4$ information bits per symbol. We are allowed

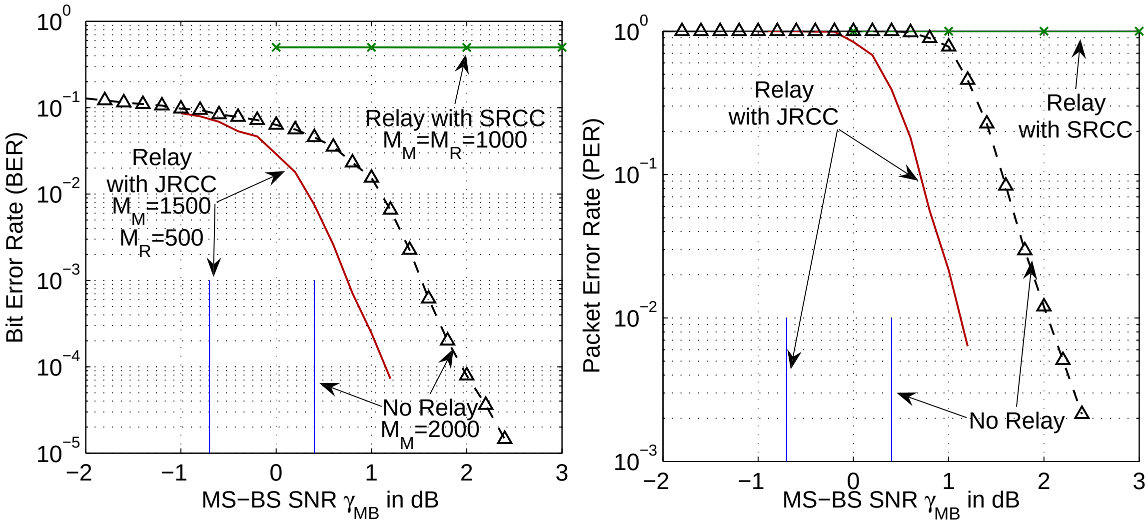


Figure 5.9: Bit and packet error rate for rate $R = 3/4$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

to transmit $M = 2000$ symbols. Figure 5.9 depicts the error rates and the theoretical limits. The communication without relay achieves a PER of 10^{-2} at an SNR of 2.1 dB. The gap to the theoretical limit is approximately 1.7 dB.
For $R = 3/4$, the optimal time-sharing parameter θ^* for the relay communication with broadcast and joint routing and channel coding is given by $\theta^* = 0.75$. That means that the mobile station transmits $M_{\mathrm{M}} = \theta^* \cdot M = 1500$ symbols and the relay $M_{\mathrm{R}} = (1 - \theta^*) \cdot M = 500$ symbols. The relay communication outperforms the point-to-point communication by more than 1.0 dB. The gap to the theoretical limit is 1.8 dB.
A reliable BPSK modulated communication with the simple relay communication without broadcast and SRCC is not possible for $R > 1/2$, because then, the rate on the MS-R and the R-BS link is larger than one.

Let us summarize the simulation results for BPSK. We considered the rates $R = 3/8$, $R = 1/2$ and $R = 3/4$. The relay communication provided gains between 1.0 dB and 5.3 dB for a PER of 10^{-2} compared to the communication without relay. The gain is larger for lower rates.
The performance with relaying benefits clearly for high rates, if the base station listens to the broadcast of the mobile station. For rates $R > 1/2$, a reliable communication is not possible over the relay, if the broadcast is not exploited. For lower rates, the benefit becomes smaller.
The system with the optimal time-sharing parameter, derived in (4.21), showed to be a good choice and provided better results than for other parameter choices. Beside the optimization of the time-sharing parameter, the theoretical results can help to answer questions for the system design with less time-consuming simulations. Such questions are for example: What is the expected gain from relaying? How does the gain depend on the

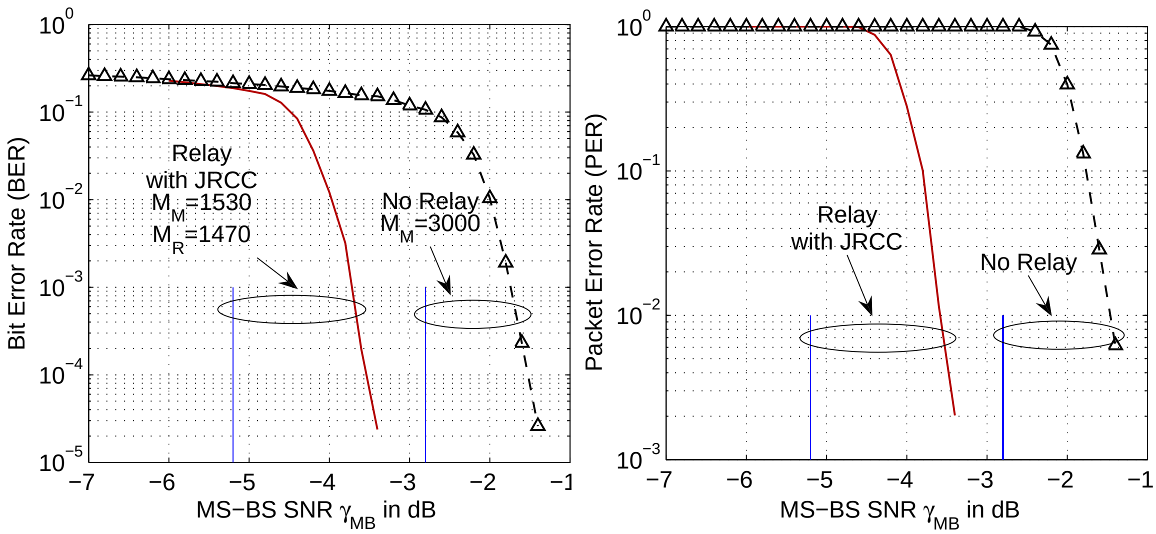


Figure 5.10: Bit and packet error rate for rate $R = 1/2$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$.

position of the relay?

We also want to show simulation results for an example where the relay is not half-way between mobile and base station. Figure 5.10 depicts the error rates for the case that the relay is positioned closer to the mobile station. The MS-R distance is one fourth and the R-BS distance is three fourth of the MS-BS distance. That means that the SNR on the MS-R link is better by 21.19 dB and the SNR on the R-BS link is better by 4.40 dB than the SNR on the MS-BS link. The rate is chosen to be $R = 1/2$. As previously in Figure 5.7, the communication without relay achieves a PER of 10^{-2} at an SNR of -1.6 dB. Next, we consider the relay communication with broadcast and the distributed turbo code as a joint routing-channel code. For $R = 1/2$, the optimal time-sharing parameter θ^* is given by $\theta^* = 0.51$. That means that the mobile station transmits $M_{\mathrm{M}} = \theta^* \cdot M = 1530$ symbols and the relay $M_{\mathrm{R}} = (1 - \theta^*) \cdot M = 1470$ symbols. The relay communication outperforms the point-to-point communication by more than 2.0 dB. The gap to the theoretical limit is 1.6 dB.

For the simple relay communication without broadcast and SRCC, it is only possible to transmit uncoded information from the mobile station to the relay and from the relay to the base station ($M_{\mathrm{M}} = M_{\mathrm{R}} = 1500$). The performance is clearly worse than without relay (not depicted in Figure 5.10).

Modulation Schemes of Higher Order

Next, we consider the distributed channel coding system with modulation schemes of higher order than BPSK modulation. In particular, we consider the use of hierarchical modulation as described in Section 5.3.

First, we consider a relay communication with 16-QAM and a rate $R = 4/3$. The optimal time allocation parameter θ^* is given by $\theta^* = 0.56$ for $R = 4/3$ under the assumption

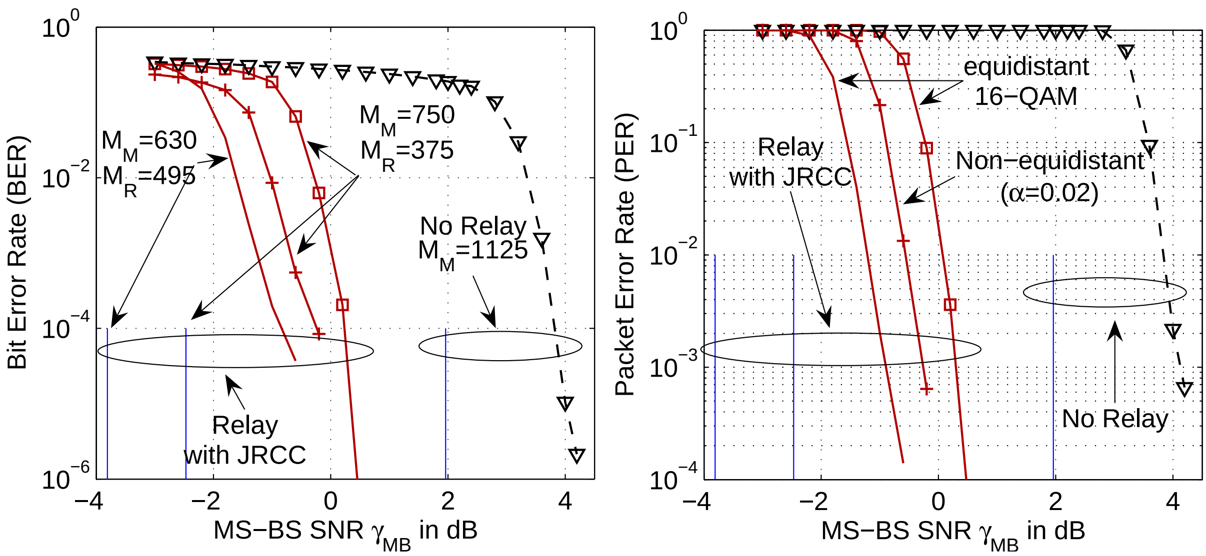


Figure 5.11: Bit and Packet Error Rate for rate $R = 4/3$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

that 16-QAM is used. That means that the mobile station transmits $M_{\mathrm{M}} = 630$ symbols and the relay transmits $M_{\mathrm{R}} = 495$ symbols. Figure 5.11 depicts the error rates for the system with $\theta^* = 0.56$ and an equidistant 16-QAM constellation. We also depict results for $\theta = 2/3$ ($M_{\mathrm{M}} = 750$ and $M_{\mathrm{R}} = 375$ symbols). The system with $\theta = 2/3$ and equidistant 16-QAM looses more than 1 dB compared to the system with $\theta^* = 0.56$. We also consider the case that a non-equidistant 16-QAM constellation with $\alpha = 0.02$ is used at the mobile station. The constellation with $\alpha = 0.02$ is depicted in Figure 5.3. For $\theta = 2/3$ the use of the non-equidistant constellation leads approximately to a gain of 0.6 dB. For the optimal time allocation, it was not possible to improve the system with a non-equidistant constellation. The best relay system outperforms the point-to-point communication with 16-QAM by more than 5 dB.
Next, we want to consider the use of hierarchical 4-QAM in 16-QAM modulation at the mobile station. That means that the base station uses a 4-QAM demodulation instead of a 16-QAM demodulation as described in Section 5.3. Figure 5.12 compares 4-QAM demodulation and 16-QAM demodulation for two relay systems. For the relay system with $M_{\mathrm{M}} = 630$, $M_{\mathrm{R}} = 495$ and equidistant 16-QAM modulation at the mobile station, the 4-QAM demodulation at the sink leads approximately to a loss of 0.08 dB compared to the 16-QAM demodulation at the sink. For the relay system with $M_{\mathrm{M}} = 750$, $M_{\mathrm{R}} = 375$ and non-equidistant modulation with $\alpha = 0.02$ at the mobile station, the 4-QAM demodulation at the sink leads approximately to a loss of 0.02 dB compared to the 16-QAM demodulation at the sink.

Next, we want to show that also for high rates, the relay communication outperforms the point-to-point communication. We consider an example with rate $R = 3.0$. The system

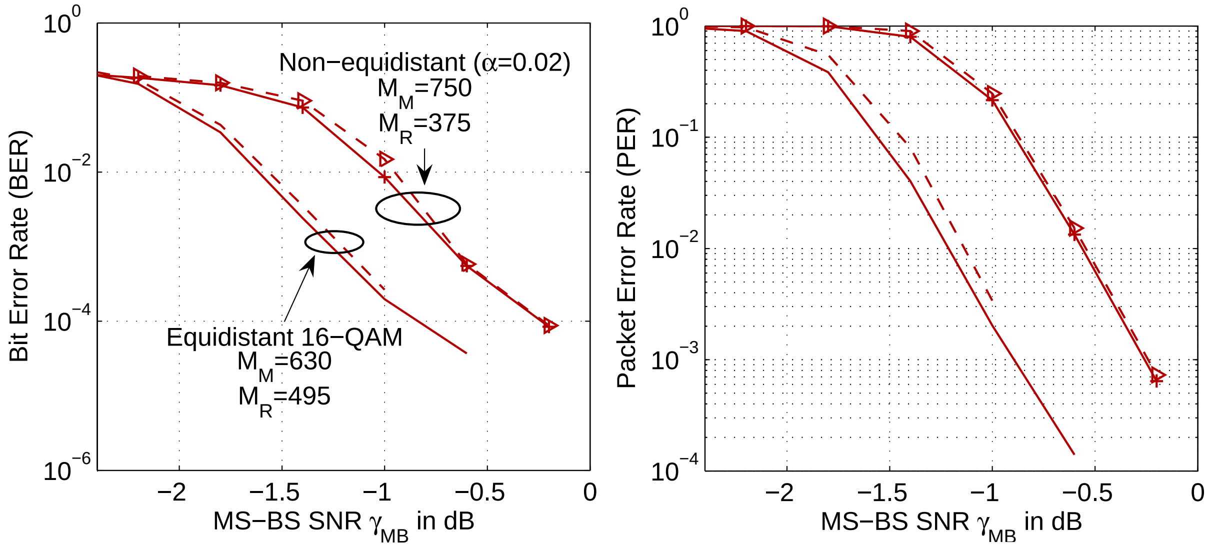


Figure 5.12: Comparison of bit and packet error rate for 16-QAM demodulation (solid curves) or 4-QAM demodulation (dashed curves) at the sink.

transmits in total $M = 500$ 64-QAM symbols to communicate $K_{\mathrm{M}} = 1500$ information bits. Figure 5.13 depicts the error rates for two relay system and the point-to-point system. The performance of the relay system is measured for $\theta = 3/4$ ($M_{\mathrm{M}} = 375$, $M_{\mathrm{R}} = 125$) and for the optimal value $\theta^* = 0.63$ ($M_{\mathrm{M}} = 315$, $M_{\mathrm{R}} = 185$). The optimized relay system gains more than 1 dB compared to the relay system with $\theta = 3/4$ and more than 3.5 dB compared to the system without relay. We also depict the performance of the relay system with $\theta^* = 0.63$, if 16-QAM demodulation is used at the sink. The performance loss to 64-QAM demodulation is small. However, an error floor can be recognized for small error rates. The performance could be probably improved with a non-equidistant constellation and optimized puncturing.

5.7.2 Results with Fading

Next, we measure the bit and packet error rate for Rayleigh-fading channels. Figure 5.14 depicts the error rates for $R = 1/2$ ($K_{\mathrm{M}} = 1500$, $M = 3000$). The relay system with $M_{\mathrm{M}} = 1680$ and $M_{\mathrm{R}} = 1320$ ($\theta = 0.56$) achieves a better performance than the system without relay. However, a diversity order of two cannot be achieved with the relay system. We could not find a θ which allows a better PER-performance for the relay system than $\theta = 0.56$. The outage rate for $M_{\mathrm{M}} = 1550$ and $M_{\mathrm{R}} = 1450$ achieves a lower outage rate for high SNRs. However, the simulated PER was higher compared to $\theta = 0.56$. According to Figure 5.6, a relay system with $\theta = 0.5$ should allow a diversity order of two for $R = R_{\mathrm{c}} = 0.5$. However, the performance of this system was worse because the transmission of the source is uncoded.

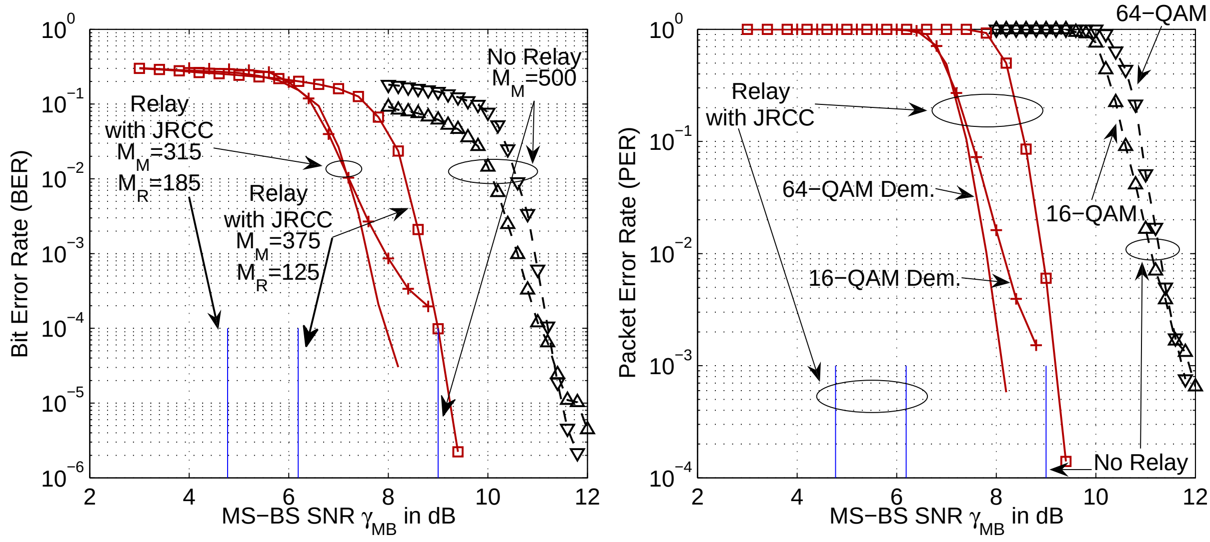


Figure 5.13: Bit and packet error rate for rate $R = 3.0$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

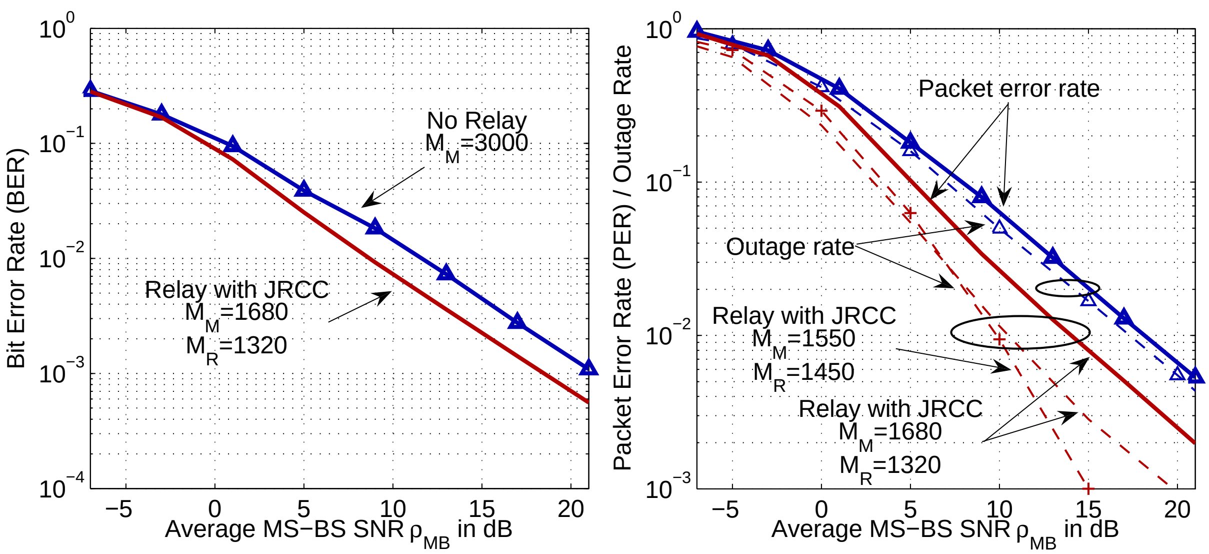


Figure 5.14: Rayleigh-fading channel: Bit and packet error rate for rate $R = 1/2$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

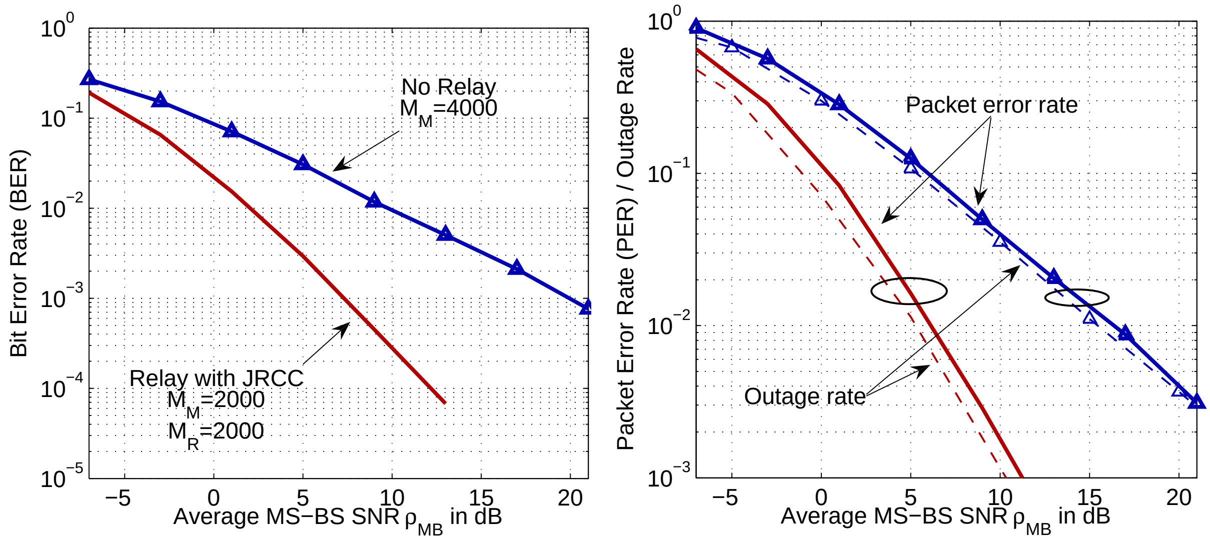


Figure 5.15: Rayleigh-fading channel: Bit and packet error rate for rate $R = 3/8$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

Figure 5.15 depicts the error rates for $R = 3/8$ ($K_{\mathrm{M}} = 1500$, $M = 4000$). The relay system with $M_{\mathrm{M}} = 2000$ and $M_{\mathrm{R}} = 2000$ achieves a better performance than the system without relay and gains clearly diversity. As predicted by the diversity region in Figure 5.6 relay systems allow a diversity order of two for $R = R_{\mathrm{c}} = 3/8$. The packet error rate follows the outage rate, even if the gap between outage rate and error rate is little bit larger than for the point-to-point communication. Contrary to the channel without fading, the relay system with $M_{\mathrm{M}} = 2000$ and $M_{\mathrm{R}} = 2000$ achieves a better performance than the relay system with $M_{\mathrm{M}} = 2160$ and $M_{\mathrm{R}} = 1840$.

Figure 5.16 depicts the error rates for 16-QAM modulated systems with rate $R = 4/3$ ($K_{\mathrm{M}} = 1500$, $M = 1125$). That means that R_{c} is given by $R_{\mathrm{c}} = R/4 = 1/3$ and that according to Figure 5.6, a diversity order of two is achievable for a broad range of the time allocation parameter θ. The relay system with $M_{\mathrm{M}} = 630$ and $M_{\mathrm{R}} = 495$ achieves a better performance than the system without relay and gains diversity.

5.8 Summary

Relaying with a broadcast transmission requires to break with the separation of routing and channel coding. In this chapter, we considered a distributed turbo code as joint routing-channel code. Simulation results confirmed that relaying allows a better error protection and increases the throughput compared to a point-to-point communication. Moreover, spatial diversity can be gained in a fading environment for low code rates ($R_{\mathrm{c}} < 0.5$).
We observed that the decision, how to allocate the transmission time to source and relay,

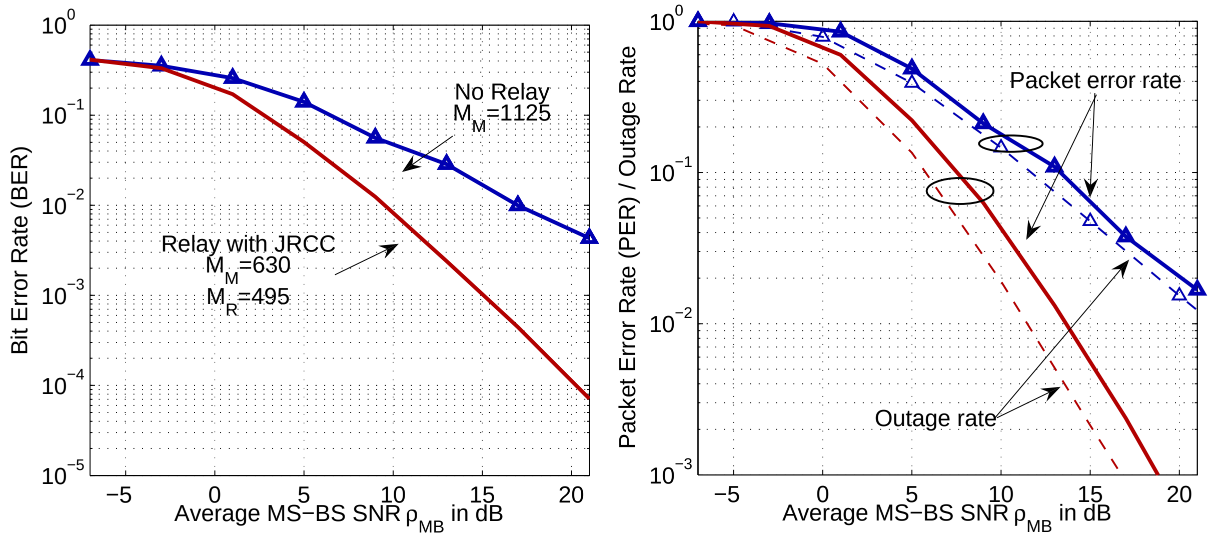


Figure 5.16: Rayleigh-fading channel: Bit and packet error rate for rate $R = 4/3$ and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

has a crucial influence on the relaying performance. We evaluated the time allocation rule, which we derived in Section 4.3 by maximizing the achievable rate, and found out that our time allocation rule achieves also in the practical coding system better simulation results than other choices for the time allocation for the considered examples. Moreover, we observed that efficient relaying requires to use high order modulation schemes. We proposed to extend distributed turbo coding with hierarchical modulation. This allows to reduce the demodulation complexity at the sink.

In the following two chapters, we will extend distributed turbo coding for the relay channel in two directions using network coding.

6

Joint Network-Channel Coding for the Multiple-Access Relay Channel

We consider the multiple-access relay channel (MARC), where two sources communicate with one sink with the help of one relay. Such a system can be used for the cooperative uplink for two mobile stations to a base station with the help of a relay. We consider time-division channels with broadcast but without simultaneous multiple-access. That means that the first mobile station broadcasts to relay and base station in the first time slot, the second mobile station broadcasts to relay and base station in the second time slot, and the relay network encodes the data of both mobile stations and transmits to the base station in the third time slot. The three time slots can have different lengths. The restriction to time-division is suboptimal compared to the general MARC model [KvW00]. However, it allows an easier realization in practical systems, because it can be realized with half-duplex relays and with less stringent synchronization constraints.
We propose a joint network-channel code design based on turbo codes (turbo network code) for the MARC and consider how to allocate the transmission time to the three slots. We compare the proposed system with a distributed turbo code for the relay channel and with a system which uses separate network-channel coding for the MARC. We consider the outage behavior for coded modulation and derive that the MARC allows to gain diversity for higher code rates compared to the relay channel. Moreover, our results show that joint network-channel coding allows to more efficiently exploit the redundancy in the transmission of the relay than separate network-channel coding.

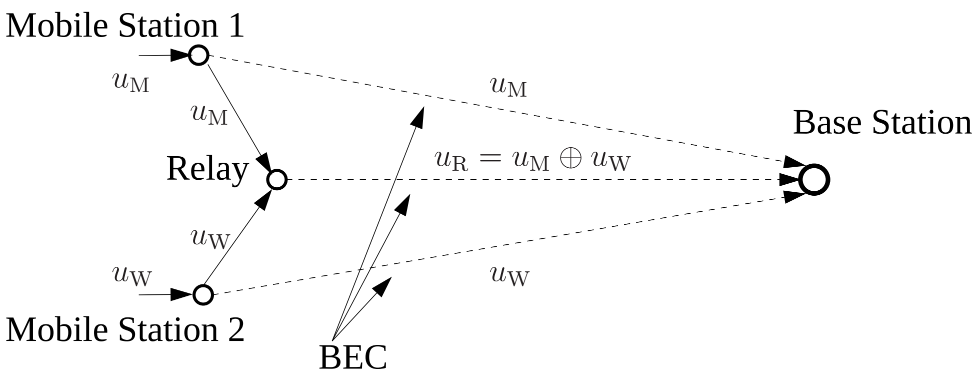


Figure 6.1: Example how network coding can help to gain diversity.

6.1 Introduction

6.1.1 Diversity Gain through Network Coding

Figure 6.1 depicts a simple example from [AIGM93, TF04] for a network with erasure channels to demonstrate how network coding can be used to gain diversity. In this example, the bits u_{M} and u_{W} have to be transferred from the two mobile stations MS1 and MS2 to the base station. It is assumed in this example that no transmission errors occur at the the links from the mobile stations to the relay. In contrast, the three incoming links at the base station are assumed to be binary erasure channels (BEC). The relay has no knowledge whether the transmissions from the mobile stations to the base station were erased or not. The bits u_{M} and u_{W} are sent from MS1 and MS2 to the base station. The relay performs network encoding and transmits $u_{\mathrm{R}} = u_{\mathrm{M}} \oplus u_{\mathrm{W}}$ to the base station. As only two out of the three incoming bits u_{M}, u_{W} and u_{R} have to be transmitted correctly to reconstruct u_{M} and u_{W} at the base station, one arbitrary erasure can be tolerated. The reference system without network coding is not able to tolerate one arbitrary erasure, if we only allow the transmission of one bit from the relay.

