Transmit Wiener Filter for the Downlink of TDDDS-CDMA Systems

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Abstract — **We derive the transmit Wiener filter for DS-CDMA systems which depends upon the noise power at the receivers. We show that the transmit Wiener filter converges to the transmit matched filter and the transmit zero-forcing filter for low and high signal-to-noise-ratio, respectively. Simulation results show the superiority of the transmit Wiener filter compared to the other two transmit filters. Moreover, we observe that the application of the three transmit filters and the respective receive filters lead to similar results.**

I. Introduction

Since the transmissions in the uplink and the downlink of a *time division duplex* (TDD) system reside in the same frequency band, the channel parameters of the uplink are also valid for the downlink, if the transmitter and the receiver are calibrated correctly [1] and the *coherence time* [2] of the channel is large enough. Therefore, it is possible to design transmit filters which are applied prior to transmission to equalize the detection signal at the output of the *a priori* known receive filter [3, 4]. This *precoding* or *pre-equalization* technique has the advantage to reduce the complexity of the receiver dramatically, because it is not necessary to adapt the receive filter to the instantaneous channel properties. As the mobile stations (MSs) in a cellular *direct sequence code divison multiple access* (DS-CDMA) system have to be low cost, transmit processing is especially attractive for the downlink, since the base station (BS) is the transmitter. Moreover, multiple antennas deployed at the BS can be exploited very easily to improve the transmission in the downlink, when transmit processing is employed. Note that we only examine *linear* transmit processing in this article to end up with MSs which are as simple as possible.

The most intuitive approach for transmit processing is to completely eliminate the *intracell interference* at the receive filter output, since the BS has no influence on the noise received by the MSs. The resulting *transmit zero-forcing filter* (TxZF, e. g. [4, 5, 6, 7, 8, 9, 10]) exhibits an extraordinary performance for high *signal-to-noise-ratio* (SNR) similar to the *receive zero-forcing filter* (RxZF).

The *receive matched filter* (RxMF) maximizes the ratio of the power of the desired signal to the noise power leading to an optimum behaviour for low SNR. Accordingly, the *transmit matched filter* (TxMF) or *prerake* [3, 11, 12] maximizes the power of the desired part of the received signal [13] and is therefore optimum, when the power of the noise at the receiver is large. Similar to the respective receive filters, the TxZF outperforms the TxMF for high SNR, whereas the TxMF is superior for low SNR [13].

The *receive Wiener filter* (RxWF) which minimizes the *mean square error* (MSE) and finds a trade-off between the signal power maximization of the RxMF and the interference suppression of the RxZF is well researched and understood. The RxWF converges to the RxMF for low SNR and to the RxZF for high SNR. However, a similar transmit filter, namely the *transmit Wiener filter* (TxWF), is missing. In a recent report [14], we showed that the TxWF can be found by minimizing a modified MSE.

In this paper, we derive the TxWF for the downlink of DS-CDMA systems. To this end, we briefly explain the system model in Section II and recall the optimizations to obtain the TxMF and the TxZF in the Sections III and IV, respectively. After discussing previous attempts to find the TxWF in Section V, we present the TxWF in Section VI. The simulation results in Section VII show the exceptional performance of the TxWF.

II. System Model

In a conventional DS-CDMA system, the m-th symbol $s_k^{(m)}$ of user k is spread with the spreading code $d_k[n]$ of length χ_k and one slot consists of $N_c = \chi_k M_k$ chips, where M_k denotes the number of symbols of the k-th user per slot. The spreading operation is *a priori* known to the receiver in the case of receive processing. Accordingly, we assume that the receive filter applied to the received signal for transmit processing is *a priori* known to the transmitter. To end up with the simplest receiver for DS-CDMA we use a correlator as receive filter and obtain for the estimate of the m-th symbol of user k:

$$\hat{s}_k^{(m)} = \sum_{n=0}^{\chi_k - 1} d_k^*[n] x_k[\chi_k m + n], \quad m = 0, \ldots, M_k - 1.$$

