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Selection and Application of Quantitative Indicators of Paths Based on Graph Theory: A Case Study of Traditional Private and Antique Gardens in Beijing

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Abstract: Chinese Traditional Gardens (CTGs) are an important part of China's cultural inheritance from the past. Today's China has experienced rapid urbanization, raising the need for a new form of contemporary gardens intended to satisfy peoples' need for traditional culture. Garden paths are important in CTGs; they are designed to show visitors changing views with each step, and to lead them to secluded, quiet places via winding paths. This enhances the ornamental interest of the gardens. Based on plane graphics, this study evaluates the characteristics of three types of garden paths in fourteen traditional gardens and a contemporary antique garden, the Daguan Garden in Beijing. The analysis uses correlation and factor analysis to integrate 28 quantitative path indicators into five aspects of average, scale, network, wide, and aggregation. The 28 indicators can be expressed by six simple indicators: average connection length, number of path sections, alpha index, average width, average tortuous angle, and concentration degree. The results show small variations of garden paths between traditional gardens, but a considerable difference between the contemporary garden and traditional gardens. The research proposes a framework for the quantification and comparison of garden path features that can be applied before and after garden path construction, for both ancient and modern garden styles. This framework generates garden path feature values and theoretical values of six indicators, and is not constrained by the garden scale. Therefore, it provides an accurate and efficient design tool for garden designers.

Keywords: Beijing private garden; traditional garden; antique garden; quantitative indicators; path design

1. Introduction

1.1. Research Background and Status

As the pioneer and prototype for urban parks [1,2], traditional gardens reflect the purest regional culture and display gardening techniques [3,4]. Several traditional gardens are listed as cultural heritage sites by their regions and some are recognised globally for their outstanding universal value [5]. As a public component of urban green spaces, traditional gardens provide value and function that cannot be replaced by ordinary parks or open green spaces [6]. Some modern urban parks and greenbelts attempt to imitate traditional garden forms [7], producing mixed gardens combining traditional and contemporary styles, but the outcome is widely criticized by users and scholars [8]. Can modern urban parks integrate the design wisdom of traditional gardens while meeting the needs of contemporary society [9,10]? To answer this question, the intentions underlying traditional garden design require research and depth of understanding [11].



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Since the 1990s, garden designers have examined the tradition of garden style [12–14]. For example, contemporary landscape designers in Britain, France, Italy, China, Japan, and Islamic regions are striving to follow the direction set out in traditional gardens, and this has produced many excellent examples [15-18]. However, initial research on the inheritance and development of CTGs has generally taken a qualitative approach [19], considering to a greater or lesser extent the feelings of researchers or designers. This allows room to improve the accuracy of the research results. Today, scholars can accurately describe the style characteristics of a garden by using a series of index parameters in the form of field measurement and questionnaires [20,21], and use these parameters to establish a quantitative evaluation model for garden characteristics [22,23], thus greatly improving the accuracy of the results. However, data acquisition is still constrained by the prerequisites for measurement (instrumentation, weather, permission of the park or garden manager, etc.) and the survey population (sample size, structure, etc.). As a branch of mathematics, graph theory considers graphs as its research object to describe specific relationships between various objects [24]. This method is not affected by objective conditions such as the environment, instrumentation, and weather, and hence overcomes the limitation of data acquisition previously mentioned. By using graph theory, reasonably specifying parameters and indicators, and applying analysis and modelling software (such as ArcGIS, Grasshopper. Available from http://www.esri.com/software/arcgis and http://www.grasshopper3d.com (accessed on 1 October 2022)), the designer can take a more scientific, accurate and efficient approach.

Chinese Traditional Gardens (CTGs), with their millennia-long history [25], have had a profound impact on the design methods of traditional gardens in Europe [26], South Korea, and Japan [27–29], and are still referred to in contemporary garden design [19,30]. In late 20th century China, rapid urbanization resulted in the speedy construction of urban parks and greenbelts that largely ignored traditional garden design [31]. Specifically, CTGs attempt to create a natural microcosm, presenting visitors with a different view at every step [32,33], leading along secluded, winding paths, and displaying grand visions through small scenes [34]. The garden path is the key element in this effect [35]. Its design is the outcome of the consideration and experience of the garden owner and the designer and was constantly adjusted through personal experiments, considering various factors ranging from body size to the psychological and spiritual needs of the garden owners [36]. Therefore, the quantification of garden paths is an important factor to accurately describe the garden owner's needs and to guide the garden designer.

CTGs reached maturity during the Qing Dynasty, when the gardening art of the southern and northern gardens attained its historical peak [37]. Due to the differences between natural conditions and social outlooks, as well as the varying political and cultural conditions between the south and the north of China, the garden path styles in the two regions exhibit considerable differences [38]. For example, the southern garden path demonstrates a slender and tortuous temperament, while the northern garden path adopts a spacious, straightforward, symmetrical, and clear official style [39]. As the political centre of the Qing Dynasty, private gardening activities in Beijing were very numerous in this period. There are several more gardens in Beijing than in Suzhou and Yangzhou, the main cities of the south where traditional gardens can be found [40]. The function and historical statues of Beijing's traditional private gardens (BTPGs) is equivalent to the private gardens [38]. Further studies of private gardens on the north will help fill these gaps.

1.2. Review of Path Indicators

At present, it has been proved that calculations using road quantitative indicators can effectively guide planning and design [41]. Through the design of road alignment and the control of indicators, the designer can reduce the occurrence of traffic accidents, reduce road construction costs, and improve the user's experience [42–44]. On the basis of

path/road studies, the relevant parameters can be categorised as scale indicators, shape indicators, and network indicators.

First, the scale indicators are calculated based on European geometry to describe the size and dimensions of the road. These include the length, width, quantity, and curvature, as well as other indicators developed from the these, such as the average length, maximum length, curvature change degree, etc. [45]. In Japan, a study compared the path widths of four traditional gardens and two modern gardens in Tokyo. The data show that the average width of traditional garden roads (1.6-2.2 m) is much less than that in contemporary gardens (3.4–6.0 m) [46]. In addition, some scholars measured the curvature of the garden path in thirty traditional Japanese gardens. The results show that with an increase of the garden area, the degree of road tortuosity decreases until it approaches a fixed value [47]. In China, scholars used basic indicators such as length, area, width, and quantity to analyse and summarize the road characteristics of southern and northern gardens, obtaining some scattered data characteristics, such as the road width of northern gardens varying around two m and not being affected by the garden area [36,48–50]. In addition to the indicator characteristics of the garden road itself, the change of the visitors' sight line with one step is another important indicator to determine the opening and closing changes of the garden space. By means of fixed-point measurement or space syntax software (Depthmap) calculations, the periodic change in the range of vision can be detected [50–52]. However, due to the different nature of the various indicators and research objects, it is difficult to link and compare the research results. Nonetheless, it can be concluded that all studies have identified a certain connection between the garden path characteristics and the garden scale [53].

