Efficient Implementation of Successive Encoding Schemes for the MIMO OFDM Broadcast Channel

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***Abstract*— In the work at hand relevant issues concerning implementation of optimal and nearly optimal transmission approaches for the MIMO OFDM broadcast channel are discussed.**

In particular, algorithms proposed to compute optimum covariance matrices are efficiently extended to the multicarrier setting. Furthermore, a method is proposed to transform the resulting vector channels into a set of scalar subchannels over which information can be independently transmitted without incurring any capacity loss. This effective diagonalization of the broadcast channel is most convenient for practical purposes as, so far, existing techniques for coding with side information have exclusively been conceived for scalar subchannels.

Finally, we discuss the practical advantages of a suboptimum technique such as the cooperative zero-forcing with successive encoding and successive allocation method (CZF-SESAM). This technique exhibits a nearly optimum performance and significantly simplifies both computation of transmit covariance matrices and downlink signaling.

I. Introduction

Aiming at the design of future wireless broadband systems combination of multiple input multiple output (MIMO) techniques and orthogonal frequency division multiplex (OFDM) seems to be very promising. On the one hand, OFDM greatly simplifies signal processing at transmitter and receiver as it decomposes the multipath frequency-selective channel into a set of decoupled flat fading channels. On the other hand, MIMO techniques deliver the high spectral efficiencies needed to face the scarcity of spectrum and the increasing demand for high rate services.

In the present work we consider a MIMO OFDM system in which a base station or access point equipped with $t \geq 1$ antennas sends information to a set of K users. Thereby, each user k is considered to have a number $r_k \geq 1$ of antennas. The channels between the transmit unit and each of the users are assumed to be static, and noise at the receivers is assumed to be Gaussian distributed. This scenario corresponds to a general Gaussian broadcast channel (BC) setting for which the capacity region has recently been found [1].

Provided that the transmit unit has perfect knowledge of all channels, the points at the boundary of the capacity region can be reached by successively encoding users and suppressing, at each step, the known interference applying dirty paper coding [2]. Boundary achieving transmit covariance matrices can be computed by solving a convex optimization problem and exploiting duality results with the multiple access channel (MAC) [3]. In [4] a gradient based algorithm is proposed to compute these covariance matrices. This algorithm proves to be highly efficient for usual numbers of antennas but turns out highly inefficient if directly applied to an OFDM transmission scheme even for low numbers of subcarriers. For computation of sum capacity optimum covariance matrices, an efficient algorithm has been presented in [5] based on iterative waterfilling. We extend the latter algorithm to OFDM and propose an efficient algorithm for the computation of general boundary achieving covariance matrices in a multicarrier setting. This algorithm is based on the iterative application of the algorithm in [4] and a waterfilling-like algorithm.

Existing coding techniques that efficiently cancel known interference, have, so far, been developed for scalar channels, i.e. single input single output (SISO) channels [6] [7] [8]. However, if a user has more than one receive element it is possible that the effective channel over which it receives information has more than one input. This is the case if the rank of its transmit covariance matrix is larger than one. Here, we present a method that transforms the vector channel of each user into a set of orthogonal scalar channels without incurring capacity loss. This makes possible the straightforward application of existing coding techniques to the MIMO broadcast channel.

In the last part of this paper we turn our attention to a suboptimum technique called CZF-SESAM [9] [10] [11]. This technique, which has been shown to exhibit a nearly optimum performance, significantly simplifies the computation of transmit covariance matrices. More important, optimum detection simply requires the application of matched filters to the precoded channel, which leads to a very efficient downlink signaling scheme.

The paper is structured as follows. In Section II the system model is introduced that will be considered along this paper. In Section III algorithms are presented to compute optimum

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covariance matrices for the MIMO OFDM broadcast channel based on those proposed for single carrier systems. In Section IV a method is proposed to transform vector channels into a set of decoupled scalar subchannels. In Section V the practical advantages of CZF-SESAM are discussed and a simple downlink signaling approach is presented. Finally, in Section VI the content of this paper is summarized and conclusions are drawn.

A. Notation

In the following, vectors and matrices are denoted by lower case bold and capital bold letters, respectively. We use $(\bullet)^*$ for complex conjugation, $(\bullet)^{\mathrm{T}}$ for matrix transposition and $(\bullet)^{\mathrm{H}}$ for conjugate transposition. $\mathrm{Tr}\{\bullet\}$ is the trace and $\mathrm{E}\{\bullet\}$ is the expectation operator. The identity matrix of dimension q is denoted by $\boldsymbol{I}_q$ and $\boldsymbol{e}_{q,s}$ denotes its sth column. Given a matrix $\boldsymbol{A}$, $|\boldsymbol{A}|$ represents its determinant. Finally, $\{\boldsymbol{A}_i\}_{i=1,\ldots,I}$ is the set of all matrices indexed by the variable i and $\mathrm{diag}[\boldsymbol{A}_1,\ldots,\boldsymbol{A}_I]$ represents a block diagonal matrix with matrices $\{\boldsymbol{A}_i\}_{i=1,\ldots,I}$ as blocks on the main diagonal.