The received signal $x_k[n]$ is the output of the FIR channel to the k-th MS perturbed by the complex Gaussian noise $\eta_k[n]$:

$$x_k[n] = \sum_{q=0}^{Q_k} \boldsymbol{h}_{k,q}^{\mathrm{T}} \boldsymbol{y}[n-q] + \eta_k[n],$$

where Q_k and $(\bullet)^{\mathrm{T}}$ denote the maximum delay and transpose, respectively. Note that the BS is equipped with N_a antenna elements. Therefore, the channel coefficients $\boldsymbol{h}_{k,q}$ and the transmitted signal $\boldsymbol{y}[n]$ are N_a-dimensional vectors. To get a compact formulation we put the estimated symbols $\hat{s}_k^{(m)}$, the noise $\eta_k[n]$, and the transmitted signal $\boldsymbol{y}[n]$ for

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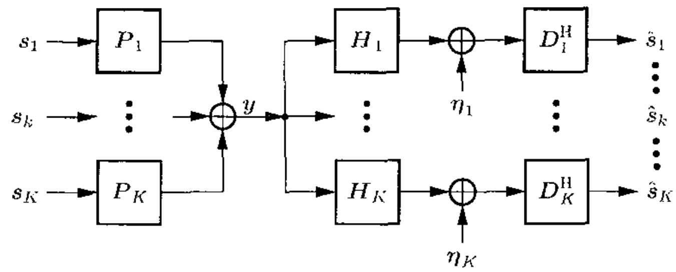

Figure 1: Multi-User Downlink Transmit Processing

one slot into the vectors:

$$\hat{\boldsymbol{s}}_k = \left[\hat{s}_k^{(0)}, \ldots, \hat{s}_k^{(M_k-1)}\right]^{\mathrm{T}} \in \mathbb{C}^{M_k},$$
$$\boldsymbol{\eta}_k = [\eta_k[0], \ldots, \eta_k[N_c-1]]^{\mathrm{T}} \in \mathbb{C}^{N_c}, \text{ and}$$
$$\boldsymbol{y} = \left[\boldsymbol{y}^{\mathrm{T}}[0], \ldots, \boldsymbol{y}^{\mathrm{T}}[N_c-1]\right]^{\mathrm{T}} \in \mathbb{C}^{N_a N_c},$$

respectively. With the Hermitian operator $(\bullet)^{\mathrm{H}}$ the estimated symbols of MS k for one slot can be written as

$$\hat{\boldsymbol{s}}_k = \boldsymbol{D}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{y} + \boldsymbol{D}_k^{\mathrm{H}} \boldsymbol{\eta}_k, \qquad (1)$$

where we introduced the block Toeplitz channel matrix

$$\boldsymbol{H}_k = \sum_{q=0}^{Q_k} \boldsymbol{\Gamma}_{N_c}^q \otimes \boldsymbol{h}_{k,q}^{\mathrm{T}} \in \mathbb{C}^{N_c \times N_a N_c}$$

with the Kronecker product '$\otimes$' and the nilpotent shift matrix

$$\boldsymbol{\Gamma}_N = \begin{bmatrix} \boldsymbol{0}_{N-1}^{\mathrm{T}} & 0 \\ \boldsymbol{1}_{N-1} & \boldsymbol{0}_{N-1} \end{bmatrix} \in \mathbb{R}^{N \times N}, \quad \boldsymbol{\Gamma}_N^0 = \boldsymbol{1}_N.$$

