COVARIANCE BASED LINEAR PRECODING FOR FULL RANK CHANNELS

Benno Zerlin, Michael Joham, Wolfgang Utschick, Josef A. Nossek

Institute for Circuit Theory and Signal Processing
Arcisstrasse 21, Munich University of Technology, 80290 Munich, Germany
Email: {zerlin,joham,utschick,nossek}@nws.ei.tum.de

ABSTRACT

We make two contributions to the field of FDD transmit processing for multi-user MISO systems: first the concept of covariance based linear precoding is extended to spatio-temporal channels with full rank covariance matrices. Within, the variance true approximation of all signal components allows the formulation of the receive signal exclusively basing upon covariance knowledge and thus the derivation of optimal linear precoding solutions. Second, the derived chip level signal model demonstrates, that the spatial transmit processing on the pilot channel has pivotal influence on the performance of precoding approaches for data channels in general. Through the consideration of the receive filter in the mobile devices, the effects of transmitter side processing on pilot channels can be included into the optimization of precoders for data channels yielding drastic performance enhancements with respect to state of the art techniques.

1. INTRODUCTION

For broadcast channels, see e.g. [1], receive processing has proven clearly suboptimum due to the lack of cooperation among the receive terminals. In the case of cooperative transmitters, precoding can be employed to jointly transform the different users' data signals prior to transmission. This way, the channels act as equalizers of the precoding filters, rendering complex receiver structures dispensable. In complete analogy to spatio-temporal receive processing, linear precoding methods for multi-user MISO systems can be classified into three categories: Approaches that aim at the maximization of the desired signal power at the receiver, in the sequel called *transmit matched filters* (TxMF) as introduced in e.g. [2] or [3], interference suppression schemes called *transmit zero forcing* techniques (TxZF) [4, 5, 6] and the *transmit Wiener filter* (TxWF) [7, 8, 9], that trades the objectives of TxMF and TxZF in an *mean square error* (MSE) optimal sense. Note, that the following investigations will spare nonlinear precoding techniques such as *Tomlinson Harashima precoding*, *lattice basis reduction* or *vector precoding*.

A general precondition of the above mentioned techniques is the full *channel state information* (CSI) at the transmitter. This assumption is usually non-critical in TDD systems, where downlink CSI can be obtained from uplink measurements, i.e. via the pilot information transmitted by the *mobile station* (MS). If up- and downlink are separated in the frequency domain though (FDD), the corresponding channels no longer share the favorable property of full reciprocity with respect to the complex fading coefficients. Thus full transmitter side CSI only can be obtained through the feedback of the complete channel information from the mobile units which in the regarded standards (e.g. HSDPA [10]) is not considered. The design of linear precoding filters therefore must be based on partial CSI. In opposite to the complex channel coefficients, a set of longterm parameters including the angle of departure of all paths, the path delays, and the average path gains are almost independent of the carrier frequency [11]. Thus these parameters are reciprocal even in FDD systems and can be obtained from uplink measurements. While the path delays can be adopted directly, the path covariance matrices that are characterizing the spatial channels and the mean path powers can be obtained through a conversion of uplink measurements [12]-[17]. This partial reciprocity of FDD channels can serve as a basis for the design of linear precoding techniques.

Thus we investigate a system with partial CSI at the transmitter and *maximum ratio combining* (MRC) rake receivers at the mobile units, which adapt to the channel via measurements on a pilot channel. The linear precoding thus has to adapt to the resulting combination of channel and receive filter. For such a system [18] introduced a TxZF and [19] investigated the design of GSM filters in the frequency domain under comparable circumstances. The following sections extend the theory introduced in [20] or [21] to channels with full rank covariance matrices and gives solutions for the TxMF, the TxZF and the TxWF. To this end Section 2 derives an analytical expression for the chip level representation of the rake filter output signal of the system depicted in Fig. 1. Introducing variance true approximations for all signal components Section 3 prepares the derivation of the mentioned linear filters for the system with partial CSI at the transmitter in Section 4 where the different linear filters are derived through the closed analytical optimization of the corresponding objective functions. As the rake receiving mobile station will adapt its filter coefficients to the performed channel estimation, the precoding solution through the consideration of the receive filter will inherently depend upon the spatial processing of the pilot channel. Therefore, Section 5 investigates three prominent examples of pilot transmission schemes in multi-user MISO systems with special focus on the *common pilot channels* (CPICH) introduced in [22]. Finally evaluations demonstrate the significant performance enhancement obtained by full rank processing and provides comparisons of different spatial pilot channels.

2. SIGNAL MODEL

With χ, L, Q, F denoting the spreading factor, the orders of the precoding filters, the channels, and the rake receivers respectively, this section establishes a signal model for $\hat{s}_k[n]$ in the K user CDMA system with N_a transmit antennas as

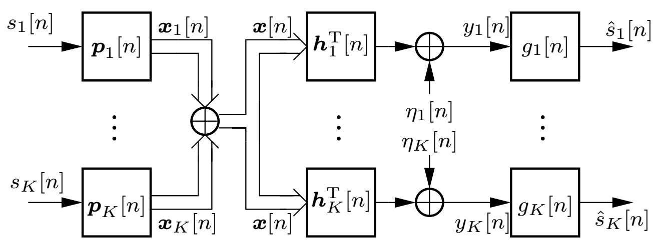

Fig. 1. Block diagram of a MU MISO system

sketched in Fig. 1. With $\boldsymbol{p}_k[n] = \sum_{l=0}^{L} \boldsymbol{p}_{k,l}\delta[n-l] \in \mathbb{C}^{N_\mathrm{a}}$ describing the vector valued impulse response of the precoding filter of user k, the transmit signal $\boldsymbol{x}[n]$ results as:

$$\boldsymbol{x}[n] = \sum_{i=1}^{K}\sum_{l=0}^{L} \boldsymbol{p}_{i,l} s_i[n-l]. \tag{1}$$