Second, the shape indicators are also calculated based on European geometry [54]. This aspect is concerned with the shape or morphological characteristics of path/road areas instead of scale, shape, and more complex formulations and theories, such as perimeter area ratio, shape index, fractal dimension, etc. Such indicators are mostly used to determine the geometric characteristics of green space, and there are few applications of these indicators in the study of the shape of garden roads. In the study of geometry, many indicator parameters can be used to calculate the boundary characteristics of green space [55], but these are prone to confusion and misuse due to the huge and chaotic nature of morphological indicators systems [56,57]. To overcome this problem, Riitters used the methods of correlation and principal component analysis to compress 55 morphological indicators into six representative indicators (average perimeter-area ratio, contagion, standardized patch shape, patch perimeter, area scaling, number of attribute classes, and large-patch density-area scaling [58]). The condition and the number of attribute classes are applicable to many types of patch elements. The path is a single element and is difficult to use. The other indicators are derived from the perimeter and area of the patch. Moreover, fractal geometry can effectively describe the complexity of road and path morphology [59,60].

Finally, network indicators are based on topological geometry. They mainly focus on connectivity and compactness between points and lines of road alignment, such as connectivity, aggregation, dispersion, etc. [61]. In terms of connectivity, traffic topological networks are commonly studied using spatial syntax and complex network theory [62,63]. The latter theory involves converting a large number of objects into points and lines to quantify the path's network connectivity through the average degree and average path length path [64]. Alpha (α), beta (β), and gamma (γ) indicators describe path network complexity conveniently and quickly [65,66], and provide a wide reference for the analysis and evaluation of urban paths [67] and ecological corridors [68]. Similarly, spatial syntax is not only applied to the spatial opening and closing of the line of sight, but is also involved in the computation of network connectivity and depth value [69,70]. However, the two indicators used in spatial syntax have the same theory as connectivity indicators such as α , β and γ , so it is sufficient to choose one set. In terms of aggregation or dispersion, although the centrality indicator is used to explain the degree of aggregation or dispersion of green or building blocks [71,72], the principle is still applicable to path patches.

To sum up, in the study of medium-scale (block) and large-scale (city) path networks [73–76], several indicators are used to describe the geometric shape of paths [77,78] and their topological network characteristics [79]. In contrast, the indicators for small-scale areas (such as gardens or parks) are rare. A quantitative indicator system specifically designed for garden paths is required. In addition, indicators may have different practical degrees when dealing with different sizes and types of roads and paths. The practical indicators for small-scale garden roads can be formulated by combining the calculation principles of indicators and using repair calculation methods.

1.3. Research Objective

The Section 1.2 expands the number and dimension of quantitative indicators of traditional garden paths by integrating existing quantitative studies of path characteristics at different scales (large: city; middle: block; and small: garden/park). Therefore, this research considers the traditional private gardens of Beijing in the Qing Dynasty as the research objects, combines the plane image information of the garden path and the larger existing quantitative indicator set, and applies factor analysis (FA) and correlation analysis (CA) to pursue the following three purposes:

- 1. Identification of representative indicators of the small-scale garden path network system.
- 2. Quantification and clarification of the path characteristics of private gardens in the Qing Dynasty and contemporary antique gardens in Beijing, and of the similarities and differences between them.
- 3. Development of suggestions for the design of garden paths of contemporary antique gardens/parks in Beijing.

2. Methodology

2.1. Study Method

The research method (Figure 1) included the following steps. (1) In Beijing, the research object is defined: historical records containing path data, satellite images, the descriptions of the garden owner, and on-site investigation are collated, and an electronic representation of the garden path is then made (Section 2.2). (2) Path types are subdivided according to the research objects (Section 2.3). (3) The quantitative research indicators (the largest indicator set) are selected, on as wide a basis as possible (Section 2.4). (4) The maximum indicator set of garden paths is compressed, and the degree of similarity between traditional and contemporary gardens is judged, along with the linear relationship with the site scale.



Figure 1. Research framework.

2.2. Study Case

Here, "private garden" refers to the residence or villa gardens used by individuals or their families over a long period [40]. In the Qing Dynasty (1616–1912), the gardens of the royal family and nobles were usually gifted by the emperor to reward their service.

However, the construction, transformation and use of the garden depended on the opinions of the royal family. Therefore, such gardens are also included in the study of private gardens. Historical events (military, environmental, and economic) since the end of the Qing Dynasty led to most of the gardens being destroyed. Only Chunwangfu Garden and Gongwangfu Garden have been preserved [40]. Three criteria exist for the designation of private gardens as described above:

- 1. The construction period of the garden was during the Qing Dynasty, and its style and craftsmanship are also from this dynasty.
- 2. Basic information (garden size, owner's identity, and garden utilization) is detailed and reliable.
- 3. There are clear path elements in the garden that provide opening and closing changes of viewpoint as visitors walk along paths.

Only 14 existing private gardens in Beijing meet these three criteria.

In order to clarify the degree to which contemporary antique gardens have inherited the garden path construction techniques of the Qing Dynasty, Daguan Garden, a contemporary antique garden built in Beijing in 1984, was selected for comparative purposes and to complement the 14 ancient gardens mentioned above (Figure 2). This garden not only restored the magnificent scenery described in the Qing Dynasty book *A Dream of Red Mansions*, but also had northern garden characteristics [80]. Using satellite images captured between March 2010 and March 2020, an electronic plan of Daguan Garden was drawn up (Figure 3), in combination with the plan and surveys of the garden conducted during this period.



Figure 2. Distribution of research objects in Beijing.



Figure 3. Plans of the research objects: (a-p) traditional private gardens, and (q) Daguan Garden.

2.3. Dividing the Type of Path

Different types of paths have different morphological characteristics, which are best extracted by classifying and analysing them one by one. This study considers three types of paths: HP (Household Paths), MP (Mountain Paths) and CP (Courtyard Paths: structures connected by covered paths). The buildings, pavilions, corridors, and garden paths share the same connecting function, because they are all covered by roofs, and walking on or through them has a different spatial feeling from external garden paths. Therefore, such covered structures are excluded from this study.

The service scope (SS) of MP is the mountainous area in the garden, whereas the SS of CP is the courtyard area, and that of HP is the garden's horizontal land. These differences may affect quantitative indicators such as the number, length, and area of paths, so they also need to be distinguished. In the garden outside the courtyard, the path mainly serves the needs of visitors viewing the garden landscape. It is often lengthy and curved to extend the viewing time. Path elevation, in such cases, is higher than that of paths of the horizontal land surface (HP), which provide a shorter sight distance.

2.4. Establishment of Quantitative Indicators

Based on the study of existing path indicators (Section 1.2) and in combination with the characteristics of BTPGs' paths, 28 indicators were selected to quantify the characteristics of paths (Table 1). Three categories, based on data types, were employed: path centreline (PC), path boundary (PB), and visible space on a path (PV).

