Recent Advances in Reduced-Rank Adaptive Filtering With Application to High-Speed Wireless Communications

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ABSTRACT

The *Wiener filter* (WF) estimate of a desired signal from a vector observation $\mathbf{x}_0[n]$ is optimal in the *Minimum Mean Square Error* (MMSE) sense, and is employed in many applications because it is easily implemented and only relies on second order statistics. If the observation $\mathbf{x}_0[n]$ is of high dimensionality, though, a reduced-rank approach is needed in order to reduce computational complexity and lessen sample support requirements. In the *Principal Components* (PC) method, the observation signal is transformed to lower dimensionality by a matrix composed of the principal eigenvectors of the autocorrelation matrix of $\mathbf{x}_0[n]$. However, the PC method is suboptimum as it only relies on the autocorrelation matrix and does not factor in the cross-correlation vector between the desired signal and the data $\mathbf{x}_0[n]$ in choosing the basis vectors for the reduced dimension subspace. Goldstein, Reed, & Scharf recently developed the *Multi-Stage Nested Wiener Filter* (MSNWF) in which the reduced dimesion subspace is inherently the Krylov subspace generated by the autocorrelation matrix and the cross-correlation vector. The MSNWF provides better peformance than the PC method at a substantially reduced computational cost. We here provide an overview of the MSNWF and a number of recent results related to both our conceptual understanding of the MSNWF and efficient implementations of the MSNWF. An application of the MSNWF to space-time equalization for the CDMA Forward Link for Third Generation cellular communications is presented demonstrating its efficacy.

Keywords: adaptive filtering, conjugate gradients, reduced-rank equalization, space-time processing, wireless communications, CDMA, multipath propagation.

1 Introduction

The *Wiener filter* (WF) estimate of an unknown signal $d_0[n]$ from a vector observation $\mathbf{x}_0[n]$ is optimal in the *Minimum Mean Square Error* (MMSE) sense, and optimal in the *Bayesian sense* if the signals $d_0[n]$ and $\mathbf{x}_0[n]$ are jointly Gaussian random variables. The WF is employed in many applications because it is easily implemented and only relies on second order statistics. However, the resulting filter depends upon the inverse of the covariance matrix, $\mathbf{R}_{\mathbf{x}_0}$. If the observation $\mathbf{x}_0[n]$ is of high dimensionality, a reduced-rank approach is needed in order to reduce computational complexity and lessen sample support requirements. The current strong need for reduced-rank adaptive filtering arises from the growing disparity between the large number of degrees of freedom in the next generation of wireless communications systems, radar systems, sonar systems, etc., and limitations on sample support size due to high mobility, high sensitivity to small movements/perturbations, etc.

In the *Principal Components* (PC) method [Hot33], the observation signal is transformed to lower dimensionality by a matrix composed of the principal eigenvectors of $\mathbf{R}_{\mathbf{x}_0}$. However, the PC method only takes into account the statistics of the observation signal and does not consider the relation to the desired signal. The *Cross-Spectral Metric* (CSM) of Goldstein et. al. [GR97b] is an alternative reduced-rank method that selects those eigenvectors that maximize a metric based on the cross-correlation vector between the observation and the desired signal and does not choose the principal eigenvectors, in general. However, Goldstein, Reed, & Scharf ultimately presented the *Multi-Stage Nested Wiener Filter* (MSNWF) [GRS98] which showed that rank reduction based on the eigenvectors is suboptimum. The MSNWF does not require the computation of eigenvectors and is thus computationally advantageous as well.

We note several applications where the MSNWF has been applied with great success:

1. Goldstein and Reed have sucessfully applied the MSNWF to a broad spectrum of radar signal processing problems [GR97b, GRZ99]

2. Honig applied MSNWF to Multi-User Access Interference (MAI) suppression for asynchronous CDMA operating in code-space where the weight vector dimensionality can be quite high [HG00, HX99]. Honig et. al. showed that the number of necessary MSNWF stages even for a heavily loaded CDMA system is a mere fraction of the subspace dimension required by the eigen-space based methods.

3. Willsky has not applied MSNWF per se, but has applied Krylov subspace estimation principles to the problem of error variance estimation in multi-resolution image processing [SW99]. The Krylov subspace basis framework makes this work inherently related to the MSNWF.

4. Myrick, Zoltowski, and Goldstein have applied the MSNWF to interference suppression for GPS receivers [Zol00aa, Zol00a, Zol00b, Zol00c] and equalization for the forward-link CDMA with long code [Zol00d, Zol00e, Zol00f]

5. Autoregressive (AR) spectral estimation is inherently related to linear prediction (LP), thereby establishing well known ties between LP based spectral estimation and adaptive filtering. This motivated the recent development of a reduced-rank based spectral estimation scheme based on the MSNWF by Witzgall, Goldstein, and Zoltowski [Zol00h, Zol00i]. In this application, the MSNWF effects power minimization under a unity tap constraint on the "weight" vector. Initializing the forward recursion of the MSNWF with the standard basis vector $\mathbf{e}_1$, where $\mathbf{e}_i$ contains all zeros except for a one in the i-th position (so that the first value of each data block effectively serves as a "desired" signal), simulations reveal the MSNWF to rapidly converge to a "weight" vector that lies in the noise subspace. The reciprocal of the magnitude square of the Fourier Transform of this "weight" vector is the spectral estimate and has been observed to exhibit a low background level with very low probability of false alarm peaks. The initial simulations are quite astounding: stopping the MSNWF at stage 3 facilitated reliable estimation of the directions of 40 signals impinging upon a linear array of 128 antenna elements [Zol00h, Zol00i]. The performance achieved is similar to MUSIC without the need for the computating eigenvectors or the need to estimate the number of sources prior to forming the spectral estimate.

There have been a number of recent fundamental advances relative to the MSNWF. These include three recent discoveries:

1. Ricks and Goldstein [Ricks00] showed that the MSNWF can be implemented without blocking matrices as required in the original algorithmic formulation. This further reduces the computational complexity of the MSNWF relative to full-rank RLS or PC based reduced-rank adaptive filtering.

2. Ricks and Goldstein [Ricks00] developed a lattice/modular MSNWF structure facilitating an efficient data-level implementation as an alternative to the original covariance level implementation. Avoiding the need to form a covariance matrix is advantageous since (i) it reduces computationally complexity, (ii) it facilitates real-time implementation, and (iii) because there may not be enough sample support to form a reliable covariance matrix estimate, especially when the data vector is high-dimensional and/or the signal statistics are rapidly time-varying.

3. Honig and Xiao [HX99] have proven an inherent relationship between MSNWF and Krylov subspace estimation: stopping the MSNWF at stage D constrains the weight vector to lie in the D-dimensional subspace spanned by $\{\ \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}, \mathbf{R}_{\mathbf{x_0}}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}, ..., \mathbf{R}_{\mathbf{x_0}}^{D-1}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\ \}$, where $\mathbf{R}_{\mathbf{x_0}}$ is the correlation matrix of the observed data and $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}$ is the cross-correlation vector between the observation data and the desired signal. This is a very important discovery relative to the theoretical underpinnings of the MSNWF and its relation to other techniques employed in numerical analysis that operate in a Krylov subspace

2 Brief development of original MSNWF.

Referring to Figure 1, the desired signal $d_0[n] \in \mathbf{C}$ is estimated by applying the linear filter $\mathbf{w} \in \mathbf{C}^N$ to the observation signal $\mathbf{x}_0[n] \in \mathbf{C}^N$. The variance of the estimation error $\varepsilon_0[n] = d_0[n] - \hat{d}_0[n] = d_0[n] - \mathbf{w}^H\mathbf{x}_0[n]$ is the *mean squared error* $\mathrm{MSE}_0 = \mathcal{E}\{|\varepsilon_0|^2\} = \sigma_{d_0}^2 - \mathbf{w}^H \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} - \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}^H \mathbf{w} + \mathbf{w}^H \mathbf{R}_{\mathbf{x_0}} \mathbf{w}$, where the covariance matrix of the observation $\mathbf{x}_0[n]$ is $\mathbf{R}_{\mathbf{x_0}} = \mathcal{E}\{\mathbf{x}_0[n]\mathbf{x}_0^H[n]\} \in \mathbf{C}^{N\times N}$. The variance of the desired signal $d_0[n]$, $\sigma_{d_0}^2 = \mathcal{E}\{|d_0[n]|^2\}$, and the cross-correlation between $d_0[n]$ and $\mathbf{x}_0[n]$ is denoted $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} = \mathcal{E}\{\mathbf{x}_0[n] d_0^*[n]\}$.

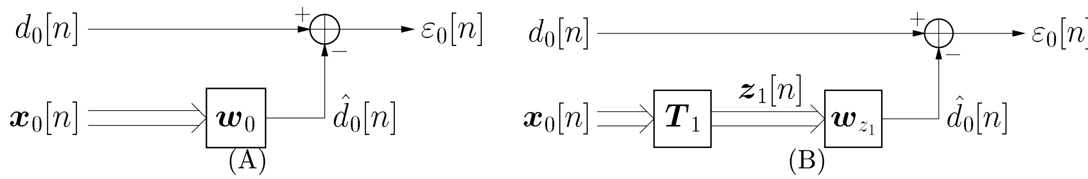

Figure 1: (A) Weiner Filter. (B) Same but with Full Rank (Square) Matrix Pre-Filtering.