Without loss of generality, let all users face $Q+1$ temporal paths, each characterized by a complex vector valued channel coefficient $\boldsymbol{h}_{k,q}$, whose components are complex Gaussian distributed. The extremely large number of phenomena influencing $\boldsymbol{h}_{k,q}$ motivates the stochastic modeling of the channel coefficients. The eigenbasis of the covariance matrix $\boldsymbol{R}_{k,q}$ of the qth channel path of the user k is given as:

$$\boldsymbol{R}_{k,q} = \mathrm{E}\Big[\boldsymbol{h}_{k,q}\boldsymbol{h}_{k,q}^\mathrm{H}\Big] = \sum_{\zeta=1}^{N_\mathrm{a}} \lambda_{k,q,\zeta}\boldsymbol{u}_{k,q,\zeta}\boldsymbol{u}_{k,q,\zeta}^\mathrm{H}. \tag{2}$$

With this eigenbasis, the channel vectors $\boldsymbol{h}_{k,q,}$ can be written as linear combinations of the eigenvectors $\boldsymbol{u}_{k,q,\zeta}$ introducing the complex scalar fast fading coefficients $\rho_{k,q,\zeta}$:

$$\boldsymbol{h}_k[n] = \sum_{q=0}^{Q}\sum_{\zeta=1}^{N_\mathrm{a}} \rho_{k,q,\zeta}\boldsymbol{u}_{k,q,\zeta}\delta[n-q] \quad \in \mathbb{C}^{N_\mathrm{a}}. \tag{3}$$

Note, that the variance of the random variable $\rho_{k,q,\zeta}$ is given by Eq. (2) as $\sigma^2_{k,q,\zeta} = \mathrm{E}\big[|\rho_{k,q,\zeta}||^2\big] = \lambda_{k,q,\zeta}$ which later will allow the variance true approximation in Section 3. In combination with the additive white Gaussian noise $\eta_k[n] \sim \mathcal{N}_\mathbb{C}(0,\sigma_\eta^2)$ the signal at the receiver input k reads as (cf. Fig. 1:

$$\begin{aligned} y_k[n] &= \sum_{q=0}^{Q}\sum_{\zeta=1}^{N_\mathrm{a}} \rho_{k,q,\zeta}\boldsymbol{u}_{k,q,\zeta}^\mathrm{T}\boldsymbol{x}[n-q] + \eta_k[n], \qquad (4)\\ &= \sum_{q=0}^{Q}\sum_{\zeta=1}^{N_\mathrm{a}} \rho_{k,q,\zeta}\boldsymbol{u}_{k,q,\zeta}^\mathrm{T} \sum_{i=1}^{K}\sum_{l=0}^{L}\boldsymbol{p}_{i,l}s_i[n-l-q] + \eta_k[n]. \end{aligned}$$

The receiver structure consists of two stages. Before the correlation with the spreading code $c_k[n]$, that can be modeled by a convolution with

$$g_k^{(c)}[n] = \sum_{j=0}^{\chi-1} c_k^*[j]\delta[n+j], \tag{5}$$

the mobile station performs an MRC receiving, i.e. a matched filtering on the effective scalar pilot channel. Assuming pure spatial processing with $\boldsymbol{b}_k$ of the pilot channel, the coefficients of this scalar channel substitute always can be described as $\sum_{\xi=1}^{N_\mathrm{a}} \nu_{k,f,\xi}\rho_{k,q,\xi}$ for $q = 0,\ldots,Q$, where $\nu_{k,f,\xi} = \boldsymbol{b}_k^\mathrm{T}\boldsymbol{u}_{k,q,\xi}$ is the inner product of the channel eigencomponent and the vector $\boldsymbol{b}_k$ used for pilot beamforming. The impulse response of the kth user's rake receiver therefore reads as:

$$g_k^{(\rho)}[n] = \sum_{f=0}^{F}\sum_{\xi=1}^{N_\mathrm{a}} \nu_{k,f,\xi}^*\rho_{k,f,\xi}^*\delta[n+f].$$

The overall signal model thus inherently depends on the beamforming of the pilot channels as Eq. (6) demonstrates:

$$\begin{aligned} \hat{s}_k[n] = &\sum_{j=0}^{\chi-1} c_k^*[j]\sum_{f=0}^{F}\sum_{\xi=1}^{N_\mathrm{a}} \nu_{k,f,\xi}^*\rho_{k,f,\xi}^* \sum_{q=0}^{Q}\sum_{\zeta=1}^{N_\mathrm{a}}\rho_{k,q,\zeta}\boldsymbol{u}_{k,q,\zeta}^\mathrm{T}\\ &\sum_{i=1}^{K}\sum_{l=0}^{L}\boldsymbol{p}_{i,l}s_i[n-l-q+f+j]\\ &+\sum_{j=0}^{\chi-1} c_k^*[j]\sum_{f=0}^{F}\sum_{\xi=1}^{N_\mathrm{a}} \nu_{k,f,\xi}^*\rho_{k,f,\xi}^*\eta_k[n+f+j]. \end{aligned} \tag{6}$$

3. COVARIANCE BASED DESIGN MODEL

Enabling the derivation of optimum precoding solutions via the optimization of objective functions like the mean squared error, this section formulates a signal model that describes the decision signal and its components independent of the fading coefficients $\rho_{k,q,\zeta}$ in contrast to Eq. (6). Recalling from Eqs. (2) and (3) that the channel covariance matrix provides knowledge about the path powers $\sigma^2_{k,q,\zeta}$, the objective is to formulate variance true approximations for the noise free signal components $\hat{s}_{k,q,\zeta,f}$, i.e. the signal that traveled over the ζth eigencomponent of the qth channel path and over the fth rake finger to the user k. With this heuristic of a variance true approximation, the following paragraphs derive a power equivalent signal model independent of the realization of the unknown random variables $\rho_{k,q,\zeta}$.