		Measurement	Interpretation	Reference
	Nodes (N)	Number of all path nodes Nodes Intersection(Nodes)	A node/vertex is the intersection of a minimum of two paths. Visitors can adapt or modify their itinerary here.	[81]
	Lines (L)	Number of all path links	Line as the path section between two nodes.	[81]
	Total Length (TL)	$TL = \sum_{i=1}^{N} l_i$ $l_i = \text{Length of the } i \text{th links}$	TL is the sum of the path centreline length in the garden.	[36]
	Average Length (AL)	$AL = \frac{TL}{L}$	AL reflects the average length of all paths within the garden.	[36]
	Density (D)	$D = \frac{1}{5} \sum_{i=1}^{N} l_i$ S = Garden Area (m ²)	D represents the density level of paths within the garden. A greater D value signifies a denser path	[82]
	Average Longer Paths (ALP)	$ALP = \frac{1}{n_a} \sum_{a=1}^{n_a} l_a$ l_a = Links with a length greater than the AL. n_a = The number of l_a .	ALP is the average of all path lengths greater than the AL. The greater the ALP, the longer the path appears.	[73]
	Average Shorter Paths (ASP)	$ASP = \frac{1}{n_b} \sum_{b=1}^{n_b} l_b$ $l_b = \text{Links with a length less than the AL.}$ $n_b = \text{The number of } l_b.$	ASR is the average of all path lengths less than the AL. The smaller the value of ASP, the shorter the path appears.	[73]
RC	Variation of Path Length (VPL)	$VPL = \sqrt{rac{\sum_{i=1}^{n} \left(l_i - \overline{l} ight)^2}{n}}$	The VPL is calculated as the square root of variance of each path length. A greater value signifies a more obvious change in the path length.	[73]
	Simple Paths (SP)	Number of simple straight lines cut into the path centreline. d_1	Through the Douglas-puke algorithm, the curve path centreline is decomposed into simple straight lines, and the SP is the total number of straight lines counted.	[73]
	Circuitous Angle of One Step (CAS)	$CAS = \frac{1}{n_c} \sum_{i=1}^{n_c} \theta_a$ $\theta_a = \text{Change angle between step } a$ and $a + 1$. $n_c = \text{The number of steps.}$ 0.4m θ_a θ_a 0.4m θ_a	Taking the average step length of 0.4 m as the unit, the centreline of the path is segmented. The average circuitous angle is the angle changed by each step. A larger CAS value signifies a larger angle that visitors need to change with each step.	[47]
	Circuitous Angle of One Path (CAP)	$CAP = \frac{n_c}{L}CAS$	CAP records the visitors' direction angle changes after walking from one path to another. A greater value of CAP indicates a more tortuous overall path garden path.	[47]
	Intersection Deflection Angle (IDA)	$IDA = \frac{1}{n_d} \sum_{b=1}^{n_d} \theta_b$ $\theta_b = \text{Minimum angle at the bth}$ intersection. $n_d = \text{The number of intersections}$ θ_b	Measures all angles of each intersection and finds the average value of the minimum angles of all intersections (IDA). A larger value equates to the increased strength of the turn that is required at the intersection.	[73]

 Table 1. Description of path quantitative indicators.

	Table	e 1. Cont.				
		Measurement	Interpretation	Reference		
	Average Number of Connections (ANC)	$ANC = \frac{1}{N} \sum_{\substack{i \neq 1 \\ i \neq 1}}^{N} n_e$ $n_e = \text{The number of lines connected by the node.}$ (2) (3) (2) (3) (4) $AD = 4$ (3) (3) (4)	Degree represents the number of edges directly connected to a node in the network. ANC is the arithmetic average of the degrees of all nodes. A larger value, infers a more complex network relationship of the path.	[64]		
	Average Connection Length (ACL)	$ACL = \frac{1}{N} \sum_{i \neq 1}^{N} l_c$ $l_c =$ The total length of links connected by the <i>c</i> th nodes.	ACL indicates the average length of the path linked by the nodes of a garden. The larger the ACL, the longer the path that is connected by this node.	[64]		
	Alpha (α)	$\alpha = \frac{(L-N)+1}{2N-5}$	This measures the circuitry of a network, or the degree to which it provides alternative paths for traveling from one node to another. A greater value increases the connectivity of a network.	[65]		
	Beta (β)	$eta = rac{L}{N}$	This reflects the complexity and completeness of a network by expressing the ratio of links to nodes. $\beta < 1$ indicates a disconnected network; $\beta = 1$ a single circuit; $\beta > 1$ implies a greater complexity of network connectivity.	[65]		
	Gamma (γ)	$\gamma = rac{L}{3(N-2)}$	This is a measure of the extent to which the nodes are connected, called connectivity. Gamma index values range between 0 and 1. A value of 1 denotes a completely connected network.	[65]		
	Centrality (CE)	$CE = \frac{\sum_{i=1}^{N} D_i}{N\sqrt{S/\pi}}$ $D_i =$ The distance of node <i>i</i> to the centroid of the garden. High $\overbrace{\bullet}^{\bullet}$ Low	The centrality indicator measures the arctice of the nodes to the garden centre. To minimize the bias of the garden scale, the average distance was divided by the radius of a circle with the total garden area.	[71]		
	Area (A)	Spatial extent of the entire path (m ²).	The absolute extent of the path area indicates the size of the path within its boundaries.	[36]		
	Perimeter (P)	Measurement of the whole path edge (m).	The perimeter of a shape is the total measurement of all the edges of the shape.	[36]		
	Average Width (AW)	$AW = \frac{A}{TL}$	Total path area divided by the sum of each path centreline length. A larger AW signifies a wider path.	[47]		
KΒ	Perimeter-Area Rate (PAR)	$PAR = \frac{P}{A}$	Area-perimeter ratio is defined by dividing the area by the perimeter of a convex space, to identify the 'fattest' convex when a convex map is drawn.	[58]		
	Shape Index (SHA)	$SHAPE = \frac{0.25P_i}{\sqrt{a_i}}$ $P_i = \text{Perimeter of patch } i.$ $a_i = \text{Area of patch } i.$	Shape index corrects for the size problem of the perimeter-area ratio indicator by adjusting for a square standard and, as a result, is the simplest and potentially most straightforward measure of shape complexity.	[72]		

