Institut für Informatik
der Technischen Universität München
Lehrstuhl für Informatik XVIII

Pricing and Bidding Strategies in Iterative Combinatorial Auctions

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Vollständiger Abdruck der von der Fakultät für Informatik der Technischen Universität München zur Erlangung des akademischen Grades eines

Doktors der Naturwissenschaften (Dr. rer. nat.)

genehmigten Dissertation.

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Die Dissertation wurde am 30.01.2008 bei der Technischen Universität München eingereicht und durch die Fakultät für Informatik am 07.07.2008 angenommen.

For Vita Pikovskaya

Abstract

Auctions have been getting increasing attention in computer science and economics, as they provide an efficient solution to resource allocation problems with self-interested agents. E-Commerce and finance have emerged as some of their largest application fields. The need for new auction mechanisms that allow for complex bids such as bundle bids and multi-attribute bids has been raised in many situations. In addition to strategic problems, the design of these multidimensional auctions exhibits hard computational problems. For example, the winner determination typically leads to NP-hard allocation problems in combinatorial auctions. More recently, researchers have focused on the pricing and information feedback in combinatorial auctions.

Iterative combinatorial auctions (ICAs) are IT-based economic mechanisms in which bidders submit bundle bids iteratively and the auctioneer computes allocations and ask prices in each auction round. Several ICA designs have been proposed in the literature, but very little was known about their behavior in different settings. The multi-item and discrete nature of ICAs and complex auction rules defy much of the traditional game theoretical analysis in this field. The literature provides merely equilibrium analysis of ICAs with non-linear personalized prices under strong assumptions on bidders' strategies. In contrast, ICAs based on linear prices have performed very well in the lab and in the field.

Computational methods and laboratory experiments can be of great help in

exploring potential auction designs and analyzing the virtues of various design options. The goal of our research was to benchmark different existing ICA designs and to propose new, improved auction rules. We focused on *linear-price auctions*, but also included one ICA design with non-linear personalized prices.

In the computational simulations we compared three selected linear price ICA designs and the VCG auction based on the *allocative efficiency*, *revenue distribution*, and *speed of convergence* using different bidding strategies and bidder valuations. We found that ICA designs with linear prices performed very well for different value models even in case of high synergies among the valuations. There were, however, significant differences in the efficiency and revenue distribution of the three ICA designs. Even heuristic bidding strategies in which bidders submit bids for only a few of the best bundles led to high levels of efficiency. We have also identified a number of auction rules for the ask price calculation, bidder activity, and auction termination that have shown to perform very well in the simulations.

In the laboratory experiments we compared the same auction designs and one ICA design with non-linear personalized prices in respect to the same performance measures. We were able to identify several similarities to the computational results, but also quite heterogeneous bidding behavior, which did not correspond to the pure myopic best-response bidding strategy in any of the auction designs. Nevertheless, we achieved high efficiency levels in all auction designs. Furthermore, we identified significant differences in the auctioneer revenue depending on the auction design, but not on the number of the auctioned items (3, 6, and 9). We also observed a very low speed of convergence of the ICA design with non-linear personalized prices, which makes it (at least in its current form without using proxy agents) hardly suitable for practical applications.

Zusammenfassung

Auktionen haben in den letzten Jahren zunehmende Aufmerksamkeit in Informatik und Wirtschaftswissenschaften gewonnen, da sie zur effizienten Allokation von Ressourcen eingesetzt werden können. Zu ihren größten Einsatzgebieten gehören unter anderem die Finanzbranche und E-Commerce. In vielen Fällen wurden dabei neue Auktionsmechanismen nachgefragt, die komplexe Gebote auf mehrere Güter oder unterschiedliche Eigenschaften eines Gutes ermöglichen. Abgesehen von der strategischen Komplexität, wird die Konstruktion solcher Verfahren zusätzlich durch die Komplexität der dort auftretenden Berechnungsprobleme erschwert. Zum Beispiel gehört das Allokationsproblem bei kombinatorischen Auktionen zur Klasse der NP-schweren Probleme. In jüngster Zeit haben sich Wissenschaftler vorwiegend auf der Preissetzung und auf den Arten der als Feedback übermittelten Informationen fokussiert.

Iterative kombinatorische Auktionen (ICAs) sind IT-basierte ökonomische Mechanismen, in denen Bieter die Möglichkeit haben, Gebote auf untrennbare Güterbündel in mehreren Runden iterativ abzugeben. Nach jeder Runde erhalten die Bieter vom Auktionator Informationen zu aktuellen Preisen und/oder der aktuellen Zwischenallokation. In der Literatur wurden mehrere Auktionsverfahren vorgeschlagen, aber ihr Verhalten unter unterschiedlichen Rahmenbedingungen wurde zu wenig untersucht. Wegen ihrer diskreter Struktur und komplexer Bietregeln, wird die spieltheoretische Analyse von ICAs

wesentlich erschwert. In der Literatur findet man lediglich Equilibrium-Analysen von ICAs mit nichtlinearen personalisierten Preisen unter sehr strengen Annahmen über die verfolgten Bietstrategien. Im Gegenteil, ICAs mit linearen anonymen Preisen wurden erfolgreich in Feldstudien und im Labor eingesetzt.

Rechensimulationen und Laborexperimente können bei der Analyse und Vergleich von Auktionsformaten und bei der Untersuchung der Auswirkungen von unterschiedlichen Konfigurationsparametern eine große Hilfe leisten. In unseren Forschungsprojekten wollten wir diverse existierende Auktionsmechanismen vergleichen und neue, verbesserte, Auktionsregeln entwickeln. Dabei haben wir uns auf Auktionen mit linearen anonymen Preisen konzentriert, aber auch ein Auktionsformat mit nicht-linearen personalisierten Preisen untersucht.

In den Rechensimulationen haben wir drei ausgewählte ICA Formate mit linearen anonymen Preisen und die VCG-Auktion verglichen, indem die allokative Effizienz, die Gewinnverteilung und die Konvergenzgeschwindigkeit unter Annahmen von unterschiedlichen Wertemodellen und Bietstrategien gemessen wurden. Dabei haben die untersuchten ICAs mit linearen Preisen bei unterschiedlichen Wertemodellen und sogar mit hohen Synergien zwischen einzelnen Gütern sehr gute Ergebnisse geliefert. Wir haben allerdings signifikante Unterschiede in der Effizienz und Gewinnverteilung zwischen den einzelnen Auktionsformaten festgestellt. Auch bei heuristischen Bietstrategien, bei denen die Bietagente nur auf eine zufällig ausgewählte Untermenge der momentan besten Bündel Gebote abgeben, haben wir hohe Effizienzgrade beobachtet. Wir haben auch mehrere neue Preisberechnungs-, Aktivitäts- und Abschlussregeln entwickelt, mit denen die Auktionsergebnisse deutlich verbessert werden konnten.

In den Laborexperimenten haben wir dieselben Auktionsformate und ein ICA mit nicht-linearen personalisierten Preisen im Bezug auf dieselben Kriterien verglichen. Wir haben viele ähnliche Phänomena wie bei unseren Simulationsergebnissen identifiziert, aber auch ein sehr heterogenes Bietverhalten beobachtet, wobei bei keinem der Auktionsformate eine Analogie zur "myopic best-response" Strategie ersichtlich war. Nichtsdestotrotz haben wir bei allen Auktionsformaten hohe Effizienzgrade erzielt. Außerdem haben wir signifikante Unterschiede in der Gewinnverteilung beobachtet, die vom Auktionsformat, aber nicht von der Güterzahl (3, 9 und 9) beeinflusst wurden. Beim untersuchten ICA mit nicht-linearen personalisierten Preisen war die Konvergenzgeschwindigkeit sehr niedrig, was dieses Auktionsformat (zumindest in der aktuellen Form ohne Proxy-Agenten) für praktische Anwendungen kaum brauchbar macht.

Acknowledgments

This dissertation is a result of many years of work and several important decisions and events, some carefully planned, and others happened purely by chance. On this way, I received help and support from many people, who directly or indirectly contributed to this work.

Above all, I would like to thank my adviser Prof. Dr. Martin Bichler for giving me the chance to do my doctoral research at his Chair of Internet-based Information Systems at the TU München. He was not only always available when I needed his help and input, but also helped in finding a highly interesting research direction and constantly encouraged and guided me through the ups and downs during my work. I could not have imagined a better adviser and mentor for my thesis than him.

A special thanks is also addressed to all who worked in our research project. With my college Pasha Shabalin we were able to extend our research area considerably. The results presented in this dissertation would also not be possible without help of our former students, who invested much time, creativity, and effort into our project. In particular, I wish to thank Tobias Scheffel, Stefan Schneider, Bernd Laqua, Zlatina Savova, Branimir Mirtchev and Frank Schmaus for their great involvement. I am glad that Tobias Scheffel and Stefan Schneider decided to continue working with us as research assistants at our university department.

I address further thanks to my other colleagues, who provided help and good times for me during the last four years. In particular, I wish to thank Dr. Christine Kiss, Thomas Setzer and Oliver Hühn for all of the interesting discussions we had and for their valuable suggestions. I would also like to thank Prof. Dr. Alfons Kemper for co-supervising my dissertation.

As I came from Ukraine in 1995 and continued my studies of Financial and Economic Mathematics at the TU München in Germany, I had difficult times learning German and adapting to a quite different education system. I highly appreciate the kind and support I received from Dr. Christian Kredler and Maria Präßl at that time. Dr. Christian Kredler was also the one, who recommended me to Prof. Dr. Martin Bichler, and so predetermined my way.

Last but most important, I want to thank my whole family, especially my wife, who fully supported my decision and tolerated my time consuming work throughout all those years, to my three years old daughter, who made my work a little bit longer but my life much more meaningful, and to my parents, who enabled and encouraged me to go this way.

I dedicate this dissertation to Vita Pikovskaya, my grandmother, who died two years ago and could not see me finishing this work. She was a great person, a loved grandmother, my closest confidant and adviser for many years. She always wanted the best for me, and, in particular, encouraged me for this work. She was a professional journalist and a wise person, but, unfortunately, she has written no book by herself. Though I can not fill this gap with my highly technical dissertation, I dedicate it to her in sign of love and memory.

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Chapter 1

Introduction

Auctions have been found to be efficient economic mechanisms for resource allocation in distributed environments with self-interested agents (Klemperer, 1999). They have found numerous applications in finance and e-commerce, and provide a promising coordination technique for many computational environments such as agent-based systems. Whereas ***forward auctions*** are used for selling, ***reverse auctions*** are used for procurement of goods or services.[1] The competitive process of auctions serves to aggregate the scattered information about bidders' valuations and to dynamically set prices of a trade.

A fundamental shortcoming of traditional auction mechanisms is their inability to allow for complex bid structures which exploit complementarities and economies of scale in valuation structures of bidders. As many organizations have begun to realize the efficacy of auctions, interest has emerged to extend basic auction types to support negotiations beyond the price and communicate bids with a more complex set of preferences. For example, the procurement of direct inputs is usually very large and requires the use of special price negotiation schemes that incorporate appropriate business practices. Typically, bids

[1]There is also a third kind of auctions called *exchanges* with multiple sellers and multiple buyers. This kind of auctions is out of the scope of this thesis.

in these settings have the following properties:

- The transaction volume tends to be large and suppliers often provide volume discounts.
- Bidders often provide all-or-nothing bids on a set of items with a special discounted price.
- Items may have multiple, non-price attributes to be traded off against price attributes.

Forward auctions often require similar bid structures as well, for example when selling frequency licenses for multiple geographical areas, pieces of land or starting and landing slots at an airport. ***Multi-unit auctions*** facilitate negotiations on large quantities of an item (Davenport and Kalagnanam, 2000), ***combinatorial auctions*** (a.k.a. ***multi-item auctions***) allow bids on bundles of different items (Nisan and Segal, 2001; Rothkopf et al., 1998), whereas ***multi-attribute auctions*** facilitate negotiations on multiple attributes of an item (Bichler, 2000). These "multidimensional" auction formats have performed well in the lab, but also in a number of real-world applications. In our research we focused on combinatorial auctions.

1.1 Combinatorial Auctions

Multi-item auctions are common in industrial procurement and logistics, where suppliers are able to satisfy the buyer's demand for several items or lanes. Often purchasing managers package these items into pre-defined bundles the suppliers can bid on (Schoenherr and Mabert, 2006). Throughout the past few years, the study of Combinatorial Auctions (CAs) has received much academic attention (Anandalingam et al., 2005; Cramton et al., 2006a). CAs are multi-item auctions in which the bidders can define their own combinations of items called "packages" or "bundles" and place bids on them, rather than just on individual items or pre-defined bundles. This allows the bidders to better express their preferences and ultimately increases the economic efficiency in the presence of superadditive and subadditive valuations (complementarities and substitutabilities respectively). Allowing for package bids helps to overcome the ***exposure problem***, in which bidders can occasionally get an unwanted combination of items with a negative payoff. CAs have already found application in various domains ranging from transportation to industrial procurement and allocation of spectrum licenses for wireless communication services (Cramton et al., 2006a).

Combinatorial auction design is a difficult task. Besides of achieving main design goals like economic efficiency, the auction design has to deal with computational, communicational and cognitive complexity and overcome several other problems like the *exposure problem* and ***threshold problem***[2] (Bichler et al., 2005). There have been multiple proposals on design of efficient combinatorial auctions. The *Vickrey-Clarke-Groves* (VCG) auction is a single-round design that, uniquely on a wide class, has a dominant-strategy property, leads to efficient outcomes, and takes only a zero payment from the losing bidders (Ausubel et al., 2006, p. 93). Though the VCG auction takes a central place in the mechanism design literature, it can produce allocations outside of the core if goods are not substitutes. In this case, the auctioneer revenue is often uncompetitively low. This opens up non-monotonicity problems and possibilities for collusion and shill-bidding (Ausubel and Milgrom, 2006b). There are also many other reasons, why the VCG auction is hardly used in practical applications (see Section 2.5.1 for details).

In comparison to single-round designs, multi-round or iterative CAs (ICAs) have been selected in a number of industrial applications, since they help bidders to express their preferences by providing feedback, such as provisional

[2]In threshold problem several small bidders do not manage to overbid one large bidder, though the allocative efficiency would be higher if they would win (Bichler et al., 2005).

pricing and allocation information, in each round (Bichler et al., 2006; Cramton, 1998). "Experience in both the field and laboratory suggest that in complex economic environments iterative auctions, which enhance the ability of the participant to detect keen competition and learn when and how high to bid, produce better results than sealed bid auctions" (Porter et al., 2003). ICA's have several advantages over sealed-bid auctions. First, bidders don't have to reveal their true preferences on all possible bundles in one round as would be necessary in Vickrey-Clarke-Groves (VCG) mechanisms (Ausubel and Milgrom, 2006b). Second, prices and other feedback received by bidders in ICAs help to reduce the amount of potentially interesting bundles. Third, Milgrom and Weber (1982) have shown for single-item auctions that if there is affiliation in the values of bidders, then sealed-bid auctions are less efficient than iterative auctions. Even in cases where sealed-bid CAs have been used, people have decided to run after-market negotiations to overcome the inefficiencies (Elmaghraby and Keskinocak, 2002).

Much research on ICAs is based on so called primal-dual auction algorithms. In their seminal paper, Bikhchandani and Ostroy (2002) use dual information based on the results of a winner determination integer program as ask prices in an ICA. The solution to the LP relaxation of the winner determination problem (WDP) suggested in their paper is integral, and the dual ask prices lead to the competitive equilibrium, maximizing the allocative efficiency. Unfortunately, they need to introduce a variable for every feasible integer solution so that the number of variables needed for the WDP is exponential in the number of bids. The formulation then results in personalized non-linear ask prices and is not a feasible approach for larger combinatorial auctions (see section Chapter 6). Nevertheless, the paper provided very useful insights for practical auction designs. There have been multiple proposals on how to design ICAs including approximate linear, non-linear, and personalized non-linear prices (Ausubel and Milgrom, 2002; Day, 2004; Drexl et al., 2005; Kelly and Steinberg, 2000; Kwasnica et al., 2005; Kwon et al., 2005; Parkes and Ungar, 2000a; Porter et al., 2003; Wurman and Wellman, 2000). As of now, there is no general consensus on a single "best" design, and it seems that several auction designs will prove useful for different applications and different valuation structures.

In our research we focused on ICA designs with ***linear ask prices*** in which each item is assigned an individual ask price, and the price of a package of items is simply the sum of the single-item prices. Although, it can be shown that exact linear prices are only possible in restricted cases (Kelso and Crawford, 1982), several authors approximate these prices with so called ***pseudo-dual linear prices*** (Kwasnica et al., 2005; Kwon et al., 2005; Rassenti et al., 1982). Such prices are easy to understand for bidders in comparison to non-linear ask prices, where the number of prices to communicate in each round is exponential in the number of items (Xia et al., 2004). Linear prices give a good guidance to the bid formation for the new entrants and losing bidders, who can use them to compute the price of any bundle even if no bids were submitted for it so far. Pseudo-dual prices have shown to perform surprisingly well in laboratory experiments, and also the US Federal Communications Commission (FCC) has examined their use within the Modified Package Bidding (MPB) auction design (Goeree et al., 2007). Unfortunately, as of now, there is little theory about the economic properties of ICAs using pseudo-dual linear ask prices, and initial evidence is restricted to a few laboratory experiments testing selected auction designs and treatment variables.

1.2 Research Goals and Methodology

In our research, we used computational and laboratory experiments as a tool to compare the relative performance of selected ICA designs primarily based on the *allocative efficiency* and *revenue distribution*, and several other characteristics including the *price monotonicity* and *speed of convergence*. The main goal was to evaluate selected ICA designs and elicit auction rules that work well with a wide range of bidder valuations and bidding strategies.

Traditionally, game theory and laboratory experiments have been used to analyze bidding in single-item auctions. For *combinatorial* auctions, equilibrium analysis has been only performed for so called *primal-dual auctions* with personalized non-linear prices under the *best-response bidding strategy* assumption (see Section 2.6.1). Computing equilibria in combinatorial auctions is hard, since the space of bidding strategies can be very large (Anandalingam et al., 2005; Sureka and Wurman, 2005). Various ask price calculation schemes, bidder decision support tools, and activity and price increment rules make it extremely complex to admit much theoretical analysis at a greater level of detail. On the other hand, laboratory experiments are costly, and typically restricted to relatively few treatment variables. Computational experiments can be of great help in exploring potential auction designs and analyzing the virtues of various design options, whereas laboratory experiments are an excellent method to observe human bidding behavior and an important complement to theoretical and computational models.[3]

In the computational experiments, we focused on three promising linear-price auction designs, namely the Combinatorial Clock (CC) auction, Resource Allocation Design (RAD), and Approximate Linear PriceS (ALPS) with its modified version ALPSm and analyzed their performance in discrete event simulations. In the first set of simulations, we did not try to emulate real-world bidding behavior, but rather used *myopic best-response* and simple *power-set* bidders (see section Section 4.1). This enabled us to compare different ICA designs and estimate the efficiency losses that can be attributed to the auction rules, but not to the bidding strategies. In the second set of simulations we analyzed the impact of selected bidding strategies on the auction outcome. This analysis is relevant, since real-world bidders typically do not follow one specific bidding strategy, but use different types of bundling heuristics (see

[3]"Computer simulations are useful for creating and exploring theoretical models, while experiments are useful for observing behavior" (Roth, 1988).

Section 6.2). Our analysis was based on different value models, in order to achieve more general results.

In the laboratory experiments we compared the same auction designs and one ICA design with non-linear personalized prices (iBundle) in respect to the same performance measures. We analyzed valuations that satisfy the buyer-submodularity conditions, for which theory predicts straightforward bidding and Vickrey payoffs in iBundle, and more general valuations, for which theory has little to say as of yet for all of the above auction designs. We also analyzed the bidding behavior on an aggregate as well as individual level.

1.3 Related Work

An et al. (2005) have also used computational experiments, which studied the impact of bidding strategies on sealed-bid CAs to ICAs using linear ask prices. Our *Pairwise Synergy* value model and *best-chain* bidder were built following the INT agent described in their paper. Our results confirm the finding, that bundling is a useful strategy for the bidders and auctioneer alike, and that auctioneers should encourage their bidders to use bundle bids.

Recently, Dunford et al. (2007) have described a set of simulations comparing various ICA designs, similar in spirit to ours. While the authors also used simulations, the study was focused on the FCC setting without evaluating different value models or bidding strategies. The authors assumed *best-response* bidders and compared different versions of RAD and some derivations thereof, which have been developed for the FCC auctions, to the Ascending Proxy Auction. On the contrary, we compared other auction designs and focused on generic market structures with CATS value models and different bundling strategies.

Dunford et al. (2007) introduce additional methods for price calculation based on RAD. Smoothed Anchoring uses a quadratic program to reduce the non-monotonicity in RAD using an exponential smoothing formula in the objective function. Some of the linear price designs described in the paper share ideas with ALPS in sequentially minimizing the prices. The authors found that for different types of valuations (with or without the BSC or BSM properties) all the linear pricing schemes perform relatively well compared to the Ascending Proxy Auction, implemented with non-linear personalized pricing. There is a significant reduction in the average number of auction rounds by using linear pricing schemes as compared to the Ascending Proxy Auction. Among the linear pricing schemes there was no clear cut winner, and the authors demand further research in this field with larger test sets. The Ascending Proxy Auction assumes "trusted" artificial proxy agents, with which bidders need to submit all their valuations before the auction. The advantages of such an iterative CA design over the VCG auction are less obvious. The performance of the Ascending Proxy Auction with non-best-response bidders is unknown as of yet.

A number of laboratory experiments have focused on combinatorial auctions and their comparison to simultaneous or sequential auctions. Banks et al. (1989) analyzed various mechanisms including the AUSM combinatorial auction mechanism and found CAs to exhibit higher efficiency than traditional auctions in the presence of complementarities. In line with this research, Ledyard et al. (1997) compared the Simultaneous Ascending Auction (SAA), sequential ascending auctions, and AUSM and found that in case of exposure problems AUSM led to a significantly higher efficiency than the other two designs. Banks et al. (2003) did another analysis on the SAA and ascending auctions having package bidding and also found package bidding to achieve a higher efficiency with complementarities.

Porter et al. (2003) compared the SAA against a design by Charles River and Associates and the CC auction and found the CC auction to achieve the highest efficiency, plus being simple for bidders. Kwasnica et al. (2005) defined the RAD auction design and compared it to SAA. They found that in environments with complementarities RAD significantly increased the efficiency and had a lower number of rounds. In additive environments without complementarities package bidding rarely occurred, and no significant differences in the efficiency and auctioneer revenue could be identified.

Kazumori (2005) analyzed the SAA, the VCG mechanism, and the *Clock-Proxy* auction. He conducted experiments with students and professional traders and confirmed the previous studies, namely that given significant complementarities bundle bidding leads to a higher efficiency than SAA. He also found, however, that in case of coordination problems package bidding may be less powerful. The Clock-Proxy auction outperformed both the SAA and the VCG auction, whereas the SAA outperformed the Clock-Proxy auction for additive value structures. He also found professional traders to have higher payoffs than students on average. In another recent study, Chen and Takeuchi (2005) compared the VCG auction and iBEA in experiments in which humans competed against artificial bidders. Here, the sealed-bid VCG auction generated a significantly higher efficiency and auctioneer revenue than iBEA. The participants in the VCG auction either underbid or bid their true valuations.

1.4 About this Thesis

This thesis includes the results of my research during the years 2003-2007 at the Chair of Internet-based Information Systems at the TU München, Germany. The main contributions of the thesis are the results of the computational and laboratory experiments that I and my colleagues conducted using our software framework *MarketDesigner* (Chapter 4 and Chapter 6). Additionally, we analyzed a lot of literature on combinatorial auctions and built a consistent terminology, which was further extended and made more precise, as we implemented the software and conducted experiments (Chapter 2).

Though the focus of my research was primarily on laboratory experiments, I have also included the computational results for the purpose of completeness. As most results were achieved in a team, I usually use "we" to emphasize this fact. The thesis contains several text modules from our joint publications: Chapter 2 partially contains the results from Pikovsky and Bichler (2005), Chapter 4 is based on Bichler et al. (2007), and Chapter 6 is based on Pikovsky et al. (2007).

The remainder of this thesis is organized as follows:

Chapter 2 builds a consistent terminology to be used further throughout the thesis, mentions most important theoretical findings and discusses their impact on the auction mechanism design. The chapter is mainly based on our own work and on the introduction to the theory of iterative combinatorial auctions written by David Parkes in the book *Combinatorial Auctions* in 2006 (Parkes, 2006).

Chapter 3 briefly describes the ICA designs discussed in this thesis. Since we focused on *linear-price* auctions, most considered auction designs are based on linear prices. The *Combinatorial Clock* (*CC*) auction, *Resource Allocation Design* (*RAD*) and *Approximate Linear PriceS* (*ALPS*) auction (developed by us) are discussed. Additionally, one member of the *primal-dual* auctions family, *iBundle*, is introduced. The description of the *Vickrey-Clarke-Groves* auction can be found in Chapter 2.

Chapter 4 presents the setup and results of our computational experiments. We compared the *CC* auction, *RAD*, *ALPS* and the *Vickrey-Clarke-Groves* auction in different settings under various assumptions about value models and bidding strategies. We discovered some interesting facts regarding the allocative efficiency, revenue distribution, price monotonicity, and speed of convergence of different designs and analyzed their robustness against selected pure and mixed bidding strategies.

Chapter 5 explains the design of our laboratory experiments. It describes the economic environment, some known common phenomena, our a priori assumptions about the bidding behavior, and summarizes a set of hypotheses for our study. It further defines and motivates the used value models, treatments, reward mechanism and experiment conduction scheme.

Chapter 6 presents the results of our laboratory experiments. We compared the *CC* auction, *ALPS*, *iBundle* and the *Vickrey-Clarke-Groves* auction using different value models. We were able to identify several similarities to the computational results, but also observed some other interesting phenomena specific for the behavior of human bidders.

Finally, **Chapter 7** draws conclusions and proposes some future research topics in this area.

The **Appendix** contains a detailed description of the *ALPS* and *ALPSm* auction designs, an overview of the software platform *MarketDesigner* and additional details on the design and results of the laboratory experiments.

Chapter 2

Iterative Combinatorial Auctions

The purpose of this chapter is to build a consistent terminology to be used throughout this thesis, mention the most important theoretical findings and discuss their impact on the auction mechanism design. The chapter is mainly based on our own work and on the introduction to the theory of iterative combinatorial auctions by David Parkes in the book *Combinatorial Auctions* (Parkes, 2006). The definitions that currently represent common knowledge are provided without citation. The definitions developed at our university department are marked with "IBIS". The sources of all theorems are explicitly specified. Though there is a lot of literature on combinatorial auctions, I mostly reference the above article of David Parkes, as it provides a good overview of related theory, based on a terminology mostly consistent with the one of this thesis.

	Bidders (Suppliers)			
Items	Bidder 1	Bidder 2	Bidder 3	Bidder 4
10 HD A 10GB	x		x	x
20 HD B 40GB		x	x	x
20 HD C 60GB	x	x	x	
Bid Price	€4000	€5800	€6700	€3500

Table 2.1: Combinatorial auction example

2.1 Introduction

Combinatorial auctions (***CAs***) are multi-item auctions in which bidders can define their own combinations of items called ***packages*** or ***bundles*** and place bids on them, rather than just on individual items or pre-defined bundles. This allows the bidders to better express their valuations and ultimately increases the economic efficiency in the presence of synergistic values, often called economies of scope. CAs have already found application in various domains ranging from transportation to industrial procurement and allocation of spectrum licenses for wireless communication services (Cramton et al., 2006a).

Table 2.1 illustrates an example of a combinatorial reverse (procurement) auction for computer hard drives and the bids from 4 suppliers. Each supplier has provided a bundled "all-or-nothing" bid and a price for the bundle. Notice that as the number of items increases, the number of bids can grow exponentially. After receiving bids on bundles of goods in a combinatorial procurement auction, the auctioneer (buyer) needs to identify the set of bids that minimizes the total procurement cost subject to the given business rules such as limits on the number of winning bidders or the amounts purchased from certain bidders or groups of bidders. Identifying the cost-minimizing bid set subject to these side constraints is a hard optimization problem. Therefore, automated winner determination is central to most combinatorial auctions.

The competitive process of auctions serves to aggregate the scattered information about bidders' valuations and to dynamically set the prices of a trade. The typical flow of an auction process is illustrated by the Figure 2.1. In a ***single-round auction*** (***sealed-bid auction***) bids are collected over a period of time, after which the auction closes, the winning bids are determined (this step is a.k.a. ***winner determination***, ***market clearing***, or ***resource allocation***) and the prices to be payed for the winning bids are calculated. In an ***iterative auction*** (***open-cry auction***) the steps of bid submission and bid evaluation are executed multiple times, whereby after each iteration some information feedback is communicated to the bidders. Iterative auctions close either at a fixed point in time or after a certain ***termination rule*** becomes satisfied (e.g., no new bids were submitted). Although in most iterative combinatorial auctions the winner determination and calculation of prices to pay is done after each iteration to compute provisional allocations (this belongs to the bids evaluation step), in some auction designs this is only done after the auction closes.

Iterative auctions are further divided in ***continuous auctions*** and ***multi-round auctions*** (***round-based auctions***). In *continuous* auctions bids are evaluated on arrival of every new bid, whereas in *multi-round* auctions bids are collected over a period of time, called ***round***, before the bid evaluation is performed. Continuous auctions contribute to a more dynamic environment, since the feedback information is kept up to date at every point in time throughout the auction. However, continuous *combinatorial* auctions are usually considered impractical, since they lead to high computational costs for the auctioneer (the winner determination must be done whenever a new bid is submitted) and to high monitoring and participation costs for bidders. All *iterative* auction designs discussed in this thesis are round-based.

Most of the desirable economic properties of auctions have been analyzed in the context of mechanism design theory (Jackson, 2000). The mechanism design approach to solving distributed resource allocation problems with self-interested bidders formulates the design problem as an optimization problem.

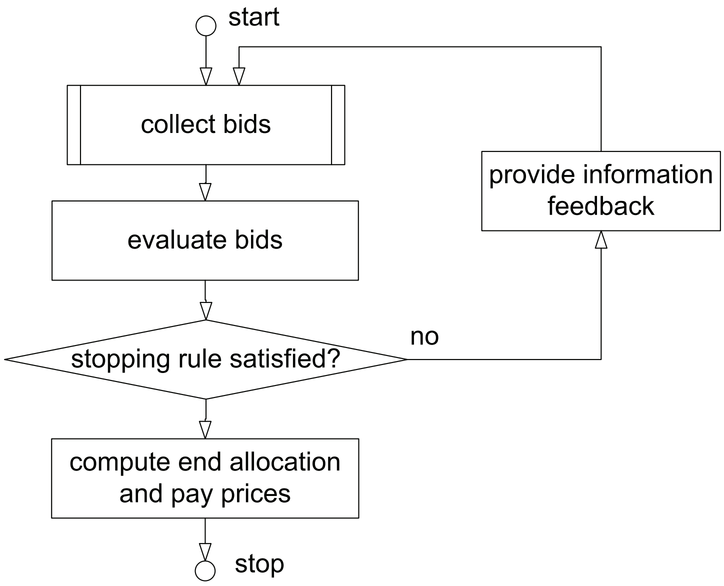


Figure 2.1: Process of an iterative auction

Bidders have private information about the quality of different solutions, they are self-interested and willing to misrepresent their private information if that can improve the solution in their favor. A mechanism takes information from the bidders and makes a decision about the outcome and payments that are implemented. This analysis assumes a certain solution concept, for example, a Nash, or a dominant solution, as well as a certain domain of bidder preferences, for example, quasi-linear, monotonic, etc.

There are two primary design goals in the application of mechanism design to auctions and markets, which are concerned with the solution of an auction. One goal is the ***allocative efficiency*** in which the auction mechanism implements a solution that maximizes the total payoff across *all* agents. Another goal is the ***revenue maximization*** in which the auction achieves a solution that maximizes the payoff to a *particular* participant, usually the auctioneer. For example, in reverse case such an auction would minimize the buyer's cost.

In addition, the ***budget-balance*** assures that there is no net payment made from the auctioneer to the bidders. In other words, the auctioneer does not loose money. The allocative efficiency and budget-balance together imply the ***Pareto optimality*** of a solution.

In mechanism design, concrete assumptions about the private valuations of all participants, called ***value model***, need to be made. In the ***private value model*** each bidder values each package of items independently from the valuations of other bidders. Each bidder knows her valuations, but not the valuations of other bidders. In the ***common value model***, all bidders have the same valuations for same packages, but these are uncertain and depend on the private information of all other bidders. The ***affiliated value model*** (***correlated value model***) contains elements of both private and common value models. Each bidder's valuations depend directly on the private information of all other bidders.

For each of the three value models there exist environments in which it is more appropriate then the others. For example, the private value model assumption is typical when auctioning pieces of artwork, whereas common value model assumption is commonplace when auctioning financial products on the stock exchange; in case of wireless spectrum auctions the correlated value model best replicates private valuations partially driven by the underlying population demographics and shared technological basis. The common and affiliated value models are, however, much younger and less studied than the private value model (Cramton et al., 2006b). Therefore, most existing work on combinatorial auctions is based on the private value model assumption (Cramton et al., 2006a).

The utility of the bidders for the various bundles is private information and not known to the auctioneer. The ***auction design*** can be described as a set of rules that need to motivate the bidders to reveal their true preferences to the extent that makes it possible to solve for the optimal allocation with respect to the true utilities of all bidders for all possible bundles. A specific auction design is defined by the following components:

- the ***auction protocol***, i.e. the sequence, syntax and semantics of messages exchanged throughout the auction
- the ***allocation rules***, which include constraints ensuring the overall objective of the allocation (i.e. efficiency vs. revenue maximization), as well as additional allocation constraints
- the ***payment rules***, which determine the payment from or to the winner(s)

As in *traditional* auction design, the allocation rules, auction protocol and payment rules impact bidders' strategies. Auction designers try to construct ***incentive-compatible*** mechanisms in which bidders are self-interested in reporting truthful information about their preferences. ***Strategy-proof mechanisms*** are even stronger in that truthful bidding is a dominant strategy. Second-price sealed-bid (Vickrey) mechanisms are an example of strategy-proof mechanisms.

The large design space (as shown in Section 2.3) and the possibility of package bidding significantly increase strategic and computational complexity of combinatorial auctions in comparison to their single-item counterparts. There are many difficulties to deal with, for detailed discussion see Bichler et al. (2005). The three main problems are the following:

- The ***computational complexity*** is due to the ***Winner Determination Problem*** (***WDP***), i.e., the problem of determining the winning bids by maximizing the total payoff subject to the given constraints and additional allocation rules. The WDP is an integer optimization problem, and even in its simplest form (if no additional allocation rules exist) it can be interpreted as a well-known *weighted set packing problem* (*SPP*). Therefore, it is $\mathcal{NP}$-complete and no polynomial time algorithm can be expected to exist (Lehmann et al., 2006; Rothkopf et al., 1998). More details on the WDP can be found in Section 2.4.

- The ***Preference Elicitation Problem*** (***PEP***) includes the ***valuation problem***, i.e., the selection and valuation of the bundles to bid for from an exponentially large set of possible bundles. In addition, the ***strategy problem*** of determining the optimal bid prices in various auction designs has been a main focus in the classic game-theoretic auction research, but turns out to be an even more difficult problem in *iterative* combinatorial auctions. For example, it is possible that a losing bid becomes winning in a subsequent round without changing the bid. The bidders face the problem of choosing appropriate bundles to bid for (i.e., bundle selection) and, if the auction design allows *jump bidding*, of choosing the bid prices.
- The ***communication complexity*** is related to the PEP and deals with the question, how many valuations need to be transferred to the auctioneer, in order for her to calculate an efficient allocation. Nisan (2000) shows that an exponential communication is required in the worst case. This problem might be addressed by designing careful bidding languages that allow for compact representation of the bidders' preferences. In addition, there is much recent research on preference elicitation in combinatorial auctions through querying, which can provide an alternative to the auction designs discussed in this thesis (Sandholm and Boutilier, 2006).

In the first decades after CAs appeared in the literature, they were considered intractable due to the WDP. Nowadays, however, in many practical cases the WDP can be solved to optimality by modern computers using sophisticated integer optimization algorithms in an adequate period of time.[1] Also, good approximation algorithms for the WDP have been developed. On the contrary, "PEP has emerged as perhaps the key bottleneck in the real-world application of combinatorial auctions. Advanced clearing algorithms are worthless if one cannot simplify the bidding problem facing bidders" (Parkes, 2006).

Iterative combinatorial auctions (***ICAs***) are to date the most promising way of addressing the PEP. "Experience in both the field and laboratory suggest that in complex economic environments iterative auctions, which enhance the ability of the participant to detect keen competition and learn when and how high to bid, produce better results than sealed bid auctions" (Porter et al., 2003). In contrast, *sealed-bid auctions* require bidders to determine and report their valuations upfront.

ICAs have emerged as the predominant form of combinatorial auctions in practice. In comparison to sealed-bid designs, ICAs have been selected in a number of industrial applications, since they help bidders to express their preferences by providing feedback, such as provisional pricing and allocation information, in each round (Bichler et al., 2006; Cramton, 1998). Even in cases in which sealed-bid CAs have been used people often decided to run after-market negotiations to overcome the generated inefficiencies (Elmaghraby and Keskinocak, 2002).

ICAs have several advantages over sealed-bid auctions. First, bidders don't have to reveal their true preferences on all possible bundles in one round. Second, prices and other feedback received by the bidders in ICAs help to reduce the amount of potentially interesting bundles. Third, Milgrom and Weber (1982) have shown for single-item auctions that if there is affiliation in the values of bidders, then sealed-bid auctions are less efficient than iterative auctions, and there are good reasons to expect similar behavior also in case

[1] The problem sizes that can be solved to optimality depend on the valuation structure. In many applications with up to 20-50 and sometimes more items the optimal solution can usually be found in less than 2 minutes.

of *combinatorial* auctions (Parkes, 2006). For more detailed discussion of the advantages of ICAs see Parkes (2006).

However, designing such iterative auctions has turned out to be a challenging task. One of the main problems is establishing a pricing rule that would provide enough information to the bidders, to lead the auction to an efficient equilibrium solution. In the following, I first describe the design space of ICAs in detail. I then give the mathematical definition of the Winner Determination Problem and discuss some of its properties. Next, I introduce the ICA pricing concepts and review some important facts from equilibrium and game theory. The rest of the chapter describes common properties of price-based combinatorial auction designs and compares the linear pricing scheme to the bundle pricing scheme.

2.2 Preliminaries

I first introduce some necessary notation. Let $\mathcal{K}$ denote the set of items to be auctioned ($|\mathcal{K}| = m$) and $k \in \mathcal{K}$ (also $l \in \mathcal{K}$) denote a specific item. Similarly, let $\mathcal{I}$ denote the set of bidders participating in the auction ($|\mathcal{I}| = n$) and $i \in \mathcal{I}$ (also $j \in \mathcal{I}$) denote a specific bidder.

Definition 1. *A* ***bundle*** *(or* ***package****)* S *(also* T*) is a subset of the item set* $\mathcal{K}$ *(*$S \subseteq \mathcal{K}$*). The empty set (*$|S| = 0$*), single-item sets (*$|S| = 1$*) and the all-items set (*$S = \mathcal{K}$*) are all considered bundles.*

Definition 2. *A* ***round*** $t = 1, 2, 3, \ldots$ *is a period of time during which bidders can submit their bids. After the round is* closed*, no more bids can be submitted, and bidders have to wait until the next round is opened or the auction closes. The auction can not be closed in the middle of a round, the current round must be closed before.*

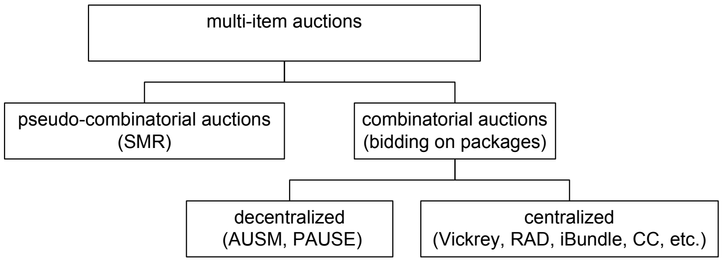


Figure 2.2: Classification of multi-item auctions

A (round-based) iterative combinatorial auction consists of one or more *rounds*. Many concepts like prices, bids, provisional allocations, etc., refer to a specific round. Since in most cases only the current round is to be considered, I shall usually omit the round index t. For example, $b_i(S)$ means the same as $b_i^t(S)$, $\mathcal{B}$ means the same as $\mathcal{B}^t$, etc., whereby t stands for the current round.[2]

2.3 Auction Design Space

Before exploring the auction design space, it is important to clarify, what kind of combinatorial auctions is discussed in this thesis. Figure 2.2 illustrates a classification of *multi-item auctions*. First, we distinguish between combinatorial auctions (auctions that allow real package bidding) and *pseudo-combinatorial* auctions. One well known example of a pseudo-combinatorial auction is the ***Simultaneous Multi-Round Design*** (***SMR***) used by the FCC to auction spectrum licenses (Cramton et al., 1998). The SMR runs multiple single-item auctions simultaneously. Though the bidders are able to utilize some synergies of the simultaneous bidding, the auction suffers from the *exposure problem* because of the inability to bid for packages.