With the above heuristic, the equivalent longterm signal model originates from the second order moments of all relevant noise free signal components $\hat{s}_{k,q,\zeta,f}[n]$. By rearranging the noise free part of (6) these signal components are obtained as

$$\hat{s}_{k,q,\zeta,f}[n] = \sum_{j=0}^{\chi-1}\sum_{\xi=1}^{N_\mathrm{a}}\sum_{i=1}^{K}\sum_{l=0}^{L} c_k^*[j]\nu^*_{k,f,\xi}\rho^*_{k,f,\xi} \times \rho_{k,q,\zeta}\boldsymbol{u}^{\mathrm{T}}_{k,q,\zeta}\boldsymbol{p}_{i,l}s_i[n-l-q+f+j].$$

Aiming for a variance true but $\rho_{k,q,\zeta}$ independent representation of these components we investigate the expected value of $|\hat{s}_{k,q,\zeta,f}[n]|^2$ with respect to $\rho_{k,q,\zeta}$ that is:

$$\mathrm{E}\left[|\hat{s}_{k,q,\zeta,f}[n]|^2\right] = \mathrm{E}\left[|\rho_{k,q,\zeta}|^2\left|\sum_{\xi=1}^{N_\mathrm{a}}\nu^*_{k,f,\xi}\rho^*_{k,f,\xi}\right|^2\right] \times \left|\sum_{j=0}^{\chi-1}\sum_{i=1}^{K}\sum_{l=0}^{L} c_k^*[j]\boldsymbol{u}^{\mathrm{T}}_{k,q,\zeta}\boldsymbol{p}_{i,l}s_i[n-l-q+f+j]\right|^2.$$

As the second term in the above equation already is independent of $\rho_{k,q,\zeta}$, the derivation of covariance based design model can focus on the first term which can be written as:

$$\mathrm{E}\left[|\rho_{k,q,\zeta}|^2\left|\sum_{\xi=1}^{N_\mathrm{a}}\nu_{k,f,\xi}\rho^*_{k,f,\xi}\right|^2\right] = \\ = \begin{cases} 2|\nu_{k,q,\zeta}|^2\sigma^4_{k,q,\zeta} + \sum\limits_{\substack{\xi=1\\ \xi\neq\zeta}}^{N_\mathrm{a}} \sigma^2_{k,q,\zeta}|\nu_{k,f,\xi}|^2\sigma^2_{k,f,\xi} & q=f, \\ \sum_{\xi=1}^{N_\mathrm{a}} \sigma^2_{k,q,\zeta}|\nu_{k,f,\xi}|^2\sigma^2_{k,f,\xi} & \text{else.} \end{cases}$$

Note, that ρ_{k,q,ξ_1} and ρ_{k,f,ξ_2} for $q \neq f$ or $\xi_1 \neq \xi_2$ are stochastically independent and that the fourth order moment of a complex Gaussian variable is given by $\mathrm{E}\left[|\rho_{k,q,\zeta}|^4\right] = 2\sigma^4_{k,q,\zeta}$. In order to simplify the upcoming notation, we introduce the variable

$$\kappa = \begin{cases} 2|\nu_{k,q,\xi}|^2 & \text{for } q=f \wedge \xi=\zeta, \\ |\nu_{k,q,\xi}|^2 & \text{else}. \end{cases}.$$

Within we leave the indices q, f, ζ, ξ of κ to the surrounding context. With this substitution the above expectation reads:

$$\mathrm{E}\left[|\rho_{k,q,\zeta}|^2\left|\sum_{\xi=1}^{N_\mathrm{a}}\nu_{k,f,\xi}\rho^*_{k,f,\xi}\right|^2\right] = \sum_{\xi=1}^{N_\mathrm{a}}\kappa\sigma^2_{k,q,\zeta}\sigma^2_{k,f,\xi},$$

which allows to formulate the power of all relevant components based on covariance knowledge and thus is independent of $\rho_{k,q,\zeta}$. With this derivation we can write a longterm signal model for the noise free component $\tilde{s}_{k,q,\zeta,f}[n]$, that traveled over the ζth eigenspace of the qth channel path and the fth rake receiver coefficient as:

$$\tilde{s}_{k,q,\zeta,f}[n] = \sum_{j=0}^{\chi-1} c_k^*[j]\frac{\nu^*_{k,q,\zeta}}{|\nu_{k,q,\zeta}|}\sqrt{\sum_{\xi=1}^{N_\mathrm{a}}\kappa\sigma^2_{k,q,\zeta}\sigma^2_{k,f,\xi}}\,\boldsymbol{u}^{\mathrm{T}}_{k,q,\zeta} \times \sum_{i=1}^{K}\sum_{l=0}^{L}\boldsymbol{p}_{i,l}s_i[n-l-q+f+j].$$

Comparing the variances of this longterm model with the expression derived above easily proves the variance truth of the proposed model, i.e. both signal components have equal power. Preparing the derivation of linear precoding solutions in the upcoming section, the stacking of all relevant samples of $s_i[n]$ allows [23, 24] to write the linear model in vector matrix notation as:

$$\tilde{\mathsf{s}}_{k,q,\zeta,f}[n] = \sum_{i=1}^{K}\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{X}_{k,q,\zeta,f}\boldsymbol{s}_i[n] + \tilde{\eta}_{k,f}. \qquad (7)$$

Within, $\boldsymbol{p}_i$ is obtained by stacking the $L+1$ filter vectors $\boldsymbol{p}_{i,l}$. Note, that the matrix $\boldsymbol{X}_{k,q,\zeta,f}$ depends on longterm parameters only and thus is known to the transmitter even in FDD systems.