		Measurement	Interpretation	Reference
	Fractal (FRA)	$FRAC = \frac{2\ln(0.25P_i)}{\ln a_i}$	Fractal reflects shape complexity across a range of spatial scales (patch sizes).	[72]
	Field of Vision Area(FVA)	Spatial extent of whole vision (m ²). Point of Sight Barrier Visible Area Sight Distance (max)	The area that can be seen on the path is the FVA. The larger the FVA, the more open the space.	[51]
RV	Average Visual Distance (AVD)	$AVC = \frac{1}{N} \sum_{a=1}^{N} D_a$ D_a = The length of sight line at node <i>a</i> .	AVD is the average value of sight length when looking at both sides of the path at nodes. The larger the AVD, the greater the distance that can be seen on the path.	[51]
	Maximum Visual Distance (MVD)	$\begin{split} MVD &= \frac{1}{L}\sum_{i=1}^{L}D_{(\max)i}\\ D_{(\max)i} &=\\ \end{split}$ The maximum sight distance length in the i path.	MVD is the maximum sight distance on a given section of path. A greater value equates to a greater visible range on the path.	[51]
	Variation of Visual Distance (VVD)	$SDVD = \sqrt{rac{\sum_{i=1}^{N} (D_a - AVD)^2}{N}}$	The VVD represents the magnitude of fluctuation of the sight distance on the path. The larger the VVD value, the more obvious the spatial opening and closing changes present on the path.	[51]

Table 1. Cont.

The PC line category includes 18 indicators to describe the scale and network characteristics of the path. These characteristics are based on the centreline of the path, and statistically calculated using Grasshopper in Rhino 7 (for example, No. 2: Lines). The PB class includes six indicators that mainly describes the path shape. They are calculated using ArcGIS.10.2 and Fragstats software; the boundary line of the path serves as the data source (for example, No. 19: Area). The PV includes four indicators and fully describes the line-of-sight features on the path. It is constructed and calculated using the viewing area as data (for example, No. 25: Field of Vision Area).

2.5. Data Acquisition and Processing

According to the 15 gardens established in Section 2.2, Autodesk CAD. 2015 software's polyline function redraws their plans. For the data type used by the indicator, the path boundary is determined by the boundary of the walkable area on the path. The centreline of the path is assisted and determined by its boundary path (Figure 4); the visible area (Figure 5) is defined by the water body, flat land, square, path, and other areas lower than the height of human sight [51].

Results were statistically analysed using Excel software, according to the parameters set in Section 2.4. To compress the number of indicators, SPSS 25.0 statistical software was used in factor analysis and correlation analysis. The Euclidean distance (ED) was used to determine shape similarity [83] between the gardens. The calculation formula of similarity is:

$$ED = (x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2$$
(1)

where *x* and *y* respectively refer to the names of the two gardens to be compared, and $1, 2, 3 \cdots n$ refers to the indicators used to compare the two gardens (all indicators are normalized). ED is the similarity value between the two gardens: the smaller the value, the higher the degree of similarity.



Figure 4. Determination of the path sideline and centreline, taking the Chunwangfu Garden as an example (left: historical original data; middle: determination of the path sideline; right: determination of the path centreline).



Figure 5. Schematic diagram of the scope of view (again, the example is the Chunwangfu Garden: Left: view of HP; Middle: view of MP; Right: view of CP).

Finally, an analysis of the correlation between the representative indicators and the service scale was conducted, and the indicators related to the service scale were preferentially used to predict the parameters of paths in Daguan Garden. For the indicators unrelated to the service scale, the range of the parameters was determined in combination with the average value of the traditional garden.

3. Results

3.1. Determination of Principal Indicators

In this study, 28 indicators of the 14 traditional gardens were extracted using the FA in SPSS 25.0 software. Before finalising results, Kaiser–Meyer–Olkin (KMO) and Bartley Spherical Test (BST) are used to check whether the data are suitable for the FA method. The results show that the KMO is 0.659, which satisfies the condition (KMO > 0.5) to establish the factor analysis model. The p-value determined using BST is 0.00 (less than 0.05), indicating that there is correlation between the variables and that the FA results are effective. As the results show that, after the fifth factor (Table 2), the eigenvalue declines to less than 1 (0.937), the overall extraction rate reaches more than 85% (85.017%). This better preserves the integrity of the 28 indicators. Therefore, the first five factors are selected as the final component.

In order to promote the interpretation and naming of the extracted factors without changing the mutual relationship between the factors, the varimax rotation method was used to change the factor load, readjusting the relationship between the factors and the original variables [72]. The commonalities of all indicators are shown in the last column of Table 3, which reflects the extraction degree of indicator data on all factors. The closer the value is to 1, the better the extraction is. According to the results, except for the intersection deflection angle and centrality, the extraction rate of all indicators is higher than 70%, providing a good interpretation for all five factors.

Component		Initial Eigenv	alues	Rotation Sums of Squared Loadings						
L	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %				
1	11.454	40.906	40.906	9.056	32.341	32.341				
2	5.885	21.017	61.924	5.825	20.805	53.146				
3	2.881	10.289	72.213	4.484	16.015	69.160				
4	2.214	7.906	80.119	2.276	8.129	77.289				
5	1.371	4.898	85.017	2.164	7.727	85.017				
6	0.937	3.345	88.362							

Table 2. Total variance explained.

Extraction Method: Principal Component Analysis.

Indicators Component												
Indicators	1	2	3	4	5	Extraction						
ACL	0.981	0.046	0.026	-0.010	-0.031	0.966						
AL	0.980	0.103	0.008	0.004	-0.028	0.972						
ASP	0.940	0.009	-0.093	-0.083	-0.069	0.904						
ALP	0.928	0.134	-0.036	0.126	-0.073	0.902						
VPL	0.918	0.263	0.096	0.116	-0.031	0.935						
FVA	0.850	0.343	-0.078	-0.014	0.004	0.846						
VVD	0.796	0.379	-0.266	-0.013	0.222	0.898						
MVD	0.775	0.462	-0.165	0.005	0.116	0.855						
AVD	0.727	0.253	-0.289	-0.192	0.285	0.794						
CAP	0.628	-0.309	-0.058	0.055	0.507	0.753						
L	0.021	0.912	0.205	0.087	-0.160	0.908						
Ν	0.017	0.903	0.091	0.091	-0.217	0.880						
TL	0.559	0.771	0.190	0.091	-0.069	0.956						
Р	0.569	0.766	0.193	0.064	-0.050	0.955						
SP	0.484	0.761	0.280	0.138	0.061	0.914						
А	0.556	0.752	0.179	-0.136	-0.034	0.926						
SHA	0.447	0.669	0.328	0.140	0.189	0.811						
α	-0.059	0.198	0.963	0.019	-0.025	0.970						
β	-0.033	0.392	0.900	0.002	-0.042	0.966						
ANC	-0.042	0.420	0.881	0.014	-0.008	0.954						
γ	-0.109	-0.140	0.871	-0.014	0.108	0.801						
IDA	-0.074	0.299	0.558	-0.049	-0.400	0.568						
AW	-0.026	-0.152	-0.028	-0.932	0.092	0.902						
PAR	-0.058	0.055	-0.074	0.929	0.086	0.883						
FRA	0.436	0.196	0.432	0.492	0.450	0.859						
CAS	0.006	-0.349	-0.053	0.340	0.771	0.835						
D	-0.248	-0.044	0.349	-0.035	0.704	0.681						
CE	0.089	0.052	-0.180	-0.141	0.383	0.209						

Rotation Method: Variance maximizing orthogonal rotation. See Table 1 for abbreviations of indicators. The grey covering part shows the indicators of high load value in five factors.