II. System Model

For any subcarrier $c \in \{\ 1,\ldots,C\ \}$ the usual model for the flat fading broadcast channel is considered, i.e.

$$\boldsymbol{y}_c = \boldsymbol{H}_c\boldsymbol{x}_c + \boldsymbol{n}_c,$$

with

$$\begin{aligned} \boldsymbol{y}_c &= [\ \boldsymbol{y}_{c,1}^{\mathrm{T}} \ \cdots \ \boldsymbol{y}_{c,K}^{\mathrm{T}}\ ]^{\mathrm{T}}, \\ \boldsymbol{n}_c &= [\ \boldsymbol{n}_{c,1}^{\mathrm{T}} \ \cdots \ \boldsymbol{n}_{c,K}^{\mathrm{T}}\ ]^{\mathrm{T}}, \\ \boldsymbol{H}_c &= [\ \boldsymbol{H}_{c,1}^{\mathrm{T}} \ \cdots \ \boldsymbol{H}_{c,K}^{\mathrm{T}}\ ]^{\mathrm{T}}, \end{aligned}$$

where K is the number of users, $\boldsymbol{H}_{c,k} \in \mathbb{C}^{r_k \times t}$ is the channel matrix, $\boldsymbol{y}_{c,k} \in \mathbb{C}^{r_k}$ the received signal and $\boldsymbol{n}_{c,k} \in \mathbb{C}^{r_k}$ a realization of a zero-mean circularly symmetric complex Gaussian distributed random variable $\mathbf{n}_{c,k}$ representing noise with covariance matrix $\mathrm{E}\{\mathbf{n}_{c,k}\mathbf{n}_{c',k}^{\mathrm{H}}\} = \boldsymbol{I}_{r_k}\delta_{c,c'}$. This model describes transmission over any subcarrier of an ideal OFDM system without intercarrier or intersymbol interference. For the vectors of transmit signals a transmit power constraint applies,

$$\frac{1}{C}\sum_{c=1}^{C}\mathrm{E}\left\{\mathrm{Tr}\left\{\boldsymbol{x}_c\boldsymbol{x}_c^{\mathrm{H}}\right\}\right\} \leq P_{\mathrm{Tx}}.$$

III. Extension of Optimum Approaches to OFDM

A MIMO-OFDM system can be viewed as a MIMO system where blocks of transmit and receive antennas are decoupled from each other. Specifically, if we define

$$\begin{aligned} \tilde{\boldsymbol{H}}_k &= \mathrm{diag}\ [\ \boldsymbol{H}_{1,k} \ \cdots \ \boldsymbol{H}_{C,k}\ ] \in \mathbb{C}^{r_kC \times tC}, \\ \tilde{\boldsymbol{y}}_k &= [\ \boldsymbol{y}_{1,k}^{\mathrm{T}} \ \cdots \ \boldsymbol{y}_{C,k}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{C}^{r_kC \times r_kC}, \\ \tilde{\boldsymbol{n}}_k &= [\ \boldsymbol{n}_{1,k}^{\mathrm{T}} \ \cdots \ \boldsymbol{n}_{C,k}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{C}^{r_kC \times r_kC}, \\ \tilde{\boldsymbol{x}} &= [\ \boldsymbol{x}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{x}_C^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{C}^{tC \times tC}, \end{aligned}$$

we can write

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{n}}, \tag{1}$$

with

$$\begin{aligned} \tilde{\boldsymbol{y}} &= [\ \tilde{\boldsymbol{y}}_1^{\mathrm{T}} \ \cdots \ \tilde{\boldsymbol{y}}_K^{\mathrm{T}}\ ]^{\mathrm{T}}, \\ \tilde{\boldsymbol{n}} &= [\ \tilde{\boldsymbol{n}}_1^{\mathrm{T}} \ \cdots \ \tilde{\boldsymbol{n}}_K^{\mathrm{T}}\ ]^{\mathrm{T}}, \\ \tilde{\boldsymbol{H}} &= [\ \tilde{\boldsymbol{H}}_1^{\mathrm{T}} \ \cdots \ \tilde{\boldsymbol{H}}_K^{\mathrm{T}}\ ]^{\mathrm{T}}. \end{aligned}$$

Now, using (1) and considering, without loss of generality, a set of priorities $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_K$ such that $\sum_k \mu_k = 1$ the corresponding point on the boundary of the capacity region can be obtained by solving the following optimization problem in the dual MAC [4],

$$\begin{aligned} &\max_{\{\tilde{\boldsymbol{Q}}_k\}_{k=1,\ldots,K}} \sum_{k=1}^{K}\eta_k \log\left|\boldsymbol{I}_{tC} + \sum_{m=1}^{k}\tilde{\boldsymbol{H}}_m^{\mathrm{H}}\tilde{\boldsymbol{Q}}_m\tilde{\boldsymbol{H}}_m\right|, \qquad (2) \\ &\text{s. t. } \frac{1}{C}\sum_k \mathrm{Tr}\{\tilde{\boldsymbol{Q}}_k\} \leq P_{\mathrm{Tx}},\ \tilde{\boldsymbol{Q}}_k \succeq \boldsymbol{0},\ \forall k, \end{aligned}$$

where $\eta_k = \mu_k - \mu_{k+1}$ and $\mu_{K+1} = 0$. After solving this problem, the optimum transmit covariance matrices in the BC can be obtained from the optimum covariance matrices $\{\tilde{\boldsymbol{Q}}_k\}_{k=1,\ldots,K}$ in the MAC using the duality transformations described in [3].