4. FULL RANK COVARIANCE BASED LINEAR PRECODING

With the above formulated longterm signal model and the optimization driven derivation of linear precoders [9], means are available to express the three linear precoding filters TxMF, TxZF, and TxWF based upon covariance knowledge. To this end the following paragraphs derive the corresponding optimization problems and their solutions. Within, the optimization techniques developed for linear precoding with full CSI [9] can be adopted directly.

4.1. Transmit Matched Filter (TxMF)

Aiming at the maximization of the desired signal power at the rake output, the TxMF is defined as the solution to the following optimization problem:

$$\{\boldsymbol{p}_{\mathrm{MF},1},\ldots,\boldsymbol{p}_{\mathrm{MF},K}\} = \underset{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K}{\operatorname{argmax}} \sum_{k=1}^{K} \mathrm{Re}\left\{\mathrm{E}\left[\boldsymbol{w}_k^{[m],\mathrm{H}}\tilde{\boldsymbol{s}}_k[\chi m]\right]\right\}$$
$$\text{s.t.} \quad \sum_{k=1}^{K}\sigma_s^2\|\boldsymbol{p}_k\|_2^2 = E_{\mathrm{tr}}, \tag{8}$$

where we define the power of the desired signal component as the cross correlation of the signal components at the receiver output and the desired signal component $w_{k,q,\zeta,f}^{[m]}$ at symbol number m:

$$w_{k,q,\zeta,f}^{[m]} \begin{cases} \pi_{k,q,\zeta}s_k[\chi m] & q=f \\ 0 & q\neq f, \end{cases} \tag{9}$$

The more convenient notation in (8) can be obtained when defining the vectors $\boldsymbol{w}_k^{[m]}$ and $\tilde{\boldsymbol{s}}_k[\chi m]$ by stacking $w_{k,q,\zeta,f}^{[m]}$ and $\tilde{s}_{k,q,\zeta,f}[\chi m]$ for $q,\zeta,f = \{0,1,0\},\ldots,\{Q,N_{\mathrm{a}},F\}$. Note, how the factors $\pi_{k,q,\zeta} = \sqrt{\sum_{\xi=1}^{N_{\mathrm{a}}}\kappa\sigma_{k,q,\zeta}^2\sigma_{k,q,\xi}^2}$ include the influence of the pilot beamforming through κ. Moreover with $M = 2\lfloor\frac{L+Q+F+\chi-1}{\chi}\rfloor + 1$ let $\boldsymbol{e}_\mu$ be the $\frac{M+1}{2}$th column of $\mathbf{1}_M$, i.e. the vector that selects the chip of interest from the impulse response of the complete system of precoder, channel, rake, and code correlation. With these notational conventions and the expression for $\tilde{s}_{k,q,\zeta,f}[\chi m]$ in (7) the Lagrangian function can be written as:

$$L(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_k,\lambda) = \sum_{k=1}^{K}\mathrm{Re}\left(\sigma_s^2\boldsymbol{p}_k^{\mathrm{T}}\sum_{q=0}^{Q}\sum_{\zeta=1}^{N_{\mathrm{a}}}\pi_{k,q,\zeta}\boldsymbol{X}_{k,q,\zeta,q}\boldsymbol{e}_\mu\right) - \lambda\left(\sum_{k=1}^{K}\sigma_s^2\boldsymbol{p}_k^{\mathrm{H}}\boldsymbol{p}_k - E_{\mathrm{tr}}\right).$$

The solution to the optimization therefrom directly results through complex valued derivatives with respect to $\boldsymbol{p}_1,\ldots,\boldsymbol{p}_k$ and λ. Solving the equation system resulting from setting the obtained derivatives to zero yields:

$$\boldsymbol{p}_{\mathrm{MF},k} = \beta_{\mathrm{MF}}\sum_{q=0}^{Q}\sum_{\xi=1}^{N_{\mathrm{a}}}\pi_{k,q,\zeta}\boldsymbol{X}_{k,q,\zeta,q}^{*}\boldsymbol{e}_\mu, \tag{10}$$
$$\beta_{\mathrm{MF}} = \left(\frac{E_{\mathrm{tr}}}{\sum_{l=1}^{K}\sigma_s^2\boldsymbol{e}_\mu^{\mathrm{T}}\sum_{i=0}^{F}\sum_{\xi=1}^{N_{\mathrm{a}}}\pi_{k,i,\xi}\boldsymbol{X}_{k,i,\xi,i}^{\mathrm{T}}\cdots\sum_{j=0}^{F}\sum_{\zeta=1}^{N_{\mathrm{a}}}\pi_{k,q,\zeta}\boldsymbol{X}_{k,j,\zeta,j}^{*}\boldsymbol{e}_\mu}\right)^{\frac{1}{2}}$$

Note, that the optimization in (8) inherently results in a spatial filter, i.e. the covariance based TxMF does not employ temporal filter components. As the MRC receive filter already maximizes the desired signal power over the temporal domain, the precoding scheme can contribute to this goal only through spatial processing. Covariance based TxMF thus is beamforming.