The Wiener Filter $\mathbf{w}_0$ minimizing the *mean squared error* (MSE) is the solution to the Wiener-Hopf equation

$$\mathbf{R}_{\mathbf{x_0}} \mathbf{w}_0 = \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} \quad \Rightarrow \quad \mathbf{w}_0 = \mathbf{R}_{\mathbf{x_0}}^{-1} \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} \in \mathbf{C}^N. \tag{1}$$

The *minimum mean squared error* (MMSE) achieved with with the WF is

$$\mathrm{MMSE}_0 = \sigma_{d_0}^2 - \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}^H \mathbf{R}_{\mathbf{x_0}}^{-1} \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}. \tag{2}$$

As discussed previously, the *Multi-Stage Nested Wiener Filter* (MSNWF) was developed by Goldstein et. al. [GR97a, GRS98] as a means for computing an approximate solution of the Wiener-Hopf equation (cf. Equation 1) that does not require the inverse or the eigenvalue decomposition of the covariance matrix. The approximation for the Wiener filter is found by stopping the recursive algorithm after D steps, hence, the approximation lies in a D-dimensional subspace of $\mathbf{C}^N$. To briefly develop the *Multi-Stage Nested Wiener Filter* (MSNWF), we first note the following theorem which is well-known and easy to prove.

Theorem 1 *If the observation $\mathbf{x}_0[n]$ to estimate $d_0[n]$ is pre-filtered by a full-rank matrix $\mathbf{T} \in \mathbf{C}^{N\times N}$, i. e., $\mathbf{z}_1[n] = \mathbf{T}\mathbf{x}_0[n]$, the Wiener filter $\mathbf{w}_{\mathbf{z_1}}$ to estimate $d_0[n]$ from $\mathbf{z}_1[n]$ leads to the same minimum MSE.*

Applying a full rank pre-filtering matrix of the form

$$\mathbf{T}_1 = \begin{bmatrix} \mathbf{h}_1^H \\ \mathbf{B}_1 \end{bmatrix} \in \mathbf{C}^{N\times N} \tag{3}$$

we obtain the new observation signal

$$\mathbf{z}_1 = \mathbf{T}_1 \mathbf{x}_0[n] = \begin{bmatrix} \mathbf{h}_1^H \mathbf{x}_0[n] \\ \mathbf{B}_1 \mathbf{x}_0[n] \end{bmatrix} = \begin{bmatrix} d_1[n] \\ \mathbf{x}_1[n] \end{bmatrix} \in \mathbf{C}^N \tag{4}$$

which does not change the estimate $\hat{d}_0[n]$ when the MSE is minimized as indicated previously. The rows of $\mathbf{B}_1$ are chosen to be orthogonal to $\mathbf{h}_1^H$ so that $\mathbf{B}_1$ is referred to as a Blocking Matrix.

$$\mathbf{B}_1 \mathbf{h}_1 = \mathbf{0} \qquad \text{or} \quad \mathbf{B}_1 = \mathrm{null}(\mathbf{h}_1^H)^H. \tag{5}$$

The intuitive choice for the first row, $\mathbf{h}_1^H$, is the vector which, when applied to $\mathbf{x}_0[n]$, gives a scalar signal $d_1[n]$ that has maximum correlation with the desired signal $d_0[n]$. Constraining $\|\mathbf{h}_1\|_2 = 1$ and forcing $d_1[n]$ to be "in-phase" with $d_0[n]$, i. e. the correlation between $d_0[n]$ and $d_1[n]$ is real-valued, without loss of generality, leads to following optimization problem $\mathbf{h}_1 = \arg\max_{\mathbf{h}} \mathcal{E}\{\mathrm{Real}(d_1[n]d_0^*[n])\}$ or $\mathbf{h}_1 = \arg\max_{\mathbf{h}} \frac{1}{2}(\mathbf{h}^H \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} + \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}^H \mathbf{h})$, subject to $\mathbf{h}^H\mathbf{h} = 1$. The solution is the normalized matched filter

$$\mathbf{h}_1 = \frac{\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}}{\|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2} \in \mathbf{C}^N. \tag{6}$$

The solution to the Wiener-Hopf equation associated with the transformed system in Figure 1 (B) is

$$\mathbf{w}_{\mathbf{z_1}} = \mathbf{R}_{\mathbf{z_1}}^{-1} \mathbf{r}_{\mathbf{z_1},\mathbf{d_0}} \in \mathbf{C}^N, \quad \text{where:} \quad \mathbf{R}_{\mathbf{z_1}} = \begin{bmatrix} \sigma_{d_1}^2 & \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}}^H \\ \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}} & \mathbf{R}_{\mathbf{x_1}} \end{bmatrix} \in \mathbf{C}^{N\times N} \tag{7}$$

is the covariance matrix of $\mathbf{z}_1[n]$, $\sigma_{d_1}^2 = \mathcal{E}\{|d_1[n]|^2\} = \mathbf{h}_1^H \mathbf{R}_{\mathbf{x_0}} \mathbf{h}_1$, $\mathbf{r}_{\mathbf{x_1},\mathbf{d_1}} = \mathcal{E}\{\mathbf{x}_1[n] d_1^*[n]\} = \mathbf{B}_1 \mathbf{R}_{\mathbf{x_0}} \mathbf{h}_1 \in \mathbf{C}^{N-1}$, and $\mathbf{R}_{\mathbf{x_1}} = \mathcal{E}\{\mathbf{x}_1[n]\mathbf{x}_1^H[n]\} = \mathbf{B}_1 \mathbf{R}_{\mathbf{x_0}} \mathbf{B}_1^H \in \mathbf{C}^{(N-1)\times(N-1)}$. By design, the cross-correlation between $\mathbf{z}_1[n]$ and $d_1[n]$ is a scalar multiple of the standard basis vector $\mathbf{e}_1$, where $\mathbf{e}_i$ denotes a unit norm vector with a one in the i-th position and zeroes elsewhere.

$$\mathbf{r}_{\mathbf{z_1},\mathbf{d_0}} = \mathbf{T}_1 \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} = \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2 \ \ \mathbf{e}_1 \in \mathbf{R}^N, \tag{8}$$

Thus, the Wiener filter $\mathbf{w}_{\mathbf{z_1}}$ of the pre-filtered signal $\mathbf{z}_1[n]$ is just a weighted version of the first column of the inverse of the covariance matrix $\mathbf{R}_{\mathbf{z_1}}$ in Equation (7). Applying the matrix inversion lemma for partitioned matrices [GR97a, GRS98] yields

$$\mathbf{w}_{\mathbf{z_1}} = \alpha_1 \begin{bmatrix} 1 \\ -\mathbf{R}_{\mathbf{x_1}}^{-1} \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}} \end{bmatrix} \in \mathbf{C}^N, \qquad \text{where:} \ \ \alpha_1 = \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2 (\sigma_{d_1}^2 - \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}}^H \mathbf{R}_{\mathbf{x_1}}^{-1} \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}})^{-1}. \tag{9}$$

Equation (9) is the key equation to understanding the basic concept underlying the MSNWF.

The most important observation in Equation (9) is that the vector in brackets, when applied to $\mathbf{z}_1[n]$, gives the error signal $\varepsilon_1[n]$ of the Wiener filter that estimates $d_1[n]$ from $\mathbf{x}_1[n]$. That is,

$$\varepsilon_1[n] = d_1[n] - \hat{d}_1[n] = d_1[n] - \mathbf{w}_1^H \mathbf{x}_1[n] = \left[1, -\mathbf{w}_1^H\right] \mathbf{z}_1[n] \tag{10}$$

achieved with the Wiener filter below (again, $\mathbf{r}_{\mathbf{x_1},\mathbf{d_1}} = \mathcal{E}\{\mathbf{x}_1[n] d_1^*[n]\} = \mathbf{B}_1 \mathbf{R}_{\mathbf{x_0}} \mathbf{h}_1$ and $\mathbf{R}_{\mathbf{x_1}} = \mathcal{E}\{\mathbf{x}_1[n]\mathbf{x}_1^H[n]\} = \mathbf{B}_1 \mathbf{R}_{\mathbf{x_0}} \mathbf{B}_1^H$):

$$\mathbf{w}_1 = \mathbf{R}_{\mathbf{x_1}}^{-1} \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}} \in \mathbf{C}^{N-1}. \tag{11}$$

Referring to Figure 2, another key observation is that α_1 may be interpreted as a scalar Wiener filter for estimating

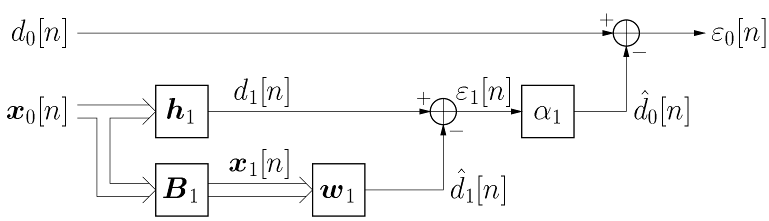

Figure 2: MSNWF after the First Step.