6.1.2 Motivation for Joint Network-Channel Coding

Motivated by the previous example, we show in this work how the redundancy provided by network coding can be used to support the channel code for a better error protection over noisy channels. This requires to design network coding and channel coding jointly. The principle of joint network-channel coding is similar to the principle of joint source-channel coding [Hag95], where the redundancy remaining after the source encoding helps the channel code to combat noise. We know from [EMH^{+}03] and [RK06] that in general, capacity can only be achieved by treating network and channel coding jointly, if we consider the communication in wireless relay networks with broadcast. We will describe how joint network-channel coding can be realized with turbo codes [HD06].

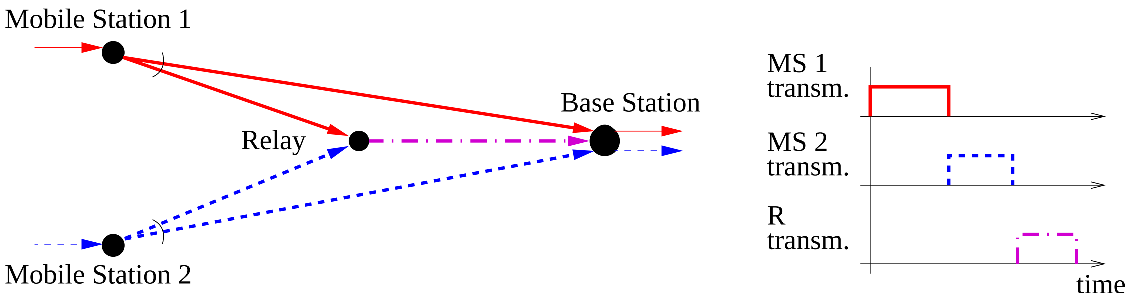


Figure 6.2: Time division multiple-access relay channel.

6.1.3 Overview over Related Work

The References [KvW00, KGG05] considered capacity bounds of the general MARC where the sources and the relay are allowed to access the channel simultaneously. Such a model requires a full-duplex relay. Capacity bounds for the MARC with a half-duplex constrained relay and partial simultaneous multiple-access were investigated in [SKM04]. The resource allocation for this model was considered in [SLPM07]. Contrary to [SLPM07], we do not assume that the SNRs depend on the resource allocation parameter θ. Moreover, our model is simpler and uses TDMA and does not allow simultaneous transmissions from the nodes. We refer to the discussion in Chapter 4 about the dependence of the SNRs on θ and on the classification of the different relaying strategies. Information-theoretic strategies and the diversity-multiplexing tradeoff for the MARC are studied in [CL06, Yil06, YE07, AGS06].
The MARC is an extension of the multiple-access channel (MAC) ([CT91] and references therein). Whereas the MAC falls back to two point-to-point channels in case of time-divided channels, we will see that the time-division MARC allows an advantageous outage behavior compared to two relay channels.
Code design for the time-division MARC are described based on LDPC codes in [HSOB05, MKR07] and based on convolutional/turbo codes in [HD06, YK07, DSCKGG07]. The diversity gain of network coding for the MARC and the outage probability are considered in [CKL06, WK07b, WK07a]. Code design for systems where several sources cooperate and relay information for each other is considered in [BL05] and in [XFKC06b, XFKC06a, XFKC07a, XFKC07b].

6.2 System Description

In a cellular based mobile communication system (Figure 6.2) two mobile stations MS1 and MS2 want to transmit statistically independent bits which are segmented in packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{W}}$ of length K_{M} and K_{W} to the base station BS[1]. We assume without loss of generality that $K_{\mathrm{M}} \geq K_{\mathrm{W}}$. The packets include cyclic redundancy check (CRC) bits as

[1]The index W refers to the second mobile station. We use the index W, because M is used for the first mobile station and W resembles an upturned M.

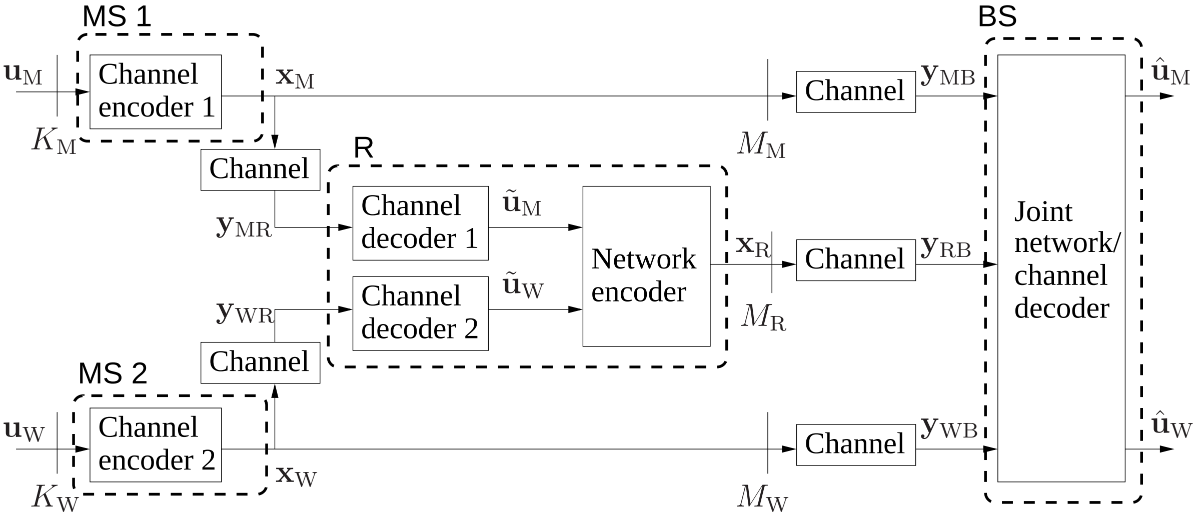


Figure 6.3: Block diagram of the system.

an error detection code. A block diagram of the system is depicted in Figure 6.3.
The information bits $\mathbf{u}_\mathrm{M}$ and $\mathbf{u}_\mathrm{W}$ are protected against transmission errors with channel codes and modulators which output the blocks $\mathbf{x}_\mathrm{M}$ and $\mathbf{x}_\mathrm{W}$ containing M_M and M_W symbols, respectively. MS1 transmits $\mathbf{x}_\mathrm{M}$ with power P in the first of the three time phases, MS2 transmits $\mathbf{x}_\mathrm{W}$ with power P in the second time phase.
The demodulator at the relay R receives the disturbed versions $\mathbf{y}_\mathrm{MR}$ and $\mathbf{y}_\mathrm{WR}$ of the symbols $\mathbf{x}_\mathrm{M}$ and $\mathbf{x}_\mathrm{W}$ to obtain the hard estimates $\tilde{\mathbf{u}}_\mathrm{M}$ about $\mathbf{u}_\mathrm{M}$ based on $\mathbf{r}_\mathrm{MR}$ and the hard estimate $\tilde{\mathbf{u}}_\mathrm{W}$ about $\mathbf{u}_\mathrm{W}$ based on $\mathbf{r}_\mathrm{WR}$. If the CRC indicates that the relay decoded both packets $\mathbf{u}_\mathrm{M}$ and $\mathbf{u}_\mathrm{W}$ successfully, the estimates $\tilde{\mathbf{u}}_\mathrm{M}$ and $\tilde{\mathbf{u}}_\mathrm{W}$ are network encoded and modulated to the block $\mathbf{x}_\mathrm{R}$ containing M_R symbols. The relay transmits $\mathbf{x}_\mathrm{R}$ to the base station with power P in the third time phase to provide additional error protection.
The base station receives the disturbed version $\mathbf{y}_\mathrm{MB}$ of $\mathbf{x}_\mathrm{M}$ in the first time phase, the disturbed version $\mathbf{y}_\mathrm{WB}$ of $\mathbf{x}_\mathrm{W}$ in the second time phase and the disturbed version $\mathbf{y}_\mathrm{RB}$ of $\mathbf{x}_\mathrm{R}$ in the third time phase. Based on $\mathbf{y}_\mathrm{MB}$, $\mathbf{y}_\mathrm{WB}$ and $\mathbf{y}_\mathrm{RB}$, the joint network-channel decoder outputs the hard estimate $\hat{\mathbf{u}}_\mathrm{M}$ about $\mathbf{u}_\mathrm{M}$ and the hard estimate $\hat{\mathbf{u}}_\mathrm{W}$ about $\mathbf{u}_\mathrm{W}$.
The number of total transmitted symbols is given by $M = M_\mathrm{M} + M_\mathrm{W} + M_\mathrm{R}$. We define the rate for MS1 as $R_\mathrm{M} = K_\mathrm{M}/M$ and the rate for MS2 as $R_\mathrm{W} = K_\mathrm{W}/M$. The rate of the system is given by $R = R_\mathrm{M} + R_\mathrm{W} = (K_\mathrm{M} + K_\mathrm{W})/M$.

If the CRC indicates that the relay decoded both packets $\mathbf{u}_\mathrm{M}$ and $\mathbf{u}_\mathrm{W}$ wrongly, there are the same options as described in Section 5.2 for the relay channel:

- The relay remains silent and does not transmit any symbols.
- The relay ignores the CRC indication and reencodes the decoded information bits anyway.

- If the relay is able to notify to the mobile or base station that it decoded wrongly, using one bit in a feedback control channel, then the system can be extended in the following way. In case the relay decoded both packets with error, the mobile stations are notified that they both have to transmit additional $M_{\mathrm{R}}/2$ symbols to the base station.
- The system can be extended with soft relaying as it was done in [YK07].

If the CRC indicates that the relay only decoded one of the packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{W}}$ wrongly, the system could be further extended by allowing that the system falls back to one relay channel and one point-to-point channel. The comparison of these schemes is beyond the scope of the thesis. We assume in the rest of the thesis that the relay remains silent, if the CRC indicates that the relay decoded one or both packets wrongly.

All channels are assumed to be block Rayleigh fading channels as described in Section 2.2 and thus, the received samples after the matched filter are

$$\mathbf{y}_{\mathrm{MR}} = h_{\mathrm{MR}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MR}} \qquad \text{(MS1 transmits)} \qquad (6.1)$$

$$\mathbf{y}_{\mathrm{MB}} = h_{\mathrm{MB}} \cdot \mathbf{x}_{\mathrm{M}} + \mathbf{z}_{\mathrm{MB}} \qquad \text{(MS1 transmits)} \qquad (6.2)$$

$$\mathbf{y}_{\mathrm{WR}} = h_{\mathrm{WR}} \cdot \mathbf{x}_{\mathrm{W}} + \mathbf{z}_{\mathrm{WR}} \qquad \text{(MS2 transmits)} \qquad (6.3)$$

$$\mathbf{y}_{\mathrm{WB}} = h_{\mathrm{WB}} \cdot \mathbf{x}_{\mathrm{W}} + \mathbf{z}_{\mathrm{WB}} \qquad \text{(MS2 transmits)} \qquad (6.4)$$

$$\mathbf{y}_{\mathrm{RB}} = h_{\mathrm{RB}} \cdot \mathbf{x}_{\mathrm{R}} + \mathbf{z}_{\mathrm{RB}}, \qquad \text{(Relay transmits)} \qquad (6.5)$$

with $h_i = a_i \cdot \sqrt{P_r(d_i)/P}$ for $i \in \{\mathrm{MR, MB, WR, WB, RB}\}$. The variables are defined analogously to the denotation in the previous chapters. The transmissions of MS1, MS2 and the relay are divided in time and thus, no interference occurs.
The fading coefficients a_i which are constrained by $E[|a_i|^2] = 1$, are Rayleigh distributed and represent the fading due to multipath propagation or shadowing behind obstacles. The fading characteristics depend on many parameters, e.g. on the motion of the transmitter and on the multiple-access scheme. We assume that the fading coefficients of the spatially distributed channels are statistically independent. The rapidity of the fading on one channel is determined by the coherence time. We assume frequency-nonselective, slow fading channels and that the coherence time has a length such that the fading does not change for M symbols. As each source starts to send one block of symbols every M symbols, the fading coefficients of one block are not assumed to be correlated with the ones of the previous block. Our channel model allows to consider the capability of the proposed strategy to gain spatial diversity. The benefit from spatial diversity depends on the practical system and will become less substantial, if other forms of diversity can be exploited, e.g. due to a shorter coherence time.
The average signal-to-noise ratios (SNR) ρ_i are given by $\rho_i = P_r(d_i)/(N_0 \cdot W)$, the instantaneous SNR by $\gamma_i = |a_i|^2 \cdot \rho_i$. The relation between the five SNRs depends on the distances d_i between the nodes and on the path-loss exponent n. We make the common assumption that for each channel the transmitters only know the average SNRs, but not the fading coefficients, and that the receivers know the instantaneous SNRs.

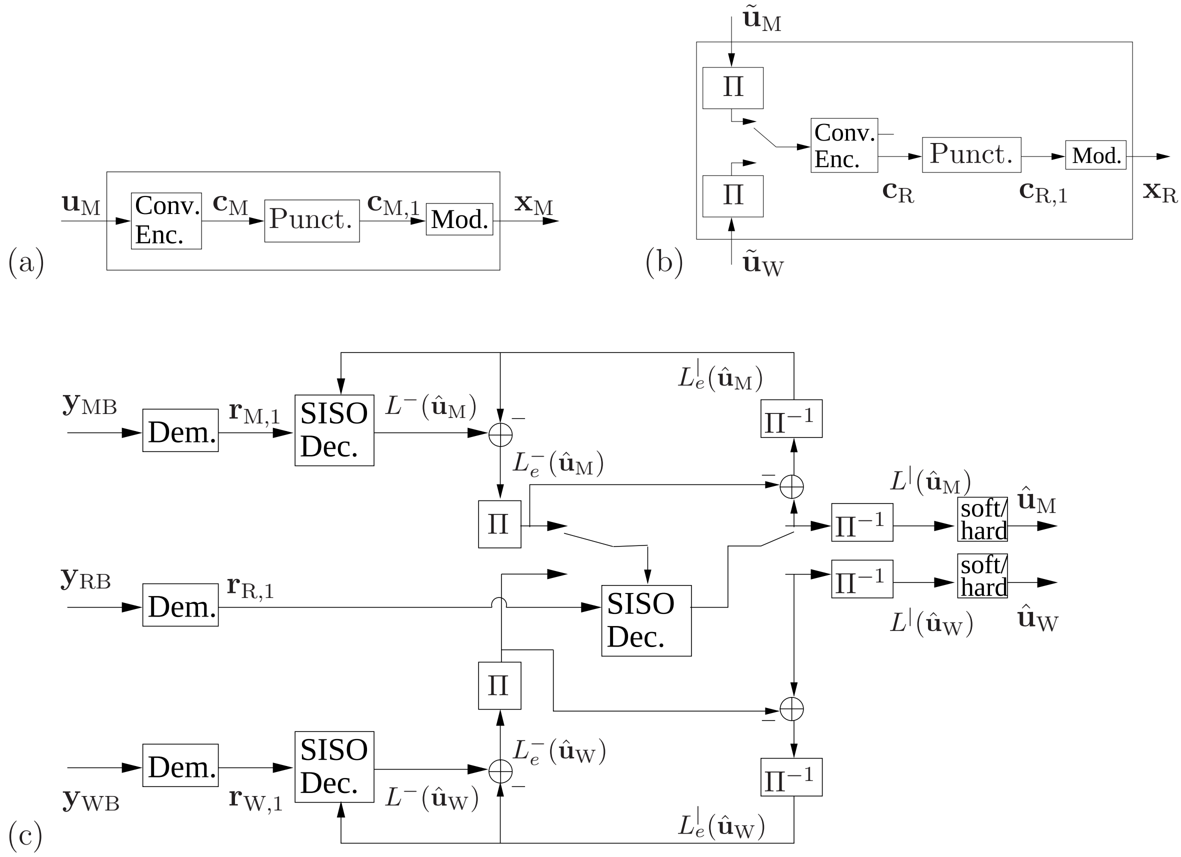


Figure 6.4: (**a**): Channel encoder 1 at mobile station 1. (**b**): Network encoder at the relay. (**c**): Iterative network and channel decoder at the base station.

6.3 Design for Joint Network-Channel Code: Turbo Network Code

We explain how joint network-channel coding can be performed with a turbo network code [HH06, HD06] for the multiple-access relay channel (MARC).

6.3.1 Channel Coding

The channel codes at the mobile stations contain either a recursive systematic convolutional encoder or a turbo encoder. For the simulation results, we will choose as convolutional code a rate 1/2 encoder with constraint length 4, with the feedforward generator 15 and with the feedback generator 13, both in octal. This code is depicted in Figure 2.5. We will choose as turbo code the UMTS turbo encoder as described in Section 2.4.2. We refer to Section 6.3.4 for more details on the required changes, if a turbo code is used instead of a convolutional code.
The channel encoder at MS1 encodes the packet $\mathbf{u}_{\mathrm{M}}$ with information bit block length K_{M} and outputs the code bits $\mathbf{c}_{\mathrm{M}}$. The code bits $\mathbf{c}_{\mathrm{M}}$ contain $\mathbf{u}_{\mathrm{M}}$ and additional parity bits

$\mathbf{p}_{\mathrm{M}}$. The puncturing block chooses from $\mathbf{c}_{\mathrm{M}}$ a subset $\mathbf{c}_{\mathrm{M},1}$ which contains $N_{\mathrm{M}} = M_{\mathrm{M}} \cdot L$ code bits. The puncturing is done in a regular way, which is similar to the rate matching strategy described in [Eur00]. We do not puncture systematic and tail bits. As the convolutional and the turbo encoder (including puncturing) are linear encoders, the encoding can be expressed as

$$\mathbf{c}_{\mathrm{M},1} = \mathbf{u}_{\mathrm{M}} \cdot \mathbf{G}_{\mathrm{M},1}. \tag{6.6}$$

The code bits $\mathbf{c}_{\mathrm{M},1}$ are modulated to the block $\mathbf{x}_{\mathrm{M}}$ containing M_{M} symbols whereas L is the number of code bits carried by one symbol. MS1 broadcasts $\mathbf{x}_{\mathrm{M}}$ to relay and base station in the first time slot. A block diagram of channel encoder 1 (including the modulator) at MS1 is depicted in Figure 6.4 (a).

The channel encoder at MS2 encodes the packet $\mathbf{u}_{\mathrm{W}}$ with information bit block length K_{W} and outputs the code bits $\mathbf{c}_{\mathrm{W}}$. The code bits $\mathbf{c}_{\mathrm{W}}$ contain $\mathbf{u}_{\mathrm{W}}$ and additional parity bits $\mathbf{p}_{\mathrm{W}}$. Again, the puncturing block chooses from $\mathbf{c}_{\mathrm{W}}$ a subset $\mathbf{c}_{\mathrm{W},1} = \mathbf{u}_{\mathrm{W}} \cdot \mathbf{G}_{\mathrm{W},1}$ which contains $N_{\mathrm{W}} = M_{\mathrm{W}} \cdot L$ code bits. The puncturing is done in a regular way, which is similar to the rate matching strategy described in [Eur00]. For $K_{\mathrm{M}} = K_{\mathrm{W}}$, the encoding matrices $\mathbf{G}_{\mathrm{M},1}$ and $\mathbf{G}_{\mathrm{W},1}$ are identical. The code bits $\mathbf{c}_{\mathrm{W},1}$ are modulated to the block $\mathbf{x}_{\mathrm{W}}$ containing M_{W} symbols. MS2 broadcasts $\mathbf{x}_{\mathrm{W}}$ to relay and base station in the second time slot. The channel encoders at the mobile stations do not change compared to conventional channel codes for the point-to-point communication, except that it is possible to reduce the number of transmitted symbols because it is not necessary that the base station decodes successfully before the transmission of the relay.

6.3.2 Network Coding

The relay demodulates and decodes the data of both mobile stations as described in Section 2.4 and obtains the estimates $\tilde{\mathbf{u}}_{\mathrm{M}}$ and $\tilde{\mathbf{u}}_{\mathrm{W}}$. If the CRC indicates that the relay decoded the data of both mobile stations successfully, the relay network-encodes the estimates $\tilde{\mathbf{u}}_{\mathrm{M}}$ and $\tilde{\mathbf{u}}_{\mathrm{W}}$ and transmits the modulated network code bits $\mathbf{x}_{\mathrm{R}}$ to the base station to provide additional redundancy for the uplink of both mobile stations.

A block diagram of the network encoder (including the modulator) is depicted in Figure 6.4 (b). Both estimates are interleaved according to [Eur00]. If K_{M} and K_{W} are equal, the interleaved bits appear alternately as the input of a convolutional encoder. If K_{M} and K_{W} are not equal, we can easily extend the method by writing the interleaved bits row-by-row into a matrix with $K_{\min} = \min\{K_{\mathrm{M}}, K_{\mathrm{W}}\}$ columns and reading the bits out column-by-column as input for the convolutional encoder. We first write all the bits from the smaller block and then all the bits from the larger block into the matrix. This permutation rule is called periodic block interleaver in [CC81]. It is not necessary that the last row is filled completely. If K_{M} and K_{W} are equal, the described periodic block interleaver falls back to the method of taking the interleaved bits alternately.

The convolutional encoder has the same parameters as the convolutional code used as channel encoders. However, the output $\mathbf{c}_{\mathrm{R}}$ of the convolutional encoder contains only the parity bits $\mathbf{p}_{\mathrm{R}}$ of the convolutional encoder. The systematic bits are punctured, because they have already been included in the transmission of the mobile stations. The puncturing block chooses from $\mathbf{c}_\mathrm{R} = \mathbf{p}_\mathrm{R}$ a subset $\mathbf{c}_{\mathrm{R},1}$ which contains $N_\mathrm{R} = M_\mathrm{R} \cdot L$ code bits. The puncturing of the parity bits is done in a regular way. As the convolutional encoder (including puncturing) is a linear encoder, the network encoding can be expressed as

$$\mathbf{c}_{\mathrm{R},1} = \tilde{\mathbf{u}}_\mathrm{M} \cdot \mathbf{G}_{\mathrm{R},1} \oplus \tilde{\mathbf{u}}_\mathrm{W} \cdot \mathbf{G}_{\mathrm{R},2}. \tag{6.7}$$

The network encoded bits $\mathbf{c}_{\mathrm{R},1}$ are modulated to the block $\mathbf{x}_\mathrm{R}$ containing M_R symbols whereas L is the number of code bits carried by one symbol. The relay transmits $\mathbf{x}_\mathrm{R}$ to the base station in the third time slot. The network code of rate $R_\mathrm{R} = (K_\mathrm{M} + K_\mathrm{W})/M_\mathrm{R}$ provides additional parity bits which support the decoding at the sink.
Although the different coding operations are processed spatially distributed, we will treat them as one network-channel code with the system rate $R = (K_\mathrm{M}+K_\mathrm{W})/(M_\mathrm{M}+M_\mathrm{W}+M_\mathrm{R})$.

The joint network-channel code consists of three constituent encoders which are illustrated graphically in Figure 6.8 (a). Channel encoder 1 and 2 at the mobile stations form two of the three constituent encoders. As they process the information bits in its original order, they are depicted in horizontal direction. The third constituent encoder is the network encoder at the relay. As it processes the interleaved information bits, it is depicted in vertical direction. The network encoder combines the information bits of MS1 and MS2. Therefore, the encoder at the mobile station and at the relay form one spatially distributed code.

6.3.3 Iterative Network and Channel Decoding

The base station receives the disturbed version $\mathbf{y}_\mathrm{MB}$ of $\mathbf{x}_\mathrm{M}$ in the first time phase, the disturbed version $\mathbf{y}_\mathrm{WB}$ of $\mathbf{x}_\mathrm{W}$ in the second time phase and the disturbed version $\mathbf{y}_\mathrm{RB}$ of $\mathbf{x}_\mathrm{R}$ in the third time phase. The channel output $\mathbf{y}_\mathrm{MB}$ is demodulated to LLRs $\mathbf{r}_{\mathrm{M},1}$ about the code bits $\mathbf{c}_{\mathrm{M},1}$. The channel output $\mathbf{y}_\mathrm{WB}$ is demodulated to LLRs $\mathbf{r}_{\mathrm{W},1}$ about the code bits $\mathbf{c}_{\mathrm{W},1}$. The channel output $\mathbf{y}_\mathrm{RB}$ is demodulated to LLRs $\mathbf{r}_{\mathrm{R},1}$ about the code bits $\mathbf{c}_{\mathrm{R},1}$. The joint network-channel decoder at the base station outputs the estimates $\hat{\mathbf{u}}_\mathrm{M}$ and $\hat{\mathbf{u}}_\mathrm{W}$ based on $\mathbf{r}_{\mathrm{M},1}$, $\mathbf{r}_{\mathrm{W},1}$ and $\mathbf{r}_{\mathrm{R},1}$. Figure 6.4 (c) depicts the decoder. There are three SISO decoders. The upper and the lower SISO decoder correspond to the channel encoders at MS1 and MS2, respectively. The middle SISO decoder corresponds to the convolutional encoder at the relay. The SISO decoder exchange iteratively soft information similar as in a conventional turbo decoder. First, the upper and lower SISO decoder calculate a posteriori LLRs $L^-(\hat{\mathbf{u}}_\mathrm{M})$ and $L^-(\hat{\mathbf{u}}_\mathrm{W})$. They use the LLRs $\mathbf{r}_{\mathrm{M},1}$ and $\mathbf{r}_{\mathrm{W},1}$, respectively. We include a value of zero for the punctured bits before the decoding starts. Initially, no a priori information is available ($L^|_e(\hat{\mathbf{u}}_\mathrm{M}) = L^|_e(\hat{\mathbf{u}}_\mathrm{W}) = 0$). Then, the middle SISO decoder calculates a posteriori LLRs $L^|(\hat{\mathbf{u}}_\mathrm{M})$ and $L^|(\hat{\mathbf{u}}_\mathrm{W})$ based on $\mathbf{r}_{\mathrm{R},1}$ and on a priori information about $\mathbf{u}_\mathrm{M}$ and $\mathbf{u}_\mathrm{W}$ from the other two SISO decoders. In order to obtain the required a priori information, the extrinsic information $L^-_e(\hat{\mathbf{u}}_\mathrm{M}) = L^-(\hat{\mathbf{u}}_\mathrm{M}) - L^|_e(\hat{\mathbf{u}}_\mathrm{M})$ and $L^-_e(\hat{\mathbf{u}}_\mathrm{W}) = L^-(\hat{\mathbf{u}}_\mathrm{W}) - L^|_e(\hat{\mathbf{u}}_\mathrm{W})$ has to be interleaved and mixed in the same way as it was done at the relay. Then, extrinsic information $L^|_e(\hat{\mathbf{u}}_\mathrm{M}) = L^|(\hat{\mathbf{u}}_\mathrm{M}) - L^-_e(\hat{\mathbf{u}}_\mathrm{M})$ and $L^|_e(\hat{\mathbf{u}}_\mathrm{W}) = L^|(\hat{\mathbf{u}}_\mathrm{W}) - L^-_e(\hat{\mathbf{u}}_\mathrm{W})$ is passed back to the upper and lower SISO decoder. There, it can be exploited as a priori information in the next decoding round. After several iterations, the middle SISO decoder outputs the hard estimates $\hat{\mathbf{u}}_{\mathrm{M}}$ and $\hat{\mathbf{u}}_{\mathrm{M}}$. Note that the upper and lower SISO decoder could deliver the estimates as well.

6.3.4 Alternative Turbo Network Codes

In the previously described system the transmissions from the mobile stations to the relay are only protected with a convolutional code. In certain scenarios it is advantageous to protect these links with a PCCC. The described code design can be easily extended by replacing the convolutional codes at the mobile stations with PCCCs. The encoder at the relay remains a convolutional code. Two of the three SISO decoders at the base station also have to be replaced by turbo decoders. The turbo decoders output soft information after several inner iterations and exchange soft information with the third decoder in several outer iterations. The performance of the code benefits, if the interleaver in the turbo codes is chosen different than the interleaver at the relay. For the simulation results in Section 6.9 with turbo codes at the mobile stations, we use the regular UMTS interleaver at the relay and a modified interleaver at the mobile stations. The modified interleaver results from flipping the UMTS interleaver from left to right. Joint network-channel codes with turbo codes at the mobile stations are compared in [Dup05, Sch07, Sch08].

6.4 Information-Theoretic Limits

Next, we consider the achievable decode-and-forward rate with the considered time-division MARC model with optimized allocation of the transmission time. Both the data of mobile station 1 and 2 can be decoded reliably at the base station, if the following five inequalities hold:

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) \tag{6.8}$$
$$K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WR}}) \tag{6.9}$$
$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) \tag{6.10}$$
$$K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) \tag{6.11}$$
$$K_{\mathrm{M}} + K_{\mathrm{W}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) \tag{6.12}$$

The Conditions (6.8)-(6.12) follow from [SKM04, Equation (12)]. The detailed derivation is given in the Appendix A.2. For $K_{\mathrm{W}} = 0$ the Conditions (6.8)-(6.12) for the MARC fall back to the Conditions (5.6) and (5.7) for the relay channel.
As an error detection code is included, the fulfillment of both of the two conditions $K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}})$ and $K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}})$ allows also a reliable communication alternatively to the fulfillment of (6.8) to (6.12).
Figure 6.5 depicts as illustration the corresponding cuts to the Conditions (6.10)-(6.12) for reliable decoding at the base station. The Conditions (6.8) and (6.9) guarantee that the relay decodes the information reliably.