4.2. Transmit Zero Forcing Filter (TxZF)

Aiming at the complete suppression of interference, the TxZF precoder results from a constrained optimization problem, where signal components from other users (*multiple access interference*, MAI) as well as signal components caused by other symbols (*inter symbol interference*, ISI) are demanded to be zero. The remaining degrees of freedom are employed to minimize the attenuation β^{-2}:

$$\{\boldsymbol{p}_{\mathrm{ZF},1},\ldots,\boldsymbol{p}_{\mathrm{ZF},K}\} = \underset{\{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K\}}{\operatorname{argmin}}\ \beta^{-2} \tag{11}$$
$$\text{s.t.:}\quad \boldsymbol{p}_k^{\mathrm{T}}\boldsymbol{X}_{k,q,\zeta,q} = \beta\pi_{k,q,\zeta}\boldsymbol{e}_\mu^{\mathrm{T}},$$
$$\boldsymbol{p}_k^{\mathrm{T}}\boldsymbol{X}_{i,q,\zeta,f} = \mathbf{0}^{\mathrm{T}} \text{ for } q\neq f \text{ or } i\neq k$$
$$\text{and}\quad \sum_{k=1}^{K}\sigma_s^2\|\boldsymbol{p}_k\|_2^2 = E_{\mathrm{tr}}.$$

For a more convenient representation of the following optimization, the first two constraints can be written in vector matrix notation as:

$$\boldsymbol{p}_k^{\mathrm{T}}\boldsymbol{X} = \beta\boldsymbol{b}_k^{\mathrm{T}},$$

where the vector $\boldsymbol{b}_k^{\mathrm{T}}$ and the matrix $\boldsymbol{X}$ are obtained by stacking

$$b_{i,q,\zeta,f} = \begin{cases} \pi_{i,q,\zeta} & \text{for } i=k \text{ and } q=f \\ 0 & \text{else,} \end{cases}$$

and $\boldsymbol{X}_{k,q,\zeta,f}$ horizontally, so that the right hand side yields the desired values of the corresponding left hand side coefficients. Note, that the order within is not critical, as long as the stacking is consistent among $\boldsymbol{b}_k^{\mathrm{T}}$ and $\boldsymbol{X}$. For a specific stacking order we refer to [24, 21] where the TxZF was derived for rank-1 channels. Including the so transformed constraint into the objective function via Lagrangian multipliers yields:

$$L(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_k,\beta,\boldsymbol{\lambda}_1,\ldots,\boldsymbol{\lambda}_K,\lambda_0) = \beta^{-2} - \mathrm{Re}\left(\sum_{k=1}^{K}\left(\boldsymbol{p}_k^{\mathrm{T}}\boldsymbol{X}-\beta\boldsymbol{b}_k^{\mathrm{T}}\right)\boldsymbol{\lambda}_k\right) - \lambda_0\left(\sum_{k=1}^{K}\sigma_s^2\boldsymbol{p}_k^{\mathrm{H}}\boldsymbol{p}_k - E_{\mathrm{tr}}\right),$$

whose derivatives must vanish. The TxZF precoder thus finally results as:

$$\boldsymbol{p}_{\mathrm{ZF},k} = \boldsymbol{X}^{\dagger,\mathrm{T}}\boldsymbol{b}_k, \tag{12}$$

$$\beta_{\mathrm{ZF}} = \sqrt{\frac{E_{\mathrm{tr}}}{\sum_{k=1}^{K}\sigma_s^2 \boldsymbol{b}_k^{\mathrm{T}}\boldsymbol{X}^{\dagger,*}\boldsymbol{X}^{\dagger,\mathrm{T}}\boldsymbol{b}_k}} \tag{13}$$

where $(\cdot)^{\dagger}$ denotes the Moore-Penrose pseudo inverse. Through the inversion of the Gramian of $\boldsymbol{X}$ within the pseudo inverse expression, ill conditioned scenarios might cause extremely large denominators and thus very small values for β_{ZF}. The signal thus is received free of interference but with extremely low power making it very sensitive towards noise adulterations in some settings. Nevertheless, even partial CSI at the transmitter allows for interference suppression as the plots in section 6 confirm.

4.3. Transmit Wiener Filter (TxWF)

With the formulation of the *mean squared error* (MSE), the TxWF optimizes a global cost function, that inherently allows to trade the heuristics of noise and interference suppression optimally. Thus the optimization (14) minimizes the modified mean squared error through:

$$\begin{aligned}&\{\boldsymbol{p}_{\mathrm{WF},1},\ldots,\boldsymbol{p}_{\mathrm{WF},K}\} = \\ &\qquad \underset{\{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K\}}{\operatorname{argmin}} \sum_{k=1}^{K} \mathrm{E}\left[\left\|\boldsymbol{w}_k^{[m]} - \beta^{-1}\tilde{\boldsymbol{s}}_k[\chi m]\right\|_2^2\right] \\ &\text{s.t.: } \sum_{k=1}^{K}\sigma_s^2\|\boldsymbol{p}_k\|_2^2 = E_{\mathrm{tr}},\end{aligned} \tag{14}$$

where the scalar β allows for a scaled version of received signals, paying tribute to the transmit power constraint, linear precoding solutions face. With the covariance based signal model from (7) the MSE

