LABORATORIUM FÜR DEN KONSTRUKTIVEN INGENIEURBAU (LKI)
TECHNISCHE UNIVERSITÄT MÜNCHEN

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RELIABILITY OF REDUNDANT DYNAMIC SYSTEMS WITH BRITTLE COMPONENTS

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ABSTRACT

The reliability of redundant systems under stationary Gaussian white noise excitation is investigated with emphasis on the dynamics of redistribution of internal forces after brittle failure of one or more system components. The non–stationary upcrossing approach is used together with FORM/SORM integration techniques for an approximation valid for high reliabilities. It is found that consideration of the dynamics of redistribution may be important especially if the system damping is small. Two simpler, limiting calculation schemes are also proposed. The methods are illustrated at a simple Daniels system.

Keywords: system reliability, non–stationary upcrossings, multiple crossings, dynamic stress redistribution

1. Introduction

Redundant structural systems designed according to the same rules are considered to provide higher reliability than non–redundant systems especially when the degree of redundancy is high. This is certainly true for systems where the components fail in a ductile manner. Much less redundancy, however, is available if componental failure is brittle. In this case redundancy is further reduced because dynamic effects are present during internal redistribution of the loads. If the system reacts dynamically such effects should be particularly relevant. Although these presumptions have been expressed repeatedly in the literature detailed studies on this subject are virtually non–existent even in a purely deterministic context. In this paper a first attempt is made to quantify the reliability of redundant dynamic structural systems based on previous formulations for the time–variant reliability of redundant structural systems under quasi–static but time–variant random loading [1,2,3]. The reliability is determined by the upcrossing approach to time–variant structural reliability. The applied methods are particularly

accurate if structural reliability is high. They are developed and illustrated at a simple Daniels–system.

2. Mechanics and Failure Process of the Daniels System to System Collapse

Figure 1 shows a Daniels system with n linear–elastic, viscously damped, brittle components subject to stationary Gaussian white noise $W(\tau)$ with one–sided spectral density of intensity W_0. The system has one degree of freedom. The displacements $X(\tau)$ are measured from the undeformed position of the intact system. Each component has the same but random stiffness S and the same but random damping C. Also, the mass M of the system is assumed to be uncertain.

The equation of motion of the system in state i–1, i.e. with n–(i–1) unfailed and i–1 failed components is [4]:

$$\ddot{X}(\tau) + 2\,\zeta_{i-1}\,\omega_{0,i-1}\,\dot{X}(\tau) + \omega_{0,i-1}^2\,X(\tau) = \frac{W(\tau) + M\,g}{M} \qquad (2.1)$$

where

$$\omega_{0,i-1} = \omega_{0,0}\,((n-(i-1))/n)^{1/2}$$
$$\omega_{0,0} = (nS/M)^{1/2}$$
$$\zeta_{i-1} = \zeta_0(n/(n-(i-1)))^{1/2}(C_{i-1}/C_0)$$
$$\zeta_0 = C_0/(2\omega_{0,0}M)$$
$$C_{i-1} = (n-(i-1))\,C$$

Two cases for the direction of the load $W(\tau)$ will be studied (see cases A and B in figure 2). Also, two models are considered for the damping coefficient C_{i-1}.

$C_{i-1} = (n-(i-1))\,C$ (Model 1)
$C_{i-1} = n\,C$ (Model 2)

Model 1 implies that with the failing element the damping dashpot also vanishes so that total structural damping decreases with system degradation. In model 2 all dashpots remain intact and structural damping increases as damage progresses. Note that response features can change significantly in time because initially overdamped or underdamped systems may evolve into underdamped (model 1) or overdamped (model 2) systems [4].