$d_0[n]$ from the error $\varepsilon_1[n]$. To see this, the scalar Wiener-Hopf Equation is $\mathcal{E}\{|\varepsilon_1[n]|^2\}\alpha_1 = \mathcal{E}\{\varepsilon_1^*[n] d_0[n]\}$. From previous definitions and the blocking property in (5), it is easily shown that

$$\mathcal{E}\{|\varepsilon_1[n]|^2\} = \sigma_{d_1}^2 - \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}}^H \mathbf{w}_1 = \sigma_{d_1}^2 - \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}}^H \mathbf{R}_{\mathbf{x_1}}^{-1} \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}}; \qquad \mathcal{E}\{\varepsilon_1^*[n] d_0[n]\} = \mathbf{h}_1^H \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}} = \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2 \tag{12}$$

Thus, we have $\alpha_1 = \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2 (\sigma_{d_1}^2 - \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}}^H \mathbf{R}_{\mathbf{x_1}}^{-1} \mathbf{r}_{\mathbf{x_1},\mathbf{d_1}})^{-1}$, which agrees with (9).

These observations relative to stage 1 of the decomposition, particularly Equations (10) and (11), lead naturally to the next stage of the MSNWF decomposition. In the second stage, the output of the Wiener filter $\mathbf{w}_1$ with dimension $N-1$ is replaced by the weighted error signal $\varepsilon_2[n]$ of a Wiener filter which estimates the output signal $d_2[n]$ of the matched filter $\mathbf{h}_2$ from the blocking-matrix output $\mathbf{x}_2[n] = \mathbf{B}_2 \mathbf{x}_1[n]$. Following this through N stages, we have the original formulation of the MSNWF depicted in Figure 3. The reduced-rank MSNWF of rank D is easily obtained by stopping the MSNWF decomposition after $D-1$ steps and replacing the last Wiener filter $\mathbf{w}_{D-1}$ by the appropriate matched filter.

To understand the importance of the innovations proposed herein relative to the MSNWF, it is important to keep in mind two key drawbacks of the original algorithmic formulation of the MSNWF depicted in Figure 3. First, the nested matched filters $\mathbf{h}_i$ and blocking matrices $\mathbf{B}_i$ are computed sequentially through the forward recursion. Only after the forward recursion is truncated at some stage D to effect rank-reduction can one then subsequently execute the backwards recursion to compute the scalar Wiener filters w_i in reverse order. If one wanted to determine the MSE as each new stage is added, to decide which stage to terminate at, for example, one had to ostensibly execute both the forwards and backwards recursion on a per stage basis since the backwards recursion coefficients completely change with each new stage that is added. Second, formation of the blocking matrices represents a significant

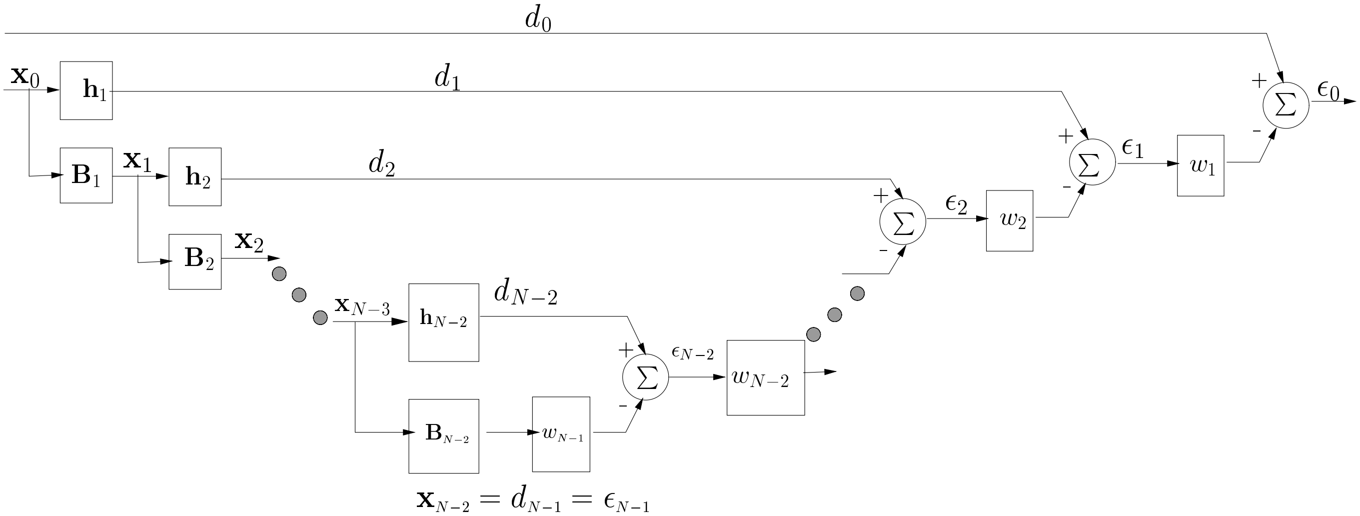

Figure 3: Structure of initial conception of the Multistage Nested Weiner Filter.

computational task. Both of these drawbacks are being eliminated with the innovations proposed herein. Note that Goldstein and Ricks [Ricks00] recently developed a data-level modular/lattice structure for the MSNWF that also avoids the formation of blocking matrices, but still requires a backwards recursion after the forward recursion. Note that the filter bank underlying the MSNWF can be synthesized without actually forming the covariance matrix $\mathbf{R}_{\mathbf{x_0}}$. This is because at the i-th stage the Wiener filter is replaced by a normalized matched filter that is simply the cross-correlation between the new observation $\mathbf{x}_i[n]$ and the new desired signal $d_i[n]$. Thus, only an estimation of this cross-correlation is needed with each new stage that is added. Observing Figure 3, it is straightforward to

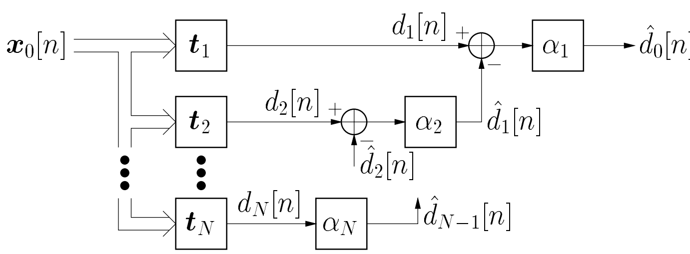

Figure 4: MSNWF as a Filter Bank

see that each new desired signal $d_i[n], i = 1, \ldots, N$, is the output of a length N filter

$$\mathbf{t}_i = (\prod_{k=1}^{i-1} \mathbf{B}_k^H)\mathbf{h}_i \in \mathbf{C}^N. \tag{13}$$

That is, the chain of nested Weiner Filters in Figure 3 may be replaced by the simple filter bank in Figure 4, where the N length filters $\mathbf{t}_i$ are computed in terms of the matched filters $\mathbf{h}_i$'s and the blocking matrices $\mathbf{B}_i$'s in Figure 3 according to Equation (13). Referring to Figure 4, a very important property is that the pre-filtered observation vector

$$\mathbf{d}[n] = [d_1[n], \ldots, d_N[n]]^T, \tag{14}$$

has a tri-diagonal covariance matrix [GRS98]. This is because the matched filter $\mathbf{t}_i$ is designed to retrieve all information of $d_{i-1}[n]$ that can be found in $\mathbf{x}_{i-1}[n]$. The output of $\mathbf{t}_i$, $d_i[n]$, is thus correlated with $d_{i-1}[n]$ and with $d_{i+1}[n]$, because $\mathbf{t}_{i+1}$ is the matched filter to find $d_i[n]$. But $d_{i+1}[n]$ includes no information about $d_{i-1}[n]$, since the input of $\mathbf{t}_{i+1}$ was pre-filtered by the blocking matrix $\mathbf{B}_{i+1}$. Consequently, $d_i[n]$ is only correlated with its neighbors $d_{i-1}[n]$ and $d_{i+1}[n]$.

3 Illustrative 3G CDMA Simulation

This illustrative example serves to demonstrate that the MSNWF converges much more quickly than LMS, and more quickly than RLS as well, with reduced computational complexity relative to RLS. There are no eigenvectors to compute or track. Thus, the MSNWF offers both improved performance and reduced computational complexity relative to PC based reduced-rank filtering as well. [GRZ99, HG00, HX99, Zol00aa, Zol00a]. The example also demonstrates that the MSNWF can be used to efficiently solve the Weiner-Hopf Equations in the case of block-mode processing.

A CDMA **forward link** was simulated similar to one of the options in the US cdma2000 proposal. The chip rate was 3.6864 MHz ($T_c = 0.27\mu s$), 3 times that of IS-95. Simulations were performed for a "saturated cell": all 64 channel codes were "active" with equal power. For each user, each BPSK data symbol was spread with one of 64 Walsh-Hadamard sequences of length 64. The frequency selective nature of the multipath channel in a high-speed (wideband) 3G CDMA link destroys the advantage of employing orthogonal Walsh-Hadarmard sequences relative to avoiding multi-user access interference. The RAKE receiver thus performs poorly, especially in a saturated case. Chip-level equalization is thus effected at the receiver in order to estimate the synchronous sum signal transmitted from the base station and thereby effectively exploit the orthogonality of the Walsh-Hadamard codes [1, 2, 3].