According to Hadamard's inequality [12] the determinants in (2) are maximized if the matrix arguments are chosen to be block diagonal, which can be enforced by choosing covariance matrices $\tilde{\boldsymbol{Q}}_k$ to have block diagonal structure matching the structure of their respective channels $\tilde{\boldsymbol{H}}_k^{\mathrm{H}}$, i.e.

$$\tilde{\boldsymbol{Q}}_k = \mathrm{diag}\ [\ \boldsymbol{Q}_{1,k} \ \cdots \ \boldsymbol{Q}_{C,k}\ ] \in \mathbb{C}^{r_kC \times r_kC}.$$

This result allows us to rewrite the above optimization problem as

$$\max_{\substack{\{\boldsymbol{Q}_{c,k}\}_{c=1,\ldots,C} \\ k=1,\ldots,K}} \sum_{\substack{k=1 \\ c=1}}^{K,C}\eta_k \log\left|\boldsymbol{I}_t + \sum_{m=1}^{k}\boldsymbol{H}_{c,m}^{\mathrm{H}}\boldsymbol{Q}_{c,m}\boldsymbol{H}_{c,m}\right|, \tag{3}$$

subject to $\frac{1}{C}\sum_{c,k}\mathrm{Tr}\{\boldsymbol{Q}_{c,k}\} \leq P_{\mathrm{Tx}}$ and $\boldsymbol{Q}_{c,k} \succeq \boldsymbol{0}\ \forall c,k$.

A. Sum Capacity

The points at the boundary of the capacity region that achieve the sum capacity of the BC are obtained by solving any of the above optimization problems for $\mu_1 = \mu_2 = \cdots = \mu_K$. In order to find the solution, the algorithm presented in [5] can directly be applied to (2). Equivalently, this algorithm can straightforwardly be adapted to a multicarrier formulation of the problem as given by (3). The resulting algorithm is shown in Table I.

B. Other Boundary Points

To compute the optimum covariance matrices for any other boundary points an algorithm has been proposed in [4] that at each step improves the choice of covariance matrices by searching on the line defined by the eigenvector associated to the largest eigenvalue of the gradients of the objective function. As before, this algorithm can directly be applied to (2) or be adapted to the multicarrier formulation given by (3). In the latter case the gradient obtained by deriving the objective

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initialization : $\ell = 1, \quad \boldsymbol{Q}^{0}_{c,k} = \boldsymbol{0}, \ \forall c, k$

repeat :

1. $\boldsymbol{H}^{\text{eff},\ell}_{c,k} = \boldsymbol{H}_{c,k}\left(\boldsymbol{I}_t + \sum_{\substack{k'=1\\k'\neq k}}^{K} \boldsymbol{H}^{\text{H}}_{c,k}\boldsymbol{Q}^{\ell-1}_{c,k}\boldsymbol{H}_{c,k}\right)^{-1/2}, \ \forall c, k$

2. $\{\boldsymbol{M}^{\ell}_{c,k}\}_{\substack{c=1,\cdots,C\\k=1,\cdots,K}} =$

$$\underset{\{\boldsymbol{A}^{\ell}_{c,k}\}_{\substack{c=1,\cdots,C\\k=1,\cdots,K}}}{\arg\max} \sum_{c=1}^{C}\sum_{k=1}^{K} \log\left|\boldsymbol{I}_t + \left(\boldsymbol{H}^{\text{eff},\ell}_{c,k}\right)^{\text{H}} \boldsymbol{A}^{\ell}_{c,k}\boldsymbol{H}^{\text{eff},\ell}_{c,k}\right|$$

$$\text{subject to } \frac{1}{C}\sum_{c=1}^{C}\sum_{k=1}^{K} \text{Tr}\left\{\boldsymbol{A}^{\ell}_{c,k}\right\} \leq P_{\text{Tx}}, \ \boldsymbol{A}^{\ell}_{c,k} \geq 0, \ \forall c, k$$

3. $\boldsymbol{Q}^{\ell}_{c,k} = \left((K-1)\boldsymbol{Q}^{\ell-1}_{c,k} + \boldsymbol{M}^{\ell}_{c,k}\right)/K, \ \forall c, k, \quad \ell = \ell + 1$

until convergence

TABLE I

MAXIMIZATION OF SUM CAPACITY IN MIMO-OFDM BROADCAST CHANNELS

function with respect to any covariance matrix $\boldsymbol{Q}_{c',k'}$ can be written as,

$$\boldsymbol{G}_{c',k'} = \sum_{k=k'}^{K} \eta_k \boldsymbol{H}_{c',k'}\left(\boldsymbol{I}_t + \sum_{m=1}^{k}\boldsymbol{H}^{\text{H}}_{c',m}\boldsymbol{Q}_{c',m}\boldsymbol{H}_{c',m}\right)^{-1}\boldsymbol{H}^{\text{H}}_{c',k'}.$$