Since each factor contains more than two indicators, the naming of the new factor should not only signify all the indicators in the group as much as possible, but it should also differ and be independent from the other factors. If the new name cannot include all the indicator information making up the factor, the factor is renamed by referring to the indicators with the larger absolute load value in the factor. If the indicators in the same factor have a high correlation, this indicates that they can replace each other. Hence, the indicator with the highest load can be selected to refer to other indicators, so as to reduce the number of indicators [58]. Combining the results of FA (Table 3) and CA (Table 4), the naming of five factors and the selection of representative indicators are as follows:

Land **2022**, 11, 2304

Factors	ACL	AL	ASP	ALP	1 VPL	FVA	VVD	MVD	AVD	CAP	L	N	TL	2 P	SP	А	SHA	α	β	3 ANC	γ	IDA	AW	4 PAR	FRA	CAS	5 D	CE		Factors
	0.57	0.60	0.34 *	0.65	0.68	0.81	0.57	0.58	0.27	0.09	0.47	0.50 **	0.75 **	0.74	0.58	0.74	0.54	0.06	0.16	0.16	-0.13	0.18	-0.16	0.01	0.34 *	-0.24	-0.34	-0.01	SS	
	1.00	1.00	0.86	0.86	0.61	0.86	0.60	0.64	0.51	0.77	-0.02	-0.03	0.41	0.41	0.36 *	0.42	0.34 *	0.04	-0.12	-0.14	-0.13	-0.16	0.14	-0.19	0.30	0.02	-0.27	-0.16	ACL	
		1.00	0.87	0.86	0.62	0.89 **	0.62	0.66	0.52	0.75	0.01	0.01	0.44	0.45 **	0.38 *	0.45	0.37 *	0.03	-0.12	-0.13	-0.15	-0.17	0.12	-0.18	0.31	0.01	-0.27	-0.17	AL	
			1.00	0.60	0.18	0.71	0.34 *	0.42	0.48	0.78	-0.15	-0.16	0.12	0.12	0.07	0.15	0.11	0.07	-0.30	-0.32	-0.19	-0.34	0.30	-0.29	0.02	0.01	-0.29	-0.23	ASP	
				1.00	0.86	0.81	0.74	0.74	0.42	0.56	0.14	0.14	0.58	0.59	0.50	0.57	0.46	-0.05	0.00	-0.02	-0.20	0.00	-0.09	0.08	0.37 *	-0.03	-0.22	-0.03	ALP	
					1.00	0.65	0.74	0.68	0.26	0.25	0.35*	0.35*	0.77	0.78	0.71	0.74	0.62	0.01	0.28	0.28	-0.10	0.23	-0.27	0.17	0.55	-0.06	-0.08	0.07	VPL	1
3	α	1				1.00	0.75	0.79 **	0.61	0.55	0.18	0.18	0.60	0.60	0.48	0.62	0.48	0.01	-0.09	-0.09	-0.16	-0.16	0.12	-0.15	0.32 *	-0.02	-0.29	-0.14	FVA	
	β	0.70	1				1.00	0.94	0.68	0.37 *	0.25	0.25	0.63	0.64	0.55	0.65	0.54	-0.21	-0.06	-0.03	-0.33	-0.08	-0.04	0.03	0.40 *	0.05	-0.12	0.08	VVD	
	ANC	0.67	0.99 **	1.00				1.00	0.78	0.39 *	0.29	0.29	0.64	0.65	0.58	0.66	0.54	-0.05	-0.05	-0.03	-0.25	-0.07	0.08	-0.01	0.31	-0.03	-0.19	-0.05	MVD	
	γ	0.85 **	0.71	0.70 **	1.00				1.00	0.41	-0.09	-0.10	0.21	0.22	0.16	0.26	0.19	-0.05	-0.36	-0.34	-0.31	-0.39 *	0.50 **	-0.29	0.04	0.06	-0.18	-0.07	AVD	
	IDA	0.33 *	0.72	0.72	0.20	1				1.00	-0.27	-0.28	-0.01	0.00	0.08	0.02	0.07	-0.04	-0.27	-0.28	-0.06	$^{-0.40}_{*}$	0.20	-0.14	0.25	0.48	0.00	-0.18	CAP	
4	AW	0.07	$^{-0.35}_{*}$	-0.37	0.01	-0.36	1.00				1.00	0.99 **	0.76	0.75	0.79 **	0.71	0.61	0.29	0.62	0.63	0.03	0.49	$^{-0.36}_{*}$	0.16	0.33 *	$^{-0.34}_{*}$	-0.02	-0.07	L	
	PAR	-0.11	0.12	0.15	-0.08	0.14	-0.78	1.00				1.00	0.73	0.72	0.74	0.68	0.54	0.18	0.54 **	0.55	-0.09	0.50	-0.37	0.17	0.27	-0.36	-0.08	-0.06	Ν	
	FRA	0.26	0.52	0.54	0.30	0.28	-0.54	0.43	1.00				1.00	1.00	0.93 **	0.95 **	0.87	0.23	0.47	0.48	0.02	0.33 *	-0.28	0.12	0.51	-0.25	-0.11	-0.01	TL	
5	CAS	-0.13	-0.19	-0.18	0.11	-0.35	-0.11	0.30	0.41	1.00				1.00	0.94 **	0.96 **	0.87 **	0.23	0.47	0.48	0.02	0.33 *	-0.25	0.12	0.50 **	-0.24	-0.08	-0.02	Р	2
	D	0.18	0.36 *	0.39 *	0.25	0.27	-0.07	0.18	0.38 *	0.44	1				1.00	0.87	0.88	0.29	0.56	0.57	0.08	0.36 *	-0.31	0.18	0.60	-0.16	0.04	-0.04	SP	
	CE	-0.09	-0.07	-0.09	-0.13	-0.16	-0.04	0.00	-0.05	0.01	0.06	1				1.00	0.81	0.22	0.44	0.45	0.02	0.33 *	-0.09	-0.06	0.41	-0.26	-0.08	-0.03	А	
Factors		α	β	ANC	γ	IDA	AW	PAR	FRA	CAS	D	CE					1.00	0.34 *	0.51	0.52	0.21	0.26	-0.26	0.16	0.65	-0.08	0.09	0.03	SHA	
				3				4			5																			

Table 4. Correlations among path indicators and service scope.

* p < 0.5. ** p < 0.01; See Tables 1 and 3 for abbreviations of indicators and factors.