Here, $\boldsymbol{0}_N$, $\boldsymbol{1}_N$, and $\boldsymbol{\Gamma}_N^q$ denote the $N \times 1$ zero vector, the $N \times N$ identity matrix, and the q-th power of $\boldsymbol{\Gamma}_N$,respectively. The block Toeplitz modulator matrix can be expressed as

$$\boldsymbol{D}_k = \boldsymbol{1}_{M_k} \otimes \boldsymbol{d}_k \in \mathbb{C}^{N_c \times M_k},$$

by using $\boldsymbol{d}_k = [d_k[0], \ldots, d_k[\chi_k - 1]]^{\mathrm{T}}$. The transmitted signal $\boldsymbol{y}$ results from the summation of the signals $\boldsymbol{s}_k$ dedicated to the K MSs transformed by the respective transmit filters $\boldsymbol{P}_k \in \mathbb{C}^{N_a N_c \times M_k}$ as depicted in Figure 1:

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{P}_k \boldsymbol{s}_k = \boldsymbol{P}\boldsymbol{s}, \qquad (2)$$

where $\boldsymbol{s}_k$ is defined equally as $\hat{\boldsymbol{s}}_k$. The transmit filters $\boldsymbol{P}_k$ and the symbols $\boldsymbol{s}_k$ are collected in:

$$\boldsymbol{P} = [\boldsymbol{P}_1, \ldots, \boldsymbol{P}_K] \in \mathbb{C}^{N_a N_c \times M_{\mathrm{tot}}} \quad \text{and}$$
$$\boldsymbol{s} = \left[\boldsymbol{s}_1^{\mathrm{T}}, \ldots, \boldsymbol{s}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{M_{\mathrm{tot}}},$$

respectively. Here, $M_{\mathrm{tot}} = \sum_{k=1}^{K} M_k$ denotes the number of symbols of all K users per slot. When we put the estimated symbols $\hat{\boldsymbol{s}}_k$ in a vector $\hat{\boldsymbol{s}}$ similar to $\boldsymbol{s}$, the whole transmission can be expressed as (cf. Figure 2)

$$\hat{\boldsymbol{s}} = \boldsymbol{D}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{P} \boldsymbol{s} + \boldsymbol{D}^{\mathrm{H}} \boldsymbol{\eta} \in \mathbb{C}^{M_{\mathrm{tot}}}, \qquad (3)$$

where we defined:

$$\boldsymbol{H} = \left[\boldsymbol{H}_1^{\mathrm{T}}, \ldots, \boldsymbol{H}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{KN_c \times N_a N_c},$$
$$\boldsymbol{D} = \mathrm{blockdiag}\left(\boldsymbol{D}_1, \ldots, \boldsymbol{D}_K\right) \in \mathbb{C}^{KN_c \times M_{\mathrm{tot}}}, \quad \text{and}$$
$$\boldsymbol{\eta} = \left[\boldsymbol{\eta}_1^{\mathrm{T}}, \ldots, \boldsymbol{\eta}_K^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{KN_c}.$$

Furthermore, we presume that the matrix $\boldsymbol{D}^{\mathrm{H}}\boldsymbol{H}$ is wide and has full rank, i. e. $\mathrm{rank}(\boldsymbol{D}^{\mathrm{H}}\boldsymbol{H}) = M_{\mathrm{tot}}$, to be able to design also transmit filters with zero-forcing constraints. Note that the symbols $\boldsymbol{s}$ and the noise $\boldsymbol{\eta}$ are uncorrelated:

$$\mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{s}^{\mathrm{H}}\right] = \boldsymbol{0}_{KN_c \times M_{\mathrm{tot}}}, \qquad (4)$$

where '$\mathrm{E}[\bullet]$' and $\boldsymbol{0}_{M \times N}$ denote the expectation operator and the $M \times N$ zero matrix, respectively.