Let $\lambda^{\ell}_{c,k}$ denote the principal eigenvalue of the gradient matrix $\boldsymbol{G}^{\ell}_{c,k}$ obtained in the ℓth iteration. Then, similar to [4], we consider the one-dimensional subspace defined by the unit norm eigenvector $\boldsymbol{v}^{\ell}_{c',k'}$ associated with the maximum principal eigenvalue $\lambda^{\ell}_{c',k'} = \max_{c,k}\{\lambda^{\ell}_{c,k}\}_{\substack{c=1,\ldots,C\\k=1,\ldots,K}}$ in order to search for an improved set of covariance matrices. Accordingly, the new set of covariance matrices are computed as

$$\boldsymbol{Q}^{\ell+1}_{c,k} = t\boldsymbol{Q}^{\ell}_{c,k} + (1-t)CP_{\text{Tx}}\boldsymbol{v}^{\ell}_{c',k'}\boldsymbol{v}^{\ell,\text{H}}_{c',k'}\delta_{c,c'}\delta_{k,k'},$$

where $0 \leq t \leq 1$. As indicated in [4], the optimum value of t along this segment can be found through bisection.

Although theoretically both, direct application of the original algorithm to (2) and the multicarrier version described above, converge to the optimum, in practice, for typical numbers of subcarriers, convergence becomes very slow.

C. Divide and conquer

In order to speed up the computation of optimum matrices, we propose to divide problem (3) into a number of smaller problems. To this end, for each subcarrier, we factorize $\boldsymbol{Q}_{c,k} = p_c\bar{\boldsymbol{Q}}_{c,k}$ such that $\sum_k \text{Tr}\{\bar{\boldsymbol{Q}}_{c,k}\} \leq 1$ and $\sum_c p_c \leq CP_{\text{Tx}}$. Taking this factorization into account, optimum covariance matrices are found iterating the following two steps.

First, for given $\boldsymbol{p} = [\ p_1 \ \ \cdots \ \ p_C\ ]$, solve

$$\max_{\{\bar{\boldsymbol{Q}}_{c,k}\}_{k=1,\ldots,K}} \sum_{k=1}^{K}\eta_k \log\left|\boldsymbol{I}_t + p_c\sum_{m=1}^{k}\boldsymbol{H}^{\text{H}}_{c,m}\bar{\boldsymbol{Q}}_{c,m}\boldsymbol{H}_{c,m}\right|,$$

subject to $\sum_k \text{Tr}\{\bar{\boldsymbol{Q}}_{c,k}\} \leq 1$ and $\bar{\boldsymbol{Q}}_{c,k} \succeq \boldsymbol{0}\ \forall k$, for every c.

Second, for a given set $\{\bar{\boldsymbol{Q}}_{c,k}\}_{\substack{c=1,\ldots,C\\k=1,\ldots,K}}$, solve

$$\max_{\boldsymbol{p}} \sum_{c=1}^{C}\sum_{k=1}^{K}\eta_k \log\left|\boldsymbol{I}_t + p_c\sum_{m=1}^{k}\boldsymbol{H}^{\text{H}}_{c,m}\bar{\boldsymbol{Q}}_{c,m}\boldsymbol{H}_{c,m}\right|,$$

subject to $\sum_c p_c \leq CP_{\text{Tx}}$ and $p_c \geq 0$.

Both problems are convex. In the second step, an optimum power allocation over subcarriers $\boldsymbol{p}$ is found for a given set of normalized covariance matrices. In the first, given the optimum power allocation $\boldsymbol{p}$ obtained in the previous iteration, an optimum set of normalized covariance matrices is found for every subcarrier. It is clear that each step improves the value of the objective function in (3) and hence convergence is guaranteed.

In the first step, optimization of normalized covariance matrices can be done applying the algorithm presented in [4]. In the second step, the Karush-Kuhn-Tucker conditions of the optimization problem yield the following set of equations,

$$\sum_{k=1}^{K}\eta_k \,\text{Tr}\left\{(\boldsymbol{I}_t + p_c\boldsymbol{A}_{c,k})^{-1}\boldsymbol{A}_{c,k}\right\} - \nu + \xi_c = 0, \ \forall c; \quad (4)$$

$$CP_{\text{Tx}} - \sum_{c=1}^{C} p_c \geq 0; \ \nu \geq 0; \ p_c \geq 0, \ \xi_c \geq 0, \ \forall c;$$

$$\nu\left(CP_{\text{Tx}} - \sum_{c=1}^{C} p_c\right) = 0; \ \xi_c\, p_c = 0, \ \forall c;$$

where $\boldsymbol{A}_{c,k} = \sum_{m=1}^{k}\boldsymbol{H}^{\text{H}}_{c,m}\bar{\boldsymbol{Q}}_{c,m}\boldsymbol{H}_{c,m}$. Considering the eigenvalues $\{\lambda^{n}_{c,k}\}_{n=1,\cdots,t}$ of matrix $\boldsymbol{A}_{c,k}$, (4) can be rewritten as

$$\sum_{k=1}^{K}\sum_{n=1}^{t}\frac{\eta_k\lambda^{n}_{c,k}}{1 + p_c\lambda^{n}_{c,k}} - \nu + \xi_c = 0, \ \forall c. \quad (5)$$

An efficient algorithm can be implemented that computes the power allocation $\boldsymbol{p}$ satisfying these conditions based on the following two observations.