Assume that the system is initially intact but after some time has degraded into state i–1 due to previous failures of components 1, 2, ..., i–1. Eventually, there will be a crossing of the temporarily lowest level R_i by the response process $X(\tau)$. If there is a crossing of R_i (primary crossing) the effective stiffness and damping experience an abrupt change due to the assumption of brittle elemental behavior. Therefore, the crossing is followed by a transient phase of motion until the motions become stationary again. During that transient phase further strength levels can be crossed depending on the velocity of the response process at the crossing of the lowest level and the relative spacing of the higher strength levels. More precisely, the crossing of the lowest level i can be followed either by no further crossing, by a crossing of the next level i+1 but not of level i+2, by a crossing of the levels i+1 and i+2 but not of i+3, etc. The event of a primary crossing followed by further secondary crossings during the transient phase will be denoted as progressive failure or a multiple crossing event. Hence, the life time of the system is a sequence of stationary phases each of which ends with a primary crossing event followed by a transient phase in which secondary crossings can occur. The system fails if the last component(s) fail(s). Since it can reasonably be assumed that the transient states of motion are short as compared with the times between primary crossings the life time of the structure is well approximated by the sum of the times T^ℓ, $\ell = 1,2,....$, between primary crossings. Due to the randomness in the crossing events there usually are many such sequences (failure paths) differing by the types and the composition of contributing events. As in other time–variant reliability problems the failure event, finally, is not the event that the structure is in a failed state at some arbitrary point in time but the event that it reaches the failed state for the first time within some pre–specified service time.

The initial values of mean and covariances of the response $(X(\tau), \dot{X}(\tau))$ in the ℓ–th time step of system degradation are:

$$X^\ell(0) = X^{\ell-1}(T^{\ell-1})$$
$$\dot{X}^\ell(0) = \dot{X}^{\ell-1}(T^{\ell-1})$$
$$\Sigma^\ell_{XX}(0) = \Sigma^\ell_{X\dot{X}}(0) = \Sigma^\ell_{\dot{X}\dot{X}}(0) = 0 \qquad (2.2)$$

where $T^{\ell-1}$ is the failure time of one or more components in the previous system state. $X^\ell(0)$ equals the strength level crossed in the previous time step.

For $W(\tau)$ being stationary Gaussian white noise the means and the covariance matrix of the response $(X(\tau), \dot{X}(\tau))$ can be calculated analytically:

$$\mathbf{m}^{\ell}(\tau) = \begin{bmatrix} a^{\ell}(\tau) & b^{\ell}(\tau) \\ \dot{a}^{\ell}(\tau) & \dot{b}^{\ell}(\tau) \end{bmatrix} \mathbf{m}^{\ell}(0) + \frac{m_W + Mg}{M} \begin{bmatrix} \int_0^{\tau} b^{\ell}(u)du \\ \int_0^{\tau} \dot{b}^{\ell}(u)du \end{bmatrix}$$

$$\Sigma^{\ell}(\tau) = \begin{bmatrix} a^{\ell}(\tau) & b^{\ell}(\tau) \\ \dot{a}^{\ell}(\tau) & \dot{b}^{\ell}(\tau) \end{bmatrix} \Sigma^{\ell}(0) \begin{bmatrix} a^{\ell}(\tau) & \dot{a}^{\ell}(\tau) \\ b^{\ell}(\tau) & \dot{b}^{\ell}(\tau) \end{bmatrix} +$$

$$+ \frac{\pi W_0}{M^2} \begin{bmatrix} \int_0^{\tau} b^{\ell}(u)^2 du & \int_0^{\tau} b^{\ell}(u)\, \dot{b}^{\ell}(u)du \\ \int_0^{\tau} b^{\ell}(u)\, \dot{b}^{\ell}(u)du & \int_0^{\tau} \dot{b}^{\ell}(u)^2 du \end{bmatrix} \qquad (2.3)$$

The various auxiliary functions a, $\dot{a}$, b, $\dot{b}$ in these formulae and their integrals are given in the appendix for easy reference. Without loss of generality we shall assume that $E[W(\tau)] = m_W = 0$. Then, the stationary mean response will be zero in case A whereas it is equal to the static response in case B . The covariance matrix in the stationary state is:

$$\Sigma^{\ell}(\tau \to \infty) = \begin{bmatrix} \Sigma^{\ell}_{XX} & 0 \\ 0 & \Sigma^{\ell}_{\dot{X}\dot{X}} \end{bmatrix}$$

with $\Sigma^{\ell}_{XX} = \frac{\pi W_0}{4 M^2 \zeta_{i-1} \omega^3_{0,i-1}}$ and $\Sigma^{\ell}_{\dot{X}\dot{X}} = \Sigma^{\ell}_{XX} \omega^2_{0,i-1}$. (2.4)