Factor 1, Average, comprises ten indicator parameters (Table 3): Average Connection Length (ACL), Average Length (AL), Average Shorter Paths (ASP), Average Longer Paths (ALP), Variation of Path Length (VPL), Field of Vision Area (FVA), Variation of Visual Distance (VVD), Maximum Visual Distance (MVD), Average Visual Distance (AVD), and Circuitous Angle of One Path (CAP). These parameters are closely related to each other (Table 4), and collectively reflect the characteristics of average values of paths in all aspects. Therefore, the Average Connection Length (0.981) with the highest load is selected as the representative indicator of factor 1.

Factor 2, Scale, comprises seven indicators: Lines (L), Nodes (N), Total Length (TL), Perimeter (P), Simple Paths (SP), Area (A), and Shape Index (SHA) (Table 3). This factor is the most basic and common indicator when defining the path scale. Here, the Lines section with the highest value (0.912) is selected as the representative indicator.

Factor 3, Network, reflects the topology of the path network, and has five indicators, Alpha (α), Beta (β), Average Number of Connections (ANC), Gamma (γ), and Intersection Deflection Angle (IDA) (Table 3). There is a high correlation between these indicators (Table 4). Therefore, the Alpha (0.963) with the highest value is selected as the representative.

Factor 4, Wide, comprises three interrelated indicators: Average Width (AW), Perimeter-Area Rate (PAR), and Fractal (FRA) (Table 4). As the paths are mostly narrow and long strips, the length should be focused on (and width can be ignored), so the ratio of area and perimeter is essentially equivalent to half of the path width, and the values of both are very similar and far greater than the value of Fractal (Table 4). As the Average Width (-0.949) is easy to understand and implement, it is selected as the representative indicator.

Factor 5, Aggregation, comprises three indicators: Circuitous Angle of One Step (CAS), Density (D), and Centrality (CE). Circuitous Average Angle is related to Density, but Centrality exists independently (Table 4). When only the average bending degree increases, the direction of each step will change greatly, making the overall layout of the path curl up. In addition, when the degree of Centrality becomes smaller, the path will focus on the centre of the courtyard. When the Density increases, the path will fill the whole garden. The independently existing Centrality (0.383) and CAS (0.771) with the highest load are selected as the representatives of this factor.

Finally, the 28 indicator parameters were compressed into six indicators that are highly representative but only weakly correlated with each other (Table 4): Average Connection Length, Lines, Alpha, Average Width, Circuitous Angle of One Step, and Centrality. They constitute the most basic parameters with which to describe the characteristics of BTPG paths.

3.2. Similarity and Application of BTPG Paths

After normalizing the data of each representative indicator, the Euclidean distance of different garden path schemes was calculated. The greater the Euclidean distance, the lower the similarity of garden views. The results show that the 14 BTPGs have high similarity (mean ED 1.30), whereas the Daguan Garden has low similarity (mean ED 3.42) (Table 5).

The Euclidean distance between traditional gardens is less than 2. Among them, the degree of similarity between Gunbeizi Garden and Liwang Garden is the highest (ED 0.57), and the similarity between Langrun Garden and Banmu Garden is the lowest (ED 2.11). According to the garden plan (Figure 2), there are many quadrangles in Gunbeizi Garden and Liwang Garden. Their path networks are in a circular layout and the structure is similar; In contrast, Langrun Garden and Ke Garden are not consistent in terms of garden scale, architectural layout, or spatial form. The quantitative and qualitative results are highly consistent, proving that the representative indicators are scientific and effective.

The ED between the contemporary Daguan Garden and the BTPGs is far greater than the ED between the BTPGs: Daguan Garden is most similar to Gongwangfu Garden (ED 2.79) and Gunbeizifu Garden (ED 2.89), and most different from Banmu Garden (ED 3.94). By comparing the path plans of Daguan Garden and 14 BTPGs, it can be seen that the path network of Daguan Garden is very complex. The paths of Gongwangfu Garden and Gunbeizifu Garden are also the most complex among the 14 gardens (Figure 2), which is again consistent with the quantitative results.

Table 5. Similarity between gardens.

No.	Average	a	b	с	d	e	f	g	h	i	j	k	m	n	р	Average
а	1.26	0.00														
b	1.19	1.19	0.00													
с	1.60	1.43	1.05	0.00												
d	1.60	1.74	1.61	1.74	0.00											
e	1.21	1.26	1.11	1.42	1.61	0.00										
f	1.38	1.23	1.51	1.97	1.53	1.47	0.00									
g	1.14	1.12	1.04	1.45	1.54	0.73	1.38	0.00								
h	1.12	1.21	0.81	1.30	1.34	0.98	1.04	1.05	0.00							
i	1.09	0.97	0.68	1.24	1.53	1.04	1.05	1.02	0.57	0.00						
j	1.60	1.41	1.67	2.11	1.96	1.67	1.67	1.55	1.63	1.50	0.00					
k	1.26	1.09	1.32	1.88	1.65	1.23	0.88	1.09	1.01	1.06	1.62	0.00				
m	1.26	1.33	1.22	1.76	1.48	1.14	1.39	0.87	1.33	1.27	1.37	1.14	0.00			
n	1.42	1.20	1.34	1.86	1.96	1.20	1.72	1.03	1.49	1.35	1.38	1.41	1.25	0.00		
р	1.08	1.24	0.97	1.57	1.22	0.93	1.11	0.90	0.78	0.91	1.22	1.03	0.83	1.32	0.00	
q	1.30	3.52	2.79	3.14	3.64	3.48	3.52	3.47	3.07	2.89	3.94	3.59	3.60	3.72	3.47	3.42

See Figure 1 for abbreviations of garden name.

In order to eliminate the influence of different site areas on the results, the linear relationship between the service area of various types of paths in 14 traditional gardens and the representative indicators was tested. Table 6 shows that the Average Connection Length (ACL) of HP and MP, and the Links (L) of CP, are linearly related to the garden area served (p < 0.05) and meet the conditions of linear correlation. The correlations between other indicators and the garden area are not significant (p > 0.05), with the standard deviation between indicators being low and the fluctuation small. Therefore, the average value of traditional garden paths can be directly used and there is no need for correlation analysis. According to the linear regression prediction, the area of Daguan Garden, the average connecting length of the horizontal path in the standard traditional garden should be 49.82 m (actual: 21.80 m), the mountain path should be 44.80 m (actual: 19.06 m), and the number of the courtyard path should be 26.62 (actual: 19 paths) (Figure 6).

Table 6. Reference range of Daguan Garden.