We further divide combinatorial auctions into *centralized* and *decentralized* auctions. ***Decentralized auctions*** were originally developed for small prob-

[2]The symbols $b_i(S)$, $b_i^t(S)$, $\mathcal{B}$, and $\mathcal{B}^t$ will be defined later.

lems in which bidders can cooperate in order to find a better allocation by themselves in each round. Two well known members of this family are the ***Adaptive User Selection Mechanism*** (***AUSM***) (Banks et al., 1989) and the ***Progressive Adaptive User Selection Environment*** (***PAUSE***) (Kelly and Steinberg, 2000). Though these auctions avoid the exposure problem, they are still vulnerable to the *threshold problem*, require full information revelation and introduce high complexity at the bidder side.

In ***centralized auctions*** the auctioneer solves the winner determination problem after the bids are collected. She then provides some kind of feedback to support the bidders in improving their bids in the next round. Usually the bidder's current winning bids and *ask prices* are used as the feedback.

Due to the disadvantages of pseudo-combinatorial and decentralized auctions referenced above, *centralized* auctions are currently considered most promising in the literature. In this thesis, I only discuss centralized auction designs. Due to their very different structure, pseudo-combinatorial and decentralized auctions are out of the scope of this thesis.

Overall, I only consider auctions that confirm to all of the following characteristics:

- truly combinatorial (allow package bidding)
- round-based (single-round or multiple-round)
- centralized

Even given the these restrictions, the design space of ICAs remains extremely large. To be mentioned, all these different rules were not developed to make the auctions unnecessary complex, but rather emerged from various attempts to overcome several auction design problems and avoid unwanted bidder strategies. From our experience in research and implementation of ICAs, we propose the following categorization of the auction design space[3]:

- bidding languages
- bidding rules
- allocation rules
- timing issues
- information feedback and pricing schemes
- proxy agents

2.3.1 Bidding Languages

A bid in an auction is an expression of the bidder's willingness to pay particular monetary amounts for various outcomes. Bidders formulate bids according to their private preferences and bidding strategies. A ***bidding language*** defines the way (the format of the communicated messages and the interpretation rules) in which bidders are allowed to formulate their bids.

In *combinatorial* auctions every auction outcome corresponds to a particular allocation. From the point of view of a particular bidder, the auction outcome is defined by the set of items allocated to her and the monetary amount she has to pay for it.[4] Therefore, the most direct way for the bid formulation is to let each bidder attach a bid price to each possible bundle. This allows one to express any kind of preferences, but in worst case requires an exponential number ($2^m - 1$) of bundles to be evaluated and monitored by every bidder and the same amount of messages to be communicated to the auctioneer. Although

[3]This categorization slightly differs from Parkes (2006).

[4]This assumes the absence of anti-social bidders, whose preferences also depend on the satisfaction of other bidders. To avoid the direct possibility of anti-social bidding, bidding languages only allow to base bids on the bidder's own outcome.

in many cases not every possible combination of items has a positive value for every bidder, the number of interesting bundles can quickly become cognitive intractable.

Bidding languages for combinatorial auctions are typically built of *atomic bids* and *logical rules* that either allow several atomic bids to win simultaneously, or not.

Definition 3. *An* ***atomic bid*** $b_i(S)$ *is a tuple consisting of a bundle* S *and a bid price* $p_{bid,i}(S)$ *submitted by the given bidder* i $(b_i(S) = \{S, p_{bid,i}(S)\})$*. A set of atomic bids is called* ***non-intersecting*** *if the intersection of their bundles is empty.*

The two most popular and intuitive bidding languages are *exclusive-OR* (*XOR*) and *additive-OR* (*OR*).

Definition 4. *The bidding language* ***exclusive-OR (XOR)*** *allows bidders to submit multiple atomic bids. For each bidder at most one of her atomic bids can win. This means that the bidder either gets* all *items contained in the bundle listed in* exactly one *of her atomic bids, or she gets nothing. By submitting her atomic bids, the bidder expresses her willingness to pay at most the amount specified in her winning atomic bid (if any).*

Definition 5. *The bidding language* ***additive-OR (OR)*** *allows bidders to submit multiple atomic bids. For each bidder any non-intersecting combination of her atomic bids can win. This means that the bidder either gets* all *items contained in the bundles listed in* some non-intersecting set *of her atomic bids, or she gets nothing. By submitting her atomic bids, the bidder expresses her willingness to pay at most the sum of amounts specified in her winning atomic bids (if any).*

The bidding language XOR lets each bidder define a bid price for each possible combination she can win, exactly as described above. From this point of view, it can be considered the most powerful of all possible bidding languages for CAs. However, it suffers from the cognitive and communicative complexity, caused by the exponential number of bundles to be evaluated and monitored. Consequently, auction designers try to use the information about the structure of the bidders' valuations to construct easier-to-handle bidding languages, still allowing bidders for complete representation of their preferences. For example, the bidding language OR is sufficient if no subadditive valuations exist. Unfortunately, this is often not the case, e.g., in the presence of budget restrictions (if the bidder can not afford every combination of bundles she bid for) or when auctioning substitute goods.

Though several more complex bidding languages have been proposed (Nissan, 2006), the research in this area is still in its early stage. No proposed combinatorial auction design really deals with the question, which bidding languages are appropriate for it: the authors mostly only mention what bidding language is considered – and this is always either OR or XOR. Furthermore, no practical applications of any other bidding languages are known to me.

Moreover, the term *bid* is very often used in place of *atomic bid*, in particular this is the case in the book "Combinatorial Auctions" (Cramton et al., 2006a). This is understandable, since *atomic bids* are actually what the bidder communicates to the auctioneer if either OR or XOR bidding language is used. Since no other bidding languages are supported by any auction design described in this thesis and to keep the common notation, I will also use the term ***bid*** instead of *atomic bid* in the following.

Though every (atomic) bid is submitted in some specific round, it is usually not important, in which round the bid was submitted.[5] In contrast, it is important, which bids are valid at the end of the given round t. This is due to the fact that some bids can be "kept" for the following rounds and other bids can be "thrown away". The decision, which bids to keep, is done due to the

[5] An important exception are activity rules, see Section 2.3.3.

rules of the specific auction design (see Section 2.3.3). Therefore, the following definitions of *active*, *displaced*, *inactive* and *revoked* bids are crucial for any real-world implementation of ICAs:

Definition 6 (IBIS)**.** *An (atomic) bid is called* ***active*** *at the end of the round t if it is allowed to be selected as a winning bid (participates at the winner determination) in case if the auction is closed after the round t.*

A bid, submitted in the round t, is always active at the end of this round. In further rounds it can be *displaced*, *deactivated* or *revoked*:

Definition 7 (IBIS)**.** *An (atomic) bid is called* ***revoked*** *if it was explicitly revoked by the bidder (which is only possible if the auction design allows bids revocations).*

Definition 8 (IBIS)**.** *An (atomic) bid submitted in some round is called* ***displaced*** *if another bid was submitted by the same bidder for the same bundle in some later round.*

Definition 9 (IBIS)**.** *An (atomic) bid submitted in some round is called* ***inactive*** *if it was deactivated in some later round due to the auction design rules.*

Displaced, inactive and revoked bids are not active, i.e., they do not participate at the winner determination as the auction closes. In real-world implementations of ICAs, bidders are usually informed about the current state of their bids. At least, they usually know, which bids are still active.

In the following, in most cases only *active* bids have to be considered. Let $b_i^t(S) = \{S, p_{bid,i}^t(S)\}$ denote the bid submitted by the bidder i for the bundle S active at the end of the round t. The set of all bids active at the end of the round t is denoted by $\mathcal{B}^t = \{b_i^t(S)\}$. For displaced, inactive and revoked bids no special notation is needed.

For more information on bidding languages the reader is referred to Nissan (2006) and Boutilier and Hoos (2001).

2.3.2 Information Feedback and Pricing Schemes

The key challenge in the iterative combinatorial auction design is providing information feedback to the bidders after each auction iteration to guide bidding towards an efficient solution. Information feedback about the state of the auction can contain the provisional allocation (if any), the list of bids submitted by other bidders, the number of active bidders and/or bids, etc. Information hiding (e.g., bid price rounding) can also be used to to limit the possibilities of signaling between bidders.

Pricing (assigning ask prices to items and/or item bundles) has been adopted as the most useful mechanism of providing feedback, to lead the auction to an efficient equilibrium solution. ***Ask prices*** are mostly used as the lower bound for possible bids, but sometimes also as a non-binding indicator of the current competition on the corresponding items or bundles. In fact, to our knowledge, every existing *centralized* multi-round ICA design uses pricing.

In contrast to single-item auctions, pricing is not trivial in combinatorial case. The main difference is the lack of natural single-item ask prices. With bundle bids, setting independent ask prices for individual items is not obvious and often even impossible (Bikhchandani and Ostroy, 2002). Additionally, ask prices may need to be *personalized*, i.e., different bidders get different prices for the same items or bundles, as opposed to traditional *anonymous* prices.

Let $p^t_{ask,i}(S)$ denote the personalized ask price for the bidder i and bundle S valid during the round t (i.e., this price was calculated after the round $t-1$ was closed) and $\mathcal{P}^t_{ask}$ denote the set of all ask prices valid during the round t. As already mentioned, I will omit the round index to refer to the current round.

Definition 10. *A set of ask prices* $\{p_{ask,i}(S)\}$ *is called* ***linear (additive)*** *if* $\forall i, S : p_{ask,i}(S) = \sum_{k \in S} p_{ask,i}(k)$

Definition 11. *A set of ask prices* $\{p_{ask,i}(S)\}$ *is called* ***anonymous*** *if* $\forall i, j, S : p_{ask,i}(S) = p_{ask,j}(S)$

In other words, the prices are *linear* if the price of a bundle is always equal to the sum of the prices of its items, and the prices are *anonymous* if the prices of the same bundle are equal for every bidder. Non-linear ask prices are also called ***bundle*** ask prices; non-anonymous ask prices are also called ***discriminatory*** or ***personalized*** ask prices. For a shorter and clearer notation, let $p^t_{ask}(S)$ denote the anonymous bundle ask price for the bundle S and $p^t_{ask}(k)$ denote the anonymous linear ask price for the item k.

The following hierarchical structure of pricing schemes can be derived from the above definitions:[6]

1. linear anonymous prices
2. non-linear anonymous prices
3. non-linear non-anonymous prices

The first pricing scheme is obviously the simplest one. Linear anonymous prices are easily understandable and usually considered fair by bidders. The communication costs are also minimized, since the amount of information to be transferred is linear in the number of items. Linear anonymous prices can sometimes be efficient even with super- or subadditive valuations (Bichler et al., 2007; Pikovsky and Bichler, 2005).

The second pricing scheme introduces non-linearity, which is often necessary to express strong super- or subadditivity in bidders' valuations (Pikovsky and Bichler, 2005). Unfortunately, non-linear prices are often considered too complex by bidders. Communication costs also increase, since in the worst case an exponential number of prices need to be exchanged.

Sometimes, even non-linear anonymous prices are not sufficient to lead the auction to competitive equilibrium. In this case the theory proposes the pricing scheme 3, which introduces discriminatory pricing. Due to Bikhchandani

[6]To our knowledge, linear discriminatory prices have hardly been considered in the context of combinatorial auctions.

and Ostroy (2002), non-linear discriminatory competitive equilibrium prices do always exist and support the efficient allocation. However, discriminatory pricing results in additional complexity and is often considered unfair by bidders.

The pricing scheme selection is one of the key decisions in the ICA design (Parkes, 2006; Pikovsky and Bichler, 2005). In Section 2.5 I discuss the impact of the different pricing schemes on the auction result from a theoretical point of view. Chapter 3 describes pricing mechanisms used in selected auction designs. The main part of the thesis starting with Chapter 4 deals with an experimental study of the influence of pricing and other factors on the auction outcome.

2.3.3 Bidding Rules

Bidding rules define, what bids can be submitted/revoked in the current auction state, and how the auction state evolves throughout the auction. Following bidding rules are common to several ICA designs:

Binding ask prices oblige bidders to bid either above or exactly at the current ask prices. Sometimes, the price to bid is splitted into the ask price and the ***price increment***, denoted by Δ^t. In latter case the price increment can either apply to the whole bundle price or to the item prices. If ***jump bidding*** is allowed, the bidders can bid above the prices, otherwise they must bid exactly at the prices. Allowing jump bidding can significantly decrease the auction duration (especially in combination with small price increments), but also allows for more complex bidding strategies and for some degree of signaling between the bidders. Additionally, the so called ***last-and-final bids*** (final bids for a bundle at a bid price slightly lower than the current ask price, see Parkes (2001)) are sometimes allowed to provide more flexibility for the bidders expressing their valuations in the latter auction rounds. For details on the effects of jump bidding and last-and-final bidding see Laqua (2006).

Price update rules determine the evolution of ask prices throughout the auction. The ***pricing scheme*** together with the price update rule usually build the kernel of an auction design. In one family of ICA designs, ask prices of selected (based on the current competition) items or bundles are increased from the current round t to the next round $t + 1$ by a fixed (absolute or relative) *price increment* Δ^t. In this case the price increment either refers to the whole bundle price or to the item prices. In another ICA family, ask prices are calculated on the basis of submitted bids and provisional allocation using a design-specific price calculation algorithm.

Bid validity determines which bids remain active from the current to the next round. In some auction designs *all* bids remain active throughout the auction (the so-called ***old-bids-active rule***). In others, only provisionally winning bids remain active, whereas all provisionally losing bids are deactivated in the next round. Holding all bids active can significantly improve the auction efficiency, since more bidders' preferences are available for the winner determination (Bichler et al., 2007). On the other hand, it can increase the complexity of the winner determination problem and sometimes confuse bidders, as every old losing bid can occasionally win. In combination with the old-bids-active rule, the ***improve-old-bids rule*** can additionally prohibit underbidding previous own bids, even if the current ask price would allow it.

Bid revocation rules allow or prohibit explicit bid revocations by bidders. The practical importance of bid revocations is often undervalued or ignored, though it is often indispensable due to typos or occasional wrong preference elicitation. Whereas revocation of provisionally winning bids is usually prohibited, revocations of provisionally losing bids are often allowed in practice. Nevertheless, to our knowledge, very low is known about the impact of bid revocations on the auction outcome.

Activity rules (a.k.a. ***eligibility rules***) enforce active bidding throughout the auction as opposed to the *wait-until-auction-end-and-snipe* strategy loved by *eBay* users. Activity rules were introduced in the early FCC wireless spectrum auctions and proved important.[7] Decisions about appropriate activity rules are often guided by a tradeoff between allowing for straightforward bidding strategies and encouraging early bidding (Parkes, 2006). More details on selected activity rules are provided in the context of the CC auction, ALPS, RAD and iBundle in Chapter 3. For an extended discussion of activity rules see Ausubel et al. (2006).

2.3.4 Allocation Rules

Allocation rules regulate the way of selecting the winning bids from the bid set $\mathcal{B}$, i.e., they determine the formulation of the winner determination problem. Typically, the *auctioneer revenue* is maximized subject to the bidding language rules and the inability to sell the same item more than once. For details on the WDP see Section 2.4.

Beyond the standard rules of the WDP, additional allocation rules, called ***business constraints*** or, more generally, ***side constraints***, are often of practical importance. Especially in industrial procurement following constraints are often requested by procurement managers:

- The number of winning suppliers should be greater than a certain number (to avoid depending too heavily on just a few suppliers), but smaller than another certain number (to avoid too much administrative overhead).
- The maximum/minimum amount purchased from each supplier is bounded to a certain limit.
- At least one supplier(s) from a target group (e.g., minority) needs to be chosen.

[7] The form of activity rule used in the FCC spectrum auctions is due to Paul Milgrom and Robert Wilson. Similar rules have since become standard in ICAs.

Sometimes even more flexibility is needed, e.g., forcing some specific bids to be winning or losing (either in a provisional or in the end allocation). Moreover, business constraints may need to be defined or removed dynamically throughout the auction.

In spite of their practical importance, there is a gap in the theory of ICAs in regard to business constraints. Whereas the impact of business constraints on the solution and complexity of the WDP have been analyzed in Kalagnanam et al. (2001), Sandholm and Suri (2006), and Collins et al. (2002), no studies of their influence on the outcome and pricing in *iterative* combinatorial auctions is known to us.

2.3.5 Timing Issues

For a round-based auction design two time units are of importance: the round duration and the auction duration. With ***round closing*** we denote the point in time at which a specific auction round is declared closed, and no more bids are accepted until the start of the next round. After the round is closed, the bid evaluation process, called ***round clearing***, starts. According to the round clearing results and to the auction ***termination rules***, the auction either moves to the next round or terminates. In latter case the auction is first ***closed*** (the bidders are informed that no more bids can be submitted) and the final bid evaluation, called ***auction clearing***, starts. After the auction clearing is finished, the end allocation and prices to be payed are communicated to the bidders. We call the time period between the round start and the round closing ***round duration*** and the time period between the auction start and the auction closing ***auction duration***.

Round closing rules control the round duration. The round duration is usually set to a fixed time period. Often, however, selecting a fixed round duration is a difficult task. On the one hand, bidders should be given enough time for the preference elicitation and bid submission in every round, especially if activity rules are used. On the other hand, one should avoid boring bidders by to long rounds, since this costs time, reduces bidders' concentration, and, therefore, can distort preference elicitation. A fixed round duration is especially problematic, as bidders usually need more time at the beginning of the auction than in its latter rounds (see the results of our laboratory experiments, Chapter 6). To mitigate the problem, we let bidders communicate their ***ready-in-round state*** to the auctioneer. The round is then prematurely closed as soon as all participating bidders have indicated their readiness.[8]

Auction closing rules (a.k.a. ***termination rules***) control the auction closing time point. Auctions may close at a fixed deadline and/or be limited in duration and/or the maximal number of rounds. Alternatively, auctions can have a rolling closure with the auction kept open while one or more losing bidders continue to submit competitive bids or the allocation does not change for a given number of rounds.

Fixed deadlines are useful in settings in which bidders are impatient and unwilling to wait a long time for an auction to terminate. However, fixed deadlines tend to require stronger activity rules to prevent the auction from reducing to a sealed-bid auction with all bids delayed until the final round. In comparison, rolling closure rules have been shown to encourage early and sincere bidding.[9] All ICA designs considered in this thesis use rolling closure termination rules.

2.3.6 Proxy Agents

With ***proxy agents*** bidders can provide direct value information to an automated bidding agent that bids on their behalf. The bidder-to-proxy language

[8] This mechanism can only be used if the auctioneer knows all participating bidders. In auctions with activity rules this is always the case starting with the second round.

[9] Roth and Ockenfels (2002) have studied the use of deadlines versus rolled closures, on eBay and Amazon Internet auctions respectively. Bidders on Amazon bid earlier than on eBay, and many bidders on eBay wait until the last seconds of the auction to bid.

should allow bidders to express partial and incomplete information, to be refined during the auction, in order to realize the elicitation and price discovery benefits of an *iterative* auction.

Proxy agents can query bidders actively when they have insufficient information to submit bids. They can also facilitate faster convergence with rapid automated proxy rounds, interleaved with bidder rounds. Mandatory proxy agents can be useful in restricting the strategy space available to bidders.

One concern in the design of proxy auctions is to determine, when to allow proxy information to be revised and to determine the degree of consistency to enforce across revisions. As an alternative to the use of "full-time" proxy agents, the iterative, non-proxy part of the auction is sometimes followed by a final second-price sealed-bid round, called ***proxy round***. An additional concern is that of trust and transparency, since the bidding activity is transferred to automated agents.

Studying effects of proxy bidding is out of the scope of this thesis and is certainly one of the very interesting ways to go in the future research. For this thesis, no considered auction design uses proxy agents. For more information on the topic see Parkes and Ungar (2000b) and Ausubel and Milgrom (2002).

2.4 Winner Determination Problem

At this point some additional notation is required. According to the *private value model* assumption, I denote the private valuation of the bidder i for the bundle S by $v_i(S)$. The valuations of different bidders are assumed independent and satisfying the ***free disposal*** condition[10], i.e., if $S \subset T$ then $v_i(S) \leq v_i(T)$.

Definition 12. *A **value model** $\mathcal{V} = \{v_i(S)\}$ is a set of the private valuations of all bidders for all bundles.*

[10]The free disposal assumption is common for the literature on combinatorial auctions.

Definition 13 (IBIS). *The price to be payed by the bidder i for the (allocated to her) bundle S is called* ***pay price*** *and is denoted by $p_{pay,i}(S)$. The set of all pay prices is denoted by $\mathcal{P}_{pay}$.*

Definition 14. ***Bidder utility*** *(a.k.a.* ***bidder payoff****) $\pi_i(S, \mathcal{P}_{pay})$ expresses the bidder's i satisfaction of getting the bundle S at the pay prices $\mathcal{P}_{pay}$. We assume quasi-linear bidder utilities $\pi_i(S, \mathcal{P}_{pay}) := v_i(S) - p_{pay,i}(S)$, $\pi_i(\emptyset, \mathcal{P}_{pay}) := 0$.*

Definition 15. *An* ***allocation*** *X is a tuple $(S_1, \ldots, S_n)$ that assigns a corresponding (possibly empty) bundle to every bidder. The allocated bundles may not intersect: $\forall i, j : S_i \cap S_j = \emptyset$. Some items can remain not allocated: $\cup_{i \in \mathcal{I}} S_i \subseteq \mathcal{K}$. The set of all possible allocations is denoted by $\mathcal{X}$.*

An allocation can also be defined by a set of binary variables $\{x_i(S)\}$, $x_i(S) \in \{0; 1\}$ where $(\forall i, S : x_i(S) = 1 \Leftrightarrow S_i = S)$ and $\left(\forall i : \sum_{S \subseteq \mathcal{K}} x_i(S) \leq 1\right)$ and $\left(\forall i : \sum_{S \subseteq \mathcal{K}} x_i(S) = 0 \Rightarrow S_i = \emptyset\right)$. In other words, $x_i(S) = 1$ means that the bidder i gets exactly the bundle S (and nothing else).

Given an allocation X and pay prices $\mathcal{P}_{pay}$, let $\pi_i(X, \mathcal{P}_{pay}) := \pi_i(S_i, \mathcal{P}_{pay})$ denote the utility of the bidder i for the bundle she gets in the allocation X and $\pi_{all}(X, \mathcal{P}_{pay}) := \sum_{i \in \mathcal{I}} \pi_i(X, \mathcal{P}_{pay})$ denote the ***total bidder utility*** (a.k.a. ***total bidder payoff***) of all bidders. Further, let $\Pi(X, \mathcal{P}_{pay}) := \sum_{S \subseteq \mathcal{K}, i \in \mathcal{I}} x_i(S) p_{pay,i}(S)$ denote the ***auctioneer revenue***. The auctioneer revenue is usually considered to be the auctioneer's gain, since the auctioneer's costs are assumed to be 0.

It can easily be shown that the ***overall gain*** (the total gain of the auctioneer and all bidders) does not depend on the pay prices, but is equal to the sum of the winning bundle valuations:

$$\begin{aligned}
&\Pi(X, \mathcal{P}_{pay}) + \pi_{all}(X, \mathcal{P}_{pay}) = \\
&\quad \sum_{S \subseteq \mathcal{K}, i \in \mathcal{I}} x_i(S) p_{pay,i}(S) + \sum_{i \in \mathcal{I}} \pi_i(X, \mathcal{P}_{pay}) = \\
&\quad \sum_{S \subseteq \mathcal{K}, i \in \mathcal{I}} x_i(S) p_{pay,i}(S) + \sum_{S \subseteq \mathcal{K}, i \in \mathcal{I}} x_i(S)(v_i(S) - p_{pay,i}(S)) = \\
&\quad \sum_{S \subseteq \mathcal{K}, i \in \mathcal{I}} x_i(S)(p_{pay,i}(S) + v_i(S) - p_{pay,i}(S)) = \\
&\quad \sum_{S \subseteq \mathcal{K}, i \in \mathcal{I}} x_i(S) v_i(S)
\end{aligned}$$

Definition 16. *An* ***efficient allocation*** *is an allocation that maximizes the overall gain. There can exist multiple efficient allocations. An efficient allocation is denoted by* $X^* = (S_1^*, \ldots, S_n^*) = \{x_i^*(S)\}$.

Obtaining an *efficient allocation* is a typical auction design goal. Given the private bidder valuations for all possible bundles, an efficient allocation can be found by solving the ***Combinatorial Allocation Problem*** (***CAP***) (also sometimes referred as *Winner Determination Problem*):

$$\max_{X=(S_1,\ldots,S_n)\in\mathcal{X}} \sum_{i \in \mathcal{I}} v_i(S_i) \qquad \textbf{(CAP)}$$

The CAP has a straightforward integer linear programming formulation using the binary decision variables $\{x_i(S)\}$ (see Figure 2.3). The objective function maximizes the overall gain. The first set of constraints guarantees that at most one bundle can be allocated to each bidder. The second set of constraints ensures that each item is not sold more than once.

It is well known that the CAP can be interpreted as a weighted set packing problem (SPP) (Lehmann et al., 2006) and is therefore NP-hard. The problem has been attracting intense research efforts. Integer programming techniques

$$\max \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) v_i(S)$$

s.t.

$$\begin{array}{rcll} \sum_{S \subseteq \mathcal{K}} x_i(S) & \leq & 1 & \forall i \in \mathcal{I} \\ \sum_{S: k \in S} \sum_{i \in \mathcal{I}} x_i(S) & \leq & 1 & \forall k \in \mathcal{K} \\ x_i(S) & \in & \{0; 1\} & \forall i \in \mathcal{I}, S \subseteq \mathcal{K} \end{array}$$

Figure 2.3: Combinatorial Allocation Problem (CAP) as ILP

can be used to handle the winner determination in combinatorial auctions with a "small enough" number of items. On the other hand, various heuristics and approximation algorithms are likely to produce solutions, which, in most cases, are optimal or close to optimal (Sandholm, 2006). However, suboptimality may not be adequate if a market designer aims for economic efficiency. Rothkopf et al. (1998) discuss limiting biddable combinations which can make the winner determination problem tractable. A survey by de Vries and Vohra (de Vries et al., 2003) addresses the literature of the last few years on algorithmic approaches. Similar computational problems can be found in volume discount auctions (Davenport and Kalagnanam, 2000) and multi-attribute auctions (Bichler and Kalagnanam, 2004).

Additional *business constraints* (see Section 2.3.4) can be easily integrated into the ILP formulation of the CAP, whereas they may be difficult to manage by special algorithms and heuristics. Nevertheless, even when using standard integer optimization techniques (e.g. branch-and-cut algorithms), they can affect the solver running time Kalagnanam et al. (2001).

There might exist multiple optimal solutions of the CAP with the same objective function value, in which case multiple efficient allocations exist. ***Tie-***

$$
\begin{aligned}
\max \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) p^t_{bid,i}(S) \\
\text{s.t.} \\
\sum_{S \subseteq \mathcal{K}} x_i(S) &\leq 1 \qquad \forall i \in \mathcal{I} \\
\sum_{S : k \in S} \sum_{i \in \mathcal{I}} x_i(S) &\leq 1 \qquad \forall k \in \mathcal{K} \\
x_i(S) &\in \{0; 1\} \qquad \forall i \in \mathcal{I}, S \subseteq \mathcal{K}
\end{aligned}
$$

Figure 2.4: Winner Determination Problem (WDP)

breaking rules determine, which of the optimal solutions is selected by the solver. For example, earlier realized allocations or allocations with the maximal number of bidders can be preferred. Tie-breaking rules can be implemented by an arrangement of the bids in the ILP depending on the used solving algorithm, by adding special tie-breaking constraints, or by solving the CAP multiple times excluding already found solutions. Often no special tie-breaking is done, so that an optimal solution is selected coincidentally.

It is important to be aware of the difference between the CAP and winner determination in a real auction. The auctioneer does not know bidders' private valuations needed to solve the CAP. Instead, she selects the winning *provisional* or *end* allocation on the basis of the submitted bids, which may or may not truly reflect the bidders' valuations. In addition, bidding languages define the possible combinations of multiple bids of the same bidder. In case of the *XOR bidding language*, the ***Winner Determination Problem*** (***WDP***) formulation is very similar to the CAP (see Figure 2.4). The only difference is the use of bid prices instead of valuations in the objective function. In case of the *OR bidding language*, the first family of constraints must be removed, since multiple bids of the same bidder can win.

For the following, let X^t denote the provisional allocation calculated on the basis of the bids active in the round t. Further let $W^t \subseteq \mathcal{B}^t$ and $L^t \subseteq \mathcal{B}^t$ denote the sets of provisionally winning and provisionally losing bids in the allocation X^t respectively, with $W^t \bigcap L^t = \emptyset$, $W^t \bigcup L^t = \mathcal{B}^t$. In other words, $b_i(S) \in W^t \Leftrightarrow x_i(S) = 1$.

The end allocation of a combinatorial auction is not always efficient. ***Allocative efficiency*** (or simply ***efficiency***) is commonly used as a primary measure to benchmark auction designs. Allocative efficiency in CAs can be measured as the ratio of the overall gain of the end allocation X to the overall gain of an efficient allocation X^*(Kwasnica et al., 2005):

$$E(X) := \frac{\Pi(X, \mathcal{P}_{pay}) + \pi_{all}(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} \in [0, 1]$$

Another important measure is the ***revenue distribution***, which shows how the overall gain is distributed between the auctioneer and bidders. The revenue distribution is affected by the deviation of the submitted bids from the true valuations and by the pay price calculation, which is also a part of the winner determination. In cases in which the auction is not 100% efficient, still another part of the overall utility is simply lost. Given the end allocation X and the pay prices $\mathcal{P}_{pay}$, the ***auctioneer utility share*** is measured as the ratio of the auctioneer revenue in the end allocation to the overall gain of an efficient allocation:

$$R(X) := \frac{\Pi(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} \in [0, E(X)] \subset [0, 1]$$

The ***total bidder utility share*** corresponds to the ratio of the total bidder utility in the end allocation to the overall gain of an efficient allocation:

$$U(X) := \frac{\pi_{all}(X, \mathcal{P}_{pay})}{\Pi(X^*, \mathcal{P}_{pay}) + \pi_{all}(X^*, \mathcal{P}_{pay})} \in [0, E(X)] \subset [0, 1]$$

Altogether, it follows that $E(X) = R(X) + U(X)$ and the lost part of the overall utility corresponds to $1 - E(X)$. Note that the auction *efficiency* does not depend on the pay prices, thus it is possible for two auction outcomes with equal efficiency to generate significantly different *auctioneer revenues*. The revenue distribution behavior of a particular auction design, if familiar to the bidders, can affect their bidding strategies. The following section deals with strategic considerations of the combinatorial auction design and the impact of pricing on the bidding strategy.

2.5 Equilibrium Prices and the Core

Coalitional game theory and equilibrium theory are strongly related to the combinatorial auction design. Both theories help to understand the structure and outcomes of CA designs and are often used as a guideline for constructing efficient price-based auctions. Game theory studies a system of self-interested players under strategical interaction considerations. The theory of the *core* in the coalitional game can be transferred to the auction theory by interpreting bidders as players and the auctioneer as either one of the players or as the "bank". As for the game theory, bidders follow different strategies, which may result in a desired equilibrium. Auction design rules may restrict bidders to certain strategies to enforce a certain outcome. Information about possible bidding strategies and their weaknesses is an important research field for bidders and auctioneers. Certain strategies can lead to efficient outcomes, and an important auction design goal is to encourage such strategies. In particular, straightforward bidding (truthful preference revelation in response to ask prices) is desirable, since it is cognitive simple for bidders and allows for finding an efficient allocation in a natural way.

Nash-equilibrium is a fundamental concept in game theory. It states that in the equilibrium every player selects the payoff maximizing strategy given the strategies of other players. However, it makes strong assumptions on the knowledge of the other players's strategies and loses its advantages in games with multiple equilibria (Parkes, 2001). A stronger concept is the one of the ***dominant strategy equilibrium***. A ***dominant strategy*** is given if the player's payoff maximizing strategy is independent from the strategies of the other players. Mechanisms with the dominant strategy equilibrium are called ***strategy-proof***. In a strategy-proof mechanism no assumptions about the information available to the agents about each other are made, and every bidder selects her own optimal strategy without requiring the others to act rational. Strategy-proofness is an important auction design goal. Another goal is to end up with an outcome for which no coalition of players is willing to renege once the result is announced - the *core* outcome.

Definition 17. *The* ***coalitional value function*** *is defined as*

$$w(C_I) := \max_{X=(S_1,\dots,S_n)\in\mathcal{X}} \sum_{i\in I} v_i(S_i)$$

for any coalition C_I consisting of the bidders $I \subseteq \mathcal{I}$ and the auctioneer[11].

Thus, the value of the coalitional value function is equal to the maximum overall gain that can be generated by the bidders contained in the coalition, which is also equal to the value of the efficient allocation computed by the CAP.

Definition 18. *The set of* **core payoffs** *is defined as*

$$Core\,(\mathcal{I}, w) = \left\{ (\Pi, \pi) : \Pi + \sum_{i\in\mathcal{I}} \pi_i = w(C_{\mathcal{I}}) \text{ and } \forall I \subset \mathcal{I} : w(C_I) \leq \Pi + \sum_{i\in I} \pi_i \right\}$$

If any payoff vector (Π, π) is not in the *core*, then there exists a coalition C_I for which the total payoff $w(C_I)$ is higher than the members' total payoff in (Π, π).

[11] The auctioneer must always be a member of the coalition since no allocation can be realized without her participation.

That is, there exists some way to redistribute the total payoff that makes all members of C_I strictly better off. In this case the payoff vector (Π, π) is said to be *blocked* by the coalition C_I.

Altogether, the following properties are considered desirable in combinatorial auction design:

- *strategy-proofness*
- *budget-balance*
- *core outcomes*
- *allocative efficiency*

In the following sections I discuss the famous sealed-bid *Vickrey-Clarke-Groves auction*, which is known to fulfill all above properties except it does not always generate core outcomes. I then introduce the notions of prices compatible with an allocation and competitive equilibrium prices and discuss their properties and impact on bidding strategies. I also summarize some important theoretical results regarding the existence of these kinds of prices in different pricing schemes.

2.5.1 Vickrey-Clarke-Groves Auction

The ***first-price sealed-bid auction*** (the sealed-bid auction in which bidders pay exactly what they bid) has been used as a model for some combinatorial auctions in practice (Elmaghraby and Keskinocak, 2002). Similar to the *single-item* sealed-bid auctions, it is resistant to collusion, but exhibits a high strategic complexity for bidders. A famous alternative to the first-price auction mechanisms in the sealed-bid family are ***Vickrey mechanisms***. They are *strategy-proof*, i.e., they make truthful bidding a dominant strategy; they do so by refunding bidders the increase in the overall gain caused by their bids. In other words, they let each bidder pay the social opportunity cost of her winnings, rather than her bid. Provided the bidders' truthful valuations, the auctioneer is able to compute an efficient allocation.[12]

The counterpart of Vickrey auctions in *combinatorial* case is called the ***Vickrey-Clarke-Groves auction*** (***VCG auction***) (a.k.a. ***generalized Vickrey auction***). The auction utilizes XOR-bidding. The allocation is calculated by solving the WDP in XOR-form (see Figure 2.4). The winning bidders pay their bids reduced by the ***VCG discount*** $w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus i})$:

$$p_{pay,i}(S) = p_{bid,i}(S) - \big(w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus i})\big)$$

The pay price in a VCG auction is also called ***VCG payment***.

Example 1. *Let $\mathcal{K} = \{A; B\}$, $\mathcal{I} = \{1, 2\}$ and the bidder valuations be defined by Table 2.2. The efficient allocation is indicated by asterisks, bidder 1 getting*

	A	B	AB
b_1	8^*	9	12
b_2	6	8^*	14

Table 2.2: VCG auction example

item A and bidder 2 getting item B. Bidder 1 has to pay $6 = 8 - (16 - 14)$ for A and bidder 2 has to pay $4 = 8 - (16 - 12)$ for B.

The VCG auction exhibits several appealing theoretical properties. It is *strategy-proof* and *budget-balanced* and finds an efficient allocation. Achieving efficiency in a *strategy-proof budget-balanced* mechanism is remarkable. Nevertheless, there are serious shortcomings:

[12]Assuming the WDP is computationally tractable.

- The outcome of the VCG auction is not always in the core, thus it is vulnerable to collusion by a coalition of losing bidders. Moreover, if the outcome is not in the core, the auctioneer revenue can be very low or zero, even if the sold items are valuable. For a detailed discussion of core selecting auctions see Day and Milgrom (2007).

- It is important that bidders reveal their entire utility function, i.e., they need to submit bids for all $2^m - 1$ possible bundles. This leads to a high *valuation complexity* for the bidders, but also to a large input size of the winner determination problem.

- The determination of the *VCG payments* itself becomes a computationally hard problem, since one instance of an NP-hard problem similar to the WDP must be solved for every winning bidder.

- The monotonicity problem: increasing the competition by adding more bidders might reduce the auctioneer revenue.

- The transparency about the *dominant strategy* is often not given to bidders. Truthful bidding in the VCG auction is not intuitive and bidders often falsify their valuations.

- The need of a trusted auctioneer. A winner in a second price auction has to doubt, whether the pay price is actually the second-highest price. Additionally, bidders need to worry, whether the auctioneer gives their valuations away to other bidders. Cryptographic approaches have been proposed to solve this problem (Brandt, 2003).

The drawbacks of the VCG auction are strong enough for it to be hardly used in practice. Nevertheless, it is an important theoretical construct that provides insights into fundamental properties of auction mechanisms in general. The *VCG auction* is often used as a reference point to derive meaningful statements about other auction designs. For further details on the VCG auction the reader is referred to Ausubel and Milgrom (2006b).

2.5.2 Compatible Prices

The concept of ask prices compatible with an allocation is not common in the auction literature. We introduced this property as it is intuitive and mostly desirable when constructing price-based ICAs. In particular, with linear pricing, compatible prices often do not exist, but can be approximated.

Definition 19 (IBIS)**.** *A set of ask prices $\mathcal{P}_{ask}$ is called **compatible** with the allocation X and bids $\mathcal{B}$ if $\forall i \in \mathcal{I}, S \subseteq \mathcal{K}$:*

$$(x_i(S) = 0 \Leftrightarrow p_{ask,i}(S) > p_{bid,i}(S)) \text{ and } (x_i(S) = 1 \Leftrightarrow p_{ask,i}(S) \leq p_{bid,i}(S))$$

The interpretation is quite intuitive: the set of ask prices is compatible with the given allocation and the given bids if and only if all winning bids are not lower than the ask prices, and all losing bids are lower than the ask prices. This is best visualized by the following example.

Example 2. *Compatible prices.*

There are 2 bidders and 2 items, current bids are given by Table 2.3 (the revenue-maximizing allocation bids are marked by an asterisk).