All users were of equal power, and their signals were summed synchronously and then multiplied with a QPSK scrambling code of length 32678. The channels were modeled to have four equal-power multi-paths, the first one arriving at 0, the last at $10\mu s$ (corresponding to about 37 chips) and the other two delays picked at random in between. The multipath coefficients are complex normal, independent random variables with equal variance. The receiver was assumed to have a dual antenna. The arrival times at antenna 1 and 2 are the same, but the multipath coefficients are independent.

In the two base-station case, the channels are scaled so that the total energy from each of the two base-stations is equal at the receiver. The 4 multi-path arrivals from the 2nd base-stations are random, with maximum delay spread of $10\mu s$. SNR is defined to be the ratio of the sum of the average power of the received signals over all the channels, to the average noise power, after chip-matched filtering. The abscissa is the **post**-correlation SNR for *each* user which includes a processing gain of 10log(64) $\approx$ 18 dB.

Figure 5 plots the Mean-Square Error for the different reduced-rank methods as a function of the subspace dimension, D. The channel statistics and noise power are assumed to be known (i.e. perfect channel estimation). In the single base-station case, 5(a), the dimension of the full space is 114 (the equalizer length is 57 at each of 2 antennas, as multipath delay spread is 37 chips and the chip pulse waveform is cut off after 5 chips at both ends). The MSE for MSNWF is seen to drop dramatically with D, and achieves the performance of the full-rank Wiener filter at dimension approximately 7! In contrast, the dimensionality required for Principal Components method to achieve near optimum MMSE is more than twice the delay spread, and the required dimensionality for the Cross-spectral method is also high.

Figure 6(a) displays the BER curves obtained with the MSNWF for different sizes of the reduced-dimension subspace. The channel statistics are assumed to be known perfectly, so these curves serve as an informative upper bound on the performance. It is observed that even a 2-stage reduced-rank filter outperforms the RAKE at all SNR's and only a small number of stages of the MSNWF are needed in order to achieve near full-rank MMSE performance over a practical range of SNR's.

Figures 5(b) and 6(b) display similar plots, but for the "edge of cell" scenario corresponding to soft hand-off. Here we effect 4 channels at the receiver by sampling the received signal at twice the chip-rate at each antenna. The dimension of the full space is 228 which makes full rank processing quite cumbersome. Amazingly, the MSE for MSNWF still goes down very steeply with rank and achieves the full-rank value for a subspace dimension of only 8 or so. In the BER plots of Figure 6(b), the bit error is calculated for the "soft handoff" mode. With perfect channel estimation, the MSNWF can achieve uncoded BER's similar to the full-rank MMSE over a practical SNR range after stopping at stage as low as 5!

These plots suggest that MSNWF can achieve rapid adaptation in the case where the chip-level MMSE equalizer is adapted based on a pilot channel. Figure 7 plots the output SINR for different chip-level equalizers vs. time in symbols, at a fixed SNR. The MSNWF at stages 5 and 10 yields very good performance with low sample-support. The convergence rate is significantly better than that of full-rank RLS which even asymptotically does not beat the MSNWF of rank only 10! The LMS algorithm converges much more slowly.

For the two base-station case, the asymptotic SINR is lower for all the equalizers due to the added interference

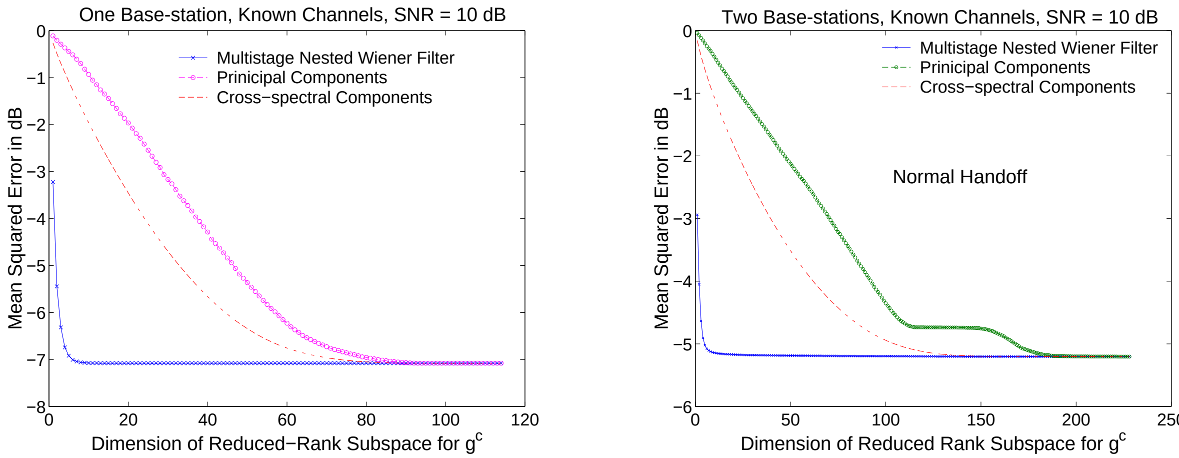

Figure 5: MSE vs Rank of Reduced Dimension Subspace

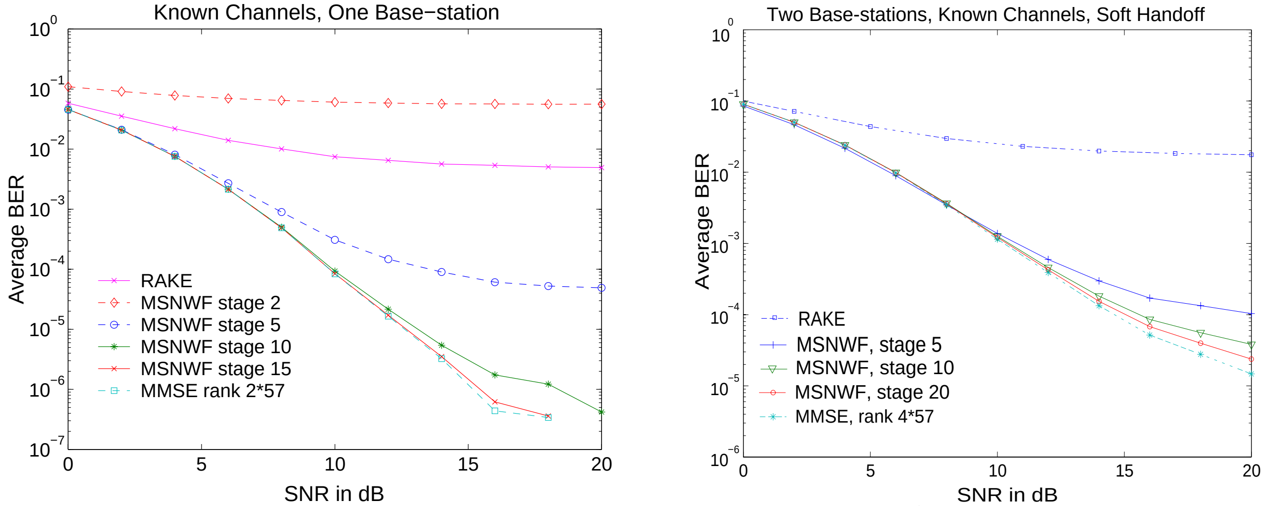

Figure 6: BER for Different Chip-level Equalizers for CDMA Downlink.

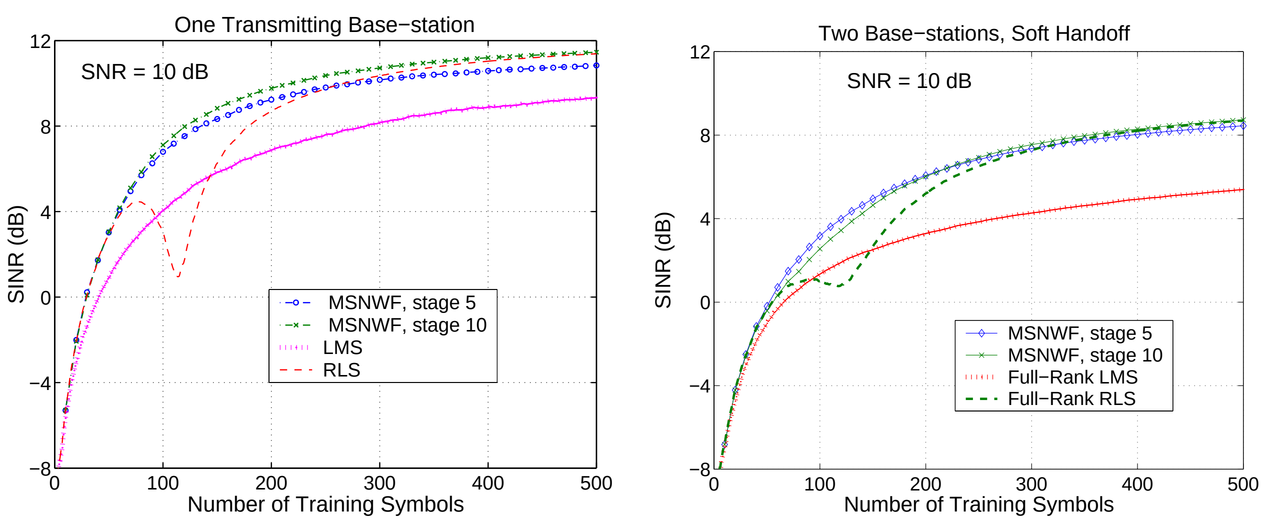

Figure 7: Output SINR vs Time for Adaptive Chip-level Equalizers for CDMA downlink.