We define the time-allocation parameters $\theta_{\mathrm{M}} = M_{\mathrm{M}}/M$, $\theta_{\mathrm{W}} = M_{\mathrm{W}}/M$ and $\theta_{\mathrm{R}} = M_{\mathrm{R}}/M$

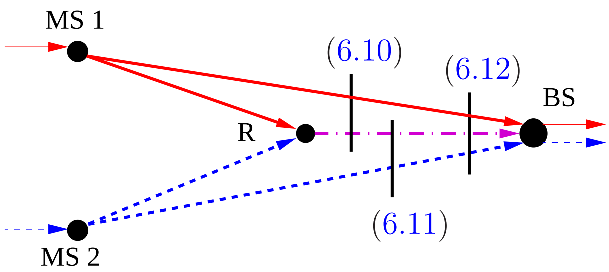


Figure 6.5: Illustration of the corresponding cuts to the Conditions (6.10)-(6.12) for reliable decoding at the base station.

with $\theta_{\mathrm{M}} + \theta_{\mathrm{W}} + \theta_{\mathrm{R}} = 1$. As for the relay channel, we want to choose the time allocation such that the rate is maximized. We maximize R_{M} under the restriction that the rate ratio $\sigma = R_{\mathrm{W}}/R_{\mathrm{M}}$ is fulfilled:

$$[\theta_{\mathrm{M}}^*, \theta_{\mathrm{W}}^*] = \arg \max_{[\theta_{\mathrm{M}},\theta_{\mathrm{W}}]} R_{\mathrm{M}} \quad \text{subject to} \quad \sigma = R_{\mathrm{W}}/R_{\mathrm{M}},\ \theta_{\mathrm{R}} = 1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}} \tag{6.13}$$

with

$$\begin{aligned} R_{\mathrm{M}} = \min\{&\theta_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MR}}), \theta_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WR}})/\sigma, \theta_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1-\theta_{\mathrm{M}}-\theta_{\mathrm{W}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}), \\ &(\theta_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + (1-\theta_{\mathrm{M}}-\theta_{\mathrm{W}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}))/\sigma, \\ &(\theta_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + \theta_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + (1-\theta_{\mathrm{M}}-\theta_{\mathrm{W}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}))/(1+\sigma)\}. \end{aligned} \tag{6.14}$$

The expression in (6.14) follows from the Conditions (6.8)-(6.12).

If the conditions $\mathcal{C}(\gamma_{\mathrm{MB}}) \le \mathcal{C}(\gamma_{\mathrm{RB}})$, $\mathcal{C}(\gamma_{\mathrm{WB}}) \le \mathcal{C}(\gamma_{\mathrm{RB}})$, $\mathcal{C}(\gamma_{\mathrm{MB}}) \le \mathcal{C}(\gamma_{\mathrm{MR}})$ and $\mathcal{C}(\gamma_{\mathrm{WB}}) \le \mathcal{C}(\gamma_{\mathrm{WR}})$ are valid, we obtain closed expressions for the optimal time allocation and the achievable rate. As shown in the Appendix A.3, the optimal time-allocation parameters are given by

$$\boxed{\theta_{\mathrm{M}}^* = \frac{\mathcal{C}(\gamma_{\mathrm{RB}})}{(1+\sigma\cdot\kappa)\cdot\mathcal{C}(\gamma_{\mathrm{RB}}) + (1+\sigma)\cdot\mathcal{C}(\gamma_{\mathrm{MR}}) - \mathcal{C}(\gamma_{\mathrm{MB}}) - \sigma\cdot\kappa\cdot\mathcal{C}(\gamma_{\mathrm{WB}})}} \tag{6.15}$$

$$\boxed{\theta_{\mathrm{W}}^* = \theta_{\mathrm{M}}^* \cdot \sigma \cdot \kappa} \tag{6.16}$$

with $\kappa = \mathcal{C}(\gamma_{\mathrm{MR}})/\mathcal{C}(\gamma_{\mathrm{WR}})$ and the achievable rates R_{M} and R_{W} are given by

$$\boxed{R_{\mathrm{M}} = \frac{\mathcal{C}(\gamma_{\mathrm{MR}})\cdot\mathcal{C}(\gamma_{\mathrm{RB}})}{(1+\sigma\cdot\kappa)\cdot\mathcal{C}(\gamma_{\mathrm{RB}}) + (1+\sigma)\cdot\mathcal{C}(\gamma_{\mathrm{MR}}) - \mathcal{C}(\gamma_{\mathrm{MB}}) - \sigma\cdot\kappa\cdot\mathcal{C}(\gamma_{\mathrm{WB}})}} \tag{6.17}$$

$$\boxed{R_{\mathrm{W}} = \sigma \cdot R_{\mathrm{M}}.} \tag{6.18}$$

If MS2 does not want to transfer any information to the sink ($\sigma = 0$), then the optimal time allocation θ_{M}^* and the achievable rate R_{M} in (6.15) and (6.17) for the MARC fall back to the allocation and rate in (4.21) and (4.22) for the relay channel. For the symmetric MARC regarding the rate ($\sigma = 1$) and the SNRs ($\kappa = 1$, $\mathcal{C}(\gamma_{\mathrm{MB}}) = \mathcal{C}(\gamma_{\mathrm{WB}})$), we obtain the same sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{W}}$ as for the relay channel in (4.22). We will show in Section 6.7 that the MARC system achieves in general the same sum-rate as the relay channel system. It also can be shown that the optimal time allocation is the same for the MARC and two time-division relay channels. Although the MARC does not increase the rate compared to the relay channel, we will see in the next section the improvements regarding the outage behavior in a fading environment.

6.5 Outage Behavior

In this section, we consider the outage behavior for the multiple-access relay channel. First, we summarize the outage behavior for the reference systems, a system with two point-to-point channels and a system with two relay channels. It depends on all five fading coefficients, whether a communication system is in outage or not. We will define the outage event for all systems separately. All definitions of the outage event OUT correspond to the case that no reliable communication from one or both of the sources to the sink is possible. The event $\overline{\mathrm{OUT}}$ is defined as the complement of the event OUT. Again, we assume the probability of erroneous error-detection through the CRC code to be much smaller than the probability of erroneous error-correction and neglect error-detection errors for the definition of the outage events.

6.5.1 Two Point-to-Point Channels

We first consider the system without relay. Source 1 encodes and modulates K_{M} information bits to $\hat{M}_{\mathrm{M}}$ symbols. Source 2 encodes and modulates K_{W} information bits to $\hat{M}_{\mathrm{W}}$ symbols. Both sources transmit their symbols to the sink with power P. In total $M = \hat{M}_{\mathrm{M}} + \hat{M}_{\mathrm{W}}$ symbols are transmitted. The system rate R is given by $R = (K_{\mathrm{M}} + K_{\mathrm{W}})/(\hat{M}_{\mathrm{M}} + \hat{M}_{\mathrm{W}})$. The instantaneous SNRs on the source-sink links are given by $\gamma_{\mathrm{MB}} = |a_{\mathrm{MB}}|^2 \cdot \rho_{\mathrm{MB}}$ and $\gamma_{\mathrm{WB}} = |a_{\mathrm{WB}}|^2 \cdot \rho_{\mathrm{WB}}$, respectively. The capacities C_1 and C_2 in bits per complex channel use are given by $C_1 = \mathcal{C}(\gamma_{\mathrm{MB}}) = \log_2(1 + |a_{\mathrm{MB}}|^2 \rho_{\mathrm{MB}})$ and by $C_2 = \mathcal{C}(\gamma_{\mathrm{WB}})$ under the assumption of a Gaussian distributed channel input variable (compare Section 2.3).
Both the data of source 1 and 2 can be decoded reliably at the sink, if the fading random variables a_{MB} and a_{WB} have values such that the following two inequalities hold [Pro95]:

$$K_{\mathrm{M}} \leq \hat{M}_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) \tag{6.19}$$

$$K_{\mathrm{W}} \leq \hat{M}_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) \tag{6.20}$$

We define the event $\overline{\mathrm{OUT}}$ for the system with two point-to-point channels as

$$\boxed{\left[K_{\mathrm{M}} \leq \hat{M}_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}})\right] \wedge \left[K_{\mathrm{W}} \leq \hat{M}_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}})\right].} \tag{6.21}$$

For a Gaussian distributed input, a closed form for the outage probability can be derived [OSW94, LTW04].

6.5.2 Two Relay Channels

Next, we consider the achievable rate with a decode-and-forward strategy for the relay channel. Source 1 encodes and modulates K_{M} information bits to M_{M} symbols. Source 2 encodes and modulates K_{W} information bits to M_{W} symbols. The communications from source 1 and from source 2 are both supported by the relay. However, the relay does not mix the data of source 1 and 2 and treats both data streams separately. Therefore, the system consists of two relay channels. Source 1 and 2 broadcast their symbols to the sink and to the relay. The relay decodes and reencodes (and remodulates) the K_{M} information bits from source 1 to $M_{\mathrm{R},1} = \alpha \cdot M_{\mathrm{R}}$ symbols $(0 \leq \alpha \leq 1)$[2]. Then, the relay decodes and reencodes (and remodulates) the K_{W} information bits from source 2 to $M_{\mathrm{R},2} = \bar{\alpha} \cdot M_{\mathrm{R}}$ symbols $(\bar{\alpha} = 1 - \alpha)$. Both sources and the relay transmit with power P. The number of transmitted symbols is given by $M = M_{\mathrm{M}} + M_{\mathrm{W}} + M_{\mathrm{R},1} + M_{\mathrm{R},2} = M_{\mathrm{M}} + M_{\mathrm{W}} + M_{\mathrm{R}}$.
Both the data of source 1 and 2 can be decoded reliably at the sink, if the fading random variables a_{MB}, a_{WB}, a_{MR}, a_{WR} and a_{RB} have values such that the following four inequalities hold (compare (5.6) and (5.7)):

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right) \tag{6.22}$$
$$K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right) \tag{6.23}$$
$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + \alpha \cdot M_{\mathrm{R}} \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) \tag{6.24}$$
$$K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right) + \bar{\alpha} \cdot M_{\mathrm{R}} \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) \tag{6.25}$$

For $\gamma_{\mathrm{MR}} < \gamma_{\mathrm{MB}}$, the fulfillment of the condition $K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)$ allows also a reliable communication alternatively to the fulfillment of (6.22) and (6.24). For $\gamma_{\mathrm{WR}} < \gamma_{\mathrm{WB}}$, the fulfillment of the condition $K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right)$ allows also a reliable communication alternatively to the fulfillment of (6.23) and (6.25).
The sink can decode the data of source 1 reliably, either if the Conditions (6.22) and (6.24) hold or if $K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)$ holds (compare (5.9)). The data of source 2 can be decoded reliably, if the corresponding equations fulfill the corresponding condition.
We define the event $\overline{\mathrm{OUT}}$ for the system with two relay channels as

$$\boxed{\begin{aligned} &\Big(\Big(\big[K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MR}})\big] \wedge \big[K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + \alpha \cdot M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\big]\Big) \\ &\vee \big[K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}})\big]\Big) \\ \wedge &\Big(\Big(\big[K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WR}})\big] \wedge \big[K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + \bar{\alpha} \cdot M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\big]\Big) \\ &\vee \big[K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}})\big]\Big). \end{aligned}} \tag{6.26}$$

[2] $M_{\mathrm{R},1} = \alpha \cdot M_{\mathrm{R}}$ has to be a non-negative integer.

6.5.3 Multiple-Access Relay Channel

Next, we consider the achievable rate with the time-division multiple-access relay channel as described in Section 6.2. As previously shown, both the data of source 1 and 2 can be decoded reliably at the sink, if the fading random variables a_{MB}, a_{WB}, a_{MR}, a_{WR} and a_{RB} have values such that the five Conditions (6.8)-(6.12) hold. The fulfillment of the two conditions $K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)$ and $K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right)$ allows also a reliable communication alternatively to the fulfillment of (6.8) to (6.12).
The sink can decode the data of the sources reliably, either if (6.8) - (6.12) hold or if both of the two conditions $K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)$ and $K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right)$ hold.
We define the event $\overline{\mathrm{OUT}}$ for the MARC as

$$\boxed{\begin{aligned} &\Big(\big[K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MR}})\big] \wedge \big[K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WR}})\big] \\ &\quad \wedge \big[K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\big] \wedge \big[K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\big] \\ &\quad \wedge \big[K_{\mathrm{M}} + K_{\mathrm{W}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})\big]\Big) \\ &\vee \Big(\big[K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}})\big] \wedge \big[K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}})\big]\Big). \end{aligned}} \tag{6.27}$$

6.5.4 Multiple-Access Relay Channel with Separate Network-Channel Coding

Next, we consider the MARC with separate network-channel coding (SNCC). This means, that first, on the physical layer one performs (local) channel coding for each transmission. Depending on the fading coefficients, the local channel code is either in outage or not. An outage is equivalent to an erasure for the higher layers and thus, we obtain a network of erasure-based links. Second, on the network layer one performs network coding for the resulting network. SNCC for MARCs was described in the example in Section 6.1.1 and was applied in [AIGM93, TF04, BL05, CKL06].
Source 1 encodes and modulates K_{M} information bits to M_{M} symbols. Source 2 encodes and modulates K_{W} information bits to M_{W} symbols. Again, source 1 and 2 broadcast their M_{M} and M_{W} symbols in time-orthogonalized channels to the sink and to the relay. The relay decodes the K_{M} information bits from source 1 to and the K_{W} information bits from source 2. Then, the relay network-encodes the bits from source 1 and source 2 to $\mathbf{u}_{\mathrm{R}} = \mathbf{u}_{\mathrm{M}} \oplus \mathbf{u}_{\mathrm{W}}$. For $K_{\mathrm{M}} > K_{\mathrm{W}}$ or for $K_{\mathrm{W}} > K_{\mathrm{M}}$, the remaining bits from the K_{M} or K_{W} bits, which do not have a partner for encoding, are attached to $\mathbf{u}_{\mathrm{R}}$. The network encoder output $\mathbf{u}_{\mathrm{R}}$ is protected against the noisy channel by a local channel encoder and modulator which belongs to the physical layer and outputs M_{R} symbols. Both sources and the relay transmit with power P. The number of total transmitted symbols is given by $M = M_{\mathrm{M}} + M_{\mathrm{W}} + M_{\mathrm{R}}$.
Both the data of source 1 and 2 can be decoded reliably at the sink, if the fading random

variables a_{MB}, a_{WB}, a_{MR}, a_{WR} and a_{RB} have values such that the following holds:

$$K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}\left(\gamma_{\text{MR}}\right) \tag{6.28}$$

$$K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}\left(\gamma_{\text{WR}}\right) \tag{6.29}$$

$$\begin{aligned} &\left(\left[K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}\left(\gamma_{\text{MB}}\right)\right] \wedge \left[K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}\left(\gamma_{\text{WB}}\right)\right]\right) \\ \vee &\left(\left[K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}\left(\gamma_{\text{MB}}\right)\right] \wedge \left[K_{\text{W}} \leq M_{\text{R}} \cdot \mathcal{C}\left(\gamma_{\text{RB}}\right)\right]\right) \\ \vee &\left(\left[K_{\text{M}} \leq M_{\text{R}} \cdot \mathcal{C}\left(\gamma_{\text{RB}}\right)\right] \wedge \left[K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}\left(\gamma_{\text{WB}}\right)\right]\right) \end{aligned} \tag{6.30}$$

The Conditions (6.28) and (6.29) guarantee that the relay is able to decode the information of source 1 and 2 reliably. The sink is able to decode reliably if two out of the three links to the sink are not erased (see (6.30)). The fulfillment of the two conditions $K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}\left(\gamma_{\text{MB}}\right)$ and $K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}\left(\gamma_{\text{WB}}\right)$ allows also a reliable communication alternatively to the fulfillment of (6.28) to (6.30).
We define the event $\overline{\text{OUT}}$ for the MARC with separate network-channel coding as

$$\boxed{\begin{aligned} &\Bigg(\left[K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}(\gamma_{\text{MR}})\right] \wedge \left[K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}(\gamma_{\text{WR}})\right] \\ \wedge &\Bigg(\left(\left[K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}(\gamma_{\text{MB}})\right] \wedge \left[K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}(\gamma_{\text{WB}})\right]\right) \\ &\quad \vee \left(\left[K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}(\gamma_{\text{MB}})\right] \wedge \left[K_{\text{W}} \leq M_{\text{R}} \cdot \mathcal{C}(\gamma_{\text{RB}})\right]\right) \\ &\quad \vee \left(\left[K_{\text{M}} \leq M_{\text{R}} \cdot \mathcal{C}(\gamma_{\text{RB}})\right] \wedge \left[K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}(\gamma_{\text{WB}})\right]\right)\Bigg)\Bigg) \\ \vee &\left(\left[K_{\text{M}} \leq M_{\text{M}} \cdot \mathcal{C}(\gamma_{\text{MB}})\right] \wedge \left[K_{\text{W}} \leq M_{\text{W}} \cdot \mathcal{C}(\gamma_{\text{WB}})\right]\right). \end{aligned}} \tag{6.31}$$

Although a Gaussian distributed channel input can achieve the largest capacity, in practical channel encoded and modulated systems, the channel input variable will normally not follow a Gaussian distribution, but will be a variable from a discrete alphabet, like BPSK, 4-QAM or 16-QAM, where each symbol carries L code bits. The outage events for all systems under the constraint that we use the discrete modulation alphabet $\mathcal{S}_k$ are given by (6.19) - (6.31), when we replace $\mathcal{C}(\cdot)$ by $\mathcal{C}_k(\cdot)$ in (2.9).

6.6 Allocation of Transmission Time

The optimal allocation of the transmission time to MS1, MS2 and the relay is derived in Section 6.4 for given channel SNRs γ_{MR}, γ_{MB}, γ_{WR}, γ_{WB} and γ_{RB}.
This allocation is suitable for noisy channels without fading. For channels with fading, it is important to choose the allocation such that diversity can be gained. We want to know how to choose the time allocation such that a diversity order of two can be gained. As previously, we consider the system with coded modulation where each symbol carries L code bits. The system has a diversity order of two, if it can tolerate that one of the

five links is in a very deep fade. That means that no outage occurs for either $\gamma_{\mathrm{MR}} \to 0$, $\gamma_{\mathrm{MB}} \to 0$, $\gamma_{\mathrm{WR}} \to 0$, $\gamma_{\mathrm{WB}} \to 0$ and $\gamma_{\mathrm{RB}} \to 0$.
If either the MS1-R link, the MS2-R link or the R-BS link is in a very deep fade, we can observe from the outage definition in (6.27) that it is necessary that the information is transferred over the direct links from the mobile stations to the base station. As $\mathcal{C}(\gamma_{\mathrm{MB}})$ and $\mathcal{C}(\gamma_{\mathrm{WB}})$ are limited by L, this requires that

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot L = \theta_{\mathrm{M}} \cdot M \cdot L \quad \text{and} \quad K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot L = \theta_{\mathrm{W}} \cdot M \cdot L \tag{6.32}$$

are fulfilled. If the MS1-BS link is in a very deep fade ($\gamma_{\mathrm{MB}} \to 0$), we can observe from the outage definition in (6.27) that the information of MS1 has to be transferred over the relay route. Beside the condition in (6.32), it is necessary that

$$K_{\mathrm{M}} \leq M_{\mathrm{R}} \cdot L = \theta_{\mathrm{R}} \cdot M \cdot L \tag{6.33}$$

holds. Otherwise (6.10) can never be fulfilled for $\gamma_{\mathrm{MB}} \to 0$. The corresponding condition for MS2 is necessary, if the MS2-BS link is in a very deep fade ($\gamma_{\mathrm{WB}} \to 0$):

$$K_{\mathrm{W}} \leq M_{\mathrm{R}} \cdot L = \theta_{\mathrm{R}} \cdot M \cdot L \tag{6.34}$$

Due to the assumption $K_{\mathrm{M}} \geq K_{\mathrm{W}}$, Condition (6.34) is not relevant, because it is implied by (6.33). From (6.32), it follows that $R_{\mathrm{c}} \leq \theta_{\mathrm{M}} + \theta_{\mathrm{W}}$ is required for a diversity order of two. Moreover, it follows from (6.33) and from $K_{\mathrm{W}} = \sigma \cdot K_{\mathrm{M}}$ that $R_{\mathrm{c}}/(1+\sigma) \leq \theta_{\mathrm{R}}$ is required for a diversity order of two. Therefore, we obtain the following condition for $\theta = \theta_{\mathrm{M}} + \theta_{\mathrm{W}}$ to allow a diversity order of two dependent on the code rate R_{c}:

$$\boxed{R_{\mathrm{c}} \leq \theta \leq 1 - \frac{R_{\mathrm{c}}}{1+\sigma}} \tag{6.35}$$

It is possible to fulfill the above condition, if the rate R_{c} fulfills

$$\boxed{R_{\mathrm{c}} \leq \frac{1+\sigma}{2+\sigma}.} \tag{6.36}$$

Figure 6.6 depicts the region for values of θ which allow a diversity order of two dependent on the code rate $R_{\mathrm{c}} = R/L$ for the symmetric MARC with $\sigma = 1$. For comparison the region for the relay channel from Figure 5.6 is also depicted. Contrary to the relay channel, the symmetric MARC allows a diversity order of two beyond $R_{\mathrm{c}} = 0.5$ up till $R_{\mathrm{c}} \leq 2/3$.

6.7 Comparison of Relay and Multiple-Access Relay Channel

6.7.1 Achievable Sum-Rate

We compare the achievable sum-rate $(K_{\mathrm{M}}+K_{\mathrm{W}})/M$ for the two relay channels according to (6.22)-(6.25) with the achievable sum-rate for the multiple-access relay channel (MARC)

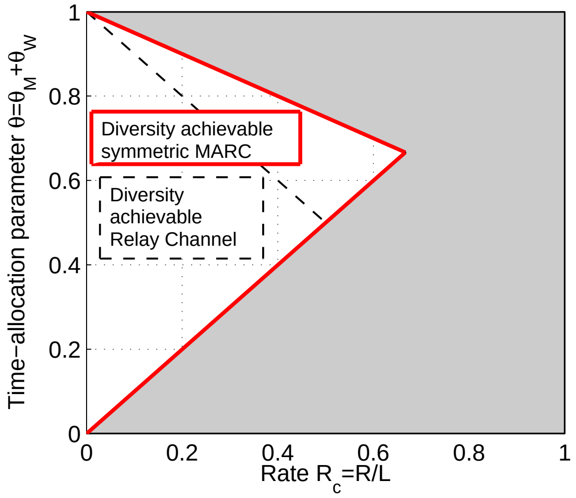


Figure 6.6: Values of $\theta = \theta_{\mathrm{M}} + \theta_{\mathrm{W}}$ which allow a diversity order of two dependent on the code rate $R_{\mathrm{c}} = R/L$ for the symmetric MARC.

according to (6.8)-(6.12). The Conditions (6.10) and (6.11) would allow higher rates for the MARC compared to (6.24) and (6.25). However, we also have to consider the additional sum-constraint on $K_{\mathrm{M}} + K_{\mathrm{W}}$ for the MARC in Equ. (6.12). As the sum of (6.24) and (6.25) results in (6.12), the relay channel and the MARC allow the same sum-rate $(K_{\mathrm{M}} + K_{\mathrm{W}})/M$.

6.7.2 Outage Behavior

We can recognize from the (6.24) and (6.25), that it is necessary for the system with two relay channels that

$$K_{\mathrm{M}} \leq L \cdot \alpha \cdot M_{\mathrm{R}} \quad \text{and} \quad K_{\mathrm{W}} \leq L \cdot \bar{\alpha} \cdot M_{\mathrm{R}} \tag{6.37}$$

are fulfilled to allow a diversity order of two. Otherwise (6.24) and (6.25) can never be fulfilled for $\gamma_{\mathrm{MB}} \to 0$ and $\gamma_{\mathrm{WB}} \to 0$, respectively. From (6.10) and (6.11), we can recognize that it is necessary for the MARC that

$$K_{\mathrm{M}} \leq L \cdot M_{\mathrm{R}} \quad \text{and} \quad K_{\mathrm{W}} \leq L \cdot M_{\mathrm{R}} \tag{6.38}$$

are fulfilled to allow a diversity order of two. It is obvious that the requirements for a diversity order of two are easier to fulfill for the MARC than for the relay channel with a fixed parameter α. As only the average channel SNRs ρ_{MB}, ρ_{WB} and ρ_{RB} are available at the relay, it is not possible to adjust α optimally.
We conclude, that the MARC offers an advantageous outage behavior compared to the relay channel for systems with a discrete modulation alphabet, if no instantaneous channel knowledge (a_{MB}, a_{WB} and a_{RB}) is available at the relay. The MARC allows to gain diversity for $R_{\mathrm{c}} \leq (1+\sigma)/(2+\sigma)$ whereas the relay channel allows to gain diversity for $R_{\mathrm{c}} \leq 1/2$ (compare Figure 6.6).

Note that the system with the relay channels could obtain the same outage behavior as the MARC system, if the parameter α were adjusted at the relay according to the instantaneous fading coefficients a_{MB}, a_{WB} and a_{RB} with the following rule. If $\hat{\alpha} = (K_{\mathrm{M}} - M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}))/(M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}}))$ is smaller than zero, we set α to $\alpha = 0$. If $\hat{\alpha}$ is larger than one, we set α to $\alpha = 1$. If $\hat{\alpha}$ is not smaller than zero and not larger than one, we set α to $\alpha = \hat{\alpha}$. With this choice for α we try to find the smallest value for α such that (6.24) is fulfilled. Note that a feedback of two bits in a control channel from the sink to the relay, whether $\mathbf{u}_{\mathrm{M}}$ and/or $\mathbf{u}_{\mathrm{W}}$ are decoded without error at the sink after the first two time phases, is not sufficient for the relay channel, because it often can happen that neither $\mathbf{u}_{\mathrm{M}}$ nor $\mathbf{u}_{\mathrm{W}}$ is decoded without error after the first two time phases. In such cases the transmission of the relay still may help to recover both $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{W}}$, however, the parameter α could only be optimized, if a_{MB}, a_{WB} and a_{RB} were known at the relay, which would require more than two bits feedback.

6.8 Relation to Hybrid ARQ/FEC with Cross-Packet Channel Coding

Joint network-channel coding for the MARC is related to a point-to-point communciation with hybrid ARQ/FEC with cross-packet channel coding (compare Section 2.5.3). The relation can be recognized, if we integrate the relay and the two mobile stations into one node ($\mathcal{C}(\gamma_{\mathrm{MR}})/M_{\mathrm{M}} \to \infty$, $\mathcal{C}(\gamma_{\mathrm{WR}})/M_{\mathrm{W}} \to \infty$, $\mathrm{E}\{\gamma_{\mathrm{RB}}\} = \mathrm{E}\{\gamma_{\mathrm{MB}}\}$) and remove the transmission from MS2[3] ($M_{\mathrm{W}} \to 0$). Then, there is no difference whether the mobile station or the relay sends the retransmission and the conditions for reliable communication over the MARC in (6.8)-(6.12) fall back to the conditions for reliable point-to-point communication with cross-packet coding in (2.18)-(2.19) whereas the notation has changed ($\gamma_1 = \gamma_{\mathrm{MB}}$, $\gamma_2 = \gamma_{\mathrm{RB}}$, $M_1 = M_{\mathrm{M}}$, $M_2 = M_{\mathrm{R}}$, $K_1 = K_{\mathrm{M}}$, $K_2 = K_{\mathrm{W}}$).
Both methods take advantage from the joint coding of two packets (cross-packet coding). This allows to achieve a diversity order of two with a larger rate than the systems without cross-packet coding (communication with relay channel or with conventional H-ARQ). Diversity can be gained for $R_{\mathrm{c}} \leq (1+\sigma)/(2+\sigma)$ whereas the systems without cross-packet coding gain diversity for $R_{\mathrm{c}} \leq 1/2$.
As the link from relay to base station is assumed on average stronger than the link from the mobile stations to the base station ($\mathrm{E}\{\gamma_{\mathrm{RB}}\} > \mathrm{E}\{\gamma_2\}$), the system with relay gains from the higher SNR of the retransmission.
Figure 6.7 depicts the conditions for reliable communication for the point-to-point communication with conventional H-ARQ (compare (2.16)) and with H-ARQ with cross-packet coding (compare (2.18)-(2.19)), for the communication over the relay channel (compare (5.6)-(5.7)) and over the MARC (compare (6.8)-(6.12)) and illustrates the relation between the four systems. For specific assumptions the system for the MARC can fall back to each of the three other systems.

[3] If we integrate the relay and the two mobile stations into one node, it is guaranteed that the relay knows both packets. Therefore, there is no need for the transmission from MS2.

Advantage of relaying: $\mathrm{E}\{\gamma_{\mathrm{RB}}\} > \mathrm{E}\{\gamma_2\}$ $\Longrightarrow$

Conventional H-ARQ

$K_1 \leq M_1 \cdot \mathcal{C}(\gamma_1) + M_2 \cdot \mathcal{C}(\gamma_2)$

Relay Channel

$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MR}})$
$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$

Advantage of cross-packet coding:
Diversity gain for $R_{\mathrm{c}} \leq \frac{1+\sigma}{2+\sigma}$
instead of $R_{\mathrm{c}} \leq 1/2$.

$\Downarrow$

$\frac{\mathcal{C}(\gamma_{\mathrm{MR}})}{M_{\mathrm{M}}} \to \infty, \quad \frac{\mathcal{C}(\gamma_{\mathrm{WR}})}{M_{\mathrm{W}}} \to \infty, \quad M_{\mathrm{W}} \to 0$

$K_{\mathrm{W}} = K_2 = 0$

H-ARQ with Cross-Packet Coding

$K_2 \leq M_2 \cdot \mathcal{C}(\gamma_2)$
$K_1 + K_2 \leq M_1 \cdot \mathcal{C}(\gamma_1) + M_2 \cdot \mathcal{C}(\gamma_2)$

Multiple-Access Relay Channel

$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MR}})$
$K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WR}})$
$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$
$K_{\mathrm{W}} \leq M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$
$K_{\mathrm{M}} + K_{\mathrm{W}} \leq M_{\mathrm{M}} \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) +$
$\quad + M_{\mathrm{W}} \cdot \mathcal{C}(\gamma_{\mathrm{WB}}) + M_{\mathrm{R}} \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$

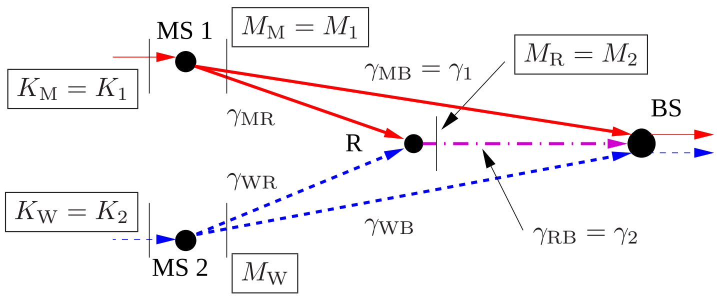


Figure 6.7: The system for the multiple-access relay channel falls back to several other systems for specific assumptions.