The system leaves the ℓ–th time step when the response $X^{\ell}(\tau)$ exceeds the temporarily lowest threshold R_i. The corresponding (stationary) mean upcrossing rate ν_i^{ℓ} for this primary upcrossing event is [5]:

$$\bar{\nu}_i^{\ell}(R_i) = \frac{\omega_{0,i-1}}{\sqrt{2\pi}} \varphi\left(\frac{R_i - g/\omega^2_{0,i-1}}{\sqrt{\Sigma^{\ell}_{XX}}}\right) \qquad (2.5)$$

The notation $\bar{\nu}_i^{\ell}(R_i)$ is used for stationary upcrossing rates. Later we shall also need the non–stationary rate for a secondary upcrossing of a level $R_j > R_i$ [5]:

$$\nu_j^{\ell+1}(R_j,\tau) = \left[\alpha(\tau)\, \Phi\left(\frac{\alpha(\tau)}{\beta(\tau)}\right) + \beta(\tau)\, \varphi\left(\frac{\alpha(\tau)}{\beta(\tau)}\right)\right] \varphi(\xi(\tau)) \qquad (2.6)$$

where

$$\alpha(\tau) = \left\{\dot{X}^{\ell}(0) + \frac{\Sigma^{\ell+1}_{X\dot{X}}(\tau)}{\Sigma^{\ell+1}_{XX}(\tau)} (R_j - X^{\ell}(0))\right\} \Sigma^{\ell+1}_{XX}(\tau)^{-1/2}$$

$$\beta(\tau)^2 = \Sigma^{\ell+1}_{\dot{X}\dot{X}}(\tau) / \Sigma^{\ell+1}_{XX}(\tau) - (\Sigma^{\ell+1}_{X\dot{X}}(\tau) / \Sigma^{\ell+1}_{XX}(\tau))^2$$

$$\xi(\tau) = (R_j - X^{\ell}(0))\, \Sigma^{\ell+1}_{XX}(\tau)^{-1/2}$$

and the superscript $\ell+1$ in $\nu_j^{\ell+1}(R_j,\tau)$ refers to the next time step. Since there cannot be any confusion these superscripts will be omitted in the following.

In figure 3 the means and covariances of the response, the mean outcrossing rate and its integral are shown for a system which initially is at rest and one component fails at time T^1.

3. Reliability Analysis

3.1 General Considerations

An exact reliability analysis of the problem just described appears hopelessly complicated if possible at all. In particular, the probabilistic modeling and mathematical manipulations of the multiple crossing events appear to be extremely difficult because the details of the sample paths of the response process must be known. For the properties of the sample paths after crossings at most asymptotic results are available [6] which turn out to be of not much use. Here, an approximate approach not directly based on sample path properties is proposed following in part the arguments in references [1,2,3].

Assume that the structure initially is intact and is in a stationary state of motion. A certain sequence of component failures, a failure path, will eventually lead to system collapse. If there are K possible paths to the system collapse, the overall failure probability is the probability of the union of the path failure events $\mathcal{F}_k(t)$.

$$P_f(t) = P(\mathcal{F}) = P\left(\bigcup_{k=1}^{K} \mathcal{F}_k(t)\right) \le \sum_{i=1}^{K} P(\mathcal{F}_k(t)) \qquad (3.1)$$

t is some pre–specified reference period.

In order to determine the probability of the event $\mathcal{F}_k(t)$ the constituting events must be carefully written down. Define by:

$\mathcal{T}_k(\underline{r})$: the event that the sum of failure times in path k is smaller than some pre–specified reference time.

$\mathcal{Q}_k(\underline{r})$: the event that failure occurs in path k.

$\mathcal{A}_k(\underline{r})$: the event that the resistances are arranged or ordered such that failure path k is entered.

The event $\mathcal{A}_k(\underline{r})$ is, in fact, a conditioning event for $\mathcal{Q}_k(\underline{r})$. And $\mathcal{A}_k(\underline{r})$ and $\mathcal{Q}_k(\underline{r})$ are conditioning events for $\mathcal{T}_k(\underline{r})$. All events are conditioned on the vector $\underline{R} = \underline{r}$ of random resistances. Obviously, it is

$$\mathcal{F}_k(\underline{r}) = \{\mathcal{T}_k(\underline{r}) \cap \mathcal{Q}_k(\underline{r}) \cap \mathcal{A}_k(\underline{r})\}$$

and the conditional failure probability can determined from:

$$\begin{aligned} P(\mathcal{F}_k(\underline{r})) &= P(\mathcal{T}_k(\underline{r}) \cap \mathcal{Q}_k(\underline{r}) \cap \mathcal{A}_k(\underline{r})) \\ &= P(\mathcal{T}_k(\underline{r}) | \mathcal{Q}_k(\underline{r}) \cap \mathcal{A}_k(\underline{r}))\, P(\mathcal{Q}_k(\underline{r}) | \mathcal{A}_k(\underline{r}))\, P(\mathcal{A}_k(\underline{r})) \end{aligned}$$

