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Minimal Flavour Violation in the Quark and Lepton Sector and beyond

Dissertation

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Abstract
We address to explain the matter-antimatter asymmetry of the universe in a framework that generalizes the quark minimal flavour violation hypothesis to the lepton sector. We study the impact of CP violation present at low and high energies and investigate the existence of correlations among leptogenesis and lepton flavour violation.
Further we present an approach alternative to minimal flavour violation where the suppression of flavour changing transitions involving quarks and leptons is governed by hierarchical fermion wave functions.

Vorwort
Ziel dieser Arbeit ist es, die Materie-Antimaterie Asymmetrie des Universums innerhalb eines Szenarios zu erklären, in dem die Hypothese der minimalen Flavour Verletzung vom Quark auf den Leptonen Sektor erweitert wurde. Wir untersuchen den Einfluss von CP Verletzung bei hohen und niederen Energien und ob Korrelationen zwischen Leptogenese und Lepton Flavour Verletzung existieren.
Desweiteren präsentieren wir einen Ansatz alternativ zur minimalen Flavour Verletzung, der die Unterdrückung von Flavour ändernden Übergängen durch hierarchische fermionische Wellenfunktionen gewährleistet.

Contents

1 Introduction

In the absence of new dynamics, the electro-weak scale would receive enormous contributions from radiative corrections. In order to explain this hierarchy problem, new physics should appear at the TeV scale. Quark masses break the electro-weak symmetry and therefore are necessarily connected to this new physics which implies that the new dynamics that stabilize the electro-weak scale lead to new flavour physics.
The Standard Model (SM) of particle physics can be regarded as the low-energy limit of a general effective Lagrangian. The flavour structure of the quark sector of the SM is very specific: The two Yukawa matrices are quasi-aligned in flavour space with the only misalignments parametrized by the CKM matrix and the eigenvalues of the Yukawa matrices are very hierarchical. These features govern a strong suppression of flavour changing neutral current (FCNC) transitions due to the GIM mechanism which renders this kind of processes small such that they are in agreement with data.
Going beyond the SM however, there could be several additional flavour structures appearing in the tower of higher dimensional operators that belongs to the non-renormalizeable part of the effective Lagrangian. However, if we assume the effective scale of new physics in the TeV range, experiments leave only a very limited room for new flavour structures. A natural solution to this problem which is known as the (quark) flavour problem is provided by the Minimal Flavour Violation (MFV) hypothesis.

In the lepton sector, a flavour problem exists as well. The discovery of neutrino oscillations provides evidence for non-vanishing neutrino masses leading to lepton flavour violation. However, lepton flavour violating processes such as $\mu \to e\gamma$ have not been observed so far implying a strong suppression of such transitions. In the SM, neutrinos are strictly massless since Dirac masses cannot be constructed due to the absence of right-handed neutrinos while left-handed Majorana masses are not present due to exact $(B - L)$ conservation.

Another clear signal for beyond SM physics has been obtained from cosmological observations. The existence of the baryon asymmetry of the universe (BAU) is experimentally proven and its magnitude has precisely been determined by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite as well as from Big Bang Nucleosynthesis.

Interestingly, the smallness of the neutrino masses as well as the generation of the BAU by means of leptogenesis can be explained in the context of see-saw models in which neutrinos are assumed to be Majorana particles and heavy right-handed Majorana neutrinos are introduced. Furthermore, if the see-saw mechanism is indeed the source of the light neutrino masses, leptogenesis is qualitatively unavoidable and the question whether this mechanism is responsible for the BAU reduces to a quantitative problem. Unfortunately, even in the simplest realization of the see-saw model, the theory has too many parameters. Indeed, extending the SM by three heavy right-handed Majorana neutrinos, the high-energy sector has eighteen parameters and nine of those enter into the effective neutrino mass matrix measurable at low energies making it difficult but desirable to establish a direct link between leptogenesis and low-energy processes.

MFV in the quark sector renders FCNC small by allowing for new physics governed solely by the CKM matrix. The appealing virtue that arises in this context is the high predictivity of this framework that offers the possibility to be tested in low-energy experiments and is universal to all models belonging to this class. However, the MFV scenario in the quark sector, although simple and elegant, suffers from the following problem. In the absence of new complex phases beyond the CKM phase, it cannot accommodate the observed size of the BAU. The question then arises, whether one could still explain the right size of the BAU within the MFV context by considering simultaneously the lepton sector, where the BAU can in principle be explained with the help of leptogenesis. Assuming the quark sector to be minimal flavour violating, it would be reasonable that for leptons there exists a similar mechanism. While this is the most natural possibility, other directions could be explored in principle. In the first part of this thesis, we will discuss in detail whether the BAU can be accommodated in the Minimal Lepton Flavour Violation (MLFV) framework and if a predictivity similar to the quark sector among low-energy lepton flavour violating processes can be achieved. Furthermore we study whether there exist correlations between phenomena at low and high energies.

In the field of leptogenesis there has been tremendous progress in the recent years. The "one flavour" approximation used to describe the dynamics of leptogenesis is rigorously only correct when the interactions mediated by charged lepton Yukawa couplings are out-of-equilibrium. This is not true below a certain temperature and the inclusion of distinguishable flavours leads to important effects on the dynamics of leptogenesis. This impact can as well be observed in the framework of MLFV with CP violation present at low and high energies. Moreover, it was commonly concluded, that the future observation of leptonic low-energy CP violation does not automatically imply that there exists a non-vanishing BAU. This conclusion however does not universally hold in temperature regimes where flavour effects play a role. With the flavour specific approach it has been found that there exists the possibility to generate the BAU from low-energy CP violation alone, in the MLFV framework with quasi-degenerate heavy Majorana masses as well as in scenarios with hierarchical heavy

Majorana neutrinos. Therefore, in the second part of this thesis, we will concentrate on the case in which the MLFV framework is CP conserving at high energies and investigate conditions for a successful leptogenesis.

MFV both in the quark and lepton sector is based on a symmetry principle. The effective Lagrangian maintains a flavour symmetry which is broken only by the quark and lepton Yukawa couplings. All new contributions such as those from higher dimensional operators are then suppressed by these couplings and the hierarchy of the eigenvalues of the Yukawa matrices plays an important role in this suppression. But is this the only possibility to render FCNC small? Obviously not and several extensions and variations to the MFV hypothesis have been proposed in the literature. In the last part of this thesis we will present an alternative solution to the flavour problem in which the hierarchy of the Yukawas is shifted to the fermion kinetic terms by a simple rescaling of the fermion wave functions. This rescaling is not relevant for SM contributions due to renormalizability, but enters the effective couplings of higher dimensional operators such as dimension-six operators contributing to FCNCs in the quark and lepton sector. Therefore, hierarchical fermion wave functions could govern the suppression of those contributions. On a dynamical level, such profiles could emerge in models with extra dimensions where the hierarchies could reflect a more complicated wave function profile in the extra dimension. We will investigate whether this approach, despite its simplicity, can be consistent with the existing data on FCNCs in the quark and lepton sector assuming the scale of new physics in the TeV range.

This thesis is organized as follows: In chapter 2 and 3 we will introduce MFV in the quark and in the lepton sector. In the chapters 4 and 5 we will provide basic features relevant for leptogenesis. As flavour effects are very important for the analyses presented, we will show at the level of the most simple Boltzmann equations how these effects come into play even if we used a more sophisticated computer code for our numerical analyses. In chapter 6 we will motivate the generation of the BAU within MLFV by means of radiative resonant leptogensis and derive explicit formulae for the quantities relevant for leptogenesis such as mass splittings and Yukawa matrices. Then we present our analysis of MLFV with CP violation present at high and low energies in chapter 7. In chapter 8 we will derive conditions for a successful leptogenesis when the MLFV scenario is assumed to be CP conserving at high energies. In chapter 9 we will motivate, develop and test a framework in which FCNCs are rendered small by hierarchical fermion wave functions where the Yukawa matrices are non-hierarchical instead. We will work out clear distinctions to MFV. In chapter 10 we will conclude.

2 Minimal Flavour Violation in the Quark Sector

2.1 Introducing MFV

The first definition of MFV has been given in [1] where new physics contributions have been absorbed into a redefinition of SM electro-weak parameters. If only one Higgs doublet is involved in the spontaneous breaking of the underlying gauge symmetry, all flavour changing charged and neutral current processes are governed in the MFV framework by the CKM matrix and the relevant local operators are only those present in the SM. Then all new contributions due to the exchange of new virtual particles are encoded in the corresponding Wilson coefficients. Examples of models belonging to the MFV class are the Littlest Higgs Model without T parity or the Two Higgs Doublet Model (2HDM) II at low and moderate $\tan\beta$. The nice features of MFV are that there exist quantities that are universal in the whole class of MFV models. As demonstrated in [2], the existing data on $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing, ε_K, $B \to X_s\gamma$, $B \to X_s l^+ l^-$ and $K^+ \to \pi^+\nu\bar{\nu}$ and the value of the angle β in the unitarity triangle from the mixing induced CP asymmetry in $B \to \psi K_S$ imply within this framework very stringent bounds on all rare K and B decay branching ratios. Consequently, substantial departures from the SM predictions are not expected if MFV occurs with just one Higgs doublet. This has explicitely be confirmed by direct calculations of rare decays e.g. in the minimal flavour violating Littlest Higgs model without T parity [3, 4] and in models with universal extra dimensions [5].

Another important virtue of MFV in the quark sector is the existence of relations [1] between the ratios of various branching ratios and the CKM parameters measured in low-energy processes that have universal character and are independent of the details of the specific MFV model. An example is the universal unitarity triangle common to all MFV models [1]. But also the fact that each branching ratio can be expressed in terms of the CKM parameters and quark masses measured at the electro-weak scale or lower energy scales makes this scenario to be a very predictive and falsifiable framework. Moreover, neither fine tuning nor resorting to unnatural scales of new physics are required to make this scenario consistent with the available data.

2.2 Effective Field Theory Approach

Leading to equivalent results, MFV can as well be formulated in an effective field theory framework [6, 7]. In the absence of quark masses, the SM Lagrangian exhibits a large exact flavour symmetry group

$$G_F = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} \otimes U(1)_B \otimes U(1)_Y \otimes U(1)_{PQ} \tag{2.1}$$

that arises from independent unitary rotations of the left-handed quark doublets Q_L and right-handed singlets U_R and D_R. The $U(1)$ charges can be identified with baryon number (B), hypercharge (Y) and the the Peccei-Quinn (PQ) symmetry of 2HDMs [8]. G_F is exclusively broken by the SM Yukawa couplings being the only sources of quark flavour violation. One can formally recover the invariance under G_F by introducing the dimensionless Yukawa matrices Y_D and Y_U as auxiliary fields transforming under $SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R}$ as

$$Y_U \sim (3, \bar{3}, 1), \qquad Y_D \sim (3, 1, \bar{3}). \tag{2.2}$$

This leads to the standard Yukawa Lagrangian

$$\mathcal{L} = \bar{Q}_L Y_D D_R H + Q_L Y_U U_R H_c + h.c. \tag{2.3}$$

with $H_c = i\tau_2 H^*$, which is consistent with the flavour symmetry. All additional terms such as operators that break G_F have to transform under G_F the same way as (2.3).

If two Higgs doublets, like in the MSSM, are involved and the ratio of the corresponding vacuum expectation values $v_2/v_1 \equiv \tan\beta$ is large, significant departures from the SM predictions for certain decays are still possible within the MFV framework [7] in spite of the processes being governed solely by the CKM matrix. The prime reason for these novel effects is the appearance of new scalar operators that are usually strongly suppressed within the SM and MFV models at low $\tan\beta$, but can become important and even dominant for large $\tan\beta$. The improved data on $B_{d,s} \to \mu^+\mu^-$, expected to come in this decade from Tevatron and LHC, will tell us whether MFV models with large $\tan\beta$ are viable.

3 Minimal Flavour Violation in the Lepton Sector

3.1 Lepton Flavour Violation

Neutrinos are massless in the SM. Therefore within the SM, lepton flavour is conserved and flavour mixing occurs solely in the quark sector. With the discovery of neutrino masses and mixing, it has been clearly established that lepton flavour is not a conserved quantum number. The smallness of the neutrino masses also provides a strong indication for the existence of lepton number violation, although this information cannot be extracted from data yet. In analogy to the quark sector, the non-conservation of lepton flavour points towards the existence of lepton flavour violating processes (LFV) involving charged leptons such as $l_i \to l_j\gamma$. Those however have not been observed so far implying a high suppression of such decays. For example the transition $\mu \to e\gamma$ maintains an upper bound of

$$B(\mu \to e\gamma) < 10^{-11}. \tag{3.1}$$

New physics models that allow for flavour-dependent interactions in the lepton sector have to face these stringent constraints. Such models should keep LFV processes automatically small without fine tuning or the introduction of unnaturally high scales of new physics in order to solve this existing "Lepton Flavour Problem".

If one assumes that in the quark sector the flavour problem is solved by MFV, it would be reasonable that there exists a similar mechanism also in the lepton sector. This would be desirable also because with MFV in the quark sector alone one cannot explain the baryon asymmetry of the universe due to the absence of new complex phases. The CP violation required for a successful baryogenesis could then be governed by the leptons and the BAU could be explained by baryogenesis through leptogenesis.

The simplest extension of the SM which allows for non-vanishing but naturally small neutrino masses, contains right-handed neutrinos added to the spectrum of the SM. This extension has the nice feature of establishing on the one hand a lepton quark symmetry and on the other hand being naturally embedded in a grand unified theory like $SO(10)$. Since right-handed neutrinos are singlets under $U(1) \otimes SU(2) \otimes SU(3)$, Majorana neutrino masses

M_R should be included, with a mass scale M_ν which can be much larger than the scale v of the electro-weak symmetry breaking. Apart from M_R, Dirac neutrino mass terms m_D are generated through leptonic Yukawa couplings upon gauge symmetry breaking. The presence of these two neutrino mass terms leads, through the see-saw mechanism [9], to three light neutrinos with masses of order v^2/M_ν and three heavy neutrinos with masses of order M_ν. The decays of these heavy neutrinos can play a crucial role in the creation of the baryon asymmetry of the universe (BAU) through the elegant mechanism of baryogenesis through leptogenesis [10,11]. Unfortunately, the masses of the Majorana neutrinos can be very heavy which implies that these particles will not be accessed in the present and upcoming collider experiments and being gauge singlets, Yukawa interactions are the only existing processes involving both, heavy Majorana neutrinos and SM particles. The exploration of indirect methods to test the see-saw model and to determine the see-saw scale are therefore of utmost importance. So far however, all existing links between the low and high energy lepton sector -if existent- turned out to be strongly dependent on the particular model considered.

In the presence of neutrino masses and mixing, one has, in general, both CP violation at low energies which can be detected through neutrino oscillations and CP violation at high energies which is an essential ingredient of leptogenesis. Connections between these two manifestations of CP violation can be established in frameworks of specific lepton flavour models.

3.2 Minimal Lepton Flavour Violation

How the MFV mechanism can be established in the lepton sector has been proposed in [12]. In analogy to the quark sector, a consistent class of SM extensions is defined in which the sources of lepton flavour violation are minimally linked to the neutrino and charged lepton mass matrices. Minimal Lepton Flavour Violation can be formulated in an effective field theory framework holding the flavour symmetry

$$G_{LF} = SU(3)_L \otimes SU(3)_E \otimes SU(3)_{\nu_R} \otimes U(1)_{\rm LN} \otimes U(1)_E \otimes U(1)_{\nu_R}, \tag{3.2}$$

which reflects the fact that gauge interactions treat all flavours equally. The $U(1)$ symmetries can be identified with total lepton number (LN) and weak hypercharge. Similar to the quark sector, the lepton Yukawa couplings Y_E, Y_ν, being 3×3 matrices in lepton flavour space, are the only sources of flavour violation[1]

$$\mathcal{L}_Y = -\bar{e}_R Y_E H^\dagger L_L - \bar{\nu}_R Y_\nu \tilde{H} L_L + h.c. \tag{3.3}$$

[1]In the lepton sector we will use the convention $\tilde{H} = -i\tau_2 H^T$.

The exact analogy to the quark sector would imply that neutrinos are Dirac particles. In order to additionally explain the smallness of neutrino masses with the help of the see-saw mechanism, the MFV hypothesis in the lepton sector requires lepton number violation at some high scale, and three heavy right-handed Majorana neutrinos being introduced.

$$\mathcal{L}_M = -\frac{1}{2}\bar{\nu}^c_R M_R \nu_R + h.c. \ , \tag{3.4}$$

where the Majorana mass matrix exhibits a trivial structure $M_R = M_\nu \mathbb{1}_{3\times 3}$. This is due to the fact that the MLFV proposal [12] consists of the assumption that the physics which generates lepton number violation, leading to M_R, is lepton flavour blind, thus leading to an exactly degenerate eigenvalue spectrum for M_R. With the heavy Majorana neutrinos present, the flavour symmetry $SU(3)_{\nu_R}$ is reduced to $O(3)_{\nu_R}$.

In the limit $\mathcal{L}_Y = \mathcal{L}_M = 0$ the Lagrangian of this minimal extension of the SM respects the flavour symmetry (3.2). A transformation of the lepton fields

$$L_L \to V_L L_L, \quad e_R \to V_E e_R, \quad \nu_R \to V_{\nu_R} \nu_R, \tag{3.5}$$

leaves the full Lagrangian invariant, provided the Yukawa couplings and the Majorana mass terms transform as:

$$Y_\nu \to Y_\nu' = V_{\nu_R} Y_\nu V_L^\dagger, \tag{3.6}$$

$$Y_E \to Y_E' = V_E Y_E V_L^\dagger, \tag{3.7}$$

$$M_R \to M_R' = V_{\nu_R}^* M_R V_{\nu_R}^T, \tag{3.8}$$

which means that there is a large equivalent class of Yukawa coupling matrices and Majorana mass terms which have the same physical content.

At this stage it is worth mentioning that there may be other equally reasonable definitions of MLFV. In the analyses presented in this thesis we will only consider the conservative generalization of the initial proposal for MLFV [12] defined above. However, it is clear that one may have other well motivated but different proposals for MLFV. In particular one should keep in mind that within the see-saw mechanism neutrinos acquire a mass in a manner which differs significantly from the one in the quark sector. Indeed it has been suggested [13] that the fact that neutrino masses arise from the see-saw mechanism is the key point in understanding why leptonic mixing is large, in contrast with small quark mixing. Therefore a reasonable definition of MLFV may differ from MFV in the quark sector. The question of different definitions of MLFV has been recently addressed in an interesting paper by Davidson and Palorini [14].

3.3 CP Violation at low and high Energies

Without loss of generality, one can choose a basis for the leptonic fields, where Y_E and M_R are diagonal and real. In this basis, the neutrino Dirac mass matrix $m_D = vY_\nu$ is an arbitrary complex matrix with 9 real parameters and 9 phases. 3 of these phases can be eliminated by a rephasing of L_L. One is then left with 6 CP violating phases. Together with the 3 heavy Majorana masses we arrive at 18 parameters for the see-saw model. There are various classes of phenomena which depend on different combinations m_D, m_D^T, $m_D^\dagger$ or equivalently Y_ν, Y_ν^T and $Y_\nu^\dagger$.

3.3.1 Leptonic Mixing and CP Violation at low Energies

In the basis where the charged lepton mass matrix is diagonal and real, leptonic mixing and CP violation at low energies are controlled by the PMNS matrix U_ν [15, 16], which diagonalizes the effective low energy neutrino mass matrix:

$$U_\nu^T (m_\nu)_{eff} U_\nu = d_\nu, \tag{3.9}$$

where $d_\nu \equiv \mathrm{diag}(m_1, m_2, m_3)$, with m_i being the masses of the light neutrinos [9],

$$(m_\nu)_{eff} = -v^2 Y_\nu^T D_R^{-1} Y_\nu, \tag{3.10}$$

where $D_R = \mathrm{diag}(M_1, M_2, M_3)$ denotes the diagonal matrix of M_R and $v = 174$ GeV is the vacuum expectation value of the SM Higgs doublet.

In the case of MLFV, $D_R = M_\nu \mathbb{1}$ and one obtains at $M_\nu \approx \Lambda_{\mathrm{LN}}$

$$(m_\nu)_{eff} = -\frac{v^2}{M_\nu} Y_\nu^T Y_\nu. \tag{3.11}$$

Consequently $Y_\nu^T Y_\nu$ is the quantity that matters here.

3.3.2 Lepton Flavour Violation

The charged LFV depends on the other hand on the combination $Y_\nu^\dagger Y_\nu$ with Y_ν again normalized at the high energy scale M_ν. In the context of MLFV, the normalized branching fractions are given by [12]

$$B(l_i \to l_j \gamma) = \frac{\Gamma(l_i \to l_j \gamma)}{\Gamma(l_i \to l_j \nu_i \bar{\nu}_j)} \equiv r_{ij} \hat{B}(l_i \to l_j \gamma), \tag{3.12}$$

where $\hat{B}(l_i \to l_j \gamma)$ is the true branching ratio and $r_{\mu e} = 1.0$, $r_{\tau e} = 5.61$ and $r_{\tau\mu} = 5.76$ and

$$B(l_i \to l_j \gamma) = 384 \pi^2 e^2 \frac{v^4}{\Lambda_{\mathrm{LFV}}^4} |\Delta_{ij}|^2 |C|^2. \tag{3.13}$$

$\Lambda_{\rm LFV}$ is the scale of charged lepton flavour violation and C summarizes the Wilson coefficients of the relevant operators that can be calculated in a given specific model. They are naturally of $\mathcal{O}(1)$ but can be different in different MLFV models. We will set $|C| = 1$ in what follows. Thus the true $B(l_i \to l_j\gamma)$ can be different from our estimate in a given MLFV model, but as C is, within MLFV, independent of external lepton flavours, the ratios of branching ratios take a very simple form

$$\frac{B(l_i \to l_j\gamma)}{B(l_m \to l_n\gamma)} = \frac{|\Delta_{ij}|^2}{|\Delta_{mn}|^2}. \tag{3.14}$$

The most important objects in (3.13) and (3.14) that govern the quantities considered are

$$\Delta_{ij} \equiv (Y_\nu^\dagger Y_\nu)_{ij}. \tag{3.15}$$

3.3.3 CP Violation relevant for Leptogenesis:

Generating the BAU through leptogenesis, a lepton asymmetry which is proportional to the CP asymmetry in the decays of heavy Majorana neutrinos is produced first. This CP asymmetry involves the interference between the tree level amplitude and the one loop vertex and self-energy contributions. The dynamics of leptogenesis are described in more detail in the following chapters. It has been shown [17] that the CP asymmetry depends on the neutrino Yukawa couplings through the combination $Y_\nu Y_\nu^\dagger$. Again Y_ν is evaluated here at the scale M_ν. When flavour effects in the Boltzmann equations become important, the final CP asymmetry is more sensitive to the individual entries $(Y_\nu)_{i\alpha}(Y_\nu)^*_{j\alpha}$ corresponding to different lepton flavours α.

3.4 A useful Parametrization

Addressing the study of correlations among low- and high-energy phenomena, it is convenient to choose an appropriate basis for Y_ν. We use the following one known as the Casas-Ibarra parametrization [18]

$$(\sqrt{D_R})^{-1}\, Y_\nu = \frac{i}{v}\, R\, \sqrt{d_\nu}\, U_\nu^\dagger\,, \tag{3.16}$$

where R is an orthogonal complex matrix ($R^T R = R\, R^T = \mathbb{1}$).

Since the left-hand side of (3.16) is an arbitrary 3×3 complex matrix with 9 real parameters and 6 phases (3 of the initial 9 phases can be removed by rephasing L_L), it is clear that the right-hand side of (3.16) also has 9 real parameters and 6 phases. Therefore R, d_ν and U_ν have each 3 real parameters and moreover R and U_ν have in addition each 3 phases.