6.9 Simulation Results

We compare the proposed system using joint network-channel coding (JNCC) for the MARC to three reference systems.
We evaluate the outage rate by generating samples of the random fading coefficients and counting the number of outages according to the definitions in Section 6.5. We generate the outage rate for BPSK ($L = 1$) and for a Gaussian distributed channel input variable. Moreover, we simulate the coding system described in Section 6.3 and measure the bit error rate (BER), the packet error rate (PER) and the common packet error rate (CPER) for BPSK. To obtain the packet error rate, we measure the packet error rates for MS1 and

MS2 separately. Then, we take the mean of the two error rates. A common packet error occurs, if one or both of the two packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{W}}$ is decoded with error at the sink. The common packet error rate corresponds to the outage rate. The outage rate can be seen as information-theoretic benchmark for the CPER. The outage rate can be generated with much less computational effort than the bit and packet error rates (by a factor of more than 100000 with our computer system). As relay channels have more parameters (e.g. channel SNRs) than a point-to-point communication, the consideration of the outage rate allows a much more efficient system design than time-consuming simulations of bit and packet error rate. We do not depict the PERs in this work. However, the trend of the PER is the same as for the CPERs.

The information bits are grouped in packets of size $K_{\mathrm{M}} = K_{\mathrm{W}} = 1500$ bits. First, we will consider the case that the sources transmit blocks of $M_{\mathrm{M}} = M_{\mathrm{W}} = 2000$ symbols. The relay transmits either a block of $M_{\mathrm{R}} = 2000$ or a block of $M_{\mathrm{R}} = 4000$ symbols, resulting in a system rate of either $R = 1/2$ or $R = 3/8$. We choose these two possibilities for M_{R} because according to (6.37) and (6.38), $M_{\mathrm{R}} = 2000$ BPSK symbols allow a diversity order of two only for the MARC but not for the relay channel, whereas $M_{\mathrm{R}} = 4000$ BPSK symbols with $\alpha = 0.5$ allow a diversity order of two for both the MARC and the relay channel. This can be also recognized in Figure 6.6. Whereas $(R_{\mathrm{c}} = 1/2, \theta = (M_{\mathrm{M}} + M_{\mathrm{W}})/M = 2/3)$ only is in the "diversity-achievable" region of the MARC system, the point $(R_{\mathrm{c}} = 3/8, \theta = (M_{\mathrm{M}} + M_{\mathrm{W}})/M = 1/2)$ is also in the "diversity-achievable" region of the relay channel system. We want to know whether the theoretical prediction of the diversity order is consistent with the simulation results. We also consider the point-to-point system with the same system rates R as reference system. Then, the sources transmit blocks of either $\hat{M}_{\mathrm{M}} = \hat{M}_{\mathrm{W}} = 3000$ or $\hat{M}_{\mathrm{M}} = \hat{M}_{\mathrm{W}} = 4000$ symbols.

Then, we will show that the MARC system allows to gain diversity for a code rate $R_{\mathrm{c}} > 0.5$ ($M_{\mathrm{M}} = M_{\mathrm{W}} = 1750$, $M_{\mathrm{R}} = 2000$) and that the MARC system does not lose performance for an AWGN channel model without fading.

6.9.1 Reference Systems

Distributed Turbo Code for the Relay Channel

In the first reference system the uplink of the two mobile stations is done separately with the help of a relay. This scenario represents two relay channels where distributed turbo codes as described in Chapter 5 can be applied. Figure 6.8 (b) depicts the system model and an illustration of the distributed coding scheme. Each mobile station uses a separate relay for the uplink. The same situation would occur if one relay is shared for the uplink of the two mobile stations without network coding. For simplicity, Figure 6.8 (b) illustrates a distributed turbo code with a convolutional code at the mobile station, even if for the comparison a distributed turbo code with a turbo code at the mobile station was used.

The PCCC which is used for the uplink of MS1 consists of two constituent encoders. The first constituent encoder is a convolutional code at MS1 which outputs the information bits $\mathbf{u}_{\mathrm{M}}$ and additional parity bits $\mathbf{p}_{\mathrm{M}}$. The convolutional code is identical to the one described in Section 6.3.1. We puncture the parity bits $\mathbf{p}_{\mathrm{M}}$ and transmit M_{M} BPSK

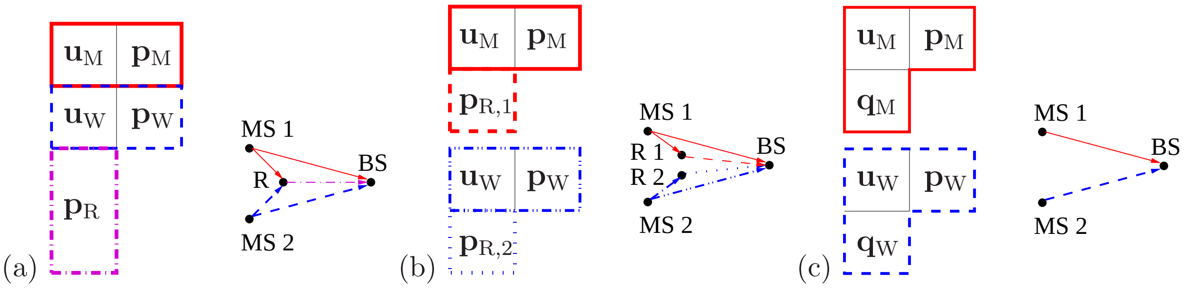


Figure 6.8: (**a**): Illustration of the turbo network code used for the multiple-access relay channel. (**b**): Illustration of the distributed turbo code used for the relay channel. (**c**): Illustration of the turbo code used for the point-to-point channel.

symbols. The second constituent code is processed at the first relay R 1. After decoding $\mathbf{u}_\mathrm{M}$, the estimates of the information bits $\mathbf{u}_\mathrm{M}$ are interleaved and channel encoded. We transmit $M_{\mathrm{R},1} = 0.5 \cdot M_\mathrm{R}$ of the resulting parity bits $\mathbf{p}_{\mathrm{R},1}$. The puncturing is done analogous to the puncturing described in Section 6.3.2. The uplink for the K_W bits from the other mobile station MS2 is done in the same way. This time a second relay R 2 processes the parity bits $\mathbf{p}_{\mathrm{R},2}$ and selects $M_{\mathrm{R},2} = 0.5 \cdot M_\mathrm{R}$ to transmit them to the base station.

Turbo Code for the Point-to-Point Channel

The second reference system does not contain a relay and the uplink for both mobile station is done with a point-to-point channel. A PCCC as described in Section 2.4.2 is used at the mobile stations. The PCCC contains the same convolutional encoders and the same interleaver as described in Section 6.3. Figure 6.8 (c) depicts an illustration of the PCCCs which are used. In this reference system, the transmission from MS1 contains the information bits $\mathbf{u}_\mathrm{M}$, the parity bits of the first constituent encoder $\mathbf{p}_\mathrm{M}$ and the parity bits of the second constituent encoder $\mathbf{q}_\mathrm{M}$.
The uplink of MS2 is done in the same way. The transmission contains K_W information bits $\mathbf{u}_\mathrm{W}$, parity bits of the first constituent encoder $\mathbf{p}_\mathrm{W}$ and parity bits of the second constituent encoder $\mathbf{q}_\mathrm{W}$.

Separate Network-Channel Coding for the MARC

We explain the system model for SNCC for the example with $K_\mathrm{M} = K_\mathrm{W} = 1500$, $M_\mathrm{M} = M_\mathrm{W} = 2000$ and $M_\mathrm{R} = 2000$. The channel codes at the mobile stations are PCCCs which consist of the convolutional encoders and the interleavers described before. Figure 6.9 (a) depicts channel encoder 1 at MS1. The output $\mathbf{c}_\mathrm{M}$ of the turbo code is punctured similar as the rate matching strategy described in [Eur00] and then modulated to $M_\mathrm{M} = 2000$ BPSK-modulated code bits $\mathbf{x}_\mathrm{M}$. Channel encoder 2 at MS2 works accordingly.
After decoding, the relay network encodes the estimates $\tilde{\mathbf{u}}_\mathrm{M}$ and $\tilde{\mathbf{u}}_\mathrm{W}$. This is done in the network layer. The network encoder is a modulo-2 addition. The network encoder output $\mathbf{u}_\mathrm{R} = \tilde{\mathbf{u}}_\mathrm{M} \oplus \tilde{\mathbf{u}}_\mathrm{W}$ is protected against the noisy channel by a local channel encoder

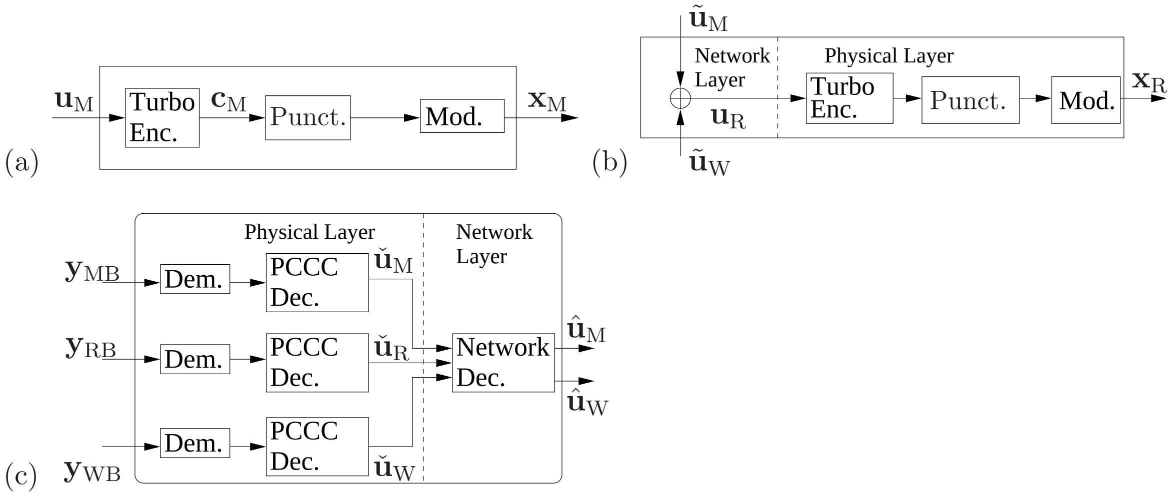


Figure 6.9: Separate Network-Channel Coding: (**a**): Channel encoder 1 at mobile station 1. (**b**): Separate network and channel encoder at the relay. (**c**): Separate network and channel decoding at the base station.

which belongs to the physical layer. This channel encoder is a punctured PCCC which is identical to the one used at the mobile stations. The output of the channel encoder can be expressed as

$$\mathbf{c}_{\mathrm{R},1} = (\tilde{\mathbf{u}}_\mathrm{M} \oplus \tilde{\mathbf{u}}_\mathrm{W}) \cdot \mathbf{G}_{\mathrm{R},1}. \tag{6.39}$$

The output of the punctured PCCC is modulated to the BPSK symbols $\mathbf{x}_\mathrm{R}$ with block length $M_\mathrm{R} = 2000$ or $M_\mathrm{R} = 4000$ and sent to the base station. A block diagram of the relay is depicted in Figure 6.9 (b).

Figure 6.9 (c) depicts the separate network and channel decoder at the base station. In the physical layer, each of the three channel outputs is demodulated and decoded with a conventional turbo decoder with several iterations. The three decoder make a hard decision and output their estimates $\check{\mathbf{u}}_\mathrm{M}$, $\check{\mathbf{u}}_\mathrm{R}$ and $\check{\mathbf{u}}_\mathrm{W}$ to the network decoder. Each channel decoder delivers an additional flag which indicates whether its estimate is error-free. This information can be obtained with the CRC code which was included in the data before the channel encoders. By delivering the hard decisions and the flags, the channel decoders provide erasure channels for the network decoder. If one of the two estimates $\check{\mathbf{u}}_\mathrm{M}$ or $\check{\mathbf{u}}_\mathrm{W}$ is not error-free and the other two estimates are error-free, the network decoder makes a modulo-2 addition of the estimates which are error-free to obtain the estimate which is not error-free. For example, if the estimates $\check{\mathbf{u}}_\mathrm{M}$, $\check{\mathbf{u}}_\mathrm{R}$ are error-free and $\check{\mathbf{u}}_\mathrm{W}$ is not error-free, the network decoder outputs the estimates $\hat{\mathbf{u}}_\mathrm{M} = \check{\mathbf{u}}_\mathrm{M}$ and $\hat{\mathbf{u}}_\mathrm{W} = \check{\mathbf{u}}_\mathrm{M} \oplus \check{\mathbf{u}}_\mathrm{R}$. If more than one of the estimates $\check{\mathbf{u}}_\mathrm{M}$, $\check{\mathbf{u}}_\mathrm{W}$ and $\check{\mathbf{u}}_\mathrm{R}$ is with error, the network decoder cannot exploit the diversity and outputs the estimates $\hat{\mathbf{u}}_\mathrm{M} = \check{\mathbf{u}}_\mathrm{M}$ and $\hat{\mathbf{u}}_\mathrm{W} = \check{\mathbf{u}}_\mathrm{W}$.

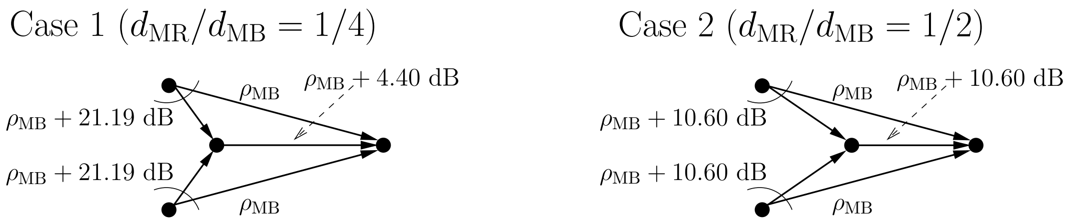


Figure 6.10: We consider the two cases $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$ and $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

6.9.2 System Scenarios

We consider two cases. It is assumed in both cases that the setup is symmetric with $d_{\mathrm{MB}} = d_{\mathrm{WB}}$ and $d_{\mathrm{MR}} = d_{\mathrm{WR}}$.
In the first case, the relay is positioned closer to the sources than to the sink. The distance between a source and the relay is one fourth of the distance between a source and the sink ($d_{\mathrm{MR}} = (1/4) \cdot d_{\mathrm{MB}}$) and the distance between relay and sink is given by $d_{\mathrm{RB}} = (3/4) \cdot d_{\mathrm{MB}}$. We assume a path-loss exponent of $n = 3.52$. Then, the average SNR $\rho_{\mathrm{MR}} = \rho_{\mathrm{WR}}$ on the source-relay links is increased by $35.2 \cdot \log_{10}(4)$ dB $= \rho_{\mathrm{MB}} + 21.19$ dB and the average SNR ρ_{RB} on the relay-sink link is increased by $35.2 \cdot \log_{10}(4/3)$ dB $= \rho_{\mathrm{MB}} + 4.40$ dB compared to the average SNR $\rho_{\mathrm{MB}} = \rho_{\mathrm{WB}}$ on the source-sink links.
In the second case, the relay is positioned half-way between the sources and the sink ($d_{\mathrm{MR}} = d_{\mathrm{RB}} = (1/2) \cdot d_{\mathrm{MB}}$). Then, the average SNRs $\rho_{\mathrm{MR}} = \rho_{\mathrm{WR}} = \rho_{\mathrm{RB}}$ are increased by $35.2 \cdot \log_{10}(2)$ dB $= \rho_{\mathrm{MB}} + 10.60$ dB compared to the average SNR $\rho_{\mathrm{MB}} = \rho_{\mathrm{WB}}$ on the source-sink links. Figure 6.10 depicts the characteristics of both cases.

6.9.3 Results with Fading

Results for BPSK

Let us first consider case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$ BPSK symbols. Figure 6.11 depicts the simulated bit error rate (BER) and common packet error rate (CPER) of the coding systems over the average signal-to-noise ratio (SNR) ρ_{MB} in dB on the source-sink link. The outage rate for BPSK is additionally depicted as theoretical benchmark for the coding system. The joint network-channel coding system uses a convolutional code at the mobile stations. The conditions for the MARC in (6.38) for a diversity order of two are fulfilled for this example with $M_{\mathrm{R}} = 2000$, whereas the conditions for the relay channel in (6.37) are not fulfilled. This can be also recognized in Figure 6.6. The point ($R_{\mathrm{c}} = 1/2, \theta = 2/3$) is in the diversity region of the MARC, but not in the diversity region of the relay channel. From the slope of the error rates curves we can recognize the diversity order of two which is provided by the JNCC scheme for the MARC. The system using SNCC achieves the same diversity gain. However, as the redundancy which is contained in the transmission of the relay cannot be efficiently exploited, SNCC has of a performance loss of more than 3 dB compared to the JNCC approach for a CPER of 10^{-2}. The systems for the relay channel and for the point-to-

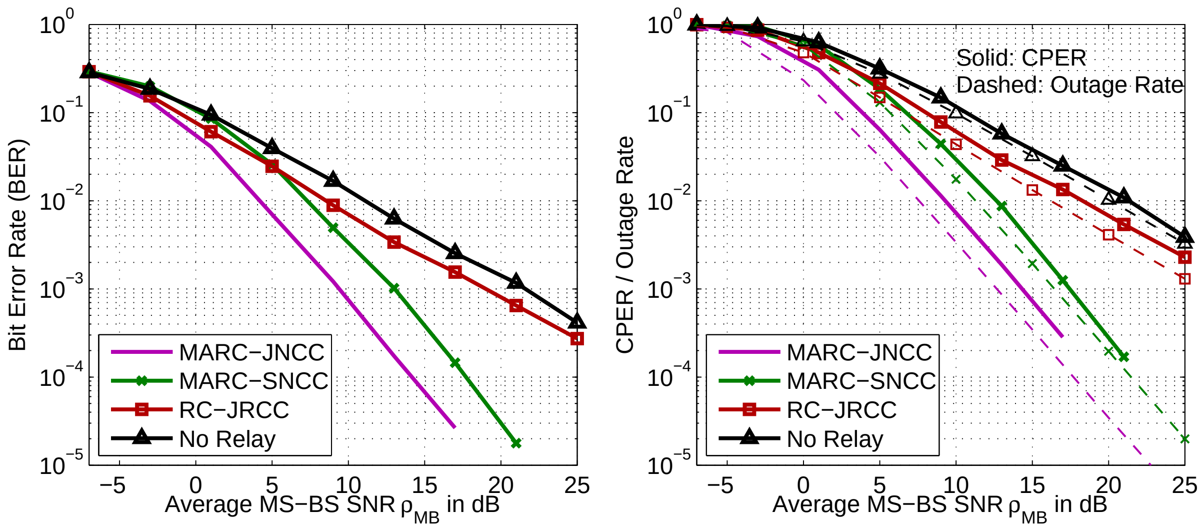


Figure 6.11: Numerical results for case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with BPSK and $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$: Bit error rate and common packet error rate (CPER) of system applying joint network-channel coding (JNCC) and separate network-channel coding (SNCC) for the multiple-access relay channel (MARC), joint routing-channel coding (JRCC) for the relay channel (RC) or a turbo code for the point-to-point channel (PtoPC).

point channel only allow a diversity order of one.

Let us consider case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with $M_{\mathrm{M}} = M_{\mathrm{W}} = 2000$ and $M_{\mathrm{R}} = 4000$ BPSK symbols. In comparison to $M_{\mathrm{R}} = 2000$, the total code rate $R = R_{\mathrm{c}}$ decreases from $R = 1/2$ to $R = 3/8$ and θ changes from $\theta = 2/3$ to $\theta = 1/2$. Figure 6.12 depicts the BER and CPER/outage rate for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$ with $M_{\mathrm{R}} = 4000$. For $M_{\mathrm{R}} = 4000$ BPSK symbols, the conditions for the MARC in (6.38) and for the relay channel in (6.37) for a diversity order of two are both fulfilled and the point $(R_{\mathrm{c}} = 3/8, \theta = 1/2)$ is in the diversity region of the MARC and the relay channel. We can observe in Figure 6.12 that the system for the relay channel achieves the same diversity than the system for the MARC. The reason is that the $M_{\mathrm{R},1} = M_{\mathrm{R},2} = M_{\mathrm{R}}/2$ symbols used by the relay for the help of the transmissions of source 1 and source 2 allow to communicate the $K_{\mathrm{M}} = K_{\mathrm{W}} = 1500$ information bits, if one of the source-relay links is in a deep fade and the relay-sink link is in a advantageous fade. The advantage of JNCC compared to SNCC increases for $M_{\mathrm{R}} = 4000$ to a gain of 4 dB, because for SNCC only the local error protection and the erasure rate of the relay-sink link benefits from the double amount of symbols $M_{\mathrm{R}} = 4000$. For JNCC, the complete distributed code benefits from using $M_{\mathrm{R}} = 4000$ instead of $M_{\mathrm{R}} = 2000$.
We conclude that the MARC system gains diversity compared to the relay channel, if (6.38) is fulfilled and (6.37) is not fulfilled.

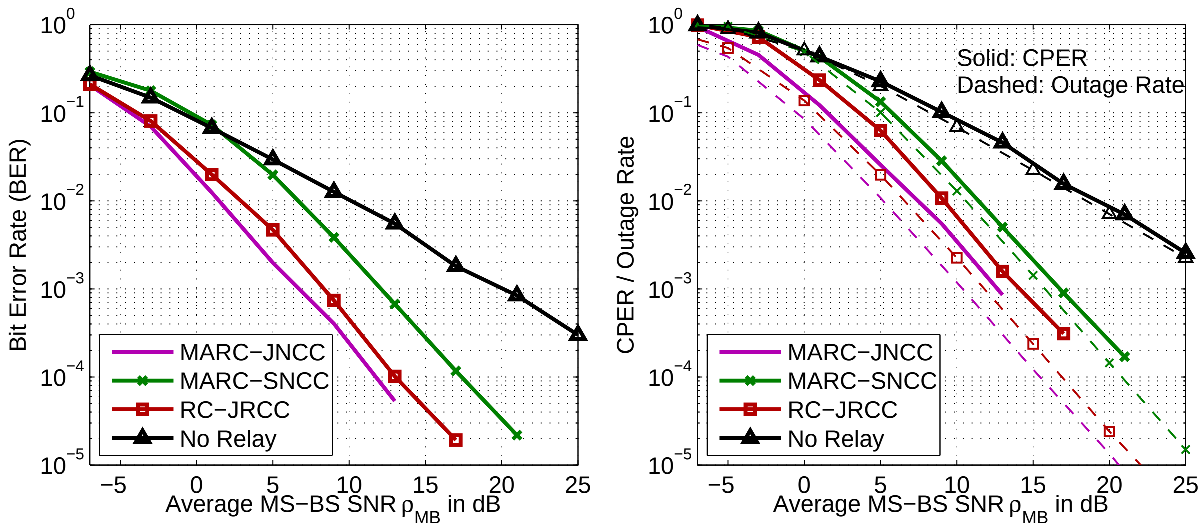


Figure 6.12: Numerical results for case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with BPSK, $M_{\mathrm{M}} = M_{\mathrm{W}} = 2000$ and $M_{\mathrm{R}} = 4000$: Bit error rate and common packet error rate (CPER) of system applying joint network-channel coding (JNCC) and separate network-channel coding (SNCC) for the multiple-access relay channel (MARC), joint routing-channel coding (JRCC) for the relay channel (RC) or a turbo code for the point-to-point channel (PtoPC).

Next, we want to show two examples where it is advantageous for the JNCC system to replace the convolutional code at the mobile stations with a turbo code.
Let us consider case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with $M_{\mathrm{M}} = M_{\mathrm{W}} = 1750$ and $M_{\mathrm{R}} = 2000$ BPSK symbols. The total code rate $R = R_{\mathrm{c}}$ increases to $R_{\mathrm{c}} = 6/11$ and θ is given by $\theta = 0.64$. Figure 6.13 depicts the BER and CPER/outage rate for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$ with $R_{\mathrm{c}} = 6/11$. As the mobile stations transmit less symbols, it is advantageous to protect the MS-R links with a turbo code. The iterative network and channel decoding still works well at the base stations. Figure 6.14 compares the BER and CPER of the JNCC system with either a turbo code or a convolutional code at the mobile stations. The turbo code increases the computational complexity because we perform four inner iterations within the channel decoders and four outer iterations between the channel and the network decoders. The number of required inner and outer iterations could be investigated in further work. For the relay channel it is not possible to achieve diversity with BPSK and $R > 1/2$ (compare Figure 6.6).
Figure 6.15 depicts the BER, the CPER and the outage rate for case 2 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$) with $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$ BPSK symbols. Compared to the results in Figure 6.11, the relay is closer to the base station. As the MS-R links are weaker than for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$, it is advantageous to protect the MS-R links with a turbo code. Figure 6.16 compares the BER and CPER of the JNCC system with either a turbo code or a convolutional code at the mobile stations.
The outage rates allowed to predict the trend of the error rates of the practical coding systems in all considered examples.

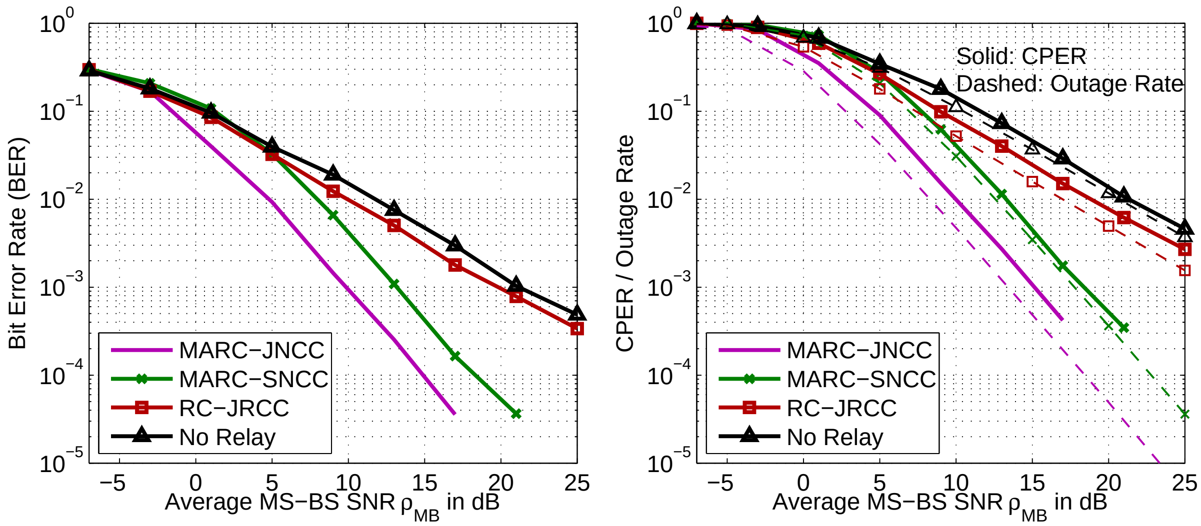


Figure 6.13: Numerical results for case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with BPSK, $M_{\mathrm{M}} = M_{\mathrm{W}} = 1750$ and $M_{\mathrm{R}} = 2000$: Bit error rate and common packet error rate (CPER) of system applying joint network-channel coding (JNCC) and separate network-channel coding (SNCC) for the multiple-access relay channel (MARC), joint routing-channel coding (JRCC) for the relay channel (RC) or a turbo code for the point-to-point channel (PtoPC).

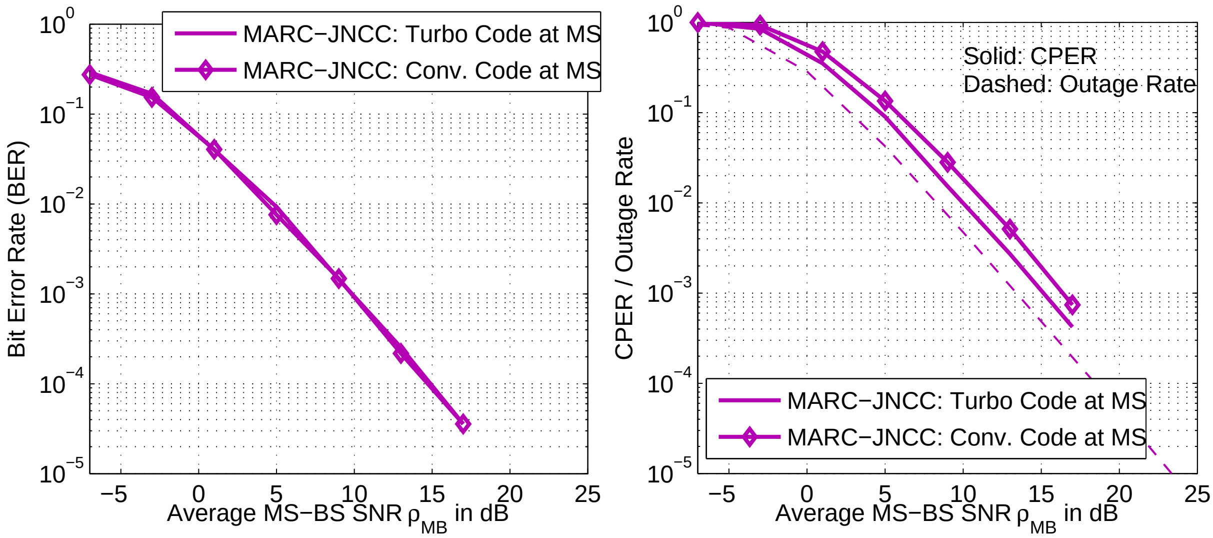


Figure 6.14: Comparison of systems with turbo code and convolutional code at mobile stations for case 1 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$) with BPSK, $M_{\mathrm{M}} = M_{\mathrm{W}} = 1750$ and $M_{\mathrm{R}} = 2000$.