The total failure probability is obtained upon integration over the vector of resistances or, possibly, other time–invariant variables.

$$\begin{aligned} P(\mathcal{F}_k(t)) &= \int_{\underline{R}=\underline{r}} P(\mathcal{T}_k(\underline{r}) \cap \mathcal{Q}_k(\underline{r}) \cap \mathcal{A}_k(\underline{r}))\, dF_{\underline{R}}(\underline{r}) \\ &= \int_{\underline{R}=\underline{r}} P(\mathcal{T}_k(\underline{r}) | \mathcal{Q}_k(\underline{r}) \cap \mathcal{A}_k(\underline{r}))\, P(\mathcal{Q}_k(\underline{r}) | \mathcal{A}_k(\underline{r}))\, P(\mathcal{A}_k(\underline{r}))\, dF_{\underline{R}}(\underline{r}) \end{aligned} \tag{3.2}$$

We shall first describe the events and their probabilities in eq. (3.2) in still vague terms and later add the necessary details. We start with the probability $P(\mathcal{Q}_k(\underline{r}) | \mathcal{A}_k(\underline{r}))$ and will omit the reference to $\mathcal{A}_k(\underline{r})$ and also to $\underline{r}$ for brevity of notation. Obviously, $\mathcal{Q}_k$ is the intersection of the various crossing events along the k–th failure path. If it is assumed that the primary crossing events are independent, i.e. are rare Poissonian events, the probability $P(\mathcal{Q}_k)$ can be given as a product of probabilities:

$$P(\mathcal{Q}_k) = \prod_{\ell=1}^{L_k} P(\mathcal{Q}_k^\ell) \tag{3.3}$$

$P(\mathcal{Q}_k^\ell)$ is the probability for the event that system degradation proceeds in the ℓ–th time

step along the k–th path. L_k is the number of time steps in the k–th failure path. We shall discuss these probabilities in detail in section 3.2.

The first probability of eq.(3.2) is the system failure probability given that the k–th path to failure is the actual one:

$$P(\mathcal{T}_k(\underline{r})) = P(\sum_{\ell=1}^{L_k} T_k^\ell(\underline{r}) \le t) \tag{3.4}$$

$T_k^\ell(\underline{r})$ is the duration of the ℓ–th time step on the k–th failure path or the time between the ℓ–1–th (composite) and the ℓ–th (composite) crossing event. These probabilities are reviewed in section 3.3.

The third probability in eq. (3.2), $P(\mathcal{A}_k(.))$ is the probability that the specific arrangement of component resistances exists which was assumed for the assessment of the probability $P(\mathcal{Q}_k)$. It can be given as:

$$P(\mathcal{A}_k) = P(\bigcap_{\ell=1}^{L_k} \bigcap_{i=1}^{I_k^\ell} R^\ell_{n_{k\ell(i)}} \le R^\ell_{n_{k\ell(i+1)}}) \tag{3.5}$$

I_k^ℓ denotes the number of components existing in the ℓ–th time step of the k–th path. $n_{k\ell(i)}$ is an integer function which assigns in ascending order the number of the existing componental resistances during the ℓ–th time step of the k–th path. In section 3.4 a special case for this probability is discussed.

3.2 The Path Probabilities $P(\mathcal{Q}_k)$ in the Dynamic Case

The most difficult term in eq. (3.2) is the probability $P(\mathcal{Q}_k)$ as it captures the special aspects of dynamic load redistribution during the transient phase. Denote by ΔT a characteristic length of the transient phase. As mentioned it is assumed that there is $\Delta T \ll T_k^\ell$. By providing some redundancy of description we consider the stationary state ℓ first. Mean and covariance function of the response process are given by the stationary parts of eqs. (2.3). In the ℓ–th state the system still has n–(i–1) components with the ordered resistances $R_i \le R_{i+1} \le \ldots \le R_j \le \ldots \le R_n$. For later notational convenience, another level $R_{n+1} = \infty$ is formally added whose upcrossing rate is zero.