$$\varepsilon(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K,\beta) = \mathrm{E}\left[\left\|\boldsymbol{w}_k^{[m]} - \beta^{-1}\tilde{\boldsymbol{s}}_k[\chi m]\right\|_2^2\right], \tag{15}$$

can be expressed in terms of $\sigma_{k,q,\zeta}$ instead of $\rho_{k,q,\zeta}$ and thus be included in the Lagrangian objective function:

$$\begin{aligned}&L(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K,\beta) = \\ &\quad \varepsilon(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K,\beta) - \lambda\left(\sum_{k=1}^{K}\sigma_{s_k}^2\boldsymbol{p}_k^{\mathrm{T}}\boldsymbol{p}_k^* - E_{\mathrm{tr}}\right),\end{aligned} \tag{16}$$

from which the *Karush-Kuhn-Tucker* (KKT) conditions can be computed. After a rather tedious derivation the TxWF solution can be obtained:

$$\boldsymbol{p}_{\mathrm{WF},k} = \beta_{\mathrm{WF},k}\left(\boldsymbol{X}^*\boldsymbol{X}^{\mathrm{T}} + \gamma\frac{\sigma_\eta^2}{E_{\mathrm{tr}}}\mathbf{1}\right)^{-1}\boldsymbol{X}^*\boldsymbol{b}_k, \tag{17}$$

$$\beta_{\mathrm{WF},k} = \sqrt{\frac{E_{\mathrm{tr}}}{\sum_{k=1}^{K}\sigma_s^2\boldsymbol{b}_k^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}}\left(\boldsymbol{X}^*\boldsymbol{X}^{\mathrm{T}} + \gamma\frac{\sigma_\eta^2}{E_{\mathrm{tr}}}\mathbf{1}\right)^{-2}\boldsymbol{X}^*\boldsymbol{b}_k}}.$$

Within it has proven convenient to introduce the factor:

$$\gamma = \sum_{k=1}^{K}(Q+1)\sum_{f=0}^{F}\sum_{\xi=1}^{N_{\mathrm{a}}}|\nu_{k,f,\xi}|^2\sigma_{k,f,\xi}^2,$$

that weights the influence of noise versus interference suppression for the MSE optimal trade-off mentioned above.

5. INFLUENCE OF THE USED PILOT CHANNELS

Given the projection of the channel impulse response onto the span of the beamforming vector $\boldsymbol{b}_k$ used on the pilot channel, the rake receivers adapt their coefficients to match the resulting scalar channel substitute

$$\begin{aligned}\boldsymbol{b}_k^{\mathrm{T}}\boldsymbol{h}_k[n] &= \sum_{q=0}^{Q}\sum_{\zeta=1}^{N_{\mathrm{a}}}\rho_{k,q,\zeta}\boldsymbol{b}_k^{\mathrm{T}}\boldsymbol{u}_{k,q,\zeta}\delta[n-q] \\ &= \sum_{q=0}^{Q}\sum_{\zeta=1}^{N_{\mathrm{a}}}\rho_{k,q,\zeta}\nu_{k,q,\zeta}\delta[n-q].\end{aligned}$$

As both components of $\nu_{k,q,\zeta}$, i.e. the pilot beamforming vector $\boldsymbol{b}_k$ and the channel eigencomponent $\boldsymbol{u}_{k,q,\zeta}$ are known to the BS, the factors $\nu_{k,q,\zeta}$ can be included in the adaptation of the linear precoders. On the background of *high speed downlink packet access* (HSDPA) the following paragraphs derive the resulting rake adulterations $\nu_{k,q,\zeta}$ for three specific spatial pilot channels:

1) The primary CPICH [22] transmits a single pilot sequence from one reference antenna only. Without loss of generality we define the first array element as reference and obtain the corresponding beamforming vector $\boldsymbol{b}_k = \boldsymbol{b} = [1,0,\ldots,0]^{\mathrm{T}} \in \{0,1\}^{N_{\mathrm{a}}}$ and the factors $\nu_{k,q,\zeta}$ as the first element of $\boldsymbol{u}_{k,q,\zeta}$:

$$\nu_{k,q,\zeta} = [\boldsymbol{u}_{k,q,\zeta}]_{(1)}\,. \tag{18}$$

2) The secondary CPICH [22] employs the attainable array gain to form N_{b} different CPICHs. The N_{b} pilot sequences thus are transmitted via a grid of N_{b} equidistant beams with the weights $\boldsymbol{b}^{(i)} = [1,\exp(-\mathrm{j}\mu_i),\ldots,\exp(-\mathrm{j}(N_{\mathrm{a}}-1)\mu_i)]^{\mathrm{T}}, i = 1,\ldots,N_{\mathrm{b}}$, from which each user picks the adequate i^* resulting in

$$\nu_{k,q,\zeta} = \boldsymbol{b}^{(i^*),\mathrm{T}}\boldsymbol{u}_{k,q,\zeta}. \tag{19}$$

Note, that the beamforming gain here compensates for the loss due to splitting the pilot power to multiple pilot channels. The estimation quality thus remains comparable.