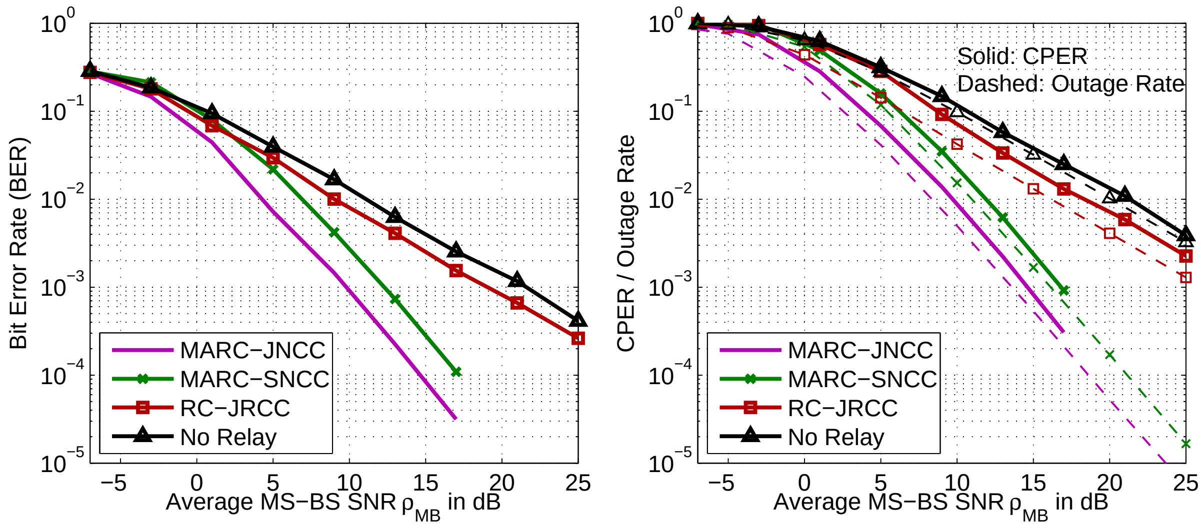


Figure 6.15: Numerical results for case 2 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$) with BPSK, $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$: Bit error rate and common packet error rate (CPER) of system applying joint network-channel coding (JNCC) and separate network-channel coding (SNCC) for the multiple-access relay channel (MARC), joint routing-channel coding (JRCC) for the relay channel (RC) or a turbo code for the point-to-point channel (PtoPC).

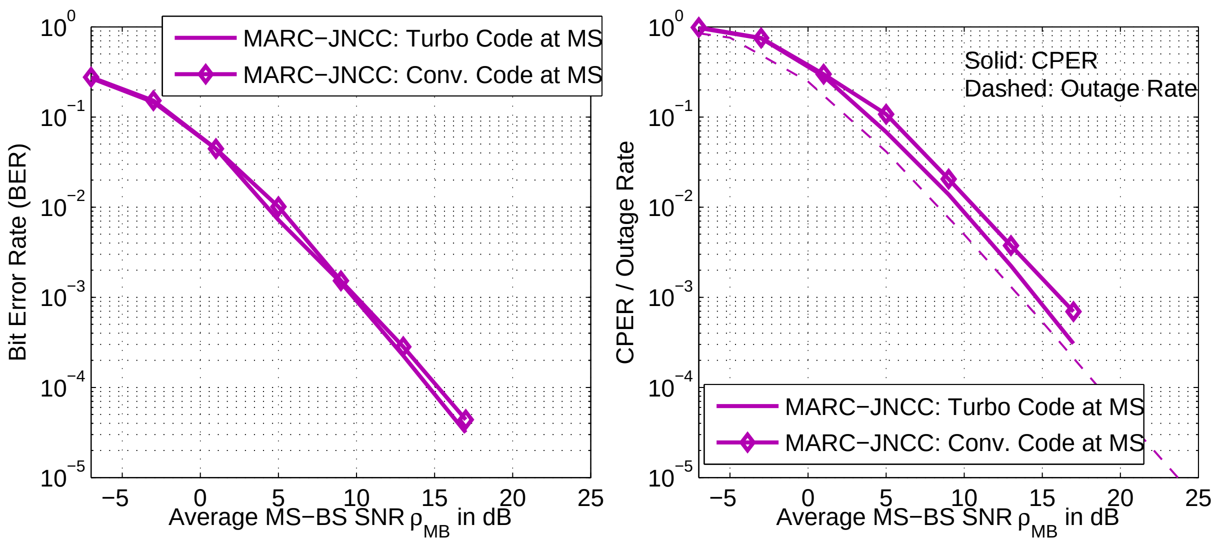


Figure 6.16: Comparison of systems with turbo code and convolutional code at mobile stations for case 2 ($d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$) with BPSK and $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$.

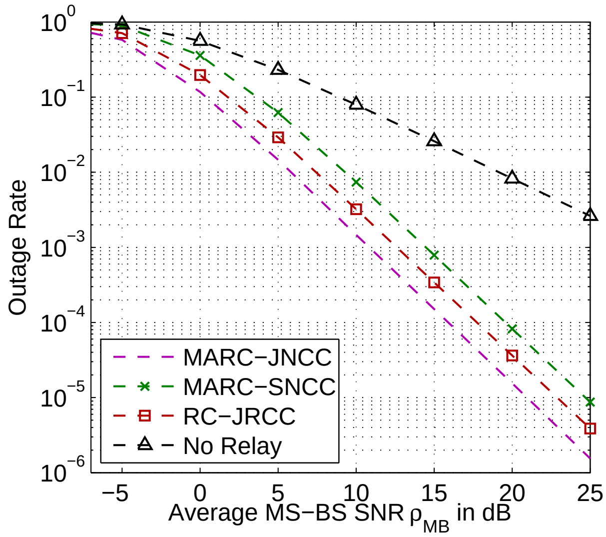


Figure 6.17: Outage rates for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ with Gaussian distributed input variable and $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$.

Results for Gaussian Distributed Input Variables

We also consider the systems under the assumption of a Gaussian distributed channel input variable. Although a Gaussian distributed channel input variable is optimal from an information-theoretic point of view, such input distributions do not occur in current real digital communication systems. Nevertheless, we will show the outage rates under such an assumption. Figure 6.17 depicts the outage rates for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $R = 1/2$ ($M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$). Under the assumption of Gaussian distributed channel input variables, the relay channel gains diversity compared to the results for BPSK. The reason is that, while for discrete modulation schemes the transmitted information of one symbol is limited by $\mathcal{C}_k(\gamma \to \infty) = L$, the transmitted information for the Gaussian case is not limited. Therefore, the conditions in (6.37) and (6.38) are not relevant and it is possible to transmit the $K_{\mathrm{M}} = K_{\mathrm{W}} = 1500$ bits from the relay to the sink over the $M_{\mathrm{R},1} = M_{\mathrm{R},2} = M_{\mathrm{R}}/2 = 1000$ Gaussian distributed symbols. This is not possible for BPSK. Note that this is not a particular problem of BPSK. Also for every other discrete modulation constellation with a maximum number of L bits mapped to one symbol, the relay channel does not provide a diversity order of two, if $M_{\mathrm{R},1} \cdot L < K_{\mathrm{M}}$ or $M_{\mathrm{R},2} \cdot L < K_{\mathrm{W}}$. As we assume that the relay does not know the instantaneous SNR γ_{RB}, it is also not possible to change the modulation constellation for every block according to γ_{RB}.
Although the MARC provides no diversity gain, Figure 6.17 shows approximately a gain of 1.6 dB of the MARC compared to the relay channel. The reason for the performance loss of the relay channel is that the instantaneous channel SNRs are not available and thus, it is not possible to optimize the parameter α and the parameter is fixed to $\alpha = 0.5$. The MARC system does not require the optimization of such a parameter.

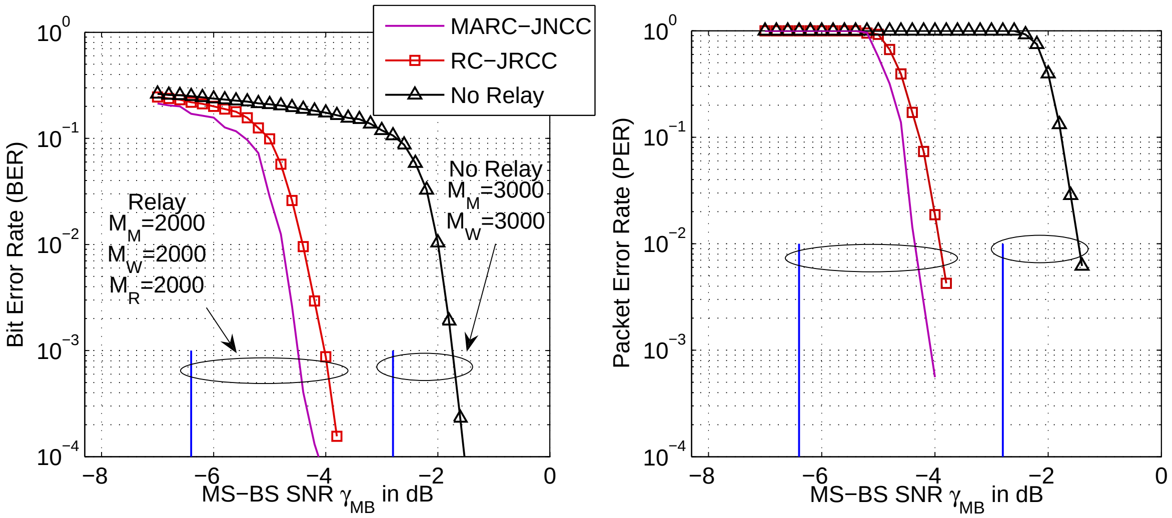


Figure 6.18: No fading: Bit and packet error rate for rate $R = 1/2$ ($M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$, BPSK) and relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$.

6.9.4 Results without Fading

We consider the performance of the proposed JNCC system for an AWGN channel model without fading. Figure 6.18 depicts the BER and PER for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ with $M_{\mathrm{M}} = M_{\mathrm{W}} = M_{\mathrm{R}} = 2000$ and BPSK. The mobile stations use convolutional codes in the JNCC system. The JNCC system for the MARC achieves a small gain compared to the JRCC system for the relay channel. The reason is the longer code length of the MARC system. As the information bits of MS1 and MS2 are jointly decoded at the base station in the MARC system, the decoder processes a code with length $K_{\mathrm{M}} + K_{\mathrm{W}} = 3000$. Contrary, the base station in the relay channel system decodes two codes where each has a length of $K_{\mathrm{M}} = K_{\mathrm{W}} = 1500$ information bits.

6.10 Summary

We proposed to use joint network-channel coding (JNCC) based on turbo codes for the multiple-access relay channel (MARC). We also analyzed the outage behavior of the proposed system. JNCC for the MARC generalizes distributed channel coding for the relay channel (RC) and requires the relay to transmit less symbols to allow cooperative diversity. Whereas the RC system allows to gain diversity for code rates R_{c} up to $R_{\mathrm{c}} \leq 1/2$, the MARC system allows a diversity gain for code rates R_{c} up to $R_{\mathrm{c}} \leq 2/3$.
Although the diversity gain can be also achieved with separate network-channel coding (SNCC), JNCC allows to more efficiently exploit the redundancy which is contained in the transmission of the relay by passing iteratively soft information between channel decoders and network decoder. Simulation results confirmed that the packet error rate of JNCC can outperform the one of SNCC by several dB.
The outage rates predicted the behavior of the practical coding systems for the considered examples. This allows a faster system design with less time-consuming simulations.

7

Joint Network-Channel Coding for the Two-Way Relay Channel

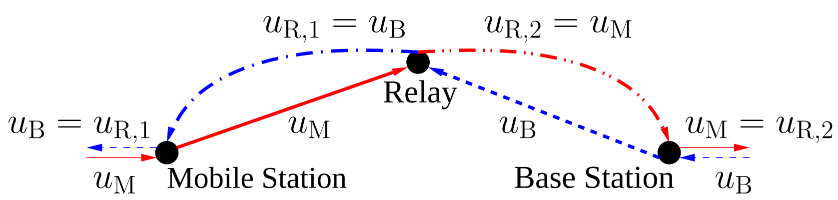
We consider the two-way relay channel (TWRC), where two terminals communicate with each other with the help of one relay. Such a system can be used for the cooperative uplink and downlink between a mobile station and a base station. As in the last two chapters, we consider time-division channels with broadcast and without simultaneous multiple-access. That means that the mobile station broadcasts to relay and base station in the first time slot, the base station broadcasts to relay and mobile station in the second time slot, and the relay broadcasts to mobile and base station in the third time slot. The three time slots can have different lengths.
We propose a joint network-channel code design for the TWRC and consider the optimal allocation of the transmission time to the three slots. Our code design supports unequal uplink and downlink rate. We derive a closed-form expression for the time allocation and the achievable decode-and-forward rate for the symmetric TWRC. We compare the proposed system with a distributed turbo code for the relay channel. Simulation results show that the proposed system achieves a rate gain compared to the relay channel system.

7.1 Introduction

7.1.1 Bidirectional Communication with Network Coding

The wireless broadcast nature allows an efficient bidirectional communication between two nodes with the help of a relay. The following simple example for noiseless channels [WCK05] [YLCZ05, Chapter 1.2] in Fig. 7.1 illustrates this idea. The example

(a) Example for Routing



(b) Example for Network Coding

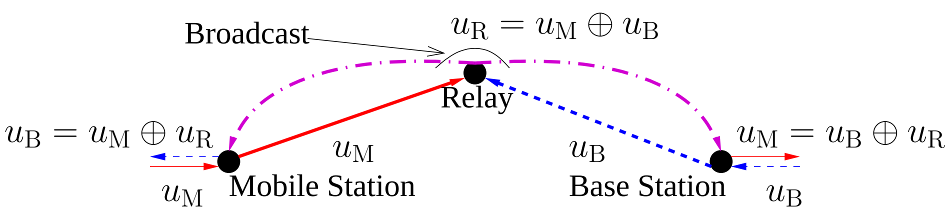


Figure 7.1: Example of [WCK05, YLCZ05] for noiseless channels which illustrates the basic idea of bidirectional relay communication with network coding: The mobile station wants to send the bit u_{M} to the base station and the base station wants to send the bit u_{B} to the mobile station. The stations can communicate over a relay. **(a) Routing:** Four bits have to be sent. **(b) Network Coding:** Three bits have to be sent.

describes an abstract model for the communication between two terminals over a relay, for example the communication between mobile and base station. It is assumed that one bit can be transferred error-free with one channel use. The mobile station wants to send the bit u_{M} to the base station and the base station wants to send the bit u_{B} to the mobile station. Fig. 7.1 (a) depicts the routing solution for this communication problem. The transmission of u_{M} and u_{B} is done separately and four bits have to be sent to solve the communication problem. In Fig. 7.1 (b) the relay network encodes the incoming bits u_{M} and u_{B} and sends in broadcast the bit $u_{\mathrm{R}} = u_{\mathrm{M}} \oplus u_{\mathrm{B}}$ to both stations. The operation $\oplus$ symbolizes a modulo-2 addition. The mobile station can decode u_{B} by performing a modulo-2 addition of u_{R} and its own bit u_{M} and the base station can decode $u_{\mathrm{M}} = u_{\mathrm{B}} \oplus u_{\mathrm{R}}$. With the help of network coding only three bits have to be sent and the data rate is increased from $R = 2/4$ bits per channel use to $R = 2/3$ bits per channel use.

The routing solution in Figure 7.1 (a) corresponds to the wireless one-way relay communication as point-to-point network (compare Section 4.2). The wireless medium allows more advanced one-way relay communications by using broadcast transmissions and/or simultaneous multiple access (compare Chapter 4). Several research groups considered the two-way relay communication for channel models with broadcast transmissions at the

stations and with simultaneous multiple access of several nodes.
The achievable rates for the two-way relay channel with broadcast transmissions at all nodes and simultaneous multiple-access of all nodes was considered in [RW06] and [Xie07]. This strategy requires three full-duplex nodes. All three terminals use the full bandwidth for the full transmission time.
Code design and achievable rates for the two-way relay channel with broadcast transmissions at all nodes and without simultaneous multiple access was considered in [LJS05, LJS06], in [HH06, Hau06] and in [KMT07]. This strategy contains three time/frequency phases. No full-duplex terminals are required. Whereas in [HH06] the allocation of the transmission time to mobile station, base station and relay was optimized and the transmission power was fixed, in [LJS06] the time allocation was fixed and the allocation of the transmission power was optimized.
The two-way relay channel with simultaneous multiple-access of mobile and base station and without broadcast transmission at mobile and base station was considered in [RW05a, RW05b, HKE+07], in [PY06b, PY06a], in [CLC06], in [DKS06], in [OB06, OSBB08] and in [SLL07]. This strategy contains two time/frequency phases. Mobile station and base station transmit simultaneously to the relay during the first phase and the relay broadcasts to mobile and base station during the second phase. No full-duplex terminals are required. Mobile and base station do not listen to each other.
Two-way relay communication over noisy channels without broadcast and simultaneous multiple access is considered in [XFKC06a].
A comparison of several two-way relaying strategies is given in [PY07] and in [KMT07]. Two-way point-to-point channels without relay were initially investigated in [Sha61].

7.1.2 Motivation for Joint Network-Channel Coding

In this chapter, we will consider the two-way relay channel with broadcast transmissions at all nodes and without simultaneous multiple access. The mobile station broadcasts to base station and relay during the first time slot, the base station broadcasts to mobile station and relay during the second time slot and the relay broadcasts to mobile and base station during the third time slot. Figure 7.2 illustrates the channel. We will focus on the code design for the considered channel. We know from [EMH+03] and [RK06] that in general, capacity can only be achieved by treating network and channel coding jointly, if we consider the communication in wireless networks with broadcast. The idea of joint network-channel coding can be easily understood if we take again a look at Fig. 7.1. In a wireless cellular based network the mobile station and the relay are mobile and randomly placed. In such a scenario the distance between mobile and base station can be only little more than the distance between one station and the relay. Therefore, the base station should also listen to the transmission of the mobile station and vice versa. If network and channel coding is performed separately, the base station will receive the bit u_{M} both from the relay and from the mobile station and thus, the network code will contain redundancy. Under these circumstances network and channel coding should not be treated separately. Analogous to joint source-channel coding [Hag95], where the remaining redundancy after the source encoding helps the channel code to combat noise, the redundancy in the network

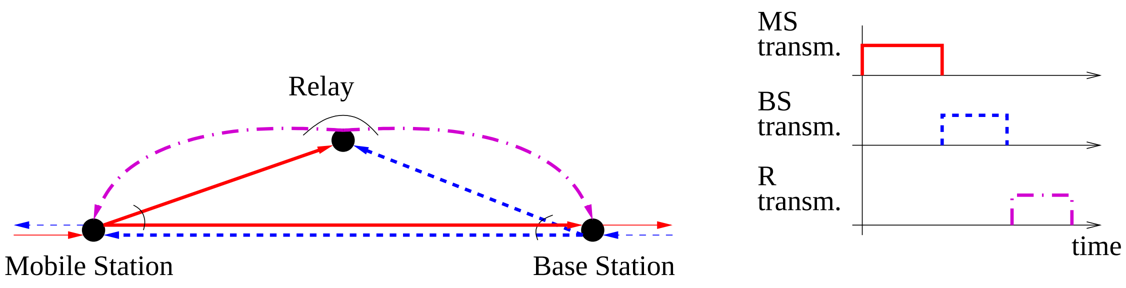


Figure 7.2: Time division two-way relay channel with broadcast transmissions at all nodes.

code should be used to support the channel code for better error protection. We will describe how joint network-channel coding can be realized based on turbo codes [Hau06]. Moreover, we will consider how to optimally allocate the transmission time to mobile station, base station and relay [HH06].
Joint network-channel coding can be seen as an extension of distributed channel codes for the relay channel as described in Chapter 5. A distributed channel code for the relay channel could be called in our context joint routing and channel coding.

7.2 System Description

We consider the uplink and downlink in a cellular based mobile communication system. The system model is illustrated in Fig. 7.2. A block diagram of the system is depicted in Fig. 7.3.
The mobile station (MS) wants to transmit information bits which are segmented in packets $\mathbf{u}_\mathrm{M}$ of length K_M to the base station (BS). The base station wants to transmit information bits which are segmented in packets $\mathbf{u}_\mathrm{B}$ of length K_B to the mobile station. We assume that the packets $\mathbf{u}_\mathrm{M}$ and $\mathbf{u}_\mathrm{B}$ carry statistically independent data and that the packets include cyclic redundancy check (CRC) bits as an error detection code. At the mobile station, the information bits $\mathbf{u}_\mathrm{M}$ are protected against transmission errors with channel codes and modulators which output the block $\mathbf{x}_\mathrm{M}$ containing M_M symbols. At the base station, the information bits $\mathbf{u}_\mathrm{B}$ are protected against transmission errors with channel codes and modulators which output the block $\mathbf{x}_\mathrm{B}$ containing M_B symbols. We assume for simplicity that both stations and the relay use the same modulation constellation and map L code bits to one symbol. MS broadcasts $\mathbf{x}_\mathrm{M}$ to relay and base station with power P in the first of the three time phases, BS broadcasts $\mathbf{x}_\mathrm{B}$ to relay and mobile station with power P in the second time phase.
The relay demodulates and decodes in the first time phase the disturbed version $\mathbf{y}_\mathrm{MR}$ of $\mathbf{x}_\mathrm{M}$ to obtain the hard estimate $\tilde{\mathbf{u}}_\mathrm{M}$ about $\mathbf{u}_\mathrm{M}$. In the second time phase, the relay demodulates and decodes the disturbed version $\mathbf{y}_\mathrm{BR}$ of $\mathbf{x}_\mathrm{B}$ to obtain the hard estimate $\tilde{\mathbf{u}}_\mathrm{B}$ about $\mathbf{u}_\mathrm{B}$. If the CRC indicates that the relay decoded both packets $\mathbf{u}_\mathrm{M}$ and $\mathbf{u}_\mathrm{B}$ successfully, the estimates $\tilde{\mathbf{u}}_\mathrm{M}$ and $\tilde{\mathbf{u}}_\mathrm{B}$ are network encoded and modulated to the block $\mathbf{x}_\mathrm{R}$ containing M_R symbols. The relay broadcasts $\mathbf{x}_\mathrm{R}$ to MS and BS with power P in the

third time phase to provide additional error protection.
The base station receives the disturbed version $\mathbf{y}_{\mathrm{MB}}$ of $\mathbf{x}_{\mathrm{M}}$ in the first time phase and the disturbed version $\mathbf{y}_{\mathrm{RB}}$ of $\mathbf{x}_{\mathrm{R}}$ in the third time phase. Based on $\mathbf{y}_{\mathrm{MB}}$, $\mathbf{y}_{\mathrm{RB}}$ and on the own information packet $\mathbf{u}_{\mathrm{B}}$, the joint network-channel decoder at the base station outputs the hard estimate $\hat{\mathbf{u}}_{\mathrm{M}}$ about $\mathbf{u}_{\mathrm{M}}$.
The mobile station receives the disturbed version $\mathbf{y}_{\mathrm{BM}}$ of $\mathbf{x}_{\mathrm{B}}$ in the second time phase and the disturbed version $\mathbf{y}_{\mathrm{RM}}$ of $\mathbf{x}_{\mathrm{R}}$ in the third time phase. Based on $\mathbf{y}_{\mathrm{BM}}$, $\mathbf{y}_{\mathrm{RM}}$ and on the own information packet $\mathbf{u}_{\mathrm{M}}$, the joint network-channel decoder at the mobile station outputs the hard estimate $\hat{\mathbf{u}}_{\mathrm{B}}$ about $\mathbf{u}_{\mathrm{B}}$.
The number of total transmitted symbols is given by $M = M_{\mathrm{M}} + M_{\mathrm{B}} + M_{\mathrm{R}}$. The uplink rate is given by $R_{\mathrm{M}} = K_{\mathrm{M}}/M$ and the downlink rate is given by $R_{\mathrm{B}} = K_{\mathrm{B}}/M$. The sum-rate of the system is given by $R = R_{\mathrm{M}} + R_{\mathrm{B}} = (K_{\mathrm{M}} + K_{\mathrm{B}})/M$.

If the CRC indicates that the relay decoded both packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{B}}$ wrongly, there are the same option as described in Section 5.2 for the relay channel:

- The relay remains silent and does not transmit any symbols.

- The relay ignores the CRC indication and reencodes the decoded information bits anyway.

- If the relay is able to notify the mobile or base station that it decoded wrongly, using one bit in a feedback control channel, then the system can be extended in the following way. In case the relay decoded both packets with error, the mobile stations are notified that they both have to transmit additional $M_{\mathrm{R}}/2$ symbols to the base station.

- The system could be extended with soft relaying similar as it was done in [YK07] for the MARC.

If the CRC indicates that the relay only decoded one of the packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{B}}$ wrongly, the system could be further extended by allowing that the system falls back to one relay channel and one point-to-point channel. The comparison of these schemes is beyond the scope of the thesis. We assume in the rest of the thesis that the relay remains silent, if the CRC indicates that the relay decoded one or both packets wrongly.

All channels are assumed to be AWGN channels as described in Section 2.2 and thus,

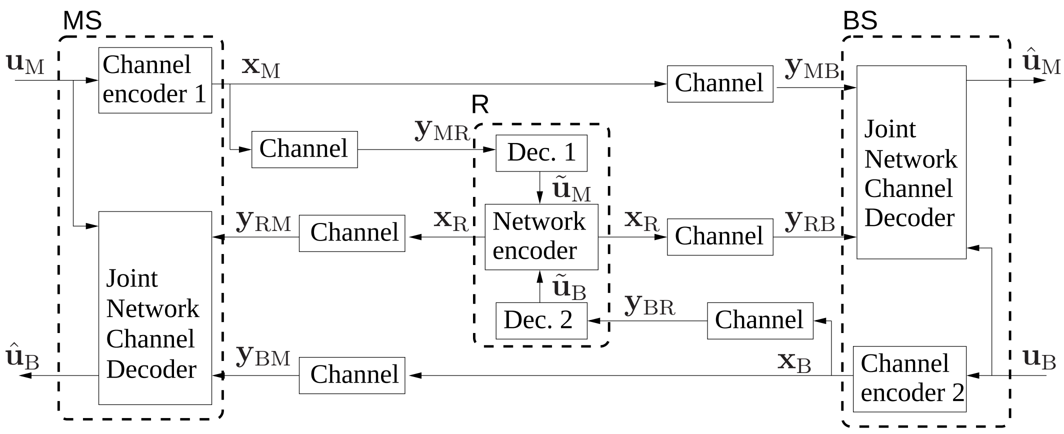


Figure 7.3: Cooperative uplink and downlink on a two-way relay channel.

the received samples after the matched filter are

$$\mathbf{y}_\mathrm{MR} = h_\mathrm{MR} \cdot \mathbf{x}_\mathrm{M} + \mathbf{z}_\mathrm{MR} \qquad \text{(MS transmits)} \qquad (7.1)$$
$$\mathbf{y}_\mathrm{MB} = h_\mathrm{MB} \cdot \mathbf{x}_\mathrm{M} + \mathbf{z}_\mathrm{MB} \qquad \text{(MS transmits)} \qquad (7.2)$$
$$\mathbf{y}_\mathrm{BR} = h_\mathrm{BR} \cdot \mathbf{x}_\mathrm{B} + \mathbf{z}_\mathrm{BR} \qquad \text{(BS transmits)} \qquad (7.3)$$
$$\mathbf{y}_\mathrm{BM} = h_\mathrm{BM} \cdot \mathbf{x}_\mathrm{B} + \mathbf{z}_\mathrm{BM} \qquad \text{(BS transmits)} \qquad (7.4)$$
$$\mathbf{y}_\mathrm{RM} = h_\mathrm{RM} \cdot \mathbf{x}_\mathrm{R} + \mathbf{z}_\mathrm{RM} \qquad \text{(Relay transmits)} \qquad (7.5)$$
$$\mathbf{y}_\mathrm{RB} = h_\mathrm{RB} \cdot \mathbf{x}_\mathrm{R} + \mathbf{z}_\mathrm{RB}, \qquad \text{(Relay transmits)} \qquad (7.6)$$

with $h_i = \sqrt{P(d_i)}$ for $i \in \{\mathrm{MR, MB, BR, BM, RM, RB}\}$. The noise values $\mathbf{z_i}$ are zero-mean and Gaussian distributed with variance $N_0 \cdot W$ (variance $N_0 \cdot W/2$ in each of the two dimensions of a complex number). The transmissions of MS, BS and the relay are divided in time and thus, no interference occurs.
The SNRs γ_i are given by $\gamma_i = P(d_i)/(N_0 \cdot W)$. The relation between the SNRs depends on the distances d_i between the nodes and on the path-loss exponent n.
We refer to [Sal08] for the treatment of two-way relay communication over fading channels.

7.3 Design for Joint Network-Channel Code

We explain how joint network-channel coding can be performed based on turbo codes [Hau06]. Compared to [Hau06], the described coding scheme in this thesis is generalized to cover unequal uplink and downlink rates as well. The described coding scheme is an extension of the distributed turbo code according to [SV04] (compare Chapter 5). We consider both uplink and downlink. At the relay the received information of the base and of the mobile station is combined (network encoded) and sent in broadcast to the base and mobile station. Taking into account the own information both the base and mobile station can use the additional redundancy which is sent by the relay.