At some time T_k^ℓ the process crosses the level R_i and the oscillations become non–stationary due to the change in the system properties. A stress redistribution also must take place. The initial values for the transient phase immediately after the crossing are $X(0) = R_i$ and $\dot{X}(0)$ where, for convenience of notation, the times for the next system state $\ell+1$ are counted starting from the primary crossing time. During that transient phase either no further crossing occurs or the levels $R_{i+1}, R_{i+2}, \ldots$, up to R_j can be crossed. After some time the oscillations become stationary again (compare also fig. 4).

The initial velocities $\dot{X}(0)$ immediately after a crossing follow a Rayleigh distribution according to [6]. The velocities in different time steps are assumed independent. In contrast, in a multiple crossing event they are assumed to be fully dependent i.e. are identical to the velocity at the primary crossing.

Given that the lowest level R_i is crossed possibly followed by secondary crossings the lowest strength level in the next system state will be $j+1$ $(i+1 \leq j+1 \leq n+1)$ in time step $\ell+1$. Denote by

$$(\nu_j - \nu_{j+1}) \tag{3.6}$$

the crossing rate of level $j \geq i+1$ but not of level $j+1$. Integration over time yields the mean number of crossings of the described type. In the stationary case one just multiplies by t. This quantity divided by the total mean number of crossings of the various types is the (long–term) probability of a transition of the system from a stationary state i–1 in time step ℓ into a stationary state j+1 where again stationarity will be reached during time step $\ell+1$. By inspection one recognizes that this sum of mean numbers of crossings is also the mean number of crossings of level i. (see [1,2,3,7] for another interpretation)

In order to quantify the required crossing probabilities we now adopt various limiting assumptions corresponding to different calculation methods. The starting point for the various approximations is the stationary upcrossing rate. In the stationary case the rates no more depend on $\dot{X}(0)$. Eq. (3.6) appropriately normalized reads:

$$P(\mathcal{Q}^\ell_{k,i\cap j}) = \frac{(\bar\nu_j - \bar\nu_{j+1})\,t}{\sum_{r=i}^{n} (\bar\nu_r - \bar\nu_{r+1})\,t} = \frac{\bar\nu_j - \bar\nu_{j+1}}{\bar\nu_i} \tag{3.7}$$

The event $\mathcal{Q}^\ell_{k,i\cap j}$ denotes the multiple crossings of the levels i through j on the k–th failure path at the beginning of the $\ell+1$–th time step.

Method 1:

It is, as an extreme limiting case, assumed that after the primary crossing the changes in the system properties take much longer than the duration of the transient phase. Therefore, we have

$$P_1(\mathcal{Q}^\ell_{k,i\cap j}) = \frac{\bar\nu_{i,j} - \bar\nu_{i,j+1}}{\bar\nu_{i,i}} \tag{3.8}$$

The additional subscript i for the mean upcrossing rates indicates the lowest level R_i in which the system properties are determined. This case may also correspond to a very large damping in the system.

Method 2:

It is now assumed that after a primary crossing the changes in the system properties take place immediately. However, the transient phase and, hence, all dynamics of stress redistribution are neglected.

$$P_2(\mathcal{Q}^\ell_{k,i\cap j}) = \frac{\bar\nu_{j,j} - \bar\nu_{j+1,j+1}}{\bar\nu_{i,i}} \tag{3.9}$$

The subscript i, j and j+1 indicate that the relevant system properties now correspond to state i, j, and j+1, respectively.

Method 3:

Following [3] the dynamic effects in the transient phase are taken into account by their maximum possible value which corresponds to zero damping in the system. More precisely, let $(n–i+1)\,R_i$ be the stress in the components at the primary crossing which has to be redistributed among the remaining components. After the crossing the stationary stresses in the remaining components must be larger by the amount $R_i/(n–i)$ or

$$\Delta\sigma_i = \frac{R_i}{n - i}$$

The maximum dynamic impulse is twice that amount if the conditions of method 1 are taken as a reference. Therefore, by assuming that multiple crossings occur immediately after the primary crossing the residual strength of the level which is no more crossed is $R_{j+1} - \Delta\sigma_j$. Then, eq. (3.9) is modified as:

$$P_3(\mathcal{Q}^{\ell}_{k,i\cap j}) = \frac{\tilde{\nu}_{j,j} - \tilde{\nu}_{j+1,j+1}}{\bar{\nu}_{i,i}} \tag{3.10}$$

where $\tilde{\nu}_{j+1,j+1}$ is the stationary upcrossing rate calculated for the level $R_{j+1} - \Delta\sigma_j$. Note that for the single crossing case the term $\tilde{\nu}_{j,j}$ has to be replaced by the term $\bar{\nu}_{i,i}$.