III. Transmit Matched Filter

The TxMF $\boldsymbol{P}_{\mathrm{MF}}$ maximizes the desired signal portion in the estimate $\hat{\boldsymbol{s}}$ due to $\boldsymbol{s}$ and uses the available transmit power E_{tr} [13]. Thus, we get:

$$\boldsymbol{P}_{\mathrm{MF}} = \arg\max_{\boldsymbol{P}} \rho(\boldsymbol{P}) \quad \text{s. t.: } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}. \qquad (5)$$

By introducing the covariance matrix $\boldsymbol{R}_s = \mathrm{E}[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}]$ of the symbols for all MSs and due to Equations (3) and (4) the objective function reads as

$$\rho(\boldsymbol{P}) = \mathrm{Re}\left(\mathrm{E}\left[\boldsymbol{s}^{\mathrm{H}}\hat{\boldsymbol{s}}\right]\right) = \mathrm{Re}\left(\mathrm{tr}\left(\boldsymbol{D}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P}\boldsymbol{R}_s\right)\right).$$

The resulting TxMF can be written as [13, 3]

$$\boldsymbol{P}_{\mathrm{MF}} = \beta_{\mathrm{MF}} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{D} \in \mathbb{C}^{N_a N_c \times M_{\mathrm{tot}}} \quad \text{and} \qquad (6)$$
$$\beta_{\mathrm{MF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{D}\boldsymbol{R}_s\boldsymbol{D}^{\mathrm{H}}\boldsymbol{H}\right)}}.$$

IV. Transmit Zero-Forcing Filter

Usually, the TxZF is found by minimizing the MSE [5, 6, 7] or by minimizing the transmit power under a zero-forcing constraint [8, 9, 10]. Since the transmit power is constrained to E_{tr}, the heuristic approach to meet this requirement is to scale the resulting filter (e. g. [10]). We follow a different strategy which is similar to the one used in [6], has been discussed in [10], and will be clear with the Appendix:

$$\{\boldsymbol{P}_{\mathrm{ZF}}, \beta_{\mathrm{ZF}}\} = \arg\min_{\boldsymbol{P},\beta} \beta^{-2} \qquad (7)$$
$$\text{s. t.: } \boldsymbol{D}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{P} = \beta \boldsymbol{1}_{M_{\mathrm{tot}}} \quad \text{and} \quad \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}.$$

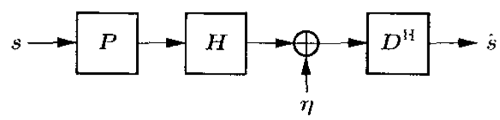

Figure 2: Downlink Transmit Processing

The first constraint is the zero-forcing condition to suppress the interference and the second constraint defines the available transmit power E_{tr}. Note that the gain of the chain of transmit filter $\boldsymbol{P}$, channels $\boldsymbol{H}_k$, and correlators $\boldsymbol{D}_k^{\mathrm{H}}$ is $\beta > 0$. Consequently, the TxZF maximizes the gain β of the whole transmission. The solution of the optimization in Equation (7) is the TxZF for the k-th user [14, 4]:

$$\boldsymbol{P}_{\mathrm{ZF}} = \beta_{\mathrm{ZF}} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{D} \left(\boldsymbol{D}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{D} \right)^{-1} \in \mathbb{C}^{N_s N_c \times M_{\mathrm{tot}}} \text{ and}$$
$$\beta_{\mathrm{ZF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\left(\boldsymbol{D}^{\mathrm{H}} \boldsymbol{H} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{D} \right)^{-1} \boldsymbol{R}_s \right)}}. \tag{8}$$

V. Previous Attempts to Obtain the TxWF

The TxZF is outperformed by the TxMF for low transmit power E_{tr}, whereas the TxMF is interference limited (e.g. [9, 13]). Thus, both filters cannot be optimum for all values of E_{tr}. For receive processing, the RxWF outperforms the RxZF and the RxMF and is found by minimizing the MSE. We can do the same to find the according transmit filter:

$$\boldsymbol{P}'_{\mathrm{WF}} = \arg\min_{\boldsymbol{P}} \mathrm{E}\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_2^2 \right].$$