7.3.1 Channel Coding

The channel codes at mobile and base station contain both the UMTS turbo code which we described in Section 2.4.2. The turbo code at the mobile station encodes the packet $\mathbf{u}_\mathrm{M}$ with information bit block length K_M and outputs the code bits $\mathbf{c}_\mathrm{M}$. The puncturing block chooses from $\mathbf{c}_\mathrm{M}$ a subset $\mathbf{c}_{\mathrm{M},1}$ which contains $N_\mathrm{M} = M_\mathrm{M} \cdot L$ code bits. The puncturing is done in a regular way, which is similar to the rate matching strategy described in [Eur00]. We do not puncture systematic and tail bits. As the turbo encoder (including puncturing) is a linear encoder, the encoding can be expressed as

$$\mathbf{c}_{\mathrm{M},1} = \mathbf{u}_\mathrm{M} \cdot \mathbf{G}_{\mathrm{M},1}. \tag{7.7}$$

The code bits $\mathbf{c}_{\mathrm{M},1}$ are modulated to the block $\mathbf{x}_\mathrm{M}$ containing M_M symbols whereas L is the number of code bits carried by one symbol. The mobile station broadcasts $\mathbf{x}_\mathrm{M}$ to relay and base station in the first time slot. A block diagram of channel encoder 1 (including the modulator) at the mobile station is depicted in Figure 7.4 (a).
The turbo code at the base station encodes the packet $\mathbf{u}_\mathrm{B}$ with length K_B and outputs the code bits $\mathbf{c}_\mathrm{B}$. The puncturing block chooses from $\mathbf{c}_\mathrm{B}$ a subset $\mathbf{c}_{\mathrm{B},1} = \mathbf{u}_\mathrm{B} \cdot \mathbf{G}_{\mathrm{B},1}$ which contains $N_\mathrm{B} = M_\mathrm{B} \cdot L$ code bits. The puncturing is done in a regular way, which is similar to the rate matching strategy described in [Eur00]. For $K_\mathrm{M} = K_\mathrm{B}$, the encoding matrices $\mathbf{G}_{\mathrm{M},1}$ and $\mathbf{G}_{\mathrm{B},1}$ are identical. The code bits $\mathbf{c}_{\mathrm{B},1}$ are modulated to the block $\mathbf{x}_\mathrm{B}$ containing M_B symbols. The base station broadcasts $\mathbf{x}_\mathrm{B}$ to relay and mobile station in the second time slot.

7.3.2 Network Coding

The relay demodulates and decodes the data of mobile station and base station as described in Section 2.4 and obtains the estimates $\tilde{\mathbf{u}}_\mathrm{M}$ and $\tilde{\mathbf{u}}_\mathrm{B}$. If the CRC indicates that the relay decoded the data of mobile and base station successfully, the relay network-encodes the estimates $\tilde{\mathbf{u}}_\mathrm{M}$ and $\tilde{\mathbf{u}}_\mathrm{B}$ and broadcasts the modulated network code bits $\mathbf{x}_\mathrm{R}$ to MS and BS to provide additional redundancy for uplink and downlink.

A block diagram of the network encoder (including the modulator) is depicted in Fig. 7.4 (b). Both estimates are channel encoded with the turbo code used at mobile and base station. The estimate $\tilde{\mathbf{u}}_\mathrm{M}$ is channel encoded to $\tilde{\mathbf{c}}_\mathrm{M}$. The puncturing block chooses from $\tilde{\mathbf{c}}_\mathrm{M}$ the subset $\tilde{\mathbf{c}}_{\mathrm{M},2}$ which contains $N_\mathrm{R} = M_\mathrm{R} \cdot L$ code bits. The subsets $\mathbf{c}_{\mathrm{M},1}$ and $\tilde{\mathbf{c}}_{\mathrm{M},2}$ are chosen to be disjoint.
The estimate $\tilde{\mathbf{u}}_\mathrm{B}$ is channel encoded to $\tilde{\mathbf{c}}_\mathrm{B}$. The puncturing block chooses from $\tilde{\mathbf{c}}_\mathrm{B}$ the subset $\tilde{\mathbf{c}}_{\mathrm{B},2}$ which contains N_R code bits. The subsets $\mathbf{c}_{\mathrm{B},1}$ and $\tilde{\mathbf{c}}_{\mathrm{B},2}$ are chosen to be disjoint. Both puncturings are done again similar to the rate matching strategy described in [Eur00]. Note that $\tilde{\mathbf{c}}_\mathrm{B}$ has to be punctured more deeply than $\tilde{\mathbf{c}}_\mathrm{M}$, if the downlink rate is larger than the uplink rate and vice versa.
Then, the punctured code bits $\tilde{\mathbf{c}}_{\mathrm{M},2}$ and $\tilde{\mathbf{c}}_{\mathrm{B},2}$ are network encoded with an element-wise modulo-2 addition to $\mathbf{c}_\mathrm{R} = \tilde{\mathbf{c}}_{\mathrm{M},2} \oplus \tilde{\mathbf{c}}_{\mathrm{B},2}$. As the turbo encoder (including puncturing) is a

linear encoder, the encoding at the relay can be expressed as

$$\mathbf{c}_{\mathrm{R}} = \tilde{\mathbf{c}}_{\mathrm{M},2} \oplus \tilde{\mathbf{c}}_{\mathrm{B},2} = \tilde{\mathbf{u}}_{\mathrm{M}} \cdot \mathbf{G}_{\mathrm{R},1} \oplus \tilde{\mathbf{u}}_{\mathrm{B}} \cdot \mathbf{G}_{\mathrm{R},2}. \tag{7.8}$$

The network-encoded code bits $\mathbf{c}_{\mathrm{R}}$ are modulated to the block $\mathbf{x}_{\mathrm{R}}$ containing M_{R} symbols. The relay broadcasts $\mathbf{x}_{\mathrm{R}}$ to mobile and base station in the third time slot.

If uplink and downlink rate are equal ($K_{\mathrm{M}} = K_{\mathrm{B}}$), then the two puncturings are identical ($\mathbf{G}_{\mathrm{R},1} = \mathbf{G}_{\mathrm{R},2}$) and the network encoder can be simplified to

$$\mathbf{c}_{\mathrm{R}} = \tilde{\mathbf{u}}_{\mathrm{M}} \cdot \mathbf{G}_{\mathrm{R},1} \oplus \tilde{\mathbf{u}}_{\mathrm{B}} \cdot \mathbf{G}_{\mathrm{R},2} = (\tilde{\mathbf{u}}_{\mathrm{M}} \oplus \tilde{\mathbf{u}}_{\mathrm{B}}) \cdot \mathbf{G}_{\mathrm{R},1}. \tag{7.9}$$

As the turbo encoder (including puncturing) is a linear encoder, it makes no difference whether network encoding is done before channel encoding or after channel encoding. Therefore, it is possible for $K_{\mathrm{M}} = K_{\mathrm{B}}$ to channel encode $\tilde{\mathbf{u}}_{\mathrm{M}} \oplus \tilde{\mathbf{u}}_{\mathrm{B}}$. This allows to reduce the computational complexity, because turbo encoding has to be performed once instead of twice.

7.3.3 Joint Network and Channel Decoding

The base station receives the disturbed version $\mathbf{y}_{\mathrm{MB}}$ of $\mathbf{x}_{\mathrm{M}}$ in the first time phase and the disturbed version $\mathbf{y}_{\mathrm{RB}}$ of $\mathbf{x}_{\mathrm{R}}$ in the third time phase. The channel output $\mathbf{y}_{\mathrm{MB}}$ is demodulated to LLRs $\mathbf{r}_{\mathrm{M},1}$ about the code bits $\mathbf{c}_{\mathrm{M},1}$. The channel output $\mathbf{y}_{\mathrm{RB}}$ is demodulated to LLRs $\mathbf{r}_{\mathrm{RB}}$ about the code bits $\mathbf{c}_{\mathrm{R}}$.
The joint network-channel decoder at the base station outputs the estimate $\hat{\mathbf{u}}_{\mathrm{M}}$ based on the LLRs $\mathbf{r}_{\mathrm{M},1}$ about $\mathbf{c}_{\mathrm{M},1}$, on the LLRs $\mathbf{r}_{\mathrm{RB}}$ about $\mathbf{c}_{\mathrm{R}}$ and on the own information packet $\mathbf{u}_{\mathrm{B}}$. This is done as follows. First, the network encoding at the relay is reversed. For this purpose we channel encode and puncture $\mathbf{u}_{\mathrm{B}}$ to obtain $\mathbf{c}_{\mathrm{B},2}$ as it was done at the relay. The sign of an element in $\mathbf{r}_{\mathrm{RB}}$ is changed, if the corresponding element in $\mathbf{c}_{\mathrm{B},2}$ is '1'. The sign of an element in $\mathbf{r}_{\mathrm{RB}}$ is not changed, if the corresponding element in $\mathbf{c}_{\mathrm{B},2}$ is '0'. The output of this operation are LLRs $\mathbf{r}_{\mathrm{M},2}$ about $\mathbf{c}_{\mathrm{M},2}$. The LLRs $\mathbf{r}_{\mathrm{M},1}$ about $\mathbf{c}_{\mathrm{M},1}$ and the LLRs about $\mathbf{c}_{\mathrm{M},2}$ are combined to LLRs $\mathbf{r}_{\mathrm{M}}$ about $\mathbf{c}_{\mathrm{M}}$ and used as input for a conventional turbo decoder as described in Section 2.4.2. The turbo decoder outputs the estimate $\hat{\mathbf{u}}_{\mathrm{M}}$ after several iterations. Fig. 7.4 (c) depicts a diagram of the decoder.
The joint network-channel decoder at the mobile station works correspondingly.

7.3.4 Comparison of Code Design for MARC and TWRC

Let us compare the joint network-and-channel decoders for the MARC (see Figure 6.4 (c)) and the TWRC (see Figure 7.4 (c)). We want to explain why different joint network-channel codes are advantageous for the MARC and the TWRC. The main difference is that the decoder for the MARC has to decode both packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{W}}$ which were network-encoded at the relay, whereas the decoders for the TWRC have to decode only

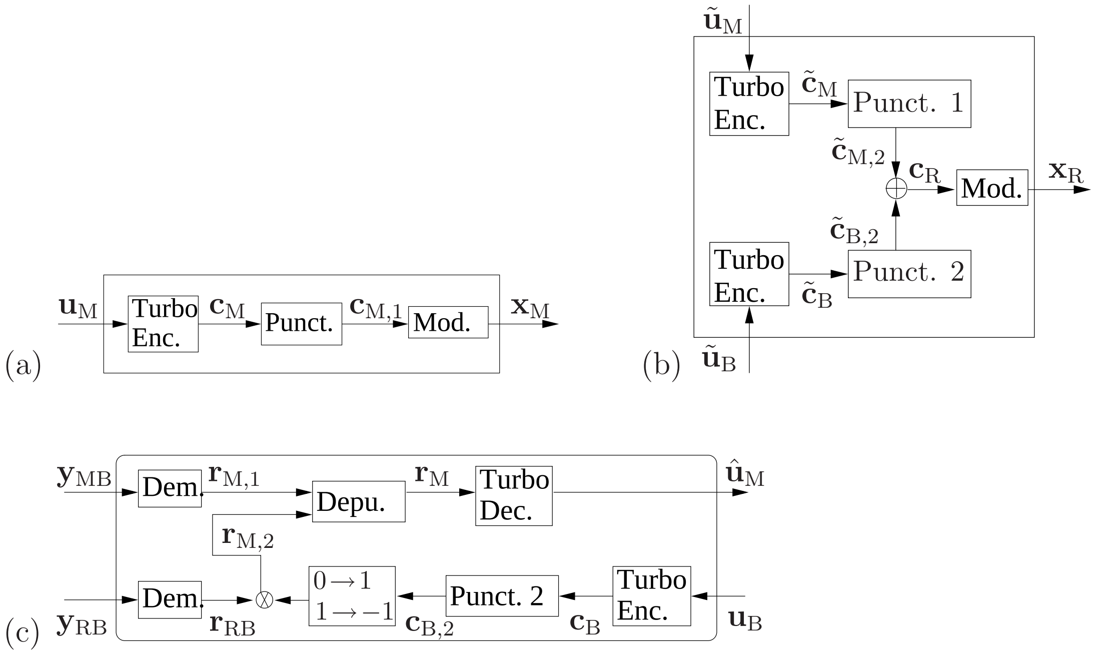


Figure 7.4: (**a**): Channel encoder at the mobile station. (**b**): Network encoder at the relay. (**c**): Joint network and channel decoder at the base station.

one of the two packets $\mathbf{u}_{\mathrm{M}}$ and $\mathbf{u}_{\mathrm{B}}$ and the other packet is perfectly known. As the decoders for the TWRC know one of the packets perfectly, the network encoding can be reversed perfectly in the first step and channel decoding can be performed in the second step. Contrary, it is not possible for the MARC-decoder to reverse the network encoding perfectly in one step, because neither of the packets is perfectly known. Therefore, it is advantageous to use the turbo principle and to repeat network decoding and channel decoding several times.
These two different decoder types require different network encoders. The network encoder in Figure 6.4 (b) for the MARC allows to improve the decoding result with the iterations in an iterative network and channel decoder. The network encoder in Fig. 7.4 (b) for the TWRC allows to reverse the encoding easily in one step.

7.3.5 Alternative Code Designs

It was proposed in [LJS06] to network encode before channel encoding at the relay ($\mathbf{c}_{\mathrm{R}} = (\tilde{\mathbf{u}}_{\mathrm{M}} \oplus \tilde{\mathbf{u}}_{\mathrm{B}}) \cdot \mathbf{G}_{\mathrm{R},1}$). In case of equal uplink and downlink rate, there is no difference compared to our approach (channel encoding before network encoding). In case of unequal uplink and downlink rate, it was proposed in [LJS06] to use zero-padding of the smaller uploaded packet at the relay. A comparison in [HHK08, Hou08] showed that this strategy achieves a worse performance for the uplink compared to our approach, because our approach allows to transfer more code bits from the relay to the base station.
It was proposed in [SFMC07] to superimpose the uplink and downlink packet after channel encoding and modulation. As a superposition of real numbers increases the power, it is

necessary to scale the transmitted signal at the relay. This approach loses performance because less useful signal power is received at mobile and base station.
A detailed comparison of several schemes can be found in [Hou08].

7.4 Information-Theoretic Limits

Next, we consider the achievable decode-and-forward rate with the time-division three-phase two-way relay channel with optimized allocation of the transmission time. The data of the mobile station can be decoded reliably at the base station and vice versa, if the following four inequalities hold [KMT07, (27) and (28)]:

$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right) \tag{7.10}$$
$$K_{\mathrm{B}} \leq M_{\mathrm{B}} \cdot \mathcal{C}\left(\gamma_{\mathrm{BR}}\right) \tag{7.11}$$
$$K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + M_{\mathrm{R}} \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) \tag{7.12}$$
$$K_{\mathrm{B}} \leq M_{\mathrm{B}} \cdot \mathcal{C}\left(\gamma_{\mathrm{BM}}\right) + M_{\mathrm{R}} \cdot \mathcal{C}\left(\gamma_{\mathrm{RM}}\right) \tag{7.13}$$

For $K_{\mathrm{B}} = 0$ the Conditions (7.10)-(7.13) for the TWRC fall back to the Conditions (5.6) and (5.7) for the relay channel.
The fulfillment of the two conditions $K_{\mathrm{M}} \leq M_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)$ and $K_{\mathrm{B}} \leq M_{\mathrm{B}} \cdot \mathcal{C}\left(\gamma_{\mathrm{BM}}\right)$ allows also a reliable communication alternatively to the fulfillment of the (7.10) to (7.13).
Figure 7.5 depicts as illustration the corresponding cuts to the Conditions (7.12) and (7.13) for reliable decoding at base and mobile station. Note that only the incoming links to the base station are relevant for Condition (7.12). For Condition (7.13) only the incoming links to the mobile station are relevant. The Conditions (7.10) and (7.11) guarantee that the relay decodes the information reliably.
We define the time-allocation parameters $\theta_{\mathrm{M}} = M_{\mathrm{M}}/M$, $\theta_{\mathrm{B}} = M_{\mathrm{B}}/M$ and $\theta_{\mathrm{R}} = M_{\mathrm{R}}/M$

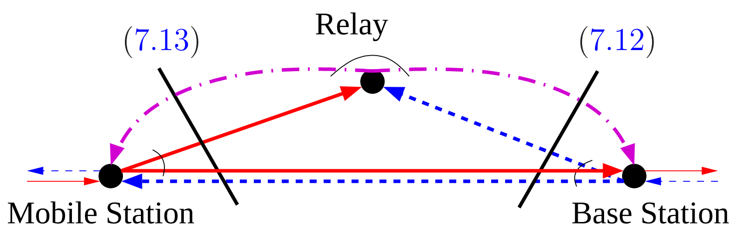


Figure 7.5: Illustration of the corresponding cuts to the Conditions (7.12) and (7.13) for reliable decoding at base and mobile station.

with $\theta_{\mathrm{M}} + \theta_{\mathrm{B}} + \theta_{\mathrm{R}} = 1$. As for the relay channel, we want to choose the time allocation such that the rate is maximized. We maximize R_{M} under the restriction that the rate ratio $\sigma = R_{\mathrm{B}}/R_{\mathrm{M}}$ is fulfilled:

$$[\theta_{\mathrm{M}}^*, \theta_{\mathrm{B}}^*] = \arg \max_{[\theta_{\mathrm{M}}, \theta_{\mathrm{B}}]} R_{\mathrm{M}} \quad \text{subject to} \quad \sigma = R_{\mathrm{B}}/R_{\mathrm{M}},\ \theta_{\mathrm{R}} = 1 - \theta_{\mathrm{M}} - \theta_{\mathrm{B}} \tag{7.14}$$

with

$$\begin{aligned} R_{\mathrm{M}} = \min\{&\theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right), \theta_{\mathrm{B}} \cdot \mathcal{C}\left(\gamma_{\mathrm{BR}}\right)/\sigma, \theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{B}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right), \\ &(\theta_{\mathrm{B}} \cdot \mathcal{C}\left(\gamma_{\mathrm{BM}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{B}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RM}}\right))/\sigma\}. \end{aligned} \tag{7.15}$$

The expression in (7.15) follows from the Conditions (7.10)-(7.13).
The optimization can be solved numerically with linear programming [KMT07].

If the setup is symmetric regarding the rate ($\sigma = 1$) and the SNRs ($\gamma_{\mathrm{MR}} = \gamma_{\mathrm{BR}}$, $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{BM}}$ and $\gamma_{\mathrm{RM}} = \gamma_{\mathrm{RB}}$) and if the condition $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq 2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$ is valid, we obtain the following closed expressions for the optimal time allocation and the achievable rate [HH06][1]:

$$\boxed{\theta_{\mathrm{M}}^* = \theta_{\mathrm{B}}^* = \frac{\mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + 2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})}.} \tag{7.16}$$

$$\boxed{R_{\mathrm{M}} = R_{\mathrm{B}} = \frac{\mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + 2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})},} \tag{7.17}$$

The sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ for uplink and downlink is given by

$$\boxed{R = \frac{2 \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})}{\mathcal{C}(\gamma_{\mathrm{MR}}) + 2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})},} \tag{7.18}$$

The derivation of (7.16)-(7.17) is given in the Appendix A.4. Moreover, it is shown in A.4 that the sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{W}}$ of the TWRC according to (7.18) is larger than the rate R of the relay channel according to (4.22), if the condition $\mathcal{C}(\gamma_{\mathrm{MB}}) < \mathcal{C}(\gamma_{\mathrm{MR}})$ is valid.

Figure 7.6 depicts the achievable sum-rate according to (7.18) for the scenario that the relay is on half-way between mobile and base station ($d_{\mathrm{MR}} = d_{\mathrm{RB}} = d_{\mathrm{MB}}/2$). The MS-BS distance is variable and the SNR on the MS-BS link γ_{MB} is in the range between -10 dB and 20 dB. We set the path-loss exponent to $n = 3.52$. Then, the SNRs on the MS-R and the R-BS link $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{BR}} = \gamma_{\mathrm{RM}} = \gamma_{\mathrm{RB}}$ are increased by $35.2 \cdot \log_{10}(2) = 10.6$ dB compared to the MS-BS link $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{BM}}$. For comparison, we also show the rate for the relay channel and the point-to-point communication without relay, which has already been shown in Figure 4.13. The two-way relay communication gains around 3.1 dB compared to the one-way relay communication for $R = 6$ bits per channel use and under the assumption of Gaussian distributed input. For $R = 3$ bits per channel use and 16-QAM, the two-way relay communication gains 4.4 dB compared to the one-way relay communication.

Figure 7.7 depicts the achievable sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ over the ratio of downlink to uplink rate σ for a path-loss exponent of $n = 3.52$. The relay is half-way between mobile and base station and γ_{MB} is given by $\gamma_{\mathrm{MB}} = 1$. The dashed-dot lines depict the rate for the one-way relay communication. The rate of the two-way relay communication approaches the rate of the one-way relay communication with growing σ.
Figure 7.8 depicts the achievable rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ over the relative MS-R distance

[1] Note that the sum of the time-allocation parameters was constrained to $\theta_{\mathrm{M}} + \theta_{\mathrm{B}} + \theta_{\mathrm{R}} = 2$ in [HH06] whereas the parameters are constrained to $\theta_{\mathrm{M}} + \theta_{\mathrm{B}} + \theta_{\mathrm{R}} = 1$ in this thesis. Therefore, the results are scaled by a factor of two.

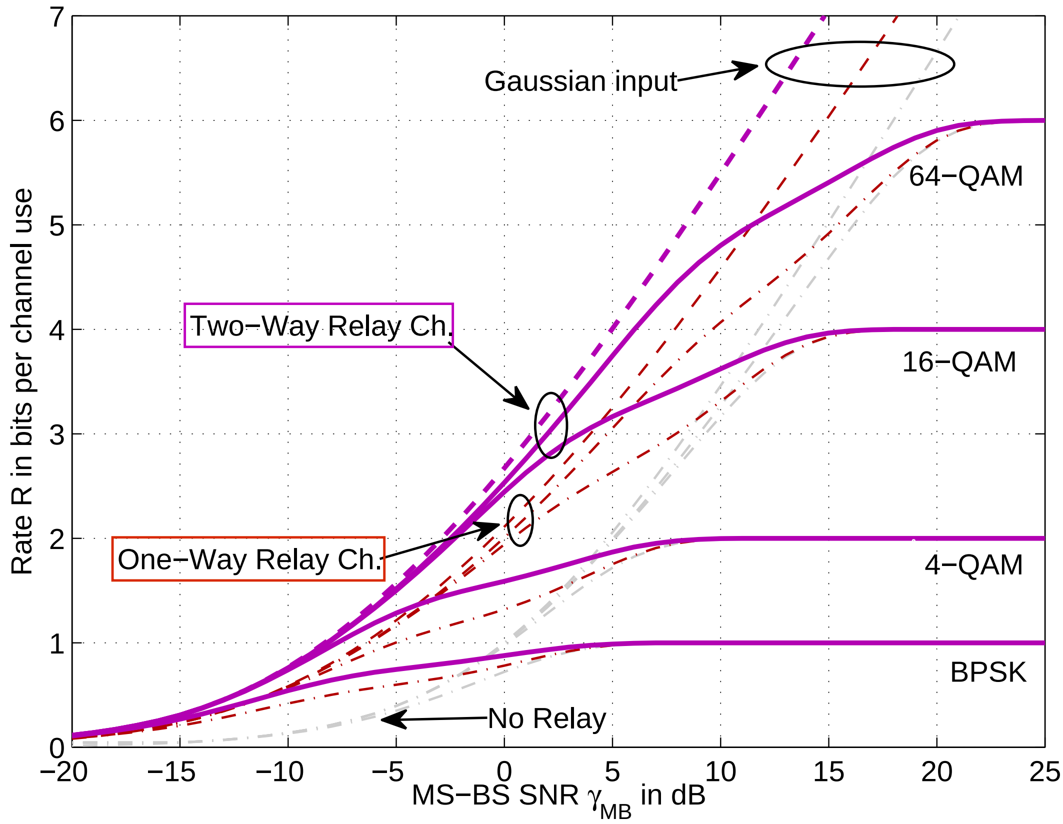


Figure 7.6: Achievable rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ for time division two-way relaying with broadcast for a path-loss exponent of $n = 3.52$. The relay is half-way between mobile and base station and ratio of uplink and downlink rate is given by $\sigma = 1$. The dashed-dot lines depict the rate for the one-way relay communication. The grey-shaded lines depict the rate for the point-to-point communication without relay.

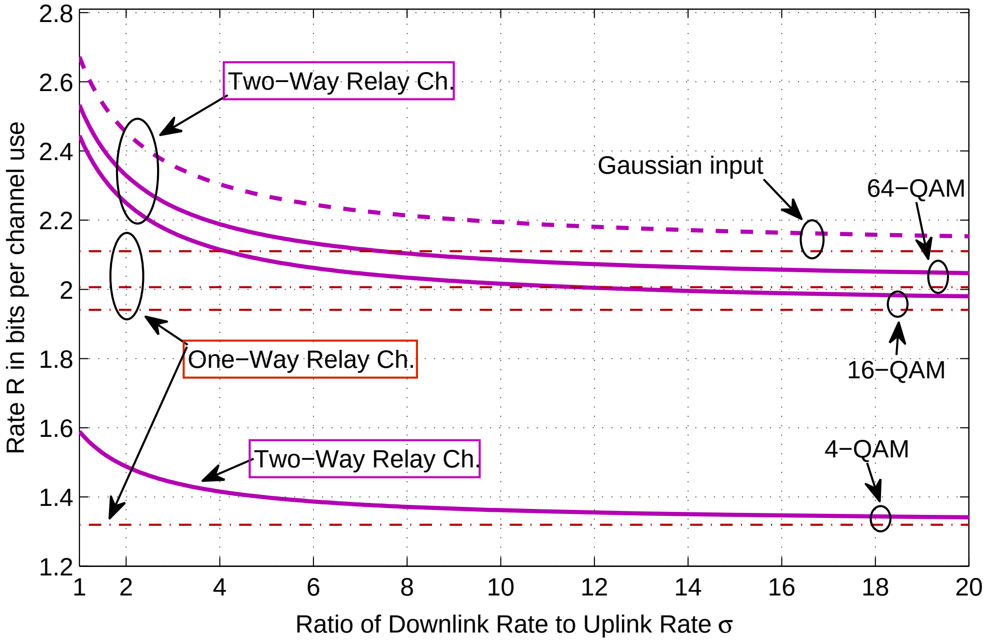


Figure 7.7: Achievable rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ over the ratio of downlink to uplink rate σ for time division two-way relaying with broadcast for a path-loss exponent of $n = 3.52$. The relay is half-way between mobile and base station and the MS-BS SNR is given by $\gamma_{\mathrm{MB}} = 1$. The dashed-dot lines depict the rate for the one-way relay communication.

$d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for a path-loss exponent of $n = 3.52$ and for a ratio of uplink to downlink rate of $\sigma = 1$. The two-way relay communication clearly outperforms the one-way relay communication. The two-way relay communication achieves the highest rate for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.5$. Figure 7.9 depicts the achievable rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ over the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for a path-loss exponent of $n = 3.52$ and for a ratio of uplink and downlink rate of $\sigma = 2$. It is advantageous to place the relay closer to the mobile station than to the base station for $\sigma = 2$. For example, the two-way relay communication achieves the highest rate approximately for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.42$ under the assumption of a Gaussian distributed channel input.

7.5 Allocation of Transmission Time

We have already seen in Chapter 5 that the performance of a relay communication depends strongly on the choice how to share the total transmission time T. For the considered two-way relay channel model the total transmission T has to be shared between the transmission time T_{M} of the mobile station, the transmission time T_{B} of the base station and the transmission time T_{R} of the relay. This is equivalent to the choice how to share the total number of symbols M between the number of symbols M_{M} transmitted from the mobile station, the number of symbols M_{B} transmitted from the base station and the number of symbols M_{R} transmitted from the relay. The time allocation parameters θ_{M} and θ_{B} defines $M_{\mathrm{M}} = \theta_{\mathrm{M}} \cdot M$, $M_{\mathrm{B}} = \theta_{\mathrm{B}} \cdot M$ and $M_{\mathrm{R}} = (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{B}}) \cdot M$.

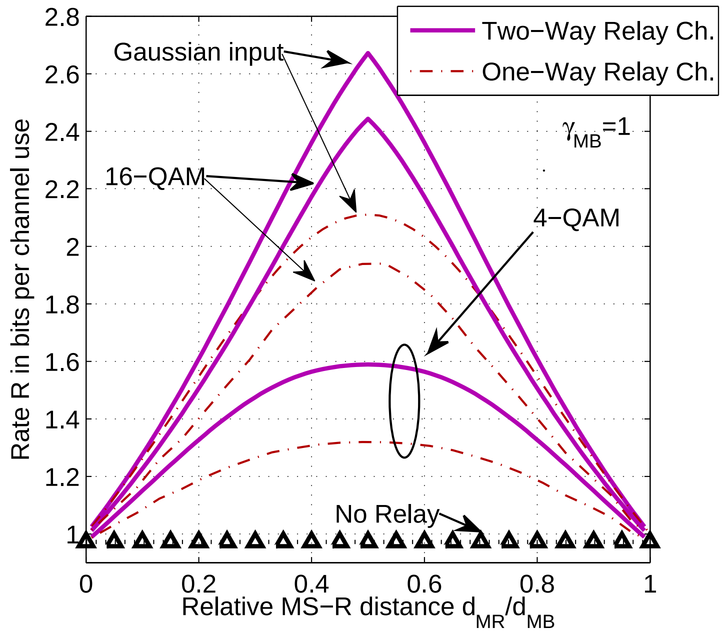


Figure 7.8: Achievable rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ for time division two-way relaying with broadcast over the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for a path-loss exponent of $n = 3.52$. The ratio of uplink and downlink rate is given by $\sigma = 1$. The dashed-dot lines depict the rate for the one-way relay communication.

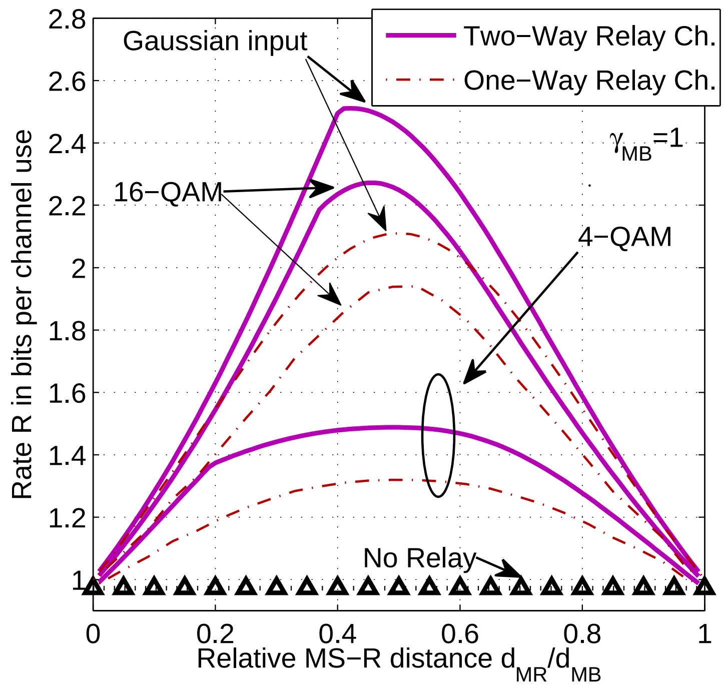


Figure 7.9: Achievable rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ for time division two-way relaying with broadcast over the relative distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ between MS and relay for a path-loss exponent of $n = 3.52$. The ratio of uplink and downlink rate is given by $\sigma = 2$. The dashed-dot lines depict the rate for the one-way relay communication.