3) Reference to the above schemes shall be the channel estimation on the dominant eigencomponent of the channel, i.e. $\boldsymbol{b}_k = \boldsymbol{u}_{k,q,1}^*$, which due to the orthonormal nature of the eigenbasis yields:

$$\nu_{k,q,\zeta} = \begin{cases} 1 & \text{for } \zeta = 1, \\ 0 & \text{else.}\end{cases} \tag{20}$$

6. EVALUATIONS

This section analyzes the presented linear precoding techniques with respect to the mean uncoded *bit error ratio* (BER) in an HSDPA-like setting with spreading factor $\chi = 16$ and 4 present users. The path powers $\sigma^2_{k,q,\zeta}$ are assumed to be constant with k and exponentially decreasing with 3 dB in q. Modeling the spatial setting by an angular spread around the direction of departure with Laplacian distribution of $2°$ standard deviation defines the relation of $\sigma^2_{k,q,\zeta}$ in the different eigensubspaces of $\boldsymbol{R}_{k,q,\cdot}$. Table 1 gives an overview of the numerical setting. Note, that the fixed spatial setting within is set to a S-CPICH worst case, demonstrating the ability of the introduced schemes to include the pilot beamforming even in scenarios where it differs significantly from the optimal spatio-temporal transmit processing. The channel is modeled as block Rayleigh fading and thus remains constant for the length of one slot, i.e. 2560 chips.

Parameter		Fig.3	Fig.2
Tx Filter Order	L	4	0
Channel Order	Q	3	2
Rx Filter Order	F	3	2
Spreading Factor	χ	16	16
User Number	K	4	4
# Tx Antennas	N_a	4	4
# Rx Antennas	—	1	1
DOD	—	$[7.5°, 15°, 22.5°, 30°]$	
Angular Spread	—	$2°$	$2°$

Table 1.

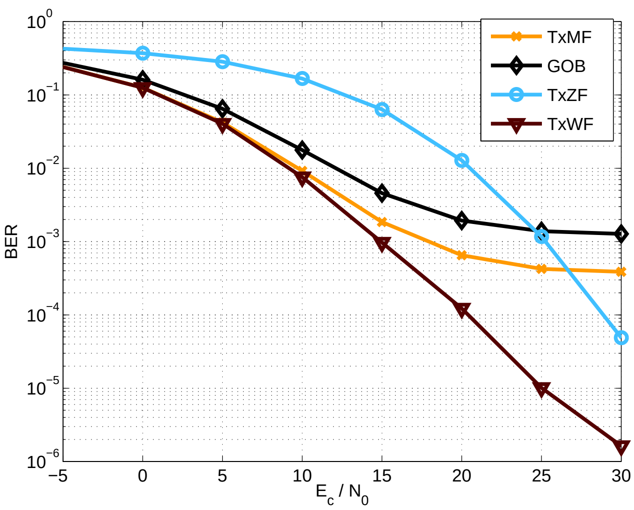

Fig. 2. Comparison of different S-CPICH schemes

Fig. 2 now compares the proposed linear precoding techniques with the state of the art approach of using the S-CPICH *grid of beam* (GOB). The drastic performance enhancements can be traced to three core phenomena: Due to the derived covariance based signal model, linear precoding is no longer restricted to the spatial sampling of the GOB. Thus, the full antenna gain of the array can be recovered, yielding an enhancement of about 3dB in this scenario. Moreover, the formulation of the optimization objectives for full rank covariance matrices enables the different schemes to serve the user through all subspaces resulting in a lower saturation level for the TxMF. Finally, the interference suppression potential of the TxZF and the TxWF completely overcome the saturation as seen compared with the other schemes. Of course, the TxWF outperforms all other linear approaches by inherently optimizing the trade-off between TxMF and TxZF.

A second simulation now evaluates the influence different *pilot channels* (PICH) have on the performance. Note, that all schemes result in comparable SNRs for the channel estimator, for which only the systematic estimation error $\nu_{k,q,\zeta}$ is taken into consideration. With this precondition, Fig. 3 compares three different pilot beamforming schemes:

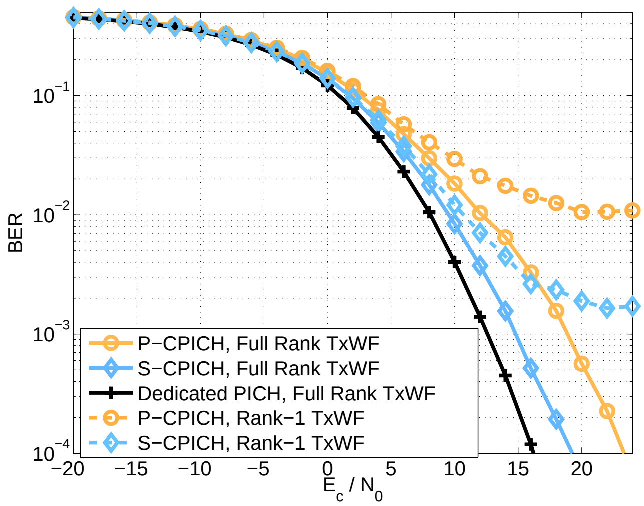

Fig. 3. Comparison of Single and Full Rank Processing

The dedicated PICH that transmits the pilot sequence over the eigenvector corresponding to the largest eigenvalue, the *primary CPICH* (P-CPICH) which radiates a single pilot sequence from one antenna only, and the S-CPICH employing a grid of 4 beams. Note, that the dedicated PICH here only serves as a reference as it does not qualify as a common PICH. Still, full rank covariance based precoding allows to perform very close to this optimum case even with S-CPICH information. The result also underlines the significance of full rank processing.

7. CONCLUSION

With the presented variance true approximation of the signal model, the covariance based optimization of the three linear objectives was enabled. Due to the stringent derivation, the resulting linear precoders inherently adapt to the used pilot channel and allows optimal spatio-temporal transmit processing even in scenarios, where the transmitter has only partial and the receiver has only scalar knowledge about the channel.

