

## **Mixed Forests reconsidered: A Forest Economics Contribution on an Ecological Concept**

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### **Abstract**

To stimulate an economically oriented discussion on mixed forest management, this paper considers economic implications of mixed investments of pure stands of the Norway spruce (*Picea abies* (L.) Karst.) and the European beech (*Fagus sylvatica* L.) on a methodological basis. It is well known that the classical economic calculus leads to an overwhelming financial superiority of single species coniferous forest management in Central Europe, especially for growing monocultures of spruce. This explains why, since the early 19<sup>th</sup> century, the range of these forest types extends far beyond their natural limits. The change in the natural vegetation cover bore severe environmental problems as it was accompanied by a loss of biodiversity and a severe reduction in the resistance against storm, snow, ice, drought and insect damage of the forest stands. In contrast to this development as early as 1886 the Bavarian silviculturist Karl Gayer claimed that the forest condition must be able to deal with the uncertainty of future development. While pointing out that this condition could only be provided by “mixed forests” he, as a pioneer, formed an ecological concept for forest management. Unfortunately, forest managers have not broadly accepted his concept up to now. In order to support the ecological idea of “mixed forests” it seems crucial to demonstrate that mixed, diverse forests also possess economic advantages. Through using the portfolio theory founded by Markowitz and Sharpe,

this paper evaluates mixed forest management and compares it to single species forest management. Its focus is on mixtures of Norway spruce and European beech. The Monte Carlo simulation method was used to simulate expected financial returns and their dispersion under risk. Mixed forests reduce the profitability but also show diversification effects due to only weak positive or even slightly negative correlated timber markets and diversified time structure of the timber harvests. Risk-averse decision-makers should therefore establish ecologically desired mixed forests with beech proportions between 10 and 50 %, even if the profitability for mixed forests decreases. They will however benefit largely due to a significant risk attenuation.

Although, effects of small scale mixtures in stands comprising of many species were not considered we are sure that the findings of this study will in part also apply to mixtures on the forest stand level. While analysing an ecological concept from an economic viewpoint, which is transferable to the general idea of natural diversity, we intend to arouse interest in the ecologically oriented reader to evoke intensified future cooperation.

**Keywords:** Mixed forest management, Diversification, Environmental Risk, Portfolio theory

## **Introduction**

It is well known that without the intervention of man, Central Europe would be almost completely covered by forests (Ellenberg, 1986, p. 20). Unlike the potential natural vegetation cover, the actual forest cover consists of mainly coniferous forests. Since the early 19<sup>th</sup> century the range of these forest types extends far beyond their natural limits because of economic reasons. According to Spiecker (2003) the change of the natural vegetation cover was accompanied by a loss of biodiversity and a severe reduction of the forest stand's resistance against storm, snow, ice, drought and insect damage (von Lüpke, 2004). Moreover, the risk involved with forest management is great, due to the uncertainty of future environmental and economic developments. Risk pervades in forest management and decision-makers have to cope with it. Scientists identify the current tree vegetation cover as an environmental problem and conclude that a conversion of pure coniferous forests into more site adapted and therefore rather low-risk mixed forests is greatly relevant from an economic and ecological point of view (von Lüpke and Spellmann, 1997, Fabian and Menzel 1998, Lindner 1999, Spiecker, 2003, von Lüpke et al. 2004). In this situation the old ecological idea "mixed forests" became popular again in Europe (Seitschek, 1991, Moser, 1995, Olsthoorn et al., 1999, Schraml and Volz, 2004).

The ecological idea "mixed forests" goes back to the Bavarian silviculturist Karl Gayer (1886). He claimed a forest condition that was able to deal with the uncertainty of future development and environmental risk (Gayer, 1886, p. 5). Gayer pointed out that this condition could be provided only by mixed forests. In order to implement and sustain the idea "mixed forests" it is crucial to analyse reasons for the development towards pure coniferous stands instead of mixed forests, despite of Gayer's early and clear recommendation.

Following Emerton (2000), one could state that Gayer and other promoters of mixed forests could not authentically demonstrate the potential economic and financial benefits of the mixed forest concept. Emerton concluded with regard to biodiversity, "People will continue to degrade and deplete biodiversity in the course of their activities because they feel it is more profitable and economically desirable to do so."

Contrary to this explanation studies in the field of environmental economics showed that there exist other aspects than profitability, like ethics or aesthetics, that influence decision-making. For example a remarkable willingness to pay for the mere existence of species or scarce ecosystems could be found (for an overview see Hampicke, 1991, p. 128-129). Furthermore,

Balmford et al. (2002)<sup>1</sup> pointed out that mankind benefits largely from wild nature in the form of non market-based services. Applying valuation techniques like hedonic pricing, contingent valuation and alternative cost methods (for a methodological explanation see Bergen et al., 2002 and also Armsworth and Roughgarden, 2001) an overall benefit:cost ratio for conserving wild nature of at least 100:1 could be calculated. Also several studies on non-industrial private forest ownership (NIPF) showed that forest owners indeed value non-market services from their forests. For example, Ask and Carlsson (2000) proved that forest owners voluntarily set aside specific forest areas from timber production in Sweden. Kant (1999) pointed out the existence of woodlot owners in Ontario who do value environmental, ecological and recreational aspects of forestry.

Nevertheless, despite the fact that people are willing to pay even for non-market benefits some forest managers adopt the reasoning of Emerton (people will only conserve biodiversity if they expect direct economic advantages) because up to now forest managers are mostly not paid for non-market services.

Therefore, looking for direct economic advantages seemed essential to promote mixed forests. The presence of environmental risk in forestry shows obvious parallels between issues of natural diversity and issues of diversification of financial assets. Figge (2004, p. 828) already pointed out these parallels on the basis of theoretical considerations and stated that this crucial finding is generally not acknowledged. The theoretical framework to evaluate aspects of natural diversity in forest management was provided by the classical portfolio theory (Markowitz, 1952). While applying the portfolio theory to mixed investments of pure spruce (*Picea abies* (L.) Karst.) and beech (*Fagus sylvatica* L.) stands, we tried to quantify economic implications of the ecologically desired mixed forests. Spruce is an exotic tree species to most sites where it is grown, while beech would dominate the natural vegetation cover in Central Europe. However, effects of small scale mixtures in stands comprising of many species are not addressed with this paper. The paper's character is rather methodologically oriented in order to stimulate discussion on "mixed forests" not only on an ecological but also on an economic foundation. But, we are sure that the findings of this study will in part also apply to small scale mixtures.

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<sup>1</sup> This citation was used because of its impressive benefit-cost relation. The estimate was carried out as a worldwide average. As one of the reviewers pointed out, this relation is of course subject to variation by location and scale.

In order to analyse the potential economic advantages of mixed forests, subsequently defined as mixed investments of pure spruce and beech stands, the following hypothesis will be examined with this paper:

“From an economic point of view the possible risk abatement for risk-averse investors to be achieved by mixed forests does not justify the decrease in the profitability involved.”

## **Theoretical background**

Markowitz (1952) founded the portfolio theory, which was then further developed among others by Sharpe (1964). The basic finding of Markowitz showed that investing in a combination of different financial assets may reduce the risk when compared to an individual investment of the same profitability. On the other hand, it may increase the profitability when compared to an alternative individual investment of the same risk. The relationship between combined investments and risk abatement or profitability increase is called “diversification effect”.

Indeed it is essential in which direction the markets of combined investments develop.

Intuitively, this becomes clear when considering the following three fictional examples:

- Firstly, imagine a combined financial investment in stocks of umbrellas and sun cream. It seems very likely that this combination will effectively reduce risk. The sun cream business will boom in sunny years with only little precipitation. The umbrella market will however decline in such years. But during a cool and wet year the umbrella market grows while the sun cream market reduces. The market development of both products has an obviously great negative correlation, hence the financial returns on both investments are also negatively correlated. This situation will yield a systematic risk reduction even up to a risk of almost zero in a theoretical case.
- Secondly, consider a combined financial investment in stocks of umbrellas and rain coats. This combination will hardly reduce risk as the markets as well as the financial returns for both products are obviously vastly positively correlated. In this case a combination will yield the same results as an individual investment.
- Thirdly, we combine financial investments in umbrella and banana stocks. There are no reasons to assume that there is any correlation between the market developments for both products. This situation will also result a risk reduction since we will randomly have years in which the umbrella market will boom while the banana market declines and vice-versa.

Among information on correlation between the financial returns of combined investments, knowledge on their individual standard deviation is a precondition to assess the effectiveness of several combinations. The standard deviation expresses the dispersion of the expected returns indicating the degree of uncertainty connected with an investment (Pflaumer, 1992).