	A	B	AB
Bidder 1	2	3	5
Bidder 2	1	4	7*

Table 2.3: Compatible prices example

Consider the optimal *(revenue-maximizing) allocation $x_2(AB) = 1$ with the total revenue of* 7*. We can easily construct* non-linear *anonymous compatible prices by setting for instance $p_{ask}(A) = 100$, $p_{ask}(B) = 100$, $p_{ask}(AB) = 7$. On* *the contrary, constructing* linear *compatible prices is not a trivial task. The price set* $p_{ask}(A) = 2$*,* $p_{ask}(B) = 4$ *(with* $p_{ask}(AB) = 6$*) is not compatible, since bidder 1 does not win item A and bidder 2 does not win item B (the compatibility conditions are violated for the bids* $b_1(A)$ *and* $b_2(B)$*). The price set* $p_{ask}(A) = 3$*,* $p_{ask}(B) = 5$ *(with* $p_{ask}(AB) = 8$*) is also not compatible, as bidder 2 does not get bundle AB (the compatibility conditions are violated for the bid* $b_2(AB)$*). A compatible* linear *price set can be found for instance at* $p_{ask}(A) = 2.5$*,* $p_{ask}(B) = 4.5$ *(with* $p_{ask}(AB) = 7$*).*

Now consider a non-optimal *allocation* $x_2(B) = 1$ *with the total revenue of* 4*, which does not allocate item A at all. In this case, (even linear) compatible prices can also be constructed by setting the price of item A high enough, for example* $p_{ask}(A) = 100$*,* $p_{ask}(B) = 4$ *(with* $p_{ask}(AB) = 104$*). Notice also that if an allocation assigns any bundle to a non-highest bidder of this bundle, no compatible prices can be constructed at all. (Such allocation can not be the revenue-maximizing one, though.)*

Compatible prices explain the winners, why they won, and the losers, why they lost. In fact, informing bidders about the allocation is superfluous if prices compatible to the current-round bids are communicated. However, the above example shows that not *every* set of compatible prices provides bidders with a meaningful information for improving their bids in the next round. Another important observation is the fact that *linear* compatible prices are harder and often even impossible (see Example 4 in the following section) to construct if bidders' valuations are super- or subadditive.

If no *linear* prices compatible with the given bids exist, one can try to approximate them by minimizing[13] the ***linear price compatibility distortions*** defined as:

$$\delta_i(S) := \begin{cases} p_{bid,i}(S) - p_{ask,i}(S) \text{ if } x_i(S) = 0 \\ p_{ask,i}(S) - p_{bid,i}(S) \text{ if } x_i(S) = 1 \end{cases}$$

[13]There are several ways of minimizing compatibility distortions (see Section 3.3).

Prices compatible with the given *valuations* can be defined similarly to the prices compatible with the given *bids*:

Definition 20 (IBIS). *A set of ask prices $\mathcal{P}_{ask}$ is called **compatible** with the allocation X and valuations $\mathcal{V}$ if $\forall i \in \mathcal{I}, S \subseteq \mathcal{K}$:*

$$(x_i(S) = 0 \Leftrightarrow p_{ask,i}(S) > v_i(S)) \ \ and \ \ (x_i(S) = 1 \Leftrightarrow p_{ask,i}(S) \leq v_i(S))$$

In case of truthful bidding, ICAs usually terminate with *final* ask prices compatible with the *end* allocation and the bidders' valuations, so that the bidders have no more incentives to continue bidding.

2.5.3 Competitive Equilibrium Prices

The idea behind the concept of *competitive equilibrium prices* is to define prices that characterize an efficient allocation.

Definition 21. *The prices $\mathcal{P}_{pay}$ and allocation $X^* = (S_1^*, \ldots, S_n^*)$ are in **competitive equilibrium (CE)** if:*

$$\begin{aligned} \pi_i(S_i^*, \mathcal{P}_{pay}) &= \max_{S \subseteq \mathcal{K}} [\pi_i(S, \mathcal{P}_{pay}), 0] \quad \forall i \in \mathcal{I} \\ \Pi(X^*, \mathcal{P}_{pay}) &= \max_{X \in \mathcal{X}} \Pi(X, \mathcal{P}_{pay}) \end{aligned}$$

*The allocation X^* is said to be **supported** by the prices $\mathcal{P}_{pay}$ in competitive equilibrium.*

The first condition is equivalent to the compatibility of the prices $\mathcal{P}_{pay}$ with the given allocation X^* and bidders' valuations $\mathcal{V}$ (note that the bundles S_i^* can also be empty). The second condition means that given the prices $\mathcal{P}_{pay}$, there exist no other allocation with a total revenue larger than the revenue of the allocation X^*. Altogether, in CE the utility of every bidder and the auctioneer revenue are maximized at the given prices, and the auction will effectively end, since the bidders will not be willing to change the allocation by submitting any further bids.

In general, one can show that the existence of competitive equilibrium prices implies the optimality of the allocation and that the opposite is also true in case of *non-linear personalized* prices:

Theorem 1 (Bikhchandani and Ostroy (2006)). *The following statements are true:*

1. *If an allocation X^* and prices $\mathcal{P}_{pay}$ are in competitive equilibrium for the given valuations $\mathcal{V}$, then this allocation is the* efficient *allocation.*
2. *For the* efficient *allocation X^* there always exist* personalized non-linear *competitive equilibrium prices $\mathcal{P}_{pay}$. This is not always true for* linear *and* anonymous non-linear *prices.*

The following examples provide a better understanding of the concept of CE prices. Example 3 extends Example 2 and illustrates, which of the constructed price sets are CE, and shows that no CE prices exist for the *non-optimal* allocation. Further, Example 4 and Example 5 demonstrate cases in which no *linear* and even no *anonymous non-linear* CE prices exist for the *optimal* allocation.

Example 3. *Competitive equilibrium prices.*

For the optimal *allocation $x_2(AB) = 1$ with the total revenue of* 7 *in Example 2 we constructed two compatible price sets, so only the second CE condition has to be verified. At the linear prices $p_{ask}(A) = 2.5$, $p_{ask}(B) = 4.5$ the most profitable possibilities are to sell the items either in a bundle for the price of* 7 *or separately for the total price of* $2.5 + 4.5 = 7$. *In both cases this is* *exactly the revenue of the considered allocation, so the prices are in competitive equilibrium. In contrast, the (non-linear) prices* $p_{ask}(A) = 100$, $p_{ask}(B) = 100$, $p_{ask}(AB) = 7$ *are not CE, since the allocation* $x_1(A) = 1$, $x_2(B) = 1$ *with the total revenue of* $100 + 100 = 200$ *would be better than the considered allocation at the current prices.*

For the non-optimal *allocation* $x_2(B) = 1$ *with the total revenue of* 4 *the linear price set* $p_{ask}(A) = 100$, $p_{ask}(B) = 4$ *is either not CE, since the auctioneer can get more revenue by additionally selling item A. Moreover, no CE prices exist for this allocation, since the price of bundle AB has to be larger than 7 to ensure compatibility, but in this case selling bundle AB would bring more revenue than the considered allocation.*

Example 4. *Linear CE prices do not always exist.*

There are 3 bidders and 3 items, the bids are given by Table 2.4 (the bids belonging to the optimal allocation are marked with an asterisk).

	A	B	C	AB	BC	AC	ABC
Bidder 1	60	50	50	**200***	100	110	250
Bidder 2	50	60	50	110	**200**	100	255
Bidder 3	50	50	**75***	100	125	**200**	250

Table 2.4: Linear CE prices example

The optimal allocation is $x_1(AB) = 1, x_3(C) = 1$ *with the total revenue of* 275. *To be compatible with this allocation, prices must satisfy the following inequalities:*

$$\begin{array}{lcr} p_{ask}(A) + p_{ask}(B) & \leq & 200 \\ p_{ask}(C) & \leq & 75 \\ \hline p_{ask}(A) + p_{ask}(B) + 2p_{ask}(C) & \leq & 350 \end{array} \qquad \begin{array}{lcr} p_{ask}(A) + p_{ask}(C) & > & 200 \\ p_{ask}(B) + p_{ask}(C) & > & 200 \\ \hline p_{ask}(A) + p_{ask}(B) + 2p_{ask}(C) & > & 400 \end{array}$$

, which is a contradiction. This proves that no linear *compatible prices (and thus no* linear *CE prices) exist for the optimal allocation in this case. The reason is the strong superadditive bidder valuations of multiple bundles.*

Example 5. *Anonymous non-linear CE prices do not always exist.*

There are 2 bidders and 2 items, the bids are given by Table 2.5 (the bids belonging to the optimal allocation are marked with an asterisk).

	A	B	AB
Bidder 1	0	0	3*
Bidder 2	2	2	2

Table 2.5: Non-linear anonymous CE prices example

The optimal allocation is $x_1(AB) = 1$ *with the total revenue of 3. To be compatible with this allocation,* anonymous *item prices* $p_{ask}(A)$ *and* $p_{ask}(B)$ *both have to be larger then 2. This implies that the auctioneer can get a total revenue of at least 4 by selling the items separately, which is larger then the total revenue of the considered allocation. This proves that no* anonymous *CE prices exist for the optimal allocation in this case. The reason is the extremely strong super- and subadditive bidder valuations of multiple bundles.*

Note that for all considered examples we can easily construct *discriminatory non-linear* CE prices for the optimal allocation. Generally, the more bundle valuations are super- or subadditive and the stronger these super- or subadditivities are, the harder is it to find *linear* or *anonymous non-linear* prices.

The following result further emphasizes the importance of CE prices:

Theorem 2 (Bikhchandani and Ostroy (2002))**.** $(\Pi, \pi) \in Core(\mathcal{I}, w)$ *if and only if there exist personalized non-linear CE prices.*

Theorem 2 states that there is an equivalence between the core of a coalitional game and the set of CE prices. All core outcomes can be priced, and all CE outcomes are in the core (see Figure 2.5).

The discussed properties of CE prices motivate for construction of ICAs that update ask prices in the direction of CE prices until there are no new bids.

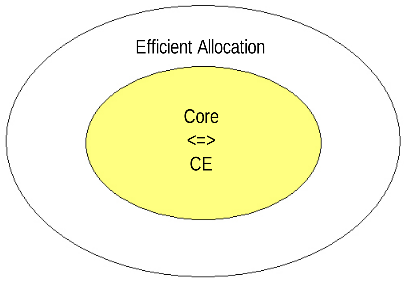


Figure 2.5: Relation between competitive equilibrium, core outcome and efficient allocation

Such an ICA will converge to so called *minimal CE prices*. Generating minimal CE prices is a desirable property, since it usually imposes the incentive compatibility of the auction design. The termination with CE prices that support VCG payments brings straightforward bidding into an ex post equilibrium (Parkes, 2006).

Definition 22 (Minimal CE Prices)**.** ***Minimal CE prices*** $\mathcal{P}_{pay}$ *minimize the auctioneer revenue* $\Pi(X^*, \mathcal{P}_{pay})$ *on an efficient allocation* X^* *across all CE prices.*

If *linear* minimal CE prices exist, they can be found by solving the dual problem of the linear relaxation of the CAP:

$$\begin{aligned}
&\min \sum_i p(i) + \sum_k p(k) \\
&s.t. \\
&p(i) + \sum_{k \in S} p(k) \geq v_i(S) \qquad \forall i \in \mathcal{I}, S \subseteq \mathcal{K} \\
&\quad p(i), p(k) \geq 0 \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}
\end{aligned} \qquad \textbf{(CAP-DLP)}$$

The values of the dual variables quantify the monetary cost of not awarding the item to whom it has been provisionally assigned. This means, the dual variables $p(k)$ can be interpreted as anonymous linear prices, the term $\sum_{k \in S} p(k)$

is then the price of bundle S and $p(i) := max_S\{v_i(S) - \sum_{k\in S} p(k)\}$ is the maximal utility of bidder i at the prices $\{p(k)\}$.

A ***Walrasian equilibrium*** is described as a vector of such item prices for which all items are sold when each bidder receives a bundle in her demand set. Unfortunately, the CAP is a binary program, i.e., a non-convex optimization problem, in which dual prices will usually overestimate the true item values. Example 4 illustrates that *linear anonymous* CE prices do not exist for certain types of bidder valuations. Kelso and Crawford (1982) show that the *goods-are-substitutes* property is a sufficient and an almost necessary[14] condition for the existence of *linear anonymous* CE prices.

Definition 23. ***Demand set*** *includes all bundles that maximize the bidder's utility at the given prices:*

$$D_i(\mathcal{P}_{pay}) := \left\{ S \subseteq \mathcal{K} : \pi_i(S, \mathcal{P}_{pay}) \geq \max_{T\subseteq\mathcal{K}} \pi_i(T, \mathcal{P}_{pay}) \text{ and } \pi_i(S, \mathcal{P}_{pay}) \geq 0 \right\}$$

Definition 24. *Valuations $\mathcal{V}$ satisfy the* ***goods-are-substitutes condition*** *(a.k.a.* ***gross substitutes condition****) if for all linear price sets $\mathcal{P}_{pay}$, $\mathcal{P}'_{pay}$ such that $\mathcal{P}'_{pay} \geq \mathcal{P}_{pay}$ (component-wise) and all $S \in D_i(\mathcal{P}_{pay})$ there exists $T \in D_i(\mathcal{P}'_{pay})$ such that $\{k \in S : p_{pay,i}(k) = p'_{pay,i}(k)\}$.*

Intuitively, this condition implies that the bidder will continue to demand the items which do not change in price, even if the prices on other items increase. However, the *goods-are-substitutes* condition is very restrictive, as most known practical applications of combinatorial auctions rather deal with complementary goods.

By adding additional constraints for each set partition of items and each bidder to the CAP the formulation can be strengthened, so that *personalized*

[14]For details see Parkes (2006).

non-linear prices can be derived from the respective dual problem. Such a formulation describes every feasible solution to an integer problem and is solvable with linear programming resulting in personalized non-linear CE prices (Bikhchandani and Ostroy, 2002). Although, such prices do always exist, such an approach is not practical for larger CAs.

Several ICA designs attempt to result in VCG payments. Minimal CE prices and VCG payments typically differ. Bikhchandani and Ostroy (2002) show that the *bidders-are-substitutes condition* is necessary and sufficient to support VCG payments in competitive equilibrium.

Definition 25. *The* ***bidders-are-substitutes condition (BSC)*** *is satisfied if* $\forall I \subseteq \mathcal{I}$*:*

$$w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus I}) \geq \sum_{i \in I} \left[w(C_{\mathcal{I}}) - w(C_{\mathcal{I}\setminus i}) \right]$$

If the BSC fails, VCG payments are not supported in any price equilibrium and truthful bidding is not an equilibrium strategy. A bidder's payment in the VCG mechanism is always less than or equal to her payment at any CE price. Furthermore, the BSC is not sufficient for an ascending auction to terminate with VCG prices and Ausubel and Milgrom (2006a) show that it requires the slightly stronger *bidder-submodularity condition* for an ascending proxy auction to implement VCG payments.

Definition 26. *The* ***bidder-submodularity condition (BSM)*** *is satisfied if* $\forall I \subseteq I' \subseteq \mathcal{I}$ *and* $\forall i \in \mathcal{I}$*:*

$$w(C_{I \cup i}) - w(C_I) \geq w(C_{I' \cup i}) - w(C_{I'})$$

de Vries et al. (2007) show that under the BSM their *primal-dual auction* (see Section 2.6.1) yields VCG payments. If the BSM condition does not hold, the property breaks down, and the *myopic best-response strategy* is likely to lead bidders to pay more than the optimal prices for the winning packages (Dunford et al., 2007).

2.6 Price-Based Iterative Combinatorial Auctions

Many *iterative* combinatorial auction designs utilize ask prices to guide bidding. In fact, every *centralized* ICA design familiar to me is price-based.[15]. Ask prices define the lower bound or the exact value (if *jump bidding* is not permitted) for bid prices, inform bidders about the current competition and help the auction to end up with an efficient allocation. In most price-based ICAs bidders pay what they bid (pay prices are equal to the bid prices) and the auction is designed to converge to either exact or approximated minimal CE prices.

Price-based ICA designs differ mostly by the *pricing scheme* and *price update rules*. Price update rules can be categorized as follows:

Greedy increase: Ask prices are increased on some arbitrary set (perhaps all) of the over-demanded items or bundles.

Minimal increase: Ask prices are increased on a *minimal set of overdemanded items*, or based on the bids from a set of *minimally undersupplied bidders* (de Vries et al., 2007; Parkes, 2006).

Bid-based update: Ask prices are calculated on the basis of submitted bids by some price calculation algorithm, which tries to find a good approximation to CE prices given current bids. Ask prices may sometimes fall in this case.

The pricing scheme is a key element of an ICA price-based design. On the one hand, *personalized non-linear* prices are used in *primal-dual auctions*, which try to use the result of the Theorem 1 and converge to exact minimal

[15] For more details on some decentralized ICA designs see Parkes (2006).

CE prices.[16] On the other hand, *linear-price auctions*, which end up with approximate minimal CE prices, avoid several shortcomings of primal-dual auctions and produce impressive efficiency results.

2.6.1 Primal-Dual Auction Designs

The fundamental work of Bikhchandani and Ostroy (Bikhchandani and Ostroy, 2002) demonstrates the strong interrelationship between iterative auctions and primal-dual linear programming algorithms. A strengthened form of the CAP is used as the primal problem, whereas the provisional allocation is interpreted as a feasible primal solution, and personalized non-linear prices are interpreted as a feasible dual solution. Bids provide sufficient information to formulate and solve *restricted* primal and dual problems, the *winner determination* and *price update* problems respectively. ***Primal-dual*** algorithms maintain a feasible allocation and a feasible price set and terminate as the efficient allocation and CE prices are found.

Primal-dual auctions assume the ***(myopic) best-response bidding strategy*** or some similar concept as it is straightforward and is justified by the termination with minimal CE prices or *VCG payments*. With this strategy, every bidder locally maximizes her utility by bidding for all bundles contained in her current *demand set* exactly at current ask prices (Parkes, 2001).

The competitive equilibrium conditions (see Definition 21) exactly correspond to the *complementary-slackness conditions* in duality theory and, therefore, indicate the algorithm termination with an optimal solution. Note that they can be directly verified by the auctioneer if bidders follow the myopic best-response bidding strategy. In fact, given an allocation and a price set, the auctioneer can verify the second CE condition without knowing the bidders'

[16] There are also similar designs that converge to other kinds of CE prices, e.g., to so called *universal* CE prices. For details see Parkes (2006).

valuations. The first CE condition is satisfied (assuming myopic best-response bidding) if all current loosing bids are lower than the prices, all current winning bids are not lower than the prices, and no new bids are submitted in the next iteration.

The general constructive scheme of primal-dual auctions[17] can be outlined as follows:

1. Choose minimal initial prices (usually set them to 0).
2. Provide the bidders with a bidding language that is expressive for straightforward bidding. Announce current ask prices and collect bids (*jump bidding* is sometimes allowed, but usually prohibited).
3. Compute the current dual solution by interpreting the bid prices as dual variables. Try to find a feasible allocation (a feasible primal solution) that minimizes the violation of the *complementary-slackness conditions*.
4. Terminate if the *complementary-slackness conditions* are satisfied (and, therefore, with CE prices) and use the last provisional allocation as the end allocation. Otherwise adjust prices to make progress towards an optimal dual solution that satisfies these conditions and go back to 2.

The *price update rule* is a key design feature, it differs considerably among the auction designs. In some auctions, prices are increased on a *minimal set of overdemanded items* or based on the bids from a set of *minimally undersupplied bidders* (de Vries et al., 2007; Parkes, 2006). Usually, a fixed price increment is used. The prices are usually increased in a way that the second CE-condition (see Definition 21) remains valid throughout the auction. In this case the auction finds CE prices as soon as no more overdemanded bundles exist. (All

[17]de Vries and Vohra (2003) further distinguish between "pure primal-dual" and "subgradient" auction algorithms. I refer to both auction families as to primal-dual auctions.

bids that are still valid at the current ask prices belong to the provisional allocation.) This condition is often used as the *termination rule*.

Some *primal-dual auction* implementations use *anonymous* bundle prices to reduce the computational, communicational, and cognitive complexity. Though *anonymous* bundle CE prices do *not always* exist, such auctions were shown to produce good efficiency results. Additionally, Parkes (2001) derives the ***bid safety condition***, which makes it possible to determine the necessity of the price personalization dynamically and to switch to personalized prices only as it is required.

There is a lot of theory on *primal-dual auctions*, which is out of the scope of this thesis. The interested reader is referred to Parkes (2001), de Vries and Vohra (2003) and Bikhchandani and Ostroy (2006). However, primal-dual auctions are hardly used in practice[18] for several reasons:

- The very low speed of convergence, which can require hundreds of rounds for the auction to complete, as our own[19] and other experiments have shown (Dunford et al., 2007).
- The BSC can often fail in realistic settings (Parkes, 2001, chap. 7). de Vries et al. (2007) show that when at least one bidder has a non-substitutes valuation, an ascending CA cannot implement the VCG outcome. In these cases VCG payments are not supported in any price equilibrium, and truthful bidding is not an equilibrium strategy (Parkes, 2006).
- The *best-response bidding strategy* can be hardly realized by bidders if *activity rules* are used. The performance of primal-dual auction designs

[18]A notable exception is the indirect use of primal-dual auctions in some implementations of proxy agents, e.g., in the Clock-Proxy Auction.

[19]The results of the referred experiments are not yet published.

for general valuations and non-myopic bidding strategies is not well studied yet. Our own computational experiments (see Chapter 4) have shown that with heuristic bidding behavior (e.g., bidders selecting randomly 3 out of the 10 best bundles in each round) the efficiency of primal-dual auctions can be very low, while *linear-price auctions* are robust against these and other bundle bidding strategies.

- Almost all bidders' valuations have to be revealed throughout the auction to end up with an efficient allocation.

Both, the large number of auction rounds and the need for the best-response bidding strategy require proxy agents. All valuations need to be provided to the proxy agent up-front or throughout the auction. The proxy agents need to be hosted by a trusted third party, which can be a considerable disadvantage in many settings. Also, the use of personalized prices might be perceived as unfair and also confusing by bidders, since they lack the information about the current competition.

2.6.2 Linear-Price Auctions Designs

Although the existence of exact *linear* CE prices is limited, there are several proposals for auction designs with linear prices. Currently, no formal equilibrium analysis for such prices exists, but they exhibit a number of very useful properties and have performed well in the laboratory:

- Linear prices are easy to understand for bidders. Simplicity of the feedback given to bidders is very important to many practical application domains.
- Only a linear number of prices has to be communicated in each round.

- One can use linear prices to compute the value of any other bundle, even if no bid was submitted for this bundle in previous rounds (Kwon et al., 2005). This gives bidders an indication, which items and bundles will be expensive and for which there is low competition.

- Overall, dual prices in linear programming are only valid within bounds under ceteris paribus conditions, when no new bids are submitted. A single new bid can completely change the allocation, and the previously losing bids may become winning. Therefore, such pricing information is best viewed as a guideline for bidders, informing them about what it would take for a bid to have some possibility of winning in the next round.

- Problems of approximate linear prices occur when ask prices violate the price compatibility condition. While this can be confusing for bidders, if ask prices are viewed as a guideline and minimum bid price this does not necessarily have to impact the efficiency of the auction.

These arguments motivate further analysis of ICA designs with linear prices, which was the central topic of our research.

Chapter 3

Selected Auction Designs

This chapter briefly describes the combinatorial auction designs discussed in this thesis. Since we focused on *linear-price auctions*, most considered auction designs are based on linear prices. The well-known *Combinatorial Clock* (*CC*) auction and *Resource Allocation Design* (*RAD*) belong to this family. Additionally, *Approximate Linear PriceS* (*ALPS*) auction, developed by us, is discussed. *ALPS* is an extension of the *RAD* design that significantly improves several auction characteristics, in particular the allocative efficiency, robustness, etc. (See Chapter 4 and Chapter 6). Furthermore, one member of the *primal-dual auctions* family, *iBundle*, is introduced. The *Vickrey-Clarke-Groves auction* has already been discussed in Section 2.5.1.

3.1 Combinatorial Clock (CC) Auction

The ***Combinatorial Clock*** (***CC***) auction has been proposed in Porter et al. (2003) and later reused by a similar ***Clock-Proxy*** auction design in (Ausubel et al., 2006). From the auction designs discussed in this thesis, this is the only one that supports auctioning multiple units of the same item. Bidders are allowed to bid on bundles containing specified quantities of every contained item.

The auction uses anonymous linear prices called *item clock prices*. In each round bidders submit bids, which express their desired packaged item quantities at the current ask prices. Jump bidding is not allowed, thus bidders always bid at the current prices.

As long as the overall demand exceeds supply for at least one item, the price clock "ticks" upwards for those items (the item prices are increased by a fixed price increment), and the auction moves on to the next round. No *provisional allocation* is calculated and *all* bids remain active throughout the auction. We call bids submitted in the current round and bids for which the ask price of the corresponding bundle did not increase ***standing bids***. In other words, a bid remains *standing* for the next round, if there is no (more) competition of the underlying items. We call a bidder ***standing*** if she has at least one standing bid.

As soon as there is no excess demand (but there can be excess supply if several bidders simultaneously reduced their demands on some item), the auctioneer solves the *winner determination problem* considering all bids submitted during the auction runtime. If the computed allocation does not generate *excess demand* or does not displace any *standing bidder* (depending on the *termination rule*, see below), the auction terminates with the computed allocation. Otherwise ask prices of the respective items are increased and the auction continues.

The advantages of the CC auction are its cognitive, computational, and communicative simplicity. However, this can be a tradeoff for efficiency losses. One kind of inefficiencies can be visualized by the following example:

Example 6. *There are 3 bidders and 3 items, the bidder valuations are given by Table 3.1.*

	A	B	C	AB	BC	AC	ABC
Bidder 1							**5***
Bidder 2				2			
Bidder 3			2				

Table 3.1: CC auction inefficiencies example - Valuations

The efficient allocation is to sell bundle ABC to bidder 1 for the total revenue of 5. The progress of this auction is illustrated in Table 3.2.

	Ask Prices			Bids		
	Item A	Item B	Item C	Bidder 1	Bidder 2	Bidder 3
Round 1	1	1	1	(ABC, 3)	(AB, 2)	(C, 1)
Round 2	2	2	2	—	—	(C, 2)
End allocation	—	—	—	—	(AB, 2)	(C, 2)

Table 3.2: CC auction inefficiencies example - Progress

The auction would allocate package AB to bidder 2 and item C to bidder 3 for the total revenue of 4, which is not efficient. This happens due to the price update rule, since the price of package ABC rapidly increases from 3 to 6 in the second round, so that bidder 1 has no chance to reveal her true valuation of 5.

Neither Porter et al. (2003) nor (Ausubel et al., 2006) examine the question, which bidding languages are appropriate for the CC auction design. Porter et al. (2003) briefly mentions that multiple bids of the same bidder cab simultaneously win, whereas (Ausubel et al., 2006) does not discuss the bidding language question explicitly, but, in our understanding, expects XOR bidding. In fact, the CC auction is capable of handling both OR and XOR bidding languages, even in the multi-unit case. However, the overall demand calculation, price update, and termination rules have to be fine-tuned to avoid a couple of potential design problems.

First of all, the demand generated by a single bidder has to be calculated in different ways in OR and XOR cases. With OR bidding, demands produced by different bids on the same item by the same bidder have to be aggregated, as long as they do not exceed the total supply on that item. In contrast, in XOR case only the maximum demand for every item has to be considered.

Furthermore, in case when some *non-standing* bids are selected as *winning* by the winner determination algorithm, they have to be included in the overall demand calculation. Otherwise the auction can get stalled since ask prices will not increase and the termination rule can not be properly verified.

Finally, the termination rule has to be carefully defined. Especially in the XOR case, the "all standing bids must win" rule is clearly inappropriate. With the correct overall demand calculation (as described above) the termination rule "no excess demand is generated by the winner determination" does well in all cases. An alternative is to terminate as soon as every standing bidder wins at least one bid (either standing or not). This termination rule is equivalent to the previous one in XOR case. In OR case, the auction can close even if there are some OR-bidders winning some bundles, but willing to continue bidding to win more bundles.

The most common *activity rule* in clock auctions is monotonicity in quantity. As ask prices rise, quantities cannot increase. A weaker activity requirement is monotonicity in *aggregate* quantity across all items. This allows full flexibility in shifting quantity among different items (Ausubel et al., 2006). We used this rule in the laboratory experiments for two reasons. First, in single-unit case it corresponds to the activity rules of RAD and ALPS and, therefore, improves the comparability of different designs. Second, we did not find an appropriate way to explain the more complex *revealed preference* activity rule proposed in Ausubel et al. (2006) in the context of the Clock-Proxy Auction to bidders. This rule is also hard to implement, since the whole bidding history (not only the last round) has to be considered.

3.2 Resource Allocation Design (RAD)

The ***Resource Allocation Design*** (***RAD***) proposed in Kwasnica et al. (2005) also uses anonymous linear *ask prices*. However, instead of increasing ask prices incrementally, the auction lets bidders submit priced bids (*jump bidding* is allowed) and calculates so called *pseudo-dual prices* based on the LP relaxation of the CAP (Rassenti et al., 1982). The dual price of each item measures the cost of not awarding the item to whom it has been allocated in the last round. Unless the LP relaxation is integral, RAD uses a restricted dual formulation to derive approximate or pseudo-dual prices *compatible* with the current provisional allocation after each auction round. In the next round losing bidders have to bid not less than the sum of ask prices for a desired bundle plus a fixed price increment. A detailed discussion of the price calculation rules in RAD and ALPS can be found in Appendix A.

RAD suggests OR *bidding language* and only winning bids remain active in its original design. In our work we have enforced all the original RAD rules, but used the XOR bidding language for comparability reasons and to avoid side effects caused by the exposure problem[1].

The strength of the RAD design lies in its cognitive simplicity for bidders and its dynamic ask price computation algorithm. Pseudo-dual ask prices are usually compatible with the provisional allocation, reflect the current competition in the market, and lead the auction to approximate *minimal CE prices*. Additionally, prices can be fine-tuned to reduce the threshold problem, etc. However, the original RAD price calculation algorithm has a couple of pitfalls, see Section 3.3 and Appendix A.

Since ask prices may sometimes fall in RAD, the auction termination relies only on its *activity rules* defined as in the *Simultaneous Multi-Round Design* (*SMR*).

[1]With OR bidding a bidder can win several bids and receive items with sub-additive valuations. This makes the bidding strategy more complex and can cause inefficiencies.

Most notably, the rules enforce monotonicity in aggregate quantity, i.e., a bidder is not allowed to bid on an increasing number of items in subsequent rounds.

For further details on RAD the reader is referred to Kwasnica et al. (2005).

3.3 Approximate Linear PriceS (ALPS)

The ***Approximate Linear PriceS*** (***ALPS***) auction design and its modification ***ALPSm*** were developed at our university department. ALPS is largely based on, but extends the RAD design. The strength of RAD lies in its simplicity and flexibility for bidders. Ask prices serve as a guideline to discover new and interesting bundles and allow for submission of bid prices. Also for novice bidders, linear prices are straightforward to use and intuitive. However, RAD faces a few design problems. Most importantly, activity and termination rules can lead to premature termination and inefficiencies. Also, there are ways to further decrease ask prices to better approximate *minimal CE prices* in the end allocation. ALPS is based on similar auction rules with a number of modifications:

Calculation of linear ask prices: ALPS calculates pseudo-dual prices, but modifies the rules specified in RAD to better minimize and balance prices and *price compatibility distortions*. We found this to have a modest, but positive impact on the auction efficiency and to better approximate *minimal CE prices* in the end allocation. Additionally, the price calculation algorithm was adopted to better support XOR bidding.

Termination rule: The termination rule has been adapted, since it is a potential cause of inefficiency in RAD. The auction terminates if there are no new bids submitted in the last round. To ensure the auction progress, ALPS increases ask prices if the provisional allocation does not change in two consecutive rounds, whereas in ALPSm every bidder has to outbid her bids from previous rounds on the same bundle.

Surplus eligibility: Many auction scenarios suffer from the problem that the RAD activity rule does not allow for an increase in the number of distinct items a bidder is bidding on. In particular, when auctioning transportation services, it can become beneficial to bid on a longer route in later rounds. We have modified the RAD activity rule to allow active bidders to increase the number of items they bid on.

ALPS supports both OR and XOR bidding languages. A detailed description of the ask price calculation, termination rules, and activity rules is given in Appendix A.

In addition to above extensions, we found the *old-bids-active rule* (see also Section 2.3.3) to have a significant effect on the auction outcome:

ALPSm old-bids-active rule: In RAD and ALPS, only provisionally winning bids remain active in the subsequent round. In a modified version of ALPS, called ***ALPSm***, *all* bids submitted throughout the auction remain active even if they are provisionally losing. This rule was shown to provide a significant positive effect on the allocative efficiency.

We have also experimented with *last-and-final bids* as described in Section 2.3.3 and with *per-bundle* price increments (as opposite to *per-item* price increments), but could not find a significant positive impact on the efficiency in the computational experiments.

3.4 iBundle

Several authors have proposed auction designs based on non-linear, usually personalized prices. These designs are mostly based on the primal-dual approach. The ***Ascending Proxy*** auction has been proposed in the context of the FCC spectrum auction design (Ausubel and Milgrom, 2006a). It uses personalized non-linear prices and is similar to the ***iBundle*** design by David Parkes (Parkes, 2001), although Ausubel and Milgrom (2006a) emphasize proxy agents, which essentially lead to a sealed-bid design. Both designs achieve an efficient outcome with *minimal CE prices* and Vickrey payments if the BSM condition is satisfied. The ***dVSV*** auction design by de Vries et al. (2007) is also similar to iBundle, but differs in the *price update rule*.

de Vries et al. (2007) also show that there cannot be an ascending combinatorial auction with Vickrey outcomes for private valuation models without restrictions. Newer approaches, such as the one by Mishra and Parkes (2007) try to overcome this negative result by extending the definition of ascending price auctions, e.g., by multiple price paths or discounts on the quoted bid prices upon termination. Most problems discussed in Section 2.6.1, however, remain. In addition, Vickrey outcomes are not in the core for general valuations.

We exemplary selected *iBundle*, developed by David Parkes, as a member of the primal-dual auction family, because it does not require proxy bidding, and the price update rule is easy to understand and to implement. *iBundle* follows the general primal-dual auction scheme described in Section 2.6.1 with the auction rules defined as follows:

- **Bidding languages:** The *XOR bidding language* is used.
- **Pricing scheme:** Non-linear ask prices are used. Three different modifications are proposed: one with personalized prices, another with anonymous prices, and the third introduces price personalization dynamically as it is required. Ask prices are used as minimum or exact bid prices (depending on the *jump bidding* option). *Last-and-final bids* can also be allowed.

- **Price update rules:** The price of every bundle contained in some bid of some provisionally losing bidder is increased to the corresponding (with personalized prices) or highest unsuccessful (with anonymous prices) bid price plus the price increment Δ. Additionally, the prices are hold consistent with the free disposal assumption, so that $p_{ask,i}(S) \geq p_{ask,i}(T)\ \forall S, T : T \subseteq S$.

- **Bid activity rules:** Provisionally winning bids are kept active for the next round. Provisionally losing bids are deactivated.

- ***Activity rules*:** No activity rules apply.

- ***Termination rules*:** The auction closes as soon as every standing (having new bids in the current round or previous-round-winning) bidder wins a bundle.

As the standard version of iBundle is a theoretical construct not intended to be used in practical applications, we contacted David Parkes and together selected the following additional set of rules, to adjust the iBundle mechanism to be used with human bidders:

- We used **iBundle with personalized prices** to reduce the cognitive complexity for bidders without risking to cause efficiency losses.

- ***Jump bidding*** was allowed.

- **Bid activity rules:** The *all-bids-active rule* was used, though it is not considered in the standard version.

- ***Activity rules*:** We introduced the following activity rule, not considered in the standard version: a bidder has to submit at least one new bid in a round (except she provisionally wins) to be able to submit further bids in following rounds.

3.5 Summary

An overview of the key properties of the ICA designs discussed in this chapter is given in Table 3.3. The auctions describe vastly different approaches, and their comparison is a difficult task. In addition to allocative efficiency, ICAs need to fulfill a number of criteria to be applicable in a wide range of domains. Above all, the design should be simple in the sense of easy to understand rules, easy to interpret information feedback, as well as a reasonable number of auction rounds. These requirements pose a number of engineering challenges to auction designers, and ICAs described so far have pros and cons with respect to these goals. In our research, we compared selected auction designs in different settings by means of computational experiments under various assumptions about value models and bidding strategies, as well as in laboratory experiments with human bidders. The results of this work are presented in Chapter 4 and Chapter 6.

There is a couple of further centralized iterative auction designs that were not considered, in particular auctions that involve proxy agents. For example, the *Clock-Proxy* auction (Ausubel et al., 2006) is an interesting design, that extends the CC auction by a last-and-final ascending proxy auction round. We did not specifically consider these auction designs in our analysis, since bidding strategies of bidders in *iterative auctions with proxy agents* are theoretically less understood and cognitive more complex for bidders. On the other hand, the results of this work are also highly valuable for constructing auctions with proxy agents, since they are mostly based on non-proxy designs.

	CC auction	**RAD**	**ALPS(m)**	**iBundle**
Bidding lang.	OR and XOR	OR and restricted XOR	OR and XOR	XOR
Pricing scheme	anonymous linear	anonymous linear	anonymous linear + opt. overbid old bids	personalized non-linear (personalization can be introduced dynamically)
Feedback	ask prices	ask prices and own winning bids	ask prices and own winning bids	ask prices and own winning bids
Price used as	exact bid price	minimal bid price (price increment excluded)	minimal bid price (price increment excluded)	[minimal bid price] or [exact bid price]
Price updates	increase on all overdemanded items	compute based on submitted bids, can decrease	compute based on submitted bids, can decrease	increase on all bids for all losing bidders
Bid validity	all bids active	current round + provisionally winning	[current round + provisionally winning] or [all bids active]	[current round + provisionally winning] or [all bids active]
Activity rules	monotonicity in aggregate quantity	monotonicity in aggregate quantity	monotonicity in aggregate quantity + opt. activity bonus	[no activity rules] or [at least one new bid per round]
Termination rules	[no overdemand generated by WD] or [no standing bidder displaced]	based on activity rules	no new bids	allocation contains every standing bidder

Table 3.3: Iterative combinatorial auctions overview

Chapter 4

Computational Experiments

This chapter presents the setup and results of the computational experiments, conducted using our *simulation framework*. We compared the CC auction, RAD, ALPS, and the Vickrey-Clarke-Groves auction[1] in different settings under various assumptions about value models and bidding strategies. We discovered some interesting facts regarding the allocative efficiency, revenue distribution, price monotonicity, and speed of convergence of different designs and analyzed their robustness against selected pure and mixed bidding strategies.

4.1 Experimental Setup

A simulation instance is configured by a combination of a value model, bidding agent and auction processor. A *value model* defines the set of valuations of every bundle for every bidder. A ***bidding agent*** implements a *bidding strategy* adhering to the given value model and to the restrictions of the specific auction design. An ***auction processor*** implements the auction logic, enforces

[1]Computational experiments with iBundle are not included, since the results are still being analyzed.

auction protocol rules, and calculates allocations and ask prices. Different implementations of value models, bidding agents, and auction processors can be combined, which allows to perform sensitivity analysis by running a set of simulations while changing only one component and preserving all other parameters. The architecture of the *simulation framework* is described in detail in Appendix D.

4.1.1 Value Models

The type of bidder valuations is an important treatment variable for the analysis of different auction designs (see Section 2.5). The performance of an auction design can significantly depend on the valuation properties, in particular on the *bidders-are-substitutes* and *bidder-submodularity* conditions, which are often not satisfied in practical settings. Since there are hardly any real-world CA data sets available, we have adopted the ***Combinatorial Auctions Test Suite*** (***CATS***) value models, which have been widely used for the evaluation of winner determination algorithms (Leyton-Brown et al., 2000).

In the following, a *value model* is defined as a function that generates realistic, economically motivated combinatorial valuations on all possible bundles for all bidders. For example, a transportation network, real estate lots, and an airport slot occupancy timetable provide the underlying rationale. In addition to CATS value models, we used the Pairwise Synergy value model from An et al. (2005). In all value models we assume *free disposal* (see Section 2.4).