In the case where the right-handed neutrinos are exactly degenerate, i.e. $D_R = M_\nu \mathbb{1}$ we will show that the 3 real parameters of R can be rotated away. Note that any complex orthogonal matrix can be parameterized as

$$R = e^{A_1} e^{iA_2}, \tag{3.17}$$

with $A_{1,2}$ real and skew symmetric. Now in the degenerate case an orthogonal rotation of $\nu_R \to O_R \nu_R$ leaves the Majorana mass proportional to the unit matrix and defines a physically equivalent reparametrization of the fields ν_R. Choosing $O_R = e^{A_1}$ we see immediately that

$$Y_\nu \to O_R^\dagger Y_\nu = \frac{\sqrt{M_\nu}}{v}\, e^{-A_1}\, R \sqrt{d_\nu}\, U_\nu^\dagger = \frac{\sqrt{M_\nu}}{v}\, e^{iA_2} \sqrt{d_\nu}\, U_\nu^\dagger\,, \tag{3.18}$$

which shows that the physically relevant parameterization is given by $R_{deg} = e^{iA_2}$, therefore R_{deg} contains 3 physical parameters which are complex. In general, the R matrix can be parametrized as follows:

$$R = \begin{pmatrix} \hat{c}_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12} & \hat{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \hat{c}_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23} & \hat{c}_{23} \end{pmatrix} \begin{pmatrix} \hat{c}_{13} & & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13} & 0 & \hat{c}_{13} \end{pmatrix}, \tag{3.19}$$

with $\hat{s}_{ij} \equiv \sin \hat{\theta}_{ij}$ and $\hat{\theta}_{ij}$ in general complex:

$$\hat{\theta}_{ij} = x_{ij} + i\, y_{ij}, \tag{3.20}$$

where in the degenerate case the angles x_{ij} can be made to vanish.

The complex parameters y_{ij} of R parametrize the high-energy CP violation while the 3 phases of the PMNS matrix represent low-energy CP violation which can be derived as follows: Using the parameterization in (3.16) one finds that the matrix $Y_\nu^T Y_\nu$ which controls low-energy CP-Violation and mixing can be written as

$$Y_\nu^T Y_\nu = -\frac{1}{v^2}\, (U_\nu^\dagger)^T \sqrt{d_\nu}\, R^{\mathrm{T}}\, D_R\, R \sqrt{d_\nu}\, U_\nu^\dagger = -\frac{M_\nu}{v^2} (U_\nu^\dagger)^T d_\nu U_\nu^\dagger, \tag{3.21}$$

where in the last step we have set $D_R = M_\nu \mathbb{1}$.

On the other hand, the matrix $Y_\nu^\dagger Y_\nu$ which controls charged LFV, can be written as follows (see also [19])

$$Y_\nu^\dagger Y_\nu = \frac{1}{v^2}\, U_\nu \sqrt{d_\nu}\, R^\dagger\, D_R\, R \sqrt{d_\nu}\, U_\nu^\dagger = \frac{M_\nu}{v^2}\, U_\nu \sqrt{d_\nu}\, R^\dagger R \sqrt{d_\nu}\, U_\nu^\dagger\,. \tag{3.22}$$

Finally, the matrix $Y_\nu Y_\nu^\dagger$ which enters in leptogenesis when flavor effects are not relevant is given by (see also [19]):

$$Y_\nu Y_\nu^\dagger = \frac{1}{v^2} \sqrt{D_R}\, R\, d_\nu\, R^\dagger \sqrt{D_R} = \frac{M_\nu}{v^2}\, R\, d_\nu\, R^\dagger. \tag{3.23}$$

We note that $Y_\nu^T Y_\nu$ depends only on U_ν and d_ν, while $Y_\nu Y_\nu^\dagger$ relevant for the leptogenesis only on d_ν and R. This means that CP violation at low energy originating in the complex U_ν and the CP violation relevant for leptogenesis are then decoupled from each other and only the mass spectrum of light neutrinos summarized by d_ν enters both phenomena in a universal way.

In this respect the charged LFV, represented by (3.22), appears also interesting as it depends on d_ν, U_ν and R and consequently can also provide an indirect link between low-energy and high-energy CP violation and generally a link between low- and high-energy phenomena.

4 The Baryon Asymmetry of the Universe

It is observed that in our patch of the universe, there is an excess of baryons (visible matter) over anti-baryons which must be true for the whole universe since we do not see gamma rays from matter-antimatter annihilation. This visible matter excess implies that there exists a baryon asymmetry. The amount of baryonic matter in the universe affects the fluctuations in the cosmic microwave background (CMB). From the data provided by the measurements of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite, the baryon asymmetry of the universe denoted by η_B [20] normalized to the photon density amounts to

$$\eta_B = \frac{N_B - N_{\bar{B}}}{N_\gamma} = (6.10 \pm 0.21) \cdot 10^{-10}. \tag{4.1}$$

This number is consistent also with the observed light element abundances which is important because the amount of the BAU influences the production of the light nuclei by Big Bang Nucleosynthesis.

For to explain cosmological problems such as the horizon problem given by the CMB fluctuations, an accelerated expansion period in the very early universe is required which is called inflation. However, since such a period erases initial conditions, a primordial matter-antimatter asymmetry would have been diluted. The conclusion is that the BAU has to be generated dynamically after inflation, which requires three conditions given by Sakharov [21] to be fulfilled:

(1) Baryon number (B) violation, which is required for a dynamic change in the baryon number,

(2) C and CP violation since particles and antiparticles need to behave different and

(3) departure from thermal equilibrium because if CPT is conserved, particles and anti-particles have the same mass and therefore the same abundances in thermal equilibrium.

There are plenty of possibilities to create the BAU that correspond to different realizations of the Sakharov conditions.

In the SM, baryon and lepton number are accidental symmetries and therefore it is not possible to violate these symmetries in tree level processes. However, at the quantum level, there are non-vanishing Adler-Bell-Jackiw triangular anomalies [22, 23], with electro-weak gauge fields contributing in the diagrams, which do not vanish. In particular, $(B + L)$ is violated while $(B - L)$ is conserved and anomaly-free.
In the 1980s it has been shown [24] that already in the SM, the non-conservation of $(B + L)$ leads to an effective 12-fermion process at high temperatures ($T > 100$ GeV), called sphaleron. Sphaleron processes satisfy condition (1).

The SM cannot explain the BAU [25, 26], electro-weak baryogenesis fails to reproduce the BAU by many orders of magnitude: In the SM, there exists only a single complex phase in the CKM matrix which represents the only source of CP violation. It had turned out, that this phase does not offer enough CP violation to fulfill condition (2). The magnitude of BAU within the SM can be estimated as $\eta_B \approx 10^{-28}$ [27].

Sphaleron processes imply that any lepton asymmetry would in part be converted into a baryon asymmetry which suggests baryogenesis through leptogenesis: Lepton number might be violated by some new physics (e.g. a see-saw scenario), giving rise to a lepton asymmetry which is then converted into a baryon asymmetry by sphaleron processes.

There are many ways to produce a baryon asymmetry in the context of see-saw models which differ in the cosmological scenario and the number of undetermined see-saw parameters present. One of the most successful possibilities is thermal leptogenesis with hierarchical or quasi-degenerate heavy right-handed Majorana neutrinos.

5 Thermal Leptogenesis

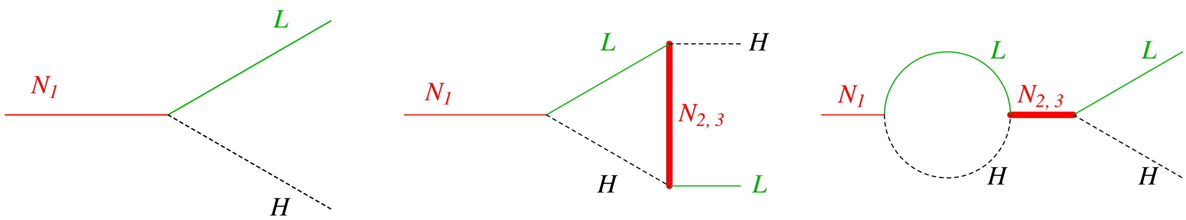


Figure 5.1: CP violating decay of the N_1 neutrino [28].

Leptogenesis [10] can be considered as the direct cosmological consequence of the see-saw mechanism. As initial condition for thermal leptogenesis, one supposes that after inflation the universe reheats to a thermal bath composed of particles with gauge interactions. The heavy right-handed Majorana neutrinos N_i are then produced by thermal scatterings like $q_L t_R \to H \to LN_i$ and inverse decays $HL \to N_i$ and a thermal number density of N_i is produced if the temperature $T > M_i$ and if the production timescale $1/\Gamma_{\text{prod}}$ is shorter than the age of the universe $\sim 1/H$. The latter condition is required because the production interaction rates have to be faster than the Hubble expansion rate of the universe $H(T)$ in order to equilibrate particle distributions.

At lower temperatures $T \sim M_i$, the heavy Majorana neutrinos decay out-of-equilibrium due to the expansion of the universe. These decays proceed in a CP and lepton number violating way, thus satisfying Sakharov's conditions. The lepton asymmetry produced is turned into a baryon asymmetry by sphaleron processes and its amount is determined by the CP asymmetry, which for the Majorana neutrino N_i and the lepton flavour α is defined as

$$\varepsilon_{i\alpha} = \frac{\Gamma(N_i \to L_\alpha H) - \Gamma(N_i \to \bar{L}_\alpha \bar{H})}{\sum_\alpha \left[\Gamma(N_i \to L_\alpha H) + \Gamma(N_i \to \bar{L}_\alpha \bar{H})\right]} \,. \tag{5.1}$$

A non-vanishing CP asymmetry arises due to the interference of the tree level amplitude with its vertex and its self-energy correction (Figure 5.1). This can be expressed in terms of the neutrino Yukawa matrices and a function containing the Majorana masses and decay widths

$$\varepsilon_{i\alpha} = \frac{1}{(Y_\nu Y_\nu^\dagger)_{ii}} \sum_j \text{Im}((Y_\nu Y_\nu^\dagger)_{ij} (Y_\nu)_{i\alpha} (Y_\nu^\dagger)_{\alpha j}) \cdot g(M_i^2, M_j^2, \Gamma_{j\alpha}^2) \,. \tag{5.2}$$

The expression of $g(M_i^2, M_j^2, \Gamma_{j\alpha}^2)$ depends on the hierarchy of the Majorana neutrino masses and is given in [29] and is non-vanishing in the case of non-degenerate masses of the heavy right-handed Majorana neutrinos. Further, equation (5.2) implies that for $\varepsilon_{i\alpha} \neq 0$, a complex ingredient of the Yukawa matrices is necessary.

5.1 Efficiency and Wash-out Regimes

For each Majorana neutrino N_i one introduces the decay parameter K_i which is defined as the ratio of the total decay width to the expansion rate

$$K_i = \frac{\sum_\alpha (\Gamma(N_i \to L_\alpha H) + \Gamma(N_i \to \bar{L}_\alpha \bar{H}))}{H(T = M_i)}. \qquad (5.3)$$

This is the key quantity for the thermodynamical description of the neutrino decays in the expanding universe [26] (in the single flavour treatment).

The inverse decays $LH \to N_i$ are relevant not only for the production of the right-handed neutrinos in the thermal bath but also for the wash-out of parts of the lepton asymmetry produced by the decays. The efficiency factors κ_i describe the effects of production and wash-out simultaneously such that the final $(B - L)$ asymmetry can be expressed as a sum of the contribution of each heavy right-handed Majorana neutrino N_i

$$N_{B-L} = \sum_i \varepsilon_i \kappa_i, \qquad \varepsilon_i = \sum_\alpha \varepsilon_{i\alpha} \qquad (5.4)$$

Including sphaleron conversion factors, the produced baryon asymmetry can be calculated as

$$\eta_B \approx 10^{-2} \sum_i \varepsilon_i \kappa_i. \qquad (5.5)$$

The parameter K_i is connected to the effective light neutrino mass ($K_i \propto (m_D^\dagger m_D)_{ii}/M_i$). One finds that the strong wash-out regime ($K_i \gg 1$) can easily satisfy low-energy experimental neutrino data while this is only true for particular classes of neutrino mass models in the weak wash-out regime ($K_i < 1$). In the strong wash-out regime only a small fraction of N_i compared to its initial ultra-relativistic thermal abundance decays out-of-equilibrium and the corresponding efficiency factors are small but large enough to allow for successful leptogenesis in a wide parameter range. However, the most appealing point is that the final asymmetry does not depend on the initial conditions while the weak wash-out regime requires a precise description of the production of the right-handed neutrinos that is sensitive to many poorly known effects. Therefore, we will concentrate on the strong wash-out regime in what follows.

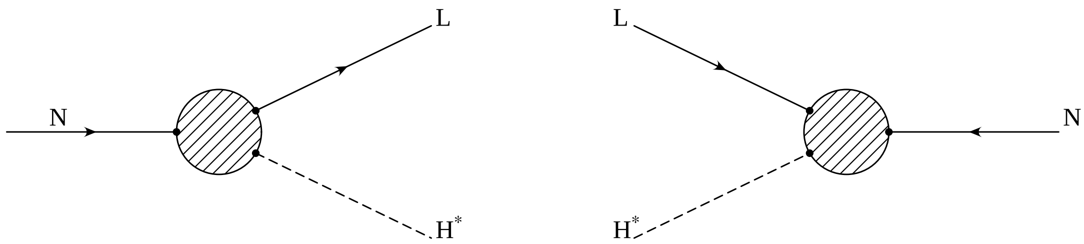


Figure 5.2: Decay and inverse decay of the right-handed heavy Majorana neutrino [30].

5.2 Boltzmann Equations

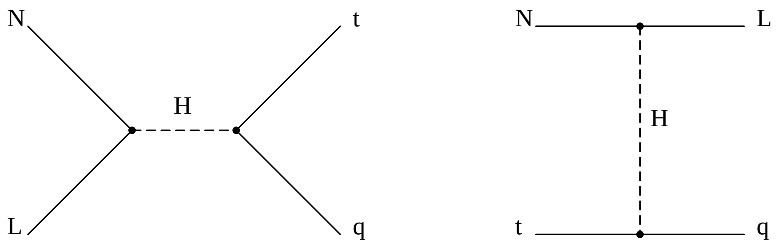


Figure 5.3: $\Delta L = 1$ scattering processes [30].

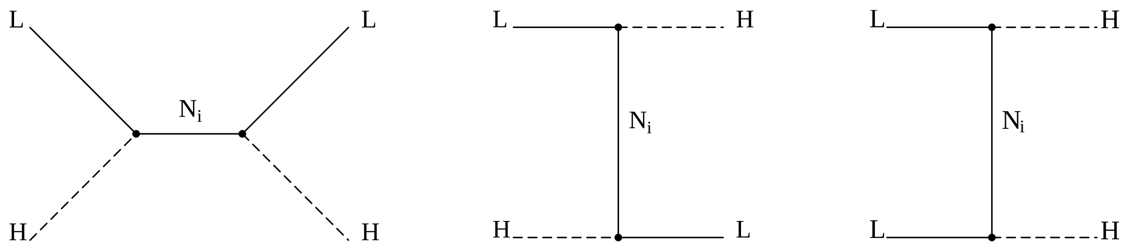


Figure 5.4: $\Delta L = 2$ scattering processes [30].

The generation of a baryon asymmetry with the help of leptogenesis is an out-of-equilibrium situation which is generally treated by means of Boltzmann equations. A detailed discussion and most subtleties are given in [26].

The main processes in the thermal bath are the decays and inverse decays of the heavy neutrinos (Figure 5.2), the lepton number violating $\Delta L = 1$ scattering processes involving the t quark (Figure 5.3) and $\Delta L = 2$ scatterings (Figure 5.4).
The basic picture is as follows: at temperatures $T > M_i$, the $\Delta L = 1$ and $\Delta L = 2$ processes have to be strong to keep N_i in equilibrium. At $T < M_i$ an asymmetry in lepton number has to be generated which requires these processes to be weak enough.
The Boltzmann equations for the number densities of the heavy neutrinos N_{N_i} and for the

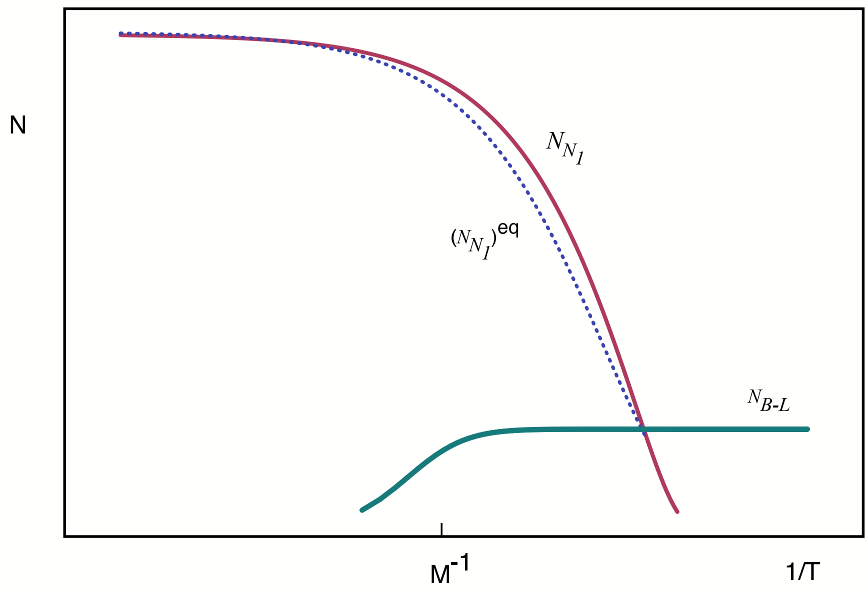


Figure 5.5: Generic behavior of the solutions to Boltzmann equations. Here the functions N_{N_1} (red solid curve) and N_{B-L} (green solid curve) are solutions to equations (5.6) and (5.7). The function $(N_{N_1})^{eq}$ (blue dotted curve) is the equilibrium particle distribution [30].

$(B-L)$ number density are given by [31, 32]:

$$\frac{dN_{N_i}}{dz} = -(D+S)\,(N_{N_i} - N_{N_i}^{\mathrm{eq}}) \tag{5.6}$$

$$\frac{dN_{B-L}}{dz} = -D\,\varepsilon_i(N_{N_i} - N_{N_i}^{\mathrm{eq}}) - W N_{B-L} \tag{5.7}$$

where

$$z = \frac{M_i}{T}, \qquad (D, S, W) = \frac{1}{Hz}(\Gamma_D, \Gamma_S, \Gamma_W). \tag{5.8}$$

Γ_D includes decay and inverse decay, Γ_S the $\Delta L = 1$ processes and Γ_W both $\Delta L = 1$ and $\Delta L = 2$ scatterings. It is clear from equation (5.7) that the source for the $(B-L)$ number density is the decay of the N_i neutrino while the inverse decay and the scatterings wash out the asymmetry. The generic behavior of solutions of the Boltzmann equations (5.6) and (5.7) is depicted in Figure 5.5.

5.3 Mass Hierarchies and Constraints

The heavy Majorana neutrinos are rather unconstrained in the sense that their masses in principle can range from a few TeV up to 10^{16} TeV depending on the model considered. For leptogenesis the hierarchy of the masses of the heavy right-handed Majorana neutrinos plays an important role and leads to different implications and constraints for each particular case. In the presence of three neutrino generations, one can distinguish the following mass hierarchies:

(A) Hierarchical heavy right-handed Majorana neutrinos: $M_1 \ll M_2, M_3$

(B) Two quasi-degenerate heavy right-handed Majorana neutrinos: $M_1 \approx M_2 \ll M_3$

(C) Three quasi-degenerate heavy right-handed Majorana neutrinos: $M_1 \approx M_2 \approx M_3$

Hierarchical Masses

In the case of the hierarchical masses (A) a great simplification occurs since both the wash-out as well as the inverse decays from the two heavier neutrinos can be neglected. Then the dominant contribution to the CP asymmetry (5.2) stems from ε_1 ($\varepsilon_1 = \sum_\alpha \varepsilon_{1\alpha}$) [17]

$$\varepsilon_1 \approx -\frac{3}{16\pi} \sum_{k=2,3} \frac{1}{\sqrt{M_k^2/M_1^2}} \frac{\mathrm{Im}(Y_\nu Y_\nu^\dagger)_{k1}^2}{(Y_\nu Y_\nu^\dagger)_{11}}. \tag{5.9}$$

Without considering flavour effects, expression (5.5) simplifies to

$$\eta_B \approx 10^{-2} \kappa_1 \varepsilon_1. \tag{5.10}$$

In the hierarchical case (A), there exist strong constraints on the mass of the lightest heavy Majorana neutrino. This can be derived from the model-independent bound on the CP asymmetry as

$$|\varepsilon_1| \lesssim \frac{3}{16\pi} \frac{M_1 \sqrt{\Delta m_{atm}^2}}{v^2} \tag{5.11}$$

with $\langle H \rangle = v = 174$ GeV. Since the reference value for a successful generation of the BAU in this scenario is $\varepsilon_1 \gtrsim 10^{-6}$ [31], the constraint (5.11) implies a lower bound on the lightest heavy Majorana neutrino mass [33]

$$M_1 \gtrsim 10^9 \mathrm{GeV} \tag{5.12}$$

given the light neutrinos are hierarchical.

Gravitino Bound

A rather large reheating temperature has important consequences for the production of gravitinos in supersymmetric theories. The production rate of gravitinos in the thermal bath is $\sim T^3/M_{\mathrm{Pl}}^2$ and the lifetime $\sim M_{\mathrm{Pl}}^2/m_{\tilde{G}}^3$. Their decays can disassociate light elements which influences the successful predictions of Big Bang Nucleosynthesis [34]. Therefore in supersymmetric theories, there exists an upper bound on the reheating temperature and so on the Majorana scale obtained from gravitino overproduction [35–37]

$$T_{reh} \lesssim 10^9 - 10^{12}\,\mathrm{GeV} \qquad \text{(gravitino production)} \tag{5.13}$$

depending on the mass of the gravitino and on the question whether the gravitino or the neutralino is the lightest supersymmetric particle. Note that this bound can collide with the bound on M_1 from successful leptogenesis (5.12) which is known as the gravitino problem.

Quasi-degenerate Masses

If Majorana masses are quasi-degenerate as in the cases (B) and (C), the CP asymmetry (5.1) receives a resonant enhancement due to the self-energy contribution [38]. This is called resonant leptogenesis and the resonant enhancements become effective if the mass splittings are comparable to the decay widths. The CP asymmetry ε_i without considering flavour effects reads

$$\varepsilon_i = \frac{1}{8\pi} \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\mathrm{Im}(Y_\nu Y_\nu^\dagger)_{ij}^2}{(Y_\nu Y_\nu^\dagger)_{ii}} \left(1 + \frac{M_i^2 M_j^2 (Y_\nu Y_\nu^\dagger)_{jj}^2}{64\pi^2 (M_i^2 - M_j^2)^2} \right)^{-1}, \tag{5.14}$$

where the last term is the regularization factor of the decay width of the corresponding heavy Majorana neutrino. Due to the resonant effect when the mass splittings are small, the lower bound on the Majorana scale obtained from a sufficiently large CP asymmetry (5.11) can be significantly lower ($M \sim \mathcal{O}(\mathrm{TeV})$) [29] and the gravitino problem can be evaded. Alternative methods to overcome the gravitino problem are soft and non-thermal leptogenesis.

Bound on the lightest neutrino mass

The decays of the heavy Majorana neutrinos are out-of-equilibrium if the decay rate is slower than the expansion of the universe, therefore $\Gamma_i < H(T \sim M_i)$. This translates into a constraint on the light effective neutrino mass (without considering flavour effects)

$$\tilde{m}_i \equiv v^2 \frac{(YY^\dagger)_{ii}}{M_i} \lesssim 10^{-3}\,\mathrm{eV} \tag{5.15}$$

and consequently $m_1 \lesssim 0.1$ eV.

Some of the above bounds obtained from successful leptogenesis can be modified when flavour effects are taken into account. We will return to this point below.

5.4 Flavour Effects

It has recently been realized that the flavour composition of the leptons produced in the decays of the heavy Majorana neutrinos plays an important role in the context of leptogenesis [32, 39–47]. These investigations all find that distinguishing flavours in the Boltzmann equations can have sizeable effects on the generation of the BAU in contrast to the former common belief which assigned effects due to flavour to be of order one.

The basic picture is as follows. In the very early universe ($T \gtrsim 10^9 - 10^{12}$ GeV) the universe has no notion of flavour since lepton Yukawa couplings are very small and not many related processes have occurred so far. Later, below some temperature ($T \lesssim 10^9 - 10^{12}$ GeV), the interactions associated with the μ and τ charged lepton Yukawa couplings are much faster than the expansion of the universe and so are in equilibrium. The interactions connected with the τ Yukawa coupling are in equilibrium below about 10^{12} GeV, where two flavours can be distinguished, followed by the ones of the μ Yukawa coupling below approximatively 10^{10} GeV, where three distinguishable flavours exist. In these regimes flavour specific solutions to the Boltzmann equations are required.