This measure is commonly used in order to quantify the risk of an investment. For the combination of two investments the following equation (Eq. 1) can be used to compute the standard deviation  $S$  of the total financial return  $R$  of a combined investment (e.g., Spremann, 1996).

$$S(a_1 \cdot r_1 + a_2 \cdot r_2) = \sqrt{a_1^2 \cdot s_{r_1}^2 + a_2^2 \cdot s_{r_2}^2 + 2 \cdot k_{r_1, r_2} \cdot a_1 \cdot a_2 \cdot s_{r_1} \cdot s_{r_2}} \quad (1)$$

$a_1$ or $a_2$	Proportions of each financial investment ( $a_1 + a_2 = 1$ )
$S(a_1 \cdot r_1 + a_2 \cdot r_2)$	Standard deviation of the financial return $R$ (classically computed as the net present value of net revenue flows) on the combined investment
$s_{r_1}$ or $s_{r_2}$	Standard deviation of the individual investment financial returns $r_1$ or $r_2$
$k_{r_1, r_2}$	Correlation coefficient between the financial returns on investment 1 and 2

The last term in (1) contains the covariance  $\text{cov}_{r_1, r_2}$  between the financial returns:

$$\text{cov}_{r_1, r_2} = k_{r_1, r_2} \cdot a_1 \cdot s_{r_1} \cdot a_2 \cdot s_{r_2}$$

The total financial return  $R_{1,2}$  of the portfolio consisting of two investments is:

$$R_{1,2} = a_1 \cdot r_1 + a_2 \cdot r_2 \quad (2)$$

The effect of different correlations between the financial returns of two investments combined in several compositions is illustrated with Figure 1.

[Figure 1]

Figure 1 demonstrates two investments (inv. 1 and inv. 2) characterised by their financial returns at the x-axis and their risks (standard deviation) at the y-axis. Usually the investment with the greater financial return is involved with a higher risk, like investment 2. The straight dashed line connects both investments directly. It indicates several combinations of both investments when no diversification effects occur. Starting with investment 1 on the left hand side, the proportion of investment 2 is enlarged stepwise in 10%-steps (indicated by the dots). Following the straight line, with increasing proportion of investment 2 the risk of the portfolio grows proportionally. A composition comprised of 50 % investment 2 and 50 % investment 1 results a risk of  $0.5 \cdot s_{r_1} + 0.5 \cdot s_{r_2}$ . This curve forms a baseline for a given coefficient of correlation of  $k_{r_1, r_2} = +1$ . If, however there is no correlation between the financial returns ( $k_{r_1, r_2} = 0$ ) an increase of investment 2 leads to a decrease in the risk and simultaneously an

increase in the financial return. This takes place up to an proportion of investment 2 of 30 %. With more than 30 % of investment 2, the risk increases but less than proportional to the admixture of investment 2. With  $k_{r1,r2}=-1$  this effect is further strengthened, and theoretically a risk reduction to almost zero can be achieved.

**The forest plantation investment in a deterministic world**

Predominantly, in order to determine the value of the soil and that of immature forest stands at a very early state forest scientists developed an investment calculus (e.g. Faustmann, 1849). For this purpose the net revenue flows involved with a forest plantation were all discounted to the point in time when the plantation is established. In order to compute the financial return two basic versions of the calculus are distinguished. At first, only one production period  $T$  is considered (Eq. 3) and secondly, all subsequent production periods are integrated (Eq. 4).

$$R_{Limited} = \sum_{t=0}^T (p_t \cdot v_t - c_t) \cdot e^{-i \cdot t} \quad (3)$$

$$R_{Infinite} = \frac{\sum_{t=0}^T (p_t \cdot v_t - c_t) \cdot e^{i \cdot (T-t)}}{e^{i \cdot T} - 1} \quad (4)$$

$R_{Limited}, R_{Infinite}$	Financial return computed as the net present value of all future net revenue flows for a limited and an unlimited time horizon
$p_t$	Price per cubic meter net of logging expenses at a specific point in time
$v_t$	Harvestings per hectare in cubic meters at a specific point in time
$c_t$	Expenses per hectare (e.g., reforestation or pre-commercial thinning) at a specific point in time
$e$	Euler's figure
$t$	Stands age
$i$	Interest rate (decimal)
$T$	$T$ is the time at which the stand will be harvested (production time)

Harvestings and average net timber prices were adopted from Knoke (2004) who employed data of growth simulations from two studies (Knoke, 2002, Felbermeier, not published).

[Table 1]



All growth simulations and financial calculations were based on hypothetical stands of an area of 1 ha. Hence, 1 ha can be considered the minimum size of a species block to achieve the financial results on which the study is based. Rather conservative silvicultural treatments were assumed because growth modelling still created problems for projecting modern treatments. In the case of spruce thinning from below was applied concluding in a clear cut at the end of the production time. For beech the random selection treatment in Knoke (2002) was used, which also finished with a clear cut. Data on the amount of harvested timber are provided in Table 1.

As for example Valsta (1998) and Knoke et al. (2001) pointed out, the majority of comparative studies in forest economics ignore the uncertainty of future net revenue flows and therefore the risk involved with forest management. Ignoring risk, typical net revenue flows for spruce and beech (Figure 2) lead to a great financial superiority of spruce.

[Figure 2]

Eq. 3 and these sequences of net revenue flows produce a financial return for spruce of 5,800 Euro (Eq. 4: 6,750 Euro), while beech merely yields 2,750 Euro (Eq. 4: 2,050 Euro). The relation of the financial return in favour of spruce is 2.1 : 1 or even 3.3 : 1, when applying Eq. 4. The reported results are obtained when applying only a small interest rate of 0.02 to evaluate the long-term investment in a forest stand. For example, Beltratti et al. (1996) demonstrated why a small interest rate is adequate for extremely long-term investments (consider also Heal, 1985, Hampicke, 1991, Price, 1997). Assuming greater interest rates, the relation in favour of spruce would be even more pronounced. Hence, the result of this calculus easily explains why the range of spruce forests was extended far beyond their natural limits in Central Europe.

### **Sources of risk in forest management as an environmental problem**

Growing trees is an extremely long-term investment in central Europe. Production times of 100 years and more are common. During its lifetime a forest stand is exposed to several risks.

Windfall, snow breakage and insect attacks are the most important natural hazards. During 1990 to 2001, 43 % of the timber cut in the Bavarian State Forests was motivated by natural hazards (Bavarian State Forest Service, 2001). Generally, native and therefore site adapted tree species are less susceptible to natural hazards. The sensitivity of tree species to natural hazards may be described by means of survival probabilities (Figure 3), which express the probability of a planted forest stand to survive a given time horizon. Survival probabilities allow the calculation of hazard risks for concrete stands at specific ages (Figure 4). This quantifies for a

given period the probability with which the actual stand will be destroyed by windfall, snow breakage or insect attacks.

[Figure 3]

[Figure 4]

The hazard risks and the survival probability until a specific period  $j$ , form the following relationship (Eq. 5):

$$SP_j = (1 - risk_1) \cdot (1 - risk_2) \cdot \dots \cdot (1 - risk_{j-1}) \quad (5)$$

$SP_j$  Survival probability up to period  $j$

$risk_1, risk_2, risk_{j-1}$  Hazard rates in several periods

$(1-risk_i)$  Probability with which a forest stand will survive period 1 (transition probability)

Figures 4 and 5 were based on survival probabilities of either pure spruce or pure beech stands. So far data on survival probabilities of stands, where species are mixed throughout the stands with varying proportions of the species, are scarce. Looking at the problem from the perspective of a mixed investment of pure spruce and pure beech stands in this paper allows for applying the single species survival probabilities. Also, it is not explicitly considered that thinnings might increase the risk of damages shortly after the intervention. However, merely moderate thinnings were simulated, which were common in bygone forest management and were also applied to those stands from which data were gained for deriving the survival probabilities.

Empirical analyses showed that once a stand has been destroyed by natural hazards only 50 % of the scheduled net revenue flow will be achieved when selling the wood (Dieter, 1997).

Besides natural hazards also the volatility of the timber prices is a source of risk in forest management. Generally, timber prices may fluctuate dramatically over time (Brazee and Mendelsohn, 1988, Haight, 1990, Knoke et al., 2001). Figure 5 shows this effect for trunk wood prices achieved for spruce and beech in Bavaria.

[Figure 5]

A synchronous development of timber prices for both species can be observed until 1979. But from 1980 onwards the markets for beech and spruce timber obviously developed in different directions. Analysing the correlation between the timber prices for those two different periods

results in a moderately strong positive correlation until 1979 (called Market Model 1) and a weak negative correlation from 1980 (called Market Model 2) onwards (Figure 6). The reason for the shifted timber price regimes may be seen in the discussions and agreements on the protection of tropical rainforests around 1980. In 1983 the International Tropical Timber Agreement (ITTA) was signed. In 1986 the import of timber from tropical rainforests was boycotted. These trends increased the demand for German beech timber grown in sustainable managed forests, which presumably led eventually to the increasing timber prices for beech, while spruce timber prices decreased.

[Figure 6]

A regression analysis for a linear function, which estimates the beech timber prices on the basis of the spruce timber prices:

$$p(\text{beech}) = a + b \cdot p(\text{spruce}) \quad (6)$$

yielded the parameters indicated in Table 2.

[Table 2]

The analysis excluded the year 1991 as an outlier, as these prices were severely influenced by an exceptional hurricane in 1990.

Because the correlation between achievable timber prices can be relevant for the financial returns, a differentiation between the two Market Models seemed essential for the following analysis.