	Type		- 1		BTI	PGs		C:- (., V-1., .)	
Indicators	Type	Daguan Garden	Reference	MAX	MIN	AVE	SD	Correlations	Sig. (<i>p</i> -value)
	HP	21.80	45.0-50.0	46.93	7.66	19.20	11.99	0.827 **	0.0006
ACL (m)	MP	19.06	40.0-45.0	64.82	5.20	17.92	16.72	0.947 **	0.0000
	CP	11.00	No	11.30	4.46	8.42	2.01	0.38	0.2864
	HP	183.00	100.0-150.0	51.00	4.00	22.50	14.56	0.28	0.3353
L (pcs)	MP	21.00	10.0 - 15.0	18.00	1.00	6.31	4.75	-0.18	0.3934
	CP	19.00	25.0-30.0	43.00	1.00	16.08	11.47	0.719 **	0.0076
	HP	0.06	-0.05 - 0.00	0.01	-0.13	-0.06	0.05	-0.07	0.7969
α	MP	-0.05	No	0.11	-0.36	-0.11	0.14	0.16	0.5129
	CP	-0.13	No	0.00	-0.17	-0.07	0.05	-0.40	0.2392
	HP	2.27	1.5-2.0	2.67	0.91	1.28	0.42	-0.04	0.8790
AW (m)	MP	1.12	2.0-2.5	3.84	1.09	2.06	0.75	0.37	0.2630
	CP	2.22	No	2.29	0.69	1.58	0.47	-0.02	0.9549
	HP	0.76	1.5-2.0	8.60	0.10	1.92	2.00	-0.34	0.2304
CAS (deg)	MP	1.28	2.0-3.0	8.60	0.50	3.13	1.94	-0.04	0.7011
	CP	0.48	No	1.50	0.00	0.34	0.45	0.08	0.9076
CE	HP	0.71	No	0.71	0.47	0.58	0.09	0.27	0.3441
	MP	0.64	No	0.91	0.24	0.60	0.17	-0.31	0.0339
	СР	0.83	No	1.02	0.27	0.60	0.21	0.02	0.9429

See Table 1 and Figure 1 for abbreviations of indicators and garden name. ** p < 0.01.



Figure 6. Relationship between Average Connection Length of HP (left) and MP (middle), number of CP (right) and service area of paths (Red: Theoretical value of Daguan Garden; Yellow: Actual value of Daguan Garden.) (See Figure 1 for abbreviations of garden names).

3.3. Daguan Garden: Suggestions for Path Design

There are large differences in several indicators between Daguan Garden and its traditional predecessors. First, in terms of path network, the path of Daguan Garden is more complex than that of traditional gardens, and it is easier to lose direction while walking around. The path network of Daguan Garden should be simplified, so as to reduce the number of difficult choices that visitors face at certain points and improve the efficiency of the visitor experience. Given that the Average Connection Length of Daguan Garden is low (21.80 m), the connection length of each node needs to be increased. The number of paths (183) and Alpha (0.6) is far higher than those of traditional gardens; thus, the number of unnecessary paths should be reduced. The HP of Daguan garden needs to be narrowed and tortuous. By adjusting the degree of path curvature in the traditional garden, the effect of a winding path leading to secluded and novel scenery can be achieved, increasing the enjoyment level of the visitors. For example, the average width of the HP (2.27 m) is much higher than that of the traditional garden (1.28 m), so it is necessary to reduce the path's Average Width. Moreover, the degree of Circuitous Angle (0.76°) in Daguan Garden is far lower than that in traditional gardens (1.91°) , indicating that the curvature of the path needs to be increased.

The mountain path (MP) of Daguan Garden shows a great difference from the traditional garden in four indicators: Average Connecting Length, Links, Average Width, and Circuitous Angle of One Step. In terms of network structure, the MP of Daguan Garden resembles that of a horizontal path (HP), and the network is more complex than a traditional garden. Daguan Garden's MP has a lower connection length (19.06 m); thus, the length of the paths connected by each node should be increased. The number of links (21.00) is unnecessarily higher than that of traditional gardens and needs to be appropriately reduced. As for the individual characteristics of the path, the MP curvature (1.28°) is also smaller than that of a traditional garden (3.13°). Unlike the HP, the MP (1.12 m) of Daguan Garden is not wider than that in a traditional garden (2.06 m), and thereby should be widened to improve traffic circulation and safety.

In terms of courtyard path (CP), only the number of path sections in Daguan Garden is significantly different from that of traditional gardens. According to the linear prediction, there should be 26.62 courtyard paths in Daguan Garden, but there are only 19; this is higher than the average value in traditional gardens (16.08). However, the courtyard of Daguan Garden is large in courtyard scale, and it still needs an appropriate number of courtyard paths to meet the demand of connectivity.

The path network of Daguan Garden is more complex than that of the BTPGs and must be simplified. Visitors need to able to choose their next direction for sightseeing at intersections node after walking a short distance. If frequent and repeated choices fail to meet the visitors' demand for sightseeing, their satisfaction will be greatly reduced [84]. In addition, the pedestrian experience on the paths is affected by the curvature change of the centre line [85], wherein the increase in the tortuous angle improves the sightseeing

performance of the garden and provides a better visitor experience [47]. However, the shape curvature of the path of Daguan Garden is less than that of traditional gardens, and this path does not achieve the twists and turns of traditional gardens.

4. Discussion

4.1. The Principle of Preparing Suitable Indicators by Screening

Section 2.4 introduced the statistical selection procedure of path quantitative indicators in this study. This procedure is based on the following three principles:

Principle 1: the maximum indicator set entering the preliminary screening is complete for quantifying the garden path. In order to meet this condition, the indicators in the present study have a wide coverage, including large (city), medium (block), and small (garden/park) scales (Section 1.2).

Principle 2: after indicator screening, the representative indicator set can scientifically and effectively replace the maximum indicator set. First, in the process of screening, the initial selection is made by selecting the indicator with the largest proportion of each factor. Second, it is determined whether the selected indicators are related to other indicators in the same group through correlation, and the irrelevant indicators are added to further improve the representativeness of all indicators. For example, Centrality is added based on this principle.

Principle 3: the selected indicators are easy to operate and implement. The designer can easily control the results of indicators such as path width, length, and number, so as to easily control the design content.

The six representative indicators obtained in this study (Average Connection Length, number of Links, Alpha, Average Width, and Circuitous Angle of One Step) have a simple calculation process and clear guidance for the design content. They not only better represent the largest indicator set, but also frame the characteristics of the garden path from different dimensions such as the overall path network and the individual form. This process achieves the expected effect in completeness, representativeness, and operability.

4.2. Innovation, Limitations, and Prospects

The results of this study show some similarities to those obtained from previous research. For example, as this study finds that the width of the paths is not directly related to the scale of the garden [36], the path width in the contemporary garden is far greater than that in the traditional garden [46], and the road length has a confidence interval [48]. However, there are also some differences. For example, the average width of the HP in this study is 1.28 m, while that of the CP and MP is 1.58 m and 2.05 m, unlike the two-metre value previously identified. This variation may be attributable to the number of samples studied and the fineness of path division. In addition, the curvature indicator in this study does not have a correlation with the garden scale, unlike in the previous research by some Japanese scholars and this may be related to the type of garden. The circuit style garden in Japan is often larger than the appreciation style garden and has different viewing modes. In circuit style gardens, the designer's intent is to encourage visitors to spend more time in the garden, with repeat visits [47]. Hence, it is appropriate for curvature to change with the garden scale. An alternative suggestion in Japan is that with the increase in garden scale, the overall space is divided into more small spaces, and this results in the road curvature not increasing [46].