However, the above optimization leads to the TxZF in Equation (8) with $\beta_{\mathrm{ZF}} = 1$ as has been demonstrated in [5, 6, 7, 14]. Vojčić et. al. [5] proposed to minimize the MSE together with a constraint for the transmit power and claimed that the resulting transmit filter outperforms the TxZF, but they also stated that the resulting optimization can only be solved numerically. Noll Barreto et. al. [10] modified the transmit power constraint to be an inequality:

$$\overline{\boldsymbol{P}}_{\mathrm{WF}} = \arg\min_{\boldsymbol{P}} \mathrm{E}\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_2^2 \right] \quad \text{s.t.: } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2 \right] \leq E_{\mathrm{tr}}.$$

The above optimization has also no closed form solution [10], but an expression depending on the Lagrangian factor $\lambda \geq 0$ can be obtained:

$$\overline{\boldsymbol{P}}_{\mathrm{WF}} = \left(\boldsymbol{H}^{\mathrm{H}} \boldsymbol{D} \boldsymbol{D}^{\mathrm{H}} \boldsymbol{H} + \lambda \mathbf{1}_{N_s N_c} \right)^{-1} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{D}.$$

If the transmit power E_{tr} is large, the constraint is inactive ($\lambda = 0$) and the resulting transmit filter is equal to the TxZF in Equation (8) with $\beta_{\mathrm{ZF}} = 1$. On the other hand, λ is large for low transmit power E_{tr} and the transmit filter has the structure of the TxMF in Equation (6). Besides the lack of a closed form solution, the transmit filter $\overline{\boldsymbol{P}}_{\mathrm{WF}}$ suffers from a disadvantage: the transmit filter is independent from the noise power at the receivers. Assume a scenario with a transmit power E_{tr} large enough to enable a TxZF-like $\overline{\boldsymbol{P}}_{\mathrm{WF}}$ ($\lambda = 0$) and high noise power at the receivers. Hence, the transmit filter uses the transmit power to suppress the interference, although signal power maximization as performed by the TxMF is better due to the high noise power. Noll Barreto et. al. [10] also reported the suboptimality of the above transmit filter and showed that it is even outperformed by the TxZF of Equation (8) in some scenarios.

VI. Transmit Wiener Filter

The key to find the TxWF is to allow the transmit filter to generate a receive signal $\hat{\boldsymbol{s}}$ whose amplitude is different from the one of the original desired signal $\boldsymbol{s}$. This property is in accordance with the behaviour of the TxMF and the TxZF. To consider the gain β of the transmit filter we have to modify the MSE [14]:

$$\varepsilon(\boldsymbol{P}, \beta) = \mathrm{E}\left[\|\boldsymbol{s} - \beta^{-1}\hat{\boldsymbol{s}}\|_2^2 \right]. \tag{9}$$

The TxWF minimizes the above modified MSE and uses the available transmit power E_{tr}:

$$\left[\boldsymbol{P}_{\mathrm{WF}}, \beta_{\mathrm{WF}} \right] = \arg\min_{\boldsymbol{P}, \beta} \varepsilon(\boldsymbol{P}, \beta) \qquad \text{s.t.: } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2 \right] = E_{\mathrm{tr}}. \tag{10}$$

With the Lagrangian factor $\lambda \in \mathbb{R}$ we form the Lagrangian function

$$L(\boldsymbol{P}, \beta, \lambda) = \varepsilon(\boldsymbol{P}, \beta) - \lambda \left(\mathrm{tr}\left(\boldsymbol{P}\boldsymbol{R}_s \boldsymbol{P} \right) - E_{\mathrm{tr}} \right),$$

whose derivations with respect to $\boldsymbol{P}$, β, and λ must vanish. From the first condition follows:

$$\boldsymbol{P}(\xi') = \beta(\xi') \left(\boldsymbol{T}^{\mathrm{H}} \boldsymbol{T} + \xi' \mathbf{1}_{N_s N_c} \right)^{-1} \boldsymbol{T}^{\mathrm{H}}, \tag{11}$$