### **Monte Carlo simulation as a methodological technique to consider environmental risk**

Harry Markowitz initiates in his famous 1952-paper by describing the fundamental structure of portfolio selection: “The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performance of available securities. The second stage starts with the relevant beliefs about future performance and ends with the choice of portfolio.” (Markowitz, 1952). The uncertainty of the future, particularly, has a great impact on long-term forestry. Forest managers’ decisions must rely on expectations on future development. But the expectations should be based on sound information. Although it is impossible to predict the future development of growing

conditions, natural hazard risks, and timber markets, data and experience of the past can be utilised in order to simulate scenarios on future developments.

Because of the already described sources of risk in forest management, the future development on financial returns will greatly be influenced stochastically. Monte Carlo simulation is a technique widely used to reflect the effects of stochastic processes (Yool, 1999, Runzheimer, 1999, Waller et al., 2003). With this technique random samples are artificially produced through generating random numbers.

Using this technique we simulated 1,000 random samples (scenarios) of possible net revenue flow sequences for plantations of spruce and beech. Each of the 1,000 scenarios comprised of 10 periods (time horizon 100 years) for the baseline simulations and 50 periods (time horizon 500 years) for others. At every period an actual stochastic timber price for spruce was simulated randomly by adding a stochastic variation to the deterministically computed mean of spruce prices. The mean of the price distribution of spruce was 79.93 Euro/cubic meter for Market Model 2 (53.60 for Market Model 1) with a standard deviation of  $\pm 11.79$  ( $\pm 11.95$  for Market Model 1). The actual timber price for beech was then estimated by the corresponding regression equation of Table 2, while employing the simulated timber price of spruce as the independent variable. The deterministically predicted beech timber price was then corrected with the generated stochastic deviation. The beech-price stochastic variability was based on the root mean square error obtained from the regression analysis (Table 2). As seen in Figure 5 b for the price scenario No. 500 the price simulation process ignored the climbing trends within the original price data for spruce and particularly beech (Figure 5 a). After simulating an actual stochastic timber price for each species the typical, risk ignoring net revenue flow illustrated in Figure 2 was adjusted by the quotient: simulated stochastic timber price divided by the expected mean timber price.

Moreover, the individual hazard rate was integrated in the Monte Carlo simulation. It was used as the expectation value of a binomial distribution in order to generate the random numbers 0 and 1. The '1' indicated the destruction of the stand, the '0' its survival, while the frequency of destruction corresponded to the hazard rate of Figure 4. Once a stand was destroyed merely 50 % of the previously simulated net revenue flow was employed to compute the financial return.

The initial payout for establishing the forest plantation was also considered a random variable subject to a standard deviation of  $\pm 20$  %.

### **Baseline simulations considering identical production times**

For both tree species an identical production period of 100 years was investigated. The integration of risk in the economic evaluation of both species produces a great dispersion of the net revenue flows (Figure 7), which describes simulations based on Market Model 1 (weak negative correlation between timber prices). Particularly the most profitable harvest at the end of the production time, yield highly variable net revenue flows. Natural hazards show a great influence here by decreasing the scheduled net revenue flow for spruce from 33,200 to on average 24,500 Euro (standard deviation  $\pm 13,800$ ). The final net revenue flow of the less risk sensitive natural tree species, beech, is merely reduced from 27,400 to 24,400 Euro (standard deviation  $\pm 8,600$ ). A combined investment of 50 % spruce and 50 % beech, yields almost the same mean net revenue flow with a more than proportionally reduced standard deviation of merely  $\pm 8,100$ .

As Table 3 shows, the integration of risk improved the relation of the financial returns from spruce and beech, which was 2.1 : 1 when ignoring risk, to 1.7 : 1. The standard deviation of an investment in beech ( $\pm 1,500$  to  $\pm 1,600$  Euro/ha) was substantially lower compared to that of an investment in spruce ( $\pm 2,300$  to  $\pm 2,400$  Euro/ha).

Considering the correlation between the net revenue flows for each period put forth a strong overlaying effect of the natural hazards (Table 4), which seriously attenuates the correlation of net revenue flows. Stochastically occurring events like natural hazards may obviously weaken the correlations between net revenue flows. When, for example, a spruce stand was destroyed by natural hazard at an age of 60 years but the beech stand survived a young spruce stand was re-established. The beech stand was then 60 years old, while the age of spruce was zero. Because spruce will not generate substantial net revenue flows for the next 40 years, timber price correlation between spruce and beech do no longer affect the net revenue flow correlation. This effect of natural hazards is the reason why the financial returns are either weakly positive (Market Model 1) or not at all correlated (Market Model 2) (Table 3). The described weak correlation of net revenue flows occurred although the timber price correlations integrated in the simulations were much stronger.

However, even a zero or a weak positive correlation between net revenue flows produce risk attenuation, when compared to proportional risk increase of several mixtures (Figure 8). In Figure 8 the black solid line describes the relationship between the financial return on the forest portfolio and risk for Market Model 2 (a weak negative correlation between timber

markets for spruce and beech). An investment in 100 % beech (left hand side of Figure 8) yields only 59 % of the financial return of an investment in 100 % spruce. The risk of merely investing in beech amounts 68 % of that of a pure spruce investment. Enlarging the proportions of spruce leads to a simultaneous increase of financial return and decrease of risk until a proportion of 30 % spruce is reached. Hence, portfolios with less than 30 % of spruce are obviously inefficient, as other combinations yield increased financial returns at the same risks. Increasing the proportions of spruce further involves an inflated risk. However, risk inflation is far less than the proportional risk. For example, 80 % of the maximum financial return can be achieved at 60 % of the maximum risk (mixture 50 % spruce and 50 % beech). While combining 75 % of spruce and 25 % of beech, 90 % of the maximum financial return can be obtained at merely 77 % of the maximum risk.

Even if there is a positive correlation between the timber markets for spruce and beech (Market Model 1, grey curve in Figure 8) diversification effects are obvious. However, at the same financial return the risk is up to 4 percent points higher compared to the results, when applying Market Model 2.

### **Effect of time diversification**

In central European forest management it is not common to use the same production times for spruce and beech. Normally, a shorter production time is applied to spruce (mostly 100 years) and a longer time period is chosen to manage beech (usually 120 years). In the following, the effect of diversifying production times is analysed.

In order to consider the unequal production times (rotations) Eq. 4 should be applied rather than Eq. 3 to compute the financial return. But for the purpose of simulation this Equation is not convenient, as it is a static formula considering only the one period  $T$  (production time). Constant net revenue flow sequences in every period are the precondition to calculate the financial return properly according to Eq. 4. Because we are analysing especially stochastic and not constant net revenue flows, we approached an unlimited time horizon by simulating 50 periods (500 years) using Eq. 3. The difference between Equations 3 and 4 becomes negligible, when considering such a long period of time (see e.g. Dieter, 2001). Hence, during the simulation over 500 years the different production times for spruce (100 years) and beech (120 years) were considered.

Diversifying the production times between spruce and beech eliminates the effect of different Market Models. The final harvests now occur in different years and the timber prices are therefore uncorrelated. As Figure 9 demonstrates, no difference between the models can be observed when spruce is managed with a production time of 100 years, while beech is grown for 120 years. A minimum of risk can be found when combining 20 % of spruce with 80 % of beech. This forest portfolio bears only 50 % of the maximum risk, while only 61 % of the maximum financial return is achieved.

### **Effect of an increased natural hazard risk**

Climate scenario analyses in forest science show that the growing conditions for spruce will deteriorate, if the global warming continues (Pretzsch and Utschig, 2000). Worse growing conditions (for example lower water supply) will also increase the risk involved in spruce management. In this paper the effect of an increased risk is analysed based on data published by Möhring (1986, p. 65). This data is characterised by a survival probability of merely 0.53 for spruce at an age of 100 years, while the standard simulations were based on a value of 0.75. The increased hazard risk scenario was applied to the simulations with different production times (spruce 100 and beech 120 years) over 50 periods.

A higher risk diminishes the financial return of spruce from 5,500 to 4,400 Euro/ha with an increased standard deviation ( $\pm 2,450$  instead of  $\pm 2,100$  Euro/ha). This effect makes beech management more profitable in relation to spruce management. Similar to the previous simulations, a minimum of risk can be achieved with 20 % of spruce and 80 % of beech (Figure 10). With this combination it is still possible to obtain more than 70 % of the maximum financial return. Almost 80 % of the maximum financial return is reached with a mixture of 40 % spruce and 60 % beech at a risk of merely 50 % of the maximum. The increased risk for spruce management makes the combination with beech even more effective.