The ***Transportation*** value model uses the *Paths in Space* model from CATS. It builds a nearly planar transportation graph in Cartesian coordinates, in which every bidder is interested in securing a path between two randomly selected vertices (cities). The items traded are the edges (routes) of the graph. The parameters for the Transportation value model are the number of items (edges) m and the graph density ρ that defines the average number of edges per city and is used to define the number of vertices as $(m*2)/\rho$. The bidder's valuation for a path is defined by the Euclidean distance between the two nodes multiplied by a random number, drawn from a specific uniform distribution. As every profitable bundle contains a path between the two selected cities, only a limited number of bundles is valuable for the bidder. This allows to consider even larger transportation networks in a reasonable time.

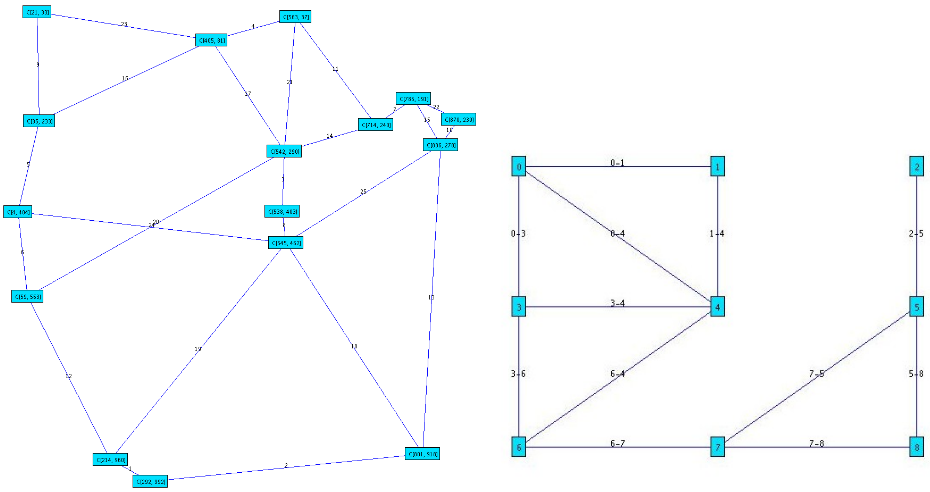
The ***Pairwise Synergy*** value model in An et al. (2005) is defined by a set of valuations of individual items $\{v_i(k)\}$ with $k \in \mathcal{K}$ and a matrix of pairwise item synergies $\{syn_{k,l} : k, l \in \mathcal{K}, syn_{k,l} = syn_{l,k}, syn_{k,k} = 0\}$. The valuation of a bundle S is then defined as:

$$v_i(S) := \sum_{k \in S} v_i(k) + \frac{1}{|S|-1} \sum_{k \in S} \sum_{l \in S, l \neq k} syn_{k,l}(v_i(k) + v_i(l))$$

A synergy value of 0 corresponds to completely independent items, and a synergy value of 1 means that the bundle valuation is twice as high as the sum of the individual item valuations. The relevant parameters for the Pairwise Synergy value model are the interval for the randomly generated item valuations and the interval for the randomly generated synergy values.

The ***Matching*** value model is an implementation of the *matching* scenario in CATS. It models the 4 largest USA airports, each having a predefined amount of starting and landing time slots. For simplicity, there is only one slot for each time unit available. Every bidder is interested in obtaining one starting and one landing slot (i.e. item) in two randomly selected airports. Her valuation is proportional to the distance between the airports and reaches its maximum as the landing time matches a certain randomly selected value. The valuation is reduced, if the landing time deviates from this ideal value, or if the time between the starting and landing slots is longer than necessary.

The ***Real Estate*** value model is based on the *Proximity in Space* model from CATS. The items sold in the auction are real estate lots k, which have



(a) Transportation (b) Real Estate 3x3

Figure 4.1: Transportation and Real Estate value models

valuations $v_i(k)$ drawn from the same normal distribution for every bidder i. The adjacency relationships between every two pieces of land k and l denoted by e_{kl} are generated randomly. The edge weights $w_i(e_{kl}) \in [0, 1]$ are then generated randomly for every bidder and used to define the bundle valuations of adjacent pieces of land as follows:

$$v_i(S) := \left(1 + \sum_{e_{kl}:k,l \in S} w_i(e_{kl})\right) \sum_{k \in S} v_i(k)$$

4.1.2 Bidding Agents

A ***bidding agent*** implements a bidding strategy adhering to the given value model and to the rules of the used auction design. In our simulations, we considered six different agent behaviors. Some of them represent extreme cases of completely bundle-unaware (*naïve*) bidders or intelligent bidders who evaluate all possible bundles (*best-response* and *power-set* bidders). Other agents implement some bundle selection heuristics, which might closer resemble real bidder behavior.

The ***naïve*** bidder does not use bundle bids at all, but rather submits singleton bids for the items that provide a positive utility at the current ask prices. In contrast to all other bidder types, this bidder uses the *OR* bidding language.

The (myopic) ***best-response*** or ***straightforward*** bidder is often assumed in the game-theoretical analysis (Parkes and Ungar, 2000a). This bidder submits bids for the bundles that maximize her surplus at the current ask prices. In other words, the bidder bids exactly for her current *demand set* (see Definition 23). Determining the demand set requires advanced computational skills.

The ***power-set*** bidder evaluates all possible bundles and submits bids for the 10 most profitable ones given the current ask prices. In contrast to the best-response bidder, the power-set bidder does not only select the bundles with the maximum profit, but also less profitable ones.

The ***heuristic*** bidder is close to the power-set bidder, but randomly selects 3 of the 10 most profitable bundles (***3of10 bidder***) or 5 of the 20 most profitable bundles (***5of20 bidder***) she can bid on.

for each $k \in \mathcal{K}$
    1) Create a single-item bundle $B_k = \{k\}$
    2) Define $\alpha = argmax_{l \in \mathcal{K} \setminus B_k} AU(B_k \cup \{l\})$
    3) if $AU(B_k \cup \{\alpha\}) > AU(B_k)$
            then $B_k = B_k \cup \{\alpha\}$, goto 2)

Figure 4.2: Best-chain bidder algorithm

The ***best-chain*** bidder is similar to the INT bidder from An et al. (2005). It implements the algorithm shown in Figure 4.2. Starting from every individual item $k \in \mathcal{K}$, the algorithm finds another item that provides the maximum increase in the average unit utility (AU) of the bundle given the current ask prices. If the new average utility exceeds the previous value, the new item is added to the bundle and the process continues until the average unit utility cannot be further increased.

4.2 Efficiency and Revenue Analysis

In the first set of the simulations our goal was to compare the performance of different ICA designs based on various value models. We were interested in the efficiency and revenue figures using only *best-response* bidders and a small price increment. The results provide an estimate of efficiency losses that can be attributed to the auction design, and, in particular, to the linearity of ask prices.

4.2.1 Efficiency of Different ICA Designs

We used seven value models to compare the CC auction, RAD, RAD without eligibility (***RADne***), ALPS, and ALPSm designs. For every value model we created 40 instances with different valuations and conducted one auction for every combination of the value model instance and auction design. All auctions used a bid increment of 0.1. The auction setup details and average results, aggregated over all instances of the same value model, are shown in Table 4.1. The left-hand column indicates the auction setup, i.e., the number of items, value model, number of bidders, and number of auctions in which the valuations fulfill the BSC (in most cases the BSC was not fulfilled).

Real Estate 3x3 describes the real-estate model with 9 lots and 5 bidders. Individual item valuations are normally distributed with a mean of 10 and variance of 2. There is a 90% probability of a vertical or horizontal edge and an 80% probability of a diagonal edge. The distribution of the edge weights has a mean of 0.5 and a variance of 0.3. 16 value model instances out of 40 fulfill the BSC. The lot valuations in the *Real Estate 4x4* model with 16 lots

ICA Design / Value Model		ALPS	ALPSm	CC	RAD	RADne	VCG
Real Estate 3x3	E(X) in %	96.5	98.81	97.13	69.9	71.21	100
9 items	R(X) in %	67.75	82.5	86.56	10.11	10.37	84.2
5 best-response bidders	U(X) in %	28.75	16.31	10.57	59.79	60.84	15.8
16 auctions BSC	Rounds	532.98	760.83	400	46.95	47.15	1
Real Estate 4x4	E(X) in %	96.84	99.82	96.24	76.13	76.09	100
16 items	R(X) in %	75.51	90.72	90.56	9.16	9.75	90.3
10 best-response bidders	U(X) in %	21.34	9.1	5.69	66.97	66.34	9.7
1 auction BSC	Rounds	440.73	641.7	247.7	28.95	30.65	1
Pairwise Synergy Low	E(X) in %	94.82	99.73	98.56	69.98	69.17	100
7 items, valued 0 to 195	R(X) in %	72.41	87.53	88.29	8.84	8.63	87.08
synergy 0 to 0.5	U(X) in %	22.42	12.19	10.27	61.14	60.54	12.92
5 best-response bidders	Rounds	369.3	816	412.82	44.42	44.4	1
20 auctions BSC							
Pairwise Synergy High	E(X) in %	92.8	99.64	99.87	72.66	71.99	100
7 items, valued 0 to 88	R(X) in %	76.28	87.97	89.18	9.82	9.6	87.5
synergy 1.5 to 2.0	U(X) in %	16.52	11.68	10.69	62.84	62.4	12.5
5 best-response bidders	Rounds	354.65	656.38	338.48	41.8	41.67	1
15 auctions BSC							
Matching	E(X) in %	97.27	99.81	97.95	90.09	90.56	100
84 items (21 slots/airport)	R(X) in %	52.01	53.81	67.9	28.26	30.45	42.33
40 best-response bidders	U(X) in %	45.26	46.01	30.04	61.83	60.11	57.67
0 auctions BSC	Rounds	671.55	186.47	93.47	23.3	27.5	1
Transportation Large	E(X) in %	93.97	99.52	96.78	82.48	83.73	100
50 items, density $\rho = 2.9$	R(X) in %	62.33	76.61	80.92	38.97	34.9	64.21
34 cities (vertices)	U(X) in %	31.65	22.91	15.86	43.5	48.83	35.79
30 best-response bidders	Rounds	193.4	161.8	180.05	31.38	28.3	1
0 auctions BSC							
Transportation Small	E(X) in %	98.26	99.78	97.73	82.98	81.31	100
25 items, density $\rho = 3.2$	R(X) in %	54.79	59.54	65	21.96	17.93	48.32
15 cities (vertices)	U(X) in %	43.48	40.23	32.74	61.02	63.38	51.68
15 best-response bidders	Rounds	409.32	327	314.62	66.17	51.1	1
0 auctions BSC							

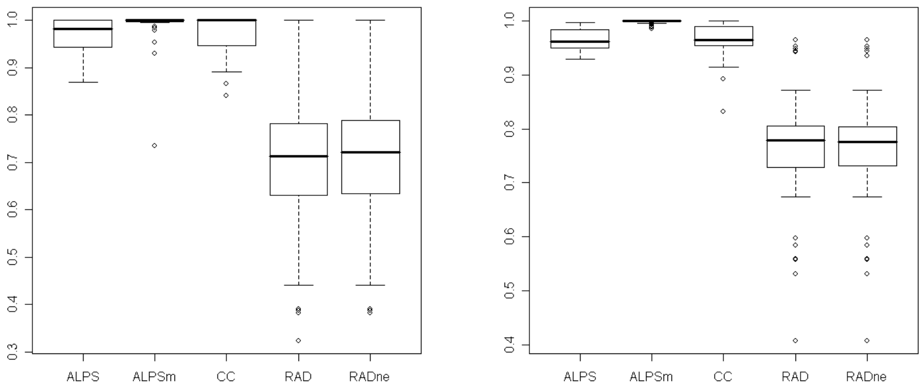
Table 4.1: Efficiency of different ICA designs with best-response bidders

and 10 bidders are distributed with a mean of 6 and variance of 1.1, whereas all other parameters are equal to the *Real Estate 3x3* value model. Only one of the instances of this value model fulfills the BSC. The *Pairwise Synergy Low* value model contains 7 items; the valuations for each auction are drawn based on a uniform distribution between the upper and lower bounds stated in the table. The synergy values lay between 0 and 0.5 in the *Pairwise Synergy Low* model and between 1.5 and 2.0 in the *Pairwise Synergy High* model, each not restricted in bundle size. For the *Transportation* and *Matching* value models the number of bidders is higher to provide sufficient competition. The *Matching* value model has 84 items, i.e., 21 time slots per airport. None of the value model instances fulfills the BSC. Finally, the *Transportation Large* models a transportation network with 50 edges, 34 vertices, and 30 bidders, while the *Transportation Small* model has only 25 edges, 15 vertices, and 15 bidders. None of the *Transportation* value model instances fulfills the BSC. All value model parameters are selected so that the efficient allocation of every auction has the same order of magnitude (200 to 250).

Overall, the efficiency of ALPS, ALPSm and the CC auction was very high over all value models and showed the same pattern. The simulations resulted in the highest efficiency levels for ALPSm. In the *Pairwise Synergy High* value model, there was no significant difference between the efficiency values of the CC auction and ALPSm (t-test, $p - value = 0.79$). RAD suffered from the premature termination. Furthermore, omitting the eligibility rules (RADne) did not show a significant improvement. In all but two value models (*Real Estate 4x4*, *Transportation Small*) the CC auction achieved higher efficiency values than ALPS.

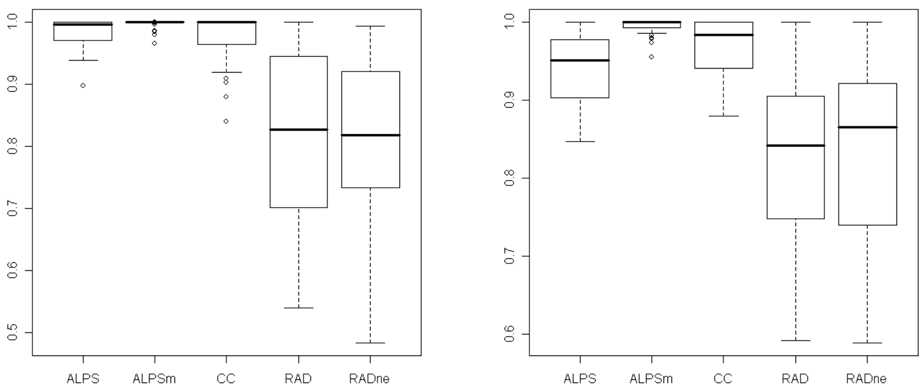
Figures 4.3 to 4.5 show box plots for the efficiency of selected value models using *best-response bidders*. We found a similar pattern in the simulations with *power-set* bidders that were restricted to submit their best 10 bids (see Figure 4.6).

In these simulations we wanted to avoid inefficiencies due to high bid increments and, therefore, used an ask price increment of 0.1. Consequently, the average number of rounds was quite high in general. A minimum bid increment of 1.0 reduced the number of rounds by a factor of 10. Note that the number of rounds is influenced by the value model, the number of bidders and their bundle selection strategy. Therefore, the figures in the table cannot easily be generalized, but only compared relative to the same setting with a



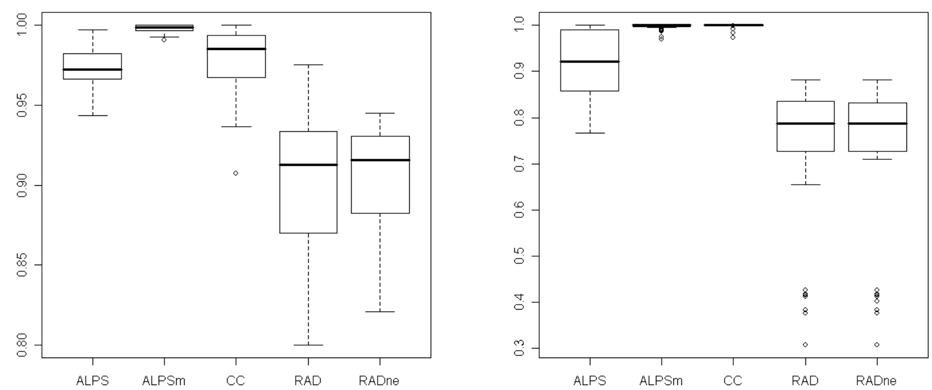
(a) Real Estate 3x3 (b) Real Estate 4x4

Figure 4.3: Box plot of allocative efficiency for the Real Estate value models with best-response bidders



(a) 25 Edges (b) 50 Edges

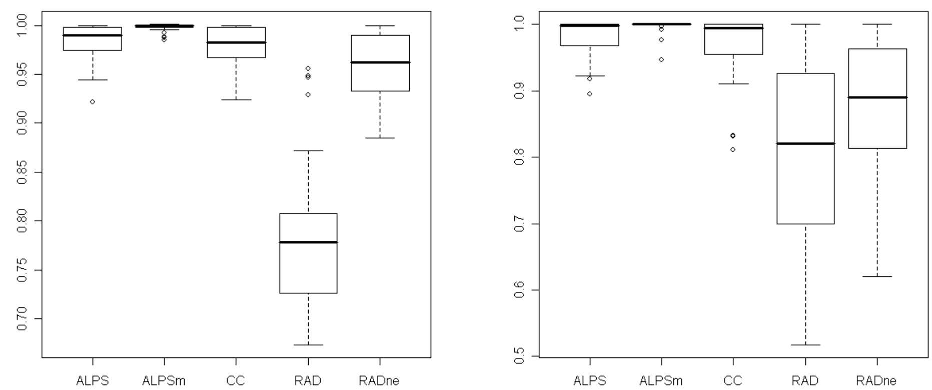
Figure 4.4: Box plot of allocative efficiency for the Transportation value models with best-response bidders



(a) Matching

(b) Pairwise Synergy High

Figure 4.5: Box plot of allocative efficiency for the Matching and Pairwise Synergy value models with best-response bidders



(a) Real Estate 4x4

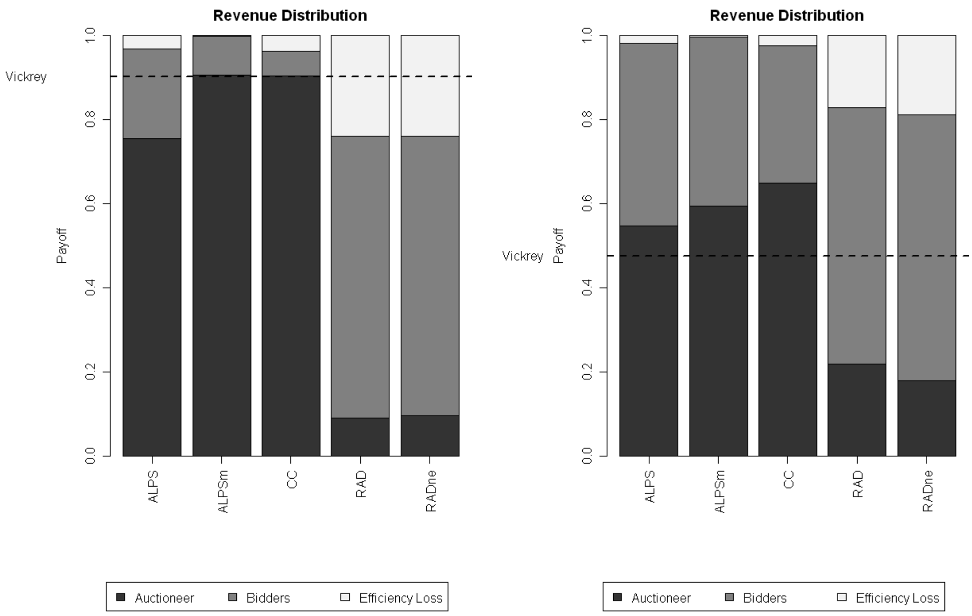
(b) Transportation 25 Edges

Figure 4.6: Box plot of allocative efficiency for a Real Estate and Transportation value model with power-set bidders

different auction design. ALPSm had the highest number of rounds, except for the Matching and Transportation value models. RAD often terminated prematurely, leading to a much lower average number of rounds, but at the cost of a significantly lower efficiency. We have also tested a dynamic version of price increments that decrease with increasing competition, and could reduce the number of rounds in ALPS and ALPSm considerably with very little or no negative impact on the efficiency.

4.2.2 Auctioneer Revenue in Different ICAs

Another performance characteristic of an auction design is the revenue distribution, i.e. the distribution of the overall utility between the auctioneer and bidders. If the auction is not 100% efficient, a part of the overall utility is lost. In theory, only *minimal CE prices* encourage *myopic best-response* bidding and lead to an efficient auction outcome, which minimizes the auctioneer revenue over all efficient allocations (Parkes, 2006). The knowledge of the revenue distribution typical for the particular ICA design can affect the bidding behavior. Our simulation results indicate significant differences in revenue distribution between different auction designs. Again, we found similar patterns across different value models (see Figure 4.7). An important observation is that the CC auction resulted in the highest average auctioneer revenue, followed by ALPSm. The dashed line in Figure 4.7 shows the average auctioneer revenue of the VCG auction. The VCG outcome can serve as an indicator for the competition in the auction, which was generally high. We have also conducted simulations with low competition (for example, the *Pairwise Synergy Low* model with only 3 bidders) and found the final ALPS ask prices to be higher than the average VCG prices, compared to the auction instances with higher competition (Real Estate 3x3 with 5 or 7 bidders).



(a) Real Estate 4x4 (b) Transportation 25 Edges

Figure 4.7: Revenue distribution of the Real Estate and Transportation Model with best-response bidders

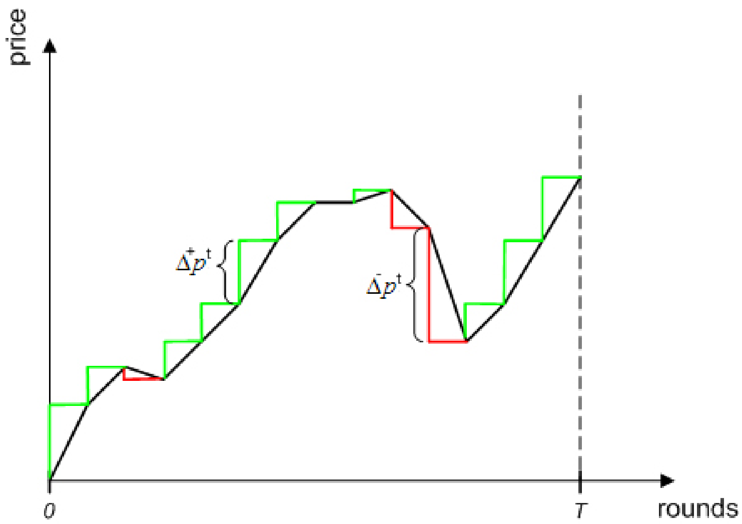


Figure 4.8: Calculation of the item price non-monotonicity

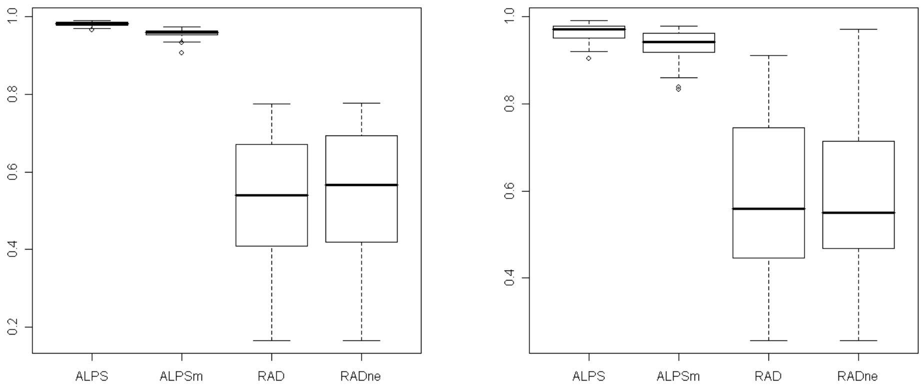
4.2.3 Price Monotonicity

Reducing item prices in the course of the auction may be necessary to reflect the competitive situation, but can also be confusing for the bidders. Price fluctuations are a phenomenon in RAD, ALPS, and ALPSm. The literature does not describe a measure for the price monotonicity. Prices in a linear-price ICA can be described as a discrete function $f : \mathbb{N} \rightarrow \mathbb{R}_0^+$ for a single item (see Figure 4.8).

We measure the price non-monotonicity as the sum of the ask price decreases $\Delta^- p_{ask}^t(k)$ divided by the sum of the ask price increases $\Delta^+ p_{ask}^t(k)$ for all items k in all rounds t. This results in the price non-monotonicity $\mu \in [0, 1]$, where $\mu = 0$ describes fully monotonic ask prices, as in the CC auction.

$$\mu = \frac{\sum_t \sum_{k \in \mathcal{K}} \Delta^- p_{ask}^t(k)}{\sum_t \sum_{k \in \mathcal{K}} \Delta^+ p_{ask}^t(k)}$$

Figure 4.9 provides a box plot for the μ values of ALPS, ALPSm, RAD, and



(a) Real Estate 4x4 (b) Transportation 25 Edges

Figure 4.9: Average price non-monotonicity μ in the Real Estate and Transportation value models

RADne for the Real Estate and Transportation value models with *best-response* bidders. Higher values of μ in cases of ALPS and ALPSm can be attributed to the fact that these auctions take more rounds and do not terminate prematurely.

There is a long tradition in economics of Walrasian tâtonnement, which allows prices both to ascend and descend (Ausubel, 2006, 604). In applications, where price fluctuations become an issue for bidders, alternative ways of calculating *pseudo-dual ask prices* can help to reduce or even eliminate this phenomenon. We have experimented with a simple rule that forces prices not to decrease across rounds (Shabalin et al., 2007). This rule ensures monotonous prices, but also causes minor efficiency losses. Dunford et al. (2007) discuss an alternative approach that uses a quadratic program to smooth price fluctuations across rounds.

4.2.4 Inefficiencies in Linear-Price ICAs

While the efficiency of the linear-price ICAs was generally high in our experiments, it is important to understand those cases, in which the final allocation was not optimal. We have analyzed all auction instances in the Real Estate and Pairwise Synergy value models, in which the efficiency was particularly low (90% and below). Additionally, we ran further simulations using best-response bidders with eligibility rules switched off, to isolate the negative impact of linear prices from the inefficiencies due to eligibility rules.

In all situations with an efficiency of less than 90%, the auctioneer did not sell all items, as opposed to the efficient allocation. These situations happened rarely in the Real Estate value model, and even less so in the Pairwise Synergy value model, as can be seen in Figure 4.3. Whenever all items were sold, the efficiency was always higher than 98%. The two small examples shown in Table 4.2 and Table 4.3 illustrate the structural characteristics of valuations that can lead to inefficiencies in ICAs with linear prices and best-response bidding.

Item	A	B	C	AB	AC	BC	ABC
Bidder1					9*		
Bidder2		2*					
Bidder3				10			
Bidder4						10	

Table 4.2: Example for inefficiencies in ALPSm

The example in Table 4.2 illustrates a scenario with 3 items A, B, C and 4 bidders. Each bidder has a valuation for one bundle only; the efficient allocation is marked with a star. In this example ALPSm selected the bid of bidder 4 on bundle BC and leaved item A unsold. The particular property of this value model is the existence of a set of mutually exclusive bundle valuations (AB and BC), none of which belongs to the efficient allocation. During the

Item	A	B	C	AB	AC	BC	ABC
Bidder1						20*	60
Bidder2	61*						
Bidder3				50	50		

Table 4.3: Example for inefficiencies in the CC auction

auction, bidders 3 and 4 drive up ask prices, which blocks other bidders from submitting their true valuations. Interestingly, the auction outcome in this case is sensitive to the start prices. The efficient allocation was found for the item start prices of 1.3 and 1.9, but for all other values from 0 to 2.0 the auction produced inefficient outcomes (with the price increment set to 0.1). The CC auction was generally efficient in this example.

The second example in Table 4.3 illustrates a set of valuations, for which the CC auction leads to an inefficient allocation. It allocates item A to bidder 2, and both items B and C remain unsold. Note that bidder 1's high valuation on bundle ABC dominates bundle BC. At the time when bidder 2 overbids bidder 1, the prices are already too high on all items, which prevents bidder 1 from submitting bids on bundle BC. Again, all bidders follow the best-response strategy. As opposed to the CC auction, ALPSm always terminates with an efficient allocation in this example.

One possibility to mitigate remaining inefficiencies in ALPS and the CC auction is to auction off the unsold goods in an after-market (sell the rest of the goods), but this still does not guarantee 100% efficiency and there might be no demand for these individual items, as in our first example in Table 4.2.

An alternative is the addition of a second phase with an ascending proxy auction, as suggested in the *Clock-Proxy* auction (Ausubel et al., 2006), with suitable eligibility rules. Without eligibility rules, using ALPS end prices as start prices for the ascending proxy auction, this will always lead to an efficient allocation under the truthful bidding assumption. However, both minimum bid prices and eligibility rules are necessary to encourage active bidding during the first linear-price auction phase. The impact of different eligibility rules on the allocative efficiency in a two-stage auction and optimal bidding strategies in these auction designs are a topic for the further research.

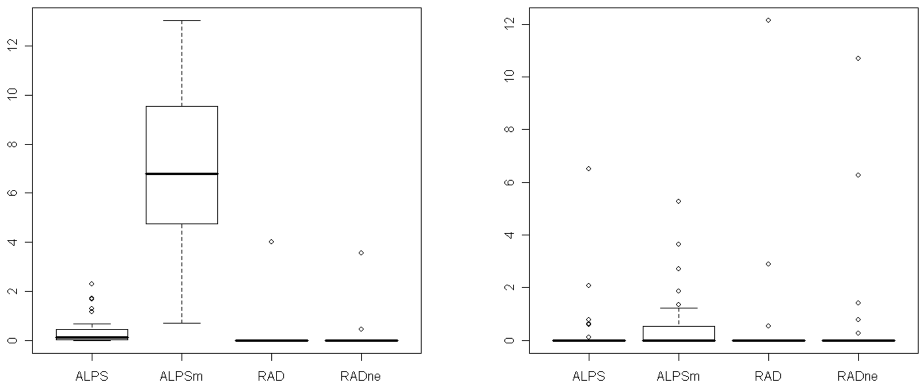
4.2.5 Linear Price Compatibility Distortions

In general, it is impossible to calculate exact linear prices except for special types of valuations in which the *goods-are-substitutes condition* holds. In other words, in both ALPS and RAD there must be cases in which the ask prices are not compatible with the provisional allocation. The *linear price compatibility distortions* (see Section 2.5.2) can be confusing for losing bidders, since their bids may be above the current ask prices, but still provisionally losing. We have measured the average percentage of individual ask prices with price distortions in each round (see Figure 4.10). Overall, price distortions occurred only for a very small part ($< 2\%$) of the ask prices. The *old-bids-active* rule led to a higher percentage of price distortions in ALPSm.

In addition to the number of price distortions, we have also analyzed the efficiency with respect to increasing levels of synergy among items. The Pairwise Synergy value model allows to increase synergy values sequentially from 0 to 3 (see Figure 4.11). Interestingly, the efficiency remains high for all auction designs even in case of high synergy values. Note that with a synergy value of 2.5 a bundle has already 3.5 times the value of its individual items.

4.3 Bidding Strategies Analysis

In the previous analysis, we have primarily used myopic *best-response* bidding agents. We have also performed the same simulations with *power-set* bidders limited to their best 10 bids and found the results to be very similar. While it is



(a) Real Estate 4x4 (b) Transportation 25 Edges

Figure 4.10: Percentage of linear ask prices with price distortions in the Real Estate and Transportation value models

useful to estimate the efficiency loss attributed to the use of a specific auction design, real world bundling and bidding strategies are often much simpler than the *power-set* or *best-response* bidding strategies, since evaluating and submitting all possible bundles is typically not practical for bidders (An et al. (2005), see also Section 6.2.2).

Moreover, according to the study of transportation CAs by Plummer (2003), only about 30 percent out of the 644 carriers submitted *package* bids. This group of carriers submitted between 2 and 7 lane combinations and the vast majority of the packages were small, containing between 2 and 4 lanes. The discounts carriers gave to the packaged lanes were around 5 percent. Apart from the novelty of CAs and the complexity of eliciting their valuations over all possible bundles, bidders face the problem of bundle selection from an exponential number of alternatives. To overcome some of these problems, bidder decision support tools have been suggested (Hoffman et al., 2005; Song and Regan, 2002), which are, however, rarely used in practice as of now. Therefore, it is interesting to see, how robust are considered ICA designs with respect to

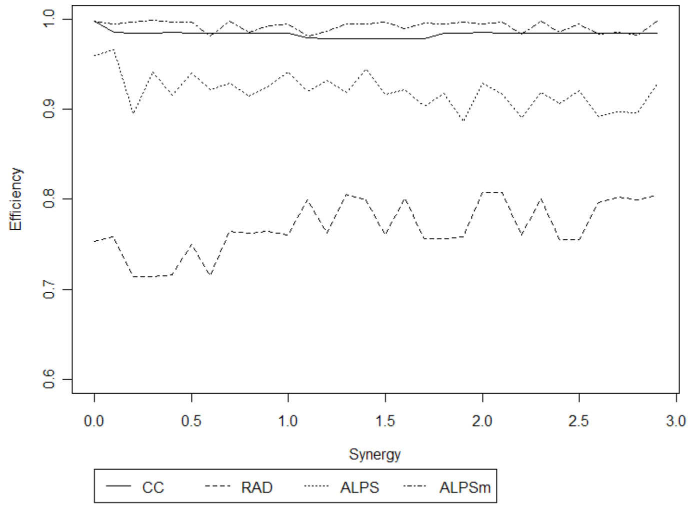


Figure 4.11: Changes in efficiency depending on synergy

other, simpler bundling strategies.

In this analysis we focused on the bundle selection in *iterative* auctions and assumed that bidders bid the minimum price only (neglecting jump bids or similar phenomena). We used 6 types of bidding agents and the ALPS auction design, since it was shown to produce high efficiency, while it converges to the minimal *pseudo-dual ask prices*. All bidders used the *XOR* bidding language with the notable exception of the *naïve* bidder, who used the *OR* language while bidding only on individual items. We have analyzed the Real Estate and Pairwise Synergy value models, in which *naïve* bidding makes sense, but did not consider the Matching and Transportation value models.

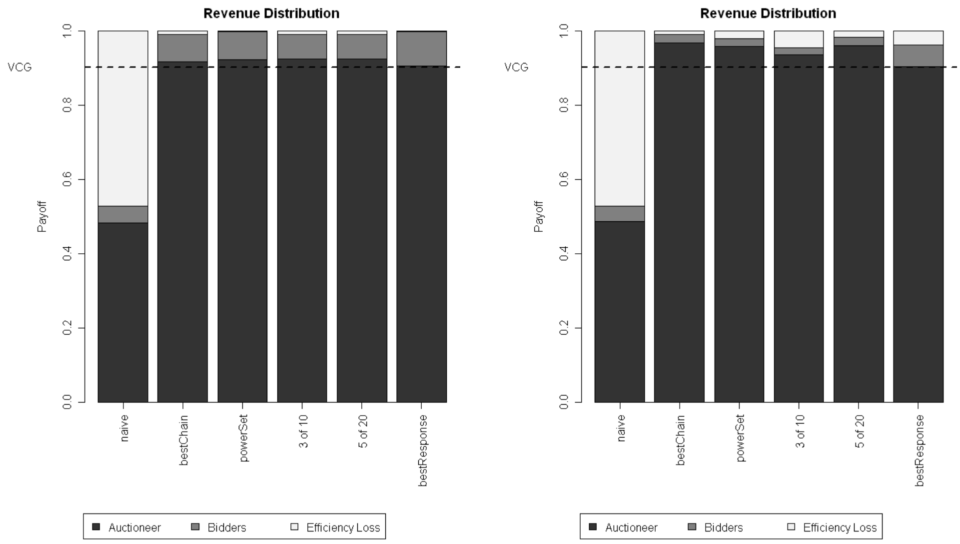
4.3.1 Efficiency of Pure Strategies

In the first set of simulations we ran the same auction with all bidders of the same type, and repeated it for different value models (see Table 4.4). The *naïve*

Setup \ Bidder Type		naïve	best-chain	power-set	3of10	5of20	best-response
Real Estate 3x3 9 items 5 bidders	E(X) in %	54.84	96.31	98.63	96.95	95.95	96.18
	R(X) in %	47.97	74.12	78.83	78.72	81	67.8
	U(X) in %	6.86	22.19	19.8	18.22	14.96	28.38
	Rounds	198.95	471	364.5	403.25	369.95	532.98
Real Estate 4x4 16 items 10 bidders	E(X) in %	52.86	97.96	98.19	96.56	96.73	96.68
	R(X) in %	48.43	84.61	86.65	85.03	87.29	75.56
	U(X) in %	4.43	13.35	11.54	11.53	9.44	21.13
	Rounds	108.55	230.43	247.5	367.23	289.7	671.95
Pairwise Synergy Low 7 items, valuations 0 to 195 synergy 0 to 0.5, 5 bidders	E(X) in %	77.21	96.25	98.09	96.99	97.7	95.64
	R(X) in %	66.63	75.68	81.83	81.56	85.3	74.07
	U(X) in %	10.59	20.57	16.25	15.43	12.4	21.57
	Rounds	259.65	461.2	369.88	395.45	382.88	541.77
Pairwise Synergy High 7 items, valuations 0 to 88 synergy 1.5 to 2.0, 5 bidders	E(X) in %	36.53	96.61	98.61	96.55	97.98	93.6
	R(X) in %	31.53	78.62	83.25	82.19	85.91	76.47
	U(X) in %	5	17.99	15.36	14.36	12.06	17.14
	Rounds	116.35	380.32	335.8	351	342.05	466.18

Table 4.4: Pure bidding strategies in ICAs

bidder only bids up to her item valuations and ignores synergistic valuations. In our simulations the *naïve* strategy was suboptimal and led to efficiency losses and low auctioneer revenues. The *power-set* bidder came out best, while also the *best-chain* and *heuristic* bidders achieved high levels of efficiency, since they focused on the best bundles. The heuristic bidders (*3of10*, *5of20*) produced efficiency and revenue values close to the *power-set* and *best-chain* bidders. For example, there was no significant difference between the *best-chain* and heuristic *3of10* bidders in ALPS (t-test, p-value of 0.65) in the Real Estate 3x3 value model. The auctions with *best-response* bidders produced high efficiency values, but the auctioneer revenue was significantly lower than the revenue in all other auctions, except with *naïve* bidders. We could find the same pattern in ALPSm, the CC auction, and in the other three analyzed value models. Figure 4.12 illustrates the revenue distributions for the RealEstate 4x4 value model with ALPSm and the CC auction.



(a) Real Estate 4x4, ALPSm (b) Real Estate 4x4, CC Auction

Figure 4.12: Revenue Distribution with pure bidding strategies in the Real Estate 4x4 value model

4.3.2 Sensitivity Analysis wrt. the Bidder Type

In the next set of simulations we measured the efficiency and revenue distribution for auctions with 9 (for the Real Estate 4x4 value model) and 4 (for all other value models) *best-response* bidders and the last bidder with a simpler bundle selection strategy (*naïve*, *best-chain*, or *heuristic*) in ALPS. For the last bidder the mean revenue over all 40 auctions was calculated. The results are shown in Table 4.5. Overall, the efficiency was not much lower, since 9 out of 10 (respectively 4 out of 5) bidders were *best-response* bidders, keeping the efficiency high. The table rows **"u(X) in %"** show the revenue share of the last bidder. Clearly, the *naïve* bidding strategy came out worst. Interestingly, either of the *power-set*, *best-chain* or *heuristic* strategies always performed better than the *best-response* strategy. One reason for this are the eligibility rules, which might have prevented some bidders from submitting the bids that should have win. The same type of sensitivity analysis was repeated with respect to *power-set* bidders in Table 4.6, where we could see a similar pattern.

In another set of simulations we tested, how much better bundle bidders perform, if they only compete with *naïve* bidders. We ran 40 auctions with 10 (or respectively 5) *naïve* bidders, and then repeated them with the last bidder playing the *best-chain* or *power-set* strategy. The results are shown in Table 4.7. Overall, the efficiency of these auctions decreased significantly, compared to the previous setups. This happened because the single smart bidder could mostly win her preferred bundle, while other bidders were restricted to bids on individual items, often leading to inefficient allocations.

In comparison to 4 *naïve* bidders, the *best-response* strategy of the 5th bidder performed slightly better than other bundling strategies (see Table 4.7). This may be attributed to the fact that bidding on more bundles drives up ask prices on individual items.