In principle, a flavour specific treatment yields two important effects. One of them is that the wash-out is reduced since in the inverse decays, the Higgs do not couple to $|l_i\rangle$ (the lepton produced by the decay of neutrino N_i) but to the flavour eigenstates $|l_\alpha\rangle$, with a reduced inverse decay rate. In order to account for this effect one introduces the flavour projectors $P_{i\alpha}$ and $\bar{P}_{i\alpha}$ defined as

$$P_{i\alpha} = |\langle l_i|l_\alpha\rangle|^2 = \frac{\Gamma_{i\alpha}}{\Gamma_i}, \qquad \bar{P}_{i\alpha} = |\langle \bar{l}_i'|l_\alpha\rangle|^2 = \frac{\bar{\Gamma}_{i\alpha}}{\bar{\Gamma}_i} \tag{5.16}$$

with $\Gamma_{i\alpha} = \Gamma(N_i \to L_\alpha H)$ being the partial decay width and $\Gamma_i = \sum_\alpha \Gamma_{i\alpha}$ the total decay rate such that $\sum_\alpha P_{i\alpha} = 1$. The index α denotes flavour ($\alpha = e, \mu, \tau$) and i the relevant heavy Majorana neutrino mass eigenstate ($i = 1, 2, 3$). Analogously $\bar{\Gamma}_{i\alpha} = \Gamma(N_i \to \bar{L}_\alpha \bar{H})$.

The evolution of the asymmetry in a flavour dependent description has then to be performed in terms of the individual asymmetries $\Delta_\alpha = B/3 - L_\alpha$ instead of $(B - L)$.

The second effect is that the state $|\bar{l}_i'\rangle$ is not the CP conjugated state of $|\bar{l}_i\rangle$ which offers an additional source of CP violation that can be described in terms of projector differences

$$\Delta P_{i\alpha} = P_{i\alpha} - \bar{P}_{i\alpha} \tag{5.17}$$

and $\sum_\alpha \Delta P_{i\alpha} = 0$. Writing $P_{i\alpha} = P^0_{i\alpha} + \Delta P_{i\alpha}/2$ and $\bar{P}_{i\alpha} = P^0_{i\alpha} - \Delta P_{i\alpha}/2$ where $P^0_{i\alpha} = (P_{i\alpha} + \bar{P}_{i\alpha})/2$ are the tree level contributions to the projectors, one finds

$$\varepsilon_{i\alpha} = \varepsilon_i P^0_{i\alpha} + \frac{\Delta P_{i\alpha}}{2} \tag{5.18}$$

with $\varepsilon_i = \sum_\alpha \varepsilon_{i\alpha}$ and the CP asymmetry $\varepsilon_{i\alpha}$ defined in (5.1).

Boltzmann Equations

The kinetic equations in terms of the decays and inverse decays (after subtracting the intermediate state contributions from $\Delta L = 2$ scatterings and neglecting $\Delta L = 1$ effects) are given by [32, 41, 43, 48]

$$\frac{dN_{N_i}}{dz} = -D_i \,(N_{N_i} - N^{\rm eq}_{N_i}) \tag{5.19}$$

$$\frac{dN_{\Delta_\alpha}}{dz} = -\textstyle\sum_i D_i \,\varepsilon_{i\alpha}(N_{N_i} - N^{\rm eq}_{N_i}) - N_{\Delta_\alpha} \sum_i P^0_{i\alpha} W^{\rm ID}_i \tag{5.20}$$

where W_i^{ID} contains the wash-out effects of the inverse decays and

$$D_i = \frac{\Gamma_{D,i}}{Hz}, \qquad \Gamma_{D,i} = \sum_\alpha (\Gamma(N_i \to L_\alpha H) + \Gamma(N_i \to \bar{L}_\alpha \bar{H})) \tag{5.21}$$

Note that in the three-flavour case, $\alpha = e, \mu, \tau$ ($\Delta_\alpha = B/3 - L_\alpha$) and in the two flavour case $\alpha = (e+\mu), \tau$ ($\Delta_\alpha = B/2 - L_\alpha$) which means that the electron and muon CP asymmetries are summed and only two equations for N_{Δ_α} have to be considered. The total $(B-L)$ asymmetry can be obtained from $N_{B-L} = \sum_\alpha N_{\Delta_\alpha}$ which can be translated into the baryon asymmetry by sphaleron conversion factors.

Single Flavour and Flavour-specific Approach

An order-of-magnitude estimate of solutions for the BAU η_B of the Boltzmann equations in the strong wash-out regime in the single-flavour approach valid at high temperatures is given by [40]

$$\eta_B \simeq -10^{-2} \sum_{i=1}^{3} e^{-(M_i - M_1)/M_1} \frac{1}{K} \sum_{\alpha=e,\mu,\tau} \varepsilon_{i\alpha} \qquad \text{(single flavour)} \tag{5.22}$$

where the final baryon asymmetry is proportional to the total CP asymmetry weightened by a single wash-out factor obtained by summing over all lepton flavours. An estimate including flavour effects reads [40]

$$\eta_B \simeq -10^{-2} \sum_{i=1}^{3} \sum_{\alpha=e,\mu,\tau} e^{-(M_i - M_1)/M_1} \varepsilon_{i\alpha} \frac{K_{i\alpha}}{K_\alpha K_i} \qquad \text{(three flavours)} \tag{5.23}$$

with the decay parameters

$$K_{i\alpha} = \frac{\Gamma(N_i \to L_\alpha H) + \Gamma(N_i \to \bar{L}_\alpha \bar{H})}{H(T = M_i)}, \tag{5.24}$$

$$K = \sum_i K_i, \qquad K_i = \sum_{\alpha=e,\mu,\tau} K_{i\alpha}, \qquad K_\alpha = \sum_{i=1}^{3} K_{i\alpha}, \tag{5.25}$$

$$H(T = M_i) \simeq 17 \frac{M_i^2}{M_{\mathrm{Pl}}}. \tag{5.26}$$

Here $M_{\mathrm{Pl}} = 1.22 \times 10^{19}$ GeV and $K_{i\alpha}$ is the decay parameter of the decay of the Majorana neutrino N_i and into a lepton flavour α. Similar estimates can be found in [44, 48]. In the strong wash-out regime the following condition for the single-flavour case has to be fulfilled:

$$K = K_1 + K_2 + K_3 \gtrsim 50 \qquad \text{(single flavour)}. \tag{5.27}$$

In the flavoured regime one has to assure that the estimate (5.23) including flavour effects is applicable [40] and that the inequality

$$K_i^l \gtrsim 1 \qquad \text{(three flavours)} \tag{5.28}$$

is always satisfied. If both (5.27) and (5.28) are satisfied, a simple decay-plus-inverse decay picture is a good description and the estimates (5.23) and (5.22) independent of the initial abundances give a good approximation of the numerical solution of the full Boltzmann equations.

6 Radiative Resonant Leptogenesis

For quasi-degenerate heavy right-handed Majorana neutrinos with mass splittings comparable to their decay widths, the CP asymmetries relevant for thermal leptogenesis are *resonantly* enhanced [29, 38, 48]. To be more specific, the enhancements become effective if the mass splittings are comparable to the decay widths:

$$|M_i - M_j| \sim \frac{\Gamma_{i,j}}{2}, \qquad \Gamma_i \sim M_i \, Y_\nu Y_\nu^\dagger. \tag{6.1}$$

The small mass splittings required can be explained by radiative resonant leptogenesis. In this context, the heavy Majorana neutrinos are assumed to be exactly degenerate at a scale higher than their decoupling scale, and the mass splittings required being induced by renormalization group effects. It has been shown, that this can lead to appropriate values for the CP asymmetries in scenarios with two quasi-degenerated Majorana neutrinos [49–51] if three generations of heavy Majorana neutrinos exist. In this section we investigate how the CP and flavour violating quantities relevant to leptogenesis and charged lepton flavour violation, respectively, can be radiatively generated. Since leptogenesis in the present framework can be considered as a generalization of the setup with two heavy singlets in [51] to the case of three degenerate flavours, we will also clarify what novelties arise in this case.

6.1 A Natural Framework

MFV in the lepton sector [12] introduces right-handed heavy Majorana neutrinos that are degenerate in mass which reflects the assumption that the new source of lepton number violation is flavour blind. However, radiative corrections spoil this mass degeneracy of the the Majorana neutrinos [46, 52]. Therefore we combine the MLFV hypothesis with a choice of a scale at which the Majorana masses are exactly degenerate [46]. A natural selection for the degeneracy scale is the GUT scale Λ_{GUT},

$$M_R(\Lambda_{\text{GUT}}) = M_\nu \mathbb{1}. \tag{6.2}$$

The mass splittings at the Majorana scale that are necessary for leptogenesis are then not put in by hand but will be induced *radiatively* which can be described by Renormalization Group Equations (RGEs) and will be derived in the next sections.

In what follows, we analyse the MLFV framework involving three quasi-degenerate right-handed Majorana neutrinos with the BAU generated by radiative resonant leptogenesis [46].

6.2 MLFV with a Degeneracy Scale

We have defined our MLFV scenario to have a scale at which the masses of the right-handed neutrinos are exactly degenerate, such that the matrix M_R has no flavour structure at all. In general, there will be additional flavoured particles in the theory. As a specific example, we consider the MSSM. Here the N_i are accompanied by heavy sneutrinos $\tilde{N}_i^c$, and there are also $SU(2)$ doublet sleptons $\tilde{l}_i$, transforming as

$$\tilde{l} \to V_L \tilde{l}, \qquad \tilde{N}^c \to V_{\nu_R}^* \tilde{N}^c \tag{6.3}$$

under the transformation (3.5). The Lagrangian then contains soft SUSY breaking terms

$$\mathcal{L}_{soft} = -\tilde{N}_i^{c*} \tilde{m}_{\nu ij}^2 \tilde{N}_j^c - \tilde{l}_i^* \tilde{m}_{lij}^2 \tilde{l}_j + \ldots, \tag{6.4}$$

where the ellipsis denotes further scalar mass matrices and trilinear scalar interactions. In general all matrices in $\mathcal{L}_{soft}$ have a non-minimal flavour structure. The simplest generalization of our degenerate scenario is then to extend the requirement of exact degeneracy to all mass matrices, similar to minimal supergravity (mSUGRA). To be specific, we require all scalar masses to have the same value m_0 at the high scale and also require the A-terms to have the mSUGRA form $A = aY$ with Y the corresponding Yukawa matrix and a a universal, real parameter of the theory. This example also provides us with a concrete value for the scale Λ_{LFV}: LFV processes such as $l_i \to l_j \gamma$ are mediated by loop diagrams involving sleptons and higgsinos or (weak) gauginos, and unless gaugino masses are very large, the scalar particles such as $\tilde{l}_i$ decouple at a scale $\Lambda \sim m_0$. Hence the operators governing charged LFV are suppressed by powers of $m_0 \equiv \Lambda_{\text{LFV}}$. As in the case of the heavy Majorana masses, the generalized degeneracy requirement is not stable under radiative corrections, and for the same reason it is not renormalization scheme independent.

6.3 Radiatively generated Flavour Structure and large Logarithms

As it will be discussed in detail in the following section, the CP asymmetries necessary for leptogenesis require the decaying particles to be non-degenerate in mass. The decaying particles are on their mass shell ($\overline{\text{MS}}$ to be definite [53]), but the degenerate initial conditions are usually specified in a massless scheme. This is likely appropriate if the degeneracy is due to some flavour symmetry of an underlying theory, that relates high-energy Lagrangian parameters and is broken at the scale $\Lambda_{\rm GUT}$.

At one loop, the two mass definitions are related by a formula of the structure

$$M_i^{os} = M_i^{\overline{\rm MS}}(\mu) + c_i M_i^{\overline{\rm MS}}(\mu) \ln \frac{M_i}{\mu} + \text{non-logarithmic corrections}, \tag{6.5}$$

where $\mu \sim \Lambda_{\rm GUT}$ is the $\overline{\rm MS}$ renormalization scale, $c_i = 2(Y_\nu Y_\nu^\dagger)_{ii}/(16\pi^2)$ in the standard see-saw, and the non-logarithmic corrections depend on our choice of a massless (or any other) renormalization scheme. The resulting scheme dependence cannot be present in physical observables such as the BAU. Since this issue is usually not discussed in the literature on lepton flavour violation, let us elaborate on how it may be resolved.

First, notice that while the non-logarithmic terms in (6.5) are scheme dependent, the logarithmic corrections proportional to c_i are actually scheme independent. If $\ln \Lambda_{\rm GUT}/M_\nu \gg 1$, the logarithmic terms must be considered to be $\mathcal{O}(1)$ and summed to all orders. This is achieved in practice by solving renormalization group equations. Similar re-summations must be performed for all other parameters in the theory (such as Yukawa couplings). Correspondingly, the dominant higher-loop corrections to LFV observables and leptogenesis are approximated by using leading-order expressions with one loop RGE-improved Yukawa couplings and masses. This is the leading logarithmic approximation. Non-logarithmic corrections such as those indicated in (6.5) are then subleading and should be dropped.

What happens when the logarithms are not large is the following. If the MLFV framework is an effective theory of some fundamental theory where the degeneracy is enforced by a flavour symmetry, for instance the group (3.2), then the degeneracy holds in *any* scheme (that respects the symmetry) in the full theory and the scheme dependence observed in (6.5) must be due to unknown threshold corrections in matching the underlying and effective theories. Since the flavour symmetry in MLFV, by definition, is broken precisely by the Yukawa matrices, this matching introduces all possible terms that are invariant under transformations (3.5) to (3.8). A list of such structures has recently been given in [52], for instance,

$$M_R = M_\nu \Big[1 + c_1 (Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger)^T) + c_2 (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T) + \dots \Big]. \tag{6.6}$$

The coefficients c_1 and c_2 have been claimed by these authors to be independent $\mathcal{O}(1)$ coefficients. Indeed these terms contain only non-logarithmic terms and (small) decoupling logarithms when M_R is taken in the $\overline{\rm MS}$ scheme, renormalized near the GUT (matching) scale.

However, when computing the (physically relevant) on-shell M_R in the case of $\Lambda_{\rm GUT} \gg M_\nu$, large logarithms dominate both c_1 and c_2. The leading logarithmic contributions are not independent, but are related by the renormalization group. The coefficient c_2 is quadratic in $L \equiv \ln \Lambda_{\rm GUT}/M_\nu$, while c_1 is linear, and the RGE for M_R implies $c_2|_{L^2} = \frac{1}{2}[c_1|_L]^2$. These logarithms are summed by RG-evolving $M_R^{\overline{\rm MS}}$ to a scale $\mu \sim M_\nu$. The additional conversion to on-shell masses is then again a subleading correction.

Finally, we note that if there is no underlying symmetry, the degeneracy condition can again be true at most for special choices of scheme/scale, and must be fine-tuned.

Numerically, the logarithms dominate already for mild hierarchies $\Lambda_{\rm GUT}/M_\nu > 10^2$, as then $2\ln \Lambda_{\rm GUT}/M_\nu \approx 10$. Let us now restrict ourselves to hierarchies of at least two orders of magnitude and work consistently in the leading logarithmic approximation. As explained above, in this case non-logarithmic corrections both of the threshold type (in the coefficients c_i in (6.6) and in physical quantities (on-shell masses, CP asymmetries, etc.) are subleading and should be dropped. In this regard our apparently "special" framework of initially degenerate heavy neutrinos turns out to be the correct choice at leading logarithmic order.

Finally, we recall that the positions of the poles of the N_i two-point functions contain an imaginary part related to the widths of these particles. While not logarithmically enhanced, these are also scheme independent at one loop (as the widths are physical), and it is unambiguous to include them in applications. In fact, these widths effects are often numerically important for the CP asymmetries in N_i decay [29, 54], and we will keep them in our numerical analysis.

6.4 Renormalization Group Evolution

The relevant renormalization group equations for the SM as well as MSSM see-saw models have been given in in [55]. Since the physical quantities considered involve mass eigenstates, it is convenient to keep the Majorana mass matrix diagonal during the evolution (see, e.g., appendix B of [55]):

$$M_R(\mu) = \mathrm{diag}(M_1(\mu), M_2(\mu), M_3(\mu)).$$

With the definitions

$$H = Y_\nu Y_\nu^\dagger, \tag{6.7}$$

and

$$t = \frac{1}{16\pi^2} \ln\left(\mu/\Lambda_{\rm GUT}\right), \tag{6.8}$$

one obtains for the running of the mass eigenvalues in the SM and MSSM with right handed neutrinos:

$$\frac{dM_i}{dt} = 2\, H_{ii}\, M_i \quad (\text{SM}), \qquad \frac{dM_i}{dt} = 4\, H_{ii}\, M_i \quad (\text{MSSM}). \tag{6.9}$$

Due to the positivity of the right-hand side of the equations (6.9), the masses at the Majorana scale will always be decreased with respect to the GUT scale.

The matrix H within th SM and MSSM extended by 3 right-handed Majorana neutrinos satisfies the renormalization group equations

$$\frac{dH}{dt} = [T, H] + 3\, H^2 - 3\, Y_\nu Y_E^\dagger Y_E Y_\nu^\dagger + 2\beta H \qquad \text{(SM)}, \tag{6.10}$$

$$\frac{dH}{dt} = [T, H] + 6\, H^2 + 2\, Y_\nu Y_E^\dagger Y_E Y_\nu^\dagger + 2\beta H \qquad \text{(MSSM)}, \tag{6.11}$$

where

$$\beta = Tr(Y_\nu^\dagger Y_\nu) + Tr(Y_E^\dagger Y_E) + 3\, Tr(Y_u^\dagger Y_u) + 3\, Tr(Y_d^\dagger Y_d) - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 \ \text{(SM)}, \tag{6.12}$$

$$\beta = Tr(Y_\nu^\dagger Y_\nu) + 3\, Tr(Y_u^\dagger Y_u) - \frac{3}{5} g_1^2 - 3 g_2^2 \quad \text{(MSSM)}, \tag{6.13}$$

$$T_{ij} = \begin{cases} -\frac{M_j+M_i}{M_j-M_i} \mathrm{Re} H_{ij} - i \frac{M_j-M_i}{M_j+M_i} \mathrm{Im} H_{ij} & (i \neq j), \\ 0 & (i = j), \end{cases} \tag{6.14}$$

and β is real and has trivial flavour structure. The matrix T satisfies $\dot{U} = TU$, where U diagonalizes the mass matrix (that is real and symmetric)

$$M_R^{(0)}(\mu) = U(\mu)^T M_R(\mu) U(\mu) \tag{6.15}$$

where $M_R^{(0)}$ satisfies the unconstrained RGEs given in [55]. Note the different relative signs in the terms involving the charged lepton Yukawas in the RGEs (6.10) and (6.11). We will return to this point below.

6.4.1 Renormalization Group and Leptogenesis

We will now investigate qualitatively the impact of these equations on leptogenesis. According to (6.9), the mass splitting induced radiatively at the Majorana scale are approximately given by

$$\frac{\Delta M}{M} \sim Y_\nu Y_\nu^\dagger \ln\left(\frac{M}{\Lambda_{\mathrm{GUT}}}\right). \tag{6.16}$$

Particularly interesting is that the radiatively generated mass splittings automatically fulfill the condition of resonant leptogenesis given in (6.1).

Ignoring flavour effects in the Boltzmann equations, the baryon asymmetry η_B is approximately proportional to

$$\eta_B \propto \mathrm{Im}((H_{ij})^2) = 2\, \mathrm{Re} H_{ij}\, \mathrm{Im} H_{ij} \qquad (i \neq j) \tag{6.17}$$

evaluated in the mass eigenbasis. At the scale Λ_{GUT}, the degeneracy of M_R allows the use of an $SO(3)$ transformation to make the off-diagonal elements of $\mathrm{Re}H$ vanish. This can be done because ReH is real and symmetric (H is hermitian) and therefore can be diagonalized by a real orthogonal (and hence unitary) transformation of the right-handed neutrinos. Now if all three neutrinos are degenerate, such a rotation affects no term in the Lagrangian besides Y_ν. Inspecting the unconstrained RGEs, one finds furthermore that the matrix T is actually zero at the initial scale, and consequently the commutator $[T, H]$ absent from (6.10), (6.11), (6.18), and (6.19).

We split (6.10) into real and imaginary parts in formal limit of vanishing charged lepton Yukawa couplings Y_E. Then one finds

$$\frac{d\mathrm{Re}H}{dt} = [\mathrm{Re}T, \mathrm{Re}H] - [\mathrm{Im}T, \mathrm{Im}H] + 3\Big\{(\mathrm{Re}H)^2 - (\mathrm{Im}H)^2\Big\} + 2\beta \mathrm{Re}H. \tag{6.18}$$

To investigate how ReH can be generated radiatively, we assume that it is zero at some scale (initial or lower). Then (6.18) reduces to

$$\frac{d\mathrm{Re}H}{dt} = -[\mathrm{Im}T, \mathrm{Im}H] - 3(\mathrm{Im}H)^2. \tag{6.19}$$

Cases with three and two neutrino flavours

We will now evaluate (6.19) for the $(2, 1)$ element ($T_{ij}, \mathrm{Im}H_{ij} = 0$ for $i = j$). If there were only two heavy singlets in the theory (case with two neutrino flavours), each term in each matrix product would require one $(2, 1)$ element and one $(1, 1)$ or $(2, 2)$ element from the two matrix factors. For example,

$$(\mathrm{Im}T\, \mathrm{Im}H)_{21} = \mathrm{Im}T_{21} \underbrace{\mathrm{Im}H_{11}}_{0} + \underbrace{\mathrm{Im}T_{22}}_{0} \mathrm{Im}H_{21} = 0, \tag{6.20}$$

and similarly for the other terms. Consequently,

$$\mathrm{Re}H_{21} = 0 \Rightarrow \frac{d\mathrm{Re}H_{21}}{dt} = 0. \tag{6.21}$$

We see that there is no radiative leptogenesis in the two-flavour case when $Y_E = 0$. This is consistent with the approximate equation (12) in [51], where ReH_{21} was found to be proportional to y_τ^2 in the two-flavour case considered and vanishes in this limit, and the total CP asymmetries for each heavy Majorana neutrino take the form [51]

$$\varepsilon_{1,2} \simeq \frac{\bar{\varepsilon}_{1,2}}{1 + D_{2,1}}, \qquad \varepsilon_3 \simeq 0, \tag{6.22}$$

and

$$\bar{\varepsilon}_j \simeq \frac{3y_\tau^2}{32\,\pi} \frac{\mathrm{Im}(H_{21})\, \mathrm{Re}\left[(Y_\nu)^*_{23}\, (Y_\nu)_{13}\right]}{H_{jj}(H_{22} - H_{11})} = \frac{3\, y_\tau^2}{64\,\pi} \frac{m_j(m_1 + m_2)\, \sqrt{m_1\, m_2}\, \sinh(2\, y_{12})\, \mathrm{Re}\,(U^*_{\tau 2}\, U_{\tau 1})}{(m_1 - m_2)(m_j^2\, \cosh^2 y_{12} + m_1 m_2\, \sinh^2 y_{12})}, \tag{6.23}$$

$$D_j \simeq \frac{\pi^2}{4} \frac{H_{jj}^2}{(H_{22} - H_{11})^2\, \ln^2 (M_\nu/M_{\mathrm{GUT}})} = \left[\frac{\pi}{2} \frac{m_j^2\, \cosh^2 y_{12} + m_2 m_1\, \sinh^2 y_{12}}{m_j(m_2 - m_1)\, \ln (M_\nu/M_{\mathrm{GUT}})}\right]^2, \tag{6.24}$$

where D_j are the regularization factors coming from the heavy Majorana decay widths. We immediately see that the total CP asymmetries only bare a very mild dependence on the heavy Majorana scale and vanish in the limit $y_\tau \to 0$ ($Y_E = 0$).

This argument however does not hold when three heavy right-handed Majorana neutrinos are present. For instance,

$$((\mathrm{Im}H)^2)_{21} = \mathrm{Im}H_{21}\mathrm{Im}H_{11} + \mathrm{Im}H_{22}\mathrm{Im}H_{21} + \mathrm{Im}H_{23}\mathrm{Im}H_{31} = \mathrm{Im}H_{23}\mathrm{Im}H_{31}, \tag{6.25}$$

which is in general not zero. The other terms in (6.19) are also proportional to $\mathrm{Im}H_{23}\mathrm{Im}H_{31}$. The conclusion is that in order to have successful leptogenesis without help from charged lepton Yukawas, three heavy right handed Majorana neutrinos are required.

Once we restore the charged lepton Yukawas, they will also contribute. The important qualitative difference is that, whereas the contribution involving the charged lepton Yukawas is only logarithmically dependent on the Majorana scale (for the two-flavour case: [51], for the three-flavour case: [29]), the contribution from Y_ν to the radiatively generated $\mathrm{Re}H_{ij}$ scales with M_ν because it contains two extra powers of Y_ν as observed in the three flavour scenario studied in [52].