where we replaced $-\lambda\beta^2$ by $\xi' \in \mathbb{R}$ and $\boldsymbol{D}^{\mathrm{H}} \boldsymbol{H}$ by $\boldsymbol{T}$. Due to the constraint of Equation (10) the weight β only depends upon ξ':

$$\beta(\xi') = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\left(\boldsymbol{T}^{\mathrm{H}} \boldsymbol{T} + \xi' \mathbf{1}_{N_s N_c} \right)^{-2} \boldsymbol{T}^{\mathrm{H}} \boldsymbol{R}_s \boldsymbol{T} \right)}}.$$

Therefore, the original constrained optimization with respect to $\boldsymbol{P}$ and β in Equation (10) can be reduced to following unconstrained optimization with respect to ξ, since we fulfill the constraint with the choice of β:

$$\xi' = \arg\min_{\xi} \varepsilon(\boldsymbol{P}(\xi), \beta(\xi)). \tag{12}$$

The optimum ξ' can be found by setting the derivation of $\varepsilon(\xi)$ with respect to ξ to zero and can be written as [14]

$$\xi' = \frac{\mathrm{tr}\left(\boldsymbol{D}^{\mathrm{H}} \boldsymbol{R}_\eta \boldsymbol{D} \right)}{E_{\mathrm{tr}}},$$

where $\boldsymbol{R}_\eta = \mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}} \right]$ denotes the noise covariance matrix. Since we have solved the optimization in Equation (12), we have obtained a closed form solution for Equation (10) which is the TxWF $\boldsymbol{P}_{\mathrm{WF}} \in \mathbb{C}^{N_s N_c \times M_{\mathrm{tot}}}$:

$$\boldsymbol{P}_{\mathrm{WF}} = \beta_{\mathrm{WF}} \widetilde{\boldsymbol{P}} \quad \text{and} \quad \beta_{\mathrm{WF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}\left(\widetilde{\boldsymbol{P}} \boldsymbol{R}_s \widetilde{\boldsymbol{P}}^{\mathrm{H}} \right)}} \quad \text{with}$$
$$\widetilde{\boldsymbol{P}} = \left(\boldsymbol{H}^{\mathrm{H}} \boldsymbol{D} \boldsymbol{D}^{\mathrm{H}} \boldsymbol{H} + \frac{\mathrm{tr}\left(\boldsymbol{D}^{\mathrm{H}} \boldsymbol{R}_\eta \boldsymbol{D} \right)}{E_{\mathrm{tr}}} \mathbf{1}_{N_s N_c} \right)^{-1} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{D}. \tag{13}$$

We note that Karimi et. al. presented a similar result in [7], but they gave no optimization nor motivation for their transmit filter. The scalar $\xi' = \mathrm{tr}(D^{\mathrm{H}} R_\eta D)/E_{\mathrm{tr}}$ depends upon the noise powers at the outputs of the receive filters. Consequently, the optimum TxWF can only be designed, if the noise powers are estimated at the MSs and fed back from the MSs to the BS. However, the simulation results in Section VII show that a suboptimum TxWF without feedback from the MSs to the BS outperforms the TxZF (cf. Figure 4).

If the receive noise power is small compared to the transmit power ($\xi' \rightarrow 0$), the weighted identity matrix in $\widetilde{P}$ vanishes and the TxWF converges to the TxZF in Equation (8) as can be seen after application of the matrix inversion lemma (e. g. [15]). On the other hand, the TxWF approaches the TxMF in Equation (6), when the transmit power is small compared to the receive noise ($\xi' \rightarrow \infty$). This behaviour of the transmit filters is in full accordance with the properties of the respective receive filters. The TxWF can be deeper understood, when we examine the optimum weight β_{WF} of the optimization in Equation (13) in terms of the singular values $\psi_b \in \mathbb{R}^+, b = 1, \ldots, M_{\mathrm{tot}}$, of the matrix $D^{\mathrm{H}} H$:

$$\beta_{\mathrm{WF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\sigma_s^2} \left(\sum_{b=1}^{M_{\mathrm{tot}}} \frac{\psi_b^2}{(\psi_b^2 + \xi')^2} \right)^{-1}},$$

where we assumed that $R_s = \sigma_s^2 \mathbf{1}_{M_{\mathrm{tot}}}$ for simplicity. It is easy to see that β_{WF} becomes larger as ξ' increases. We can follow that $\beta_{\mathrm{ZF}} = \beta_{\mathrm{WF}}(\xi' \rightarrow 0)$ is smaller than $\beta_{\mathrm{MF}} = \beta_{\mathrm{WF}}(\xi' \rightarrow \infty)$ and the TxWF uses the optimum β_{WF} which lies between the two extreme values β_{ZF} and β_{MF} depending on the noise powers at the MSs. Thus, the TxWF finds the optimum trade-off between interference cancellation (TxZF) and signal power maximization (TxMF).

VII. Simulation Results

We applied the transmit filters to the downlink of a DS-CDMA system with orthogonal spreading codes and $K = 4$ MSs. The maximum delay of the channels is $Q_k = 5, k = 1, \ldots, K$, and the powers of the channel coefficients are $\mathrm{E}[\|h_{k,q}\|_2^2] = N_a, k = 1, \ldots, K$ and $q = 0, \ldots, Q_k$, where N_a antenna elements are deployed at the BS. One TDD slot consists of $N_c = 128$ chips and with a spreading factor $\chi_k = 4, k = 1, \ldots, K$, the number of QPSK symbols $s_k^{(m)}$ per slot of user k equals $M_k = 32, k = 1, \ldots, K$. The presented results are the mean of 10000 channel realizations and we assumed that the BS perfectly knows the instantaneous channel impulse responses.

In Figure 3, we compare the uncoded *bit error ratio* (raw BER) of the different linear transmit filters versus the SNR for $N_a = 1$ antenna element. We observe that the TxWF equals to the TxMF for low SNR and outperforms the other transmit filters, namely the TxMF and the TxZF. The poor performance of the TxZF is due to the lack of degrees of freedom available to maximize the gain β_{ZF}. Therefore, we cannot see the convergence of the TxWF to the TxZF for high SNR in Figure 3.

The influence of the weight ξ' on the performance of the TxWF (cf. Equation 11) for $N_a = 1$ antenna element is studied in Figure 4. Obviously, a wrong weight ξ_0 leads to a degradation of the resulting TxWF. However, the TxZF is always worse than the TxWFs with wrong ξ_0 and the result of the suboptimum TxWF with wrong ξ_0 is equal to the performance of the optimum TxWF, when $\xi_0 = E_s/N_0$ as can be seen in Figure 4. Therefore, it is advantageous to utilize a TxWF with *a priori* fixed ξ_0 instead of a TxZF, if the noise power received by the MSs is not available at the BS.