Observation 1: For a given ν, $p_c \neq 0$ if and only if $\sum_{k=1}^{K}\sum_{n=1}^{t}\eta_k\lambda^{n}_{c,k} > \nu$. In that case, $\xi_c = 0$ and

$$\sum_{k=1}^{K}\sum_{n=1}^{t}\frac{\eta_k\lambda^{n}_{c,k}}{1 + p_c\lambda^{n}_{c,k}} - \nu$$

is a monotonically decreasing function of the transmit power p_c.

Observation 2: The optimum ν is a monotonically decreasing function of the transmit power P_{Tx}. Moreover,

$$\nu < \max_{c}\left\{\sum_{k=1}^{K}\sum_{n=1}^{t}\eta_k\lambda^{n}_{c,k}\right\},$$

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i.e. at least one subcarrier gets some power.

From observation 1 it becomes clear that for a given ν there is a unique power allocation $\boldsymbol{p}$ which can be efficiently computed. On the other hand, according to observation 2, if this power allocation exceeds the available transmit power, ν should be increased, otherwise it should be decreased. In this way, bisection can be used in order to compute ν corresponding to the particular transmit power constraint.

D. Performance

It turns out that the divide and conquer algorithm converges much faster than the multicarrier adaptation of the original algorithm described in section III-B. This is related to the fact that the improvement achieved in an iteration of the original algorithm dramatically reduces as the number of subcarriers increases. As a result, more iterations are needed to approach the optimum. On the other hand the number of iterations required by the divide and conquer algorithm to reach the optimum seems to be independent of the number of subcarriers.

Fig. 1 visualizes the different behaviour of both algorithms. The settings haven been chosen to be $K = 2$, $r_1 = r_2 = 2$, $t = 4$ and $P_{\mathrm{Tx}} = 10$ dB. The channel coefficients have been independently drawn for each subcarrier and each matrix entry according to a zero-mean circularly symmetric complex Gaussian distributed random variable with unit variance. Points in the capacity region achieved by both algorithms are represented for two different stop criteria $\epsilon = (f^{\ell} - f^{\ell-1})/f^{\ell-1}$, which define the minimum improvement of the objective function f that is required in any iteration ℓ to go for a new iteration $\ell + 1$. On the axes, rates are given in bits per subcarrier. Dots correspond to a system with $C = 64$ subcarriers, circles correspond to a system with $C = 16$ subcarriers. Each point corresponds to a different pair of priorities $(\mu_1, \mu_2 = 1 - \mu_1)$ chosen such that $\mu_1 = 0.05 \times n$ with $0 \leq n \leq 20$, $n \in \mathbb{Z}$. For every point, the initial covariance matrices have been chosen to be $\boldsymbol{Q}_{c,k} = 0.25 P_{\mathrm{Tx}} \boldsymbol{I}_2 \; \forall c, k$.

It can be seen that performance of the divide and conquer algorithm does basically depend on the stop condition and not on the number of subcarriers. Also the number of iterations is approximately the same, e.g. for $\epsilon = 10^{-3}$, 3.0 iterations in average were required for $C = 16$ and 3.2 for $C = 64$.

On the contrary, for a given stop condition, performance of the original algorithm strongly degrades for increasing number of subcarriers. Specially visible is this effect for $\epsilon = 10^{-3}$, where, in the case of $C = 64$, independently of the priorities, the algorithm already terminates in the proximity of the start value resulting in complete overlap of all points belonging to a same decoding order. As mentioned at the beginning of this section, due to the reduced improvement accomplished in one iteration, more subcarriers require more iterations in order to get an acceptable performance. As an example, for $\epsilon = 10^{-4}$, 98.5 iterations were in average needed to compute each circle in the lower left plot ($C = 16$). In average, 189.5 iterations were required to compute each dot ($C = 64$).

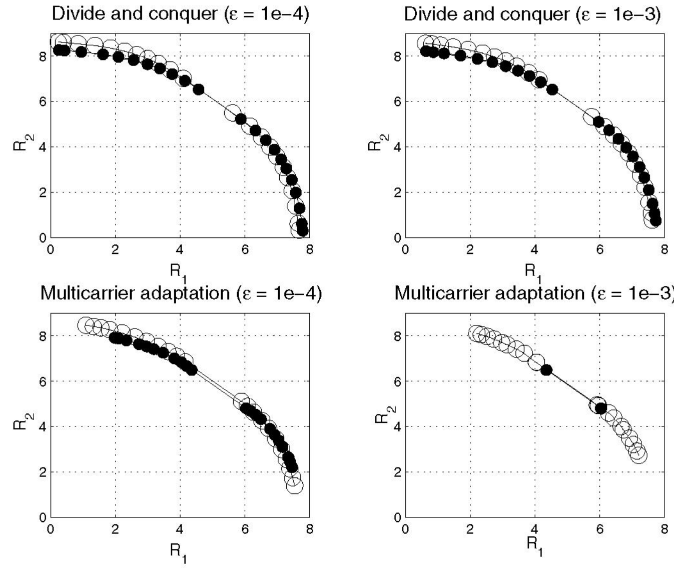

Fig. 1. Comparison of capacity regions obtained from application of the proposed algorithms.