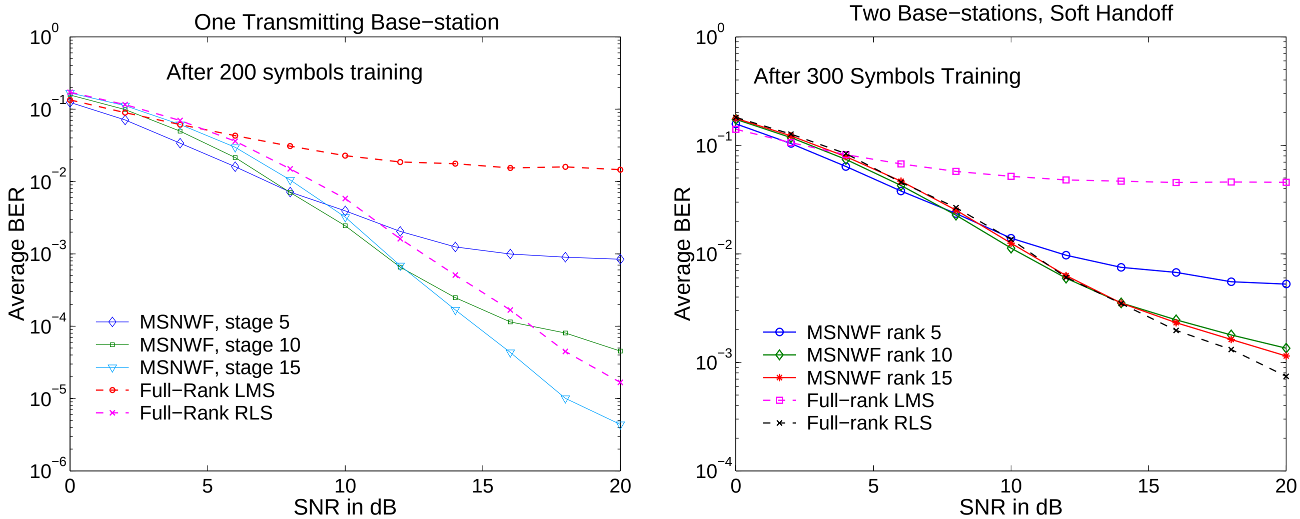

Figure 8: BER for Adaptive Chip-level Equalizers for CDMA Downlink.

from the MAI of the 2nd base-station. But the convergence speed of the low-rank MSNWF is still impressive. The BER curves in Figure 8 illustrate the performance of these equalizers. Note that graphs presented plot uncoded BER. In practice, the target uncoded BER is somewhere between 10^{-1} and 10^{-2}. Figure 8 (a) reveals that for uncoded BER's in this range, the stage 5 MSNWF performs better than the stage 10 or stage 15 MSNWF, as well as better than full-rank RLS! This improvement comes with dramatically lower computational complexity than RLS. The LMS algorithm is simpler, but performs extremely poor with slow convergence.

4 Recent fundamental advances on MSNWF.

In this section we present additional advances relative to the MSNWF, briefly summarized below.

1. We recently developed a computationally efficient scheme for generating an *orthogonal* basis for the Krylov subspace spanned by $\{\ \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}, \mathbf{R}_{\mathbf{x_0}}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}, ..., \mathbf{R}_{\mathbf{x_0}}^{D-1}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\ \}$: each successive member of the basis is generated by multiplying the previous member by $\mathbf{R}_{\mathbf{x_0}}$ and subtracting off from the resulting vector its components onto only the last two members of the basis. The resulting orthogonal basis is exactly the same as that generated via the original forward recursion of the MSNWF, which tri-diagonalizes $\mathbf{R}_{\mathbf{x_0}}$ at any stage, but it is computed without the need for blocking matrices as required in the original formulation of the MSNWF [GRS98] leading to substantially reduced computation.

2. We have developed a simple order-recursion for updating the weight vector and the MSE as each new stage is added. The original MSNWF was composed of two parts: a forward recursion followed by a backward recursion. Now, it is important to monitor the Mean Square Error (MSE) as each new stage is added (new stage $\equiv$ additional basis vector from forward recursion) since the sample support may be insufficient to support an additional stage such that the addition of such may cause the MSE to increase. Since the backwards recursion coefficients completely change each time a new basis vector is added, evaluation of its impact on the MSE previously required a backwards recursion for each new added stage. In Section 4.2, we devevelop a method which allows the MSE to be updated at each stage along with the backwards-recursion coefficients via a simple recursion.

4.1 Recursive computation of othogonal basis for Krylov subspace.

Recall that the chain of nested Weiner Filters in Figure 3 may be replaced by the simple filter bank in Figure 4, where the N length filters $\mathbf{t}_i$ are computed in terms of the matched filters $\mathbf{h}_i$'s and the blocking matrices $\mathbf{B}_i$'s in Figure 3 according to Equation (13). We here show that we can compute exactly the same set of orthonormal filters

$\mathbf{t}_i$ **without having to form the blocking matrices!** Adding the i-th stage we obtain the additional output signal $d_i[n] = \mathbf{t}_i^H \mathbf{x}_0[n]$ which is required to be maximally correlated with the output signal of the previous stage $d_{i-1}[n] = \mathbf{t}_{i-1}^H \mathbf{x}_0[n]$. Together with the orthogonality conditions this leads to following optimization problem:

$$\mathbf{t}_i = \arg\max_{\mathbf{t}} \mathcal{E}\{\mathrm{R}eal(d_i[n]d_{i-1}^*[n])\} \quad = \arg\max_{\mathbf{t}} \frac{1}{2}(\mathbf{t}^H \mathbf{R}_{\mathbf{x_0}} \mathbf{t}_{i-1} + \mathbf{t}_{i-1}^H \mathbf{R}_{\mathbf{x_0}} \mathbf{t}) \tag{15}$$

$$\text{s.t.:} \quad \mathbf{t}^H \mathbf{t} = 1 \quad \text{and} \quad \mathbf{t}^H \mathbf{t}_k = 0, k = 1, \ldots, i-1. \tag{16}$$

The solution, which is easily determined via the use of Lagrange multipliers, for example, is

$$\mathbf{t}_i = \frac{\left(\prod_{k=i-1}^{1} \mathbf{P}_k\right) \mathbf{R}_{\mathbf{x_0}} \mathbf{t}_{i-1}}{\| \left(\prod_{k=i-1}^{1} \mathbf{P}_k\right) \mathbf{R}_{\mathbf{x_0}} \mathbf{t}_{i-1} \|_2}, \quad \text{where:} \quad \mathbf{P}_i = \mathbf{I}_N - \mathbf{t}_i \mathbf{t}_i^H \tag{17}$$

$\mathbf{P}_i$ is the unique projection operator onto the orthogonal complement of the 1-D space spanned $\mathbf{t}_i$. We now show that it is not necessary to actually form $\mathbf{P}_i$! The key observation is that the recursion in Equation (17) is the Gram-Schmidt *Arnoldi algorithm* [Arn51, Saa96] for computing an orthonormal basis for the Krylov subspace $\mathbf{C}K^{(D)}$ generated by the square matrix $\mathbf{A} \in \mathbf{C}^{M\times M}$ and the column vector $\mathbf{b} \in \mathbf{C}^M$: $\mathbf{C}K^{(D)} = \text{span}\left([\mathbf{b}, \mathbf{A}\mathbf{b}, \ldots, \mathbf{A}^{D-1}\mathbf{b}]\right)$ [Saa96, vdV00]. Recall that Honig and Xiao [HX99] proved that with $\mathbf{B}_i = \mathbf{P}_i$ the filters $\mathbf{t}_i$ are an orthonormal basis for the Krylov subspace generated by $(\mathbf{R}_{\mathbf{x_0}}, \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}})$.