Quantitative indicators can also have defects or errors, and the indicators selected in this study are no exception [57]. For example, visibility field analysis (FVA, AVD, and MVD) often ignores the influence of plants on people's line of sight. In order to obtain more accurate sight distance measurement results, it is necessary to measure trees' positions, crown heights, trunk thicknesses, the light transmittance of the leaves, etc.—all of which require an increased workload. In general, research using spatial syntax theory will not carry out such high-accuracy work, so the application of similar indicators in the field of landscape architecture has limitations [51]. Another similar example is precise path classification. The separation of the three types of paths simplifies the network structure for each type of path more than that of the whole garden, resulting in nil or negative values for indicators suitable for measuring the complex path network (such as Alpha, Beta, and Gamma). It shows a very simple road network structure, which is very rare [86].

The model also has limitations in operation. Although the calculation of indicators can be generated directly using parameter software, the extraction process for image rendering generates a huge workload that is both time consuming and labour demanding. In addition, the design parameters differ for different regional environments. There are several variations in the scale of gardens in the north and south. At the start of designing, it is necessary to resample the surrounding gardens and obtain numerical information for the reference indicators, which increases the computational workload. At this time, it is particularly important to improve the efficiency of image rendering, reduce the time requirements (such as image automatic generation technology), and share data information. This is a critical process and a key step in the future implementation of this model, which will eventually be extended to more garden types and serve many designers.

In comparison with previous research, this study takes a further step to improve the comprehensiveness, quantity, and quality of indicators for the analysis of garden paths in traditional gardens. One example is in the classification of garden paths, highlighting the impact of altitude on the line of sight from the path, thereby reducing the error in the results in comparison with previous line of sight analysis in space syntax [51]. In addition, the geographical environment of private gardens in Beijing also suggests that the error in line of sight indicators has been reduced, because the plants in the Beijing gardens are mostly large trees such as locust, willow, and poplar unlike those in gardens in southern China. Moreover, their crown heights are higher level from the ground, and their leaves are dry in winter [40]. These cannot, therefore, effectively block the sightlines of visitors and can be ignored. In contrast, the compression results from the indicators do not include the indicators for spatial vision, which avoids the occurrence of errors. The results of the calculation process using representative indicators display a good relationship to the garden size and clearly and accurately represent the characteristics of the garden paths. This is a prerequisite for the smooth promotion of indicators and models in future applications.

4.3. Application of the Indicator Parameters to Guide Design

In contrast to previous studies of garden paths (Section 1.2), this study proposes a broader range of indicators to explain garden path characteristics. Although the quantitative indicators are clear, it is not advisable to implant the quantitative results into the reference sizes of traditional gardens. Garden design is a reciprocating process that is influenced, restricted, and balanced [87] by various garden elements. Garden path design should also be considered in combination with the layout of various elements in the garden or park, the functional requirements of the users, the follow-up operations and management, the climatic environment of geographical areas, etc. The quantitative results should be appropriately adjusted as a reference.

In addition, the quantitative research focuses on the physical space of the path, whereas the traditional Chinese garden is a combination of physical space and an artist's particular conception of space [88]. When the design of a path takes into account the artist's spatial concepts, this will limit the quantitative approach (Results 3.3). To highlight a solemn atmosphere, for example, the paths along the central axis should be as wide and straight as possible. To pursue the mysterious quietness and be pleasantly surprised, the road will inevitably become tortuous and varied [89]. Aesthetic conceptions of space in garden path design are too important to be put aside in pursuit of quantitative results.

Function is also a non-negotiable aspect of path design, even though some scholars suggest that the relationship between garden function and path alignment is weak [46]. However, the scale of the path should still meet the needs of the function. For example, the Daguan Garden in Beijing was built on the prototype of the traditional, Qing-era private garden described in *A Dream of Red Mansions*. Historically, traditional gardens were only

used by a limited number of people, usually families. Today, Daguan Garden is open to the public, bringing great pressure on the paths. Usually, this pressure would be eased if paths were to be widened and new paths built [90]. It cannot be denied that nowadays traditional gardens are also making a shift from privacy to openness, such as the well-preserved Chunwangfu Garden and Gongwangfu Garden, which have become green spaces for the general public to relax and be entertained. The limited number of narrow paths in these gardens does not limit public appreciation of these spaces [40].

In addition, when using the indicator parameters, the operation and management of the garden and the local climate also need to be considered. For example, in gardens in cold areas, for safe winter travel in rain and snow, the path is usually flat and straight, while in warm and hot areas, this is not such an important consideration [91]. Design, as a process of sensibility, rationality, and mutual prosperity, has no fixed pattern. Use of mathematical relationships can help improve results. Exploration of the indicators mentioned here, and their varying performances, can be enhanced by the use of a certain approach to space. Therefore, it is appropriate to formulate separate parameters for different functions, environments, and artistic styles of garden paths.

5. Conclusions

The objective of this paper is to determine a minimal indicator set describing the spatial characteristics of private traditional garden paths in Beijing and to use the results as a reference guide for path design of contemporary antique gardens. In this study, 28 quantitative path indicators were selected to analyse the characteristics of three types of paths in 14 private gardens in Beijing. Among them, the characteristics of paths in five aspects of Average, Scale, Network, Wide, and Aggregation can be expressed by six representative simple indicators: average connection length, number of path sections, alpha index (a), average width, average tortuous angle, and concentration degree. The 28 quantitative path indicators can be compressed to the six representative indicators.

The makeup of the indicator relates to the overall network and the individual form of the path, reflecting the mathematical characteristics behind the phenomenon of winding paths and the changing of views with each step that is found in traditional gardens. The results suggest that the representative indicators clearly distinguish the differences in garden paths. The selected indicators performed well for generality, representativeness, and implementation. This makes it possible to improve the contemporary antique garden by applying the wisdom from traditional gardens. Future studies can carry out in-depth research on the basis of the six indicators according to the specific control type.

Finally, the Euclidean distance between the Daguan Garden and the traditional garden path in Beijing was calculated using representative indicators, and the difference between the two was intuitively reflected. With reference to the linear and non-linear results of the service area and indicators, clear suggestions are put forward for the improvement of paths in Daguan Garden. The 28 path indicators are not affected by the design process and can be used during design and repair. This study also has limitations in terms of representativeness and operability, so future work should also consider garden samples of different types and from different regions.

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