### **Measuring the utility of mixed forest management**

The utility people obtain from specific financial returns does not only depend on their magnitudes. Also the risk involved with the investment is relevant. Often, a great dispersion (high standard deviation) of financial returns diminishes the utility, which is gained from an investment. In order to quantify this effect, a simple function (Eq. 7) can be used estimating the

certainty equivalent of a risk-averse decision maker.

$$Z = E(R) - \alpha \cdot \frac{S^2}{2} \quad (7)$$

$Z$	Certainty equivalent
$E(R)$	Expected financial return on an invested amount of money
$S^2$	Variance of financial returns
$\alpha$	Constant depending on the decision-makers attitude towards risk

The expected financial return is reduced proportionally to the dispersion of the financial return. A reduction of the financial return is correct, if a risk-averse decision-maker is considered. He would prefer a financially secure return rather than one subject to dispersion (uncertainty), when comparing identical expected values. The risk-costs derived were based on a negative exponential utility function. When introducing the risk dependent reduction of the financial return, the function becomes an approximation for the certainty equivalent of the decision-maker, valid for small  $\alpha$  (Gerber and Pafumi, 1998, p. 77). This function is often employed to characterise the preferences when comparing specific investments with a given financial return and standard deviation (see Spremann, 1996, Dieter, 1997, Weber, 2002).

Risk-aversion, implicitly presupposed in Eq. 7, can usually be observed as the choice used by most people. Gamblers, of course, show a different attitude towards risk. They generally appreciate a great dispersion of financial returns because of the chance of an unexpectedly great financial return. The constant  $\alpha$  in Eq. 7 describes the degree of risk-aversion of the decision-maker. It is considered normal that people have an  $\alpha$ -value around “1/original assets” (Spremann, 1996, p. 502). To integrate the expected attitude towards risk for the extremely long-term investments in forestry  $\alpha$ -values of “1.2/original assets” (risk-averse) and “2/original assets” (very risk-averse) were applied.

The maximum initial investment for the forest plantation was used to estimate the  $\alpha$ -value. It was 3,000 Euro/ha (for the case of a pure beech plantation), hence  $\alpha$ -values of 0.0004 (risk-averse) and 0.00066 (very risk-averse) resulted. For a better comparison the values of the certainty equivalent ( $Z$ ) were standardised to an interval between 0 and 1 and the values obtained were called “utility-index”.

Before computing the utility-indices, the problem of unequal initial investments for spruce and beech stands had to be solved. When establishing a pure spruce stand the initial investment



was 1,000 Euro/ha less than compared to a pure beech stand. To be able to compare alternatives an identical initial investment must be considered. Therefore, it was assumed that in the case of spruce only 2,000 Euro/ha are invested in forestry while 1,000 Euro are invested alternatively in a low-return, risk-free asset at an interest of 2 % (this was assumed as the internal rate of return using the best alternative for an extremely long-term investment). This assumption is discussed in the “Discussion and conclusions” section of the paper. Thus, the proportion of the forest investment is  $0.6\bar{6}$  (as  $0.6\bar{6} \cdot 3,000$  results 2,000) for the establishment of a pure spruce stand, while the proportion of the alternative investment is  $(1 - 0.6\bar{6})$ . By definition, the net present value of an investment is zero, if the initial payout is invested at the internal rate of return of the investment. This proves true because among various interest rates, the internal rate of return can be found, when the net present value of the investment becomes zero. Also the risk of the alternative investment was assumed as zero. As a consequence of this consideration, the expected financial returns and the risks remained unchanged but were always evaluated as results of an initial investment of 3,000 Euro.

The different scenarios analysed in this paper do not result in the same utility-index curves (Figure 11). When relying on the scenario with an increased risk for spruce management (50 periods, high risk) the decision-maker would obtain the maximum utility from a mixture of the forest investment with a considerable beech component (70 % spruce and 30 % beech, Figure 11, a). Considering an identical production time (10-period scenarios) with a normal risk-aversion, a combination of 90 % spruce and 10 % of beech stands would be chosen regardless of the Market Model applied.

If the decision-maker's attitude towards risk is very averse, his mixed forest investment should contain at least 30 % of beech forests (Figure 11, b). He should either establish beech forests at 30 % (10-period scenarios) or even at 50 % of the area (scenario 50 periods, high risk) to obtain the maximum utility.

## **Discussion and conclusions**

While analysing an ecological concept from an economic viewpoint, which is transferable to the general idea of natural diversity, we intended to arouse interest in the ecologically oriented reader to evoke intensified future cooperation.

The results of this paper provide strong economic arguments in favour of the ecological concept “mixed forests”, if the mixed investment problem is analysed from the perspective of a risk-averse decision-maker. A proportion of 30 % beech would by no means achieve less than 97 % of the maximum economic utility. When natural diversity is considered analogous to these positive economic results of “mixed forests”, natural diversity of ecosystems should generally also render economic advantages (e.g. Figge, 2004). Facing an uncertain future as a severe environmental problem, the ecologically desired participation of natural tree species in forest management is therefore advisable even from an economic point of view, albeit native tree species may produce substantially smaller financial returns. While the ecological concept of mixed forests is very old, so far no study could demonstrate so clearly economic advantages of mixed forests.

On a scientific basis the theory of portfolio was first applied to forestry in central Europe by Weber (2002). Weber based his study on a completely different approach. He did not consider the establishment of mixed forests. While ignoring hazard risks he preferred to consider perfectly sustainable composed model forests of each tree species, which could be bought or sold on an unlimited scale. This assumption is not realistic because firstly, no perfectly sustainable composed real forest exists (Möhring, 1986). Secondly, forests consisting of the desired tree species cannot be bought and sold on an unlimited scale. Indeed, Weber’s study was very valuable for European forest economics, whereas the approach in our paper seems to be more realistic. In forest practice a change in the tree species composition will only be achieved through the establishment of young stands.

Moreover, Weber (2002) pointed out that the forestland investment as a whole is an excellent component of a larger portfolio of investments particularly in the USA. This fact was used by one of the reviewers who criticised a low-return, risk-free investment considered as the alternative investment as not realistic. He pointed out that landowners would rather invest in a larger portfolio. Using a risk-free interest rate for calculation is, however, a method widely used in forest economics. An alternative would be to maximise the internal rate of return of mixed investments. In the German economics literature the method of calculating the internal rate of return for investment decisions is heavily criticised from the perspective of investment theory (Schmalen, 2002, p. 575). For our two species the method of internal rate of return would assume that all net revenue flows would be invested just at the internal rate of return of the individual species. An assumption also to be considered as unrealistic. Our intention was rather to provide a methodological step on applying financial theory to mixed forests in order

to stimulate discussion. We are aware that this step is not perfect, although it is, despite its imperfections, already able to support a better decision making on long-term forestry investments.

Our approach did also not consider the increase in managerial flexibility when growing more than one tree species. The timber harvesting can be intensified in beech stands when the spruce timber price is low and vice versa. In this paper we assumed a fixed time structure of the silvicultural interventions regardless of the timber price. The improved managerial flexibility can be expressed by the option value (e.g., Zinkhan, 1995). Ignoring the advantage of richer options means probably underestimating the benefits of mixed forests. Furthermore the results of this study are limited because applying conservative silvicultural concepts, it takes about 40 to 50 years until the native species stands can yield the first positive net revenue flows. Consequently, investing in more naturally composed forests is an investment for future generations. However, such investments have been carried out in Central Europe on large areas for 25 years (von Lükpe et al. 2004). Immediate diversification effects can be obtained only through purchasing older native species stands.

Basically, we feel that the findings of this paper will also apply to small scale mixtures. Although, we expect disadvantages with regards to “economies of scale” when mixing species throughout the stands, positive effects are likely. For example, reduced re-forestation costs will occur after a natural hazard, which just affected one species in a mixed forest stand.

Combining the strong economic advantages of the ecologically oriented concept “mixed forests” and its ecological advantages (e.g., Rothe, 1997, Rothe and Binkley, 2001, Rothe et al., 2002, Zhong and Makeschin, 2003), the idea “mixed forests” should become a general strategy in environmental decision-making in forestry.

Supported by the results obtained, the hypothesis:

“From an economic point of view the possible risk abatement for risk-averse investors to be achieved by mixed forests does not justify the decrease in the profitability involved.”,

can now clearly be rejected.

## **References**

- Ask, P. and Carlsson, M., 2000. Nature conservation and timber production in areas with fragmented ownership patterns. *Forest Policy and Economics* 1: 209-223.
- Armsworth, P.R. and Roughgarden, J.E., 2001. An invitation to ecological economics. *Trends in Ecology and Evolution* 16: 229-234.
- Balmford, A. et al., 2002. Economic Reasons for Conserving Wild Nature. *Science* 297: 950-953.
- Bavarian State Forest Service, 2001. Jahresbericht 2001. Bavarian State Ministry of Agriculture and Forests. Munich.
- Beltratti, Andrea, Chichilnisky, Gracelia and Heal, G., 1996. Sustainable Use of Renewable Resources. Social Science Research Network Electronic Library. <http://papers.ssrn.com>
- Bergen, V., Löwenstein, W. and Olschewski, R., 2002. Forstökonomie. Volkswirtschaftliche Grundlagen. München: Vahlen.
- Braze, R. and Mendelsohn, R., 1988. Timber Harvesting with Fluctuating Prices. *Forest Science* 34: 359-372.
- Chichilnisky, Gracelia, 1996. The economic value of the Earth's resources. *Trends in Ecology and Evolution* 11: 135-140.
- Dieter, M., 1997. Berücksichtigung von Risiko bei forstbetrieblichen Entscheidungen. Schriften zur Forstökonomie Bd. 16. Frankfurt/Main: J.D. Sauerländer's.
- Dieter, M., 2001. Land expectation values for spruce and beech calculated with Monte Carlo modelling techniques. *Forest Policy and Economics* 2: 157-166.
- Ellenberg, H., 1986. Vegetation Mitteleuropas mit den Alpen aus ökologischer Sicht. 4., verbesserte Auflage. Stuttgart: Ulmer.
- Emerton, L., 2000. Economics and the convention on biological diversity. IUCN - The World Conservation Union.
- Fabian, P. and Menzel, Annette, 1998. Wie sehen die Wälder von morgen aus - aus der Sicht eines Klimatologen. *Forstwissenschaftliches Centralblatt* 117: 339-354.