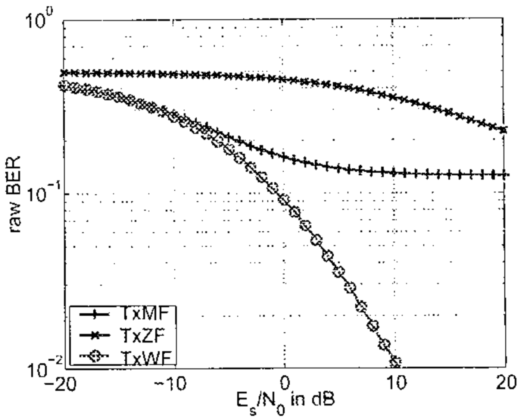

Figure 3: Transmit processing for $N_a = 1$ antenna

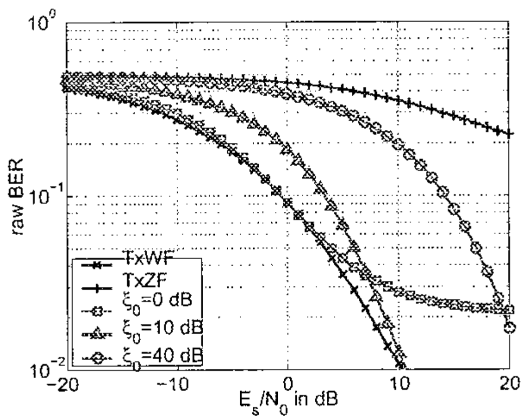

Figure 4: Wrong scalar ξ_0 for TxWF

Figure 5 shows the raw BER for a system with $N_a = 2$ antenna elements at the BS, whose signal processing is performed at the BS, i. e. transmit filters are applied in the downlink and receive filters are used in the uplink. We can see that the transmit filters lead to very similar results as the respective receive filters. Again, we observe that the TxWF converges to the TxMF for low SNR. Moreover, Figure 5 shows the convergence of the TxWF to the TxZF for high SNR. As a system with $N_a = 2$ antenna elements has many degrees of freedom, the TxZF is very close to the TxWF.

VIII. Conclusions

We have presented the TxWF for DS-CDMA systems which shows the same dependence upon the SNR as the

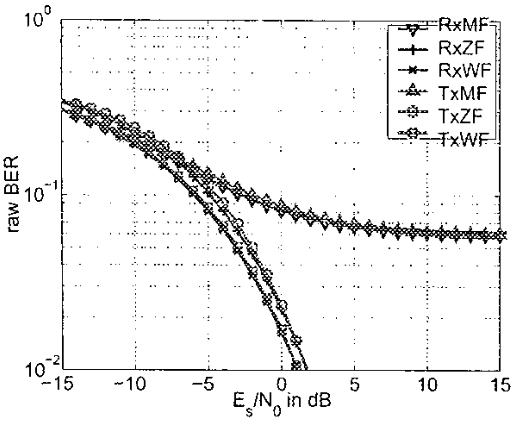

Figure 5: Receive and transmit processing for $N_a = 2$ antenna elements

RxWF. Simulation results revealed the advantages of the TxWF compared to the other transmit filters. We observed that a suboptimum TxWF which does not need feedback from the MSs is more reliable than the TxZF. Moreover, transmit processing leads to similar results as receive processing.

IX. Appendix

The RxZF minimizes the MSE as the RxWF, but also fulfills a zero-forcing constraint [16]. Accordingly, the TxZF minimizes the modified MSE with a transmit power constraint like the TxWF, but also meets a zero-forcing constraint [14]:

$$[\boldsymbol{P}_{ZF}, \beta_{ZF}] = \arg\min_{\boldsymbol{P},\beta} \varepsilon(\boldsymbol{P},\beta) \quad (14)$$

$$\text{s.t.: } \boldsymbol{D}^H \boldsymbol{H} \boldsymbol{P} = \beta \mathbf{1}_{M_{tot}} \quad \text{and} \quad \mathrm{E}[\|\boldsymbol{P}\boldsymbol{s}\|_2^2] = E_{tr}.$$

To see that the optimization in Equation (7) is equivalent to the above optimization we plug the first constraint into the definition of the modified MSE in Equation (9) and utilize Equation (3):

$$\varepsilon_{ZF}(\boldsymbol{P},\beta) = \mathrm{E}\left[\left\|\beta^{-1}\boldsymbol{D}^H\boldsymbol{\eta}\right\|_2^2\right].$$

Since the scalar $\mathrm{tr}(\boldsymbol{D}^H \boldsymbol{R}_\eta \boldsymbol{D})$ is independent from $\boldsymbol{P}$ and β, minimizing $\varepsilon_{ZF}(\boldsymbol{P},\beta)$ is equivalent to minimizing β^{-2} as in Equation (7).

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