In summary, we have the following expectations for qualitative behavior for the BAU as a function of M_ν:

(I) In the case of only two heavy Majorana neutrinos, η_B is weakly dependent on M_ν over the whole range of M_ν.

(II) For small Y_ν (small M_ν), the dominant contribution to $\mathrm{Re}H_{ij}$ and hence to η_B should be due to Y_E. η_B turns out to be weakly dependent on M_ν.

(III) For large Y_ν (large M_ν), in the case of three heavy Majorana neutrinos there is a relevant contribution proportional to $((\mathrm{Im}H)^2)_{ij}$. Since it contains two extra powers of Y_ν with respect to the contribution proportional to y_τ^2, η_B scales linearly with M_ν.

These qualitative conclusions only hold when flavour effects in the Boltzmann evolution are neglected.

6.4.2 RGE, the PMNS Matrix and Δ_{ij}

The objects Y_ν and D_R in equation (3.16) are defined at a high scale while d_ν and U_ν can be identified as the physical (light) neutrino masses and mixing matrix. However, to be orthogonal the matrix R has to be defined with all objects given at the same scale. Using low-energy inputs in d_ν can be a bad approximation because there can be significant radiative corrections between the electro-weak and the GUT scale. However, as investigated in [56], both in the SM and in the MSSM with small $\tan\beta$, the main effect below M_ν is an approximately universal rescaling of the light neutrino masses. This leads to a larger size of the elements of Y_ν extracted by means of (3.16) and in a weak running of the matrix U_ν. Above the scale M_ν one can still *define* an effective neutrino mass matrix through the see-saw relation (3.10). However, the evolution becomes more involved, as in the presence of heavy singlets there are additional contributions to the running involving Y_ν. To deal with this situation, where some of our inputs are specified at the electro-weak scale, while the matrix $R_{\rm deg}$ is defined at the scale $\Lambda_{\rm GUT}$, we employ an iterative procedure described in detail in Appendix A.

Δ_{ij} evolves above the scale M_ν and the flavour structures, such as the slepton mass matrix $m^2_{\tilde{l}}$, which are affected by Δ_{ij}, also evolve between M_ν and $\Lambda_{\rm LFV}$ (and the resulting effective operators below $\Lambda_{\rm LFV}$ also evolve). Moreover, the flavour violating piece in, for example, $m^2_{\tilde{l}}$ is not exactly proportional to Δ_{ij} at the scale M_ν beyond leading order because these objects satisfy different RGEs between M_ν and $\Lambda_{\rm GUT}$. All this running depends, beyond the operator, also on the details of the model. Below the see-saw scale the flavour non-universal contributions are governed by Y_E (although trilinear couplings such as the A-terms in the MSSM can also contribute), which is analogous to the case of the PMNS matrix. Based on the experience that the running of the PMNS angles is weak in the SM and the MSSM unless $\tan\beta$ (and hence y_τ) is large, we ignore all these details and evaluate Δ_{ij} at the scale M_ν. That Δ_{ij} has to be evaluated at the high energy scale M_ν, and hence U_ν and d_ν have to be evaluated at M_ν by means of renormalization group equations with the initial conditions given by their values at M_Z, has recently been stressed in particular in [19]. The dominant contributions to the flavour-violating pieces in the charged slepton masses matrix in the MSSM that is relevant for $l_i \to l_j\gamma$ are proportional to $Y_\nu^\dagger Y_\nu$ and come from scales above M_ν, as seen for instance in equation (30) of [57] (where charged lepton Yukawas and A-terms have been dropped and only contribute at higher orders) and the fact that right-handed neutrinos and their Yukawa couplings are absent below that scale. All other parameters of a given MLFV model, hidden in the Wilson coefficient C in (3.13), like slepton and chargino masses in the MSSM, would have to be evaluated at the electro-weak scale and lower scales if a concrete value for C was desired.

6.4.3 CP Asymmetries

We now would like to illustrate and check numerically our qualitative discussion of the CP asymmetries relevant for leptogenesis performed so far. A thorough investigation of the baryon asymmetry follows in the next chapter. Figure 6.1 shows the sum of the three CP asymmetries $|\sum_i \epsilon_i|$ defined in equation (5.1), for the generic three-flavour case (left plot) and

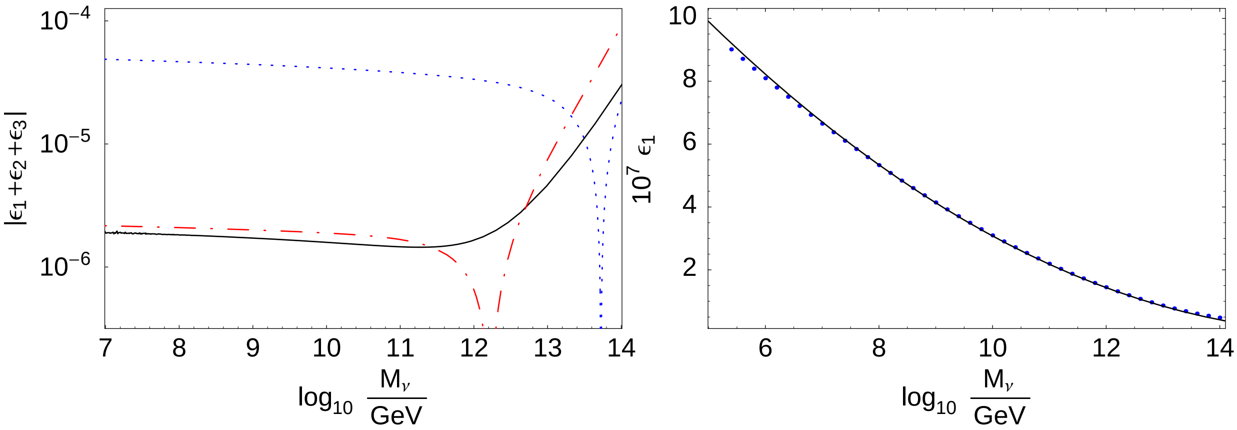


Figure 6.1: Left plot: M_ν dependence of $|\sum_i \epsilon_i|$ for the generic (3-flavour) case. Right plot: effective 2-flavour case. Normal hierarchy, $m_\nu^{\text{lightest}} = 0.02$ eV ; $y_{12} = 0.8$, $y_{13} = 0.2$, $y_{23} = 0.6$ (3-flavour case), $y_{12} = 1$ and $y_{13} = y_{23} = 0$ (effective 2-flavour case). The PMNS phases have been taken to be $\delta = \alpha = \beta = \pi/10$. Right plot: Effective two-flavour case; only ϵ_1 is shown, on a linear scale.

the CP asymmetry ϵ_1 for the effective two-flavour case where only $y_{12} \neq 0$ (right plot). One can see clearly that in the latter case the dependence on M_ν is weak and slightly reciprocal. In fact this dependence is approximately proportional to $\ln^2 \Lambda_{\text{GUT}}/M_\nu$ (*black solid line*) in agreement with expectations. The generic case is shown in the left plot for the SM (*black solid*) as well as the MSSM for $\tan\beta = 2$ (*red dot-dashed*) and $\tan\beta = 10$ (*blue dotted*), with the remaining parameters given in the Figure caption. In contrast to the two-flavour case, there exists a strong dependence on M_ν for $M_\nu > 10^{12}$ GeV, when the contribution due to Y_ν alone starts to dominate the RGEs (6.10) and (6.11). The precise form of the M_ν dependence is quite sensitive to the "angles" y_{ij}, but the roughly linear growth of $|\sum_i \epsilon_i|$ in the regime of large M_ν appears to be general. However, the figure also clearly shows a strong dependence on the MSSM parameter $\tan\beta$ particularly for small M_ν. Indeed already for relatively small $\tan\beta = 10$ the CP asymmetries can be more than an order of magnitude larger than in the SM. Moreover, in the case of the MSSM we observe a sign change at some scale $M_\nu \gtrsim 10^{12}$ GeV, which can be traced to the different relative signs between the terms on the right-hand sides of (6.10) and (6.11). This example clearly demonstrates a rather dramatic dependence on details of the model.

6.4.4 Iterative and simplified Procedure

We compare the elaborate iterative procedure of matching high- and low-energy parameters described in the appendix to a simpler procedure where we simply impose the electro-weak scale PMNS and neutrino mass parameters at the scale Λ_{GUT}. In the left plot of Figure 6.2 (corresponding to the MSSM with $\tan\beta = 2$), we investigated the impact of the iterative

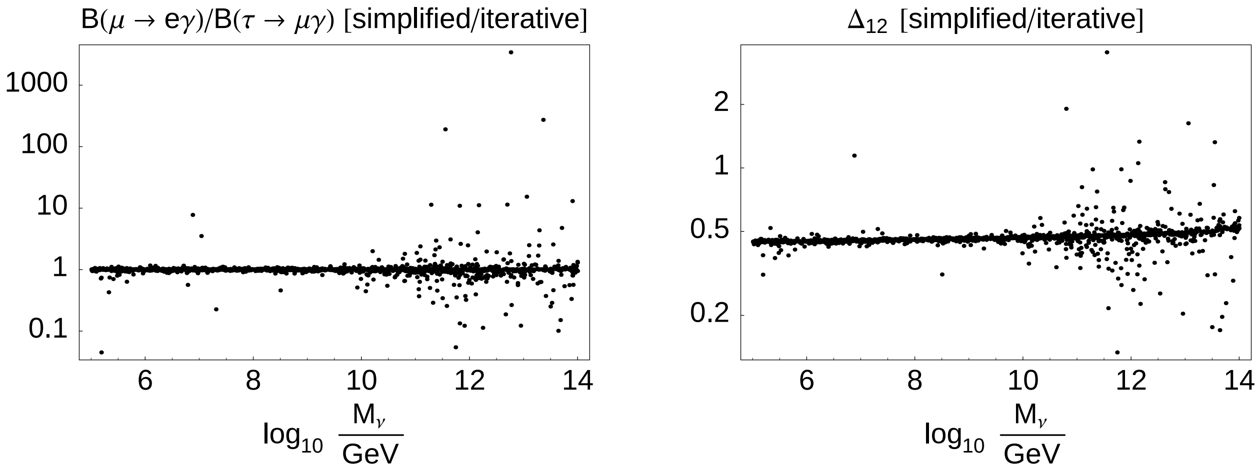


Figure 6.2: Impact of iterative vs simplified procedure. Left plot: Simplified result for the ratio of branching ratios $B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$, normalized to the one obtained with the iterative procedure. Right plot: Similarly for Δ_{12}.

procedure compared to the simplified one for the ratios $B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$. It turns out that both procedures agree well for small scales M_ν. For $\tan\beta = 10$ this agreement is slightly worse. At large values $M_\nu > 10^{11}$ GeV, deviations up to a few orders of magnitude can occur for some choices of parameters. It appears that this is usually due to accidentally small branching ratios in one of the approaches. This is supported by the right plot in the Figure 6.2, which shows a good agreement for the more fundamental flavour violating quantity Δ_{12} up to the (expected) different overall normalization. Also for the CP asymmetries we compared these two approaches and found the differences to be generically small. Hence we feel justified to use the simplified procedure in order to save computer time.

7 MLFV and Leptogenesis

We will now analyse leptogenesis and LFV processes in the framework of MLFV allowing for CP violation at low and high energies (corresponding to our paper [46]). The BAU is generated by means of radiative resonant leptogenesis with flavour effects taken into account. Which Boltzmann equation to use depends on the temperature scale at which leptogenesis takes place. We will follow a simplistic approach ignoring all subtleties generically coming into play in the intermediate regime between different mechanisms at work. Our main conclusions, however, will not be affected by this omission. We will simply divide the temperature scale into a region up to which all three lepton flavours have to be taken into account and a region above which the single flavour approximation works. Different results for T^{μ}_{eq} can be found in the literature ranging from $T^{\mu}_{\text{eq}} \simeq 10^{9}$ GeV [32, 41] to $T^{\mu}_{\text{eq}} \simeq 10^{11}$ GeV [43]. We will choose $T^{\mu}_{\text{eq}} \simeq 10^{10}$ GeV in our analysis.

We will examine whether this approach is successful and investigate the role of the flavour specific treatment. Further we will explore possible correlations among leptogenesis and low-energy observables.

We have performed the leptogenesis analysis specifically for the SM. We do not expect large deviations in the MSSM from the SM if the same $Y_\nu(M_\nu)$ and $M^i_\nu(M_\nu)$ are given. The main differences come from the CP asymmetries, which now include contributions from the supersymmetric particles, from the wash-out, and from conversion and dilution factors. The supersymmetric CP asymmetries have the same flavour structure as in the SM and using [17] one can show that $\epsilon^{\text{MSSM}} \simeq 2\,\epsilon^{\text{SM}}$ for quasi-degenerate heavy neutrinos. We also expect the correction from the decay widths to be similar in size. Next, the wash-out in the strong wash-out regime is about a factor of $\sqrt{2}$ larger [58] in the MSSM, whereas the dilution and sphaleron conversion factors stay almost unchanged. Concluding, we find that in the scenario considered the predicted values roughly satisfy $\eta_B^{\text{MSSM}} \simeq 1.5\,\eta_B^{\text{SM}}$ for the same set of input parameters $Y_\nu(M_\nu)$ and $M^i_\nu(M_\nu)$. The RGE induced values of $Y_\nu(M_\nu)$ and $M^i_\nu(M_\nu)$, however, are model dependent and lead to in general different $Y_\nu(M_\nu)$ and $M^i_\nu(M_\nu)$ for the same boundary conditions at the GUT and low-energy scale, as discussed in Section 6.4. Especially sensitive is the region $M_\nu \lesssim 10^{12}$ GeV where the CP asymmetries are dominantly generated by the tau Yukawa coupling, which is enhanced by a factor of $\tan\beta$ in the MSSM. Note also that in the MSSM, T^{μ}_{eq} and T^{τ}_{eq} should be rescaled by a factor $(1+\tan^2\beta)$ to take account of the larger Yukawa couplings [45], which should make flavour effects even more prominent.[1]

7.1 Numerical Analysis

The RGEs discussed in Section 6.4 are implemented in the package REAP [59] which we will use for our analysis. Further we take our input parameters at the electro-weak scale, except for the matrix $R_{\rm deg}$, which has to be defined at the scale $\Lambda_{\rm GUT}$ which is to a good approximation equivalent to the iterative treatment evaluating the low-energy parameters at high scales (see Appendix A) as discussed above. From these inputs we find a consistent set of parameters at the see-saw scale M_ν, where the CP asymmetries as well as $B(l_i \to l_j \gamma)$ are calculated.

For the PMNS matrix that describes leptonic low-energy mixing and CP violation, we use the convention

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot V \tag{7.1}$$

where $c_{ij} = \cos(\theta_{ij})$, $V = \mathrm{diag}(e^{i\alpha}, e^{i\beta}, 1)$, α and β denote the Majorana phases and δ denotes the Dirac phase. Apart from the phases and the CHOOZ angle $\sin(\theta_{13})$ which is restricted to the following range

$$0 \le \sin(\theta_{13}) \le 0.2, \tag{7.2}$$

the PMNS matrix is relatively well known from neutrino oscillation experiments. We use maximal atmospheric mixing $c_{23} = s_{23} = 1/\sqrt{2}$ and the solar mixing angle $\theta_{12} = 33°$. For the PMNS phases we will consider the physically relevant ranges

$$0 \le \alpha, \beta \le \pi, \qquad 0 \le \delta \le 2\pi. \tag{7.3}$$

For the light neutrinos we have the low-energy values

$$\Delta m^2_{\rm sol} = m_2^2 - m_1^2 = 8.0 \cdot 10^{-5}\ \text{eV}^2 \tag{7.4}$$

$$\Delta m^2_{\rm atm} = |m_3^2 - m_2^2| = 2.5 \cdot 10^{-3}\ \text{eV}^2 \tag{7.5}$$

with $m_{\nu,\rm lightest} = m_1(m_3)$ for normal (inverted) hierarchy, respectively, while for the lightest neutrino mass one has

$$0 \le m_{\nu,\rm lightest} \le 0.2\,\text{eV}. \tag{7.6}$$

[1] We thank S. Antusch for drawing our attention to this point.

See [60] for a detailed discussion of the neutrino masses and mixing.

The R matrix of the form R_{deg} is parameterized by three real numbers y_{ij} that are the source of high-energy CP violation as introduced in equations (3.19) and (3.20). In our analysis, y_{ij} are all taken in the range $[-1, 1]$ if not otherwise stated. For the heavy neutrino mass scale, we investigate a wide range

$$10^6\,\text{GeV} < M_\nu < 10^{14}\,\text{GeV}. \tag{7.7}$$

Perturbativity bound

In the MLFV framework the magnitudes of the Yukawa couplings Y_ν are very sensitive to the choice of M_ν, m_ν^{lightest} and the angles in the matrix R_{deg}, which is evident from (3.16). To render the framework perturbative, we impose the constraint

$$\frac{y_{\text{max}}^2}{4\pi} < 0.3, \tag{7.8}$$

where y_{max}^2 is the largest eigenvalue of $Y_\nu^\dagger Y_\nu$. By means of (3.22), this translates into a bound on $R^\dagger R = R^2$ and the angles y_{ij} that scales with M_ν^{-1} and hence is most severe for a large scale of lepton number violation. Analogous bounds apply to other dimensionless couplings whose number depends on the precise MLFV model.

7.2 Two quasi-degenerate heavy Majorana Neutrinos

At first we discuss the case of only y_{12} being non-vanishing at the GUT scale while all other $y_{ij} = 0$ which approximately corresponds to a scenario with two right-handed neutrinos being quasi-degenerate and a third right-handed neutrino decoupled ($M_1 \simeq M_2 \ll M_3$) as it was considered in [51]. This approximation holds because if only $y_{12} \neq 0$ the calculation of η_B proceeds in the same way since to a good approximation only N_1 and N_2 contribute to the CP asymmetry. However the wash-out in or approach is enhanced with respect to the explicit case because the third heavy neutrino is also produced due to the inverse decays.

We find our expectation (I) of chapter 6 confirmed. As described in Section 6.4, ignoring flavour subtleties in leptogenesis, the CP violating effects due to renormalization group effects are induced only by the charged lepton Yukawa couplings. In Figure 7.1, where the resulting η_B is shown as a function of M_ν, one can see that there is a mild dependence of the CP asymmetry on the Majorana scale. Figure 7.1 also nicely illustrates the relative importance of flavour effects in leptogenesis. If no cancellations occur, we find, that flavour effects generate an η_B which is of the same order of magnitude (*blue circles*), however almost always larger than the one calculated ignoring flavour effects (*green squares*).

Considering the unphysical limit of setting $Y_E = 0$ in the renormalization group running, we have concluded in Section 6.4 that the total CP asymmetries ε_i and η_B should vanish since no CP violation effects are induced by the RGEs. We confirm this behavior in Figure 7.1 (*red triangles*).

A very different picture emerges once one includes flavour effects. The relevant quantity for leptogenesis is then $\mathrm{Im}((Y_\nu Y_\nu^\dagger)_{ij}(Y_\nu)_{i\alpha}(Y_\nu^\dagger)_{\alpha j})$ with no summation over the lepton flavour index α (see equation (5.23)). Although no total CP asymmetries ε_i are generated via the RG evolution in the limit $Y_E = 0$, the CP asymmetries for a specific lepton flavour are non-vanishing and being weightened separately by the corresponding $K_{i\alpha}$ factors (5.23) this leads to a non-vanishing BAU. Additionally, the resulting η_B now shows a dependence on M_ν which stems from the RGE contributions due to Y_ν only, whereas this dependence is absent in the total CP asymmetries in the limit of two quasi-degenerate heavy Majorana neutrinos (*orange crosses*).

In Figure 7.2 we additionally show the dependence of η_B on y_{12} and $m_{\nu 1}$. We find that flavour effects enlarge the y_{12} range where successful baryogenesis is possible and slightly

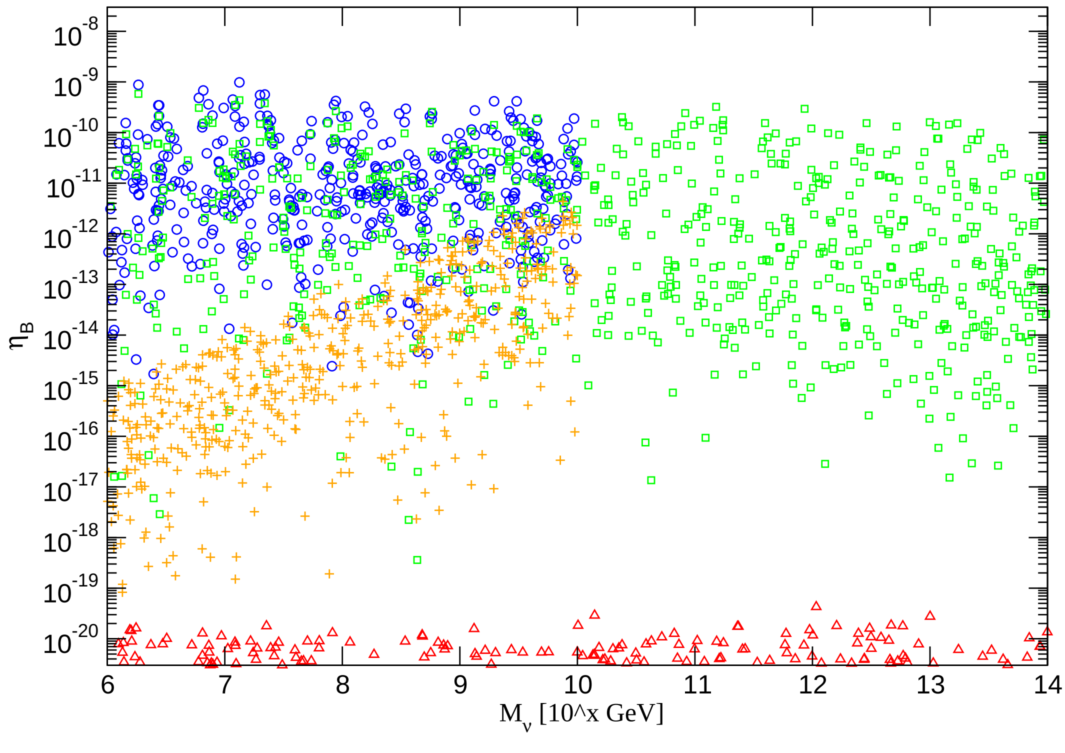


Figure 7.1: Resulting η_B for the case in which only $y_{12} \neq 0$ (approximate limit of two quasi-degenerate heavy Majorana neutrinos) as a function of M_ν for the normal hierarchy of light neutrinos: the *orange crosses* and *red triangles* show the unphysical limit $Y_E = 0$ in the RGEs with and without including lepton flavour effects in the calculation of η_B, respectively. Setting Y_E to their physical values, the *blue circles* and the *green squares* correspond to including and ignoring lepton flavour effects in the calculation, respectively.

soften the upper bound on the light neutrino mass scale. The left panel even demonstrates that leptogenesis in the MLFV scenario is possible for a real R matrix. Then lepton flavour effects are essential [41, 43].

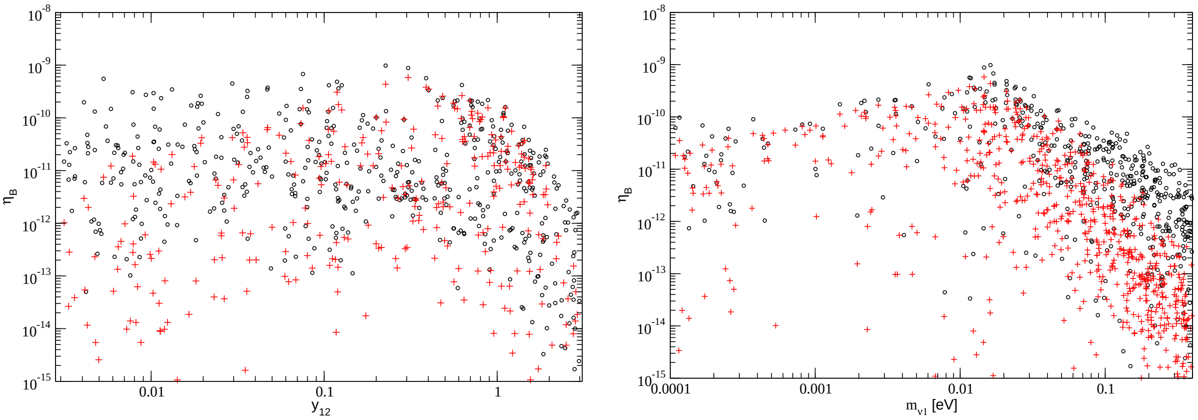


Figure 7.2: η_B for the case in which only $y_{12} \neq 0$ (approximate limit of two quasi-degenerate heavy Majorana neutrinos) as a function of y_{12} (**Left Plot**) and $m_{\nu 1}$ (**Right Plot**) for the normal hierarchy. The *black circles* are obtained including lepton flavour effects and the *red crosses* are calculated ignoring them.