- Faustmann, M., 1849. Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirtschaft besitzen. Allgemeine Forst- und Jagd-Zeitung 20: 441-455.
- Figge, F., 2004. Bio-folio: applying portfolio theory to biodiversity. Biodiversity and Conservation 13: 827-849.
- Gayer, K., 1886. Der gemischte Wald. Berlin: Parey.
- Gerber, H.U. and Pafumi, G. 1998. Utility Functions: From Risk Theory to Finance. North American Actuarial Journal 2: 74-100. Haight, R.G., 1990. Feedback Thinning Policies for Uneven-Aged Stand Management with Stochastic Prices. Forest Science 36: 1015-1031.
- Hampicke, U., 1991. Naturschutz-Ökonomie. Stuttgart: Ulmer.
- Heal, G., 1985. Depletion and Discounting: A classical Issue in the Economics of Exhaustible Resources. In: American Mathematical Society: Proceedings of Symposia in Applied Mathematics 32: 33-43.
- Kant, S., 1999. Sustainable management of uneven-aged private forests: a case study from Ontario, Canada. Ecological Economics 30: 131-146.
- Knoke, T., Moog, M. and Plusczyk, N., 2001. On the effect of volatile stumpage prices on the economic attractiveness of a silvicultural transformation strategy. Forest Policy and Economics 2: 229-240.
- Knoke, Th., 2002. Value of perfect information on red heartwood formation in beech (*Fagus sylvatica* L.). Silva Fennica 36: 841-851.
- Knoke, T., 2004. Die Begründung von Mischbeständen: Eine Möglichkeit zur Minderung von Risiko? In: Deutscher Forstverein e.V. (ed.): 61. Jahrestagung, Forstwirtschaft im Dialog ... gemeinsam die Zukunft gestalten, Mainz 2003, Kongressbericht, Göttingen: 344-355.
- Kouba, J., 2002. Das Leben des Waldes und seine Lebensunsicherheit. Forstwissenschaftliches Centralblatt 121: 211-228.
- Lindner, M., 1999. Waldbaustrategien im Kontext möglicher Klimaänderungen. Forstwissenschaftliches Centralblatt 118: 1-13.

- Lüpke, B. von, 2004. Risikominderung durch Mischwälder und naturnaher Waldbau - ein Spannungsfeld. *Forstarchiv* 75: 43-50.
- Lüpke, B. von and Spellmann, H., 1997. Aspekte der Stabilität und des Wachstums von Mischbeständen aus Fichte und Buche als Grundlage für waldbauliche Entscheidungen. *Forstarchiv* 68: 167-179.
- Lüpke B. von, Ammer, C., Bruciamacchie, M., Brunner, A., Ceitel, J., Collet, C., Deuleuze, C., Di Placido, J., Huss, J., Jankovic, J., Kantor, P., Larsen, J. B., Lexer, M., Löff, M., Longauer, R., Madsen, P., Modrzynski, J., Mosandl, R., Pampe, A., Pommerening, A., Stefancik, J., Tesar, V., Thompson, R. and Zientarski, J., 2004. Silvicultural strategies for conversion. In: Spiecker, H., Hansen, J., Klimo, E., Skovsgaard, J. P., Sterba, H. and Teuffel, K. von (eds): *Norway spruce conversion - options and consequences*. Brill, Leiden, Boston, pp 121-164.
- Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance* 7: 77-91.
- Möhring, B., 1986. Dynamische Betriebsklassensimulation - Ein Hilfsmittel für die Waldschadensbewertung und Entscheidungsfindung im Forstbetrieb. *Berichte des Forschungszentrums Waldökosysteme-Waldsterben der Universität Göttingen* Band 20.
- Moser, O., 1995. Umwandlung unstandortsgemäßer Fichtenbestände in naturnahe Mischwälder im Bereich der Forstverwaltung Bad Goisern der österreichischen Bundesforste. *Centralblatt für das gesamte Forstwesen* 112: 33-51.
- Olsthoorn, A.F.M., Bartelink, H.H., Gardiner, J.J., Pretzsch, H. und Franc, A. (eds), 1999. *Management of mixed-species forest: silviculture and economics*. Wageningen: IBNDLO Scientific Contributions.
- Pflaumer, P., 1992. *Investitionsrechnung*. München, Wien: Oldenbourg.
- Pretzsch, H. and Utschig, H., 2000. Wachstumstrends der Fichte in Bayern. *Mitteilungen aus der Bayerischen Staatsforstverwaltung* Heft 49.
- Price, C., 1997. A critical note on a long-running debate in forest economics. *Forestry* 70: 389-397.
- Rothe, A., 1997. Einfluß des Baumartenanteils auf Durchwurzelung, Wasserhaushalt, Stoffhaushalt und Zuwachsleistung eines Fichten-Buchen-Misch-Bestandes am Standort Höglwald. *Forstliche Forschungsberichte München* Nr. 163.

- Rothe, A. and Binkley, D., 2001. Nutritional interactions in mixed species forests: a synthesis. *Canadian Journal of Forest Research* 31: 1855-1870.
- Rothe, A., Kreuzer, K. and Küchenhoff, H., 2002. Influence of tree species composition on soil and soil solution properties in two mixed spruce-beech stands with contrasting history in southern Germany. *Plant and Soil* 240: 47-56.
- Runzheimer, B., 1999. *Operations Research*. Wiesbaden: Gabler.
- Schmalen, H., 2002. *Grundlagen und Probleme der Betriebswirtschaft*. 12., überarbeitete Auflage. Stuttgart: Schäffer-Poeschel.
- Schraml, U. and Volz, K.-R., 2004. Conversion of coniferous forests - social and political perspectives. Findings from selected countries with special focus on Germany. In: Spiecker, H., Hansen, J., Klimo, E., Skovsgaard, J. P., Sterba, H. and Teuffel, K. von (eds): *Norway spruce conversion - options and consequences*. Brill, Leiden, Boston, pp 97-119.
- Seitschek, O., 1991. Mischwald als Ziel des Waldbaus. *Allgemeine Forstzeitschrift* 46: 1246-1251.
- Sharpe, W.F., 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance* 14: 425-442.
- Spiecker, H., 2003. Silvicultural management in maintaining biodiversity and resistance of forests in Europe-temperate zone. *Journal of Environmental Management* 67: 55-65.
- Spremann, K., 1996. *Wirtschaft, Investition und Finanzierung*. 5., vollständig überarbeitete, ergänzte und aktualisierte Auflage. München und Wien: Oldenbourg.
- Valsta, L.T., 1998. Economy in the short and long term perspectives. In: *New Stand Types in Boreal Forestry*. Finnish Forest Research Institute, research paper 714.
- Waller, L.A., Smith, D., Childs, J.E. and Leslie, A.R., 2003. Monte Carlo assessment of goodness-of-fit for ecological simulation models. *Ecological Modelling* 164: 49-63.
- Weber, M.-W., 2002. *Portefeuille- und Optionspreis-Theorie und forstliche Entscheidungen*. Schriften zur Forstökonomie Band 23. Frankfurt a.M.: Sauerländer's.
- Yool, A., 1999. Comments on the paper: on repeated parameter sampling in Monte Carlo simulations. *Ecological Modelling* 115: 95-98.

Zinkhan, F.C., 1995. Forest Economics: The Management of Options and Values. *Journal of Forestry* 93: 25-29.

Zhong, Z. and Makeshin, F. 2003. Soil biochemical and chemical changes in relation to mature spruce (*Picea abies*) forest conversion and regeneration. *Journal of Plant Nutrition and Soil Science* 166: 291-299.