Method 4:

In this method we use the non–stationary upcrossing rates after the primary crossing. The determination of the mean number of crossings now requires integration over time. It is further necessary to treat the initial velocity as a random variable as explained above. Furthermore, in recognition of the oscillating nature of the crossing rate in this phase the terms in the numerator of the general formula (3.6) are taken as the maximum values during the transient phase because it is most likely that crossings occur at this point if crossings occur at all. The maximum point usually occurs one to three natural periods after the former crossing. Therefore, the path probability now is:

$$P_4(\mathcal{Q}^{\ell}_{k,i\cap j}) = \frac{\int_0^{\infty} \nu^{max}_{j,j}(\dot{X}(0)^{\ell+1}) - \nu^{max}_{j+1,j+1}(\dot{X}(0)^{\ell+1})\, dF_{\dot{X}(0)^{\ell+1}}}{\bar{\nu}_{i,i}} \tag{3.11}$$

where $\nu^{max}_{j+1,j+1}(\dot{X}(0)^{\ell+1})$ is the maximum value of the non–stationary upcrossing rate in time step $\ell+1$ with initial velocity $\dot{X}(0)^{\ell+1}$. For the single crossing case the first term in the numerator of eq. (3.11) has to be replaced by $\bar{\nu}_{i,i}$.

The types of upcrossing rates used in methods 1 to 4 are illustrated in fig. 5.

In methods 2, 3, and 4 $P(\mathcal{Q}^{\ell+1}_{k,i\cap j})$ can become negative. This is interpreted as crossings of the level i through j with probability one. Therefore, the events of crossings of the levels i+1, ..., j must be excluded from the failure tree of the system.

It should be clear that method 4 provides the most realistic description of system behavior. Method 3 produces upper limits for the path probabilities whereas method 2 and even more so method 1 produce lower limits. Computationally, method 4 is the most complex.

3.3 The Probabilities $P(\mathcal{T}_k(r))$

In accordance with previous assumptions the failure times for the time steps are conditionally independent and exponentially distributed. The distribution function for the first passage time $T^{\ell}_k(\underline{r})$ is:

$$P(T^{\ell}_k(\underline{r}) \le t) = 1 - \exp[-\int_0^t \nu^{\ell}_k(\tau|\underline{r})\, d\tau] \tag{3.12}$$

In methods 1 to 3 the integral over the upcrossing rate simply is:

$$\int_0^t \nu^{\ell}_k(\tau|\underline{r})\, d\tau = \bar{\nu}^{\ell}_k(\underline{r})\, t \tag{3.13}$$

Method 4, in principle, requires numerical integration. However, by observing that the oscillating upcrossing rate $\nu^{\ell}_k(\tau)$ approaches the stationary upcrossing rate $\bar{\nu}^{\ell}_k$, with growing τ the time integral in eq.(3.12) can be represented as follows.

$$\int_0^t \nu^{\ell}_k(\tau)\, d\tau = \bar{\nu}^{\ell}_k\, t + \int_0^t \left\{\nu^{\ell}_k(\tau) - \bar{\nu}^{\ell}_k\right\} d\tau$$

$$= \bar{\nu}^{\ell}_k\, t + C \qquad (t-\infty, C: \text{constant}) \tag{3.14}$$

Generally, C converges fast especially for high damping ratios and is in any case small so that its omission does not involve greater errors.

3.4 The Probability $P(\mathcal{A}(r))$

The event associated with this probability is the conditioning event for both the events $\mathcal{T}_k$ and $\mathcal{Q}_k$. In general, it is difficult to evaluate. In the case of a Daniels system its explicit evaluation can be avoided by making use of the order statistics distributions of the resistances which can even be dependent in a certain sense [8]. Then, the ordering conditions in the last probability term in eq. (3.2) are automatically fulfilled. Their probability is one.