8. REFERENCES

[1] G. Caire and S. Shamai (Shitz), "On the Achievable Throughput of a Multiantenna Gaussian Broadcast Channel," *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1691–1706, July 2003.

[2] R. E. McIntosh and S. E. El-Khamy, "Optimum Pulse Transmission Through a Plasma Medium," *IEEE Transactions on Antennas and Propagation*, vol. AP-18, no. 5, pp. 666–671, September 1970.

[3] R. Esmailzadeh and M. Nakagawa, "Pre-RAKE Diversity Combination for Direct Sequence Spread Spectrum Mobile Communications Systems," in *Proc. ICC 93*, May 1993, vol. 1, pp. 463–467.

[4] V. Weerackody, "Precoding of Signature Sequences for CDMA Systems," U. S. Patent # 5,461,610, October 1995.

[5] B. Vojčić and W. M. Jang, "Transmitter Precoding in Synchronous Multiuser Communications," *IEEE Transactions on Communications*, vol. 46, no. 10, pp. 1346–1355, October 1998.

[6] M. Meurer, P. W. Baier, T. Weber, Y. Lu, and A. Papathanassiou, "Joint transmission: advantageous downlink concept for CDMA mobile radio systems using time division duplexing," *Electronics Letters*, vol. 36, no. 10, pp. 900–901, May 2000.

[7] H. R. Karimi, M. Sandell, and J. Salz, "Comparison between transmitter and receiver array processing to achieve interference nulling and diversity," in *Proc. PIMRC'99*, September 1999, vol. 3, pp. 997–1001.

[8] R. L. Choi and R. D. Murch, "Transmit MMSE Pre-Rake Preprocessing with Simplified Receivers for the Downlink of MISO TDD-CDMA Systems," in *Proc. Globecom 2002*, Nov. 2002, vol. 1, pp. 429–433.

[9] M. Joham, W. Utschick, and J. A. Nossek, "Linear Transmit Processing in MIMO Communications Systems," *Accepted for publication in IEEE Transactions on Signal Processing*, 2004.

[10] Technical Specification Group Radio Access Network, "TS 25.308 - High Speed Downlink Packe Access, Overall Description, Stage 2 (Release 6)," Tech. Rep., 3GPP, Dezember 2004.

[11] L. Bigler, H. P. Lin, S. S. Jeng, and G. Xu, "Experimental Direction of Arrival and Spatial Signature Measurements at 900 MHz for Smart Antenna Systems," in *Proc. VTC'95*, July 1995, vol. 1, pp. 55–58.

[12] T. Asté, P. Forster, L. Féty, and S. Mayrargue, "Downlink Beamforming Avoiding DOA Estimation for Cellular Mobile Communications," in *Proc. ICASSP'98*, May 1998, vol. VI, pp. 3313–3316.

[13] J. Goldberg and J. R. Fonollosa, "Downlink beamforming for cellular mobile communications," in *Proceedings of the 47th Vehicular Technology Conference*. IEEE, May 1997, vol. 2, pp. 632 – 636.

[14] C. Farsakh and J. A. Nossek, "Spatial covraiance based downlink beamforming in an sdma mobile radio system," *IEEE Transactions on Communications*, vol. 46, no. 11, pp. 1497–1506, November 1998.

[15] B. M. Hochwald and T. L. Marzetta, "Adapting a downlink array from uplink measurements," *IEEE Transactions on Signal Processing*, vol. 49, no. 3, pp. 642–653, March 2001.

[16] Y. C. Liang and F. P. S. Chin, "Donwlink channel covariance matrix estimation and its applications in wireless ds-cdma systems," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 2, pp. 222–232, February 2001.

[17] B. K. Chalise, L. Haering, and A. Czylwik, "Robust uplink to downlink spatial covariance matrix transformation for downlink beamforming," in *Proceedings of the International Conference onCommunications*. IEEE, June 2004, vol. 5, pp. 3010–3014.

[18] P. Forster, L. Féty, and M. Le Bot, "Spatio-Temporal Filters for Downlink Processing in FDD Systems," in *Proc. ICASSP 1000*, June 2000, vol. V, pp. 2585–2588.

[19] G. Montalbano and D. T. M. Slock, "Spatio-Temporal Array Processing for FDD/CDMA/SDMA Downlink Transmission," in *Proc. VTC'99 Fall*, September 1999, vol. 3, pp. 1910–1914.

[20] M. Joham, K. Kusume, M. H. Gzara, W. Utschick, and J. A. Nossek, "Transmit Wiener Filter for the Downlink of TDD DS-CDMA Systems," in *Proc. ISSSTA 2002*, September 2002, vol. 1, pp. 9–13.

[21] B. Zerlin, M. Joham, W. Utschick, and J. A. Nossek, "Linear Precoding in W-CDMA Systems based on S-CPICH Channel Estimation," in *Proceedings of the International Symposium on Signal Processing and Information Theory (ISSPIT)*. IEEE, December 2003, Invited Paper.

[22] Technical Specification Group Radio Access Network, "TS 25.211 - Physical Channels and Mapping of Transport Channels onto Physical Channels," Tech. Rep., 3GPP, 2004.

[23] B. Zerlin, M. Joham, W. Utschick, and J. A. Nossek, "Covariance Based Linear Precoding," Submitted to the IEEE Journal of Selected Areas in Communications, 2005.

[24] M. Joham, K. Kusume, W. Utschick, and J. A. Nossek, "Transmit Matched Filter and Transmit Wiener Filter for the Downlink of FDD DS-CDMA Systems," in *Proc. PIMRC 2002*, September 2002, vol. 5, pp. 2312–2316.