The covariance matrix $\mathbf{R}_{\mathbf{d}}$ of the pre-filtered observation $\mathbf{d}[n]$ (cf. Equation 14) is tri-diagonal. Coupled with the Hermitian property of $\mathbf{R}_{\mathbf{x_0}}$, the orthogonal basis $\mathbf{t}_i$ of the Krylov subspace $\mathbf{C}K^{(D)}$ of $(\mathbf{R}_{\mathbf{x_0}}, \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}})$ can be alternatively computed using the *Lanczos algorithm* [Lan50, Saa96]. The net result is that the forward recursion of the MSNWF may be executed without the need for forming blocking matrices. Each successive member of the forward recursion basis may be efficiently computed as follows. At the i-th stage, first compute

$$\mathbf{u}_i = \mathbf{R}_{\mathbf{x_0}} \mathbf{t}_{i-1}; \tag{18}$$

the next basis vector for the forward recursion is then computed as

$$\mathbf{t}_i = \mathbf{u}_i - (\mathbf{t}_{i-1}^H \mathbf{u}_i)\mathbf{t}_{i-1} - (\mathbf{t}_{i-2}^H \mathbf{u}_i)\mathbf{t}_{i-2} \tag{19}$$

followed by scaling $\mathbf{t}_i$ to have unit norm. Thus, we have an algorithm for computing the exact same orthogonal basis as that generated by the forward recursion in the original algorithmic structure of the MSNWF depicted in Figure 3, but which does not require blocking matrices!! This is a substantial computational savings. Another computation reducing feature of our innovation is the realization that after multiplying the previous member of the forward recursion basis by $\mathbf{R}_{\mathbf{x_0}}$, we need only subtract off from the resulting vector its components onto *only* the last two members of the basis.

Note that Goldstein and Ricks [Ricks00] recently developed a data-level lattice structure for the MSNWF that also avoids the formation of blocking matrices. In contrast, we have developed a covariance-level filter bank structure for the MSNWF that does not require blocking matrices. Goldstein and Ricks' [Ricks00] algorithm requires a backwards recursion after the forward recursion is terminated. In contrast, in the next section, we develop an order-recursive form of the MSNWF through which the backwards recursion coefficients, and hence the weight vector, may be updated at each stage via a simple recursion.

4.2 Order-Recursive MSNWF

Recall that at stage D the orthogonal basis

$$\mathbf{T}^{(D)} = [\mathbf{t}_1, \ldots, \mathbf{t}_D] \in \mathbf{C}^{N\times D} \tag{20}$$

obtained through the forward recursion yields the length D observation

$$\mathbf{d}^{(D)}[n] = \mathbf{T}^{(D),H} \mathbf{x}_0[n] \in \mathbf{C}^D, \tag{21}$$

having the $D \times D$ tri-diagonal covariance matrix

$$\mathbf{R}_{\mathbf{d}}^{(D)} = \mathcal{E}\{\mathbf{d}^{(D)}[n]\mathbf{d}^{(D),H}[n]\} = \mathbf{T}^{(D),H} \mathbf{R}_{\mathbf{x_0}} \mathbf{T}^{(D)}. \tag{22}$$

If we terminate at stage D, indicated by the superscript $(\bullet)^{(D)}$, the backwards recursion coefficients are the components of the Wiener filter $\mathbf{w}_{\mathbf{d}}^{(D)}$ which estimates $d_0[n]$ from $\mathbf{d}^{(D)}[n]$:

$$\mathbf{w}_{\mathbf{d}}^{(D)} = \left(\mathbf{R}_{\mathbf{d}}^{(D)}\right)^{-1} \mathbf{r}_{\mathbf{d},\mathbf{d_0}}^{(D)} = \left(\mathbf{T}^{(D),H}\mathbf{R}_{\mathbf{x_0}}\mathbf{T}^{(D)}\right)^{-1} \mathbf{T}^{(D),H}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} \tag{23}$$

The rank D MSNWF approximation of the Wiener filter is then

$$\mathbf{w}_0^{(D)} = \mathbf{T}^{(D)}\mathbf{w}_{\mathbf{d}}^{(D)} = \mathbf{T}^{(D)} \left(\mathbf{T}^{(D),H}\mathbf{R}_{\mathbf{x_0}}\mathbf{T}^{(D)}\right)^{-1} \mathbf{T}^{(D),H}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}. \tag{24}$$

which yields the *mean squared error*

$$\mathrm{MSE}^{(D)} = \sigma_{d_0}^2 - \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}^H \mathbf{T}^{(D)} \left(\mathbf{T}^{(D),H}\mathbf{R}_{\mathbf{x_0}}\mathbf{T}^{(D)}\right)^{-1} \mathbf{T}^{(D),H}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}. \tag{25}$$

The goal is to update both the backwards recursion coefficients $\mathbf{w}_{\mathbf{d}}^{(D)}$ (which change with each stage) and the $\mathrm{MSE}^{(D)}$ for stage D in terms of $\mathbf{w}_{\mathbf{d}}^{(D-1)}$ and $\mathrm{MSE}^{(D-1)}$ from the previous stage.

To do this, recall that the observation $\mathbf{d}^{(D)}[n] = \mathbf{T}^{(D),H}\mathbf{x}_0[n]$ has the tri-diagonal covariance matrix

$$\mathbf{R}_{\mathbf{d}}^{(D)} = \mathbf{T}^{(D),H}\mathbf{R}_{\mathbf{x_0}}\mathbf{T}^{(D)} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{T}^{(D-1),H}\mathbf{R}_{\mathbf{x_0}}\mathbf{T}^{(D-1)}} & \begin{matrix}\mathbf{0} \\ r_{D-1,D}\end{matrix} \\ \hline \mathbf{0}^T & r_{D-1,D}^* & r_{D,D} \end{array}\right] \in \mathbf{C}^{D\times D} \tag{26}$$

and the cross-correlation vector with respect to the desired signal $d_0[n]$

$$\mathbf{r}_{\mathbf{d},\mathbf{d_0}}^{(D)} = \mathbf{T}^{(D),H}\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} = \begin{bmatrix} \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2 \\ \mathbf{0} \end{bmatrix} \in \mathbf{r}^D. \tag{27}$$

Given $\mathbf{R}_{\mathbf{d}}^{(D-1)}$ from stage $D-1$, the new entries of $\mathbf{R}_{\mathbf{d}}^{(D)}$ are simply

$$r_{D-1,D} = \mathbf{t}_{D-1}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{t}_D \quad \text{and} \quad r_{D,D} = \mathbf{t}_D^H\mathbf{R}_{\mathbf{x_0}}\mathbf{t}_D. \tag{28}$$

Because $\mathbf{r}_{\mathbf{d},\mathbf{d_0}}^{(D)}$ has the property that only the first element is not equal to 0, only the first column of the inverse of $\mathbf{R}_{\mathbf{d}}^{(D)}$ is needed to compute the backwards recursion coefficients via $\mathbf{w}_{\mathbf{d}}^{(D)} = \mathbf{R}_{\mathbf{d}}^{(D),-1}\mathbf{r}_{\mathbf{d},\mathbf{d_0}}^{(D)}$.

For the sake of notational simplicity, define

$$\mathbf{C}^{(D)} = \mathbf{R}_{\mathbf{d}}^{(D),-1} = [\mathbf{c}_1^{(D)}, \ldots, \mathbf{c}_D^{(D)}] \in \mathbf{C}^{D\times D}. \tag{29}$$

The backwards recursion coefficients for stage D, $\mathbf{w}_{\mathbf{d}}^{(D)}$, is then the first column, $\mathbf{c}_1^{(D)}$, of $\mathbf{C}^{(D)} = \mathbf{R}_{\mathbf{d}}^{(D),-1}$. The inversion lemma for partitioned matrices (e.g., [Sch91, **?**]) leads to

$$\mathbf{C}^{(D)} = \begin{bmatrix} \mathbf{C}^{(D-1)} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix} + \beta_D^{-1}\mathbf{b}^{(D)}\mathbf{b}^{(D),H}, \tag{30}$$

where the various quantities are defined as follows.

$$\mathbf{b}^{(D)} = \begin{bmatrix} -\mathbf{C}^{(D-1)} \begin{bmatrix} \mathbf{0} \\ r_{D-1,D} \end{bmatrix} \\ 1 \end{bmatrix} = \begin{bmatrix} -r_{D-1,D}\mathbf{c}_{D-1}^{(D-1)} \\ 1 \end{bmatrix} \in \mathbf{C}^D \tag{31}$$

and

$$\beta_D = r_{D,D} - [\mathbf{0}^T, r_{D-1,D}^*]\mathbf{C}^{(D-1)} \begin{bmatrix} \mathbf{0} \\ r_{D-1,D} \end{bmatrix} = r_{D,D} - |r_{D-1,D}|^2 c_{D-1,D-1}^{(D-1)} \tag{32}$$

with $c_{D-1,D-1}^{(D-1)}$ being the last element of the last column $\mathbf{c}_{D-1}^{(D-1)}$ of $\mathbf{C}^{(D)}$ at the previous step. Therefore, the first column $\mathbf{c}_1^{(D)}$ can be written in terms of the first column of $\mathbf{C}^{(D-1)}$ from stage $D-1$ as

$$\mathbf{c}_1^{(D)} = \begin{bmatrix} \mathbf{c}_1^{(D-1)} \\ 0 \end{bmatrix} + \beta_D^{-1} c_{1,D-1}^{(D-1),*} \begin{bmatrix} |r_{D-1,D}|^2\mathbf{c}_{D-1}^{(D-1)} \\ -r_{D-1,D}^* \end{bmatrix} \in \mathbf{C}^D, \tag{33}$$