**Table 1.** Harvestings per hectare and net timber prices employed

Period (10 years)	Age (years)	Harvestings (cubic meter/ha)		Average net timber prices (Euro/cubic meter)	
		Spruce	Beech	Spruce	Beech
P1	10	2	0	5.47	0.00
P2	20	4	0	9.41	0.00
P3	30	20	0	7.15	0.00
P4	40	34	30	12.94	7.28
P5	50	44	35	24.67	25.13
P6	60	51	40	42.20	37.48
P7	70	53	50	59.99	40.17
P8	80	49	60	67.97	38.97
P9	90	38	50	61.55	48.41
P10	100	724	40	45.83	54.66
P11	110	0	30	0.00	52.22
P12	120	2	567	5.47	56.98

**Table 2.** Parameters of regression analysis for the functional relationship between timber prices for beech and spruce ( $p(\text{beech}) = a + b \cdot p(\text{spruce})$ )

Market Model	n	Parameters		Root mean square error	r <sup>2</sup>	Pr>F
		a	b			
1 (up to 1979)	27	9.76	+0.64	4.83	0.72	<0.0001
2 (1980 onwards)	22	136.66	-0.57	8.89	0.38	0.0024

**Table 3.** Simulated financial return, standard deviation and correlation coefficient between financial returns (simulations over 10 periods)

	Market Model 1		Market Model 2	
	Spruce	Beech	Spruce	Beech
Financial return	4,387	2,570	4,398	2,599
Standard deviation	± 2,391	± 1,648	± 2,276	± 1,515
Correlation coefficient	+ 0.16		0.00	

**Table 4.** Correlation coefficients  $k$  between net revenue flows for beech and spruce with and without the influence of natural hazards (only periods in which substantial positive net revenue flows occurred where analysed)

Correlation coefficients $k$ between net revenue flows				
Period	Market Model 1		Market Model 2	
	All net revenue flows	Net revenue flows caused by hazards excluded	All net revenue flows	Net revenue flows caused by hazards excluded
5	+0.28	+0.83	-0.20	-0.63
6	+0.35	+0.83	-0.10	-0.59
7	+0.21	+0.85	-0.06	-0.60
8	+0.13	+0.85	-0.04	-0.59
9	+0.09	+0.84	-0.05	-0.63
10	+0.19	+0.86	0.00	-0.61

**Table 5.** Simulated financial return, standard deviation and correlation coefficient between financial returns for simulations over 50 periods and different production times for spruce (100 years) and beech (120 years) (n.s.: not significant)

	Market Model 1		Market Model 2	
	Spruce	Beech	Spruce	Beech
Financial return	5,526	2,813	5,538	2,793
Standard deviation	± 2,229	± 1,321	± 2,118	± 1,207
Correlation coefficient	+0.03 (n.s.)		0.00	

Figure captions

**Figure 1.** Effect of the coefficient of correlation  $k$  on the standard deviation of several compositions of two financial investments

**Figure 2.** Typical net revenue flows of spruce (*Picea abies* (L.) Karst.) and beech (*Fagus sylvatica* L.) (adopted from Knoke, 2004)

**Figure 3.** Survival probabilities of spruce and beech (according to Möhring, 1986, Dieter, 2001, Kouba, 2002, with alterations)

**Figure 4.** Hazard rates derived from survival probabilities

**Figure 5.** [a] Gross timber prices published by the Bavarian State Forest Administration (2001) for specific log sizes and qualities (spruce: mid diameter 25 to 29 cm, beech: mid diameter 35 to 39 cm, quality grade b). [b] Example of a timber price scenario (No. 500). Note that [a] was based on a resolution of 1 year while [b] shows a resolution of 10 years as thinnings were carried out after 10-year-periods in the model

**Figure 6.** Correlation between timber prices for beech and spruce for two different periods (period 1: 1953 to 1979, period 2: 1980 to 2001, the year 1991 was excluded as an outlier)

**Figure 7.** Net revenue flows considering risk and their dispersion

**Figure 8.** Effect of several combinations of spruce and beech on risk and financial return when the production time is considered identical

**Figure 9.** Effect of diversified production times on the risk of the portfolio of spruce and beech

**Figure 10.** Effect of an increased hazard risk for spruce on the portfolio of spruce and beech

**Figure 11.** Preference indices of several forest portfolio compositions for a risk-averse (a) and a very risk-averse decision-maker (b)