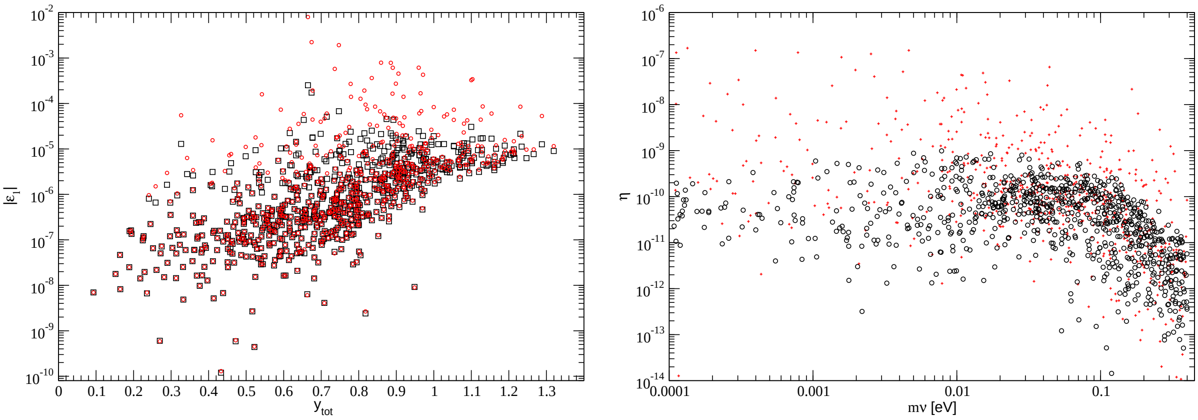


Figure 7.3: Left Plot: The total CP asymmetry $|\epsilon_1|$ for the general case with $0.01 < |y_{ij}| < 0.8$ as a function of $y_{tot} = (y_{12}^2 + y_{13}^2 + y_{23}^2)^{\frac{1}{2}}$ for input values that result in the right oder of magnitude of η_B. The *red circles* are obtained using the uncorrected CP asymmetries and the *black squares* include the corrections by the decay widths. **Right Plot:** η_B for the general case with $0.01 < |y_{ij}| < 1$ as a function of $m_{\nu 1}$. The *black circles* are obtained including lepton flavour effects whereas the *red crosses* are calculated ignoring them. The flavour blind results (*red crosses*) reach higher values due to the CP asymmetries growing as M_ν gets bigger in this regime.

7.3 Three quasi-degenerate heavy Majorana Neutrinos

Now we analyse the general case with all three phases y_{ij} non-vanishing and the parameter space considered as described in Section 7.1.

In Figure 7.4 we present η_B versus the Majorana scale. In accordance with our expectation (II) of chapter 6, for smaller values of M_ν, where the running is dominated by the lepton Yukawas, there is no significant dependence on the Majorana scale. For larger values of the Majorana scale we confirm the linear scaling of η_B with M_ν as claimed in (III). Concerning the impact of flavour effects in this scenario, we find, that the flavour specific treatment (*black circles*) generically enhances the BAU.

It is interesting to analyse the impact of the resonant enhancements of the CP asymmetries in the context of resonant leptogenesis in our framework. These enhancements are described by the regularization factors D_i given in (6.24). From Figure 7.3 one can read that these terms turn out to be important for values of $\epsilon_i \gtrsim 10^{-6}$ which corresponds to the required value of the CP asymmetries when the BAU is obtained with hierarchical neutrinos (case

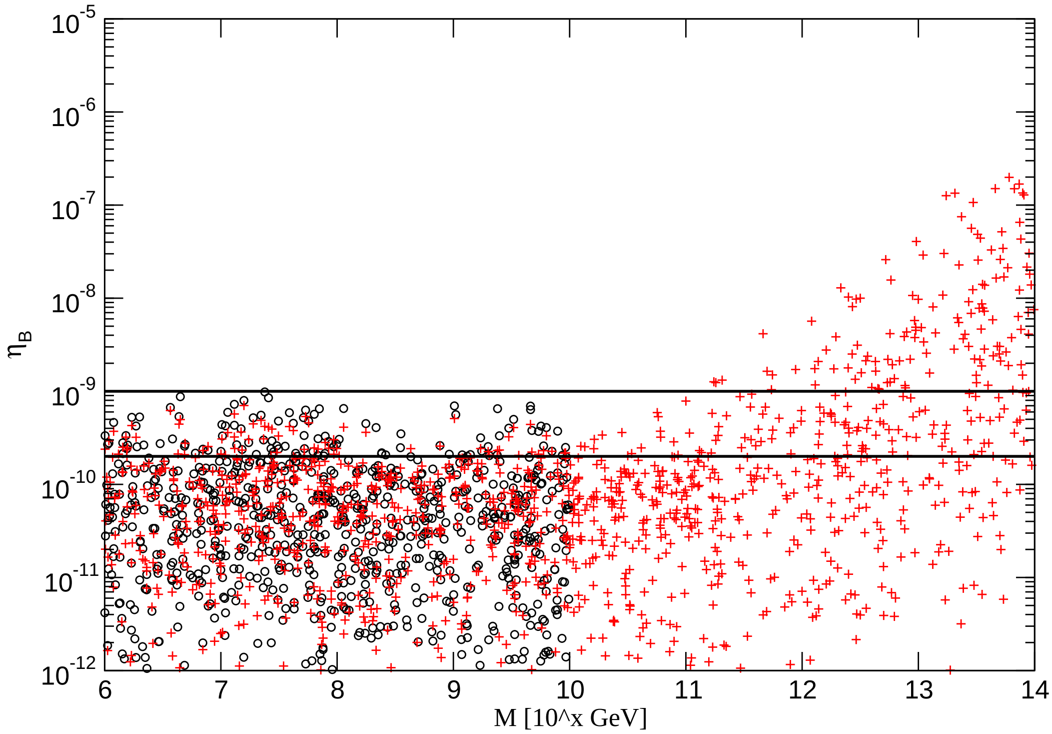


Figure 7.4: η_B for the general case with $0.01 < |y_{ij}| < 1$ as a function of M_ν. The *black circles* are obtained including lepton flavour effects and the *red crosses* are calculated ignoring them.

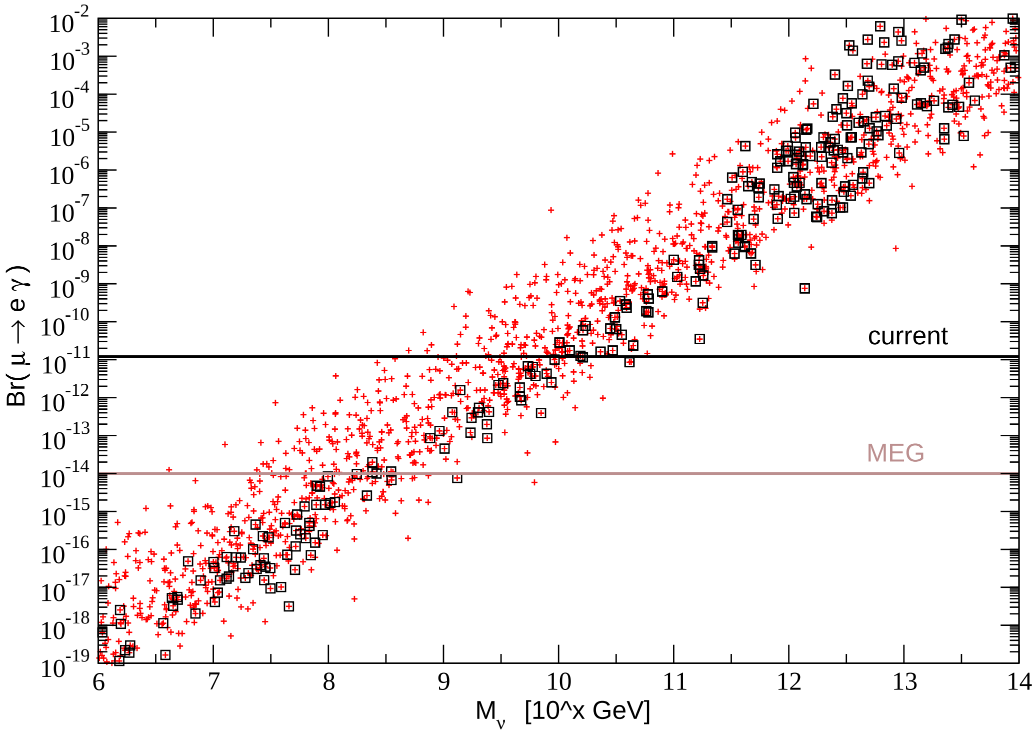


Figure 7.5: $B(\mu \to e\gamma)$ as a function of M_ν for $\Lambda_{\rm LFV} = 1$ TeV. The *black squares* show points where a baryon asymmetry in the range $2 \cdot 10^{-10} < \eta_B < 10 \cdot 10^{-10}$ is possible.

(A)).

In the regime where flavour effects are important we find an upper bound on the lightest neutrino mass of $m_{\nu 1} \lesssim 0.2$ eV from successful leptogenesis (see Figure 7.3). In the flavour insensitive regime at high temperatures, no relevant bound exists. This can be explained with our point (III) of Section 6.4.1, claiming that the enhancement of the CP asymmetry approximately grows linearly for values of $M_\nu \gtrsim 10^{12}$ GeV.

In summary, our analysis demonstrates that in the context of MLFV and the BAU generated by means of RRL, successful leptogenesis is possible independent from the magnitude of the Majorana scale ranging from 10^6 GeV $< M_\nu < 10^{14}$ GeV and that the flavour specific treatment is relevant in the flavour sensitive temperature regime. The absence of a lower bound on M_ν found here has of course an impact on the LFV processes, which we will discuss next.

7.4 LFV Processes

In Figure 7.5 we show $B(\mu \to e\gamma)$ versus the Majorana scale M_ν for the parameter ranges described above and for a lepton flavor violation scale Λ_{LFV} of 1 TeV. We highlighted the points where successful baryogenesis is possible (*black squares*). We find that $B(\mu \to e\gamma)$ can be made small enough to evade bounds from current and future experiments and one can have successful baryogenesis through leptogenesis at the same time. Based on the experience that the running of the PMNS angles is weak in the SM and the MSSM unless $\tan\beta$ (and hence y_τ) is large, we ignore details that depend on the particular model considered and evaluate Δ_{ij} at the scale M_ν.

The ratio $B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$ is shown for the case of the MSSM with $\tan\beta = 2$ in Fig. 7.6 (left). All other parameters are varied in the ranges given above. We see that this ratio varies over about six orders of magnitude and $B(\mu \to e\gamma)$ can be a factor 10^3 larger than $B(\tau \to \mu\gamma)$ in qualitative agreement with [61, 62]. We have checked that the leptogenesis constraint has no significant impact. Even when constraining the Dirac and Majorana phases in the PMNS matrix to zero and allowing only for a single non-vanishing angle y_{12} at the scale Λ_{GUT}, we can still have $B(\mu \to e\gamma) \gg B(\tau \to \mu\gamma)$. This is again in agreement with [61, 62] but contradicts the findings of [52] as we will discuss below.

7.5 Comparison of different Analyses present in the Literature

In an independent analysis Cirigliano, Isidori and Porretti [52] generalized MLFV formulation [12] to include CP violation at low and high energies. Similarly to us they found it convenient

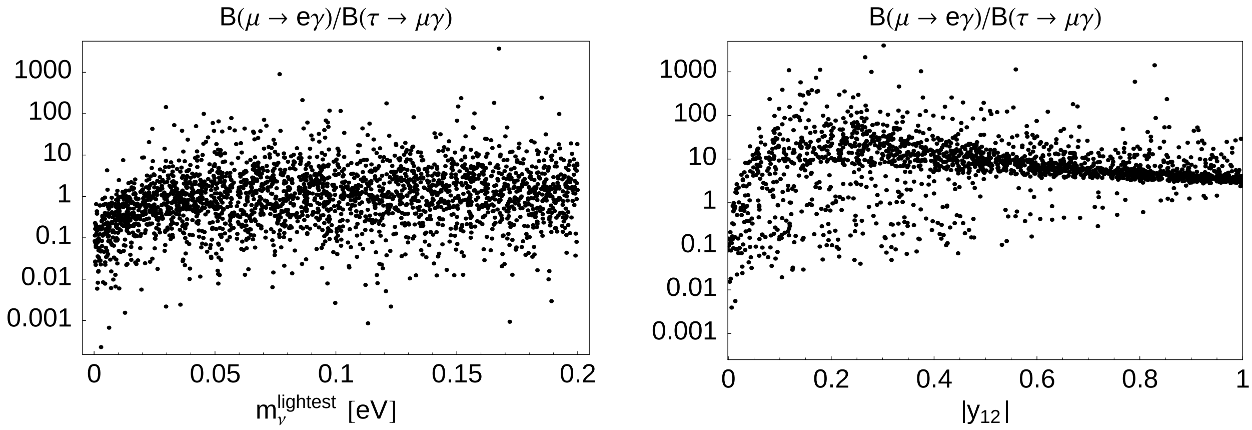


Figure 7.6: Double ratios of $l_i \to l_j\gamma$ for the MSSM with $\tan\beta = 2$. **Left Plot:** All parameters varied. **Right Plot:** no phases and only $y_{12} \neq 0$. For a discussion, see the text.

to use for Y_ν the parametrization of Casas and Ibarra. They have also pointed out that in the MLFV context, the most natural framework to generate the BAU is resonant leptogenesis.

On the other hand, flavour dependent effects were not considered in the evaluation of η_B, that we find in agreement with other authors to be important [32, 40, 41, 43, 44]. This has crucial consequences already at the qualitative level. The qualitative discussion of the splittings of the M_ν^i at the see-saw scale given therein is similar to ours and we agree with the main physical points in this context. On the other hand, while we have demonstrated explicitely by means of a renormalization group analysis that a successful RRL can be achieved, Cirigliano et al. confined their analysis to parametrizing possible radiative effects in terms of a few parameters. In this context a new point made by us is that the coefficients c_i in (6.5) are in fact not independent of each other. Indeed the leading logarithmic contribution to c_i are related by the renormalization group. This can in principle increase the predictivity of MLFV.

The three most interesting messages of [52] are that

- a successful resonant leptogenesis within the MLFV framework implies a lower bound $M_\nu \geq 10^{12}\,\text{GeV}$,
- with $\Lambda_{\text{LFV}} = \mathcal{O}(1\,\text{TeV})$, this lower bound implies the rate for $\mu \to e\gamma$ close to the present exclusion limit,
- MLFV implies a specific pattern of charged LFV rates: $B(\mu \to e\gamma) < B(\tau \to \mu\gamma)$.

For $M_\nu \geq 10^{12}\,\text{GeV}$, in spite of some differences in the numerics as discussed above, we basically confirm these findings. Unfortunately, for lower values of M_ν our results differ from theirs. In particular, as we have demonstrated in Figure 7.7, the observed value of η_B can be obtained for M_ν by several orders of magnitude below the bound in [52], in accordance with other analyses of leptogenesis. Once M_ν is allowed to be far below $10^{12}\,\text{GeV}$, $\Lambda_{\text{LFV}} = \mathcal{O}(1\,\text{TeV})$ does not imply necessarily $B(\mu \to e\gamma)$ close to the exclusion limit as clearly seen in Figure 7.5.

One of the reasons for the discrepancy between our result with regard to M_ν and the one of [52] is the neglect of flavour effects in leptogenesis in the latter paper. Figure 7.7 illustrates that flavour effects in leptogenesis matter.

Concerning $B(\mu \to e\gamma) < B(\tau \to \mu\gamma)$, we confirm the result of [52] in the limit of very small y_{12}, but as shown in Figure 7.6, this is not true in general, as also found in [61, 62]. Consequently, this hierarchy of charged LFV rates cannot be used as model independent signature of MLFV.

In Figure 7.7, we compare different calculations of η_B:

- the flavour independent estimate of [48] used in Cirigliano et al. [52] (*red boxes*),

- the numerical solution of the flavour independent Boltzmann equations using the LeptoGen package (*black circles*),
- the recent estimate by Blanchet and Di Bari [44] that includes flavour effects (*green triangles*),
- the approximate expression of [40] given in (5.23) that also includes flavour effects (*brown crosses*).

We find that the flavour blind estimate of [48] used in Cirigliano et al. [52] lies consistently below the numerical solution of the flavour independent Boltzmann equations. For $M_\nu \geq 10^{12}$ GeV this turns out to be unimportant as flavour effects in this region are small and we confirm the increase of η_B with M_ν in this region found by these authors. Potentially large flavour effects that have been left out in [52] generally enhance the predicted η_B, in particular for $M_\nu \leq 10^{12}$ GeV, in accordance with the existing literature. Both flavour estimates and

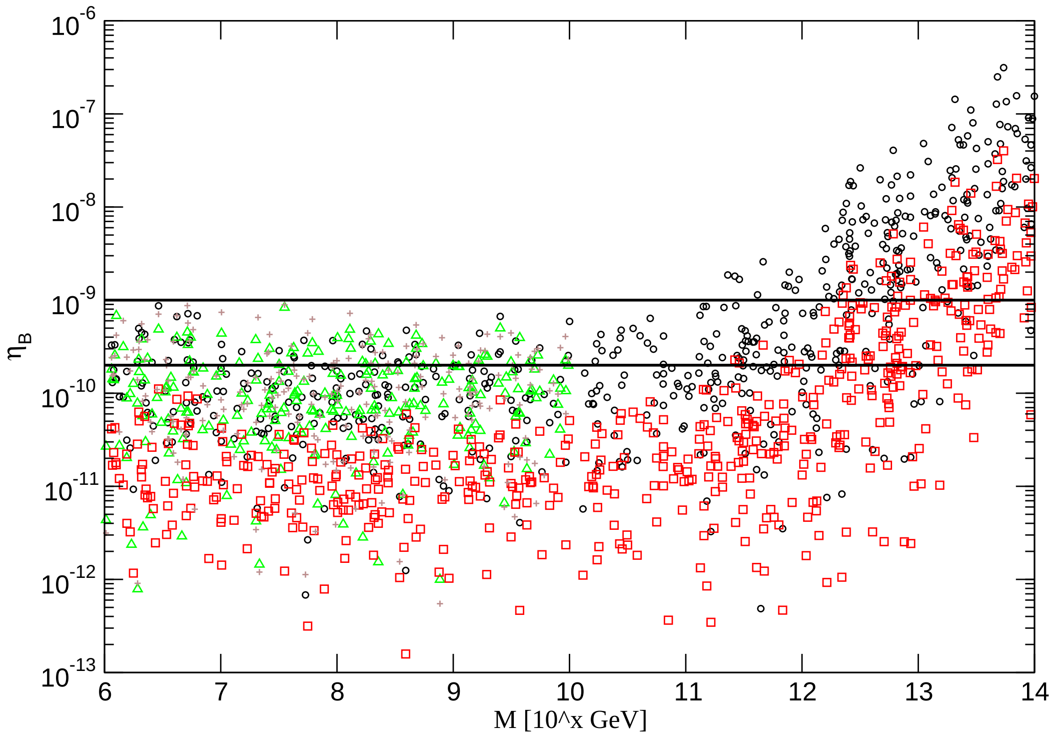


Figure 7.7: Different determinations of η_B for the general case with $0.01 < |y_{ij}| < 1$ as a function of M_ν. The *black circles* are obtained numerically solving the flavour independent Boltzmann equations using the LeptoGen package, the *green triangles* and the *brown crosses* show estimates including flavour effects of [44] and (5.23), respectively. Finally the *red boxes* show the estimate of [48] used in Cirigliano et al. [52] which ignores flavour effects.

the full numerical treatment to solve the flavour independent Boltzmann equations show solutions with η_B of the measured order of magnitude without imposing a stringent lower bound on the value of M_ν.

The last finding is in contrast to the analysis of [52] which using the flavour independent estimate of [48] finds a lower bound on M_ν of $\mathcal{O}(10^{12}\mathrm{GeV})$ as clearly represented by the red boxes in Figure 7.7. The same qualitative conclusion holds for η_B using the RGE induced CP asymmetries in the MSSM. The $\tan\beta$ enhancement of the CP asymmetries as discussed in Section 6.4.3 even facilitates the generation of an η_B of the right size.

Our analysis that includes flavour effects demonstrates that baryogenesis through leptogenesis in the framework of MLFV is a stable mechanism and allows a successful generation of η_B over a wide range of parameters. We find that $B(\mu \to e\gamma)$ can be made small enough to evade bounds from current and future experiments and one can have successful baryogenesis through leptogenesis at the same time in contrast to the findings of [52].

7.6 Final Statements

We have generalized the proposal of minimal flavour violation in the lepton sector of [12] to include CP Violation at low and high energies. While the definition proposed in [12] could be considered to be truly minimal, it appears to us too restrictive and not as general as the one in the quark sector (MFV) in which CP violation at low energy is automatically included [63] and in fact all flavour violating effects proceeding through SM Yukawa couplings are taken into account [7]. The new aspect of MLFV in the presence of right-handed neutrinos, when compared with MFV, is that the driving source of flavour violation, the neutrino Yukawa matrix Y_ν, depends generally also on physics at very high scales. This means also on CP violating sources relevant for the generation of baryon-antibaryon asymmetry with the help of leptogenesis. The first discussion of CP violation at low and high energies has been presented in [52]. Our conclusions for $M_\nu \geq 10^{12}$ GeV agree basically with these authors. However, they differ in an essential manner for lower values of M_ν. The important messages we would like to draw are the following:

- In the context of MLFV the only admissible BAU with the help of leptogenesis is the one through radiative resonant leptogenesis (RRL). Similar observations have been made in [52]. In this context our analysis benefited from the ones in [29, 49, 50, 54]. The numerous analyses of leptogenesis with hierarchical right-handed neutrinos present in the literature are therefore outside the MLFV framework and the differences between the results presented here and the ones found in the literature for $M_1 \ll M_2 \ll M_3$ can be used to distinguish MLFV from these analyses that could be affected by new flavour violating interactions responsible for hierarchical right-handed neutrinos.

- We have demonstrated explicitely within the SM and the MSSM at low $\tan\beta$ that within a general MLFV scenario the right size of η_B can indeed be obtained by means of RRL. The important property of this type of leptogenesis is the very weak sensitivity of η_B to the see-saw scale M_ν so that for scales as low as 10^6 GeV but also as high as 10^{13} GeV, the observed η_B can be found.

- Flavor effects, as addressed by several authors recently in the literature [32,40,41,43,44], play an important role for $M_\nu \lesssim 10^{10}$ GeV as they generally enhance η_B.

- With the flavour-specific treatment, it is possible to generate the BAU in the MLFV context with a real R matrix.

- As charged LFV processes like $\mu \to e\gamma$ are sensitive functions of M_ν, while η_B is not in the RRL scenario considered here, strong correlations between the rates for these processes and η_B, found in new physics scenarios with other types of leptogenesis can be avoided.

- Except for this important message, several of the observations made by us with regard to the dependence of charged LFV processes on the complex phases in the matrix R and the Majorana phases have been already made by other authors in the rich literature on LFV and leptogenesis. But most of these analyses were done in the context of supersymmetry. Here we would like to emphasize that various effects and several patterns identified there are valid also beyond low-energy supersymmetry, even if supersymmetry allows a definite realization of MLFV provided right-handed neutrinos are degenerate in mass at the GUT scale.

- One of the important consequences of the messages above is the realization that the relations between the flavour violating processes in the charged lepton sector, the low energy parameters in the neutrino sector, the LHC physics and the size of η_B are much richer in a general MLFV framework than suggested by [12, 52]. Without a specific MLFV model no general clear cut conclusions about the scale Λ_{LFV} on the basis of a future observation or non-observation of $\mu \to e\gamma$ with the rate $\mathcal{O}(10^{-13})$ can be made in this framework.

- On the other hand we fully agree with the point made in [12] that the observation of $\mu \to e\gamma$ with the rate at the level of 10^{-13}, is much easier to obtain within the MLFV scenario if the scales Λ_{LFV} and M_ν are sufficiently separated from each other. We want only to add that the necessary size of this separation is sensitive to the physics between M_Z and Λ_{GUT}, Majorana phases and CP violation at high energy. In this manner the lepton flavour violating processes, even in the MLFV framework, probe scales well above the scales attainable at LHC, which is not necessarily the case within MFV in the quark sector.