IV. Diagonalization of Vector Channels

Given a set of priorities $\{\mu_k\}_{k=1,\ldots,K}$, optimum covariance matrices can be computed employing the algorithms described in the previous section and using the duality transformations given in [5]. Thereby, the optimum encoding order is determined by the order of the priorities assigned to the users, i.e. first encoded user is the one with highest priority and so on. Without loss of generality, assume $\mu_1 \geq \cdots \geq \mu_K$ and let $\{\boldsymbol{\Sigma}_{c,k}\}_{k=1,\ldots,K}$ be the set of optimum transmit covariance matrices at subcarrier c. Then, the signal received by user k on subcarrier c can be written as

$$\boldsymbol{y}_{c,k} = \boldsymbol{H}_{c,k} \boldsymbol{B}_{c,k}^{1/2} \boldsymbol{s}_{c,k} + \boldsymbol{H}_{c,k} \sum_{m=1}^{k-1} \boldsymbol{B}_{c,m}^{1/2} \boldsymbol{s}_{c,m} + \\ + \boldsymbol{H}_{c,k} \sum_{m=k+1}^{K} \boldsymbol{B}_{c,m}^{1/2} \boldsymbol{s}_{c,m} + \boldsymbol{n}_{c,k}, \quad (6)$$

where the factorization $\boldsymbol{\Sigma}_{c,k} = \boldsymbol{B}_{c,k}^{1/2} \boldsymbol{B}_{c,k}^{\mathrm{H}/2}$ has been applied and $\boldsymbol{s}_{c,k}$ is the vector of signals transmitted to user k on subcarrier c. Optimally, these vectors are realizations of a zero-mean circularly symmetric complex Gaussian random variable with covariance $\mathrm{E}\{\mathbf{s}_{c,k} \mathbf{s}_{c,k}^{\mathrm{H}}\} = \boldsymbol{I}_{\rho(c,k)}$, where $\rho(c,k)$ is the rank of matrix $\boldsymbol{\Sigma}_{c,k}$.

In (6) the second term on the right hand side represents the interference due to those users encoded before user k, whereas the third term represents the part of interference due to users encoded after user k. The third term is not known at the time of encoding information for user k, therefore, no countermeasures can be taken at this stage. On the contrary, at this point, the second term is perfectly known and this knowledge can be used to encode information of user k so that the achievable capacity is the same as if this term were not present [2]. This theoretical result was initially shown for scalar channels, i.e. channels with a scalar input and a scalar output. More recently, several authors (see [13] and references therein) have extended this result to vector channels, i.e. channels where the output ($\boldsymbol{y}_{c,k}$) and the input ($\boldsymbol{s}_{c,k}$) have more than one dimensions. However, for the time being, practical coding techniques to cancel known interference [6] [7] [8] are available only for the scalar case. In the following, a method is presented to convert vector channels of any user into a set of scalar channels without incurring any capacity loss. In this way, capacity can be approached by using existing coding techniques for scalar channels.

First, a linear zero-forcing filter $\boldsymbol{F}_{c,k}$ will be applied at the receiver. The output signal reads

$$\begin{aligned}\boldsymbol{y}'_{c,k} &= \boldsymbol{s}_{c,k} + \boldsymbol{F}_{c,k}\boldsymbol{H}_{c,k}\sum_{m=1}^{k-1}\boldsymbol{B}_{c,m}\boldsymbol{s}_{c,m} + \\ &\quad + \boldsymbol{F}_{c,k}\underbrace{\left(\boldsymbol{H}_{c,k}\sum_{m=k+1}^{K}\boldsymbol{B}_{c,m}\boldsymbol{s}_{c,m} + \boldsymbol{n}_{c,k}\right)}_{\boldsymbol{z}_{c,k}}, \quad (7)\end{aligned}$$

and the zero forcing filter is given by

$$\boldsymbol{F}_{c,k} = \left(\boldsymbol{B}^{\mathrm{H}}_{c,k}\boldsymbol{H}^{\mathrm{H}}_{c,k}\boldsymbol{R}^{-1}_{c,k}\boldsymbol{H}_{c,k}\boldsymbol{B}_{c,k}\right)^{-1}\boldsymbol{B}^{\mathrm{H}}_{c,k}\boldsymbol{H}^{\mathrm{H}}_{c,k}\boldsymbol{R}^{-1}_{c,k},$$

being $\boldsymbol{R}_{c,k}$ the covariance matrix of the effective noise $\boldsymbol{z}_{c,k}$. Note that this linear transformation of the receive signal preserves the rate of user k and, as it only applies to the receiver of that user, it does not affect the rates of any other user.