Figure 1

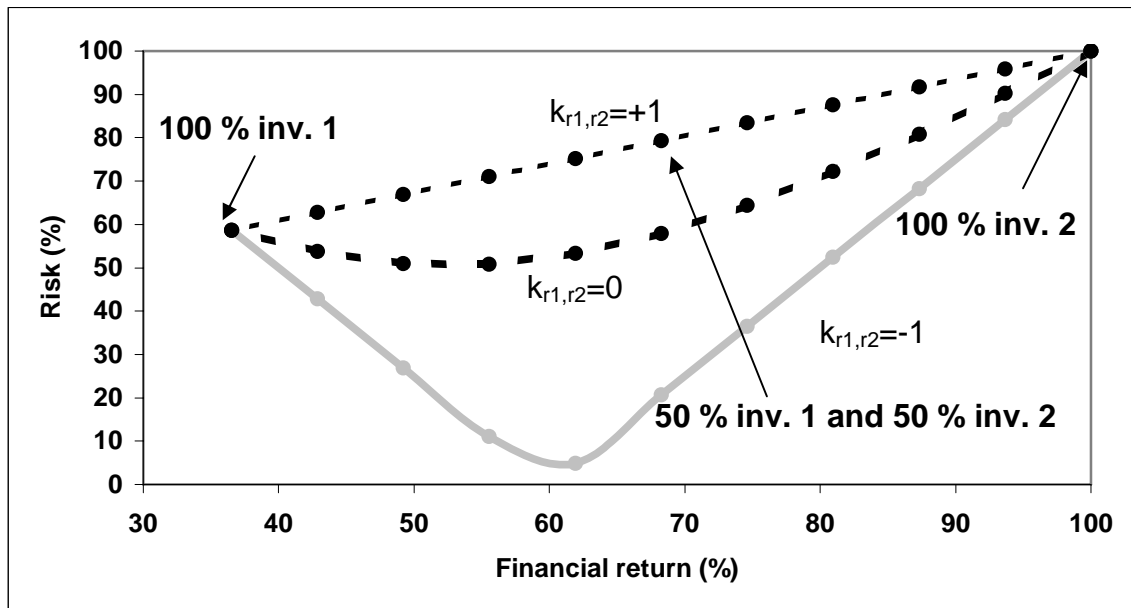


Figure 2

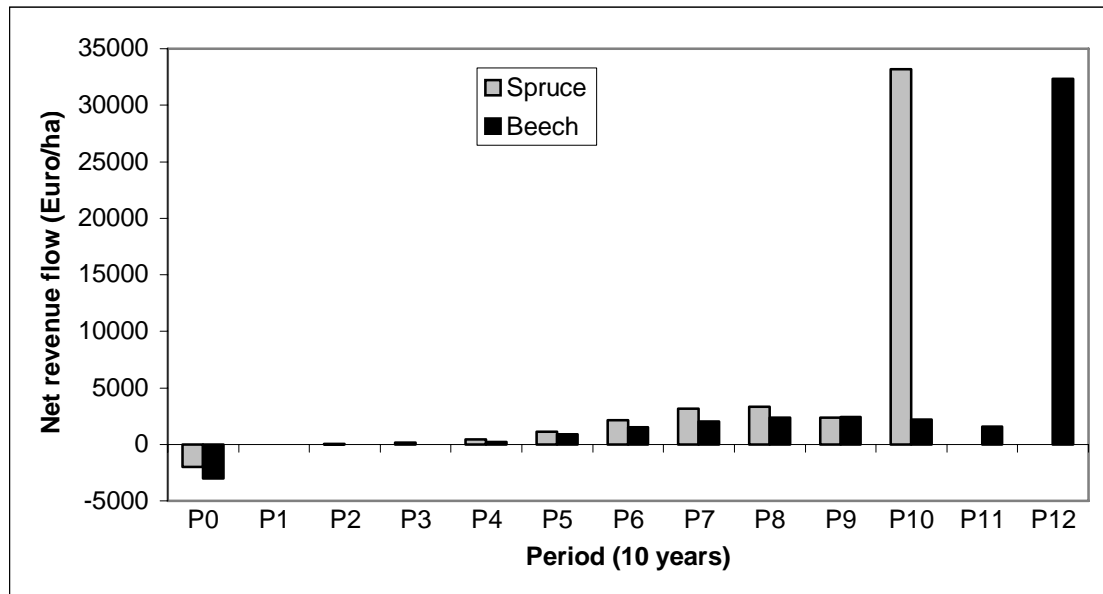


Figure 3

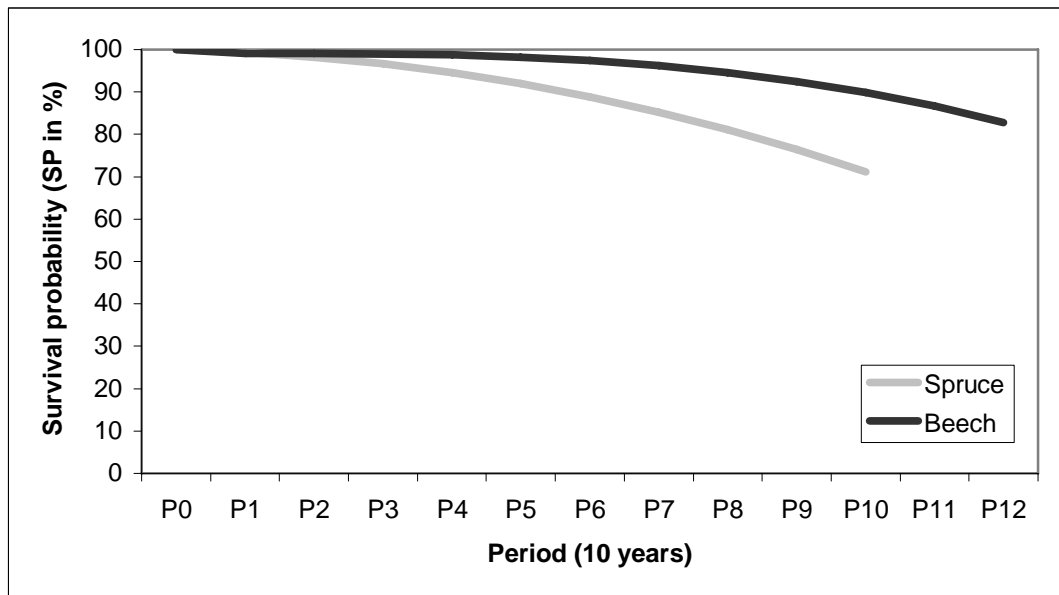


Figure 4

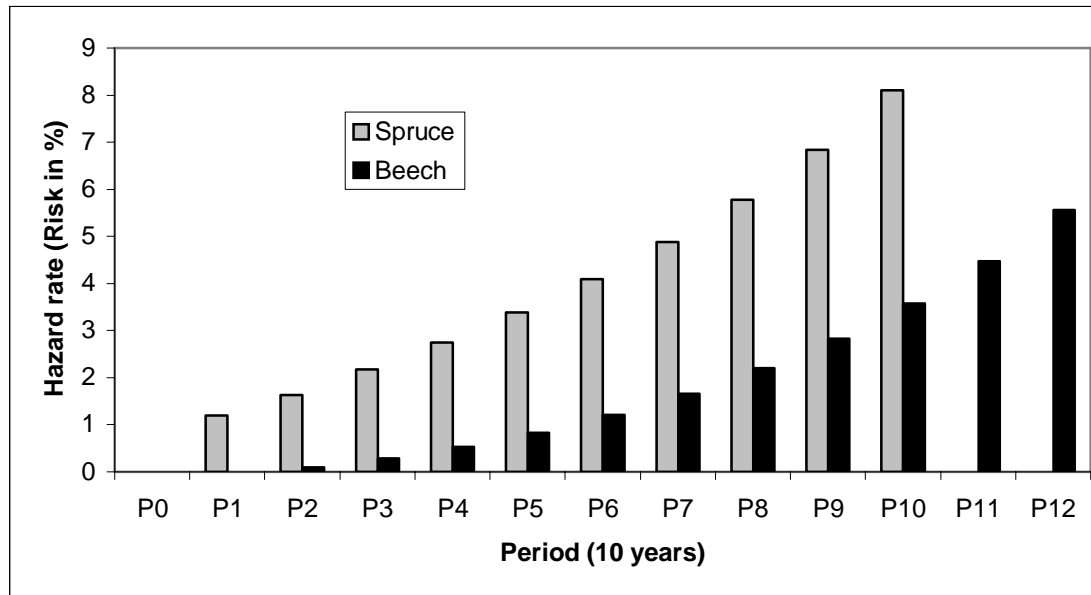




Figure 5

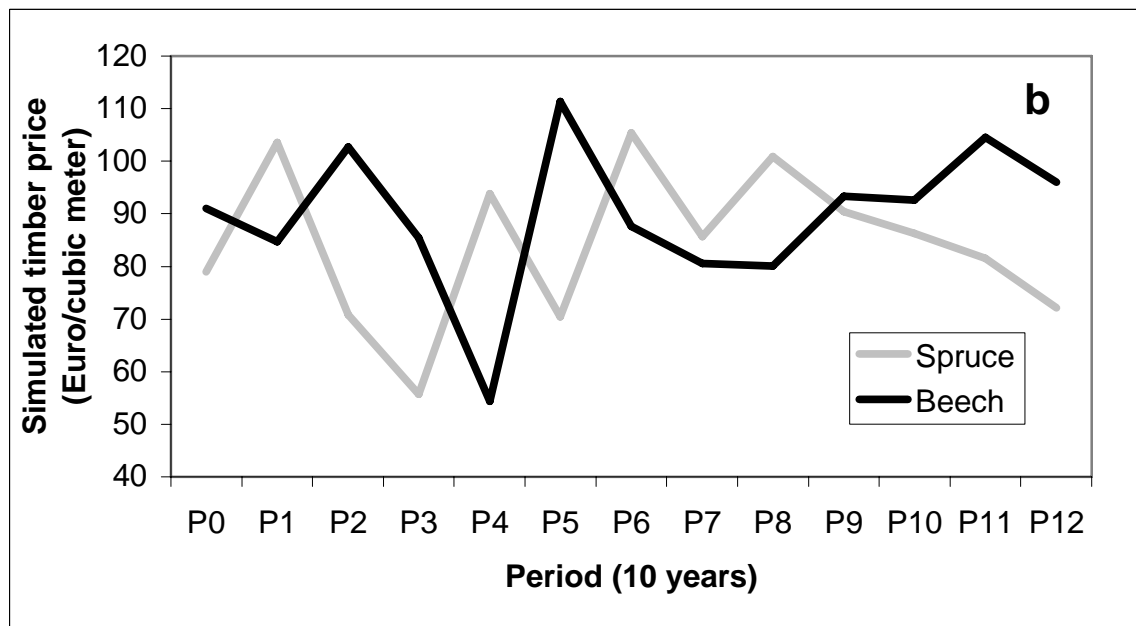
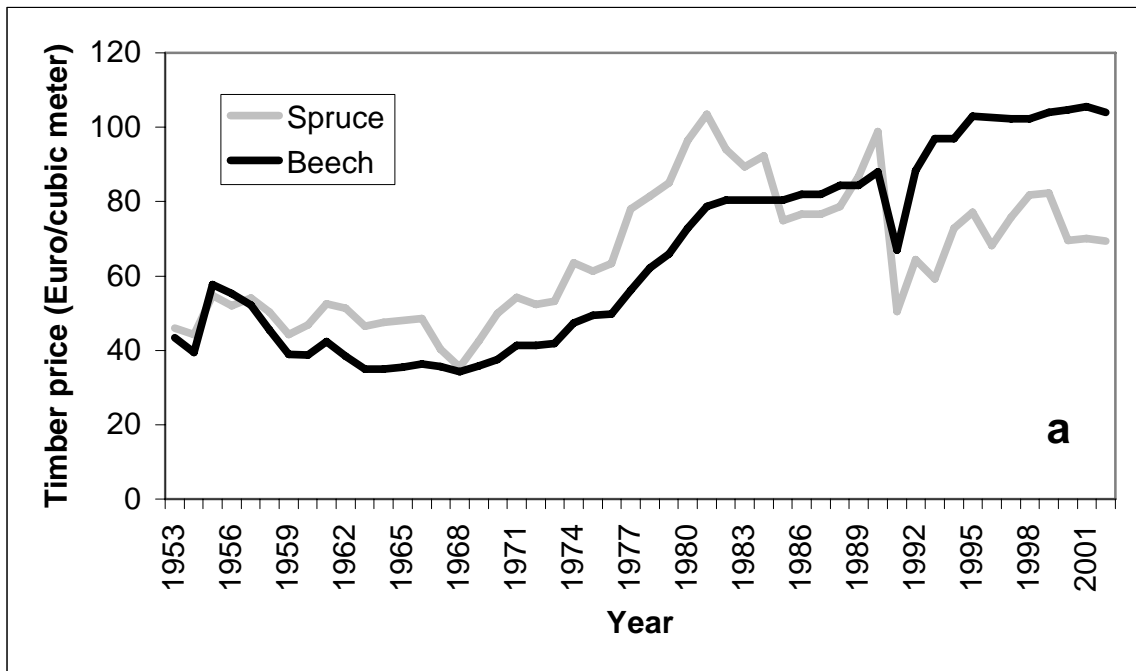