In summary, from the perspective of a bidder who is interested in maximizing her own revenue, it is favorable to bid on *bundles*. If all other bidders in the auction use the *best-response* or *power-set* strategy, the bidder is better off using the *power-set* strategy. In contrast, if all other bidders bid naïvely, the *best-response* strategy is slightly better than the *power-set* strategy. Overall, the more bidders use bundle bids, the better it is for the auctioneer.

The computational complexity of the CAP and of the ask price calculation and the time to solve realistic problem sizes are particularly important in iterative CAs, in which bidders submit bids in an interactive mode. We could solve problems of practically relevant sizes with up to 2659 bids (196 items, 230 bidders) in the Airport value model and up to 659 bids (62 items, 40 bidders) in the Transportation value model in less than 2 minutes using the open source IP solver *lp_solve*[2] under Windows XP on an Intel M (2.13 GHz) PC.

[2]http://sourceforge.net/projects/lpsolve

Setup \ Last Bidder Type		best-response	power-set	3of10	5of20	best-chain	naïve
Real Estate 3x3	E(X) in %	96.18	96.65	96.33	96.52	96.26	94.96
9 items	R(X) in %	67.88	70.99	71.67	67.82	69.41	61.87
4 best-response	U(X) in %	28.30	25.67	24.66	28.7	26.84	33.09
plus one bidder	u(X) in %	3.785	4.844	4.708	6.041	5.392	0.5227
Real Estate 4x4	E(X) in %	96.24	97.13	96.58	96.29	96.95	96.08
16 items	R(X) in %	74.39	75.77	74.54	76.38	75.41	71.83
9 best-response	U(X) in %	21.85	21.36	22.05	19.91	21.54	24.25
plus one bidder	u(X) in %	1.314	2.558	2.562	2.187	2.269	0.2110
Pairwise Synergy Low	E(X) in %	95.35	97.78	96.98	97.09	96.96	92.86
7 items, valued 0 to 195	R(X) in %	71.87	73.91	76.81	73.83	73.94	69.35
synergy 0 to 0.5	U(X) in %	23.48	23.87	20.17	23.26	23.02	23.51
4 best-response	u(X) in %	4.826	7.928	5.679	6.908	5.487	1.533
plus one bidder							
Pairwise Synergy High	E(X) in %	92.04	94.17	92.9	93.88	93.98	86.37
7 items, valued 0 to 88	R(X) in %	73.33	76.37	77.15	76.24	74.74	65.05
synergy 1.5 to 2.0	U(X) in %	18.71	17.8	15.75	17.64	19.24	21.32
4 best-response	u(X) in %	3.191	5.076	4.569	5.577	5.487	0
plus one bidder							

Table 4.5: Sensitivity with respect to best-response bidders

Setup \ Last Bidder Type		best-response	power-set	3of10	5of20	best-chain	naïve
Real Estate 3x3	E(X) in %	98.13	98.9	98.34	98.2	98.08	96.05
9 items	R(X) in %	79.34	79.67	79.87	80.14	79.67	66.76
4 power-set	U(X) in %	18.79	19.23	18.47	18.06	18.41	29.29
plus one bidder	u(X) in %	1.522	3.237	3.263	2.809	2.341	0.05379
Real Estate 4x4	E(X) in %	98.68	98.83	98.53	98.4	98.6	97.4
16 items	R(X) in %	85.67	86.88	86.57	86.53	86.76	85
9 power-set	U(X) in %	13.01	11.95	11.95	11.88	11.83	12.40
plus one bidder	u(X) in %	0.4362	0.8017	0.6333	1.009	1.123	0.002506
Pairwise Synergy Low	E(X) in %	98.42	99.6	98.3	98.84	99.25	96.33
7 items, valued 0 to 195	R(X) in %	80.57	83.78	83.85	84.06	84.62	78.41
synergy 0 to 0.5	U(X) in %	17.84	15.81	14.45	14.79	14.63	17.92
4 power-set	u(X) in %	2.502	4.171	4.104	3.899	3.883	0.2604
plus one bidder							
Pairwise Synergy High	E(X) in %	98.17	99.06	98.55	99.01	98.36	95.88
7 items, valued 0 to 88	R(X) in %	82.2	86.41	86.25	86.47	85.56	74.6
synergy 1.5 to 2.0	U(X) in %	15.97	12.65	12.29	12.54	12.80	21.27
4 power-set	u(X) in %	1.949	3.336	2.876	3.106	2.757	0
plus one bidder							

Table 4.6: Sensitivity with respect to power-set bidders

Setup \ Last Bidder Type		best-response	power-set	3of10	5of20	best-chain	naïve
Real Estate 3x3	E(X) in %	69.96	69.95	69.78	69.46	69.18	54.84
9 items	R(X) in %	48.19	48.68	48.68	48.89	48.48	47.97
4 naïve	U(X) in %	21.77	21.27	21.1	20.57	20.71	6.863
plus one bidder	u(X) in %	17.12	16.92	16.81	16.18	16.26	1.199
Real Estate 4x4	E(X) in %	62.07	61.99	61.76	61.74	61.66	52.86
16 items	R(X) in %	48.6	48.72	48.93	48.85	48.87	48.43
9 naïve	U(X) in %	13.47	13.27	12.83	12.89	12.78	4.431
plus one bidder	u(X) in %	9.939	9.809	9.471	9.452	9.295	0.4877
Pairwise Synergy Low	E(X) in %	85.13	85.1	85.08	85.08	84.66	77.15
7 items, valued 0 to 195	R(X) in %	67.63	68.34	68.46	68.46	68.28	67.62
synergy 0 to 0.5	U(X) in %	17.5	16.76	16.61	16.62	16.38	9.535
4 naïve	u(X) in %	10.96	10.55	10.57	10.61	9.984	1.827
plus one bidder							
Pairwise Synergy High	E(X) in %	61.96	61.97	61.97	61.87	60.5	36.50
7 items, valued 0 to 88	R(X) in %	31.76	32.32	32.28	32.4	32.32	32.01
synergy 1.5 to 2.0	U(X) in %	30.19	29.66	29.69	29.47	28.18	4.49
4 naïve	u(X) in %	27.01	26.72	26.74	26.59	25.37	0.8595
plus one bidder							

Table 4.7: Sensitivity with respect to naïve bidders

Chapter 5

Design of Laboratory Experiments

This chapter explains the design of our laboratory experiments. It describes the economic environment, some known common phenomena, our a priori assumptions about the bidding behavior, and summarizes a set of hypotheses for our study. It further defines and motivates used value models, treatment variables, the reward mechanism, and the experiment conduction scheme.

5.1 Economic Environment and Auction Mechanisms

5.1.1 Classification of Economic Experiments

Laboratory experiments are the main tool of experimental economics, which allows for studying the *market behavior* of human subjects under a *controlled environment*. As earlier experiments have shown, still many behavioral phenomena cannot be explained by theories, in particular the *Free Rider Problem*

(Guala, 2005; Roth, 1988), *Prisoners Dilemma* (Roth, 1988), *Risk Preferences* (Davis and Holt, 1992), *Winner's Curse* (Roth, 1988), *Preference Reversal* (Davis and Holt, 1992), and *Allais Paradox* (Allais, 1953). Laboratory experiments differ from field studies in that they take place in a *controlled environment* (Roth, 1988), so that they can be reproduced by other researchers with a high probability of getting the same results.

Laboratory experiments are typically classified by the pursued goals. There are several similar[1] classifications proposed in the literature, e.g., the classification by Davis and Holt (Davis and Holt, 1992) and the one by Sudgen (Sugden, 2005). Maybe the most famous classification can be found in (Roth, 1995). It envelops the following three types of experiments named after their primary goals:

- Speaking to Theorists
- Searching for Facts (Searching for Meaning)
- Whispering in the Ears of Princess

Experiments of type *Speaking to Theorists* are primarily designed to test well defined theories and explore possible surprising regularities for which experimental evidence can be found.

Experiments of type *Searching for Facts* are designed to investigate the impact of selected variables, not well considered in the underlying theory. Such experiments are often used to further investigate observations from earlier experiments and, given some evidence of empirical regularities, extend theories to properly reflect the observed behavior (*Searching for Meaning*).

[1] The referred classifications are similar in that they usually distinguish between experiments conducted to prove theoretical predictions, and experiments used to find new regularities that might flow into new theories.

Finally, experiments of type *Whispering in the Ears of Princess* are designed to investigate the influence of regulatory authorities and are mainly conducted to seek for new policies and their market effects. They need to be conducted under an environment that as close as possible replicates reality.

The basis for most experiments (at least for experiments of the first two types) is a proposition of one or more hypotheses to be verified. The hypotheses must not be obvious, rather, there must be a chance of refusion (Guala, 2005). Nevertheless, an experiment is not restricted to analyzing only those phenomena the experiment was designed for. For example, it is possible to use the results of an experiment designed as *Speaking to Theorists*, to exhibit new phenomena.

5.1.2 Factors

Factors are variables that can have an impact on the experiment results. Depending on the experiment goals, the factors are divided in *focus variables* and *nuisances*:

- ***Focus variables*** are factors whose impact is to be investigated in the survey.
- ***Nuisance variables*** or simply ***nuisances*** are further factors that affect the results and have to be considered.

A good experimental design sharpens the effects of the focus variables and minimizes the blurring effects related to the nuisances. The other design goal is to distinguish between the effects of the two kinds of variables. Some factors are selected as ***treatment variables*** or simply ***treatments*** in that one or more experiment instances are conducted for every possible value (level) of every treatment variable, whereby all treatments should be varied independently. Other factors are either held constant or randomized.

Focus variables are usually selected as treatments with two or more strongly separated levels. In contrast, most *nuisances* should be held constant, so that they are controlled, and complexity and costs are kept low. However, if a nuisance is suspected to interact with a focus variable, it can be controlled as a treatment. Some potential important nuisances, such as subjects' alertness and interest, are not even observable by the experimenter and much less controllable. Uncontrolled nuisances can cause inferential errors if they are correlated with focus variables. *Randomization* and *blocking* are a tool that can be used if full control is not possible (Box et al., 2005).

5.1.3 Validity and Realism of Experiments

Validity (or *relevance*) is a critical issue for all data sources. Experimental economics distinguishes between the *internal* and *external* validity:

- ***Internal validity*** deals with the question, whether the data permits for correct causal inferences for environments controlled in a similar way.
- ***External validity*** also known as ***parallelism*** deals with the generalization of inferences from the laboratory environment to the field. The general principle of induction is that behavioral regularities will persist in new situations as long as the relevant underlying conditions remain substantially unchanged (Friedman and Sunder, 1994, page 15). For example, it may be appropriate to conduct experiments with more traders or with more experienced (or professional) traders to guarantee external validity.

Realism is highly related to validity and deals with the question, how close the laboratory environment should be to the formal model and reality. Both designing the laboratory environment as close as possible to the real-world setting and replicating the formal model assumptions are misleading approaches.

The first one is very expensive and complex, whereas the second one leaves out details important when analyzing human behavior and simply reproduces existing theoretical results. An effective experimental design should be as simple as possible and should offer the best opportunity to answer important research questions. "Good experiments grow organically out of the issues they are designed to investigate and the hypothesis among which they are designed to distinguish" (Kagel and Roth, 1995).

5.1.4 Reward Mechanisms

Selecting a proper reward mechanism is crucial for a laboratory experiment, since it is the most important tool to impose internal and especially external validity. Evidently, acting in the real economical setting mostly includes monetary incentives. On the other hand, when using students as subjects, there are several other thinkable reward mechanisms, e.g., giving credits for the participation (Guala, 2005).

Guala (Guala, 2005) provides the following four basic precepts that an incentive system has to take into account:

1. Non-satiation: Design a reward mechanism in which the subject always choses the alternative having the largest reward.
2. Saliency: The reward must be adjusted to the successes an failures of the subject.
3. Dominance: Any subjective costs must be dominated by the reward.
4. Privacy: Subjects has only informations about the own payoffs.

These precepts might also be accomplished with other reward mechanisms than monetary incentives, but the mechanism must always be valid within the

Bidder	1	2		3	4	
Bundle	AB	BC	C	AB	AB	C
Value	15	14	5	9	10	4

Table 5.1: Value model VM1

experiment. Nevertheless, most experimenters use monetary incentives, since it seems rather natural using money in an economic experiment (Friedman and Sunder, 1994). In fact, monetary rewards are of similar value for every student, can be directly mapped to experiment results (e.g., in case of an auction), and replicate the economic reality, which is especially important to impose external validity. On the other hand, such experiments are costly. Additionally, there is a couple of issues that can influence the outcome, as for example the *cognitive exertion*, *motivational focus*, and *emotional triggers* (Read, 2005).

5.2 Experimental Setup

5.2.1 Value Models

We used 4 value models, two small ones with only 3 items, and two larger ones with 6 and 9 items respectively. The small value models ***VM1*** and ***VM2*** follow selected examples from Dunford et al. (2007). The individual valuations for each bidder are given in Table 5.1 and Table 5.2. The first value model, VM1, fulfills the *bidders-are-substitutes* and *bidder-submodularity* conditions, while the second one, VM2, does not.

Note that in all value models we assumed *free disposal*, i.e., the value of every bundle is not less than the value of any other contained bundle. However, the valuations are not additive for disjunct bundles. For example, in VM1 bidder 2 has a value of 14 for bundle AB and a value 5 for item C. This implies that she also has an (implicit) value of $14 = max(14; 5)$ for bundle ABC.

Bundle	A	B	C	AB	BC
Bidder 1	10	5	2		
Bidder 2	5	10	5		
Bidder 3	2	5	10		
Bidder 4				5	16

Table 5.2: Value model VM2

shoreline		
A	B	C
D	E	F

Table 5.3: Structure of the value model VM3

For the two larger value models, the value model VM3 fulfills the *bidders-are-substitutes* and *bidder-submodularity* conditions, whereas VM4 does not. ***VM3*** describes 6 pieces of land arranged in two rows at a shoreline (see Table 5.3). Bidders 1 and 2 are interested in individual items or in bundles of two items. For them, every bundle of interest contains at least one lot at the shore. Bidders 3 and 4 are interested in larger bundles of size 2, 3, and 4. For all bundles of size 3 and 4, they also must have at least two pieces of land at the shore. These valuations have both sub- and superadditivities. The individual valuations for each bidder can be found in Appendix B.

In ***VM4*** there are 9 pieces of land (see Table 5.4). There are bidders 1-3 with maximal bundle size of 3 and the bidder 4 with bundles of size 4, 5, and 6. Each of the bidders has her preferred location (marked in the table) and, consequently, different bundles of interest containing items close to it. The valuation of a single item for a bidder depends on its distance to the bidder's preferred location: $v_i(k) := \mu^\gamma * B_i, \;\; k \in \mathcal{K}$, where B_i denotes the basic value of the bidder's preferred property, and γ measures the distance to the preferred location. For bundle valuations, a markup has been added depending on the bundle size and a parameter δ: $v_i(S) := \sum\limits_{k \in S} v_i(k) + \delta * |S|$. The individual

A Bidder 1	B Bidder 1	C Bidder 3,4
D	E	F
G	H	I

Table 5.4: Structure of the value model VM4

A Bidder 1	B Bidder 2	C Bidder 3
D Bidder 1	E Bidder 2	F Bidder 3
G Bidder 1	H Bidder 2	I Bidder 3

Table 5.5: Efficient allocation in the value model VM4

valuations for each bidder can be found in Appendix B.

VM4 exhibits the threshold problem for bidders 1, 2, and 3. In the efficient allocation bidder 1 wins bundle ADG, bidder 2 wins bundle BEH, and bidder 3 wins bundle CFI. Bidder 4 has a high valuation for bundle $BCEFHI$, and bidders 2 and 3 have to coordinate their efforts to outbid her. The second best allocation is shown in Table 5.6.

For all value models we ran simulations with best-response bidding agents, as well as with more heuristic bidders that would randomly select three of their best five bundles or 3 of their best 10 bundles (see Appendix C). With best-response bidders, all value models produced efficient allocations, which served

A Bidder 1	B Bidder 4	C Bidder 4
D Bidder 1	E Bidder 4	F Bidder 4
G Bidder 1	H Bidder 4	I Bidder 4

Table 5.6: Second best allocation in the value model VM4

No.	Auction Design	Value Model
1	VCG	VM1
2	VCG	VM2
3	VCG	VM3
4	VCG	VM4
5	ALPSm	VM1
6	ALPSm	VM2
7	ALPSm	VM3
8	ALPSm	VM4
9	CC	VM1
10	CC	VM2
11	CC	VM3
12	CC	VM4
13	iBundle	VM1
14	iBundle	VM2
15	iBundle	VM3
16	iBundle	VM4

Table 5.7: Experimental design - Treatments

as a benchmark for the laboratory experiments.

5.2.2 Treatments

As we wanted to study the impact of the auction design, auction size, threshold problem, and some other factors, we selected the auction design and value model as the *focus variables*. The auction design variable has 4 values (levels): ALPSm, the CC auction, iBundle, and the VCG auction. The value model variable has also 4 levels: VM1, VM2, VM3, and VM4. Both focus variables were selected as independent *treatments*, which resulted in a 4 x 4 design with 16 treatment combinations (see Table 5.7). We conducted 4 auctions for every treatment combination, so that altogether 64 auctions were conducted.

5.2.3 Nuisances

There are several *nuisances* that often occur in economical experiments:

- **Experience and learning:** It can be controlled as a constant by using only experienced or only non-experienced subjects. Another possibility is to treat it as a blocking variable by using a balanced switchover design.
- **Intercommunication:** The communication between subjects must be forbidden during the experiment. The use of partitions between the computers, to block the subjects' view on the others' screens, is recommended. There should be also one or more monitors that control the interactions during the experiment.
- **Boredom and fatigue**: The experience of earlier experiments shows that one session should not last more than 3 to 4 hours.
- **Subject or group idiosyncrasies:** A subjects' background or temperament or unusual influences in a group of subjects may lead to unrepresentative behavior. Therefore, it is essential to replicate the same experimental situation with different subjects.

For our experiments, we identified the following nuisances: *Experience*, *Learning*, *Round Duration*, *Boredom and Fatigue*, *Intercommunication.*

Experience: Since our subjects were students from our university, we do not expect any of them to be experienced in *combinatorial* auctions. We also expect all subjects to have *general* experience in auction trading, since electronic auction platforms as eBay are open for everyone and currently very popular. Furthermore, we took only students from *technical* departments (informatics, mathematics, mechanical engineering, and physics), since combinatorial auctions require understanding of complex auction rules, bundle bidding, and pricing, and the subjects had only a few time to learn all that rules.

Though students are the main subject pool for economic experiments (Friedman and Sunder, 1994), and in many cases the results achieved with students were shown to exhibit external validity (Dyer et al., 1989), there can still be difference in behavior compared to other subject pools. For example, an experiment on combinatorial auctions conducted by Kazumori (Kazumori, 2005) showed that there are some differences between students and business professionals. He found that the efficiency of an auction was not affected by the different subject pools, while the auctioneer revenue was significantly higher with students. Moreover, even taking students from *economic* departments can produce different results (Guala, 2005). To clearly eliminate this uncertainty, further laboratory experiments are required.

Learning: Another big issue in experimental economics is learning effects during the experiment. As students are unexperienced and mostly have a steep experience curve (Guala, 2005), the effect of learning has to be taken into account. Güth et al. (2003) studied bid function adjustments in auctions and fair division games with independent private values. They found the differences in behavior rather large compared to the experience of the subjects. Chen and Takeuchi (2005) found that subjects adapted bid pricing to their success in the previous auction. Though the learning process was clearly identified, no dependency of the point on the experience curve and the subjects' performance could be shown.

To minimize the effects of learning, we conducted one training auction during the introductory presentation and two further training sessions, after which the subjects were able to ask questions and then had to fill out a questionnaire. Only those subjects who correctly answered the questions were permitted for the participation. With all that, we covered the first, steepest part of the learning curve. Furthermore, every session was conducted in the sequence VM1, VM2, VM3, VM4. We did not use randomization at this point, to let subjects deal with more complex value models sequentially, which should also have partially compensated for learning effects. This also allowed us for some analysis of learning effects based on the experiment results.

Round Duration: The round duration is another important factor in combinatorial auction experiments, since the auction environment is complex. Especially in auction designs with activity rules, subjects make important decisions in the first round. Though Kazumori (2005) did not identify significant impact of the time limit to input bids, we decided to handle this point with much care.

In our software platform "MarketDesigner" we implemented two round closing mechanisms: the timer-triggered and readiness-based round closing. The timer-triggered closing defines the round duration limit valid for every round. With readiness-based closing, the round additionally closes as soon as every bidder indicated her unwillingness to submit more bids. This rule has been shown to work very well in the experiments. With the round duration limit set to 5 minutes, the first 2-4 rounds lasted approximately 4 minutes, after which the round duration usually fall under 1-2 minutes. Only in a couple of cases some bidder did not indicated her readiness, and the round was closed by the timer-triggered mechanism. The described round closing rules allowed us to eliminate the negative impact of *to short* rounds due to cognitive complexity, but also to avoid the boredom due to *too long* rounds.

Boredom and Fatigue: Boredom is a common problem in laboratory experiments. In our case, the possibility of boredom was strongly reduced by dynamic round duration, as described in the previous paragraph. Additionally, we limited the usage of computer terminals for other purposes, in particular we disabled Internet surfing. What remains is the boredom of the experiment itself, as for example if subjects have to take part in over fifty repetitions of the prisoner's dilemma, they might change their behavior just to do something else (Friedman and Sunder, 1994). Fatigue is another aspect, which was addressed by holding the session duration within four hours including a pause of 15 minutes, whereby the effective part of the experiment (the part after the pause) did not exceed two hours. We also provided some (nonalcoholic) drinks for the participants.

Intercommunication: We avoided visual communication between the participants by using partitions between the computers. Additionally, two experimenters were present to prevent possible oral communication.

5.2.4 Reward Mechanism

We used a monetary reward mechanism, in which the subjects were rewarded both for the active participation and for the results achieved in the auctions. This encouraged the subjects to better understanding of the auction rules as well as to meaningful bidding. We had also to avoid the *bankruptcy problem* which appears if a subject makes negative earnings. It is impractical to ask the subject to net payment to the experimenters. However, making zero payment in this case may induce risk-seeking behavior.

To fulfill above requirements, we decided for the following payment scheme. Every subject was guaranteed to get at least the *show up fee* of €10 for the participation. This was also the amount payed to the subjects who were not permitted for the effective part of the experiment due to their incorrect answers in the questionnaire. The subjects who provided correct answers, but were not permitted for the effective part of the experiment due to overbooking, got the *qualification fee* of €20. The subjects who participated at the whole experiment were rewarded by $[max(10; min(80, 30 + gain * 2))]$ euro, where 10 is the *minimum payment*, 80 is the *maximum payment*, 30 is the *start deposit*, *gain* is the total bidder gain over all auctions measured in the virtual currency, and 2 is the exchange rate. This payment scheme guaranteed a minimal payment of €10 in any case, but also discouraged overbidding, since *gain* can also be negative.

Altogether, this payment scheme resulted in an average payment of €48.59 for the subjects who participated at the whole experiment (4 hours).

5.2.5 Conduction Scheme

All experiments were conducted from June to August 2007 with undergraduate students in computer science, physics, mechanical engineering, and mathematics at the TU München in their first or second year. Each auction was conducted with 4 subjects. We used our web-based software plattform "MarketDesigner" (see Appendix D) and conducted experiments in our computer lab at the Garching campus of the TU München.

Each session tested a single auction design with all 4 value models[2]. Every session started with 10 participants. At the beginning of the session every subject was given printed instructions. The instructions were then read aloud, whereby the subjects were encouraged to ask questions. Pauses were made to let the subjects try using the software platform in the first training auction. The instruction period took 50 minutes on average.

The instructions part was followed by the second training auction, whereby the subjects were still allowed to ask questions. Afterwards, they had 20 minutes to fill out a permission questionnaire designed to test their understanding of the mechanism. In the pause of 15 minutes the partitions were installed and the questionnaires evaluated. On the basis of the questionnaires, 8 subjects were permitted for the further participation. They were divided into 2 groups with 4 subjects each, so that 2 experiment runs could be conducted simultaneously.

After the pause, the subjects randomly drew a PC terminal number. The second part of the experiment started with the last training auctions (one auction per group of 4 subjects). Then, the effective auctions for the value models VM1 to VM4 were conducted for every group. Finally, the subjects filled out a feedback questionnaire.

Each session took 3.5-4.5 hours (usually under 4) inclusive 15 minutes pause

[2]We designed sessions in this way due to considerable differences in the auction design rules that make learning of different auction designs in one session almost impossible for subjects.

Treatments	Auction Design Value Model
Levels of treatment 1	VCG, CC ALPSm, iBundle
Levels of treatment 2	VM1, VM2 VM3, VM4
Total subjects	80
Total auctions	64
Total sessions	8
Runs per session	2
Subjects per run	4
Overbooking per session	2
Training auctions per run	3
Effective auctions per run	4
Bundles per bidder	1 - 27
Items per auction	3 - 9
Show up fee	€10
Qualification fee	€20
Minimum payment per subject	€10
Maximum payment per subject	€80
Average payment per subject permitted for effective part	€48.59
Session duration	3.5-4.5 hours
Average instructions duration	50 min
Questionnaire duration	20 min
Pause duration	15 min
Effective part duration	2-2.5 hours

Table 5.8: Experimental setup overview

and 2-2.5 hours effective part. Altogether, we conducted 4 repetitions of each treatment combination (i.e. 64 auctions) in 8 sessions. Every subject participated in only 1 session.

5.2.6 Summary

Table 5.8 gives an overview on the experimental setup.

5.3 Hypotheses and Response Variables

5.3.1 Behavioral Assumptions and Bidding Strategies

For the VCG auction there is a dominant strategy for bidders to bid their true valuations on all bundles. Provided that the valuations satisfy the *bidder-submodularity* condition, myopic best-response bidding is an ex-post Nash equilibrium in iBundle (de Vries et al., 2007). As already discussed, if this condition does not hold, the myopic best-response strategy is likely to lead a bidder to pay more than the optimal price for the winning package (Dunford et al., 2007). Behavioral models of bidding in multi-item auctions are rare (see for example Plott and Salmon (2002)). In Schneider et al. (2007) we analyzed the performance of primal-dual auctions and linear-price ICAs in computational experiments. Provided best-response bidding, the simulations confirmed the theory. The efficiency of iBundle was at 100% in all auctions. The prices were, however, above the Vickrey prices on average, since not all value models satisfied the BSM. Based on best-response bidding, both the CC auction and ALPSm performed significantly worse in terms of efficiency. The prices in these linear-price auction designs were mostly higher, but sometimes also lower than the Vickrey prices. When bidders followed a heuristic bidding strategy, linear-price mechanisms showed to be fairly robust, while primal-dual auctions often led to very low efficiency values.

Since in a private values experiment it is not known to the bidders, whether the BSM holds, one could expect bidders in iBundle to shade their bids in general. Also, one can often not expect bidders to follow the pure best-response strategy due to cognitive barriers, risk aversion, or some sort of strategizing. In general, deviations from the best-response strategy can have multiple reasons:

- To follow the best-response strategy, bidders need to determine their demand set from an exponential number of possible bundle bids. This might be impossible due to cognitive restrictions, but can also have strategic reasons. For example, in the early stages of FCC spectrum auctions bidders bid deliberately on bundles of lower interest, to drive up the prices on those and to maintain a high eligibility.

- Such strategic reasons, might not only be a reason for non-best-response bundle selection, but also for jump bidding. Isaac et al. (2007) describe jump bidding to take place in a large proportion of FCC spectrum auctions (up to 44% of the bids with a 5% bid increment) as well as 3G spectrum auctions in the U.K.

- Bidders might also have a biased estimate of their valuations at the start of the auction. The auction can then be seen as a way to help bidders elicit and learn about their true valuations throughout the auction. Researchers like Sargent (1993), among others, have looked into respective theories of learning.

- Finally, bidders cannot be assumed to behave perfectly rational in complex decision environments. They make mistakes and have different conceptual models of what strategy works best in a given environment.

The discrete choice behavior of a bidder can be described by an ordered logit model, characterizing how covariates such as the value model or the auction format impact the bidder's bundle selection decision, whereas possible bundles are ranked by their payoff at the ask prices in the given round. For this type of data the ordered logit model is a suitable analysis tool (Greene, 2003, p. 736-740). The model is based on the following specification:

$$y^* = \mathbf{x}^T\beta + \epsilon$$

where $\mathbf{x}^T$ is the vector of explanatory variables and ϵ is the disturbance term. As usual, y^* can not be directly observed. What we observe is the ordinal variable y, so that:

$$
\begin{aligned}
y &= 0 \quad if \qquad y^* \leq 0, \\
&= 1 \quad if \qquad 0 < y^* \leq \mu_1, \\
&= 2 \quad if \qquad 0 < y^* \leq \mu_2, \\
&\quad\;\; \vdots \\
&= J \quad if \qquad \mu_{J-1} \leq y^*.
\end{aligned}
$$

In our case y corresponds to the bundle ranking, so that bundles with the best possible payoff are assigned the value 0, second best bundles the value 1, and so on. The μ-s are unknown parameters to be estimated by the coefficients β. The model is inspired by the Quantal Response Equilibrium, or more specifically logit equilibrium models, which have been used in experimental economics to model deviations from Nash equilibrium predictions (Bajari and Hortacsu, 2005; Goeree and Hold, 2002; McKelvey and Palfrey, 1998). Here, a bidder's choice is influenced by an idiosyncratic deviation ϵ_t, which is i.i.d.

5.3.2 Hypotheses

Based on the theoretical predictions, we identified the following hypotheses:

Hypothesis 1. In VCG auctions, bidders bid on all packages.

Hypothesis 2. In VCG auctions, bidders bid truthfully.

These two hypotheses are based on the dominant strategy of the VCG auction. Furthermore, theory suggests that straightforward bidding is an ex-post equilibrium in iBundle and the auction will result in Vickrey payoffs when the bidder-submodularity condition is fulfilled.

Hypothesis 3. iBundle and the VCG auction achieve the same efficiency.

Hypothesis 4. iBundle and the VCG auction achieve the same auctioneer revenue if the BSM holds.

Hypothesis 5. The auctioneer revenue in iBundle is higher than in the VCG auction if the BSM does not hold.

While there are strong incentives for bidders to follow the best-response strategy if the BSM is satisfied, there are less reasons to assume this strategy for general valuations or in linear-price auction designs. Nevertheless, we postulate the following hypothesis on bidding behavior in ICAs.

Hypothesis 6. In all auction designs bidders follow the best-response strategy.

Hypothesis 7. Bidding behavior is homogeneous across individuals with the same treatment.

The performance of different auction designs depends on the valuations of the bidders. In our lab experiments, we have used 4 specific value models. We ran simulations with best-response bidders and also with heuristic bidders who randomly picked 3 out of their 5 best bundles or 3 out of their 10 best bundles in each round. The results can be found in Appendix C.

Hypothesis 8. With best-response bidding, the results follow the results of the simulations with best-response bidders.

In addition, we tested the following hypothesis on the number of auction rounds and the size of the auction.

Hypothesis 9. The number of auction rounds in ALPSm is significantly lower than in iBundle.

Hypothesis 10. The efficiency of combinatorial auctions with more items is lower than the efficiency in small auctions.

Chapter 6

Results of Laboratory Experiments

This chapter presents the results of our laboratory experiments. We compared the CC auction, ALPS, iBundle and the Vickrey-Clarke-Groves auction[1] using different value models. We were able to identify several similarities to the computational results, but also observed some other interesting phenomena specific for the behavior of human bidders.

6.1 Aggregate Performance Metrics

We used four different value models to compare ALPS, the CC auction, iBundle, and the VCG auction designs. The details on the auction setup and the results, averaged over 4 sessions each, are provided in Table 6.1. The left column describes the auction setup, i.e., the number of items and the value model. Note that 3 of the iBundle sessions on the value model VM4 had to

[1]RAD was not included due to its premature termination problems and since it was shown to perform worse than ALPS in the computational experiments.

be canceled, since the auction converged very slowly. For example, session 5-2 was canceled after 4.5 hours, with one iBundle auction being at round 162. Also, session 10-2 was canceled after 4.5 hours, the last auction being at round 46. In contrast, session 10-1 ended prematurely after 2 rounds, as one bidder did not submit any bid in the second round and every other bidder got a bundle in the provisional allocation. If the best-response bidding strategy is not guaranteed, the iBundle termination rule can lead to such inefficiencies.

We have added the results of the simulations using the same value models with best-response bidders as a benchmark in the columns suffixed by "-sim". The benchmark indicates, how different the human bidders behaved compared to best-response agents. Appendix C provides more simulation results, in particular the results of simulations with agents following heuristic bidding strategies. Also in the simulations iBundle was shown to produce very low revenues under heuristic bidding assumptions in cases when the auction prematurely closed due to the iBundle termination rule.

Result 1: There was no significant difference in allocative efficiency across all 4 auction designs. (Hypothesis 3)

All auctions simulated with best-response bidders achieved 100% efficiency, but varied in the auctioneer revenue. In the lab, the efficiency of all auction designs was also very high across all value models (see Figure 6.1). We did not find a significant difference in efficiency between different auction designs (see Table 6.2).

The primary conjecture for this observation is the *decision support* and the *user interface* provided to the bidders. While in previous auction experiments bidders were just provided with a list of valuations and had to determine their best bundles manually, our *MarketDesigner* platform allowed our subjects to enter their private valuations for a set of watched bundles and to monitor payoffs of that bundles at the current ask prices. Moreover, the bidders could also

Value Model \ Design		*ALPS-sim*	ALPS-21	ALPS-22	ALPS-31	ALPS-32	*CC-sim*	CC-41	CC-71	CC-72	CC-91
VM1	E(X)	1.0	1.0	1.0	0.9	1.0	1.0	1.0	1.0	0.9	1.0
3 items	R(X)	0.9	0.9	0.85	0.43	0.78	0.750	0.74	0.67	0.74	0.97
BSM	Rounds	9	4	14	4	6	9	8	7	9	11
VM2	E(X)	1.0	1.0	1.0	0.7	1.0	1.0	0.87	1.0	1.0	1.0
3 items	R(X)	0.733	0.7	0.8	0.58	0.66	0.667	0.67	0.53	0.73	0.7
not BSC	Rounds	10	6	3	8	6	10	15	6	8	8
VM3	E(X)	1.0	1.0	0.62	1.0	1.0	1.0	1.0	1.0	1.0	1.0
6 items	R(X)	0.769	0.78	0.83	0.57	0.82	0.836	0.8	0.8	0.85	0.85
not BSC	Rounds	21	7	9	10	9	15	10	10	12	12
VM4	E(X)	1.0	1.0	1.0	0.83	1.0	1.0	1.0	0.92	1.0	1.0
9 items	R(X)	0.769	0.81	0.93	0.77	0.9	0.872	0.78	0.85	0.89	0.89
not BSC	Rounds	16	6	5	13	5	11	9	13	11	15

Value Model \ Design		*VCG-sim*	VCG-61	VCG-62	VCG-81	VCG-82	*iB-sim*	iB-51	iB-52	iB-101	iB-102
VM1	E(X)	1.0	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3 items	R(X)	0.850	1.0	0.88	0.59	0.68	0.850	0.85	0.79	0.9	0.9
BSM	Rounds	1	1	1	1	1	23	10	16	14	40
VM2	E(X)	1.0	0.67	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3 items	R(X)	0.733	0.99	0.31	0.37	0.49	0.633	0.84	0.63	0.74	0.9
not BSC	Rounds	1	1	1	1	1	24	12	20	23	40
VM3	E(X)	1.0	1.0	1.0	0.69	1.0	1.0	1.0	0.84	0.76	0.22
6 items	R(X)	0.600	0.67	0.61	0.62	0.62	0.769	0.8	0.56	0.3	0.91
not BSC	Rounds	1	1	1	1	1	45	14	162	2	37
VM4	E(X)	1.0	1.0	1.0	1.0	1.0	1.0	1.0			
9 items	R(X)	0.731	0.67	0.63	0.42	0.56	0.769	0.86			
not BSC	Rounds	1	1	1	1	1	81	22			

Table 6.1: Aggregate measures of auction performance

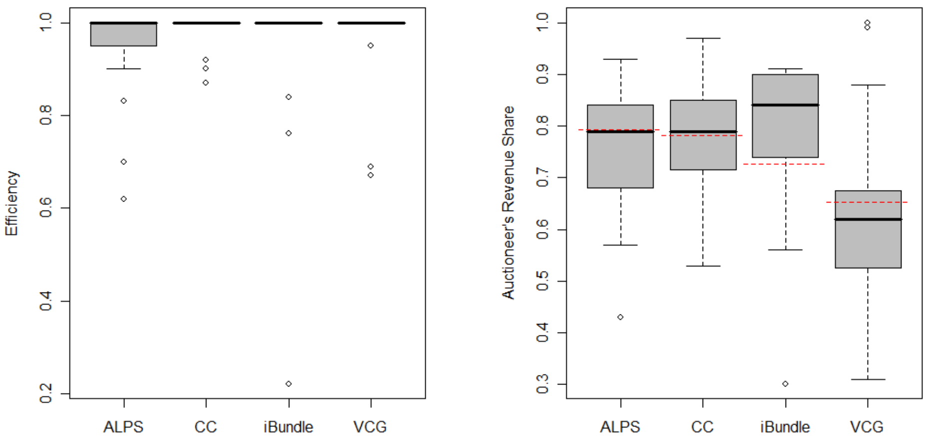
sort their watched bundles by the current payoff. In particular, as the number of interesting bundles became large, this helped bidders to avoid simple calculation errors and to focus on their bidding strategy. Clearly, the findings in these experiments can only be generalized for auction events in which bidders are provided with a similar kind of decision support. Overall, we believe that for complex economic mechanisms framing effects like bidder decision support and user interface play a significantly larger role than for simple ones. Experimental results for complex mechanisms are still valuable, but always need to be seen in the given context.

Result 2: The auctioneer revenue in the VCG auction is lower than in iBundle, the CC auction, and ALPS. (Hypothesis 5)

If the auction result is not 100% efficient, a part of the overall utility is lost. In simulations with best-response bidders we found differences in the revenue distribution for different auction designs and value models. We observed similar behaviour also in the lab experiments, as shown in Figure 6.1. The average auctioneer revenue share across all value models produced by the simulations is shown as a benchmark by a dashed red line. Note that while the auctioneer revenue in ALPS and the CC auction in the lab was very close to the simulation results, the lab results for iBundle were significantly higher, and the lab results for the VCG auction were significantly lower than the simulation results. The iBundle results can be explained by jump bidding and non-best-response bidding that has been observed (see Section 6.2.2).

Result 3: The number of rounds in iBundle is significantly higher than in ALPS and the CC auction. (Hypothesis 9)

Figure 6.2 shows the number of auction rounds for every auction design and for every value model. The average number of rounds across all value models



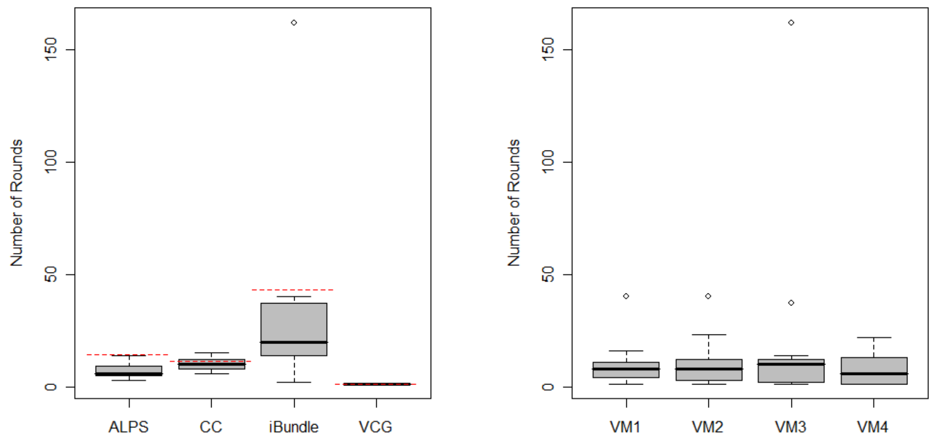
(a) Efficiency (b) Auctioneer revenue share

Figure 6.1: Revenue distribution in different auction designs

produced by the simulations is shown as a benchmark by a dashed red line. The number of rounds in ALPS and the CC auction were lower than in the simulations for two reasons. First, eligibility rules encouraged the bidders to bid on more bundles than their demand set. Second, jump bids have been used in ALPS. (The CC auction does not allow jump bidding.) In iBundle we observed a significantly higher number of rounds, even though the data does not contain the 3 sessions that had to be canceled after more than 100 rounds due to time reasons. Nevertheless, the number of rounds was lower than in the simulations due to jump bidding, which was allowed in iBundle.