- Finally, but very importantly, MLFV being very sensitive to new physics at high energy scales, does not generally solve possible CP and flavour problems. This should be contrasted with the MFV in the quark sector, where the sensitivity to new physics at scales larger than 1 TeV is suppressed by GIM mechanism.

8 MLFV and Leptogenesis without high-energy CP Violation

For a long time it was commonly believed that for a successful leptogenesis high-energy CP violation is required since the complex phases from the PMNS matrix do not offer a sufficient amount of CP violation. This statement is true in temperature regimes where the lepton flavours involved need not to be distinguished in the Boltzmann equations. However, when flavour effects are relevant, the low-energy CP phases present in the PMNS matrix can be sufficient for a successful leptogenesis [42, 43, 46, 64–66].

The difference arises from the generated CP asymmetries. As already stated, when flavours are treated specifically, the un-summed terms in $\varepsilon_{i\alpha}$ (5.2)

$$\varepsilon_{i\alpha} \sim \sum_j \mathrm{Im}((Y_\nu Y_\nu^\dagger)_{ij}(Y_\nu)_{i\alpha}(Y_\nu^\dagger)_{\alpha j}), \tag{8.1}$$

$(Y_\nu)_{i\alpha}(Y_\nu^\dagger)_{\alpha j}$, can become important. Summing over the flavours as it is performed in the single-flavour treatment, one finds from the parametrization of Y_ν given in (3.16) that the PMNS matrix cancels due to its unitarity (apart from radiative corrections between the GUT and the Majorana scale). This however is not the case in the three-flavour case (5.23), where the CP asymmetry of a single flavour is weighted separately due to the $K_{i\alpha}$ factors and the PMNS phases come into play.

Leptogenesis without high-energy CP violation has been found to be successful in frameworks with hierarchical heavy right-handed Majorana neutrinos [64–66]. But also in the case of resonant leptogenesis [29, 38, 40, 48], the BAU can be accommodated in the presence of exclusively low-energy CP violation which has been shown for a mass spectrum with two [65] and with three [46] quasi-degenerate heavy right-handed Majorana neutrinos. The analysis for three quasi-degenerate heavy right-handed Majorana neutrinos which corresponds to the Minimal Lepton Flavour Violation scenario that has been performed in the un-flavoured regime by [52], has been presented in [46] allowing for CP violation at low and hight energies with the BAU generated by RRL. We have discussed the latter one in the previous chapter.

Now we concentrate on the limit of no high-energy CP violation within the MLFV framework. We show that for a successful leptogenesis clear conditions have to be fulfilled.

Applying these constraints we find strong correlations among low-energy lepton flavour violating (LFV) decays, which are weak in the presence of high-energy CP violation [46] (see Section 7.4). Similar predictivity for ratios of LFV decays has been observed in the single-flavour case in the limit of large Majorana masses [52]. This chapter is based on our publication [67].

8.1 Leptogenesis with a real R Matrix

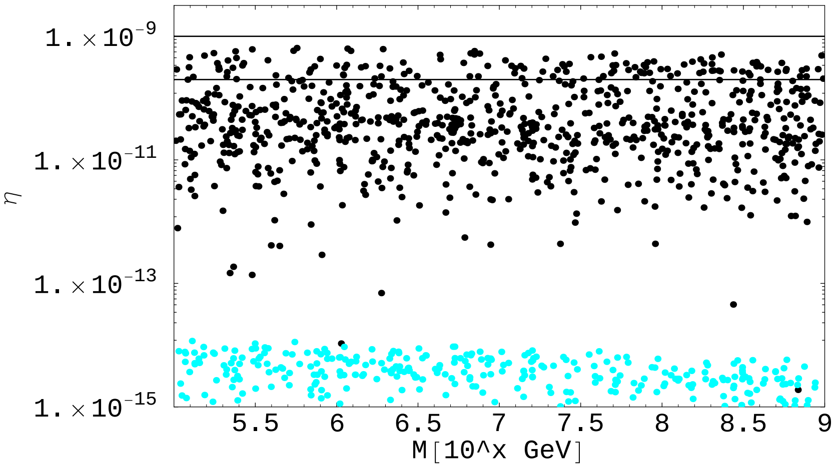


Figure 8.1: The BAU η_B over the Majorana scale up to 10^9 GeV. The *black points* correspond to the three-flavour estimate, the *light-blue points* to the single-flavour solution. The two *black lines* mark where η_B is of the right order of magnitude.

We consider the framework of MLFV with leptogenesis generated by RRL. In order to work out conditions for a successful leptogenesis without high-energy CP violation we analyse the temperature regime below 10^9 GeV, where flavour effects are relevant and the three-flavour approach is applicable. We use the RGE for the SM extended by three heavy Majorana neutrinos given in Section 6.4.

In the limit of vanishing high-energy CP violation, the matrix R of (3.16) has to be real and we set it equal to unity at the GUT scale where the heavy right-handed Majorana neutrinos are exactly degenerate.

$$R(\Lambda_{\mathrm{GUT}}) = \mathbb{1} \tag{8.2}$$

Then the only complex phases that enter the neutrino Yukawa matrix Y_ν and the BAU (apart from radiative corrections) are the ones of the PMNS matrix. It has been found that it is possible to obtain the baryon asymmetry generated by RRL with only low-energy CP violation of the right order of magnitude in the regime where the flavour-dependent estimate (5.23) is valid [46]. This can also be seen in Figure 8.1, showing that the BAU can

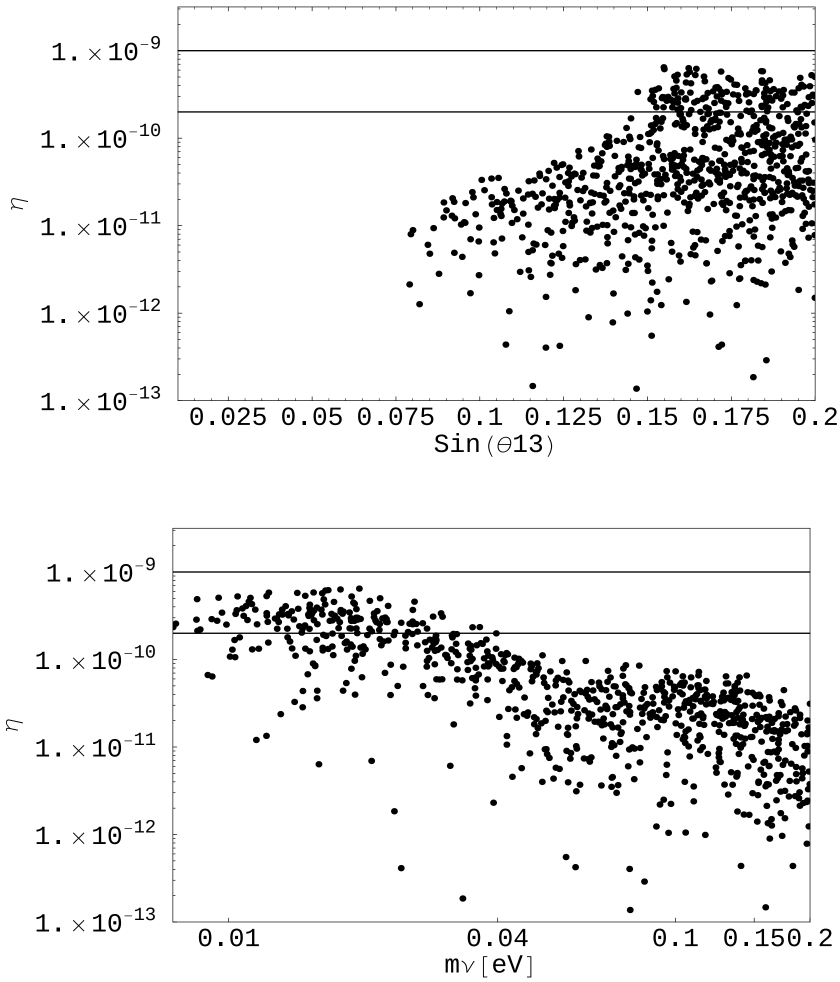


Figure 8.2: The BAU η_B over $\sin(\theta_{13})$ and the lightest neutrino mass m_ν. In our scenario with exclusively low-energy CP violation, successful leptogenesis requires $0.13 \lesssim \sin(\theta_{13})$ and $m_\nu \lesssim 0.04$ eV.

be accommodated properly independent from the Majorana scale with the estimate (5.23), whereas with a single-flavour treatment in this regime, the right size of the BAU cannot be obtained (*light-blue points*).

In the scenario presented here with exclusively low-energy CP violation, we find that successful leptogenesis implies the lower bound on the sinus of the PMNS angle θ_{13}

$$\sin(\theta_{13}) \gtrsim 0.13. \tag{8.3}$$

For the lightest neutrino mass we obtain the upper bound

$$m_\nu \lesssim 0.04 \text{ eV} \tag{8.4}$$

(see Figure 8.2). These findings imply that improvements on the experimental input for these quantities would offer the possibility to test this setup.

If not stated differently, all plots presented here correspond to *normal hierarchy* in the light neutrino masses.

For a scenario with two quasi-degenerate heavy right-handed neutrinos, it has been found [65] that it is possible to generate the BAU of the right order of magnitude for all hierarchies of the light neutrino masses.
In this setup with three quasi-degenerate heavy right-handed neutrinos, we find that for *inverted hierarchy*, it is not possible to generate the BAU of the right order of magnitude from low-energy CP violation alone. This issue is shown in Figure 8.3.

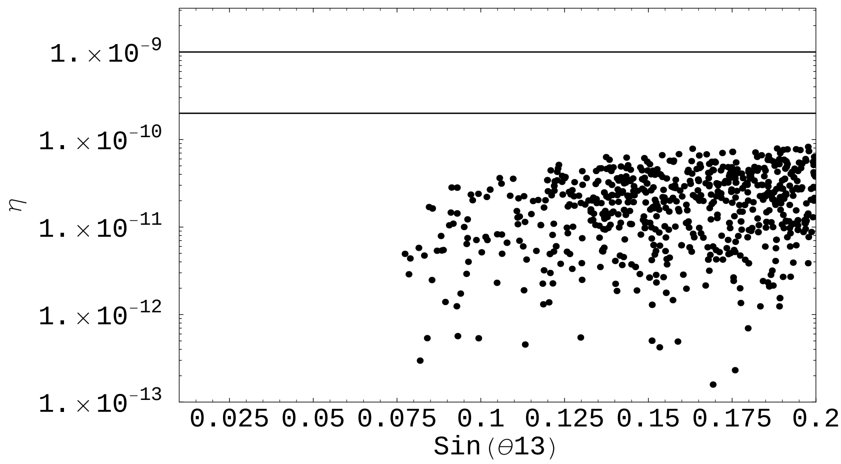


Figure 8.3: The BAU η_B over $\sin(\theta_{13})$ for *inverted hierarchy* of the light neutrino masses with exclusively low-energy CP violation. It is not possible to obtain the BAU of the right size with *inverted hierarchy*.

8.2 CP Violation governed by a single PMNS Phase

Now we would like to investigate whether single CP violating phases could be sufficient to generate the BAU. If the Dirac phase δ is the only complex phase involved (see upper plot of Figure 8.4), we find that it is not possible to fulfill the leptogenesis constraint which can be explained by the suppression of the corresponding PMNS entries due to $\sin(\theta_{13})$. This implies that the observation of CP violating neutrino oscillations alone is not sufficient to ensure successful leptogenesis in this framework.
However it is possible to successfully generate the BAU with a single Majorana phase α or β which is depicted in the lower plot of Figure 8.4 where $\beta \neq 0$, $\alpha = \delta = 0$ and $R(\Lambda_{\text{GUT}}) = \mathbb{1}$. This result is in accordance with the one of [65] where in the resonant case a strong sensitivity on the Majorana phases has been observed.
Therefore one can say that experimental observation or non-observation of Majorana phases will decide whether the setup presented could be realized in nature.

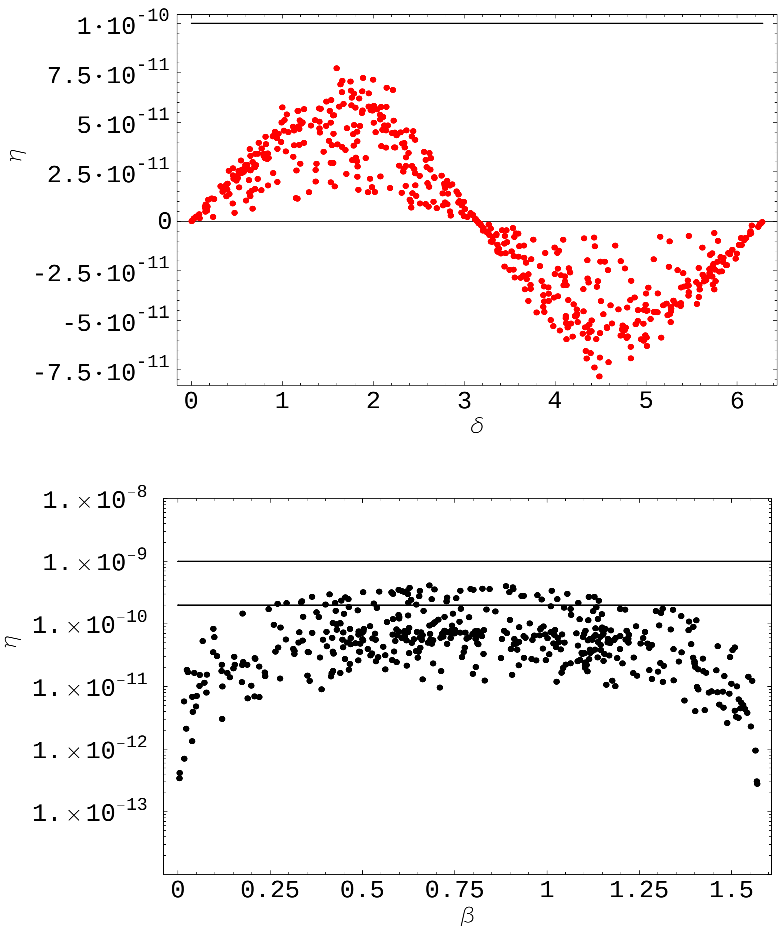


Figure 8.4: Upper Plot: The baryon asymmetry η_B (left) plotted over the Dirac phase δ for δ being the only source of CP violation which is not sufficient to obtain the right order of magnitude of the BAU indicated by the *black line* in the upper plot. The range $\pi \leq \delta \leq 2\pi$ corresponds to negative values of η_B. **Lower Plot:** The baryon asymmetry η_B with the Majorana phase β being the only source of CP violation.

8.3 LFV Processes

From equation (3.13) one can read that $B(\mu \to e\gamma)$ rises with increasing Majorana scale for a fixed scale of lepton number violation. In a scenario with low-energy CP violation the Majorana scale is bounded by up to which scale the flavour-dependent analysis is valid. An analysis that includes the two-flavoured regime 10^9 GeV $< M_\nu < 10^{12}$ GeV has to decide whether successful leptogenesis without high-energy CP violation for scales above 10^9 GeV is possible. If this is not the case and the Majorana scale is bounded to be below 10^9 GeV as in the scenario considered, Λ_{LFV} could be as low as 300-500 GeV to obtain $B(\mu \to e\gamma)$ in the reach of the PSI experiment (see Figure 8.5).

The very important point of this analysis is that with exclusively low-energy CP violation the relation $B(\mu \to e\gamma) < B(\tau \to \mu\gamma)$ is valid which has been found to be not true in the

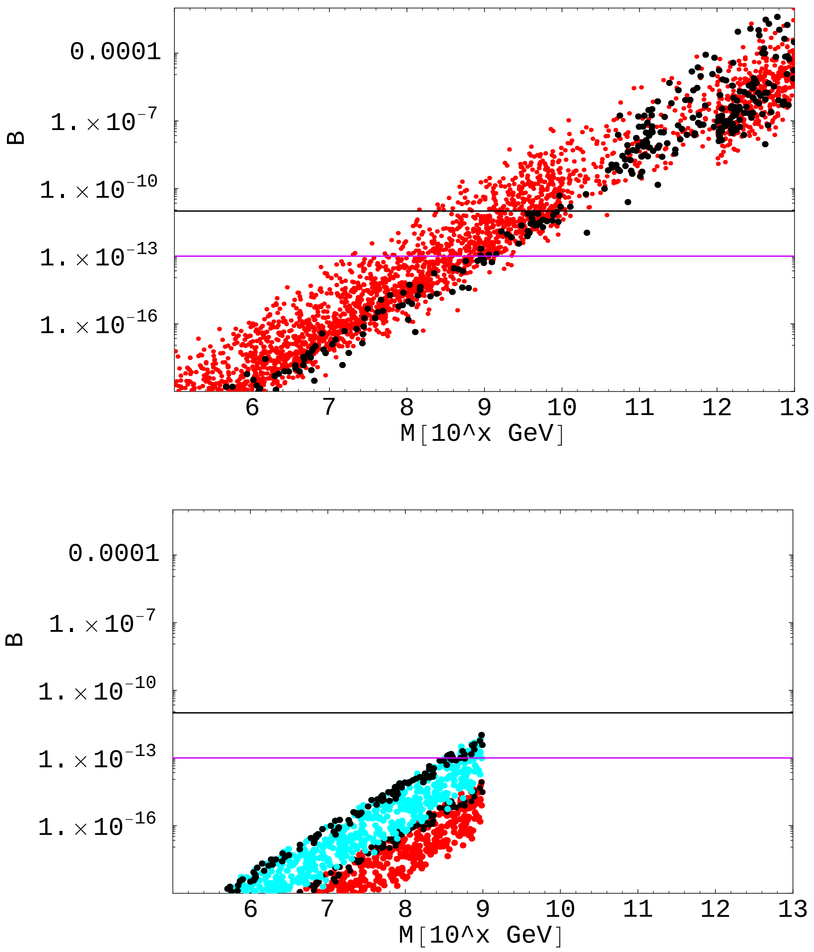
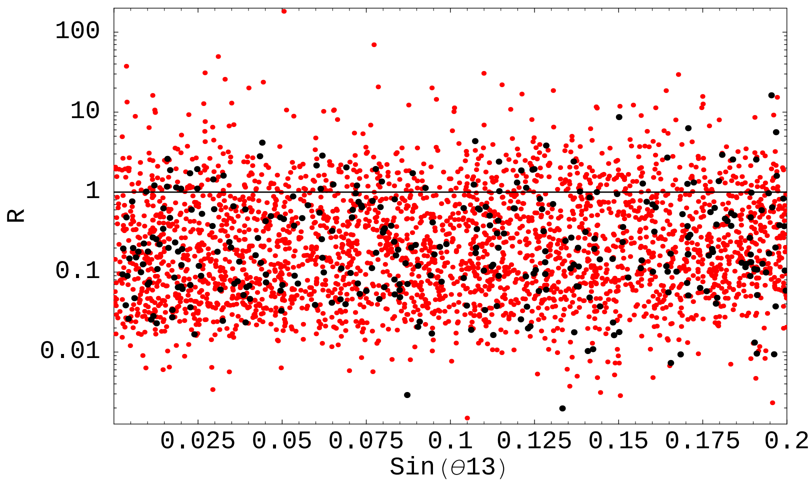


Figure 8.5: $B(\mu \to e\gamma)$ over M_ν for the general analysis [46] including high-energy CP violation and $\Lambda_{\rm LFV} = 1$ TeV (**Upper Plot**) and without high-energy CP violation (**Lower Plot**) where $\Lambda_{\rm LFV} = 1$ TeV (*red*) and $\Lambda_{\rm LFV} = 300$ GeV (light-blue). The *black points* indicate where the leptogenesis constraint is fulfilled. The *black line* corresponds to the present bound on $B(\mu \to e\gamma) < 10^{-11}$ and the lower *pink line* to the sensitivity of the upcoming PSI experiment $\sim 10^{-13}$.

case of CP violation at low and high energies [46] (see Figure 8.6). Furthermore we find $B(\tau \to e\gamma) < B(\tau \to \mu\gamma)$. Figure 8.6 nicely depicts how dramatic the situation changes when high-energy CP violation is excluded. The rich amount of possibilities for the ratio $R_{\rm LFV} = B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$ that is present in the high-energy CP violation case even if it is independent from the ratio of the scales $M_\nu^2/\Lambda_{\rm LFV}^4$, can then significantly be constrained. We find MLFV with low-energy CP violation predicts a strong correlation among these LFV decays that can be tested in the future.



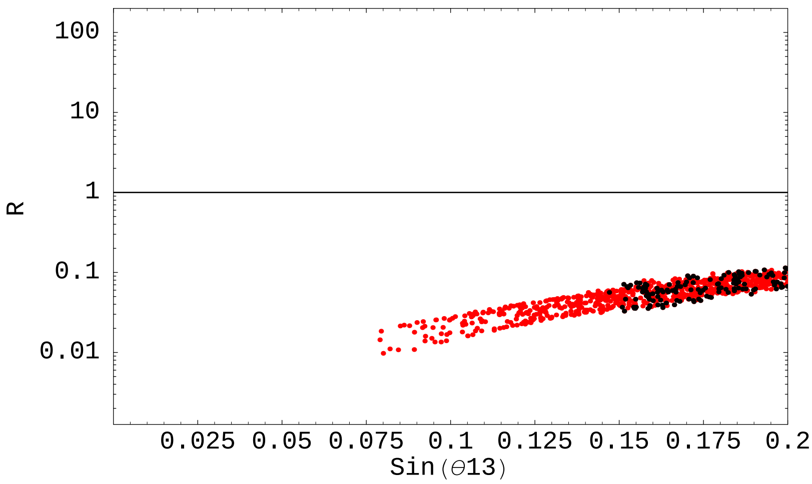


Figure 8.6: $R_{\rm LFV} = B(\mu \to e\gamma)/B(\tau \to \mu\gamma)$ over $\sin(\theta_{13})$ for the general analysis [46] including high-energy CP violation (**Upper Plot**) and without high-energy CP violation (**Lower Plot**) where the relation $B(\mu \to e\gamma) < B(\tau \to \mu\gamma)$ is satisfied. The *black points* fulfill the leptogenesis constraint.

8.4 Summary

Recent studies showed the relevance of the inclusion of flavour effects in the Boltzmann equations for the generation of the baryon asymmetry of the universe and that in the flavoured regime the existence of CP violation at high energies is no longer a necessary requirement for successful leptogenesis.

When leptogenesis is implemented in the framework of Minimal Lepton Flavour Violation by radiative resonant leptogenesis including high-energy CP violation [46], a rich spectrum of possibilities for charged LFV processes and ratios of such processes is present matching with the leptogenesis constraint. The correlations among leptogenesis and LFV were found to be very weak.

We have analysed the framework of Minimal Lepton Flavour Violation in the limit of no

CP violation at high energies, with the PMNS phases being the only complex ingredient apart from radiative corrections. We find that with radiative resonant leptogenesis with RGE of the SM, one can successfully generate the BAU provided that:

- there is a non-vanishing Majorana phase,
- the light neutrino masses have normal hierarchy,
- the lightest neutrino mass and the PMNS angle θ_{13} fulfill $m_\nu \lesssim 0.04$ eV and $0.13 \lesssim \sin(\theta_{13})$.

When these constraints are fulfilled, we find strong correlations among LFV processes and that the rich spectrum of possibilities that is present when high-energy CP violation is included [46] can significantly be reduced. In this case MLFV turns out to be much more predictive.

9 Hierarchical Fermion Wave Functions: Going beyond MFV

9.1 Introduction

Within the renormalizable part of a general effective Lagrangian there are two types of operators involving fermion fields: kinetic and Yukawa terms. In general both these terms can have a non-trivial flavour structure. Since the description of physics is invariant under field reparametrizations, it is only the relative flavour structure between kinetic and Yukawa terms that has a physical meaning.

Employing the canonical normalization of the kinetic terms, the physical flavour structure of the SM (or the renormalizable part of the effective theory) is manifest in the Yukawa matrices. As it is well known, these have a quite peculiar form: their eigenvalues are very hierarchical and the two matrices in the quark sector are almost diagonal in flavour space, with the off-diagonal entries parameterized by the CKM matrix. This structure is responsible for the great successes of the SM in the flavour sector, including the strong suppression of CP violating and flavour changing neutral current amplitudes.