For example, if the normally distributed thresholds R_j are assumed to be equicorrelated with correlation coefficient ρ, the resistances can be represented as:

$$R_j = E[R] + D[R]\,(U_o\,\rho + U_j\sqrt{1-\rho^2}) \tag{3.15}$$

where U_o and U_j's are independent standard normal variables. In order to establish the sequence of componental failures, we need the order statistics $(\dot{R}_1 \le \dot{R}_2 \le \ldots \le \dot{R}_n)$ of $(R_1, R_2, \ldots, R_n)$.

In [8] it is shown that the order statistics distribution of a vector of independent standard normal variables has the Rosenblatt–transformation:

$$\begin{aligned}\dot{U}_1 &= \Phi^{-1}\,[1-\Phi(-U_1)^{1/n}]\\ \dot{U}_j &= \Phi^{-1}\,[1-\prod_{i=1}^{j}\Phi(-U_i)^{1/(n-i+1)}]\end{aligned} \tag{3.16}$$

Therefore,

$$\dot{R}_j = E[R] + D[R]\,(U_o\,\rho + \dot{U}_j\sqrt{1-\rho^2}) \tag{3.17}$$

which has to be used in eqs.(3.8) to (3.11) and (3.14).

3.5 Numerical Analysis by FORM/SORM

An analytical analysis of eq. (3.2) even under extremely simplifying assumptions regarding the probability models appears impossible. Also, an analysis by numerical integration is prohibitive even for small Daniels systems. Further, Monte Carlo methods must be ruled out because the probability of "hits" in the interesting domain is extremely small except when special importance sampling schemes are applied. However, classical first– or second–order reliability methods (FORM or SORM) [9] are well suited for an approximate analysis as will be shown. Those just require an appropriate reformulation of the various events. At first, we write the identity:

$$P(\mathcal{Q}_k) = \prod_{\ell=1}^{L_k} P(\mathcal{Q}_k^{\ell}) = P(\,U_{p,k} - \Phi^{-1}(P(\mathcal{Q}_k)) = 0\,) \tag{3.18}$$

where $U_{p,k}$ is an auxiliary standard normal variable. Also, the following probabilities are equal:

$$\begin{aligned}P(T_k^{\ell}(\underline{r}) \le t) &= 1-\exp[-\textstyle\int_0^t \nu_k^{\ell}(\tau|\underline{r})\,d\tau]\\ &= P(U_{T,k}^{\ell} \le u_{T,k}^{\ell}) = \Phi(u_{T,k}^{\ell})\end{aligned} \tag{3.19}$$

where $U_{T,k}^{\ell}$ is another auxiliary standard normal variable. Each first passage time $T_k^{\ell}(\underline{r})$ is the solution of the following equation with respect to t.

$$\ln\Phi(-U_{T,k}^{\ell}) + \left[\textstyle\int_0^t \nu_k^{\ell}(\tau|\underline{r})\,d\tau\right] = 0 \tag{3.20}$$

From eq.(3.14) and eq.(3.20), the first passage time $T_k^{\ell}(\underline{r})$ is represented as:

$$T_k^{\ell}(\underline{r}) = -\frac{1}{\nu_k^{\ell}(\underline{r})}\left\{\ln\Phi(-U_{T,k}^{\ell}) + C\right\} \tag{3.21}$$

so that the probability $P(\mathcal{I}_k)$ can be rewritten as follows.

$$P\left(\sum_{\ell=1}^{L_k}\left[-\frac{1}{\nu_k^{\ell}(\underline{r})}\left\{\ln\Phi(-U_{T,k}^{\ell}) + C\right\}\right] - t \le 0\right) \tag{3.22}$$

The last probability $P(\mathcal{A}_k)$ in the integral of eq.(3.2) can, as mentioned, be evaluated by using the order statistics of the resistances. Since the correct ordering of the resistance variables is already taken account for in the joint distribution function of the vector $\underline{R} = \dot{\underline{R}}$ as shown in eq. (3.17) the probability $P(\mathcal{A}_k)$ needs not to be considered explicitly. Denote the probability distribution transformation for the resistance variables by $\underline{R} = \underline{T}(\underline{U})$. It is then clear that eq. (3.2) can be rewritten as:

$$\begin{aligned}P_f(t) = P\Big[\Big\{U_{p,k} - \Phi^{-1}[P(\mathcal{Q}_k(\underline{T}(\underline{U})))] = 0\Big\} \cap\\ \cap\left\{\sum_{\ell=1}^{L_k}\left[-\frac{1}{\nu_k^{\ell}(\underline{T}(\underline{U}))}\left\{\ln\Phi(-U_{T,k}^{\ell}) + C\right\}\right] - t \le 0\right\}\Big]\end{aligned} \tag{3.23}$$

One recognizes that this is the probability of the intersection of one equality and one inequality. The modifications to classical first– or second–order reliability methods as

described in [9] when equality constraints are present are given in [10]. In section 5 certain computational aspects will be discussed.