The covariance matrix of the effective noise at the output of the zero-forcing filter, $\boldsymbol{z}'_{c,k} = \boldsymbol{F}_{c,k}\boldsymbol{z}_{c,k}$, equals

$$\boldsymbol{R}'_{c,k} = \left(\boldsymbol{B}^{\mathrm{H}}_{c,k}\boldsymbol{H}^{\mathrm{H}}_{c,k}\boldsymbol{R}^{-1}_{c,k}\boldsymbol{H}_{c,k}\boldsymbol{B}_{c,k}\right)^{-1}. \quad (8)$$

Performing an eigenvalue decomposition of this matrix, $\boldsymbol{R}'_{c,k} = \boldsymbol{U}_{c,k}\boldsymbol{\Lambda}_{c,k}\boldsymbol{U}^{\mathrm{H}}_{c,k}$, the unitary matrix $\boldsymbol{U}^{\mathrm{H}}_{c,k}$ can be applied at the receiver to decorrelate the effective noise and signals can be transmitted along the column vectors of matrix $\boldsymbol{U}_{c,k}$, i.e. $\boldsymbol{s}_{c,k} = \boldsymbol{U}_{c,k}\boldsymbol{s}'_{c,k}$. As a result, the equivalent channel

$$\boldsymbol{y}''_{c,k} = \boldsymbol{s}'_{c,k} + \underbrace{\boldsymbol{U}^{\mathrm{H}}_{c,k}\boldsymbol{z}'_{c,k}}_{\boldsymbol{z}''_{c,k}} + \boldsymbol{U}^{\mathrm{H}}_{c,k}\boldsymbol{F}_{c,k}\boldsymbol{H}_{c,k}\sum_{m=1}^{K-1}\boldsymbol{B}_{c,m}\boldsymbol{s}_{c,m}$$

is obtained where the effective noise $\boldsymbol{z}''_{c,k}$ is uncorrelated and whose capacity can be achieved by separately coding over each of the scalar components. Note that correlation of the third term is not important as this term is known and each of the components can effectively be neutralized on the respective scalar channel. As the transformation applied to decorrelate the effective noise is invertible the rate achieved by user k is preserved and, as it only applies to the receiver of that user, it does not affect the rates of any other user. As the statistics of $\mathbf{s}_{c,k}$ are invariant under any unitary transformation neither the rate of user k nor the rate of any other user is affected by this kind of precoding.

V. BENEFITS OF CZF-SESAM

The cooperative zero-forcing with successive encoding and successive allocation method (CZF-SESAM) is a technique that decomposes the MIMO broadcast channel into a set of effectively decoupled scalar channels [9] [10]. The algorithm, which can be run in paralell over all subcarriers of an OFDM scheme, assigns at each step a new spatial dimension to a certain user according to some given criterion in such a way that no interference is caused on previously assigned dimensions. Thereby, a dimension is characterized by a unit-norm transmit weighting vector and a unit-norm receive weighting vector. To be specific, assume that on a particular subcarrier c the first $i-1$ dimensions have already been assigned to certain users in the system. In order to assigned the ith spatial dimension the algorithm proceeds as follows.

First, all channel matrices are projected onto the subspace complementary to that spanned by already assigned transmit weighting vectors, i.e. if $\boldsymbol{V}_{i-1} = [\ \boldsymbol{v}_1 \ \cdots \ \boldsymbol{v}_{i-1}\ ]$ is a matrix formed by the first $i-1$ weighting vectors, the projected channel matrices are given by

$$\boldsymbol{H}^{i}_{c,k} = \boldsymbol{H}_{c,k}\left(\boldsymbol{I}_t - \boldsymbol{V}_{i-1}\boldsymbol{V}^{\mathrm{H}}_{i-1}\right), \ \forall k.$$

Then, singular value decompositions of all projected matrices are performed,

$$\boldsymbol{H}^{i}_{c,k} = \boldsymbol{U}^{i}_{c,k}\boldsymbol{\Lambda}^{i}_{c,k}\boldsymbol{V}^{i,\mathrm{H}}_{c,k}, \ \forall k,$$

and, based on a certain criterion, among all pair of right and left singular vectors one is chosen that characterizes the new dimension i. For instance, if the pair associated with the sth singular value of user k_0 is chosen as the ith subchannel, $\boldsymbol{v}_i = \boldsymbol{V}^{i}_{c,k_0}\boldsymbol{e}_{t,s}$ is the new transmit weighting vector and the conjugate transpose of $\boldsymbol{u}_i = \boldsymbol{U}^{i}_{c,k_0}\boldsymbol{e}_{r_{k_0},s}$ is the new receive weighting vector. Due to the projection step it is guaranteed that signals transmitted over the ith dimension do not cause interference on the previous $i-1$ dimensions, i.e. $\boldsymbol{u}^{\mathrm{H}}_{j<i}\boldsymbol{H}_{c,u(j)}\boldsymbol{v}_i = 0$, where $u(j)$ indicates the user to which dimension j has been assigned. The converse is not true, i.e. dimension $j < i$ will in general cause interference on dimension i but this can be cancelled by coding provided that the encoding order is chosen to be the same as the allocation order.