where $c_{1,D-1}^{(D-1)}$ denotes the first element of $\mathbf{c}_{D-1}^{(D-1)}$. Obviously, the first column of $\mathbf{C}^{(D)}$ and, thus, the Wiener filter $\mathbf{w}_{\mathbf{d}}^{(D)}$ at step D depends upon the first column $\mathbf{c}_1^{(D-1)}$ at step $D-1$ and the new entries of the covariance matrix $r_{D-1,D}$ and $r_{D,D}$. However, we also observe a dependency on the previous last column $\mathbf{c}_{D-1}^{(D-1)}$. Hence, we have to determine an expression for the last column of $\mathbf{C}^{(D)}$. Invoking Equation (30) we obtain

$$\mathbf{c}_D^{(D)} = \beta_D^{-1} \begin{bmatrix} -r_{D-1,D}\mathbf{c}_{D-1}^{(D-1)} \\ 1 \end{bmatrix} \tag{34}$$

which only depends on the previous last column and the new entries of $\mathbf{R}_{\mathbf{d}}^{(D)}$. So, we have developed an iteration that only updates two vectors $\mathbf{c}_1^{(D)}$ and $\mathbf{c}_D^{(D)}$ at each stage. In addition, the *mean squared error* at stage D can be updated via the first entry $c_{1,1}^{(D)}$ of $\mathbf{c}_1^{(D)}$ (cf. Equation 25):

$$\mathrm{MSE}^{(D)} = \sigma_{d_0}^2 - \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2^2 c_{1,1}^{(D)}. \tag{35}$$

The resulting "covariance level" version of the new (proposed) order-recursive MSNWF is summarized in Table 1, where we substituted $\mathbf{c}_1^{(i)}$ and $\mathbf{c}_i^{(i)}$ by $\mathbf{c}_{\mathrm{first}}^{(i)}$ and $\mathbf{c}_{\mathrm{last}}^{(i)}$, respectively.

$\mathbf{t}_0 = \mathbf{0}, \quad \mathbf{t}_1 = \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}/\|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2$
$\mathbf{u} = \mathbf{R}_{\mathbf{x_0}}\mathbf{t}_1$
$r_{0,1} = 0, \quad r_{1,1} = \mathbf{t}_1^H\mathbf{u}$
$c_{\mathrm{first}}^{(1)} = r_{1,1}^{-1}, \quad c_{\mathrm{last}}^{(1)} = r_{1,1}^{-1}$
$\mathrm{MSE}^{(1)} = \sigma_{d_0}^2 - \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2^2 c_{\mathrm{first}}^{(1)}$
for $i = 2, \ldots, D$
$\quad \mathbf{v} = \mathbf{u} - r_{i-1,i-1}\mathbf{t}_{i-1} - r_{i-2,i-1}\mathbf{t}_{i-2}$
$\quad r_{i-1,i} = \|\mathbf{v}\|_2$
$\quad \mathbf{t}_i = \mathbf{v}/r_{i-1,i}$
$\quad \mathbf{u} = \mathbf{R}_{\mathbf{x_0}}\mathbf{t}_i$
$\quad r_{i,i} = \mathbf{t}_i^H\mathbf{u}$
$\quad \beta_i = r_{i,i} - \|r_{i-1,i}\|^2 c_{\mathrm{last},i-1}^{(i-1)}$
$\quad \mathbf{c}_{\mathrm{first}}^{(i)} = \begin{bmatrix} \mathbf{c}_{\mathrm{first}}^{(i-1)} \\ 0 \end{bmatrix} + \beta_i^{-1} c_{\mathrm{last},1}^{(i-1),*} \begin{bmatrix} \|r_{i-1,i}\|^2 \mathbf{c}_{\mathrm{last}}^{(i-1)} \\ -r_{i-1,i}^* \end{bmatrix}$
$\quad \mathbf{c}_{\mathrm{last}}^{(i)} = \beta_i^{-1} \begin{bmatrix} -r_{i-1,i}\mathbf{c}_{\mathrm{last}}^{(i-1)} \\ 1 \end{bmatrix}$
$\quad \mathrm{MSE}^{(i)} = \sigma_{d_0}^2 - \|\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}\|_2^2 c_{\mathrm{first},1}^{(i)}$
$\mathbf{T}^{(D)} = [\mathbf{t}_1, \ldots, \mathbf{t}_D]$
$\mathbf{w}_0^{(D)} = \mathbf{T}^{(D)}\mathbf{c}_{\mathrm{first}}^{(D)}$

Table 1. Covariance-Level Order-Recursive MSNWF.

Forward Recursion:
for $i = 1, ..., D$
$\quad \mathbf{t}_i = \sum_{n=0}^{M-1} d_{i-1}^*[n]\mathbf{x}_{i-1}[n], \qquad \mathbf{t}_i = \mathbf{t}_i/\|\mathbf{t}_i\|_2$
$\quad d_i[n] = \mathbf{t}_i^H\mathbf{x}_{i-1}[n], \;\; n = 0, .., , M-1$
$\quad \mathbf{x}_i[n] = \mathbf{x}_{i-1}[n] - d_i[n]\mathbf{t}_i, \;\; n = 0, .., , M-1$
$\epsilon_D[n] = d_D[n]$
Backwards Recursion:
for $i = (D-1), ..., 1$
$\quad w_{i+1} = \left\{ \sum_{n=0}^{M-1} d_i[n]\epsilon_{i+1}^*[n] \right\} / \left\{ \sum_{n=0}^{M-1} \|\epsilon_{i+1}[n]\|^2 \right\}$
$\quad \epsilon_i[n] = d_i[n] - w_{i+1}\epsilon_{i+1}[n], \;\; n = 0, .., , M-1$
$\mathbf{w}_0^{(D)} = \sum_{i=1}^{D} (-1)^{i+1} \left\{ \prod_{\ell=1}^{i} w_i \right\} \mathbf{h}_i$

Table 2. Data-Level Lattice MSNWF.
(not order-recursive)

5 Further advances relative to the MSNWF.

These are summarized here and developed briefly in later sections.

1. The order-recursive MSNWF developed in Section 4.2 works at the covariance level, thereby presuming formation of a sample covariance matrix. We propose to develop data level versions of the order-recursive MSNWF amenable to the modular/lattice structure of the MSNWF recently developed by Goldstein and Ricks [Ricks00]; the latter is not order-recursive but rather requires a backwards-recursion as well as a forward recursion. The proposed data-level, order-recursive MSNWF offers the following important benefits: (i) it is order-recursive thereby updating the backwards recursion coefficients and MSE at each stage, (ii) it avoids computation of blocking matrices, and (iii) it avoids computation of a covariance matrix (for which there may not be sufficient sample support.)

2. We recently discovered a connection between the MSNWF and conjugate gradient (CG) search methods. The inherent relationship between the two follows from the aforementioned connection between the MSNWF and

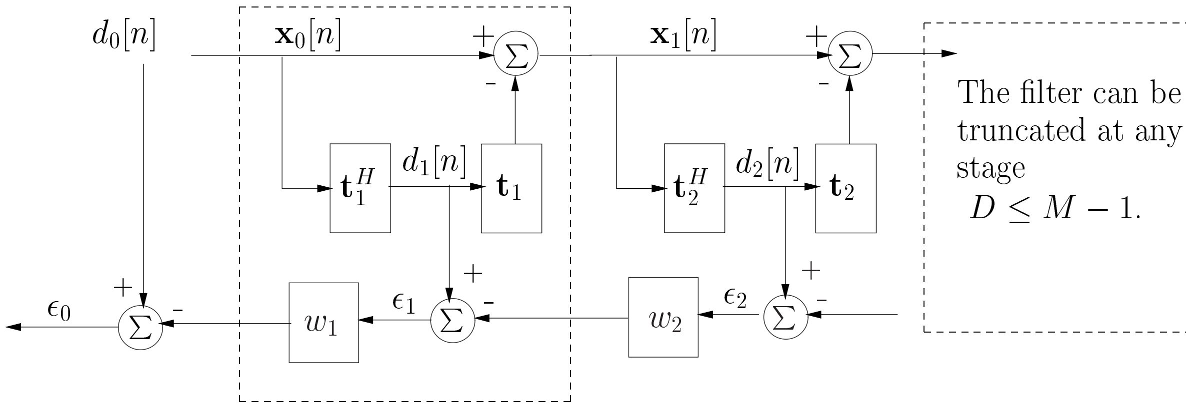

Figure 9: Lattice structure for MSNWF; dashed box is basic module for each additional stage.

Krylov subspace estimation since at each iteration the standard form of the CG search method minimizes $\mathbf{w}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{w} + \mathbf{w}^H\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} + \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}^H\mathbf{w}$ in the Krylov subspace generated by $\mathbf{R}_{\mathbf{x_0}}$ and $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}$. The fact that an iterative search algorithm is related to a reduced-rank adaptive filtering scheme is a fascinating connection.