Several additional flavour structures could appear in connection with higher dimensional operators in the non-renormalizable part of the effective Lagrangian. However, if we assume an effective scale of new physics in the TeV range (as expected by a natural stabilization of the electro-weak symmetry-breaking sector), present experimental results leave a very limited room for new flavour structures. The MFV hypothesis is a simple and effective solution to the flavour problem [1, 6, 7, 63, 68], but is far from being the only allowed possibility. Various alternatives or variations of this hypothesis have indeed been proposed in the recent literature [14, 69–72]. The question we would like to address is the viability, in general terms, of solutions to the flavour problem based not on a flavour symmetry in the low-energy effective theory, but instead on hierarchies in the kinetic terms which suppress flavour changing transitions. This could be a "democratic" alternative, where the many coupling constant matrices can be of any form, and the only restriction is that the kinetic terms should have a hierarchical normalization. The wave function normalization factors then function as a distorting lense, through which all interactions are seen as approximately aligned on the magnification axes of the lense.

From a dynamical point of view, hierarchical fermion wave functions (and corresponding hierarchical kinetic terms) naturally emerge in scenarios with extra dimensions, where the hierarchy in the four-dimensional wave functions reflect their non-trivial profile in the extra dimension [73]. Note that this is not an universal feature of models with extra dimensions. For instance, as recently showed in [74], in the context of the Randall-Sundrum scenario [75] it is possible to construct a concrete realization of the MFV hypothesis. Flavour changing processes in such models have been studied in [76–79].

It is always possible to redefine fields so as to choose a basis in which the Yukawa interactions are not hierarchical, and the SM flavour structure manifests itself through the hierarchy of the kinetic terms. As long as we look only at the renormalizable part of the low-energy effective Lagrangian, there is no way to isolate the origin of the flavour hierarchy (kinetic vs. Yukawa terms). However, the different assumptions about its origin lead to different ansätze for the flavour structure of the higher dimensional operators.

We will analyse the class of scenarios where dimensionless coupling matrices (both Yukawas and non-renormalizable operators) have $\mathcal{O}(1)$ entries. That is, they do not exhibit a specific flavour structure in a basis where the kinetic terms are hierarchical. More explicitly, we investigate whether it is possible to choose a hierarchy for the fermion wave functions such that, after moving to the canonical basis, the contributions from dimension-six FCNC operators are sufficiently suppressed. We deem the scenario to work if the dimensional suppression scale of the operators turns out to be in the TeV range (as in the MFV scenario). We analyse this problem both in the quark and in the lepton sector, discuss the corresponding bounds on the effective operators from quark and lepton FCNCs and provide a comparison with the MFV scenario.

This chapter is based on our publication [80].

9.2 Basic Setup for the Quark Sector

The operators involving quark fields in the renormalizable part of the effective Lagrangian are

$$\mathcal{L}^{d=4}_{\text{quarks}} = \overline{Q_L} X_Q \not{D} Q_L + \overline{D_R} X_D \not{D} D_R + \overline{U_R} X_U \not{D} U_R + \overline{Q_L} Y_D D_R H + \overline{Q_L} Y_U U_R H_c \ , \tag{9.1}$$

where $H_c = i\tau_2 H^*$. In a generic non-canonical basis, the three $X_{Q,D,U}$ and the two $Y_{D,U}$ are 3×3 complex matrices. The usual choice of field normalization and basis is obtained by diagonalising the kinetic terms, rescaling the fields to obtain the canonical normalization of the kinetic terms, and finally diagonalising the down-quark masses. With this choice

$$X_Q = X_D = X_U = I \ , \qquad Y_D = \lambda_D \ , \qquad Y_U = V^\dagger \lambda_U \ , \tag{9.2}$$

where I is the identity matrix, V is the CKM matrix and

$$\lambda_D = \frac{1}{v}\text{diag}(m_d, m_s, m_b) , \qquad \lambda_U = \frac{1}{v}\text{diag}(m_u, m_c, m_t) , \tag{9.3}$$

with $v = (\langle H^\dagger H \rangle)^{1/2} = 174$ GeV.

For the purposes of this analysis we express the $d = 4$ Lagrangian with a different, hierarchical, field normalization for the kinetic terms, and then add the higher dimensional operators with "democratic" flavour couplings. An example of the bases where the Yukawas are no longer hierarchical can be reached starting from the basis (9.2), by performing a unitary transformation on U_R and by an appropriate rescaling of the fields. In particular, denoting by Z_A ($A = Q, U, D$) the diagonal matrices by which we rescale the fermion fields ($Q_A \to Z_A^{-1} Q_A$) and by W_A the complex matrices describing their unitary transformations in the canonical basis ($W_U^\dagger W_U = I$), we can move to a non-canonical basis where[1]

$$Y_D \to Z_Q^{-1} \lambda_D Z_D^{-1} = I , \qquad (I)_{ij} = \delta_{ij} , \tag{9.4}$$

$$Y_U \to Z_Q^{-1} V^\dagger \lambda_U W_U Z_U^{-1} = T_U \qquad (T_U)_{ij} = \mathcal{O}(1) . \tag{9.5}$$

In this new basis the hierarchical structure usually attributed to the Yukawa couplings is hidden in the flavour structure of the (diagonal) kinetic terms:

$$X_Q = Z_Q^{-2} , \qquad X_D = Z_D^{-2} , \qquad X_U = Z_U^{-2} . \tag{9.6}$$

The conditions we have imposed on the Z_A in equations (9.4) and (9.5) do not specify completely their structure. However, assuming the maximal entries in the Z_A are at most of $\mathcal{O}(1)$ implies a hierarchical structure of the type

$$Z_A = \text{diag}(z_A^{(1)}, z_A^{(2)}, z_A^{(3)}) , \qquad z_A^{(1)} \ll z_A^{(2)} \ll z_A^{(3)} \lesssim 1 . \tag{9.7}$$

The framework we want to investigate is a scenario where the hierarchical structure in equation (9.7) is the only responsible for the natural suppression of FCNCs. More explicitly, we assume that with hierarchical normalization of the kinetic terms, equation (9.6), all the higher dimensional operators of the effective Lagrangian have a generic $\mathcal{O}(1)$ structure, such as the up-type Yukawa coupling in equation (9.5). In this framework the suppression of FCNCs arises by the rescaling the fermion fields necessary for the canonical normalization of the kinetic terms:

$$Q_A \to Z_A Q_A . \tag{9.8}$$

[1] With different unitary transformation we could have chosen $Y_U \to I$ and $Y_D \to \mathcal{O}(1)$, or $Y_{U,D} \to \mathcal{O}(1)$. As it will become clear in the next section, the choice in equations (9.4) to (9.5), is the simplest one for the phenomenological analysis of FCNC constraints.

Starting from non-hierarchical bilinear structures,

$$\bar{A} X_{AB} B \ , \qquad X^{ij}_{AB} = \mathcal{O}(1) \ , \tag{9.9}$$

the transformation into the canonical basis moves the Z_A into the effective couplings, with a corresponding suppression of the flavour changing terms:

$$X^{ij}_{AB} \to (Z_A X_{AB} Z_B)^{ij} \ \sim \ z^{(i)}_A \ z^{(j)}_B \ . \tag{9.10}$$

In the next sections we analyse which conditions the Z_A should satisfy in order to provide a suppression of FCNCs compatible with experimental data, while keeping the effective scale of new physics in the TeV range. As we will show, in the quark sector these conditions are compatible with equations (9.4)–(9.5), namely with a natural generation of the observed Yukawa couplings starting from generic $\mathcal{O}(1)$ structures. The only exceptions are the kaon constraints from ϵ_K and ϵ'/ϵ, which however can be fulfilled with a modest fine tuning ($\sim 10^{-1}$) in the coupling of the effective operators.

9.3 Bounds from Quark FCNCs

9.3.1 $\Delta F = 2$ Operators

As a first step we would like to constrain the parameters introduced in (9.7) with the help of processes involving $\Delta F = 2$ operators.

There are in principle eight dimension-six four-quark operators that can contribute to down-type $\Delta F = 2$ processes [81]. However, restricting the attention to operators which preserve the $SU(2)_L \otimes U(1)_Y$ gauge symmetry, this number reduces to four:

$$\begin{aligned} O^{ij}_{LL} &= \tfrac{1}{2}(\overline{Q_L}^i \gamma_\mu Q^j_L)^2 \ , \qquad & O^{ij}_{LR1} &= (\overline{Q_L}^i \gamma_\mu Q^j_L)(\overline{D_R}^i \gamma_\mu D^j_R) \ , \\ O^{ij}_{RR} &= \tfrac{1}{2}(\overline{D_R}^i \gamma_\mu D^j_R)^2 \ , \qquad & O^{ij}_{LR2} &= (\overline{D_R}^i Q^j_L)(\overline{Q_L}^i D^j_R) \ . \end{aligned} \tag{9.11}$$

In order to derive model independent bounds on the coupling of these operators, we introduce the following effective Hamiltonian (defined at the at the electro-weak scale with canonically-normalized fields):

$$\begin{aligned} \mathcal{H}_{\rm eff} &= \left(C^{ij}_{\rm SM} + \frac{X^{ij}_{LL}}{\Lambda^2} \right) O^{ij}_{LL} + \frac{X^{ij}_{RR}}{\Lambda^2} O^{ij}_{RR} + \frac{X^{ij}_{LR1}}{\Lambda^2} O^{ij}_{LR1} + \frac{X^{ij}_{LR2}}{\Lambda^2} O^{ij}_{LR2} + \text{ h.c.} \qquad (9.12) \\ &= C^{ij}_{LL}(M_W) O^{ij}_{LL} + C^{ij}_{RR}(M_W) O^{ij}_{RR} + C^{ij}_{LR1}(M_W) O^{ij}_{LR1} + C_{LR2^{ij}}(M_W) O^{ij}_{LR2} + \text{ h.c.} \ , \end{aligned}$$

where

$$C^{ij}_{\rm SM} = \frac{G_F^2}{2\pi^2} M_W^2 (V^*_{ti} V_{tj})^2 S_0(x_t) \ . \tag{9.13}$$

The amplitudes of the various $\Delta F = 2$ processes can be calculated renormalizing the effective Hamiltonian (9.13) at the relevant low scale μ (e.g. $\mu \approx m_b$ for $\bar{B}^0 - B^0$ mixing). The RGE running of the Wilson coefficients can be written as

$$\begin{pmatrix} C^{ij}_{LL}(\mu) \\ C^{ij}_{RR}(\mu) \\ \vec{C}^{ij}_{LR}(\mu) \end{pmatrix} = \begin{pmatrix} \eta^{ij}_{LL}(\mu, M_W) & & \\ & \eta^{ij}_{RR}(\mu, M_W) & \\ & & \hat{\eta}^{ij}_{LR}(\mu, M_W) \end{pmatrix} \begin{pmatrix} C^{ij}_{LL}(M_W) \\ C^{ij}_{RR}(M_W) \\ \vec{C}^{ij}_{LR}(M_W) \end{pmatrix} . \tag{9.14}$$

Since QCD preserves chirality, there is no mixing between the LL, RR and LR sectors and $\eta^{ij}_{LL} = \eta^{ij}_{RR}$. The only non-trivial mixing occurs among the two LR operators, where $\hat{\eta}^{ij}_{LR}(\mu, M_W)$ is a 2×2 matrix and

$$\vec{C}^{ij}_{LR} = \begin{pmatrix} C^{ij}_{LR1}(\mu) \\ C^{ij}_{LR2}(\mu) \end{pmatrix} = \hat{\eta}^{ij}_{LR}(\mu, M_W) \begin{pmatrix} C^{ij}_{LR1}(M_W) \\ C^{ij}_{LR2}(M_W) \end{pmatrix} . \tag{9.15}$$

The analytic formulae for these RGE factors as well as the relevant hadronic matrix elements can be found in [81]. Following the approach given therein, we can express the $\Delta F = 2$ amplitude for a generic neutral meson mixing as

$$\begin{aligned} \mathcal{A}^{ij} &= \langle \overline{M}^0_{ij} | \mathcal{H}_{\text{eff}}(\mu) | M^0_{ij} \rangle \\ &= \frac{2}{3} F_M^2 m_M^2 \left(P_{LL}(C_{LL} + C_{RR}) + P_{LR1} C_{LR1} + P_{LR2} C_{LR2} \right) , \end{aligned} \tag{9.16}$$

where F_M and m_M denote the decay constant and the mass of the meson, respectively. The P_A factors are appropriate combinations of RGE coefficients and hadronic matrix elements. Experimentally we have several precise constraints on this type of amplitudes: both $|\mathcal{A}^{31}|$ and $\arg(\mathcal{A}^{31})$ are constrained by ΔM_{B_d} and $S_{\psi K_S}$; $|\mathcal{A}^{32}|$ is constrained by ΔM_{B_s}; $\text{Im}(\mathcal{A}^{12})$ is constrained by ϵ_K. In the following we will impose that the non-standard contribution is within $\pm 10\%$ of the SM contribution, in magnitude, for all down-type $\Delta F = 2$ mixing amplitudes.

Let us first analyse the LL sector. Here the situation is quite simple since we can factorize the new physics contribution as a correction to the SM Wilson coefficient:

$$\mathcal{A}^{ij}\big|_{LL} = \mathcal{A}^{ij}_{\text{SM}} \left(1 + \frac{X^{ij}_{LL}}{(V^*_{ti} V_{tj})^2} \frac{F^2}{\Lambda^2} \right) , \qquad F = \left(\frac{2\pi^2}{G_F^2 M_W^2 S_0(x_t)} \right)^{\frac{1}{2}} \approx 3 \text{ TeV} . \tag{9.17}$$

Since the effective scale of the SM contribution is 3 TeV, if new physics contributes via loops and is weakly interacting (as the electro-weak SM contribution), taking $\Lambda \sim 10$ TeV corresponds to new masses of the order of $3 M_Z$. Allowing for the amplitude (9.17) to vary from its SM values by at most $\pm 10\%$, in magnitude, lead to

$$\sqrt{|X^{ij}_{LL}|} < 0.3 \, |V^*_{ti} V_{tj}| \left(\frac{\Lambda}{F} \right) \quad < \; |V_{ti}||V_{tj}| \text{ for } \Lambda < 10 \text{ TeV} . \tag{9.18}$$

Expressing the flavour structure of the X_{LL}^{ij} in terms of the corresponding hierarchical fermion wave functions, as in equations (9.8)–(9.10), we find

$$\sqrt{|X_{LL}^{ij}|} \sim |z_Q^{(i)} z_Q^{(j)}| < |V_{ti}||V_{tj}| \quad \rightarrow \quad |z_Q^{(i)}| < |V_{ti}| \, . \tag{9.19}$$

Since $\eta_{LL} = \eta_{RR}$ and $\langle O_{LL}^{ij} \rangle = \langle O_{RR}^{ij} \rangle$, the constraint on the RR operator is completely analog to the LL one:

$$\sqrt{|X_{RR}^{ij}|} \sim |z_D^{(i)} z_D^{(j)}| < |V_{ti}||V_{tj}| \quad \rightarrow \quad |z_D^{(i)}| < |V_{ti}| \, . \tag{9.20}$$

In the LR sector the situation is slightly more complicated because of the different matrix elements involved. The P_A factors introduced in equation (9.16) can be decomposed as [81]:

$$P_{LL} = \eta_{LL}(\mu, M_W) B_{LL}(\mu) \, , \tag{9.21}$$

$$P_{LR1} = -\hat{\eta}_{LR}(\mu, M_W)_{11} \left[B_{LR1}(\mu)\right]_{\text{eff}} + \frac{3}{2}\hat{\eta}_{LR}(\mu, M_W)_{21} \left[B_{LR2}(\mu)\right]_{\text{eff}} \, , \tag{9.22}$$

$$P_{LR2} = -\hat{\eta}_{LR}(\mu, M_W)_{12} \left[B_{LR1}(\mu)\right]_{\text{eff}} + \frac{3}{2}\hat{\eta}_{LR}(\mu, M_W)_{22} \left[B_{LR2}(\mu)\right]_{\text{eff}} \, . \tag{9.23}$$

In the specific case of $\bar{B}^0 - B^0$ we can further write

$$\eta_{LL}(\mu, M_W) B_{LL}(\mu) = \eta_B \hat{B}_{B_q} \, , \tag{9.24}$$

$$\left[B_{LRi}(\mu)\right]_{\text{eff}} = \left(\frac{m_{B_q}}{m_b(\mu) + m_q(\mu)} \right)^2 B_{LRi}(\mu) \, , \tag{9.25}$$

where $\hat{B}_{B_q}$ is the RGE invariant bag factor of the SM (LL) operator. Using the RGE factors in [81–83] and the $B_{LRi}(\mu)$ factors from lattice [68, 84] leads to

$$P_{LL} = 0.7 \, , \qquad P_{LR1} = -5.0 \, , \qquad P_{LR2} = 6.3 \, , \tag{9.26}$$

with negligible differences between B_s and B_d cases. The contributions of the $O_{LR1(2)}$ operator is thus enhanced by a factor $|P_{LR1(2)}/P_{LL}| \approx 7(9)$ compared to the SM one. Allowing at most $\pm 10\%$ corrections to the SM amplitude, the bounds derived from B_s and B_d mixing are

$$|X_{LR1}^{3j}| \sim |z_Q^{(3)} z_Q^{(j)} z_D^{(3)} z_D^{(j)}| < 0.2 \, |V_{tj}|^2 \, , \tag{9.27}$$

$$|X_{LR2}^{3j}| \sim |z_D^{(3)} z_Q^{(j)} z_Q^{(3)} z_D^{(j)}| < 0.1 \, |V_{tj}|^2 \, , \tag{9.28}$$

where we have set $V_{tb} = 1$. These inequalities are satisfied if we assume the hierarchy

$$z_Q^{(i)} z_D^{(i)} = (\lambda_D)_{ii} \, , \tag{9.29}$$

which follows from the definition of $z_Q^{(i)}$ and $z_D^{(i)}$ in equation (9.4).

Matrix elements and RGE factors lead to $P_{LR1(2)}$ substantially larger in the case of $\bar{K}^0 - K^0$ mixing [81]:

$$P_{LL} = 0.5, \qquad P_{LR1} = -0.7 \times 10^2 \ , \qquad P_{LR2} = 1.1 \times 10^2 \ . \tag{9.30}$$

Proceeding as in the $\bar{B}^0 - B^0$ case, this implies the stringent bound

$$|X_{LR2}^{21}| \sim |z_Q^{(2)} z_Q^{(1)} z_D^{(2)} z_D^{(1)}| < \ 0.004 \ |V_{ts}^* V_{td}|^2 \approx 0.6 \times 10^{-9} \ , \tag{9.31}$$

and similarly for $|X_{LR1}^{21}|$. In such case, using equation (9.29) the bound is not fulfilled by about one order of magnitude:

$$|z_Q^{(2)} z_Q^{(1)} z_D^{(2)} z_D^{(1)}| \sim \frac{m_d m_s}{v^2} \approx 1 \times 10^{-8} \ . \tag{9.32}$$

Note that in the case of $\bar{K}^0 - K^0$ mixing we do not have a stringent experimental constraint on the modulo of the amplitude (given the large long-distance contributions to ΔM_K) but only on its imaginary part (thanks to ϵ_K): thus the order of magnitude disagreement concerns only the CP violating part of the $\Delta S = 2$ amplitude.

9.3.2 $\Delta F = 1$ Operators

Taking into account the analysis of the $\Delta F = 2$ sector, it is clear that $\Delta F = 1$ operators with a LL (or RR) structure, such as $\overline{Q_L}^i \gamma^\mu {Q_L}^j H^\dagger D_\mu H$ do not represent a serious problem. The corresponding constraints are equivalent to those derived from LL and RR $\Delta F = 2$ operators, which are naturally fulfilled. On the other hand, a potentially interesting class of new constraints in the $\Delta F = 1$ sector arises by magnetic and chromo-magnetic operators:

$$O_{RL\gamma}^{ij} = e H^\dagger \overline{D_R}^i \sigma^{\mu\nu} Q_L^j F_{\mu\nu} \ , \qquad O_{RLg}^{ij} = g_s H^\dagger \overline{D_R}^i \sigma^{\mu\nu} T^a Q_L^j G_{\mu\nu}^a \ . \tag{9.33}$$

In the case of the $O_{RL\gamma}^{Fij}$ operators the most significant constraint is derived from $B \to X_s \gamma$. The leading effective Hamiltonian at the electro-weak scale can be written as

$$\mathcal{H}_{\text{eff}} = \left(C_{\text{SM}}^{32} + \frac{X_{RL\gamma}^{32}}{\Lambda^2} \right) O_{RL\gamma}^{32} + \frac{X_{RL\gamma}^{23}}{\Lambda^2} O_{RL\gamma}^{23} \ + \ \text{h.c.} \ , \tag{9.34}$$

where

$$C_{\text{SM}}^{32} = -\frac{G_F}{4\pi^2\sqrt{2}} \lambda_b V_{ts}^* V_{td} C_7^{\text{SM}}(M_W) \ , \tag{9.35}$$

and $C_7^{\text{SM}}(M_W^2) \approx -0.3$ is defined as in [85]. The contribution of $O_{RL\gamma}^{32}$ operator, which encodes also the SM contribution, can simply be taken into account by a shift of $C_7^{\text{SM}}(M_W^2)$ at the electro-weak scale:

$$\frac{\delta C_7(M_W)}{C_7^{\text{SM}}(M_W)} = \frac{X_{RL\gamma}^{32}}{V_{ts}^* V_{tb}} \frac{1}{\lambda_b} \frac{F_B^2}{\Lambda^2} \ , \qquad F_B = \left(-\frac{4\pi^2\sqrt{2}}{C_7^{\text{SM}} G_F} \right)^{\frac{1}{2}} \approx 5 \, \text{TeV} \ . \tag{9.36}$$

Using the approximate expression [85]

$$\frac{B(B \to X_s\gamma)}{B(B \to X_s\gamma)_{\rm SM}} \approx 1 + 2.9 \times \delta C_7(M_W) \ , \tag{9.37}$$

and allowing for a 15% departure from the SM value, leads to the bound:

$$|X^{32}_{RL\gamma}| \sim |z_D^{(3)} z_Q^{(2)}| < \ 0.5\,|V^*_{ts}V_{tb}|\lambda_b \frac{\Lambda^2}{F_B^2} < 1.2 \times 10^{-3} \ . \tag{9.38}$$

Employing the hierarchy (9.29) and assuming $|z_Q^{(i)}| \sim |V_{ti}|$ (i.e. saturating the constraint (9.19) from $\Delta F = 2$ LL operators), this bound is naturally fulfilled:

$$|z_D^{(3)} z_Q^{(2)}| \sim |\lambda_b V_{ts}/V_{tb}| \approx 7 \times 10^{-4} \ . \tag{9.39}$$

Employing the same hierarchy, the coupling of the $O^{23}_{RL\gamma}$ operator is substantially larger: $|X^{23}_{RL\gamma}| \sim |z_D^{(2)} z_Q^{(3)}| \sim |\lambda_s V_{tb}/V_{ts}| \approx 8 \times 10^{-3}$. However, since this operator does not interfere with the SM contribution, the bound on $X^{23}_{RL\gamma}$ is weaker with respect to one in equation (9.38): $|X^{23}_{RL\gamma}| < 6 \times 10^{-3}$. We then conclude that the constraints from $B \to X_s\gamma$ are essentially fulfilled without fine tuning.