If, at each step, the dimension associated with the largest singular value is chosen, simulation results have shown that the algorithm is nearly optimum in terms of sum capacity [9]. Also in terms of capacity region, this algorithm shows a nearly optimum performance for a wide range of scenarios [11]. Beside the good performance, there are two relevant issues that make this approach worth considering as an alternative to the optimum solution.

A. *Computation of transmit covariance matrices*

Given a criterion for the selection of subchannels, which, as priorities in the optimum solution, might be linked to quality of service demands from higher layers, computation of transmit and receive weighting vectors requires a maximum of tK singular value decompositions per subcarrier. Note that at most t orthogonal spatial dimensions can be allocated on a subcarrier. Subsequently, power loading can be performed over the set of decoupled channels according to some criterion of interest. Both power loading and transmit weighting vectors characterize the transmit covariance matrices of all users in the system. Thereby, complexity is comparable to that involved in one iteration of any of the optimum methods discussed in Section III for computing optimum covariance matrices. The main difference is that CZF-SESAM does not require further iterations. Also, no duality transformations are required as the solution directly applies to the broadcast channel.

B. Downlink Signaling

Downlink signaling is an important issue in systems employing precoding techniques. Indeed, in order to detect signals, receivers need to know some basic parameters related to the precoding that was applied to those signals at the transmitter.

Let $\boldsymbol{V} = [\,\boldsymbol{v}_1 \;\cdots\; \boldsymbol{v}_i \;\cdots\; \boldsymbol{v}_t\,]$ be the matrix of transmit weighting vectors obtained from application of CZF-SESAM on subcarrier c and $\boldsymbol{P}$ a diagonal power loading matrix that on that subcarrier assigns certain power to each spatial dimension. Furthermore, let $\boldsymbol{u}_i^{\mathrm{H}}$ be the receive weighting vector corresponding to transmit weighting vector $\boldsymbol{v}_i$ and assume that this dimension has been assigned to user k. The signal received by user k can be written as

$$\boldsymbol{y}_{c,k} = \boldsymbol{M}_{c,k}\boldsymbol{s}_c + \boldsymbol{n}_{c,k},$$

where $\boldsymbol{M}_{c,k} = \boldsymbol{H}_{c,k}\boldsymbol{V}\boldsymbol{P}^{1/2}$ and $\boldsymbol{s}_c$ is the vector of signals transmitted on subcarrier c, of which at least the ith component, $s_{c,i}$, is intended for user k. In order to optimally detect this signal all that user k must know is vector $\boldsymbol{u}_i^{\mathrm{H}}$ and the effective subchannel gain given by $g_i = \boldsymbol{u}_i^{\mathrm{H}}\boldsymbol{m}_{c,s}^i$, where $\boldsymbol{m}_{c,s}^i$ is the ith column of $\boldsymbol{M}_{c,k}$. By construction, it can be shown that $\boldsymbol{m}_{c,s}^i = g_i\boldsymbol{u}_i$, and hence knowledge of $\boldsymbol{m}_{c,s}^i$ suffices to detect signal $s_{c,i}$. Based on this fundamental property of CZF-SESAM a simple downlink signaling approach can be applied as follows.

First, the transmit unit broadcasts training pilots precoded with $\boldsymbol{V}\boldsymbol{P}^{1/2}$. This allows any user k to estimate its own matrix $\boldsymbol{M}_{c,k}$. At this stage, any user k does not know which of the columns of $\boldsymbol{M}_{c,k}$, if any at all, it has got assigned but it is able to compute the t potential receive weighting vectors and respective channel gains.

In the second step, in order to communicate the allocation of spatial dimensions, the transmit unit sends over each subchannel an identifier corresponding to the user to which that subchannel has been assigned. This transmission is made using a predefined signal constellation and applying successive encoding with cancellation of known interference. Each user processes the received signal in t different ways, according to the t different receive weighting vectors and gains it computed in the first step, and compares the detected signal with its own identifier.

Finally, in the third step, further parameters such as signal constellation or coding rate employed for data transmission are sent by the transmit unit on each subchannel.

Recalling (6) we note that in the optimum approach, in order to perform optimum detection, any user k must not only know its own effective channel resulting from the concatenation of physical channel and precoding matrix, i.e. $\boldsymbol{H}_{c,k}\boldsymbol{B}_{c,k}^{1/2}$, but also the covariance matrix of the effective interference caused by subsequently encoded users. This makes downlink signaling for optimum approaches certainly more involved.

VI. Conclusion

Approaches to compute optimum transmit covariance matrices for the MIMO broadcast channel have been extended to a multicarrier setting. To compute general points at the boundary of the capacity region, an algorithm has been introduced that shows better convergence properties than the straightforward extension of the known approach. In order to allow application of existing coding techniques for cancelling known interference a method has been presented to convert the vector channels resulting from application of optimum transmit strategies into a set of decoupled scalar subchannels without incurring capacity loss. Finally, the simple computation of transmit covariance matrices and the simple configuration of downlink signaling have been pointed out as two major practical aspects that, beside a good performance, make of CZF-SESAM an interesting alternative to optimum transmit strategies.

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