6.1.1 Pairwise Comparisons of Auction Designs

This section provides some statistical analysis with pairwise comparisons of selected metrics among treatment groups. A pairwise comparison entails less assumptions on the data generating process than a linear model. We utilize the *t-test* for independent samples and the *nonparamteric Wilcoxon rank sum test* to remove underlying distributional assumptions. The results are shown in



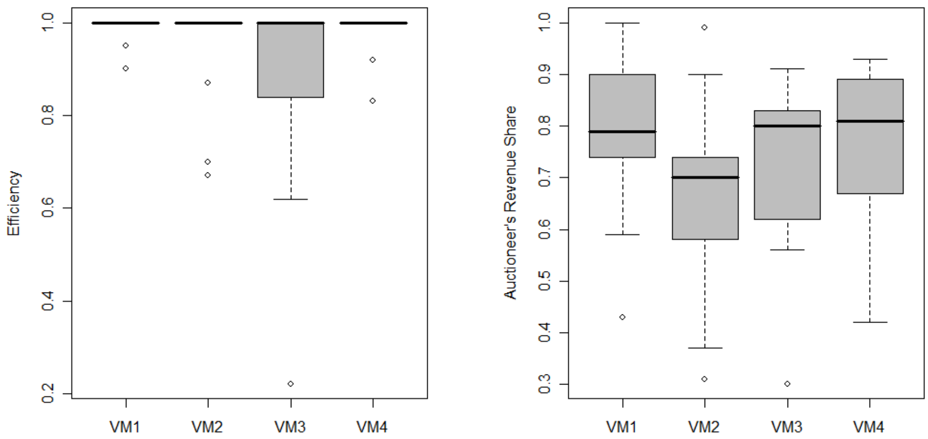
(a) Rounds per auction design (b) Rounds per value model

Figure 6.2: Number of auction rounds

Table 6.2. While we did not find significant differences in efficiency, we found the auctioneer revenue share in the VCG auction to be significantly lower than in all other auction designs, which is consistent with the simulation results.

Comparison	E(X)	R(X)	U(X)	Rounds
VCG vs. iBundle (Wilcoxon)	109 (0.8314)	60 (0.0539)	230 (0.0694)	109 (0.8314)
VCG vs. iBundle (t-test)	0.71 (0.4868)	-1.94 (0.0633)	2.47 (0.0197)	0.71 (0.4868)
VCG vs. ALPS (Wilcoxon)	136 (0.7405)	74 (0.0416)	243 (0.2534)	136 (0.7405)
VCG vs. ALPS (t-test)	0,40 (0.6915)	-2.09 (0.0450)	1.58 (0.1239)	0.40 (0.6915)
VCG vs. CC (Wilcoxon)	126.5 (0.8962)	58 (0.0072)	231 (0.4096)	126.5 (0.8962)
VCG vs. CC (t-test)	-0.81 (0.4263)	-2.63 (0.0134)	1.40 (0.1719)	-8.1 (0.4263)
ALPS vs. iBundle (Wilcoxon)	103 (0.9619)	88 (0.4944)	200 (0.3730)	103 (0.9619)
ALPS vs. iBundle (t-test)	0.46 (0.6509)	-0.18 (0.854)	1.24 (0.2238)	0.49 (0.6295)
ALPS vs. CC (Wilcoxon)	115.5 (0.4463)	124 (0.889)	176 (0.5246)	115.5 (0.4463)
ALPS vs. CC (t-test)	-1.25 (0.2253)	-0.50 (0.6191)	-0.31 (0.7569)	-1.25 (0.2253)
iBundle vs. CC (Wilcoxon)	95 (0.6)	119.5 (0.5083)	120 (0.1306)	95 (0.6)
iBundle vs. CC (t-test)	-1.15 (0.2717)	-0.20 (0.8373)	-1.62 (0.1136)	-1.15 (0.2717)

Table 6.2: Pairwise significance tests for different auction designs



(a) Efficiency (b) Auctioneer revenue share

Figure 6.3: Revenue distribution in different value models

6.1.2 Pairwise Comparisons of Value Models

Result 4: There are no significant differences in efficiency across different value models with 3, 6, and 9 items. (Hypothesis 10)

In our experimental setting we have used auctions with 3, 6, and 9 items. The bidders were interested in only 1 to 3 bundles in VM1 and VM2 (3 items), in 11-15 bundles in VM3 (6 items), and in 24-27 bundles in VM4 (9 items). We did not find any significant differences in efficiency between different value models (see Figure 6.3 and Table 6.3). Again, we assume the main reason for that to be the bidder decision support tools that made it fairly easy for bidders to find out the profitable bundles and follow a certain strategy. It is not clear, however, whether the same holds for even larger auctions with more than 9 items, in which the preference elicitation on an exponentially large number of bundles might become a problem.

Comparison	Efficiency	Revenue	Rounds
VM1 vs. VM2 (Wilcoxon)	142 (1)	211 (0.0209)	147 (0.8472)
VM1 vs. VM2 (t-test)	0.87 (0.3923)	2.21 (0.03441)	-0.19 (0.8472)
VM1 vs. VM3 (Wilcoxon)	163 (0.4352)	177 (0.2620)	129.5 (0.6125)
VM1 vs. VM3 (t-test)	1.70 (0.1071)	1.30 (0.2027)	-0.9482 (0.3556)
VM1 vs. VM4 (Wilcoxon)	102.5 (0.6591)	119 (0.7333)	115 (0.5894)
VM1 vs. VM4 (t-test)	-0.08 (0.9348)	4481 (0.6575)	0.4122 (0.6833)
VM2 vs. VM3 (Wilcoxon)	164 (0.3569)	115.5 (0.3259)	126 (0.5308)
VM2 vs. VM3 (t-test)	1.13 (0.2699)	-0.9395 (0.3545)	-0.8756 (0.3926)
VM2 vs. VM4 (Wilcoxon)	106 (0.7022)	70.5 (0.0964)	118 (0.762)
VM2 vs. VM4 (t-test)	-0.87 (0.3947)	-1.58 (0.1247)	0.5876 (0.5615)
VM3 vs. VM4 (Wilcoxon)	91 (0.2260)	89 (0.3787)	121.5 (0.6538)
VM3 vs. VM4 (t-test)	-1.696 (0.1069)	-0.7473 (0.4611)	1.09 (0.2904)

Table 6.3: Pairwise significance tests for different value models

	DF	Sum of Squares	Mean Square	F Value	Pr > F
Value Model	1	0.00664	0.00664	0.3829	0.5384
Auction Design	1	0.00535	0.00535	0.3085	0.5807
Value Model * Auction Design	1	0.00003	0.00003	0.0018	0.9661
Residuals	60	1.04066	0.01734		

Table 6.4: Impact of the value model and auction design on efficiency

6.1.3 ANOVA Analysis

This section describes the results of the *analysis of variance* (ANOVA), which identifies the main impact factors and magnitude of interaction effects. The ANOVA statistical model for our case is given by:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

where Y_{ijk} stands for the dependent variable, and the differences are explained by the auction design α_i and the value model β_j. The estimation error is denoted by ϵ_{ijk} and the expected value is denoted by μ. ANOVA assumes homoscedasticity of residuals, as well as their normal distribution.

First, the parameter estimation was performed with the allocative efficiency

	DF	Sum of Squares	Mean Square	F Value	Pr > F
Value Model	1	0.00161	0.00161	0.0596	0.8079
Auction Design	1	0.03931	0.03931	1.4592	0.2318
Value Model * Auction Design	1	0.00054	0.00054	0.0200	0.8879
Residuals	60	1.61634	0.02694		

Table 6.5: Impact of the value model and auction design on the auctioneer revenue share

as the explanatory variable Y_{ijk}. The estimation results are given in Table 6.4. The results support the null hypothesis that the auction design and value model overall did not have a significant impact on efficiency in these experiments. The same analysis was then performed with the auctioneer revenue share as the explanatory variable Y_{ijk} with similar results, see Table 6.5.

6.2 Analysis of Bidding Behavior

This section provides an individual-level analysis of the bidding behavior. In *combinatorial* auctions bidding strategies can be much more complex than in traditional single-item auctions, since bidders not only have to choose the bid price, but also to select the bundles to bid on in every round. In the VCG auction, we tested Hypotheses 1 and 2 and analyzed, whether the bidders followed the strategies predicted by theory. Since there is no equilibrium analysis of examined linear-price designs, we did not conduct any structural analysis, but estimated a *logit model* with different covariates possibly explaining the bidding behavior. In iBundle we compared the bidders' strategies to best-response bidders in our simulations.

For the following analysis the ***extended bundles phenomenon*** that we observed in many auctions is essential. The bidders sometimes submitted bids for ***extended bundles***, i.e., supersets of the bundles for which they were given explicit valuations. For example, a bidder that received a positive valuation for AB would sometimes also bid on ABC with the same or higher bid price as for AB, although she has not been given an explicit valuation for ABC. The awareness of free disposal might have led them to the conclusion that they could win more by bidding on extended bundles. On the other hand, bidding on extended bundles can be a strategy in ALPS and the CC auction to keep eligibility high, but at the risk of winning a bundle at a lower payoff. Altogether, from the 6340 bids over all auctions there were 614 bids on extended bundles, namely 84 in VM1, 151 in VM2, 233 in VM3, and 146 in VM4. From those bids more than a half were submitted in the VCG auction, so that excluding the VCG auction the numbers are 59, 105, 89, 14 respectively.

6.2.1 Bidding Behavior in the VCG Auction

Most laboratory studies of single-unit Vickrey auctions found that bidders tend to overbid in such environments (Kagel, 1995). In multi-unit uniform price auctions bidders tend to overbid on the first unit and underbid on the second unit, which is consistent with the theoretical prediction of demand reduction (Kagel and Levin, 2001). Chen and Takeuchi (2005) found that most bidders either underbid or bid at their true value, i.e., the overbidding in single-unit Vickrey auctions did not carry over to *combinatorial* VCG auctions. In contrast, we identified a clear pattern of overbidding in the VCG auction in our experiment (see Table 6.6).

Result 5: On average, 17.74% of the bids in conducted VCG auctions revealed true bidders' valuations, 25.84% can be classified as underbidders, 56.40% as bidding more than the true valuation for the bundle. (Hypothesis 2)

Table 6.6 shows the relative number of bids that were above, below, and exactly at the respective valuation in different value models. Overall, in all value

	truthful bidding	overbidding	underbidding	activity ratio 1	activity ratio 2
VM1	0.10	0.59	0.31	2.12	0.96
VM2	0.10	0.58	0.33	1.89	0.95
VM3	0.24	0.50	0.26	1.57	0.88
VM4	0.15	0.61	0.24	1.11	0.80

Table 6.6: Non-truthful bidding in the VCG auction

models the relative number of truthful bids was very low.

Result 6: In VCG auctions bidders did not bid on all bundles for which they had positive valuations. (Hypothesis 1)

The column "activity ratio 1" in Table 6.6 provides the number of bundles the bidders have bid on throughout the auction, divided by the number of positive-valued bundles that were explicitly given to the bidders. The numbers are significantly higher than 1.0, which can be explained by the *extended bundles phenomenon* (see Section 6.2). Additionally, some bids were also placed on bundles without any positive valuation, which might have been simple mistakes. Interestingly, the bidders have not bid on *all* positive-valued bundles that were explicitly provided to them. The column "activity ratio 2" in Table 6.6 shows the ratio of the bids on bundles with explicitly specified positive valuations to the total number of these bundles. Actually, the longer the list of valuations, the smaller is this ratio.

To further analyze Hypothesis 2 we used the *OLS regression* with the bundle value as the independent variable and the bid price as the dependent variable. The estimated regression coefficients for explicitly provided and extended bundles are shown in Table 6.7. The row "all VMs" contains the coefficients estimated for *all* auctions. The column "all bundles" refers to all bundles the bidders bid on, inclusive bundles with a negative payoff, which explains values less than 1.0.

VM	all bundles	given bundles	extended bundles
VM1	0.9251	1.2707	1.3698
VM2	0.8457	1.1597	1.1714
VM3	1.0389	1.0296	1.0904
VM4	0.9683	0.9474	1.0406
all VMs	0.9699	0.9635	1.0881

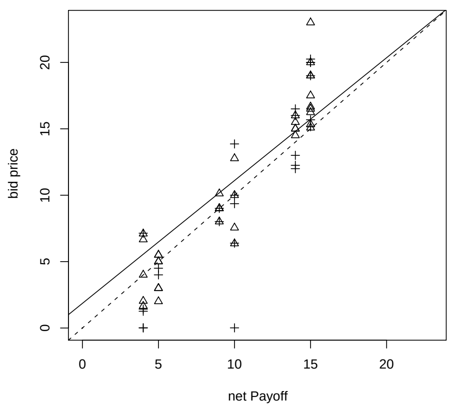
Table 6.7: Regression coefficients for bundle values

Isaac and James (2000) performed a similar regression and found a coefficient value of 0.95 and not statistically different from 1.0. Chen and Takeuchi (2005) report a coefficient of 0.962, which is close to truthful preference revelation. In Table 6.7 we found that the regression coefficient decreased with the value model from VM1 to VM4. This can be due to an increasing number of items, but also to the learning effects during the session, since the auctions were conducted in the order from VM1 to VM4. The coefficients for the small value models were significantly larger than those reported by Chen and Takeuchi (2005) and indicate overbidding. Figure 6.4 depicts the diagrams plotting the bundle valuation (or net payoff respectively) against the actual bid price of all four value models in all VCG auctions. Triangles denote bids on bundles with an explicitly given positive valuation, plus signs (+) denote extended bundles, while stars (*) denote bundles with zero valuation.

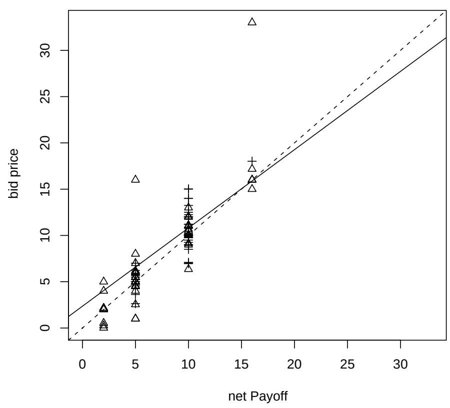
6.2.2 Best-Response Bidding Behavior in Iterative CAs

In *iterative* combinatorial auctions, an interesting question is the bundle selection behavior, i.e., which bundles the bidders bid for in different rounds given their private valuations and the current ask prices. Deviations from best-response bidding might impact the efficiency of auction designs based on this assumption, in particular for iBundle.

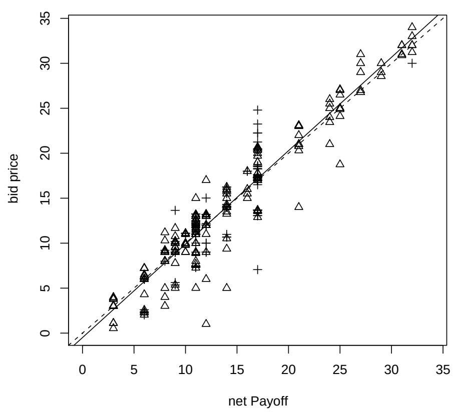
To analyze the bundle selection behavior, we define 4 groups of bundles: BB,



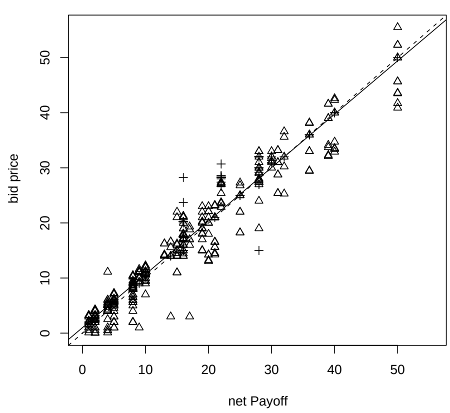
(a) VM1



(b) VM2



(c) VM3



(d) VM4

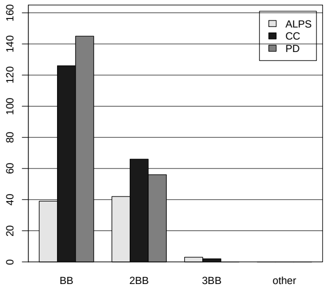
Figure 6.4: Scatter plots of bids in the VCG auction

$2BB$, $3BB$, and all other bundles. For bundles *with explicitly provided valuations* we determine the best possible, second best possible and third best possible payoffs (u_1, u_2, and u_3 respectively) at the current ask prices. We then assign these bundles to the groups according to the rank of their possible payoffs. Since possible payoffs of *extended* bundles can lay in between, we assign extended bundles by the following rule: the extended bundles with payoffs equal to u_1 are assigned to BB, with payoffs in $[u_2; u_1[$ to $2BB$, with payoffs in $[u_3; u_2[$ to $3BB$, and all further to other bundles. (Note that there were only 267 bids on extended bundles from the overall 5249 bids in the conducted *iterative* auctions.) With this notation we also refer to the *bids* submitted on corresponding bundles. We sometimes further distinguish between jump bids and non-jump bids in each of the groups, denoted by J and NJ respectively. So, for example, $2BB^{NJ}$ refers to the non-jump bids on the bundles from $2BB$. Note that in this notation BB^{NJ} contains exactly the best-response bids.

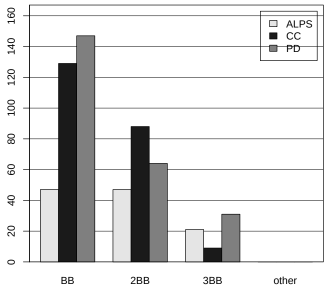
Table 6.8 provides an overview of what proportion of bids has been submitted for different kinds of bundles throughout the auctions. The column xBB^{NJ} refers to all bids from the groups BB^{NJ}, $2BB^{NJ}$, and $3BB^{NJ}$. Similarly, the column xBB refers to all bids from the groups BB, $2BB$, and $3BB$. The results are also visualized in another way in Figure 6.5 and at a more detailed level (BB, $2BB$, ..., nBB on the x-axis) in Figures 6.6 to 6.7.

Result 7: The bidders did not follow the pure best-response bidding strategy in any of the studied iterative combinatorial auction designs. (Hypothesis 6)

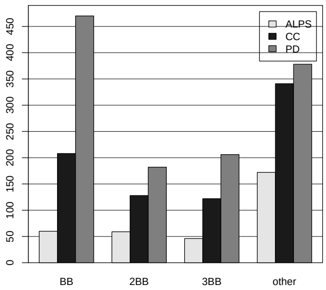
In ALPS, 59% of bids (including jump bids) were submitted on bundles from the best three groups, but only 7% were pure best-response bids. There are multiple reasons for non-best-response bidding in ALPS, such as eligibility rules and the non-monotonicity in prices. Also, the fact that all bids remain active throughout the auction might have an impact on the bidder behavior. In



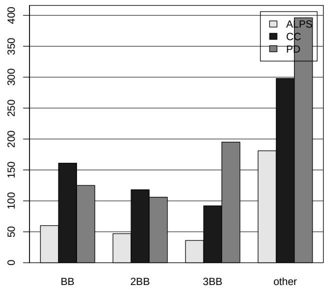
(a) VM1



(b) VM2

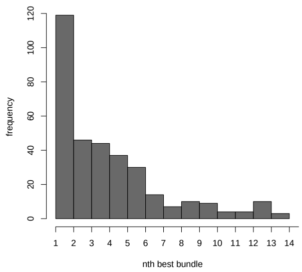


(c) VM3

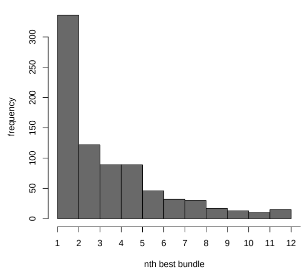


(d) VM4

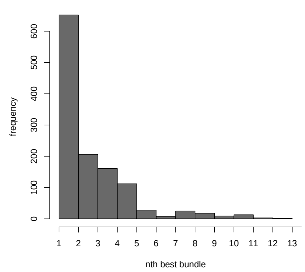
Figure 6.5: Distribution of bids in different value models



(a) VM3, ALPS



(b) VM3, CC

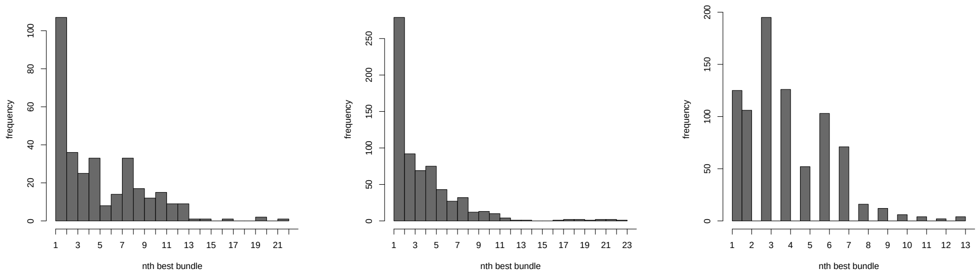


(c) VM3, iBundle

Figure 6.6: Detailed distribution of bids in VM3

	BB^{NJ}	xBB^{NJ}	xBB
ALPS	0.07	0.16	0.59
Clock	0.33	0.66	0.66
iBundle	0.13	0.24	0.69
VM1	0.33	0.54	1.00
VM2	0.30	0.51	1.00
VM3	0.16	0.28	0.62
VM4	0.09	0.24	0.47
VM1 + ALPS	0.15	0.35	1.00
VM1 + Clock	0.65	1.00	1.00
VM1 + iBundle	0.22	0.39	1.00
VM2 + ALPS	0.11	0.25	1.00
VM2 + Clock	0.57	1.00	1.00
VM2 + iBundle	0.26	0.39	1.00
VM3 + ALPS	0.05	0.11	0.49
VM3 + Clock	0.26	0.57	0.57
VM3 + iBundle	0.16	0.22	0.69
VM4 + ALPS	0.06	0.12	0.44
VM4 + Clock	0.24	0.55	0.55
VM4 + iBundle	0.03	0.18	0.52

Table 6.8: Best-response bidding in different auction designs



(a) VM4, ALPS (b) VM4, CC (c) VM4, iBundle

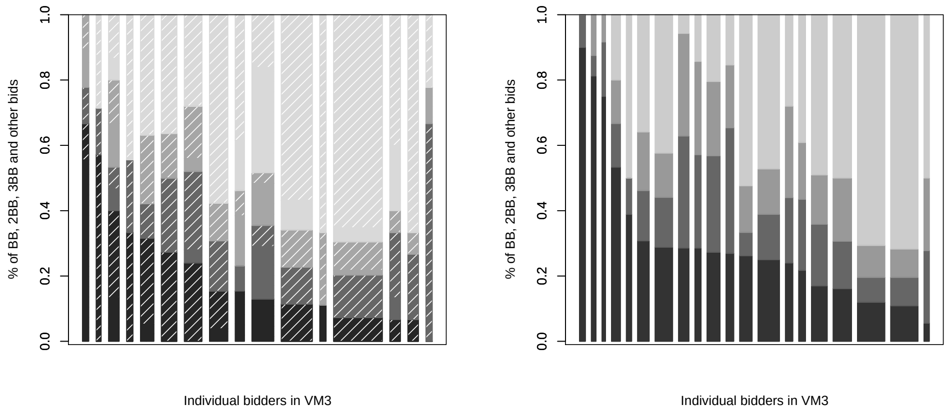
Figure 6.7: Detailed distribution of bids in VM4

the CC action bidders cannot submit jump bids, but the same eligibility rules apply. Here the percentage of bids from the top three bundle groups was 66%. Eligibility rules incent bidders to submit many bids, which accounts for many bids on bundles with a lower payoff. In contrast, the iBundle eligibility rule does not hinder best-response bidding, and theory predicts bidders to follow the best-response strategy. Nevertheless, only 13% of bids were best-response bids in our iBundle experiments. 69% of bids (including jump bids) were submitted on bundles from the best three bundle groups. The reason can be the inability of personalized bundle prices to reflect the current market competition and the large number of rounds, which both induced the bidders to jump bidding. As the computational experiments have shown, the efficiency of iBundle can be significantly below 100% for realistic value models if bidders do not follow the best-response, but heuristic bidding strategies (Schneider et al., 2007).

Result 8: We did not observe pure best-response bidding in iBundle, but the efficiency levels were close to 100% on average in this experiment. However, larger value models suffered from many auction rounds and had partially been canceled due to time reasons. (Hypothesis 8)

The iBundle results need to be interpreted with care. The small value models all achieved 100% efficiency. Already for value models with 6 or 9 items the number of auction rounds increased significantly, and some auctions had to be canceled due to time reasons. This suggests that in its original form iBundle will only be suitable for very small combinatorial auctions. Higher price increments together with proxy agents that translate these high bid increments into many small rounds, might be a remedy.

Result 9: The bidding behavior was heterogeneous across individuals with the same treatments. (Hypothesis 7)



(a) ALPS (b) CC auction

Figure 6.8: Distribution of bids on individual level in ALPS and the CC auction in VM3

The cognitive complexity of the decision environment is probably the best explanation for the fact that observed bidding behavior was different across individuals with the same treatments. This can be illustrated by a bidder-level analysis of particular auction designs, as will also be discussed in subsequent sections. Figures 6.8 to 6.9 show the distribution of bids for individual bidders in VM3. In the plots the bids submitted by every bidder are splitted in the groups BB, $2BB$, $3BB$ and other bids respectively. The plots also reveal the relative number of jump bids in every group, which is reflected by the respective shaded areas. Furthermore, the relative number of submitted bids in relation to other bidders is indicated by the width of the corresponding bar. The figures show that some bidders submitted many bids, the others only a few. There were bidders who concentrated on the best three bundle groups, whereas the others submitted more bids for lower-payoff bundles.

Figure 6.10 is just another visualization of the observed phenomenon. Each plot shows the bid distribution for 4 physical bidders (marked by 4 different colors) that had the same set of valuations in different experimental sessions.

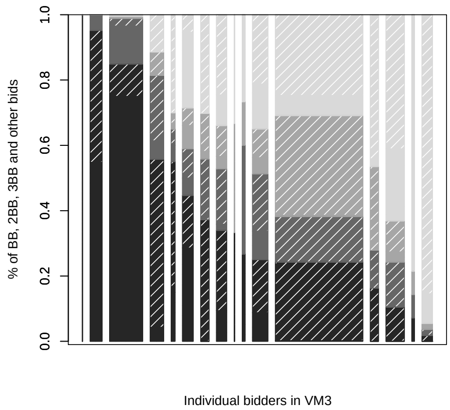
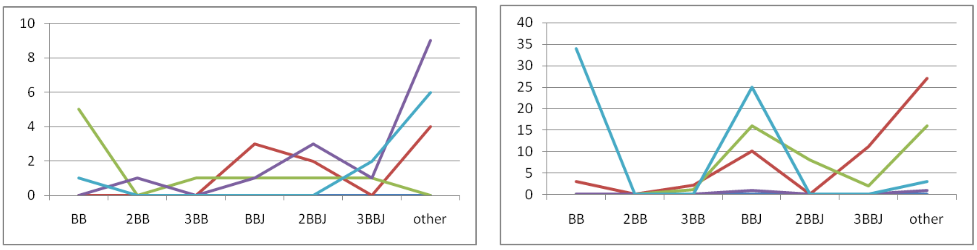


Figure 6.9: Distribution of bids on individual level in iBundle in VM3

The results suggest that even if bidders are provided with decision support tools, one cannot assume them to act according to the same "best strategy" in a complex decision environment (at least without considering learning effects over a longer time period). While there are rational explanations for ALPS, such as eligibility rules, the reasons for this behavior are less obvious in iBundle. Bidder idiosyncrasies and mixed strategies such as in trembling-hand perfect equilibria (Selten, 1975) are possible explanations of these observations. In light of these findings, robustness of auction designs against non-optimal bidding strategies emerges as an important design criterion for practical applications of combinatorial auctions.

6.2.3 Ordinal Choice Model

This section models a bidder's bundle selection decision in each round as a discrete choice problem, whereby the impact of the auction design and value model is estimated. The model follows the general structure outlined in Section 5.3.1. In the ordinal logit model the dependent variable y can take four



(a) ALPS, bidder 3, VM3 (b) iBundle, bidder 1, VM3

Figure 6.10: Distribution of bids in ALPS and iBundle for bidders with the same set of valuations (selected valuation sets, VM3)

Estimates	Y=0 (BB)	Y=1 (2BB)	Y=2 (3BB)	Y=4 (other)
CC	.0927	.0152	-.0164	-.0916
PD	.1238	.0231	-.0201	-.1268
VM2	-.0708	-.0192	.0083	.0817
VM3	-.3663	-.0728	.0393	.3998
VM4	-.4132	-.1183	.0063	.5251

Table 6.9: Estimation results for the *ordinal logit model*

different values: 0 means that the bidder submitted a bid on a bundle from BB, 1 stays for a bid on a bundle from $2BB$, 2 stays for a bid on a bundle from $3BB$, and 3 stays for a bid on any other bundle. (For the distribution of these attribute values in different value models see Figure 6.5). The independent variables $\mathbf{x}_i$ include binary variables for the auction designs (CC = CC auction, PD = iBundle) and value models (VM2, VM3, VM4). As a baseline for the categorical variables auction design and value model we used ALPS and VM1 respectively. In the ordered logit model ϵ_i has a standard logistic distribution. For details on the estimation procedure and the interpretation of the results see (Greene, 2003).

The marginal effects for the ordered probability model are given in Table 6.9. Each negative coefficient in the second column describes a negative impact on choosing one of the best bundles, the third column describes the marginal effect on the second best bundles, etc. While the CC auction (CC) and iBundle (PD) have a positive impact on selecting one of the best and second best bundles, a larger value model (VM3, VM4) has a negative impact. The last observation is especially important as it emphasizes the suboptimality of bidders' decisions in larger value models, and, therefore, the need of further research in the area of large-scale auctions and bidder support tools.

Chapter 7

Conclusions

In this final chapter, the main contributions of this thesis are summarized and discussed. I also outline potential directions of further research in this area.

7.1 A Brief Review

Iterative combinatorial auctions using linear ask prices are promising mechanisms for complex negotiation problems including multiple heterogeneous items. While game theoretical modeling is essential for the understanding of basic economic laws, the discrete nature of ICAs, the effects of eligibility rules and fine grained ask price calculations in these auction designs defy much formal analysis. Economic experiments, on the other hand, are costly and the number of treatment variables that can be analyzed in laboratory experiments is limited. In recent years, computation has become another research method, complementing theory and experiment (a.k.a. computational sciences). Computer simulations make it possible to investigate scenarios and study phenomena that have been shown to be difficult to analyze analytically. Combinatorial auctions are still a new phenomenon and, after a number of seminal contributions describing the underlying economic theory, much can be gained by testing new auction rules and different types of information feedback using computational methods and laboratory experiments.

In our work, we used both computational and laboratory experiments to compare characteristics of three established (the CC auction, RAD, and iBundle) and two new (ALPS and ALPSm) ICA designs, four of which are based on linear ask prices. The comparison was done against the Vickrey-Clarke-Groves auction, which provides at least a theoretical benchmark. All experiments have been conducted based on the same auction platform with minimal differences in the user interface across the auction designs.

In contrast to primal-dual auction designs, *linear-price auctions* follow a more heuristic approach to update ask prices and find the efficient solution. While it is easy to construct examples in which linear prices lead to inefficiencies, the allocative efficiency in ALPS, ALPSm, and the CC auction was surprisingly high for very realistic value models in our large-scale experiments. Just like exact combinatorial optimization algorithms find a feasible optimal solution at the cost of high computational effort, primal-dual auction mechanisms provide an efficient solution at the cost of many auction rounds and non-linear personalized prices. Moreover, best-response bidding is required to achieve the allocative efficiency. In analogy, similar to approximation schemes or heuristics, linear-price auctions are robust against different bidding strategies and can find very good allocations in a much lower number of rounds at the cost of minor inefficiencies.

In the computational experiments, ALPSm typically achieved a higher efficiency and lower ask prices, than the CC auction. As opposed to the CC auction, the price non-monotonicity and price distortions can be disturbing for the bidders in RAD, ALPS and ALPSm. Only a small percentage of ask prices exhibited price distortions in our experiments, but the price monotonicity was relatively low. Interestingly, even in cases with high synergy values in the

Pairwise Synergy value model, the efficiency levels in ALPS, ALPSm and the CC auction were very high. For the remaining inefficiencies in linear-price auctions, there are a few remedies, such as the Proxy phase in the *Clock-Proxy* auction (Ausubel et al., 2006) that address these inefficiencies, but these designs have not yet been thoroughly analyzed. In summary, linear-price designs bear a number of advantages:

- Only a linear number of prices needs to be communicated.
- Linear prices, if perceived as a guideline, help bidders to easily find items with high competition and allow for endogenous bidding (Kwon et al., 2005).
- The perceived fairness of anonymous prices might be of importance in some applications.
- The number of auction rounds is much lower at the expense of small inefficiencies at the end of the auction compared to primal-dual auction designs.
- ALPSm showed to be robust against different non-best-response bidding strategies. This robustness is important since human bidders might not follow the pure best-response strategy.

In the laboratory experiments, we compared the auction designs based on 4 different value models with 3, 6, and 9 items. Although the bidding behavior was heterogeneous and did not follow the pure best-response strategy in any of the auction designs, we achieved high levels of efficiency in all of them. Actually, we did not find a significant difference in efficiency across different auction designs. While the laboratory experiments suggest all examined auction designs to be robust against non-best-response bidding, the computational experiments have shown that this can actually lead to significant efficiency losses in iBundle. Interestingly, we did also find no significant difference in efficiency across value models of different size. We believe that this is largely due to the decision support tools that we provided to the bidders. Furthermore, the auctioneer revenue in the VCG auction was significantly lower than in the *iterative* designs, as predicted by the simulation results. Overall, iBundle suffered from a very large number of auction rounds even for auctions with only 6 items. On the other hand, some instances terminated after one or two rounds, as for some reason every active bidder got allocated some bundle, which caused iBundle to terminate.

7.2 Future Work

Combinatorial auctions are considerably more complex for bidders than traditional single-item auctions. The bidders have to choose one or more bundles from an exponentially large set of alternatives and select a bid price for every bid. We believe that bidder decision support tools play an important role in these auctions. The role of decision support tools, however, needs to be analyzed in more detail in future experiments. Software vendors in this field promote the use of combinatorial auctions for very large events. While our experiments suggest that combinatorial auctions of up to 9 items can achieve very high levels of efficiency with only minimal bidder decision support, it would be important to analyze the role of decision support tools and the efficiency in large-scale combinatorial auctions.

Beside of the auction size, there is also a number of other interesting aspects that require further research. For example, we did not analyze learning effects specifically in our laboratory experiments. (Nevertheless, we performed a number of training rounds and tested the subjects knowledge to make sure that they had a solid understanding of every auction design.) We expect that with multiple repetitions bidders would better learn the corresponding auction design and adjust their bidding strategy. So, for auctions with higher auctioneer revenues that do not converge to approximate minimal CE prices (like the CC auction) this can lead to speculative bidding and, hence, larger inefficiencies than we observed. Furthermore, the impact of different bidding languages can be analyzed.

Appendix A

ALPS Auction Design Reference

A.1 ALPS Ask Prices

The central ALPS auction rule focuses on the calculation of ask prices. There are a number of ways, how *pseudo-dual prices* can be calculated. The following three properties can serve as guidelines:

1. The ask prices for the next round should be *compatible* with the current provisional allocation and submitted bids, i.e., all winning bids should be higher than or equal to the ask prices, and all losing bids should be lower than the ask prices. If no such prices exist, they should be approximated as closely as possible by minimizing the *linear price compatibility distortions* (see Section 2.5.2).

2. The ask prices should be balanced across all items to be perceived as fair and to mitigate the *threshold problem*.

3. The ask prices should be minimal, enabling bidders to submit bids as long as they can and to end up with approximate *minimal CE prices*.

RAD describes a procedure to satisfy the first two properties by solving a series of linear programs (LPs), minimizing the sum of slack variables that represent the linear price compatibility distortions. The idea of *pseudo-dual prices* has been introduced already in Rassenti et al. (1982). In ALPS, we propose an extension to the RAD price calculation rules, which fulfills all above requirements and addresses some pitfalls of RAD. The overall approach can be schematically described as follows:

$$
\begin{array}{lll}
\min_{p_{ask}(k),\delta_i(S)} \{max\{\delta_i(S)\}, max\{p_{ask}(k)\}\} & & \\
\text{s.t.} & & \\
\sum_{k\in S} p_{ask}(k) = p_{bid,i}(S) & \forall\, b_i(S) \in W & \\
\sum_{k\in S} p_{ask}(k) + \delta_i(S) \geq p_{bid,i}(S) & \forall\, b_i(S) \in L & (1)\\
\delta_i(S) \geq 0 & \forall\, b_i(S) \in L & \\
p_{ask}(k) \geq 0 & \forall\, k \in \mathcal{K} &
\end{array}
$$

The first condition sets the winning bid prices equal to the ask prices, which satisfies the first price compatibility requirement. The second condition tries to satisfy the second price compatibility requirement as closely as possible, whereby the distortions $\delta_i(S)$ represent the deviations from the ideal (slack variables).

With the *XOR bidding language* the losing bids of a winning bidder are not included in (1). Since a bidder can only win one bundle at maximum, her losing bids might unnecessarily keep up prices on other items, which conflicts with the third requirement. With the *OR bidding language* all losing bids are included (as in RAD).

Note that RAD and ALPS describe only two ways to calculate pseudo-dual ask prices in each round. There are various possibilities in choosing an objective function and constraints that satisfy different criteria. For example, one might also try to minimize the non-monotonicity across rounds. Dunford et al. (2007) have explored this subject further and found high monotonicity with alternative formulations. In ALPS, we have focused on a method that satisfies all above requirements. The schematically defined objective function $\min\{max\{\delta_i(S)\}, max\{p_{ask}(k)\}\}$ stays for a balanced minimization of all distortions $\delta_i(S)$ and then a balanced minimization of the ask prices. This price calculation procedure is now described in detail.

In the first step we sequentially lower all slack variables while trying to keep them balanced. We first minimize the maximum of all slack variables, then fix those slack variables that can not be further improved, and repeat. Let $\hat{L}$ denote the set of all bids $b_i(S)$ for which $\delta_i(S)$ can not be improved any more, and initialize it with $\hat{L} = \emptyset$. Then solve the linear program (2).

$$\begin{array}{lrcll}
\min_{p_{ask}(k),Z,\delta_i(S)} Z & & & & \\
\text{s.t.} & & & & \\
& \sum_{k\in S} p_{ask}(k) & = & p_{bid,i}(S) & \forall\, b_i(S) \in W \\
& \sum_{k\in S} p_{ask}(k) + \hat{\delta}_i(S) & = & p_{bid,i}(S) & \forall\, b_i(S) \in \hat{L} \\
& \sum_{k\in S} p_{ask}(k) + \delta_i(S) & \geq & p_{bid,i}(S) & \forall\, b_i(S) \in L \setminus \hat{L} \\
& 0 \leq \delta_i(S) & \leq & Z & \forall\, b_i(S) \in L \setminus \hat{L} \\
& p_{ask}(k) & \geq & 0 & \forall\, k \in \mathcal{K}
\end{array} \tag{2}$$

Let $\{Z^*, \{\delta_i^*(S)\}, \mathcal{P}_{ask}^*\}$ be the solution of (2) and $L^* := \{b_i(S) : \delta_i^*(S) = Z^*\}$. If $Z^* = 0$, we are done. Otherwise RAD would fix the slack variables for all bids in L^* and proceed. However, if L^* contains more than one element, some of these slack variables may still be possible to improve. Moreover, if the Simplex optimization algorithm (Nemhauser and Wolsey, 1988) is used, we will very likely get some $\delta_i^*(S) = Z^*$ since it always finds some vertex of the feasible polytope. This requires additional steps. Therefore, ALPS restricts the slack variables by Z^* and minimizes the sum of all slack variables in L^* as follows:

$$
\begin{array}{lrcll}
\min_{p_{ask}(k),\delta_i(S)} \sum_{b_i(S)\in L^*} \delta_i(S) & & & & \\
\text{s.t.} & & & & \\
& \sum_{k\in S} p_{ask}(k) & = & p_{bid,i}(S) & \forall\, b_i(S)\in W \\
& \sum_{k\in S} p_{ask}(k)+\hat{\delta}_i(S) & = & p_{bid,i}(S) & \forall\, b_i(S)\in \hat{L} \\
& \sum_{k\in S} p_{ask}(k)+\delta_i(S) & \geq & p_{bid,i}(S) & \forall\, b_i(S)\in L\setminus\hat{L} \\
& 0\leq \delta_i(S) & \leq & Z^* & \forall\, b_i(S)\in L\setminus\hat{L} \\
& p_{ask}(k) & \geq & 0 & \forall\, k\in \mathcal{K}
\end{array}
\tag{3}
$$

If at least one of the slack variables in $\hat{L}$ can be improved, this will be done by (3). We now remove all bids with improved slack variables from $\hat{L}$ and repeat (3) until no more slack variables can be improved. At this point we set $\hat{L} := \hat{L}\cup L^*$, fix all non-improvable slack variables ($\forall b_i(S)\in L^*$ set $\hat{\delta}_i(S) := \delta_i^*(S)$), and continue with (2).