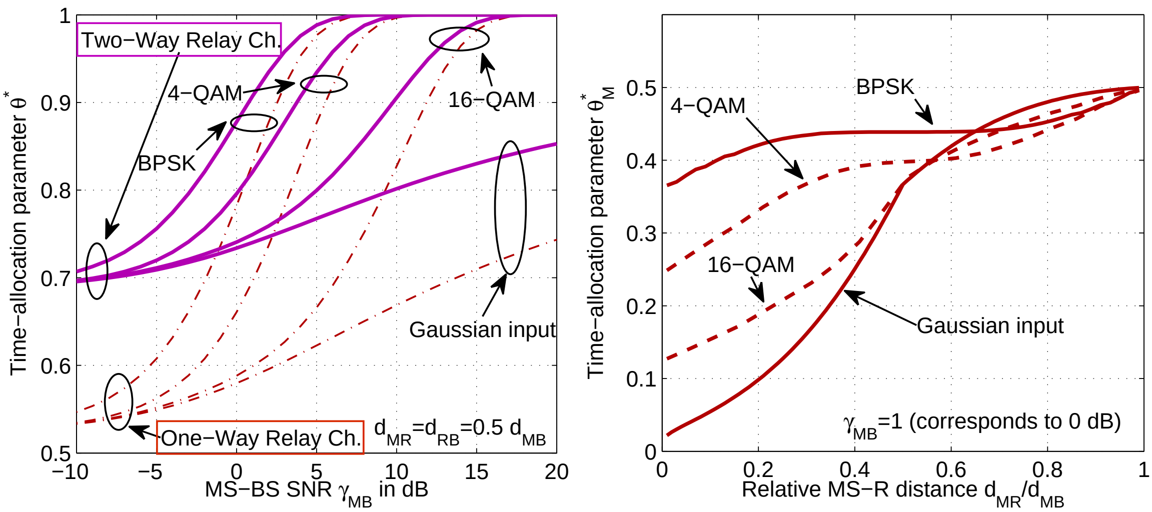


Figure 7.10: Ratio of downlink to uplink rate $\sigma = 1$: Optimal time-allocation parameter $\theta^* = \theta_{\mathrm{M}}^* + \theta_{\mathrm{B}}^*$ dependent on γ_{MB} for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.5$ (left) and optimal time-allocation parameter θ_{M}^* dependent on the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for $\gamma_{\mathrm{MB}} = 1$ (right).

For given channel SNRs and a given ratio of the downlink to uplink rate σ, the optimal value θ_{M}^* and θ_{B}^* for θ_{M} and θ_{B} can be found numerically by solving the optimization in (7.14). For the symmetric TWRC regarding the rate and the channel SNRs, a closed expression for $\theta_{\mathrm{M}}^* = \theta_{\mathrm{B}}^*$ can be found in (7.16). Figure 7.10 (left) depicts $\theta^* = \theta_{\mathrm{M}}^* + \theta_{\mathrm{B}}^*$ dependent on the MS-BS γ_{MB} for $\sigma = 1$. It is assumed that the relay is half-way between mobile and base station ($d_{\mathrm{MR}} = d_{\mathrm{RB}} = d_{\mathrm{MB}}/2$). Therefore, the SNRs are given by $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{BR}} = \gamma_{\mathrm{RB}} = \gamma_{\mathrm{RM}} = \gamma_{\mathrm{MB}} \cdot 2^n$ with path-loss exponent $n = 3.52$. Mobile station and base station have equal transmission time for this example ($\theta_{\mathrm{M}}^* = \theta_{\mathrm{B}}^* = \theta^*/2$). Mobile and base station are allowed to transmit longer compared to the system with one-way relay channels, because the relay requires less transmission time.
Figure 7.10 (right) depicts θ_{M}^* for $\sigma = 1$ dependent on the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ with $d_{\mathrm{RB}} = d_{\mathrm{MB}} - d_{\mathrm{MR}}$, $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{RM}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{MR}})^n$ and $\gamma_{\mathrm{BR}} = \gamma_{\mathrm{RB}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{RB}})^n$. The MS-BS SNR γ_{MB} is assumed to be $\gamma_{\mathrm{MB}} = 1$. The time-allocation parameter θ_{B}^* for $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ is given by θ_{M}^* for $1 - d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for this example.

Figure 7.11 depicts the corresponding curves for $\sigma = 2$. Figure 7.11 (left) depicts $\theta^* = \theta_{\mathrm{M}}^* + \theta_{\mathrm{B}}^*$ and θ_{B}^* dependent on γ_{MB} for $\sigma = 2$. Figure 7.11 (right) depicts θ_{M}^* and θ_{B}^* dependent on the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for $\sigma = 2$.

We also want to know a good choice for θ_{M} and θ_{B} for a given sum-rate $R = R_0$, a given rate ratio σ and a given relation of the channel SNRs. For example, the SNR on the link between mobile station and base station $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{BM}}$ can be variable and the SNRs $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{RM}}$ and $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{BR}}$ can be given by $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{MR}})^n$ and by $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{MB}} \cdot (d_{\mathrm{MB}}/d_{\mathrm{RB}})^n$. To find a good choice for θ_{M} and θ_{B}, we first determine (numerically) the lowest SNR $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{MB},0}$ which allows to achieve the rate $R = R_0$ according to the optimization in (7.14). If the relay is half-way between mobile and

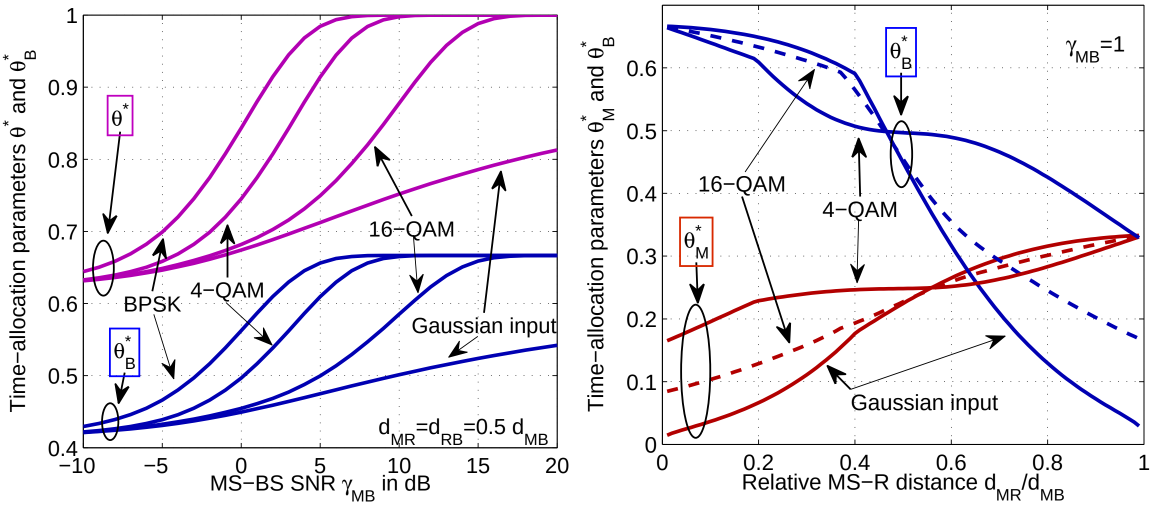


Figure 7.11: Ratio of downlink to uplink rate $\sigma = 2$: Optimal time-allocation parameter $\theta^* = \theta_{\mathrm{M}}^* + \theta_{\mathrm{B}}^*$ and θ_{B}^* dependent on γ_{MB} for $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.5$ (left) and optimal time-allocation parameter θ_{M}^* and θ_{B}^* dependent on the relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}}$ for $\gamma_{\mathrm{MB}} = 1$ (right).

base station with $n = 3.52$, $\gamma_{\mathrm{MB},0}$ can be obtained graphically from Figure 7.6. The corresponding SNRs $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{MR},0}$ and $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{RB},0}$ can be calculated from $\gamma_{\mathrm{MB},0}$. Then, we obtain the optimal values θ_{M}^* and θ_{B}^* with the optimization in (7.14) by setting the SNRs to $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{BM}} = \gamma_{\mathrm{MB},0}$, $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{RM}} = \gamma_{\mathrm{MR},0}$ and $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{BR}} = \gamma_{\mathrm{RB},0}$.
Figure 7.12 (left) depicts $\theta^* = \theta_{\mathrm{M}}^* + \theta_{\mathrm{B}}^*$ dependent on the rate $R = R_0$ under the assumption that the relay is half-way between mobile and base station with path-loss exponent $n = 3.52$ for $\sigma = 1$. Mobile station and base station have equal transmission time for this example ($\theta_{\mathrm{M}}^* = \theta_{\mathrm{B}}^* = \theta^*/2$). Figure 7.12 (right) depicts θ^* and θ_{B}^* dependent on the rate $R = R_0$ for $\sigma = 1$ under the assumption that the MS-R distance is one fourth and the R-BS distance is three fourth of the MS-BS distance.
It is advantageous to optimize θ_{M} and θ_{B} according to the given rate instead to the given SNRs, because the performance of the practical coding system will have a certain SNR gap to the theoretical limit.

Figure 7.13 depicts the corresponding curves for $\sigma = 2$. Figure 7.13 (left) depicts $\theta^* = \theta_{\mathrm{M}}^* + \theta_{\mathrm{B}}^*$ and θ_{B}^* dependent on the sum-rate R for $\sigma = 2$ and $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.5$. Figure 7.13 (right) depicts θ_{M}^* and θ_{B}^* dependent on the sum-rate R for $\sigma = 2$ and $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.25$.

7.6 Simulation Results

We compare the proposed system using joint network-channel coding (JNCC) for the TWRC to three reference systems.
We simulate the coding system described in Section 7.3 and measure the bit error rate (BER) and the packet error rate (PER) both for uplink and downlink. We compare the performance of the coding system with the information-theoretic limits described in Sec-

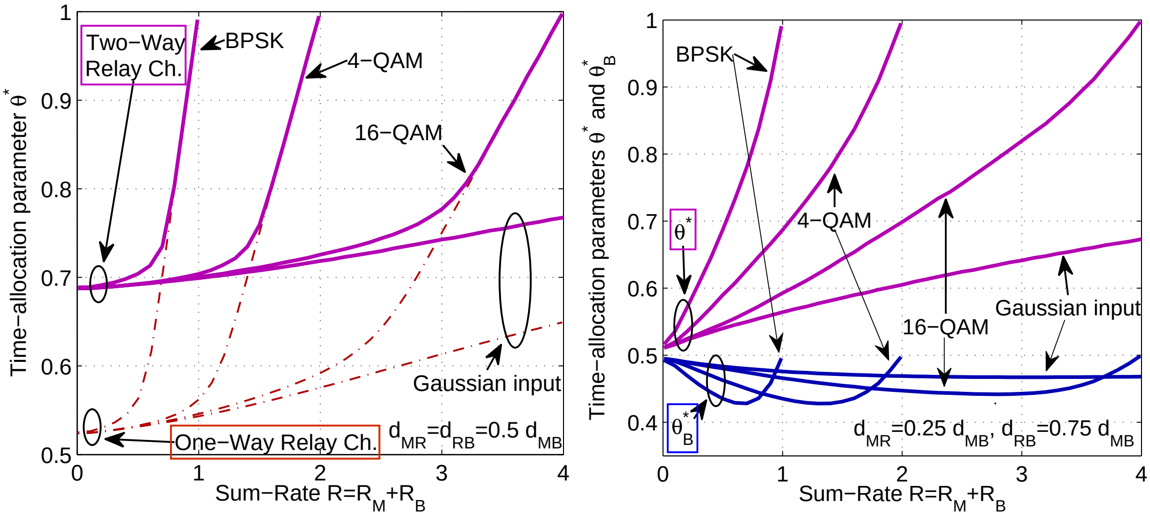


Figure 7.12: Ratio of downlink to uplink rate $\sigma = 1$: Optimal time-allocation parameter $\theta^* = \theta^*_{\mathrm{M}} + \theta^*_{\mathrm{B}}$ and θ^*_{B} dependent on the sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ for two positions of the relay: Left: Relay positioned at half distance between mobile and base station. Right: MS-R distance is one fourth and the R-BS distance is three fourth of the MS-BS distance.

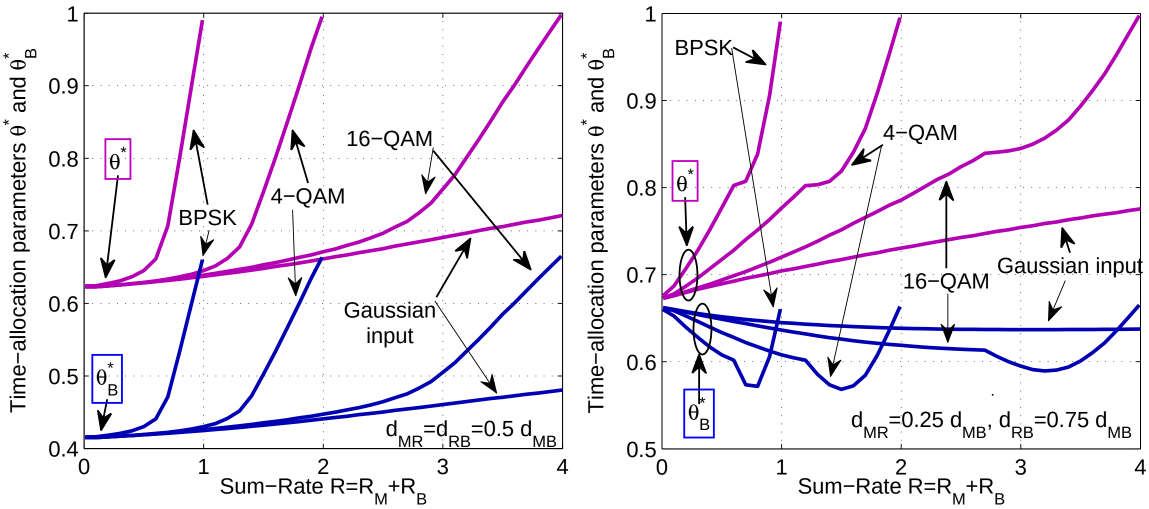


Figure 7.13: Ratio of downlink to uplink rate $\sigma = 2$: Optimal time-allocation parameter $\theta^* = \theta^*_{\mathrm{M}} + \theta^*_{\mathrm{B}}$ dependent on the sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{B}}$ for two positions of the relay: Left: Relay positioned at half distance between mobile and base station. Right: MS-R distance is one fourth and the R-BS distance is three fourth of the MS-BS distance.

tion 7.4. The allocation of the transmission time to mobile station, base station and relay is done according to Section 7.5. The SNR gain is measured at a PER of 10^{-2}.
The comparison with the reference systems is fair, because all systems require the same transmission time (i.e. total transmitted symbols), the same bandwidth, the same transmission power and the same total energy consumption.

7.6.1 Reference Systems

Distributed Turbo Code for the Relay Channel

In the first reference system the uplink and downlink transmissions are performed separately with the help of a relay. This scenario represents a relay channel where a distributed turbo code as described in Chapter 5 can be applied.

Turbo Code for the Point-to-Point Channel

The second reference system does not contain a relay and uplink and downlink transmissions are performed with a conventional turbo code as described in Section 2.4.2.

Separate Network-Channel Coding for the TWRC

This reference system describes a two-way relay communication where the direct links between MS and BS are not exploited [WCK05, YLCZ05]. During the first time phase the mobile station transmits M_M symbols to the relay. During the second time phase the base station transmits M_B symbols to the relay. These transmissions are protected by a conventional turbo code for a point-to-point communication. MS and BS do not listen to each other. After decoding, the relay network encodes the estimates $\tilde{\mathbf{u}}_\mathrm{M}$ and $\tilde{\mathbf{u}}_\mathrm{B}$. This is done in the network layer. The network encoder is a modulo-2 addition. The network encoder output $\mathbf{u}_\mathrm{R} = \tilde{\mathbf{u}}_\mathrm{M} \oplus \tilde{\mathbf{u}}_\mathrm{B}$ is protected against the noisy channel by a conventional turbo code which belongs to the physical layer. The output of the channel encoder can be expressed as $\mathbf{c}_{\mathrm{R},1} = (\tilde{\mathbf{u}}_\mathrm{M} \oplus \tilde{\mathbf{u}}_\mathrm{B}) \cdot \mathbf{G}_{\mathrm{R},1}$. The relay broadcasts the modulated codeword to mobile and base station. Mobile and base station channel decode the received symbols from the relay with a conventional turbo decoder to obtain an estimate about $\mathbf{u}_\mathrm{R}$. Then, this estimate is network decoded by performing a modulo-2 addition with the own packet.

7.6.2 System Scenarios

We consider two cases. In the first case, the relay is positioned half-way between mobile and base station ($d_\mathrm{MR} = d_\mathrm{RM} = d_\mathrm{RB} = d_\mathrm{BR} = (1/2) \cdot d_\mathrm{MB}$). Then, the SNRs $\gamma_\mathrm{MR} = \gamma_\mathrm{RM} = \gamma_\mathrm{RB} = \gamma_\mathrm{BR}$ are increased by $35.2 \cdot \log_{10}(2)$ dB $= \rho_\mathrm{MB} + 10.60$ dB compared to the SNRs $\gamma_\mathrm{MB} = \gamma_\mathrm{BM}$ on the MS-BS link. In the second case, the relay is positioned closer to the mobile station than to the base station. The distance between the mobile station and the relay is one fourth of the distance between MS and BS ($d_\mathrm{MR} = d_\mathrm{RM} = (1/4) \cdot d_\mathrm{MB}$) and the distance between relay and base station is given by $d_\mathrm{RB} = d_\mathrm{BR} = (3/4) \cdot d_\mathrm{MB}$. Then, the SNRs $\gamma_\mathrm{MR} = \gamma_\mathrm{RM}$ is increased by $35.2 \cdot \log_{10}(4)$ dB $= \rho_\mathrm{MB} + 21.19$ dB and the

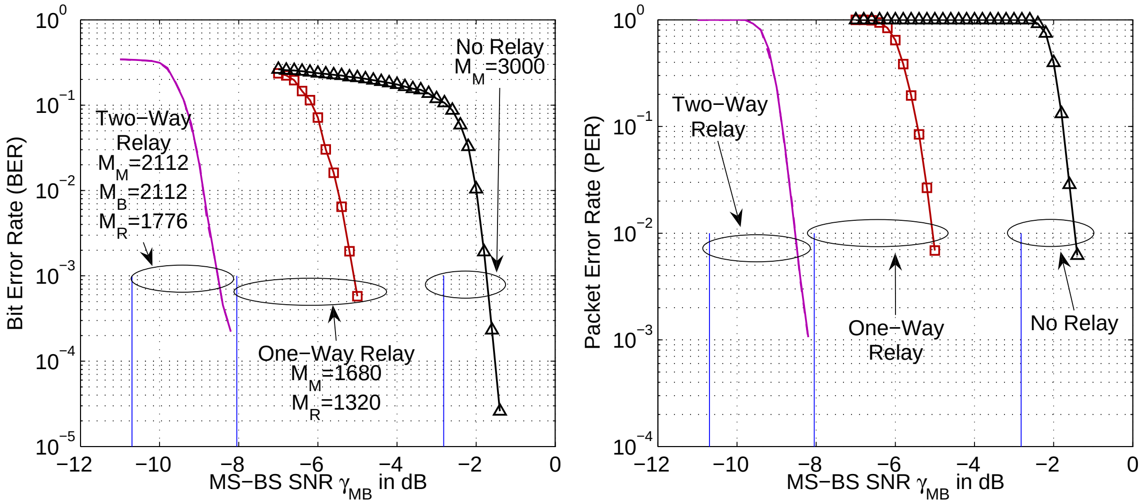


Figure 7.14: Bit and packet error rate for BPSK with rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 1$ with $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$ bits. The error rates for uplink and downlink with the two-way relay communication are identical.

SNR $\gamma_{\mathrm{RB}} = \gamma_{\mathrm{BR}}$ is increased by $35.2 \cdot \log_{10}(4/3)$ dB $= \rho_{\mathrm{MB}} + 4.40$ dB compared to the SNR $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{BM}}$ on the MS-BS links.

7.6.3 Comparison of Error Rates

Figure 7.14 depicts the BER and PER for the rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 1$ with $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$ bits. The allocation of the transmission time is optimized as described in Section 7.5. Assuming BPSK modulation and rate $R = 1/2$, we obtained $\theta_{\mathrm{M}}^* = \theta_{\mathrm{B}}^* = 0.35$ ($M_{\mathrm{M}} = M_{\mathrm{B}} = 2112$ symbols and $M_{\mathrm{R}} = 1776$ symbols). For the one-way relay communication, we chose $\theta_{\mathrm{M}}^* = 0.56$ ($M_{\mathrm{M}} = 1680$ symbols and $M_{\mathrm{R}} = 1320$ symbols). The two-way relay communication gains more than 3.0 dB compared to the one-way relay communication. The error rates for uplink and downlink with the two-way relay communication are identical.
Let us make the same comparison for the case that the relay is closer to the mobile station. Figure 7.15 depicts the BER and PER for the rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$ and $\sigma = 1$ with $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$ bits. Assuming BPSK modulation and rate $R = 1/2$, we obtained $\theta_{\mathrm{M}}^* = 0.25$ and $\theta_{\mathrm{B}}^* = 0.44$ ($M_{\mathrm{M}} = 1512$ symbols, $M_{\mathrm{B}} = 2610$ symbols and $M_{\mathrm{R}} = 1878$ symbols). The optimization of θ_{M} and θ_{B} achieves that the error rates for uplink and downlink are almost identically. The downlink is slightly better protected than the uplink. We only depict the simulation results for the uplink for the one-way relay communication with $\theta_{\mathrm{M}}^* = 0.51$ ($M_{\mathrm{M}} = 1530$ symbols and $M_{\mathrm{R}} = 1470$ symbols). The achievable rate for uplink and downlink is identical (compare Figure 7.8). The two-way relay communication gains more than 1.0 dB compared to the one-way relay communication.

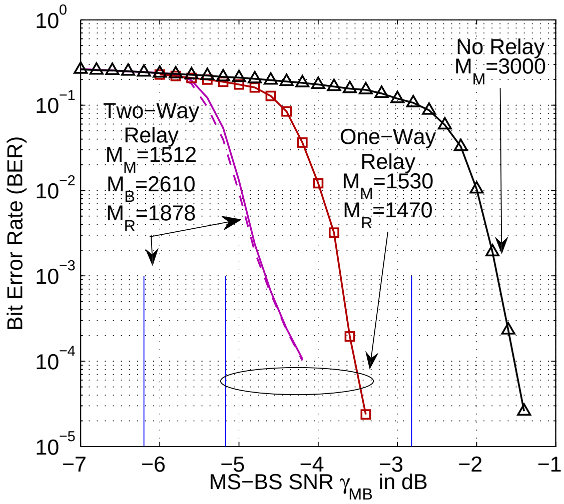


Figure 7.15: Bit and packet error rate for BPSK with rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/4$ and $\sigma = 1$ with $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$ bits. For the two-way relay communication, the error rates for the downlink (dashed curves) differ slightly from the error rates for the uplink (solid curves).

Let us make the comparison for the case that 16-QAM is used at all nodes. Figure 7.16 depicts the BER and PER for the rate $R = 2.5$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 1$ with $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$ bits. Assuming 16-QAM modulation and rate $R = 2.5$, we obtained $\theta^*_{\mathrm{M}} = \theta^*_{\mathrm{B}} = 0.57$ ($M_{\mathrm{M}} = M_{\mathrm{B}} = 447$ symbols and $M_{\mathrm{R}} = 306$ symbols). For the one-way relay communication, we chose $\theta^*_{\mathrm{M}} = 0.64$ ($M_{\mathrm{M}} = 385$ symbols and $M_{\mathrm{R}} = 215$ symbols). The two-way relay communication gains around 3.5 dB compared to the one-way relay communication.

Next, we consider the case that the downlink rate is larger than the uplink rate. Figure 7.17 depicts the BER and PER for BPSK with the rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 2$ with $K_{\mathrm{M}} = 1000$ bits and $K_{\mathrm{B}} = 2000$ bits. Assuming BPSK modulation and rate $R = 1/2$, we obtained $\theta^*_{\mathrm{M}} = 0.22$ and $\theta^*_{\mathrm{B}} = 0.43$ ($M_{\mathrm{M}} = 1291$ symbols, $M_{\mathrm{B}} = 2581$ symbols and $M_{\mathrm{R}} = 2128$ symbols). The downlink is slightly better protected than the uplink. The performance of the one-way relay communication is not affected by the choice of σ. Therefore, we choose the same parameters as for the comparison with $\sigma = 1$ in Figure 7.14. The two-way relay communication gains around 2.7/3.0 dB (uplink/downlink) compared to the one-way relay communication.
We also want to know the behavior of the proposed coding system in the case of non-equal uplink and downlink rate and non-optimal time allocation. Figure 7.18 depicts the BER and PER for BPSK with the rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 2$ with $K_{\mathrm{M}} = 1000$ bits and $K_{\mathrm{B}} = 2000$ bits. Contrary to the results in Figure 7.17, the time-allocation is chosen non-optimal to $M_{\mathrm{M}} = 1400$ symbols, $M_{\mathrm{B}} = 2800$ symbols and $M_{\mathrm{R}} = 1800$ symbols ($\theta_{\mathrm{M}} = 0.23$ and $\theta_{\mathrm{B}} = 0.47$). Due to the non-optimal time allocation, the error rate for the uplink becomes lower and the error rate for the downlink

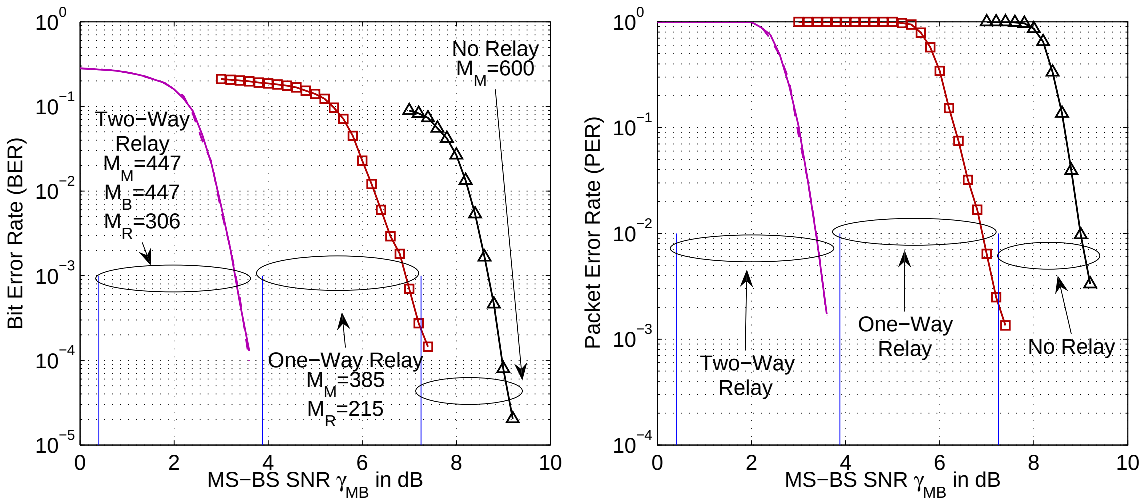


Figure 7.16: Bit and packet error rate for 16-QAM with rate $R = 2.5$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 1$ with $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$ bits. The error rates for uplink and downlink with the two-way relay communication are identical.

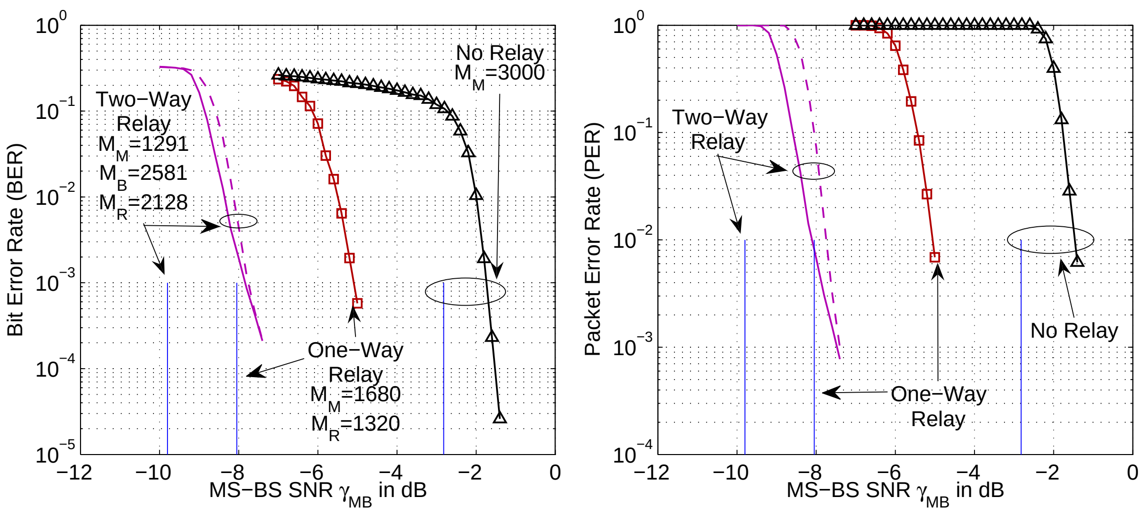


Figure 7.17: Bit and packet error rate for rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 2$ with $K_{\mathrm{M}} = 1000$ bits and $K_{\mathrm{B}} = 2000$ bits. For the two-way relay communication, the error rates for the downlink (dashed curves) differ slightly from the error rates for the uplink (solid curves).