Similarly to the $\Delta F = 2$ sector, the most serious problems arise from K physics. Here the most significant constraints are obtained from the possible impact in ϵ'/ϵ of the chromo-magnetic operators. The contribution of $\Delta I = 1/2$ operators (such as O^{12}_{RLg} and O^{21}_{RLg}) to ϵ'/ϵ can be generally written as

$$\delta{\rm Re}\left(\frac{\epsilon'}{\epsilon}\right) \approx \frac{\omega}{\sqrt{2}|\epsilon|{\rm Re}\mathcal{A}_0} \times \delta{\rm Im}\mathcal{A}_0 \ , \tag{9.40}$$

where $\mathcal{A}_I = \mathcal{A}(K^0 \to 2\pi_I)$ and $\omega = |A_2/A_0| \approx 1/22$. In the specific case of the chromo-magnetic operators, following [86–88], we have

$$\begin{aligned} |\delta{\rm Im}\mathcal{A}_0| &= \frac{|{\rm Im}(X^{12}_{RLg} - X^{21}_{RLg})|v}{\Lambda^2} \ \eta_G \ \langle 2\pi_{I=0}|g_s \bar{s}_R(\sigma \cdot G)d_L|K^0\rangle \\ &= \frac{|{\rm Im}(X^{12}_{RLg} - X^{21}_{RLg})|}{\Lambda^2} \ \eta_G B_G \sqrt{\frac{3}{2}}\frac{11}{2}\frac{m_\pi^2 m_K^2}{F_\pi \lambda_s} \ , \end{aligned} \tag{9.41}$$

where $X^{12(21)}_{RLg}$ are the couplings of the effective operators at the electro-weak scale (defined in analogy to the $X^{12(21)}_{RL\gamma}$), η_G is the RGE factor, and B_G denotes the bag parameter of the hadronic matrix element. Using the numerical values of [86] and imposing that the extra contributions to ϵ'/ϵ do not exceed 10^{-3} leads to the following bound:

$$\frac{|{\rm Im}(X^{12}_{RLg} - X^{21}_{RLg})|}{\lambda_s} \ < \ 10^{-2}\left(\frac{\Lambda}{10\ {\rm TeV}}\right)^2 \ . \tag{9.42}$$

Using, as in the previous cases, the hierarchy (9.29) and $|z_Q^{(i)}| \sim |V_{ti}|$, we obtain

$$\left|\frac{X^{21}_{RLg}}{\lambda_s}\right| \sim \left|\frac{z_D^{(2)} z_Q^{(1)}}{\lambda_s}\right| \sim \left|\frac{V_{td}}{V_{ts}}\right| \approx 0.2 \ , \qquad \left|\frac{X^{12}_{RLg}}{\lambda_s}\right| \sim \left|\frac{z_D^{(1)} z_Q^{(2)}}{\lambda_s}\right| \sim \left|\frac{\lambda_d V_{ts}}{\lambda_s V_{td}}\right| \approx 0.3 \ . \tag{9.43}$$

Similarly to the case of $\bar{K}^0 - K^0$ mixing, also in this case the suppression implied by the $z_A^{(i)}$ leads to a natural size about one order of magnitude larger than what is needed to fulfill the experimental constraints. In close analogy with the $\Delta S = 2$ case, the problem arises only from the imaginary (CP violating) part of the amplitude.

9.4 Operators involving Lepton Fields

The approach we have followed so far in the quark sector can easily be exported also to the lepton sector. Introducing the diagonal matrices Z_L and Z_E, such that

$$Z_L^{-1} \lambda_E Z_E^{-1} \simeq I \ , \qquad \lambda_E = \frac{1}{v} \mathrm{diag}(m_e, m_\mu, m_\tau) \ , \tag{9.44}$$

we proceed investigating the impact of the transformation

$$L_L \to Z_L L_L \qquad E_R \to Z_E E_R \ , \tag{9.45}$$

onto operators involving lepton fields. As already discussed, a major difference with respect to the quark sector is that lepton flavour is conserved in the SM ($d = 4$) Lagrangian. The only observed lepton-flavour changing transitions are in neutrino oscillations, whereas lepton flavour violating (LFV) transitions of charged leptons are severely constrained by experiments.

Before analysing the efficiency of the transformation (9.45) in suppressing LFV processes, it is worth stressing that it has a non-trivial impact also in quark FCNC transitions. Indeed four-fermion operators with a leptonic current, such as

$$\overline{Q_L}^i \gamma^\mu Q_L^j \overline{L_L}^k \gamma_\mu L_L^\ell \ , \qquad \overline{D_R}^i Q_L^j \overline{L_L}^k E_R^\ell \ , \tag{9.46}$$

receive lepton suppression factors in addition to those from the quark wave functions (the coefficients are respectively $\sim z_Q^{(i)} z_Q^{(j)} z_L^{(k)} z_L^{(\ell)}$ and $\sim z_Q^{(j)} z_D^{(i)} z_L^{(k)} z_E^{(\ell)}$). This implies that such operators are totally negligible. Their contributions to lepton flavour conserving processes, such as $B(K) \to \ell^+\ell^-$ or $B(K) \to \pi\ell^+\ell^-$, are well below the size expected in the MFV framework. Similarly, their contributions to neutral current processes which violate both quark and lepton flavour, such as $B_0 \to \tau\bar{\mu}$ or $K_L^0 \to \mu\bar{e}$, are well below the current experimental sensitivity.

9.5 Bounds from LFV Processes

In the lepton sector the most stringent constraints are on $\Delta F = 1$ transitions among the first two generations ($\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\mu \to e$ conversion on nuclei). $\Delta F = 2$ processes, such as muonium-anti-muonium conversion [89], are less restrictive. In our scenario the largest rates for $\Delta F = 1$ processes arise from higher dimensional operators bilinear in the lepton fields (suppressed only by two $z_{L,E}$ factors) which induce $\mu e\gamma$ and μeZ effective interactions. So we focus on electro-weak dipole operators such as

$$O^{ij}_{RL1} = g' H^\dagger \overline{E_R}^i \sigma^{\mu\nu} L^j_L B_{\mu\nu} \ , \qquad O^{ij}_{RL2} = g H^\dagger \overline{E_R}^i \sigma^{\mu\nu} \tau^a L^j_L W^a_{\mu\nu} \ . \tag{9.47}$$

and operators contributing to the $\ell\ell' Z$ vertex[2]

$$O^{ij}_{LL} = \overline{L_L}^i \gamma^\mu L^j_L H^\dagger D_\mu H \ , \qquad O^{ij}_{RR} = \overline{E_R}^i \gamma^\mu E^j_R H^\dagger D_\mu H \ . \tag{9.48}$$

We start analysing the constraints from the radiative LFV decays, which are sensitive to dipole operators only and which turn out to be the most significant processes. Introducing the effective Lagrangian

$$\mathcal{L} = \frac{1}{\Lambda^2} \sum X^{ij}_{RLx} O^{ij}_{RLx} \ + \ \text{h.c.} \tag{9.49}$$

the radiative branching ratios can be written as[3] [92]

$$B(l_i \to l_j\gamma) = \frac{\Gamma(l_i \to l_j\gamma)}{\Gamma(l_i \to l_j\nu\bar{\nu})} B(l_i \to l_j\nu\bar{\nu}) = \frac{192\,\pi^3\,\alpha}{G_F^2\Lambda^4} \frac{1}{(\lambda_E)^2_{ii}} \left[|X^{ij}_{RL\gamma}|^2 + |X^{ji}_{RL\gamma}|^2\right] b_{ij} \ , \tag{9.50}$$

where $X^{ij}_{RL\gamma} = X^{ij}_{RL2} - X^{ij}_{RL1}$ and $b_{ij} = B(l_i \to l_j\nu\bar{\nu})$, $\{b_{\mu e}, b_{\tau e}, b_{\tau\mu}\} = \{1.0, 0.178, 0.173\}$. We can already see that these branching ratios tend to be large in our scenario: the $z^{(i)}_L z^{(j)}_E$ suppression is partially compensated by the $1/(\lambda_E)^2_{ii}$ term (which appears because of the normalization to $\Gamma(l_i \to l_j\nu\bar{\nu}) \propto m_i^5$) and if the new physics gives $l_i \to l_j\gamma$ via loops, the associated $1/16\pi$ (which is absorbed in Λ^2) is compensated by the three body final state phase space of $l_i \to l_j\nu\bar{\nu}$.

The decomposition $X^{ij}_{RL} \sim z^{(i)}_E z^{(j)}_L$ leads to

$$B(\mu \to e\gamma) \approx 1.2 \times 10^{-11} \left(\frac{130 \text{ TeV}}{\Lambda}\right)^4 \left(\frac{|z^{(1)}_L z^{(2)}_E|^2}{2(\lambda_E)_{22}(\lambda_E)_{11}} + \frac{|z^{(1)}_E z^{(2)}_L|^2}{2(\lambda_E)_{22}(\lambda_E)_{11}}\right) \ , \tag{9.51}$$

$$B(\tau \to e\gamma) \approx 1.1 \times 10^{-7} \left(\frac{4.3 \text{ TeV}}{\Lambda}\right)^4 \left(\frac{|z^{(1)}_L z^{(3)}_E|^2}{2(\lambda_E)_{11}(\lambda_E)_{33}} + \frac{|z^{(3)}_E z^{(1)}_L|^2}{2(\lambda_E)_{33}(\lambda_E)_{11}}\right) \ , \tag{9.52}$$

$$B(\tau \to \mu\gamma) \approx 4.5 \times 10^{-8} \left(\frac{20 \text{ TeV}}{\Lambda}\right)^4 \left(\frac{|z^{(2)}_L z^{(3)}_E|^2}{2(\lambda_E)_{22}(\lambda_E)_{33}} + \frac{|z^{(3)}_E z^{(2)}_L|^2}{2(\lambda_E)_{33}(\lambda_E)_{22}}\right) \ , \tag{9.53}$$

[2] The complete basis of LFV operators can be found for instance in [90, 91]

[3] We have neglected the helicity-suppressed interference term between $X^{ij}_{RL\gamma}$ and $X^{ji}_{RL\gamma}$

where the scale for each mode has been chosen such that the branching ratio is close to its current experimental bound [93–95].

Using a CKM-type ansatz ($z_L^{(3)} \sim 1$, $z_L^{(2)} \sim \lambda^2$, $z_L^{(1)} \sim \lambda^3$, with $\lambda \sim 0.2$), for $\Lambda < 10$ TeV both $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are within their experimental bounds. On the contrary, $\mu \to e\gamma$ exceeds its bound by six (!) orders of magnitude. The problem of $\mu \to e\gamma$ persists with any reasonable ansatz (such as the "democratic" assignment $z_L^{(i)}/z_L^{(j)} \sim z_E^{(i)}/z_E^{(j)} \sim \sqrt{m_i/m_j}$). We thus conclude that either the scale of the LFV operators is pushed well above 10 TeV or, equivalently, the corresponding couplings are suppressed by several orders of magnitude compared to their naive expectation in this framework. For instance, if the dipole operator is generated only via an effective four-lepton interaction (with two lepton lines closed into a loop), its coupling receives an extra suppression factor which allow to set $\Lambda \sim 10$ TeV. Similarly, dipole operators are dynamically suppressed in the the RS scenario considered in [79], where the new degrees of freedom are only vector-like.

Similar (slightly less stringent) bounds on the dipole operators are obtained from their contributions to $\mu \to e$ conversion on nuclei and $\mu \to 3e$. We finally briefly consider the LL and RR operators in equation (9.48). After electro-weak symmetry breaking and integrating out the heavy Z field, these give rise to four-fermion operators of the type $\bar{f}^k\gamma^\mu f^k\overline{L_L}^i\gamma_\mu L_L^j$, where the f^k can be any of the SM fermions. These contribute to $\ell_i \to 3\ell$ decays , $\mu \to e$ conversion, and other previously discussed quark and lepton operators. For $\Lambda \sim 10$ TeV, and with "democratic" assignment $X_{LL,RR}^{ij} \sim \sqrt{(\lambda_E)_{ii}(\lambda_E)_{jj}}$, the expected rate are all below the experimental bounds.

9.6 Discussion and Comparison to MFV

The analysis of the previous sections can be summarized as follows. In the quark sector the hierarchy of the fermion kinetic terms necessary to naturally reproduce both Yukawa structures and suppress dimension-six LL operators is

$$z_Q^{(i)} \sim V_{ti} \sim \mathcal{O}(1), \mathcal{O}(\lambda^2), \mathcal{O}(\lambda^3), \qquad z_D^{(i)} \sim \frac{(\lambda_D)_{ii}}{V_{ti}} \sim \mathcal{O}(\lambda^2)\ , \mathcal{O}(\lambda^3), \mathcal{O}(\lambda^4)\ . \tag{9.54}$$

Employing this hierarchy, most of the predictions from other $\Delta F = 2$ and $\Delta F = 1$ dimension-six FCNC operators fall within the experimental bounds without fine tuning, i.e. assuming generic $\mathcal{O}(1)$ couplings in the basis where the kinetic terms are hierarchical, and an effective scale of new physics ~ 10 TeV. The only exceptions are the LR operators contributing to ϵ_K and ϵ'/ϵ, which can be sufficiently suppressed with a modest fine tuning of the hierarchies and $\mathcal{O}(1)$ coefficients. This scenario predicts that new physics in ϵ_K and ϵ'/ϵ is just around the corner: a conspiracy to suppress the LR operator coefficients by another order of magnitude would require a too severe fine tuning.

Quark Bilinears	MFV		Hierarchical Kinetic Terms	
	parametric size	comparison with exp.	parametric size	comparison with exp.
$L^i L^j$	$V_{ti}^* V_{tj}$	close to experiment	$V_{ti}^* V_{tj}$	same size as MFV
$L^i R^j$	$(\lambda_D)_{ii} V_{ti}^* V_{tj}$	negligible	$(\lambda_D)_{ii} \dfrac{V_{tj}}{V_{ti}}$	can exceed exp. bounds
$R^i R^j$	$(\lambda_D)_{ii} V_{ti}^* V_{tj} (\lambda_D)_{jj}$	negligible	$\dfrac{(\lambda_D)_{ii}(\lambda_D)_{jj}}{V_{ti}^* V_{tj}}$	comparable to $L^i L^j$

Table 9.1: Comparison of the parametrical suppression factors of the various quark bilinears, both in the MFV framework and in the general scenario with hierarchical kinetic terms.

Given the consistency of this general framework in suppressing quark FCNC amplitudes, it is useful to compare it with the more precise predictions of the MFV framework (from which the fuzzy $\mathcal{O}(1)$ factors are absent). In Table 9.1 we list quark bilinears and their suppression factors, as expected within MFV and within the framework with hierarchical fermion profiles. The differences arise for RR and LR operators. Within the MFV hypothesis these are strongly suppressed by one or two powers of the down-type Yukawa coupling. However, such as suppression is not necessary for the description of nature (especially in the case of RR operators) and is partially removed in the scenario with hierarchical fermion profiles.

The situation of the lepton sector is more problematic. The constraints on helicity conserving (LL and RR) LFV operators are satisfied assuming a hierarchy of the type (9.54) for $z_L^{(i)}$ and $z_E^{(i)}$ (with $\lambda_D \to \lambda_E$) or, equivalently, the democratic assignment $z_{L,E}^{(i)} \sim \sqrt{(\lambda_E)_{ii}}$. However, the constraints on LR operators contributing to $\mu \to e\gamma$ and $\mu \to e$ conversion, require an effective scale in the 100 TeV range. It is therefore difficult to make predictions: if the LFV rates are suppressed because the new physics scale Λ is high in the lepton sector, then $\mu \to e\gamma$ could be close to its present exclusion bound, $B(\mu \to 3e)$ and the rate for $\mu \to e$ conversion are suppressed by $\mathcal{O}(\alpha)$ with respect to $B(\mu \to e\gamma)$, and τ LFV decays are beyond the reach of future facilities. However, these predictions do not hold if LFV dipole operators are suppressed by some specific dynamical mechanism. In the latter case we cannot exclude scenarios where the $\tau \to \mu\gamma$ rate is close to its present exclusion bound.

Given the important role of dipole operators in this framework, it is worth to look at the bounds derived from the corresponding flavour-diagonal partners contributing to anomalous-magnetic and electric-dipole moments, both in the quark and in the lepton sector. As far as anomalous-magnetic moments are concerned, the most significant constraint comes from $(g-2)_\mu$. Here we could solve the current discrepancy [96] only for $\Lambda \sim 2-3$ TeV (setting $z_E^{(2)} z_L^{(2)} \sim (\lambda_E)_{22}$), a scale which is far too low compared to the $\mu \to e\gamma$ bound. We thus conclude that there is no significant contribution to $(g-2)_\mu$ in this framework. On the other hand, stringent bounds on Λ (in the ~ 100 TeV range) are imposed by the electron and the neutron electric-dipole moments (assuming $z_{E(D)}^{(1)} z_{L(Q)}^{(1)} \sim (\lambda_{E(D)})_{11}$ and $\mathcal{O}(1)$ flavour-diagonal CP violating phases). Being flavour conserving and CP violating, the coupling of these operators could easily be suppressed by independent mechanisms, such as an approximate CP invariance in the flavour conserving part of the Lagrangian. However, the fact that these bounds are close to those derived from $\mu \to e\gamma$ can also be interpreted as a further indication of a common dynamical suppression mechanism of dipole-type operators.

9.7 Summary

The absence of deviations from the SM in the flavour sector points toward extensions of the model with highly non-generic flavour structures. We have investigated the viability of models where the suppression of flavour changing transitions is not attributed to a specific flavour symmetry, but it arises only from appropriate hierarchies in the kinetic terms, a generic framework which could occur in models with extra dimensions.

We have considered in particular the class of scenarios where the Yukawa matrices and the dimensionless flavour changing couplings of the higher dimensional operators do not exhibit a specific flavour structure (i.e. have generic $\mathcal{O}(1)$ entries) in a basis where the kinetic terms are hierarchical. Choosing the scale of new physics in the TeV range, it was possible to suppress the contributions to quark FCNC transitions from dimension-six effective operators sufficiently without (or with modest) fine tuning. The only two observables where a mild tuning of the $\mathcal{O}(1)$ coefficient is needed are the CP violating parameters of the neutral kaon system: within this scenarios new physics effects in ϵ_K and ϵ'/ϵ should be detectable with improved control on the corresponding SM predictions.

In the lepton sector there exists the most serious challenge to this class of models which is to suppress $\mu \to e$ transitions. The latter requires either a large effective scale of LFV ($\Lambda_{\rm LFV} \gtrsim 100$ TeV) or an independent dynamical suppression mechanism for dipole-type operators.

10 Conclusions

A possible extension of the minimal flavour violation hypothesis to the lepton sector has been proposed recently in the literature. With the inclusion of CP violation at low and high energies we addressed to explain the matter-antimatter asymmetry of the universe by means of leptogenesis in this context and to study possible correlations between leptogenesis and lepton flavour violating transitions. In the framework of MLFV we pointed out that the only admissible BAU with the help of leptogenesis is the one through radiative resonant leptogenesis (RRL) because the truly minimal setup with exactly degenerate Majorana neutrinos is not renormalization group invariant and would imply a vanishing BAU. We showed that in the SM and the MSSM with RRL, the BAU can successfully be generated and that there exists only a very weak sensitivity of η_B to the see-saw scale M_ν so that for scales as low as $10^6\,\mathrm{GeV}$ but also as high as $10^{13}\,\mathrm{GeV}$, the observed η_B can be obtained. Furthermore we have found that flavour effects are important in this scenario and moreover offer the possibility to obtain the BAU even without high-energy CP violation. In contrast to η_B, LFV processes like $\mu \to e\gamma$ are sensitive functions of M_ν in the RRL scenario considered here, therefore strong correlations between the rates for these processes and η_B do not exist in this framework.

An important consequence is that relations between leptogenesis and the flavour violating processes in the charged lepton sector and the low-energy parameters in the neutrino sector are rich in the general MLFV framework without a particular MLFV model specified. Therefore, no general conclusions about the scale Λ_{LFV} on the basis of a future observation or non-observation of $\mu \to e\gamma$ with the rate $\mathcal{O}(10^{-13})$ can be made in this framework.

Restricting the MLFV framework to be CP conserving at high energies, the correlation between leptgenesis and low-energy parameters is much stronger. This restricted MLFV scenario achieves a much higher predictivity than the general framework. In particular, successful leptogenesis leads to constraints on low-energy neutrino parameters and on ratios of lepton flavour violating transitions that can be tested in low-energy experiments.

Further we have investigated the flavour structure of generic extensions of the SM where quark and lepton mass hierarchies and the suppression of flavour changing transitions originate only by the normalization constants of the fermion kinetic terms. We showed that in such scenarios the contributions to quark FCNC transitions from dimension-six effective operators are sufficiently suppressed without (or with modest) fine tuning in the effective scale of new physics. Interestingly, there exist clear distinctions to the minimal flavour hypothesis. To be more specific, in contrast to MFV, hierarchical fermion profiles allow for more sizeable contributions for left-right-handed and right-right-handed effective quark couplings while four-fermion operators involving light leptons are strongly suppressed.

Despite its simplicity, this construction is sufficient to suppress to a level consistent with experiments all the flavour changing transitions in the quark sector, assuming a scale of new physics in the TeV range as required for a natural stabilization of the Higgs mass. The most serious challenge to this type of scenarios appears in the lepton sector, thanks to the stringent bounds on LFV.

A Iterative Solution of the Renormalization Group Equations

The goal of our numerical analysis of chapters 7 and 8 is to determine the neutrino Yukawa matrix Y_ν and the masses of the right-handed neutrinos at the scale M_ν taking into account the constraints on the masses and mixing of light neutrinos measured at low energies and imposing the GUT condition characteristic for MLFV

$$M_1(\Lambda_{\mathrm{GUT}}) = M_2(\Lambda_{\mathrm{GUT}}) = M_3(\Lambda_{\mathrm{GUT}}). \tag{A.1}$$

As discussed in chapter 3 the latter condition implies

$$\mathrm{Re}(R(\Lambda_{\mathrm{GUT}})) = 0, \tag{A.2}$$

but $\mathrm{Im}(R(\Lambda_{\mathrm{GUT}}))$ must be kept non-zero in order to have CP violation at high energies.

The RG evolution from Λ_{GUT} down to M_ν generates small splittings between $M_i(M_\nu)$ and a non-vanishing $\mathrm{Re}(R(M_\nu))$, both required for leptogenesis. As the splittings between $M_i(M_\nu)$ turn out to be small, we integrate out the right-handed neutrinos simultaneously at $\mu = M_\nu$ imposing, up to their splittings,

$$M_1(M_\nu) \approx M_2(M_\nu) \approx M_3(M_\nu) \approx M_\nu. \tag{A.3}$$

In view of various correlations and mixing under RG between different variables we reach the goal outlined above by means of the following recursive procedure:

Step 1

We associate the values for the solar and atmospheric neutrino oscillation parameters given in (7.4)–(7.5) with the scale $\mu = M_Z$ and set θ_{13} and the smallest neutrino mass $m_\nu^{\mathrm{lightest}}$ to particular values corresponding to $\mu = M_Z$.

Step 2

For a chosen value of M_ν, the RG equations, specific to a given MLFV model, are used to find the values of the parameters of Step 1 at $\mu = M_\nu$. For instance we find $m_i^\nu(M_\nu)$ and similarly for other parameters.

Step 3

We choose a value for $\Lambda_{\rm GUT}$ and set first $m_i^\nu(\Lambda_{\rm GUT}) = m_i^\nu(M_\nu)$ and similarly for other parameters evaluated in Step 2. Setting next

$$M_1(\Lambda_{\rm GUT}) = M_2(\Lambda_{\rm GUT}) = M_3(\Lambda_{\rm GUT}) = M_\nu \tag{A.4}$$

and choosing the matrix R, that satisfies (A.2), allows also to construct $Y_\nu(\Lambda_{\rm GUT})$ by means of the parametrization in (3.16).

Step 4

Having determined the initial conditions for all the parameters at $\mu = \Lambda_{\rm GUT}$ we use the full set of the RG equations [59] to evaluate these parameters at M_ν. In the range $M_\nu \leq \mu \leq \Lambda_{\rm GUT}$ we use

$$m_\nu(\mu) = -v^2\, Y_\nu^T(\mu) M^{-1}(\mu) Y_\nu(\mu). \tag{A.5}$$

The RG effects between $\Lambda_{\rm GUT}$ and M_ν will generally shift $m_i^\nu(M_\nu)$ to new values

$$\tilde{m}_i^\nu(M_\nu) = m_i^\nu(M_\nu) + \Delta m_i^\nu \tag{A.6}$$

with similar shifts in other low energy parameters. If these shifts are very small our goal is achieved and the resulting $Y_\nu(M_\nu)$ and $M_i(M_\nu)$ can be used for lepton flavour violating processes and leptogenesis. If the shifts in question are significant we go to Step 5.

Step 5

The initial conditions at $\mu = \Lambda_{\rm GUT}$ are adjusted in order to obtain the correct values for low-energy parameters at $\mu = M_\nu$ as obtained in Step 2. In particular we set

$$m_i^\nu(\Lambda_{\rm GUT}) = m_i^\nu(M_\nu) - \Delta m_i^\nu \tag{A.7}$$

with Δm_i^ν defined in (A.6). Similar shifts are made for other parameters. If the condition (A.3) is not satisfied in Step 4, the corresponding shift in (A.4) should be made. Choosing R as in Step 3 allows to construct an improved $Y_\nu(\Lambda_{\rm GUT})$. Performing RG evolution with new input from $\Lambda_{\rm GUT}$ to M_ν we find new values for the low energy parameters at M_ν that should now be closer to the values found in Step 2 than it was the case in Step 4. If necessary, new iterations of this procedure can be performed until the values of Step 2 are reached. The resulting $Y_\nu(M_\nu)$ and $M_i(M_\nu)$ are the ones we were looking for.

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