After the set of all bids with positive slack variables $\hat{L}$ is identified, and all those slack variables are minimized and fixed to $\{\hat{\delta}_i(S)\}$, prices may still not be unique. For example, in the ideal case we get $\hat{L}=\emptyset$, and we still have a lot of freedom in setting prices. We now balance prices similar to the minimizing slack variables in the previous step. We first minimize the maximum of all prices, then fix those prices that can not be further lowered, and repeat. Let $\hat{K}$ denote the set of all items which prices can not be lowered any more, and initialize it with $\hat{K}=\emptyset$. Then solve the linear program (4).

Let $\{Y^*, \mathcal{P}^*_{ask}\}$ be the solution of (4) and let $K^* := \{k : p^*_{ask}(k) = Y^*\}$. Now RAD would fix the prices for all bids in K^* and proceed. But again, if K^* contains more than one element, some of these prices may still be lowered and this is very likely to happen when using a Simplex-based LP solver. This can be illustrated by the following examples.

$$
\begin{array}{rcll}
\multicolumn{4}{l}{\min_{p_{ask}(k),Y} Y} \\
\multicolumn{4}{l}{\text{s.t.}} \\
\sum_{k\in S} p_{ask}(k) & = & p_{bid,i}(S) & \forall\, b_i(S) \in W \\
\sum_{k\in S} p_{ask}(k) + \hat{\delta}_i(S) & = & p_{bid,i}(S) & \forall\, b_i(S) \in \hat{L} \\
\sum_{k\in S} p_{ask}(k) & \geq & p_{bid,i}(S) & \forall\, b_i(S) \in L \setminus \hat{L} \\
p_{ask}(k) & = & \hat{p}_{ask}(k) & \forall\, k \in \hat{K} \\
0 \leq p_{ask}(k) & \leq & Y & \forall\, k \in \mathcal{K} \setminus \hat{K}
\end{array}
\tag{4}
$$

Example 7. *Consider an auction with three items A, B, C and four currently active bids from different bidders* $p_{bid,1}(A) = 55$*,* $p_{bid,2}(C) = 55$*,* $p_{bid,3}(AB) = 40$*,* $p_{bid,4}(BC) = 40$*. The provisional winners are bidders* 1 *and* 2 *and* $\hat{L} = \emptyset$*. After removing redundant inequalities the linear program (4) looks like:*

$$
\begin{array}{rcl}
\multicolumn{3}{l}{\min_{p_{ask}(B),Y} Y} \\
\multicolumn{3}{l}{\text{s.t.}} \\
p_{ask}(A) & = & 55 \\
p_{ask}(C) & = & 55 \\
55 & \leq & Y \\
0 \leq p_{ask}(B) & \leq & Y
\end{array}
$$

We can get two possible solutions of this problem when using a simplex-based LP solver: $\{p^*_{ask}(B) = 55, Y^* = 55\}$ *or* $\{p^*_{ask}(B) = 0, Y^* = 55\}$*. In the first case RAD would fix all prices to* 55*, which would distort the bidder's understanding of the current competition on item B.*

Another important point is the balancing method used. RAD proposes maximizing the minimal *price instead of* minimizing the maximal *price. However, if the solver finds the second solution, RAD would fix* $\hat{p}_{ask}(A) = 55$ *and* $\hat{p}_{ask}(C) = 55$ *and then yield* $p^*_{ask}(B) = \infty$ *in the next iteration.*

Example 8. *Now consider another auction with three items A, B, C and* *two currently active bids* $p_{bid,1}(ABC) = 160$*,* $p_{bid,2}(A) = 70$*, in which the provisional winner is bidder* 1 *and, again,* $\hat{L} = \emptyset$*. The linear program (4) looks like:*

$$
\begin{array}{rcl}
\multicolumn{3}{l}{\min_{p_{ask}(A), p_{ask}(B), p_{ask}(C), Y} Y} \\
\multicolumn{3}{l}{\text{s.t.}} \\
p_{ask}(A) + p_{ask}(B) + p_{ask}(C) & = & 160 \\
p_{ask}(A) & \geq & 70 \\
0 \leq p_{ask}(A), p_{ask}(B), p_{ask}(C) & \leq & Y
\end{array}
$$

With a simplex-based solver this would yield one of two possible solutions: $\{p^*_{ask}(A) = 70, p^*_{ask}(B) = 20, p^*_{ask}(C) = 70, Y^* = 70\}$ *or* $\{p^*_{ask}(A) = 70, p^*_{ask}(B) = 70, p^*_{ask}(C) = 20, Y^* = 70\}$*. In both cases RAD would stop with this solution. However, there are no reasons, why the prices for items* B *and* C *are different.*

To avoid the pitfalls illustrated in the above examples, ALPS continues by bounding the prices to Y^* and minimizing the sum of all prices in K^* as follows:

$$
\begin{array}{rcll}
\multicolumn{4}{l}{\min_{p_{ask}(k)} \sum_{k \in K^*} p_{ask}(k)} \\
\multicolumn{4}{l}{\text{s.t.}} \\
\sum_{k \in S} p_{ask}(k) & = & p_{bid,i}(S) & \forall\, b_i(S) \in W \\
\sum_{k \in S} p_{ask}(k) + \hat{\delta}_i(S) & = & p_{bid,i}(S) & \forall\, b_i(S) \in \hat{L} \\
\sum_{k \in S} p_{ask}(k) & \geq & p_{bid,i}(S) & \forall\, b_i(S) \in L \setminus \hat{L} \\
p_{ask}(k) & = & \hat{p}_{ask}(k) & \forall\, k \in \hat{K} \\
0 \leq p_{ask}(k) & \leq & Y^* & \forall\, k \in \mathcal{K} \setminus \hat{K}
\end{array}
\tag{5}
$$

If at least one of the prices in K^* can be lowered, this will be done by (5). We now remove all items with lowered prices from K^* and repeat with (5) until no more prices can be improved. At this point we set $\hat{K} := \hat{K} \cup K^*$, fix all non-improvable prices ($\forall k \in K^*$ set $\hat{p}_{ask}(k) := p^*_{ask}(k)$), and continue with (4), unless $\mathcal{K} \setminus \hat{K} = \emptyset$. In Example 8 the algorithm terminates after one iteration with the prices $\{p^*_{ask}(A) = 70, p^*_{ask}(B) = 45, p^*_{ask}(C) = 45\}$, which better describe the provisional competitive situation.

A.2 ALPS Surplus Eligibility

ALPS *activity rules* are also based on the RAD eligibility rule. A bidders ***eligibility*** e^t_i is the number of distinct objects she is allowed to bid on in a round. In SMR and RAD a collection of bids is eligible if the new bids plus the last round winning bids are placed on no more items than the eligibility e^{t-1}_i. These rules, however, can also lead to inefficiencies. For example, if the items vary in price significantly, bidders may want to replace a single expensive item by a set of cheaper items. This is typically the case in transportation, when bidders give up bidding on the shortest route and start bidding on a detour. In ALPS we extend the RAD eligibility rules with the *surplus-eligibility*, in order to account for these cases.

The ***surplus-eligibility*** $e^t_{+,i}$ gives each bidder i a chance to increase her round t eligibility e^t_i. To retain the original purpose of enforcing activity throughout the auction, the value of the surplus-eligibility is directly bound to the bidder's market activity in the auction so far. The surplus-eligibility e^t_i for each bidder is calculated in each round and is communicated to the bidders along with the ask prices and provisional allocation. In round t a bidder is allowed to bid maximally on as many distinct items as she has bid in the last round, plus the surplus eligibility:

$$\{\text{dist. items in round } t\} \leq e^t_i := \{\text{dist. items in round } t-1\} + e^t_{+,i}$$

To determine the value $e^t_{+,i}$ we propose a measure for a bidder's market activity, in which we want to avoid pretending activity by submitting deliberately losing bids. For this purpose, we introduce the notion of the ***bid volume*** of bidder i in round t:

***round bid volume*:** $rbv_i^t := \sum_{k \in \mathcal{K}} bpe_i^t(k)$

***total bid volume*:** $tbv_i := \sum_t rbv_i^t$

The ***optimistic bid price estimator function*** $bpe_i^t(k)$ represents an optimistic estimator of bidder's i bid price for the single item k based on her bundle bids in round t. For each bid $b_i^t(S)$ the bid price is splitted between the individual items proportionally to the current item ask prices; for each item $k \in S$ the maximum over all bids is then taken. In other words, $bpe_i^t(k)$ describes, how much the item k is worth to bidder i in round t. A simple example illustrates this concept.

Example 9. *Consider an auction with three items A, B and C and linear prices in round t respectively 10, 10, and 20. If bidder i submits a bid on bundle ABC for 50, the bid price is splitted proportionally to the ask prices, resulting in the values 12.5, 12.5, and 25 for A, B and C respectively. Let her second (and last) bid in round t be 30 on bundle BC, which is splitted proportionally to the ask prices as 10 for B and 20 for C. In this case, we obtain:*

$$\begin{aligned} bpe_i^t(A) &= 12.5 \\ bpe_i^t(B) &= max(10, 12.5) = 12.5 \\ bpe_i^t(C) &= max(20, 25) = 25 \end{aligned}$$

The total bid volume tbv_i equals to the sum of rbv_i^t over all auction rounds and represents the overall bid volume that bidder i has generated in the auction so far. Further, the bidders are ranked by their tbv_i in ascending order. The ***bidder volume rank***, denoted by bvr_i, is the position index in the ordered sequence of this bidder's tbv_i minus 1 (so the rank of the most inactive bidder is 0). The surplus eligibility is then defined as:

$$e^t_{+,i} := round\left(\left(\frac{bvr_i}{|n|-1}\right) \cdot e^{max}_+\right)$$

The value $bvr_i/(|n|-1)$ is scaled to be in $[0,1]$ and serves as an indicator of the relative market activity of bidder i. The e^{max}_+ is the maximal possible surplus eligibility (an auction design parameter). The fact that a bidder's activity can be accumulated throughout the auction, sets incentives for bidders to bid actively right from the start. We found surplus eligibility to have a significant positive impact on efficiency in the transportation value model.

A.3 ALPS Termination Rules

The *termination rule* is central to an auction design. RAD has an eligibility based termination rule and enforces minimum bid increments. As illustrated below, this is not always sufficient to ensure auction termination. Additionally, RAD enforces the auction termination if the same provisional allocation is determined in two consecutive rounds. However, the approximative nature of linear ask prices in RAD in combination with this termination rule can result in inefficient allocations.

Example 10. *Consider an example auction with the bidders' valuations given in Table A.1 and a minimum increment of 2.0. The efficient outcome would be to sell A to bidder 1 and BC to bidder 2. Let two bids (Table A.2) be active in some round. Table A.3 shows the resulting ask prices.*

Bidder 2 does not win in the provisional allocation, so she has to submit another bid. She now has to choose between 27 [11.5+11.5+ 2+2] for AB and

Item	A	B	C	AB	AC	BC	ABC
Bidder1	10						35
Bidder2				32		32	

Table A.1: ALPS termination rules example - Valuations

Item	A	B	C	AB	AC	BC	ABC
Bidder1							30.5
Bidder2				23			

Table A.2: ALPS termination rules example - Some round bids

23 [11.6+7.5+2+2] for BC. As she has equal payoffs for both bundles, the second alternative is selected accidentally. The next round bids are depicted in Table A.4.

This is the second round with the same provisional allocation, and consequently the auction will be terminated with bidder 1 getting all 3 items. Obviously, this is not an efficient outcome. For bidder 2 the auction termination comes as a surprise, since she was still ready to submit higher bids.

A naïve approach of removing this termination rule and relying only on the eligibility-based principle (Kwasnica et al., 2005) causes other problems.

Example 11. *Continuing the above example, ask prices in the new round will change to the values shown in Table A.5.*

At this point bidder 2 could bid 23 on AB again, and the auction would continue

Item	A	B	C
Price	11.5	11.5	7.5

Table A.3: ALPS termination rules example - Some round ask prices

Item	A	B	C	AB	AC	BC	ABC
Bidder1							30.5
Bidder2						23	

Table A.4: ALPS termination rules example: Next round bids

Item	A	B	C
Price	7.5	11.5	11.5

Table A.5: ALPS termination rules example: Next round ask prices

without stopping at all. The reason for this infinite loop is the possibility for ask prices to fall (non-monotonicity).

In order to avoid these problems, we suggest omitting the auction termination rule based on two successive identical allocations and introduce alternative rules to prevent the auction from looping:

- Increase the minimum increment with every equal allocation, but reset the minimum increment to the original value as the allocation changes (ALPS).
- Request every bidder to outbid her own bids which were submitted previously on the same bundle by the price increment Δ (ALPSm).

If the losing bidder's valuation is high enough, both rules will eventually cause the allocation to change. Otherwise, the losing bidder will stop bidding and the auction will close.

Appendix B

Value Models for Lab Experiments

B.1 Value Model VM1

This value model contains the items A,B,C.

Bidder 1		
Bundle		Valuation
{A,B}	=	15.0

Bidder 2		
Bundle		Valuation
{A,B}	=	14.0
{C}	=	5.0

Bidder 3		
Bundle		Valuation
{A,B}	=	9.0

Bidder 4		
Bundle		Valuation
{A,B}	=	10.0
{C}	=	4.0

B.2 Value Model VM2

This value model contains the items A,B,C.

Bidder 1		
Bundle		Valuation
{A}	=	10.0
{B}	=	5.0
{C}	=	2.0

Bidder 2		
Bundle		Valuation
{A}	=	5.0
{B}	=	10.0
{C}	=	5.0

Bidder 3		
Bundle		Valuation
{A}	=	2.0
{B}	=	5.0
{C}	=	10.0

Bidder 4		
Bundle		Valuation
{A,B}	=	5.0
{B,C}	=	16.0

B.3 Value Model VM3

This value model contains the items A, B, C, D, E, F.

Bidder 1		
Bundle		Valuation
{A}	=	9.0
{B}	=	6.0
{C}	=	6.0
{D}	=	6.0
{E}	=	3.0
{F}	=	3.0
{A,B}	=	17.0
{A,D}	=	17.0
{B,C}	=	14.0
{B,E}	=	11.0
{C,F}	=	11.0

Bidder 2		
Bundle		Valuation
{A}	=	6.0
{B}	=	6.0
{C}	=	9.0
{D}	=	3.0
{E}	=	3.0
{F}	=	6.0
{A,B}	=	14.0
{A,D}	=	11.0
{B,C}	=	17.0
{B,E}	=	11.0
{C,F}	=	17.0

Bidder 3		
Bundle		Valuation
{A,B}	=	14.0
{A,D}	=	16.0
{B,C}	=	11.0
{B,E}	=	10.0
{C,F}	=	10.0
{A,B,C}	=	25.0
{A,B,D}	=	27.0
{A,B,E}	=	24.0
{B,C,E}	=	21.0
{B,C,F}	=	21.0
{A,B,C,D}	=	32.0
{A,B,C,F}	=	29.0
{A,B,D,E}	=	31.0
{B,C,E,F}	=	25.0

Bidder 4		
Bundle		Valuation
{A}	=	9.0
{A,D}	=	9.0
{A,E}	=	11.0
{A,F}	=	12.0
{B}	=	12.0
{B,D}	=	12.0
{B,E}	=	12.0
{B,F}	=	12.0
{C}	=	9.0
{C,D}	=	12.0
{C,E}	=	11.0
{C,F}	=	9.0
{D}	=	8.0
{E}	=	11.0
{F}	=	8.0

B.4 Value Model VM4

This value model contains the items A, B, C, D, E, F, G, H, I.

Bidder 1		
Bundle		Valuation
{A}	=	10.0
{B}	=	5.0
{C}	=	2.0
{D}	=	5.0
{E}	=	2.0
{F}	=	1.0
{G}	=	2.0
{H}	=	1.0
{I}	=	0.0
{A,B}	=	16.0
{A,D}	=	16.0
{B,C}	=	8.0
{B,E}	=	8.0
{C,F}	=	4.0
{D,E}	=	8.0
{D,G}	=	8.0
{E,F}	=	4.0
{E,H}	=	4.0
{F,I}	=	2.0
{G,H}	=	4.0
{H,I}	=	2.0
{A,B,C}	=	22.0
{A,D,G}	=	22.0
{B,E,H}	=	13.0
{C,F,I}	=	9.0
{D,E,F}	=	13.0
{G,H,I}	=	9.0

Bidder 2		
Bundle		Valuation
{A}	=	5.0
{B}	=	10.0
{C}	=	5.0
{D}	=	2.0
{E}	=	5.0
{F}	=	2.0
{G}	=	1.0
{H}	=	2.0
{I}	=	1.0
{A,B}	=	16.0
{A,D}	=	8.0
{B,C}	=	16.0
{B,E}	=	16.0
{C,F}	=	8.0
{D,E}	=	8.0
{D,G}	=	4.0
{E,F}	=	8.0
{E,H}	=	8.0
{F,I}	=	4.0
{G,H}	=	4.0
{H,I}	=	4.0
{A,B,C}	=	30.0
{A,D,G}	=	19.0
{B,E,H}	=	28.0
{C,F,I}	=	19.0
{D,E,F}	=	20.0
{G,H,I}	=	15.0

Bidder 3		
Bundle		Valuation
{A}	=	2.0
{B}	=	5.0
{C}	=	10.0
{D}	=	1.0
{E}	=	2.0
{F}	=	5.0
{G}	=	0.0
{H}	=	1.0
{I}	=	2.0
{A,B}	=	8.0
{A,D}	=	4.0
{B,C}	=	16.0
{B,E}	=	8.0
{C,F}	=	16.0
{D,E}	=	4.0
{D,G}	=	2.0
{E,F}	=	8.0
{E,H}	=	4.0
{F,I}	=	8.0
{G,H}	=	2.0
{H,I}	=	4.0
{A,B,C}	=	28.0
{A,D,G}	=	15.0
{B,E,H}	=	19.0
{C,F,I}	=	28.0
{D,E,F}	=	19.0
{G,H,I}	=	15.0

Bidder 4		
Bundle		Valuation
{A,B,D,E}	=	14.0
{B,C,E,F}	=	32.0
{D,E,G,H}	=	9.0
{E,F,H,I}	=	14.0
{A,B,C,D,E}	=	36.0
{A,B,C,E,F}	=	40.0
{A,B,D,E,F}	=	25.0
{A,B,D,E,G}	=	21.0
{A,B,D,E,H}	=	21.0
{A,D,E,G,H}	=	17.0
{B,C,D,E,F}	=	39.0
{B,C,E,F,H}	=	39.0
{B,C,E,F,I}	=	40.0
{B,D,E,G,H}	=	20.0
{B,E,F,H,I}	=	25.0
{C,E,F,H,I}	=	36.0
{D,E,F,G,H}	=	20.0
{D,E,F,H,I}	=	21.0
{D,E,G,H,I}	=	17.0
{E,F,G,H,I}	=	21.0
{A,B,C,D,E,F}	=	50.0
{A,B,D,E,G,H}	=	31.0
{B,C,E,F,H,I}	=	50.0
{D,E,F,G,H,I}	=	31.0

Appendix C

Simulations for Lab Experiments

The following tables illustrate simulation results we achieved for the same value models that were later used in laboratory experiments. The results under different bidding strategy assumptions are presented. VCG describes the results of a VCG mechanism, where bidders follow their dominant strategy. "2nd best allocation" describes the results of a first-price sealed-bid auction, where bidders bid their true valuations, but we do not allow the best set of bundles to win. This indicates how far away would be the 2nd best allocation from an efficient one and should provide a measure for the competition in the auction.

C.1 Best-response bidding strategy

C.1.1 Value Model VM1

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	20.0	14.39	12	B1	{A,B}	10.04	10.04
				B2	{C}	4.35	4.35
ALPS	20.0	16.74	14	B1	{A,B}	12.72	12.72
				B2	{C}	4.02	4.02
iBundle	20.0	17.0	26	B3	{}	0.0	0.0
				B1	{A,B}	13.0	13.0
				B2	{C}	4.0	4.0
				B4	{}	0.0	0.0
VCG	20.0	17.0	1	B1	{A,B}	15.0	13.0
				B2	{C}	5.0	4.0
2nd best	19.0	19.0	1	B1	{A,B}	15.0	15.0
				B4	{C}	4.0	4.0

C.1.2 Value Model VM2

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	30.0	20.0	10	B1	{A}	3.0	3.0
				B2	{B}	8.0	8.0
				B3	{C}	9.0	9.0
ALPS	30.0	22.0	10	B1	{A}	4.0	4.0
				B2	{B}	8.0	8.0
				B3	{C}	10.0	10.0
iBundle	30.0	18.0	26	B1	{A}	2.0	2.0
				B2	{B}	7.0	7.0
				B3	{C}	9.0	9.0
				B4	{}	0.0	0.0
VCG	30.0	13.0	1	B1	{A}	10.0	1.0
				B2	{B}	10.0	6.0
				B3	{C}	10.0	6.0
2nd best	26.0	26.0	1	B1	{A}	10.0	10.0
				B4	{B,C}	16.0	16.0

C.1.3 Value Model VM3

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	55.0	45.56	16	B2	{C,F}	13.96	13.96
				B3	{A,B,D}	26.0	26.0
				B4	{E}	5.6	5.6
ALPS	55.0	40.01	25	B4	{E}	5.52	5.52
				B2	{C,F}	11.47	11.47
				B3	{A,B,D}	23.02	23.02
iBundle	55.0	33.0	47	B4	{E}	4.0	4.0
				B2	{C,F}	11.0	11.0
				B3	{A,B,D}	18.0	18.0
				B1	{}	0.0	0.0
VCG	55.0	33.0	1	B2	{C,F}	17.0	11.0
				B3	{A,B,D}	27.0	18.0
				B4	{E}	11.0	4.0
2nd best	50.0	50.0	1	B1	{C}	6.0	6.0
				B2	{F}	6.0	6.0
				B3	{A,B,D}	27.0	27.0
				B4	{E}	11.0	11.0

C.1.4 Value Model VM4

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	78.0	69.03	14	B3	{C,F,I}	25.62	25.62
				B1	{A,D,G}	17.79	17.79
				B2	{B,E,H}	25.62	25.62
ALPS	78.0	66.44	26	B1	{A,D,G}	16.03	16.03
				B3	{C,F,I}	25.64	25.64
				B2	{B,E,H}	24.77	24.77
iBundle	78.0	65.0	83	B1	{A,D,G}	14.0	14.0
				B2	{B,E,H}	24.0	24.0
				B3	{C,F,I}	27.0	27.0
				B4	{}	0.0	0.0
VCG	78.0	57.0	1	B1	{A,D,G}	22.0	13.0
				B2	{B,E,H}	28.0	22.0
				B3	{C,F,I}	28.0	22.0
2nd best	72.0	72.0	1	B1	{A,D,G}	22.0	22.0
				B4	{B,C,E,F,H,I}	50.0	50.0

C.2 Heuristic *3of5* bidding strategy

C.2.1 Value Model VM1

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	20.0	18.41	10	B1	{A,B}	14.06	14.06
				B2	{C}	4.35	4.35
ALPS	20.0	18.41	10	B2	{C}	4.35	4.35
				B1	{A,B}	14.06	14.06
iBundle	20.0	16.0	21	B2	{C}	5.0	5.0
				B1	{A,B}	11.0	11.0
VCG	20.0	17.0	1	B1	{A,B}	15.0	13.0
				B2	{C}	5.0	4.0
2nd best	19.0	19.0	1	B1	{A,B}	15.0	15.0
				B4	{C}	4.0	4.0

C.2.2 Value Model VM2

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	30.0	24.0	9	B1	{A}	6.0	6.0
				B2	{B}	9.0	9.0
				B3	{C}	9.0	9.0
ALPS	30.0	24.0	9	B1	{A}	6.0	6.0
				B2	{B}	9.0	9.0
				B3	{C}	9.0	9.0
iBundle	30.0	18.0	27	B1	{A}	2.0	2.0
				B2	{B}	7.0	7.0
				B3	{C}	9.0	9.0
VCG	30.0	13.0	1	B1	{A}	10.0	1.0
				B2	{B}	10.0	6.0
				B3	{C}	10.0	6.0
2nd best	26.0	26.0	1	B1	{A}	10.0	10.0
				B4	{B,C}	16.0	16.0

C.2.3 Value Model VM3

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	55.0	49.24	13	B2	{C,F}	15.8	15.8
				B3	{A,B,D}	26.92	26.92
				B4	{E}	6.52	6.52
ALPS	55.0	36.43	18	B2	{C,F}	11.2	11.2
				B3	{A,B,D}	19.05	19.05
				B4	{E}	6.18	6.18
iBundle	55.0	37.0	114	B4	{E}	5.0	5.0
				B2	{C,F}	13.0	13.0
				B3	{A,B,D}	19.0	19.0
VCG	55.0	33.0	1	B2	{C,F}	17.0	11.0
				B3	{A,B,D}	27.0	18.0
				B4	{E}	11.0	4.0
2nd best	50.0	50.0	1	B1	{C}	6.0	6.0
				B2	{F}	6.0	6.0
				B3	{A,B,D}	27.0	27.0
				B4	{E}	11.0	11.0

C.2.4 Value Model VM4

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	78.0	69.03	11	B1	{A,D,G}	17.79	17.79
				B2	{B,E,H}	24.75	24.75
				B3	{C,F,I}	26.49	26.49
ALPS	78.0	71.1	18	B1	{A,D,G}	18.66	18.66
				B2	{B,E,H}	24.9	24.9
				B3	{C,F,I}	27.54	27.54
iBundle	72.0	60.0	300	B4	{B,C,E,F,H,I}	40.0	40.0
				B1	{A,D,G}	20.0	20.0
VCG	78.0	57.0	1	B1	{A,D,G}	22.0	13.0
				B2	{B,E,H}	28.0	22.0
				B3	{C,F,I}	28.0	22.0
2nd best	72.0	72.0	1	B1	{A,D,G}	22.0	22.0
				B4	{B,C,E,F,H,I}	50.0	50.0

C.3 Heuristic *3of10* bidding strategy

C.3.1 Value Model VM1

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	20.0	18.41	10	B1	{A,B}	14.06	14.06
				B2	{C}	4.35	4.35
ALPS	20.0	18.41	10	B2	{C}	4.35	4.35
				B1	{A,B}	14.06	14.06
iBundle	20.0	16.0	21	B2	{C}	5.0	5.0
				B1	{A,B}	11.0	11.0
VCG	20.0	17.0	1	B1	{A,B}	15.0	13.0
				B2	{C}	5.0	4.0
2nd best	19.0	19.0	1	B1	{A,B}	15.0	15.0
				B4	{C}	4.0	4.0

C.3.2 Value Model VM2

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	30.0	24.0	9	B1	{A}	6.0	6.0
				B2	{B}	9.0	9.0
				B3	{C}	9.0	9.0
ALPS	30.0	24.0	9	B1	{A}	6.0	6.0
				B2	{B}	9.0	9.0
				B3	{C}	9.0	9.0
iBundle	30.0	18.0	27	B1	{A}	2.0	2.0
				B2	{B}	7.0	7.0
				B3	{C}	9.0	9.0
VCG	30.0	13.0	1	B1	{A}	10.0	1.0
				B2	{B}	10.0	6.0
				B3	{C}	10.0	6.0
2nd best	26.0	26.0	1	B1	{A}	10.0	10.0
				B4	{B,C}	16.0	16.0

C.3.3 Value Model VM3

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	55.0	49.24	14	B3	{A,B,D}	25.08	25.08
				B2	{C,F}	16.72	16.72
				B4	{E}	7.44	7.44
ALPS	55.0	36.21	14	B4	{E}	5.14	5.14
				B3	{A,B,D}	18.64	18.64
				B2	{C,F}	12.43	12.43
iBundle	55.0	37.0	114	B2	{C,F}	13.0	13.0
				B3	{A,B,D}	18.0	18.0
				B4	{E}	6.0	6.0
VCG	55.0	33.0	1	B2	{C,F}	17.0	11.0
				B3	{A,B,D}	27.0	18.0
				B4	{E}	11.0	4.0
2nd best	50.0	50.0	1	B1	{C}	6.0	6.0
				B2	{F}	6.0	6.0
				B3	{A,B,D}	27.0	27.0
				B4	{E}	11.0	11.0

C.3.4 Value Model VM4

Auction Format	Overall Gain	Auctioneer Revenue	Rounds	Winning Bidder	Winning Bid	Bid Price	Pay Price
CC	78.0	69.9	11	B1	{A,D,G}	19.53	19.53
				B2	{B,E,H}	23.88	23.88
				B3	{C,F,I}	26.49	26.49
ALPS	69.0	66.06	16	B2	{A,D,G}	16.06	16.06
				B4	{B,C,E,F,H,I}	50.0	50.0
iBundle	72.0	61.0	297	B4	{B,C,E,F,H,I}	42.0	42.0
				B1	{A,D,G}	19.0	19.0
VCG	78.0	57.0	1	B1	{A,D,G}	22.0	13.0
				B2	{B,E,H}	28.0	22.0
				B3	{C,F,I}	28.0	22.0
2nd best	72.0	72.0	1	B1	{A,D,G}	22.0	22.0
				B4	{B,C,E,F,H,I}	50.0	50.0

Appendix D

Software Plattform Overview

Iterative combinatorial auctions would not be possible without IT-based auction platforms ensuring the correct auction protocol and solving hard computational problems in each auction round, most notably the *Winner Determination Problem* and the calculation of *ask prices*. Additionally, an easy-to-use user interface with integrated bidder support tools is crucial to handle the cognitive complexity of ICAs. This is also a reason, why combinatorial auctions have been a topic in much recent IS research (see for example Adomavicius and Gupta (2005); Fan et al. (2003); Jones and Koehler (2005); Kelly and Steinberg (2000); Xia et al. (2004)).

The computational and laboratory experiments presented in this thesis were conducted using the software framework ***MarketDesigner***, which was developed at our university department as an essential part of several research projects. In the following sections I give a short overview of the framework architecture and provide a couple of screenshots to illustrate the user interface used in our laboratory experiments.

D.1 Framework Architecture

MarketDesigner is a software platform for iterative combinatorial auctions. The platform allows for automated simulations of ICAs under specific assumptions about the value model and agents' bidding strategies and for conducting ICAs with human bidders. It was implemented as an extensible, plugin-based framework with the goal to allow easy integration of additional auction designs. All framework components were developed in *JAVA* using several current *J2EE*-related technologies like *XML*, *Spring*, *Hibernate*, *Struts*, *AJAX*, *XStream*, etc. For integer optimization the LP-Solver library "lpsolve" was used.

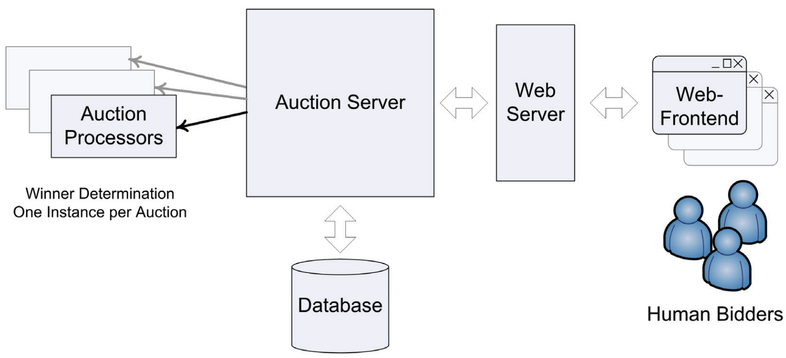


Figure D.1: Web usage view of the MarketDesigner framework

Figure D.1 shows a schematic overview of the *MarketDesigner* framework when used for conducting auctions with human bidders. As depicted, the main component of the framework is the ***auction server*** that is capable of holding several auction processor instances simultaneously, one for every registered and running auction. ***Auction processors*** implement the actual logic of different CA designs including the winner determination, price calculation and enforcement of the auction protocol. Additionally, the auction server stores all essential information generated during the auction flow in a persistent database. The auction server can be accessed via a *SOAP communication interface* using the web services technology. This interface is used by the *web server* that provides the web-based ***user interface*** of the framework. The web server enables auction managers and administrators to create, edit and control auctions as well as human bidders to participate in these auctions. In the following, the auction server is often referred to as ***backend*** and the web server as ***frontend***.

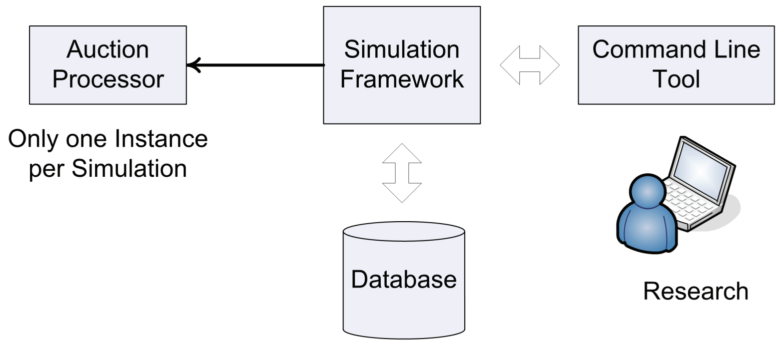


Figure D.2: Simulation view of the MarketDesigner framework

In the automated auction simulation scenario the *web server* component is replaced by the ***simulation framework*** component. Additionally, the simulation framework leaves out the time-consuming web service communication and is directly integrated into the auction server (see Figure D.2). It provides functionality to simulate auctions using artificial bidders and value models and allows for comprehensive analysis of different CA designs.

D.2 Communication Interface and Data Model

Data containers play an essential role for the MarketDesigner framework. On the one hand, they provide auction specific data structures, such as for bids, bundles, and ask prices, used for internal data representation. On the other hand, they are also used as parameters of the *SOAP communication interface* and the simulation framework interface methods.

The data containers class diagram is shown in the Figure D.3. The `AuctionDescriptorComm` object holds the static auction description. In particular, it contains the auction rules (`BiddingRulesComm`, `ClearingRulesComm`, `VisibilityRulesComm`, `AllocationRulesComm`) and the list of the auctioned lots, whereas the lot object (`LotComm`) describes the auctioned item, its start and reservation prices, etc.

A *bundle* is represented by the class `BundleComm`, which contains a list of bundle entries (`BundleEntryComm`), each referencing a specific auction lot. The class `AskPriceComm` holds the bundle (linear for trivial bundles) ask price and the class `AtomicBidComm` stores an atomic bid for a given bundle.

Another group of classes represents the dynamic state of related static objects. So, the `AtomicBidStatusComm` class represents the current atomic bid state (*active*, *inactive*, *displaced*, or *revoked*) and indicates, whether the given bid is provisionally winning or losing. The `BidderStatusComm` class holds the current status information about the given bidder, in particular her current and expected next round eligibility, the number of active, inactive, and provisionally winning bids, etc. The `AuctioneerStatusComm` holds the current status information about the auction manager. Another important class of this group is `AuctionStatusComm`, which describes the current auction state. It contains the counters of the currently active bids and bidders, current round number, and timing information about the auction and the current round. It also contains

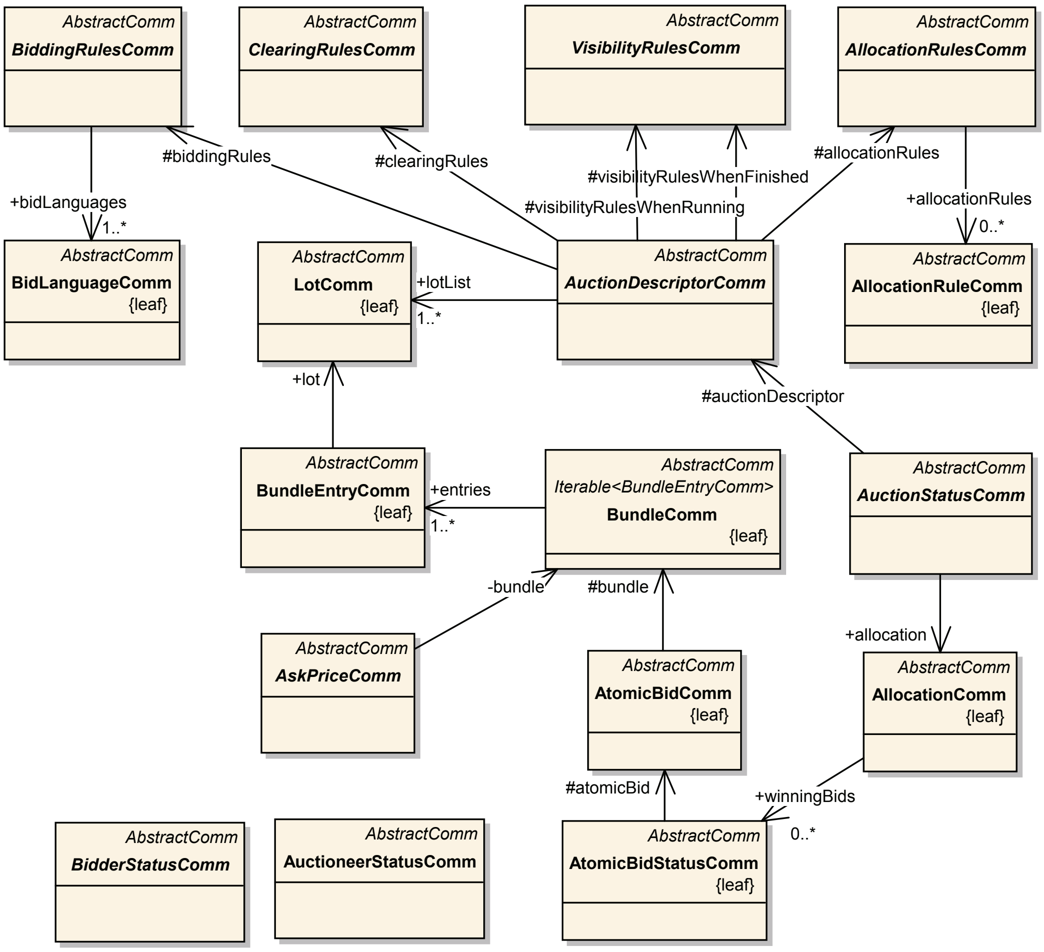


Figure D.3: MarketDesigner data model

the current provisional allocation represented by the class `AllocationComm`.

For a particular auction design, many data container classes are extended by auction design specific classes in the corresponding auction design plugin. For example, the class `AuctionStatusClockComm` extends the class `AuctionStatusComm` for the CC auction. The specialized classes are then used in place of generic classes by all framework components and by all communication interface methods.