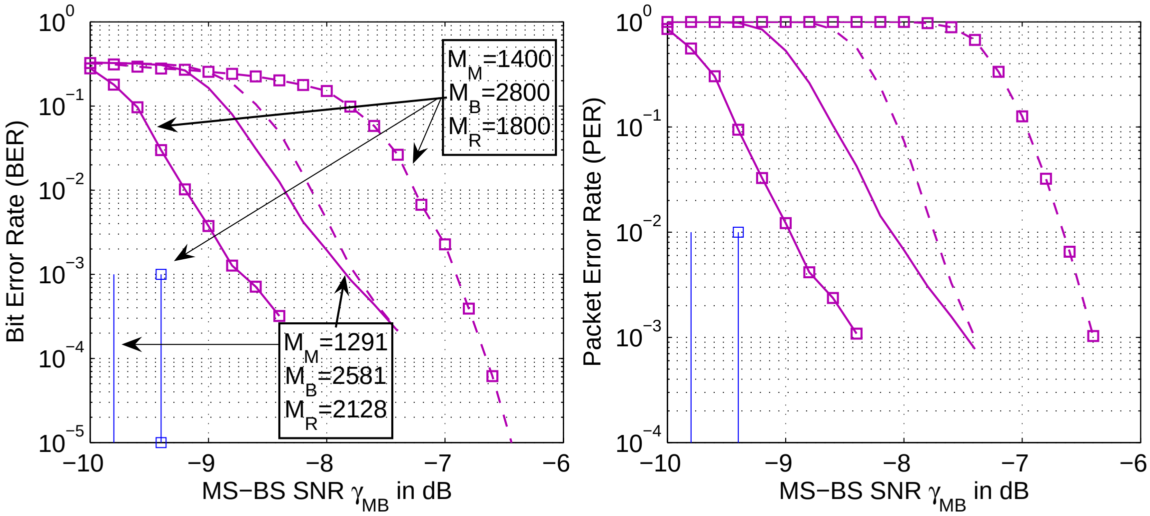


Figure 7.18: Bit and packet error rate for rate $R = 1/2$, relative MS-R distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 1/2$ and $\sigma = 2$ with $K_{\mathrm{M}} = 1000$ bits and $K_{\mathrm{B}} = 2000$ bits and non-optimal time-allocation ($M_{\mathrm{M}} = 1400$, $M_{\mathrm{B}} = 2800$ and $M_{\mathrm{R}} = 1800$ BPSK symbols). For the non-optimal time allocation, the error rates for the downlink (dashed curves) differ clearly from the error rates for the uplink (solid curves).

becomes higher. Whereas the gap between uplink and downlink performance was less than 0.4 dB for the optimal time allocation, the gap increases to more than 2.3 dB for the non-optimal time allocation. The reason is that the relay always transmits the same number of coded bits to mobile and base station. If we increase M_{R}, the uplink benefits more than the downlink, because the length of the information packet is smaller for the uplink. The average error rate is lower for the optimal time allocation.
We conclude that the optimal time allocation decreases the average error rate and makes the error rates for uplink and downlink decreasing at a similar SNR value.

7.6.4 Comparison of Throughput

Instead of measuring the bit and packet error rates for a given throughput, we want to measure the achievable error-free throughput by simulation. The measurement is done with a rate-compatible punctured code (compare Section 2.5) as described in the following: The packet lengths of the information bits is fixed to $K_{\mathrm{M}} = K_{\mathrm{B}} = 1500$. After encoding and modulation, we transmit $M_{\mathrm{M}} = 900$ 4-QAM symbols from MS to BS and $M_{\mathrm{B}} = 900$ 4-QAM symbols from BS to MS. In case of non-successful decoding, blocks of $M_{\mathrm{R}} = 300$ 4-QAM symbols are retransmitted until the decoding at the MS and BS was successful. The throughput is given by $(K_{\mathrm{M}} + K_{\mathrm{B}})/(M_{\mathrm{M}} + M_{\mathrm{B}} + i \cdot M_{\mathrm{R}})$ where i denotes the number of required retransmissions. For the relay transmission, the retransmissions are done by the relay as soon the relay decoded both packets successfully. The average throughput is determined by measuring the throughput for 300 packets.
Figure 7.19 depicts the measured throughput for the proposed joint network-channel coding scheme for the two-way relay channel. As reference systems, we consider the dis-

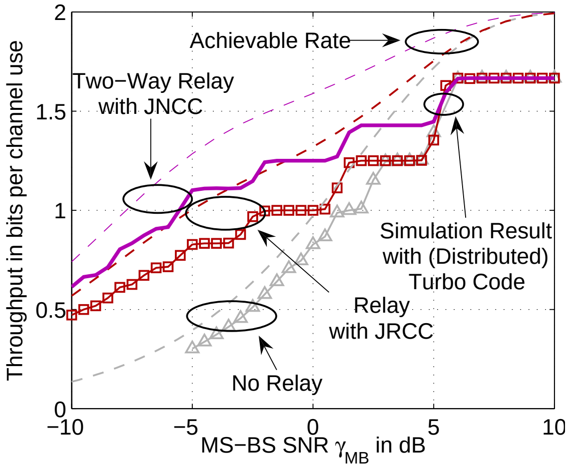


Figure 7.19: Throughput with 4-QAM for relative source-relay distance $d_{\mathrm{MR}}/d_{\mathrm{MB}} = 0.5$.

tributed turbo code for the one-way relay channel and the conventional turbo code (no relay) and the separate network-channel coding scheme for the two-way relay channel (no broadcast, direct link between MS and BS is not exploited). We apply all coding schemes with 15 iterations. For comparison, the achievable rate from Figure 7.6 for the systems is also depicted.
The two-way relay system outperforms the reference system. Note that in mobile communication systems the number of retransmissions is normally restricted to one or two due to delay constraints. However, a similar gain for low rates can be achieved with less retransmissions, if other values for M_{M}, M_{B} and M_{R} are chosen.

7.7 Summary

We proposed a joint network-channel coding (JNCC) scheme based on turbo codes for the time-division two-way relay channel (TWRC). Such a system could be used for the cooperative uplink and downlink of a mobile station in a cellular based mobile communication system with the help of one relay. We allocated optimally the transmission time to mobile station, base station and relay.
JNCC for the TWRC achieves a higher throughput than distributed channel coding for the relay channel and allows to gain up to 3.5 dB in the considered examples. The proposed system also allows to realize efficiently a two-way relay communication with a higher downlink than uplink rate.

8

Conclusions and Outlook

We considered wireless relay networks with broadcast transmissions and time-division multiple-access. The main contributions that have been achieved in the course of this work are described in the following:

- We designed a **joint network-channel code** (JNCC) for the **multiple-access relay channel** (MARC). The channel encoders at the two sources and the network encoder at the relay form one distributed code which can be decoded at the sink. We proposed a decoding strategy according to the turbo principle with an iterative exchange of soft information between the channel decoders and the network decoder. The proposed network encoder is well suited for such a decoder. Our code design outperforms distributed channel coding for the relay channel (RC) and requires the relay to transmit less symbols to gain diversity in a fading environment. Whereas the RC system allows to gain diversity for code rates R_c up to $R_\mathrm{c} \leq 1/2$, the MARC system allows a diversity gain for code rates R_c up to $R_\mathrm{c} \leq 2/3$. We compared the proposed code with a system with separate network-channel coding (SNCC), where hard decisions are passed from the channel decoders to the network decoder. The diversity gain can be also achieved with SNCC. However, JNCC allows to more efficiently exploit the redundancy which is contained in the transmission of the relay. Simulation results showed that the packet error rate of JNCC can outperform the one of SNCC by several dB.

- We designed a **joint network-channel code** for the **two-way relay channel** (TWRC). Contrary to the code design for the MARC, one of the network-encoded packets is perfectly known at the sinks. Therefore, network encoding can be reversed perfectly in the first step and an iterative exchange of soft information is not necessary. We proposed an element-wise xor-operation of the code bits at the relay. This is especially advantageous for an asymmetric setup with different uplink and downlink rate. Our code design achieves a better error protection (respectively a higher throughput) than distributed channel coding for the relay channel in a system with AWGN channels. Moreover, the proposed code outperforms the system with SNCC, where the direct link between the two sources is not exploited.

- We investigated how to allocate the transmission time to the source(s) and the relay. We derived a closed-form expression for the optimal **time-allocation** for the relay channel, the MARC and the symmetric TWRC. The optimization was done such that the achievable decode-and-forward rate for AWGN channels is maximized. Moreover, we considered how to allocate the transmission time for fading channels in order to gain diversity in a system with a discrete modulation constellation. We applied the optimal time-allocation to the practical coding systems. Simulation results showed the usefulness of the optimal time-allocation for the practical systems.

- Efficient relaying requires the use of high order modulation schemes. We proposed to extend distributed channel coding for the relay channel with **hierarchical modulation**. This allows to reduce the computational complexity at the sink. Simulation results showed that the performance loss through hierarchical modulation is in many cases very small.

- We proposed to extend current H-ARQ point-to-point communication systems with **cross-packet channel coding**. This allows to gain diversity from retransmissions with higher code rates. Contrary to conventional H-ARQ systems, it is possible to gain diversity for a code rate $R_c > 1/2$. The proposed cross-packet code design is related to the code design for the MARC. The decoding strategy uses an iterative exchange of soft information.

To conclude we describe some possibilities how the work could be extended:

- The MARC system can be extended to a system with more than two mobile stations. A possible code design for a system with three mobile station was considered in [Sch08].

- In this thesis, we considered the strategy that the relay only transmits, if it decoded correctly. An interesting topic is to find better strategies for the case that decoding failures happen at the relay. For the MARC, such strategies were proposed in [YK07, ZKBW08].

- The MARC and the TWRC system can be extended to a system with correlated sources. Such a MARC system is considered in [DSCKGG07].

- The design of the joint network-channel codes in this thesis is based on turbo codes. Similar codes could be designed based on LDPC codes. For the MARC, a first approach for the design of an LDPC code was presented in [HSOB05] and a design based on the LDPC code in the IEEE 802.16 standard was treated in [Che08]. Cross-packet channel codes based on LDPC codes were considered in [CC07]. LDPC codes are very flexible and have many degrees of freedom and there are many options how to adapt LDPC codes to the MARC, the TWRC and for cross-packet channel coding.

- The code design could be further optimized by considering other convolutional codes. An optimization of the iterative decoding procedures could be done with EXIT charts.
- The code design for the MARC and the TWRC could be extended to systems with broadcast transmissions and simultaneous multiple-access where the sources are allowed to transmit during the relay-transmit phase. For the MARC, such a strategy was proposed in [XFG08].
- Joint network-channel coding also can be helpful in other networks than the MARC or the TWRC. Specific other networks are considered in [KDH07].

A

Derivations

A.1 Derivation of (2.18) and (2.19)

We show how to derive (2.18) and (2.19) from [SKM04, Equation (12)]. The model of [SKM04] is a multiple-access relay channel (MARC) with several sources, one relay and one sink. We consider the MARC with two sources which want to send packets of K_1 and K_2 information bits to the sink, respectively. The MARC of [SKM04] is orthogonalized in two time phases with source 1 and 2 sending simultaneously M_1 symbols in the first time phase and source 1, source 2 and relay sending simultaneously M_2 symbols in the second time phase. Our point-to-point channel model with cross-packet coding after two transmissions is a special case of the MARC model of [SKM04]. We make the following simplifications for the MARC model to obtain our model: The channels between the sources and the relay are without noise and the channel-SNR between source 2 and the sink is zero. The channel-SNR between source 1 and sink is given by γ_1. The channel-SNR between relay and sink is given by γ_2. Only the relay sends during the second time phase. Based on these simplifications, we know from [SKM04, Equation (12)] the conditions that the $K_1 + K_2$ information bits in both packets can be decoded reliably at the sink. Under the assumption of Gaussian channels (Gaussian distributed channel input variable, Gaussian distributed noise), the packets can be decoded reliably at the sink, if the Conditions in (2.18) and (2.19) are fulfilled.

A.2 Derivation of (6.8)-(6.12)

We show how to derive the achievable rates K_M/M and K_W/M in (6.8)-(6.12) from [SKM04, Equation (12)]. Whereas our model is orthogonalized in three time phases, the

model of [SKM04] is orthogonalized in two time phases with source 1 and 2 sending simultaneously in the first time phase and source 1, source 2 and relay sending simultaneously in the second time phase. The idea to derive (6.8)-(6.12) is to assume two parallel two-time-phases MARCs[1], which results in a four-time-phases MARC, and to neglect transmissions such that we obtain our three-time-phases MARC model. This method has already been used in [LV05b] for the relay channel.
In order to obtain our three-time-phases MARC model, source 1 transmits M_{M} symbols $\mathbf{x}_{\mathrm{M}}$ during the first time phase of the first MARC. No symbols are transmitted in the second time phase. Source 2 transmits M_{W} symbols $\mathbf{x}_{\mathrm{W}}$ during the first time phase of the second MARC and the relay transmits M_{R} symbols $\mathbf{x}_{\mathrm{R}}$ during the second time phase of the second MARC. The other transmissions are neglected. By summing up the achievable rates from the first and the second two-time-phases MARC and under the assumption of Gaussian channels, we obtain (6.8)-(6.12).

A.3 Derivation of (6.15)-(6.18)

We want to solve the optimization in (6.13) where R_{M} has to fulfill the Conditions (6.8)-(6.12). First, we replace R_{W} by $R_{\mathrm{W}} = \sigma \cdot R_{\mathrm{M}}$ and θ_{R} by $\theta_{\mathrm{R}} = (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}})$ and obtain the following conditions for R_{M}:

$$R_{\mathrm{M}} \leq \theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right) \tag{A.1}$$
$$R_{\mathrm{M}} \leq \theta_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right)/\sigma \tag{A.2}$$
$$R_{\mathrm{M}} \leq \theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) \tag{A.3}$$
$$R_{\mathrm{M}} \leq (\theta_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right))/\sigma \tag{A.4}$$
$$R_{\mathrm{M}} \leq (\theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + \theta_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right))/(1+\sigma) \tag{A.5}$$

The optimization is done in the following two steps. First we will determine the optimal time-allocation parameters θ_{M} and θ_{W} taking into account (A.1), (A.2) and (A.5). Then, we will show that with these time-allocation parameters the Conditions (A.3) and (A.4) are not relevant for $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$ and $\mathcal{C}(\gamma_{\mathrm{WB}}) \leq \mathcal{C}(\gamma_{\mathrm{WR}})$.

Let us optimize θ_{M} and θ_{W} taking into account the Conditions (A.1), (A.2) and (A.5). For $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{RB}})$, $\theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right)$ in (A.1) is monotone increasing with θ_{M} and $(\theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + \theta_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right))/(1+\sigma)$ in (A.5) is monotone decreasing with θ_{M}. As θ_{M} is not included in (A.2), the optimal value θ_{M}^{*} can be found at the cross-over point of the right hand sides in (A.1) and (A.5). For $\mathcal{C}(\gamma_{\mathrm{WB}}) \leq \mathcal{C}(\gamma_{\mathrm{RB}})$, $(\theta_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right))/\sigma$ in (A.2) is monotone increasing with θ_{W} and $(\theta_{\mathrm{M}} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) + \theta_{\mathrm{W}} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right) + (1 - \theta_{\mathrm{M}} - \theta_{\mathrm{W}}) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right))/(1+\sigma)$ in (A.5) is monotone decreasing with θ_{W}. As θ_{W} is not included in (A.1), the optimal value θ_{W}^{*} can be found at the cross-over point of the right hand sides in (A.2) and (A.5). In order to determine the optimal values θ_{M}^{*} and θ_{W}^{*}, the following equations ((A.1)=(A.5)

[1] The terminals are identical in both MARCs.

and (A.2)=(A.5)) have to be solved:

$$\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right)=\left(\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)+\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right)+\left(1-\theta_{\mathrm{M}}^{*}-\theta_{\mathrm{W}}^{*}\right) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right)\right)/(1+\sigma) \tag{A.6}$$

$$\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right)/\sigma=\left(\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)+\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right)+\left(1-\theta_{\mathrm{M}}^{*}-\theta_{\mathrm{W}}^{*}\right) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right)\right)/(1+\sigma) \tag{A.7}$$

Solving these equations results in (6.15) and (6.16).
Next, we will show that the conditions in (A.3) and (A.4) are not relevant for $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$ and $\mathcal{C}(\gamma_{\mathrm{WB}}) \leq \mathcal{C}(\gamma_{\mathrm{WR}})$. We know from (A.6) that

$$\left(1-\theta_{\mathrm{M}}^{*}-\theta_{\mathrm{W}}^{*}\right) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right)=(1+\sigma) \cdot \theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right)-\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)-\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right). \tag{A.8}$$

Considering first (A.8) and then (6.16), we can transform the right hand side in (A.3) to

$$\begin{aligned}
\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)+\left(1-\theta_{\mathrm{M}}^{*}-\theta_{\mathrm{W}}^{*}\right) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) &= && \text{(A.9)}\\
=(1+\sigma) \cdot \theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right)-\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right) &= && \text{(A.10)}\\
=\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right) \cdot\left(1+\sigma-\sigma \cdot \frac{\mathcal{C}(\gamma_{\mathrm{WB}})}{\mathcal{C}(\gamma_{\mathrm{WR}})}\right) &\geq && \text{(A.11)}\\
\geq \theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right) \quad \text{for } \mathcal{C}(\gamma_{\mathrm{WB}}) \leq \mathcal{C}(\gamma_{\mathrm{WR}}) & && \text{(A.12)}
\end{aligned}$$

which is larger than the condition in (A.1) for $\mathcal{C}(\gamma_{\mathrm{WB}}) \leq \mathcal{C}(\gamma_{\mathrm{WR}})$. Similarly we can show that the right hand side in (A.4) is larger than the right hand side in (A.2) for $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$. We know from (A.7) that

$$\left(1-\theta_{\mathrm{M}}^{*}-\theta_{\mathrm{W}}^{*}\right) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right)=(1+\sigma) \cdot \theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right)/\sigma-\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right)-\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right). \tag{A.13}$$

Considering first (A.13) and then (6.16), we can transform the right hand side in (A.4) to

$$\begin{aligned}
\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WB}}\right)+\left(1-\theta_{\mathrm{M}}^{*}-\theta_{\mathrm{W}}^{*}\right) \cdot \mathcal{C}\left(\gamma_{\mathrm{RB}}\right) &= && \text{(A.14)}\\
=(1+1/\sigma) \cdot \theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right)-\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) &= && \text{(A.15)}\\
=\theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right)+\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MR}}\right)-\theta_{\mathrm{M}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{MB}}\right) &\geq && \text{(A.16)}\\
\geq \theta_{\mathrm{W}}^{*} \cdot \mathcal{C}\left(\gamma_{\mathrm{WR}}\right) \quad \text{for } \mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}}) & && \text{(A.17)}
\end{aligned}$$

which is larger than the condition in (A.2) for $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq \mathcal{C}(\gamma_{\mathrm{MR}})$.

A.4 Derivation of (7.16)-(7.17)

For $\sigma = 1$, $\gamma_{\mathrm{MR}} = \gamma_{\mathrm{BR}}$, $\gamma_{\mathrm{MB}} = \gamma_{\mathrm{BM}}$ and $\gamma_{\mathrm{RM}} = \gamma_{\mathrm{RB}}$, the Conditions (7.10) and (7.11) and the Conditions (7.12) and (7.13) are identical and it is optimal to choose $\theta_{\mathrm{M}} = \theta_{\mathrm{B}}$ due to symmetry arguments. Therefore, we only have to consider the Conditions (7.10) and (7.12) for the optimization of $\theta_{\mathrm{M}} = M_{\mathrm{M}}/M$. Then, the maximization of R in (4.19) can be easily solved, because (7.12) is monotone decreasing with θ_{M} for $\mathcal{C}(\gamma_{\mathrm{MB}}) \leq 2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$ and (7.10) is monotone increasing with θ_{M}. Therefore, the solution of the maximization can be always found at the cross-over point of (7.12) and (7.10) and we can find the optimal parameter θ_{M}^* solving the following equation:

$$\theta_{\mathrm{M}}^* \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) = \theta_{\mathrm{M}}^* \cdot \mathcal{C}(\gamma_{\mathrm{MB}}) + (1 - 2 \cdot \theta_{\mathrm{M}}^*) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) \tag{A.18}$$

Solving this equation leads to the optimal value for the time sharing parameter θ_{M}^* in (7.16) and to the closed expression for R_{M} in (7.17).

The sum-rate $R = R_{\mathrm{M}} + R_{\mathrm{W}}$ of the TWRC according to (7.18) is larger than the rate R of the relay channel according to (4.22), if the condition

$$\mathcal{C}(\gamma_{\mathrm{MB}}) < \mathcal{C}(\gamma_{\mathrm{MR}}) \tag{A.19}$$

is valid. This becomes obvious, if we first multiply both sides of (A.19) with $\mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})$, then add $\mathcal{C}(\gamma_{\mathrm{MR}})^2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) + 2 \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}})^2 - 2 \cdot \mathcal{C}(\gamma_{\mathrm{MR}}) \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) \cdot \mathcal{C}(\gamma_{\mathrm{MB}})$ to both sides and then divide both sides with $(\mathcal{C}(\gamma_{\mathrm{MR}}) + \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}})) \cdot (\mathcal{C}(\gamma_{\mathrm{MR}}) + 2 \cdot \mathcal{C}(\gamma_{\mathrm{RB}}) - \mathcal{C}(\gamma_{\mathrm{MB}}))$.

Nomenclature

a	Rayleigh fading coefficient, page 10
a_{ij}	Rayleigh fading coefficient for channel between node i and node j, page 10
C	Channel capacity, page 11
$\mathcal{C}_k(\cdot)$	Channel capacity of point-to-point channel with discrete input alphabet $\mathcal{S}_k$, page 11
$\mathcal{C}(\cdot)$	Channel capacity of Gaussian point-to-point channel, page 11
$\mathbf{c}_i$	Block with code bits[1]. The block $\mathbf{c}_3$ denotes the output of the CPC encoder., page 6
$\mathbf{c}_{i,p}$	Punctured version of block $\mathbf{c}_i$ with $p \in \{1, 2, \mathrm{C}, \mathrm{E}\}$, page 6
$\tilde{\mathbf{c}}_{i,p}$	Punctured version of block $\tilde{\mathbf{c}}_i$ with $p \in \{1, 2, \mathrm{C}, \mathrm{E}\}$, page 67
$\tilde{\mathbf{c}}_i$	Regenerated block with code bits at relay[1], page 67
d	Distance between source and sink, page 8
d_{ij}	Distance between node i and node j, page 8
E_i	Transmission energy[1], page 40
E_s	Received symbol energy, page 11
f_c	Carrier frequency, page 9
$\mathbf{G}$	Generator matrix of encoder, page 6
h	Channel coefficient, page 10
h_{ij}	Channel coefficient for channel between node i and node j, page 10
K_i	Number of information bits in one packet[1], page 6
L	Number of code bits carried by one symbol, page 6
$L(\cdot)$	Log-likelihood ratio, page 13
$L_{\mathrm{e}}^{|}(\cdot)$	A priori LLRs for first SISO decoder in turbo decoder, page 15
$L^{|}(\cdot)$	A posteriori LLRs after second SISO decoder in turbo decoder, page 15

[1]Meaning of index i: The indices $(\cdot)_{\mathrm{M}} = (\cdot)$, $(\cdot)_{\mathrm{W}}$, $(\cdot)_{\mathrm{B}}$ and $(\cdot)_{\mathrm{R}}$ refer to mobile station 1, mobile station 2, base station and relay, respectively. The indices $(\cdot)_1$,$(\cdot)_2$ refer to first and second packet of mobile station 1.

$L^{-}(\cdot)$	A posteriori LLRs after first SISO decoder in turbo decoder, page 15
$L_{\mathrm{e}}^{-}(\cdot)$	A priori LLRs for second SISO decoder in turbo decoder, page 15
M	Number of total transmitted symbols in system, page 7
M_i	Number of symbols in one block[1], page 7
m	Memory of convolutional code, page 13
N_0	Power spectral density of white noise, page 7
N_i	Number of code bits in one block[1], page 6
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with mean μ and variance σ^2, page 10
n	Path-loss exponent, page 8
OUT	Outage Event, page 22
$\overline{\mathrm{OUT}}$	Complement of the outage event OUT, page 23
P_i	Transmission power[1], page 8
$P_r(d)$	Received power at distance d, page 8
$P_{\mathrm{M}}^{(2)}$	Transmission power of mobile station 1 during the relay-transmit phase, page 53
$\mathbf{p}$	Block of parity bits, page 13
$\mathbf{q}$	Block of parity bits from second convolutional code in PCCC, page 15
R	Sum-rate of system (ratio of total number of information bits to total number of transmitted symbols), page 7
R_i	Rate of channel encoder and modulator ($R_i = K_i/M$), page 7
R_{c}	Code rate of channel encoder ($R_{\mathrm{c}} = K/N$), page 6
$\mathbf{r}$	Soft output of demodulator (LLRs in this thesis), page 13
$\mathbf{r}_i$	Soft information which corresponds to the code bits in $\mathbf{c}_i$, page 13
$\mathbf{r}_{i,p}$	Soft information which corresponds to the code bits in $\mathbf{c}_{i,p}$, page 13
$\tilde{\mathbf{r}}_{i,p}$	Soft information which corresponds to the code bits in $\mathbf{c}_{i,p}$ at relay, page 67
$\tilde{\mathbf{r}}_i$	Soft information which corresponds to the code bits in $\mathbf{c}_i$ at relay, page 67
$\mathcal{S}$	Alphabet for mapping, page 6
T	Total transmission time, page 11
T_i	Transmission time of one node[1], page 11
T_s	Symbol duration, page 11
T_{c}	Coherence time of channel, page 9
t	Time, page 9
$\mathbf{u}_i$	Packet with information bits[1], page 6
$\hat{\mathbf{u}}_i$	Estimate of $\mathbf{u}_i$ at sink[1], page 7
$\tilde{\mathbf{u}}_i$	Estimate of $\mathbf{u}_i$ at relay[1], page 67

W	Bandpass bandwidth, page 7
W_i	Bandpass bandwidth[1], page 42
$\mathbf{x}_\mathrm{C}$	The common bits are mapped to these symbols, page 70
$\mathbf{x}_\mathrm{E}$	The enhancement bits are mapped to these symbols, page 70
$\mathbf{x}_\mathrm{M}^{(2)}$	Transmitted symbols from mobile station 1 during the relay-transmit phase, page 53
$\mathbf{x}_i$	Block of transmitted symbols[1] (channel input), page 7
$\mathbf{y}$	Block of received symbols at sink (channel output), page 7
$\mathbf{y}_{ij}$	Block of received symbols at node j. Transmission was done at node i., page 7
$\mathbf{z}_{ij}$	Additive noise at node j., page 42
$\Pi(\cdot)$	Interleaver, page 15
$\Pi^{-1}(\cdot)$	Deinterleaver, page 16
α	This parameter determines the allocation of the transmission power in a non-equidistant 16-QAM constellation., page 70
β	This parameter defines the allocation of the total transmission energy to source and relay, page 43
β^*	Optimal choice for β, page 43
γ	Instantaneous signal-to-noise ratio, page 10
γ_0	Required SNR for reliable communication with rate R_0, page 11
γ_{ij}	Instantaneous signal-to-noise ratio for channel between node i and node j, page 10
$\hat{\gamma}_{ij}$	SINR between node i and node j, page 46
$\tilde{\gamma}_{ij}$	Normalized SNR γ_{ij} for transmission power P and bandwidth W, page 42
κ	This parameter defines the allocation of the source energy to relay-receive and relay-transmit phase, page 55
κ^*	Optimal choice for κ, page 55
φ	Correlation between the signals of MS and R during the relay-transmit phase, page 54
φ^*	Optimal choice for φ, page 54
ρ	Average signal-to-noise ratio, page 11
ρ_{ij}	Average signal-to-noise ratio for channel between node i and node j, page 11
σ	Ratio of length of second and length of first packet, page 20
θ_i	Relative transmission time of one node for TDMA[1] ($\theta_i = T_i/T$) or relative bandwidth used by one node for FDMA[1] ($\theta_i = W_i/W$), page 42
θ_i^*	Optimal choice for θ_i, page 43
$\oplus$	Modulo-2 addition of two bits, page 13

ARQ Automatic repeat request, page 17

BC Broadcast transmission, page 59

BCJR algorithm Algorithm by Bahl, Cocke, Jelinek and Raviv [BCJR74], page 14

BER Bit error rate, page 26

BPSK Binary phase-shift-keying, page 6

BS Base station, page 41

CPC Cross-packet channel coding, page 29

CPER Common packet error rate, page 104

CRC Cyclic redundancy check, page 18

DVB Digital Video Broadcast, page 14

FDMA Frequency-division multiple access, page 39

FEC Forward Error Correction, page 5

GSM Global System for Mobile Communications, page 1

H-ARQ Hybrid ARQ/FEC, page 17

JNCC Joint network-channel coding, page 4

JRCC Joint routing-channel coding, page 4

LDPC code Low-density parity-check code, page 52

LLR Log-likelihood ratio, page 13

LOG-MAP Logarithmic MAP decoder, page 14

MAP decoder Maximum a posteriori probability decoder, page 14

MARC Multiple-Access Relay Channel, page 2

MS Mobile station, page 41

OSI Open Systems Interconnection, page 31

PCCC Parallel concatenated convolutional code, page 15

PDF Probability density function, page 10

PER Packet error rate, page 26

PSMA Partial SMA, page 55

QAM Quadrature Amplitude Modulation, page 6

QPSK Quadrature phase-shift-keying, page 6

R Relay, page 41

SINR Signal-to-interference-and-noise ratio, page 46

SISO decoder Soft-input soft-output decoder, page 14

SMA Simultaneous multiple access, page 39

SNCC Separate network-channel coding, page 4

SNR Signal-to-noise ratio, page 10

SOVA Soft-output Viterbi algorithm, page 14

SRCC Separate routing-channel coding, page 4

TDMA Time-Division Multiple-Access, page 2

TWRC Two-Way Relay Channel, page 2

UMTS Universal Mobile Telecommunications System, page 1

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