Figure 6

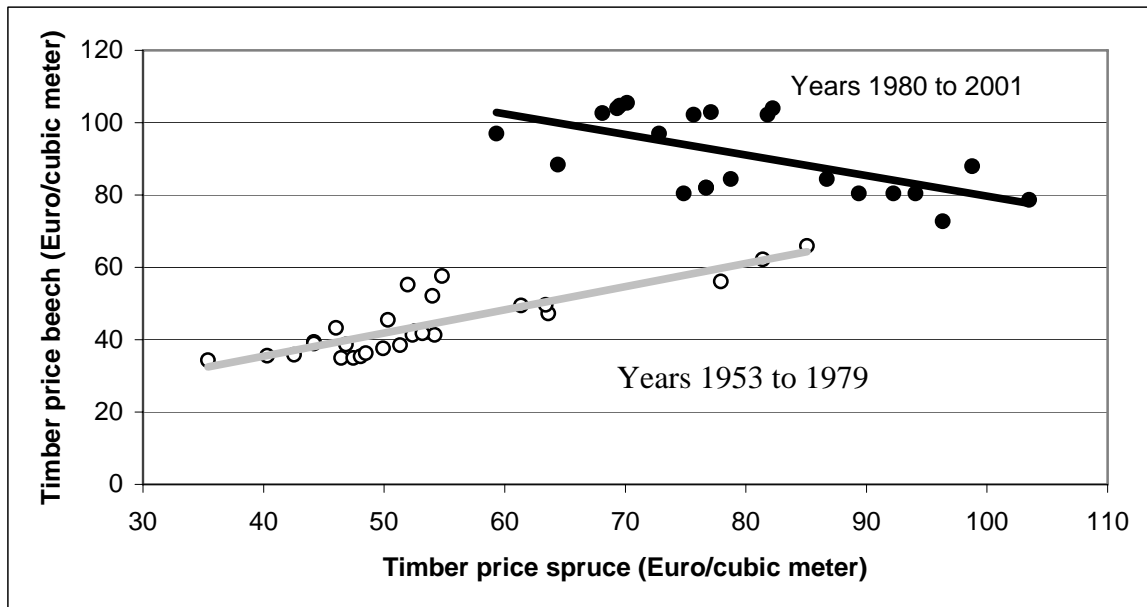


Figure 8

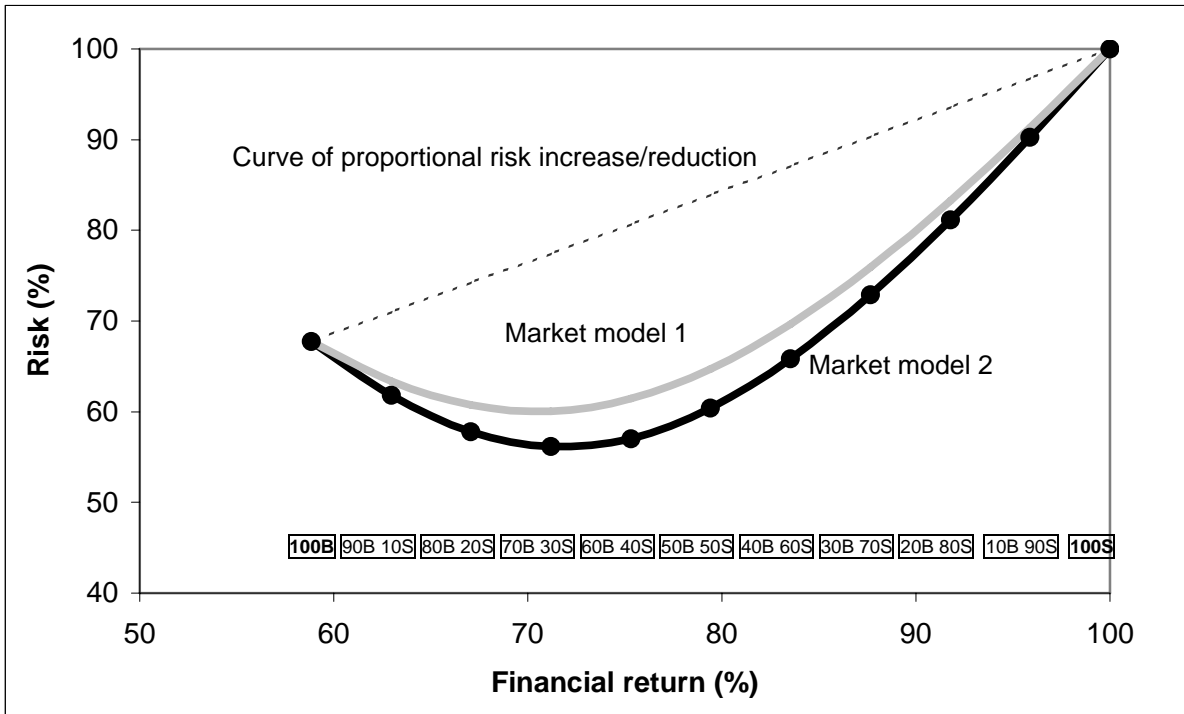


Figure 9

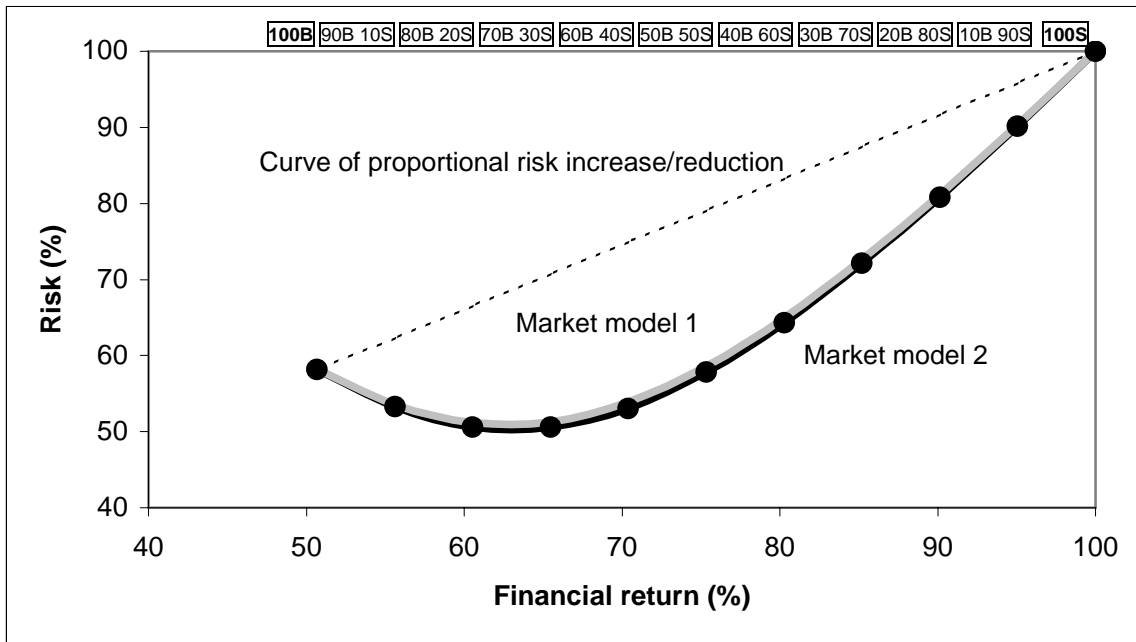


Figure 10

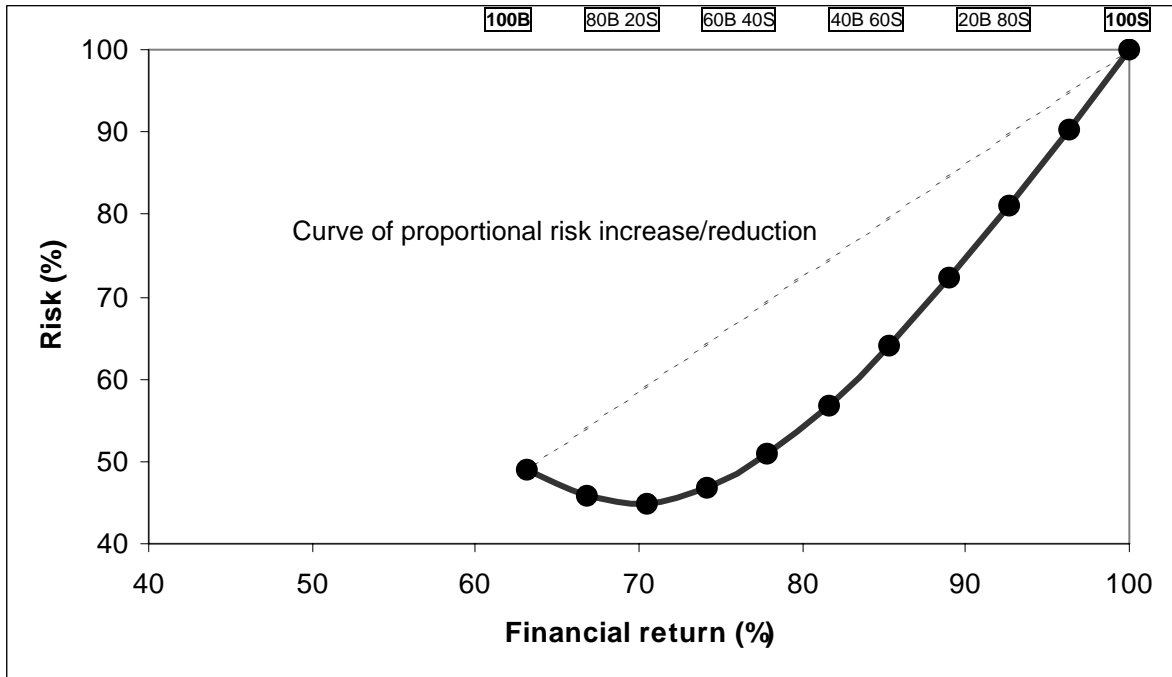


Figure 11