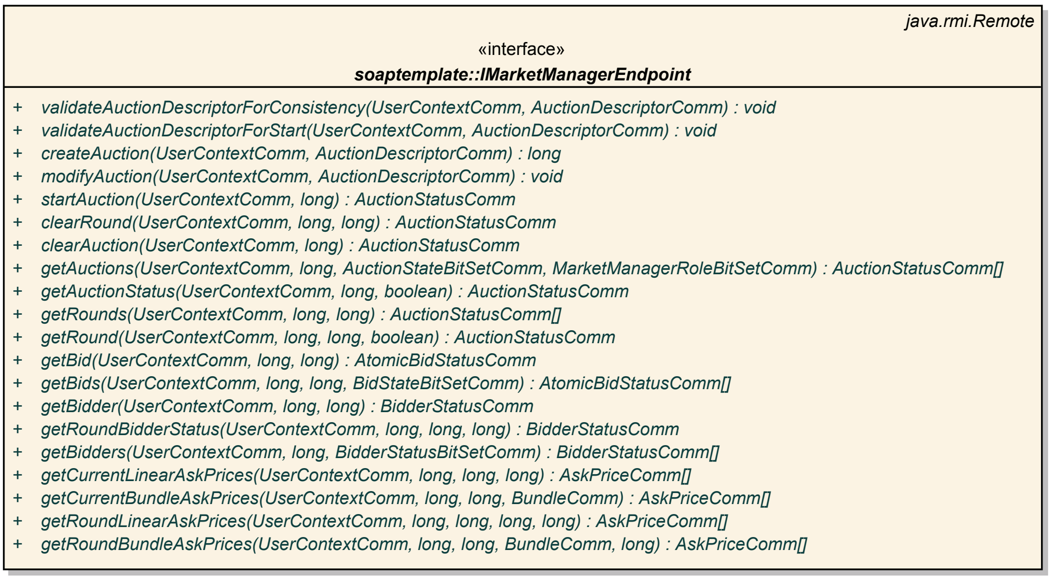


Figure D.4: MarketDesigner MarketManager interface

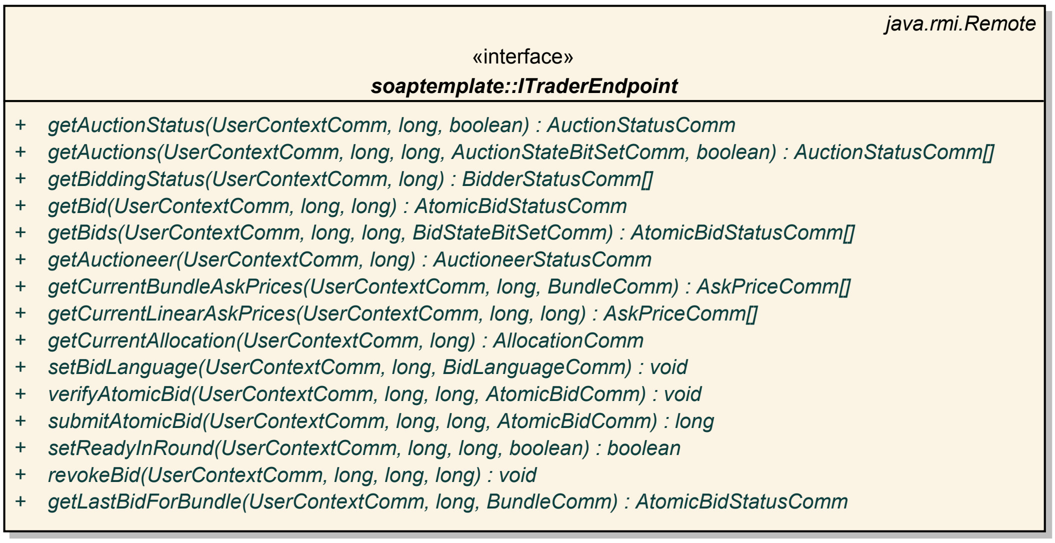


Figure D.5: MarketDesigner Trader interface

Figure D.4 and Figure D.5 show the list of methods of the auction server communication interfaces. The *MarketManager* interface can be used by auction managers to create, edit and control auctions. The *Trader* interface provides the bidding functionality for bidders. Although a detailed discussion of the interface methods is out of the scope of this thesis, the listed method signatures can give an impression of what kind of functionality is needed to conduct an iterative combinatorial auction.

D.3 Auction Server

The internal structure of the *auction server* is schematically shown in Figure D.6. The central logic component *BackendKernel* is based on the following four main classes:

- The `UserManagerServer` class is responsible for the user management, tracking of active user sessions, and management and enforcement of access rights. It exposes the `IUserVerifier` interface to other parts of the server that provides user request authorization services.
- The `CatalogueServer` class manages a hierarchical catalog of categories and items to be auctioned.
- The `MarketManagerServer` class exposes the functionality used by auction managers. The class provides an implementation of the *MarketManager* interface shown in Figure D.4. This includes creating and editing auctions, auction runtime control, auction status monitoring, etc. The class manages an instance of the `AuctionProcessorTable` class that holds references to all currently running auctions.
- The `TraderServer` class provides the functionality used by bidders, i.e., validating bids, submitting bids, and querying the auction status. It implements the *Trader* interface shown in Figure D.5.

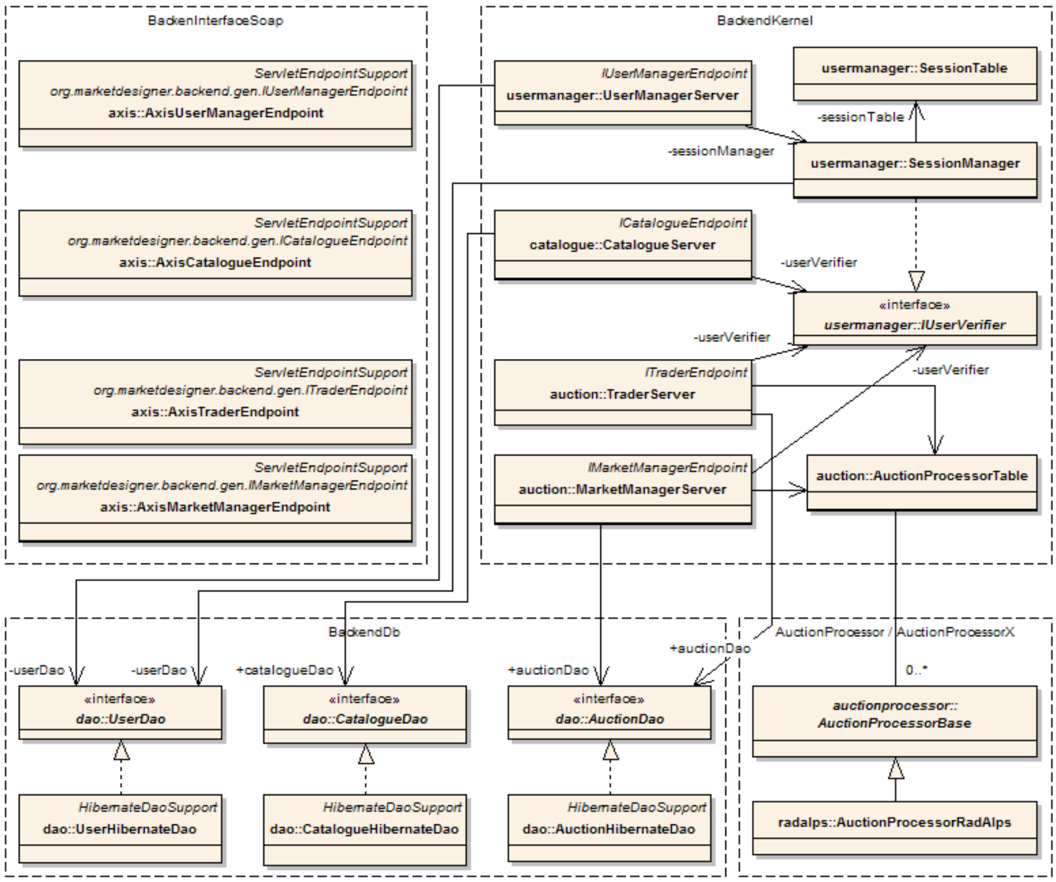


Figure D.6: MarketDesigner Auction server architecture

The *BackendDb* component is an abstraction layer for the *MySql* database used by the auction server. It implements a set of Data Access Objects (DAO) that, integrated with the open-source library *Hibernate*[1], implement an efficient mapping of the *MarketDesigner* data object classes to a relational database.

The *BackendInterfaceSoap* component implements the SOAP abstraction layer that provides a gateway to the *BackendKernel* when handling network requests. For each *BackendKernel* object, a corresponding *Axis* endpoint object

[1]http://www.hibernate.org

is created, which exposes the web service functionality. The proxy implementation is provided by the *Axis*[2] and *Spring*[3] frameworks.

Figure D.7 details the *AuctionProcessor* component and shows an overview of all classes involved in the direct auction execution. The abstract class `AuctionProcessorBase` provides a generic implementation of the functionality common to all auction designs. It has to be extended by a concrete subclass implementing a specific auction design. The class `AuctionProcessorRadAlps` exemplary shows the specialized auction processor class for *RAD* and *ALPS*.

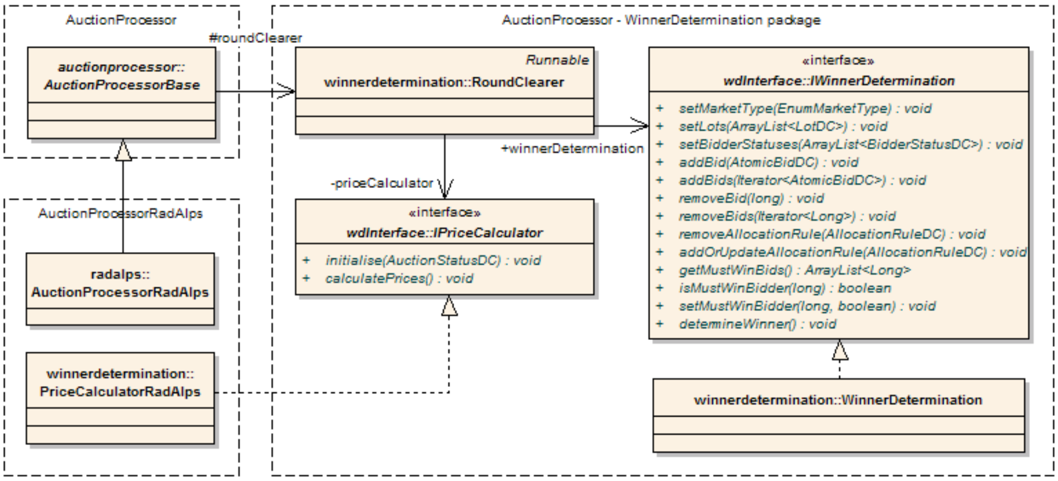


Figure D.7: MarketDesigner auction processor and round clearer

Each auction processor instance holds an instance of the `RoundClearer` class, whose primary objective is to execute the winner determination and price calculation procedures in a separate execution thread, which may take a long time to complete. The `RoundClearer` class is not auction design specific. The winner determination and price calculation execution are controlled through the `IWinnerDetermination` and `IPriceCalculator` interfaces respectively. The winner determination algorithm is common for all auction designs and im-

[2]http://ws.apache.org/axis
[3]http://www.springframework.org

plemented by the class `WinnerDetermination`. In contrast, price calculation algorithms are auction design specific. Each auction processor is therefore paired with an implementation of the `IPriceCalculator` interface. The class `PriceCalculatorRadAlps` exemplary shows the specialized price calculation class for *RAD* and *ALPS*.

D.4 Simulation Framework

The *simulation framework* allows for automated simulations of ICAs under specific assumptions about the value model and agents' bidding strategies. It integrates directly with the auction server and provides a command line interface to control simulations. The simulation runner takes a *simulation parameter set* represented by an *XML* file as input and generates output in form of text dump, database entries, and *Microsoft Excel* sheets. Scripts for sequential execution of multiple simulations using different parameter sets can be easily written in the *Python*[4] scripting language.

Internally, the simulation framework is formed by a number of interfaces and their reference implementations. Figure D.8 shows the main interfaces and available implementations. The `IKernelForSimulation` interface provides methods for communication with the associated auction processor. Its standard implementation `InProcessKernel` implements an inter-process communication, i.e., the direct communication with the associated `AuctionProcessor` class in the same Java application context. The abstract program flow controller class `TestRunner` is extended by the `CompleteAuctionFlowRunner` class, which just simulates the whole auction without interruption. This architecture allows for later implementation of other flow control mechanisms, e.g., for manual step-by-step control or real-time visualization of the bidding process.

[4] http://www.python.org

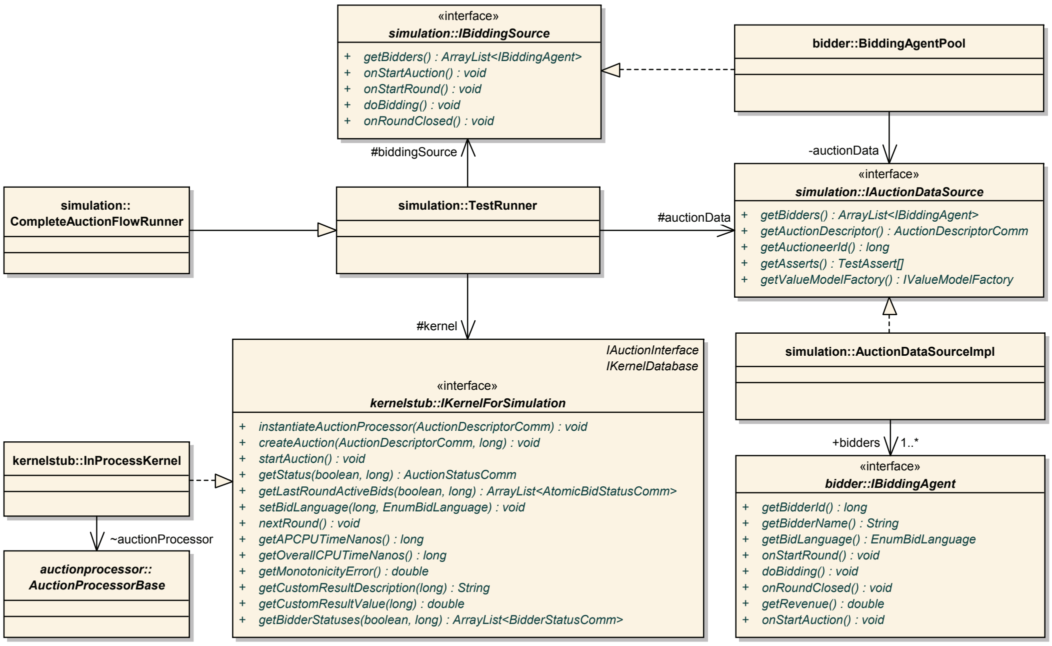


Figure D.8: MarketDesigner simulation framework

The `TestRunner` class references one implementation of the `IBiddingSource` and `IAuctionDataSource` interfaces respectively. Whereas the bidding source generates bids for every round, the auction data source provides general information about the auction design, bidding rules, and participating bidders. The bidding source interface can be implemented using various sources, e.g., by getting the bids from a database, from an XML-document, using an interactive user interface, or, as currently implemented, using automated bidding agents.

Bidding agents do not act independently in multiple threads. Instead, their methods are invoked by a central control class derived from `TestRunner`. The `BiddingAgentPool` class references multiple `IBiddingAgent` interface implementations, which are queried for their bids through the `doBidding(...)` method. Bidding agents submit bids based on their private valuations in the given *value model*, given *bidding strategy* and current auction state, i.e., the current ask prices, eligibility, etc.

D.5 Web Server

The *web server* is a J2EE web application that provides a web-based user interface for the *auction server*. It allows auction managers to create, edit, control, and monitor auctions and bidders to submit bids and observe the auction state. The web server is based on the *Struts*[5] technology that implements a Model-View-Controller architecture for Java-based web applications.

The web server is built of a number of Java Server Pages pages (JSPs) and a number of *Struts action classes* that implement the logic needed to query the auction server, generate objects to be displayed, and process user actions. Additionally, the *ClientApi* module implements proxy classes for efficient communication with the auction server.

The flow of events during processing of a typical web request is illustrated by the sequence diagram in the Figure D.9. The web request sent by a user's web browser is first processed by the servlet class `FrontendActionServlet` and request processor class `FrontendRequestProcessor`, both deeply integrated with the *Struts* framework. Processing is then dispatched to the corresponding *action class* responsible for processing of the specified URI. The action class (exemplary `BidConsoleAction` in the diagram) then performs required processing, in particular, it queries the auction server using the *ClientApi* module (exemplary the `TraderController` class in the diagram), prepares the objects to be displayed, and forwards processing to the appropriate JSP.

Many JSP pages and action classes can be directly reused by different auction designs. Another group of JSPs and action classes is either completely or partially auction design specific. The *WebPlugin technology* developed at our

[5] http://struts.apache.org

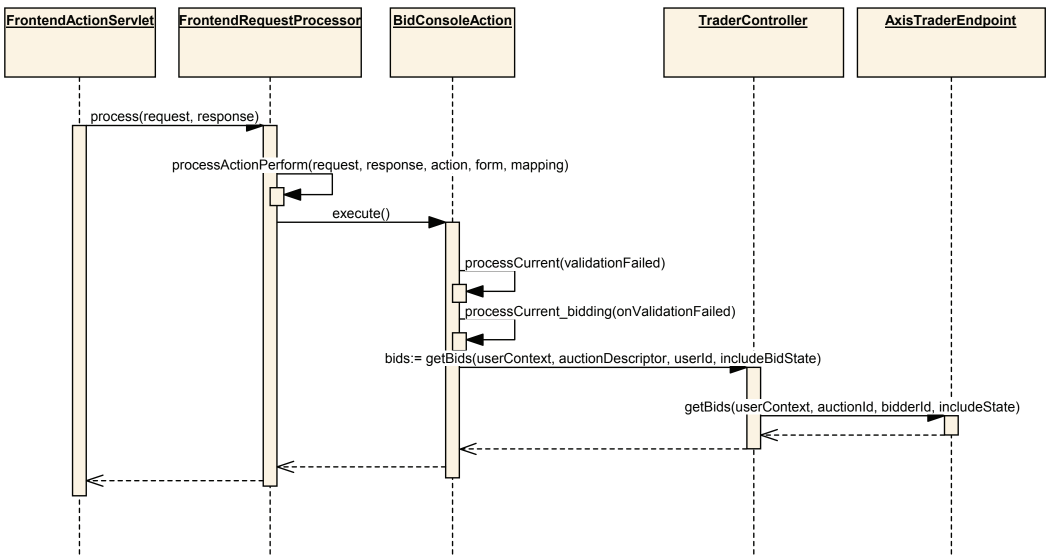


Figure D.9: MarketDesigner web request processing example

university department is used to implement different auction designs. For every auction design a module called *WebPlugin* is implemented, which contains auction design specific JSPs and action classes and can reuse common framework functionality. Plugins implement a specific *WebPlugin* interface and can be easily connected to the system to support additional auction designs. The system is capable of recognizing connected plugins, displaying the auction specific information and navigation menus on common pages, and navigating from common framework pages to plugin pages and vice versa.

D.6 User Interface

As already mentioned, an easy-to-use user interface with integrated bidder support tools is crucial to handle the cognitive complexity of ICAs. Moreover, results of laboratory experiments are highly dependent on the usability and amount of information that bidders face in their interaction with the system.

The following screenshots (Figure D.10 - Figure D.16) illustrate the most important and frequent interactions of experiment participants with the system during the bidding process.

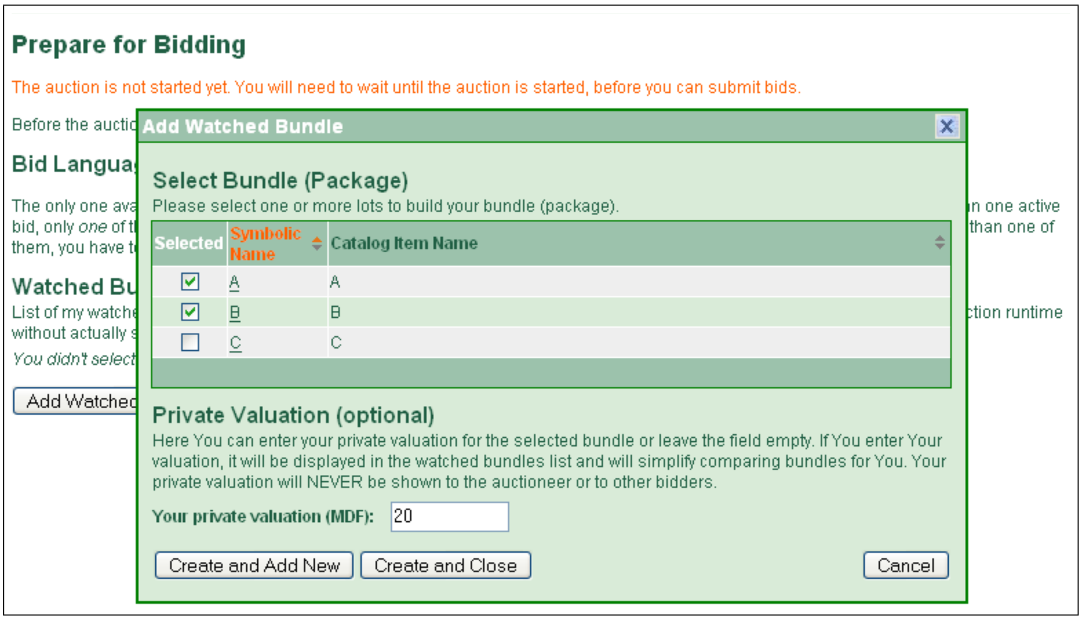


Figure D.10: Screenshot - Creating a watched bundle

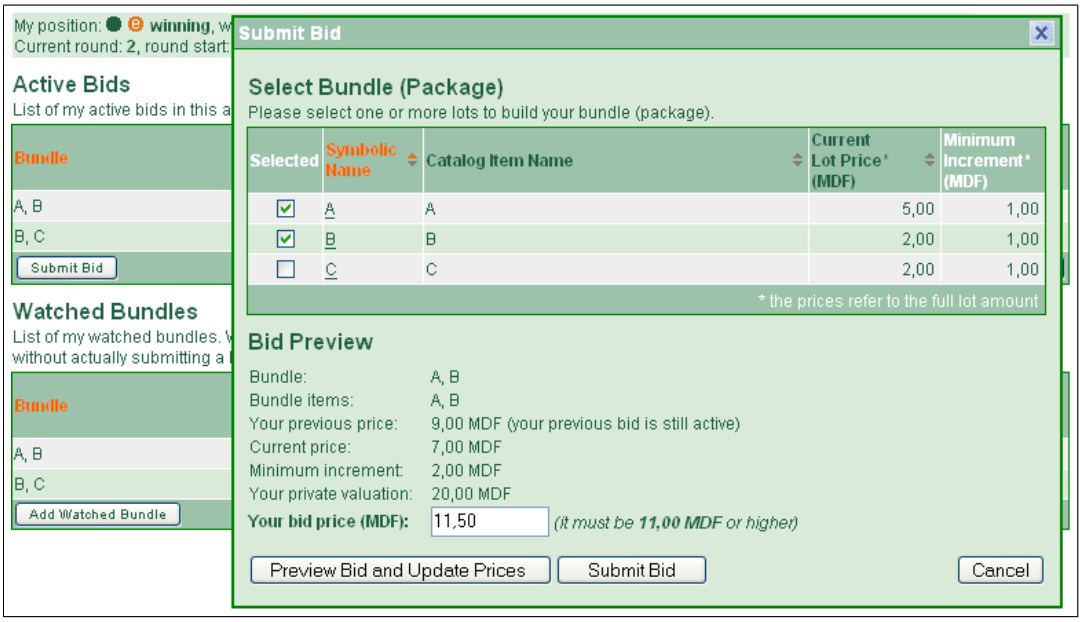


Figure D.11: Screenshot - Submitting a bid

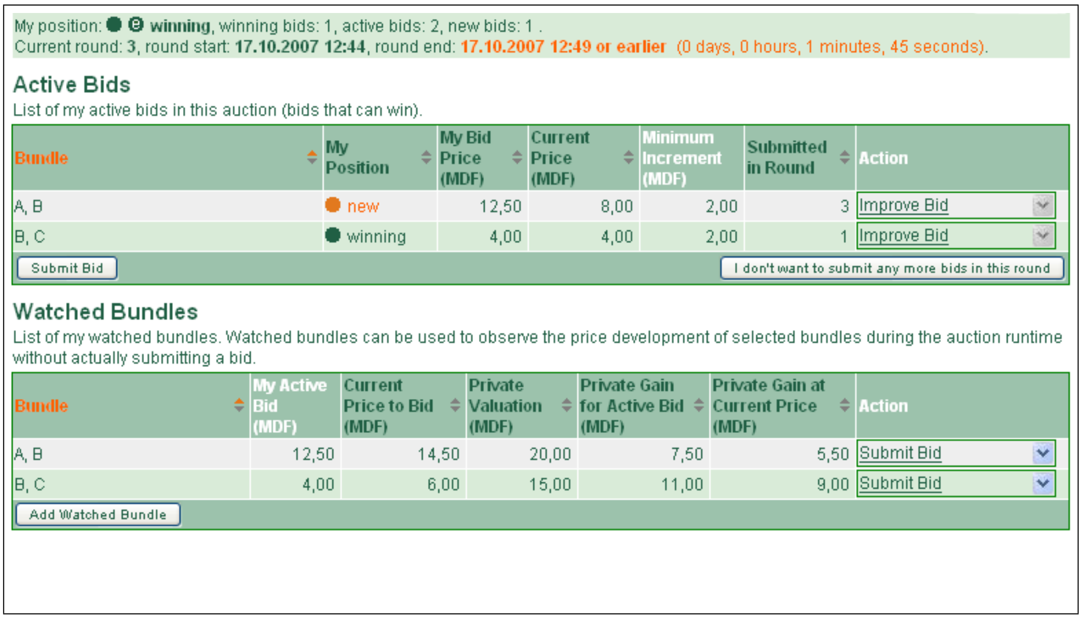


Figure D.12: Screenshot - Bids and watched bundles

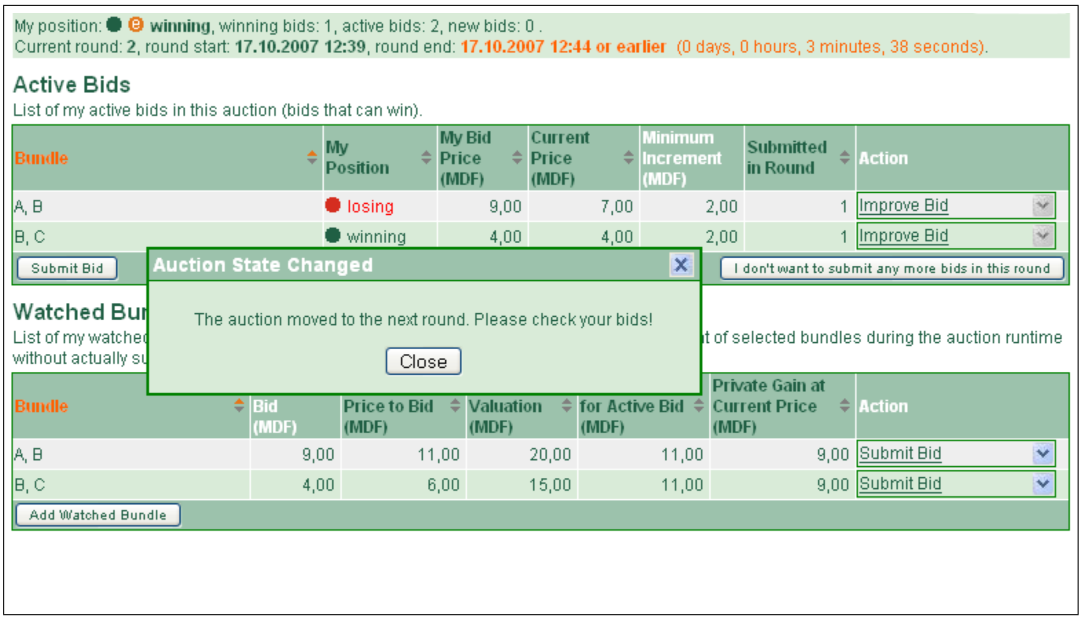


Figure D.13: Screenshot - Next round notification

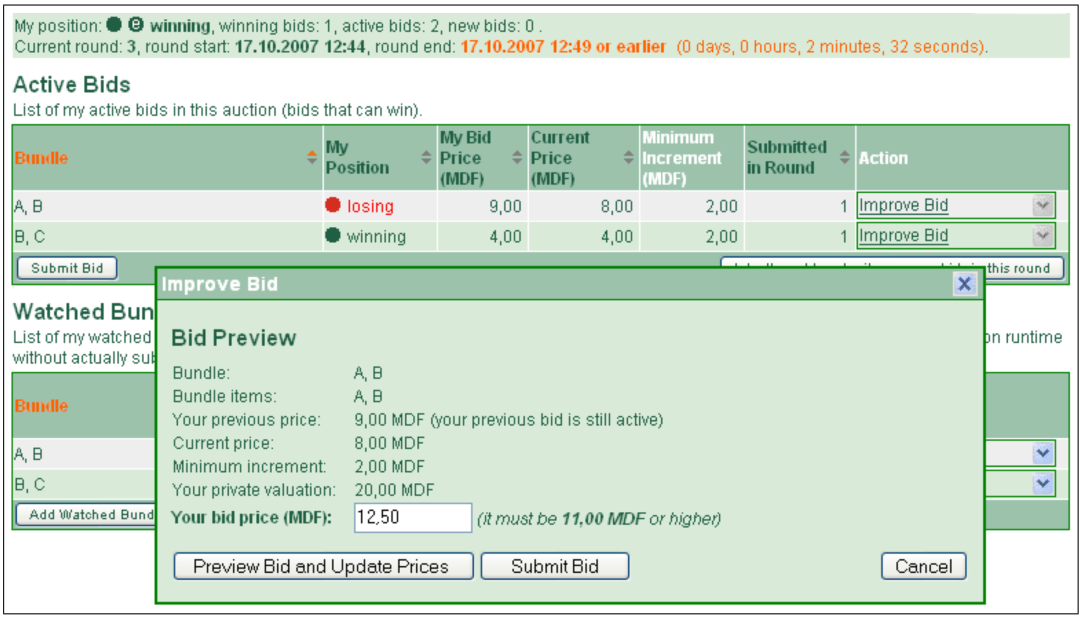


Figure D.14: Screenshot - Improving a bid

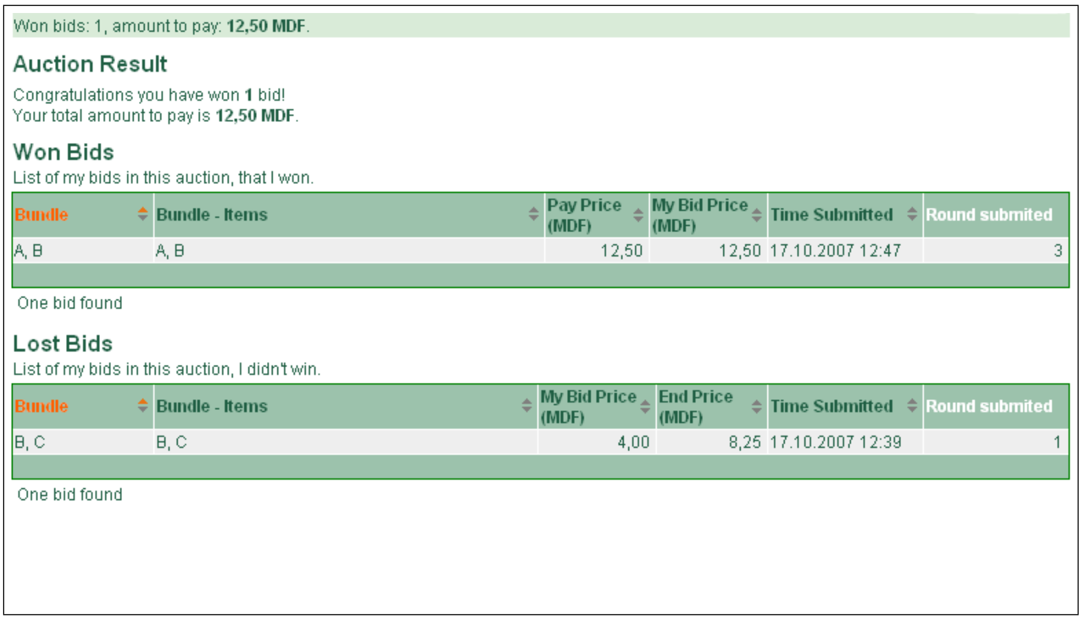


Figure D.15: Screenshot - Auction finished

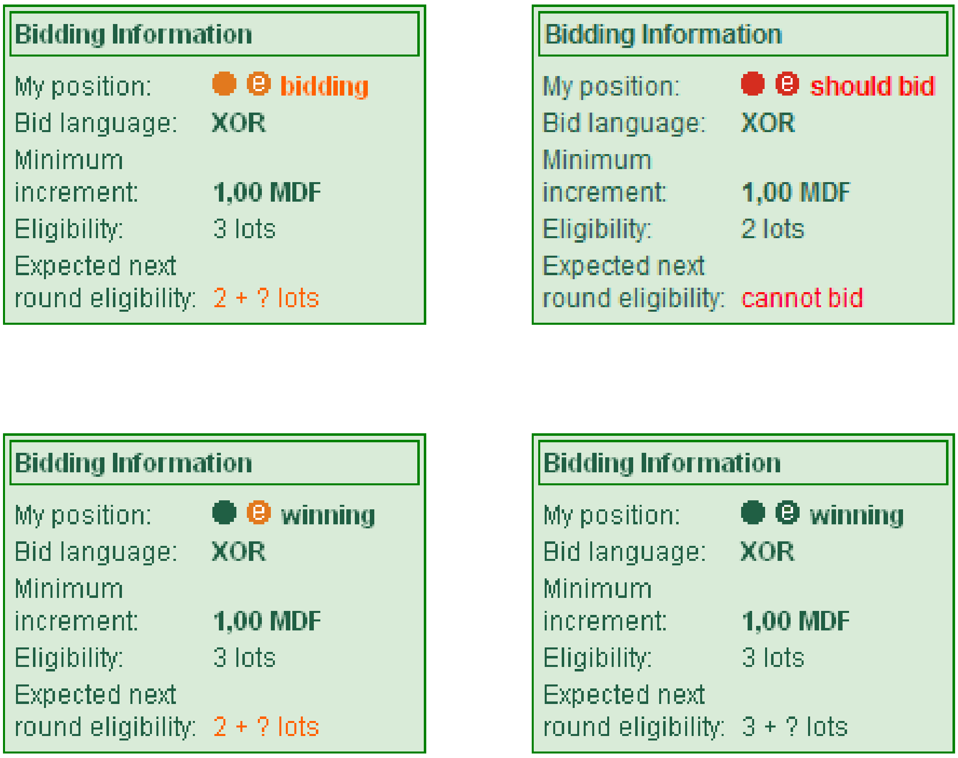


Figure D.16: Screenshot - Bidding Information (different states)

Appendix E

List of Symbols

$\mathcal{K}$ - set of items

$k \in \mathcal{K}$**, also** $l \in \mathcal{K}$ - item

m - number of items

$S \subseteq \mathcal{K}$**, also** $T \subseteq \mathcal{K}$ - subset of items (bundle, package)

$\mathcal{I}$ - set of bidders

$i \in \mathcal{I}$**, also** $j \in \mathcal{I}$ - bidder

$I \subseteq \mathcal{I}$ - subset of bidders

n - number of bidders

t - round number

$\mathcal{B}^t = \{b_i^t(S)\}$ - set of bids active after the round t

$b_i^t(S) = \{S, p_{bid,i}^t(S)\} \in \mathcal{B}^t$ - bid of the bidder i for the bundle S active after the round t

$p_{bid,i}^t(S) \in \mathcal{B}^t$ - bid price of the bid $b_i^t(S)$

$\mathcal{P}^t_{ask} = \{p^t_{ask,i}(S)\}$ **or** $\{p^t_{ask}(S)\}$ **or** $\{p^t_{ask}(k)\}$ - set of ask prices valid during the round t

$p^t_{ask,i}(S)$ - personalized bundle ask price for the bidder i and bundle S valid during the round t

$p^t_{ask}(S)$ - anonymous bundle ask price for the bundle S valid during the round t

$p^t_{ask}(k)$ - anonymous linear ask price for the item k valid during the round t

Δ^t - price increment valid during the round t or used for the price update from the round t to the round $t+1$

$v_i(S)$ - private valuation of the bidder i for the bundle S

$\mathcal{P}_{pay} = \{p_{pay,i}(S)\}$ - set of pay prices

$p_{pay,i}(S)$ - pay price for the bidder i and bundle S

$\pi_i(S, \mathcal{P}_{pay})$ - utility of the bidder i for the bundle S at the pay prices $\mathcal{P}_{pay}$

$\mathcal{X} = \{X\}$ - set of all possible allocations

$X = (S_1, \ldots, S_n) = \{x_i(S)\}$ - allocation

$S_i \subseteq \mathcal{K}$ - bundle allocated to the bidder i

$x_i(S) \in \{0; 1\}$ - binary variable which determines, whether the bidder i becomes allocated exactly the bundle S

$\pi_i(X, \mathcal{P}_{pay})$ - utility of the bidder i for the allocation X at the pay prices $\mathcal{P}_{pay}$

$\pi_{all}(X, \mathcal{P}_{pay})$ - total bidder utility for the allocation X at the pay prices $\mathcal{P}_{pay}$

$\Pi(X, \mathcal{P}_{pay})$ - auctioneer revenue for the allocation X at the pay prices $\mathcal{P}_{pay}$

$X^* = (S^*_1, \ldots, S^*_n) = \{x^*_i(S)\}$ - efficient allocation

X^t - provisional allocation calculated on the basis of the bids active in the round t

W^t - set of provisionally winning bids in the allocation X^t

L^t - set of provisionally losing bids in the allocation X^t

$E(X) \in [0,1]$ - allocative efficiency of the allocation X

$R(X) \in [0, E(X)]$ - auctioneer utility share in the allocation X

$U(X) \in [0, E(X)]$ - total bidder utility share in the allocation X

C_I - coalition consisting of the bidders $I \subseteq \mathcal{I}$ and the auctioneer

$w(C_I)$ - coalitional value function on the coalition C_I

(Π, π) - payoff vector

$Core\,(\mathcal{I}, w)$ - set of core payoffs

$\delta_i(S)$ - linear price compatibility distortion of the bid $b_i(S)$

$D_i(\mathcal{P}_{pay})$ - demand set of the bidder $i$at the prices $\mathcal{P}_{pay}$

e_i^t - eligibilty of the bidder i in the round t

$e_{+,i}^t$ - surplus eligibilty of the bidder i in the round t

e_+^{max} - maximal possible surplus eligibility

rbv_i^t - round bid volume of the bidder i in the round t

tbv_i - total bid volume of the bidder i

$bpe_i^t(k)$ - optimistic estimator of the bidder's i bid price for the single item k based on her bundle bids in the round t

Appendix F

List of Abbreviations

ALPS **A**pproximate **L**inear **P**rice**S**

AUSM **A**daptive **U**ser **S**election **M**echanism

BSC **B**idders are **S**ubstitutes **C**ondition

BSM **B**idder **S**ub**m**odularity condition

CAP **C**ombinatorial **A**llocation **P**roblem

CA **C**ombinatorial **A**uction

CC **C**ombinatorial **C**lock auction

CE **C**ompetitive **E**quilibrium

IBIS Chair of **I**nternet-**b**ased **I**nformation **S**ystems at the Technische Universität München (Munich, Germany)

ICA **I**terative **C**ombinatorial **A**uction

ILP **I**nteger **L**inear **P**roblem

IS **I**nformation **S**ystems

LP **L**inear **P**roblem

NP **N**on/deterministic **P**olynomial time

OR additive-**OR** (bidding language)

PAUSE **P**rogressive **A**daptive **U**ser **S**election **E**nvironment

PEP **P**reference **E**licitation **P**roblem

RAD **R**esource **A**llocation **D**esign

SMR **S**imultaneous **M**ulti-**R**ound Design

WDP **W**inner **D**etermination **P**roblem

XOR e**x**clusive-**OR** (bidding language)

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