3. We have worked to incorporate multiple constraints into the MSNWF. In addition to its use in implicitly solving the Weiner-Hopf equations $\mathbf{R}_{\mathbf{x_0}}\mathbf{w} = \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}$ through reduced-rank adaptation, the MSNWF can be used to solve Minimum Variance problems of the form:

$$\begin{aligned} \mathbf{w} &= \arg\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{w} \\ \text{s.t.:} \quad & \mathbf{d}^H\mathbf{w} = 1 \end{aligned} \tag{36}$$

where $\mathbf{d}$ is the signature vector for the desired user (array manifold vector, code, etc.) which is either known or estimated a-priori. In this scenario, $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} \propto \mathbf{d}$; we effectively know $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}$ to within an unknown multiplicative scalar. The solution to (36) may be computed as the solution to $\mathbf{R}_{\mathbf{x_0}}\mathbf{w} = \lambda\mathbf{d}$; λ is a scalar used to satisfy the constraint in (36). Both the covariance and data level versions of the MSNWF can be used to solve constrained minimum variance problems of this form. However, some applications involve multiple constraints (to effect smoothness, for example) in the form of a constraint matrix equation $\mathbf{C}^H\mathbf{w} = \boldsymbol{\delta}$. The incorporation of multiple constraints into the MSNWF has not yet been developed. We will also develop how to modify the recently developed "data-level" modular/lattice form of the MSNWF [Ricks00] to accommodate multiple constraints.

5.1 Data-Level Order-Recursive MSNWF.

The proposed order-recursive MSNWF summarized in Table 1 works at the covariance level, thereby presuming formation of a sample covariance matrix. We propose to develop data level versions of the order-recursive MSNWF amenable to the modular/lattice structure of the MSNWF recently developed by Goldstein and Ricks [Ricks00] and depicted in Figure 9. The algorithm accompanying Figure 9 is delineated in Table 2 and entails block-oriented processing: a block of data is extracted from the overall data stream and broken up into M blocks of length N denoted $\mathbf{x}[n]$, $n = 0, 1, ..., M-1$. The M data blocks may or may not be overlapping depending on the application.

Similar to our forward recursion based on Krylov subspace estimation, the Goldstein/Ricks algorithm in Table 2 does not require blocking matrices but it still requires a backwards-recursion once the forward recursion is terminated. We thus propose a data-level, order-recursive MSNWF: a single do-loop consisting of (in order) (I) the first three lines of the forward recursion do-loop in Table 2 (compute $\mathbf{t}_i$, $d_i[n]$, and $\mathbf{x}_i[n]$ at i-th stage), (II) compute $r_{i-1,i} = \sum_{n=0}^{M-1} d_{i-1}^*[n]d_i[n]$ and $r_{i,i} = \sum_{n=0}^{M-1} |d_i[n]|^2$, and (III) the last four lines of the do-loop in Table 1 (compute β_i, $\mathbf{c}_{\text{first}}^{(i)}$, $\mathbf{c}_{\text{last}}^{(i)}$, and $\text{MSE}^{(i)}$ at i-th stage). The quantities $r_{i-1,i}$ and $r_{i,i}$ are the new entries introduced into the tri-diagonal covariance matrix at stage i: $r_{i-1,i} = \mathbf{t}_{i-1}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{t}_i$ and $r_{i,i} = \mathbf{t}_i^H\mathbf{R}_{\mathbf{x_0}}\mathbf{t}_i$ (cf Eqn (28)), but expressed alternatively in terms of $d_i[n]$ and $d_{i-1}[n]$, quantites produced in the execution of the lattice MSNWF depicted in Figure 9. The proposed data-level, order-recursive MSNWF has all the desired benefits: (i) order-recursive thereby updating the backwards recursion coefficients and MSE at each stage, (ii) avoids computation of blocking matrices, and (iii) avoids computation of a covariance matrix (for which there may not be sufficient sample support.)

5.2 Connection to Gradient Search (CG) Techniques.

Substituting the expression for $\mathbf{c}_{\text{first}}^{(i)}$ in Table 1 into $\mathbf{w}_0^{(i)} = \mathbf{T}^{(i)}\mathbf{c}_{\text{first}}^{(i)}$. where $\mathbf{T}^{(i)} = [\mathbf{t}_1, \ldots, \mathbf{t}_i]$, yields a stage to stage direct update of the weight vector

$$\mathbf{w}_0^{(i)} = \mathbf{w}_0^{(i-1)} + \gamma_i \mathbf{g}_i + \phi_i \mathbf{t}_i, \qquad \text{where: } \mathbf{g}_i = \mathbf{T}^{(i)}\mathbf{c}_{\text{last}}^{(i)} = \eta_i \mathbf{g}_{i-1} + \zeta_i \mathbf{t}_i \tag{37}$$

where γ_i, ϕ_i, η_i, and ζ_i are all scalars whose expressions are not provided here due to space limitations. As discussed previously, the connection between CG and MSNWF is that at each iteration CG minimizes $\mathbf{w}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{w} + \mathbf{w}^H\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}} + \mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}^H\mathbf{w}$ in the Krylov subspace generated by $\mathbf{R}_{\mathbf{x_0}}$ and $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}$. An analysis of the direct MSNWF weight update in (37) will allow us to assess the equivalence between MSNWF and CG.

5.3 Incorporating Multiple Constraints into the MSNWF

We propose to incorporate multiple constraints into the MSNWF, for both the covariance level and data level versions of the MSNWF. A classic example of where multiple constraints may arise is in robust beamforming. In addition to a unity gain constraint in the desired look direction, a zero derivative constraint at the look direction is often imposed to reduce sensitivity to mismatch between the "look" direction and the actual arrival angle of the desired source. The incorporation of multiple constraints into the MSNWF has heretofore not yet been developed.

We here briefly develop MSNWF based solutions for Minimum Variance problems of the form

$$\begin{aligned} \mathbf{w} &= \arg\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{w} \\ &\text{s.t.:} \quad \mathbf{C}^H\mathbf{w} = \boldsymbol{\delta} \end{aligned} \tag{38}$$

with multiple constraints incorporated in the form of a constraint matrix equation $\mathbf{C}^H\mathbf{w} = \boldsymbol{\delta}$. The closed-form solution to (38) may be expressed as

$$\mathbf{w} = \mathbf{A}\boldsymbol{\beta} + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\boldsymbol{\delta} \; = \; \mathbf{A}\boldsymbol{\beta} + \boldsymbol{\gamma} \tag{39}$$

where $\boldsymbol{\gamma} = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\boldsymbol{\delta}$ and $\mathbf{C}^H\mathbf{A} = \mathbf{O}$, i. e., the column space of $\mathbf{A}$ spans the orthogonal complement of the column space of $\mathbf{C}$. This leads to the unconstrained optimization problem

$$\boldsymbol{\beta} = \arg\min_{\boldsymbol{\beta}} \boldsymbol{\beta}^H\mathbf{A}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{A}\boldsymbol{\beta} + \boldsymbol{\beta}^H\mathbf{A}^H\mathbf{R}_{\mathbf{x_0}}\boldsymbol{\gamma} + \boldsymbol{\gamma}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{A}\boldsymbol{\beta} + \boldsymbol{\gamma}^H\mathbf{R}_{\mathbf{x_0}}\boldsymbol{\gamma} \tag{40}$$

The optimal $\boldsymbol{\beta}$ may be computed as the solution to the Wiener-Hopf Eqns $\{\mathbf{A}^H\mathbf{R}_{\mathbf{x_0}}\mathbf{A}\}\boldsymbol{\beta} = -\mathbf{A}^H\mathbf{R}_{\mathbf{x_0}}\boldsymbol{\gamma}$. It is apparent that one may to solve for $\boldsymbol{\beta}$ via the efficient, covariance level version of the MSNWF summarized in Table 1 by replacing $\mathbf{R}_{\mathbf{x_0}}$ by $\mathbf{A}^H\mathbf{R}_{\mathbf{x_0},\mathbf{d_0}}\mathbf{A}$ and $\mathbf{r}_{\mathbf{x_0},\mathbf{d_0}}$ by $-\mathbf{A}^H\mathbf{R}_{\mathbf{x_0},\mathbf{d_0}}\boldsymbol{\gamma}$. The reduced-rank solution for $\boldsymbol{\beta}$ thus obtained is then substituted into (39).

A data-level, modular/lattice form of the MSNWF incorporating multiple constraints is facilitated by substituting $\hat{\mathbf{R}}_{\mathbf{x_0}} = \frac{1}{M}\sum_{n=0}^{M}\mathbf{x}[n]\mathbf{x}^H[n]$ into $\mathbf{A}^H\mathbf{R}_{\mathbf{x_0},\mathbf{d_0}}\mathbf{A}$ and $\mathbf{A}^H\mathbf{R}_{\mathbf{x_0},\mathbf{d_0}}$. The net result is that the structure in Figure 9, governed by the algorithm outlined in Table 2, may be employed by replacing the input data blocks $\mathbf{x}_0[n]$ by $\mathbf{x}_{0,r}[n] = \mathbf{A}^H\mathbf{x}[n]$, $n = 0, 1, ..., M-1$, and replacing the first basis vector for the forward recursion, $\mathbf{t}_1$, by $\mathbf{t}_1 = \sum_{n=0}^{M}(\mathbf{A}^H\mathbf{x}[n])(\mathbf{x}^H[n]\boldsymbol{\gamma})$ (followed by normalizing $\mathbf{t}_1$ to have unit length).

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