4. Example

As an illustrating example consider a Daniels system with 3 components. The following parameters for the normally distributed variables S, c, M and R_i are assumed. Also, all random variables are independent. The load is stationary Gaussian white noise with spectral density $W_o = 0.5$. The reference time is $t = 100$.

Variable	mean	c.o.v
Stiffness S	50	5%
Damping c	0.4 (ζ=0.05)	10%
	0.64(ζ=0.08)	10%
Mass M	1.0	1%
Strength R_i	0.5	20%

Table 1: Parameters for random variables

Figures 6, 7, 8 and 9 show the failure trees for different combinations of conditions as shown in table 2:

Example No.	Loading direction	Damping model	Damping ratio
1	Case B	1	5%
2	Case A	1	5%
3	Case B	2	5%
4	Case B	1	8%

Table 2: Characteristics of examples

In the figures each knot point has four numbers quantifying the branch probabilities $V_k^\ell = \prod_{i=1}^{\ell} P(\mathcal{Q}_k^i)$. The first value corresponds to the case of "delayed stress redistribution" (method 1), the second value to "immediate stress redistribution" (method 2), the third value to "immediate stress redistribution with dynamic impulse" (method 3) and the forth value to "non–stationary stress redistribution" (method 4).

The safety index $\beta(\Sigma T_k^\ell)$ quantifies the probability eq. (3.22), the safety index β_k the total branch probability. The system safety index calculated according to eq. (3.1) is shown under the column for the β_k's. It is possible to draw some preliminary conclusions with respect to various factors.

First of all. the conditions under which method 1 is an approximation to reality can, in some sense. be interpreted as a quasi–ductile behavior of the components at failure resulting in an extremely damped behavior of the system as the stress redistribution is concerned. It is seen that this assumption leads to the largest safety indices. One can hardly think of a more favorable system behavior.

The loading direction. of course. has a significant influence although its effect can be shown to vary to a certain extent with the relative magnitudes of the variables W_o, R_i and Mg/S. For the parameter combinations used in the example, the failure probability for horizontal loading is about an order of magnitude smaller than for vertical loading.

With the exception of method 1 which is less relevant when judging the influence of the two damping models. the damping model is not very important. This is readily explained by the fact that system failure in the other cases is caused by multiple crossings in the transient phase in which the system damping remains essentially unchanged. Only if the system degrades in several time steps the effect of different damping models can be important.

The comparison of the various methods for (dynamic) stress redistribution appears most interesting. The system failure probabilities increase from method 1 to 2 to 4 and to 3 in all cases. This is not surprising because the same ordering is valid for the upcrossing rates. Certainly. method 4 yields the most realistic results. Then, it is seen that method 3 produces conservative results close to those of method 4 for small damping ratios. For larger damping ratios method 2 yields good approximations although those are always slightly on the unconservative side.

It is remarkable that only for method 1 (delayed stress redistribution) the system degrades most likely in several time steps. There is also a tendency that the probabilities of the failure paths $k = 1$ or sometimes $k = 3$ are the largest for that method, or better, the assumptions underlying it. Consideration of dynamic "overshooting" during stress redistribution leads to progressive failure, i.e. successive crossings of the levels 1, 2 and 3, in all examples and path $k = 4$ is also the most likely one to system collapse.

5. Remarks on Calculation Methods

The complete mathematical formulation is given in eq. (3.23) where it is also proposed to use first– or second– order reliability methods for the numerical analysis. In order to verify the numerical accuracy of those approximate methods several attempts have been undertaken . A first approach consisted in applying crude FORM, that is individual linearization of the two failure boundaries of the events in eq.(3.23). It was found that crude FORM produces only rough approximations. A second approach consisted in searching for the joint beta point of the two events (see [9] for a detailed account of definition and notions). Given that point the probability of eq.(3.23) was then calculated as the product of the probability $\Phi_k(.)$ at that point times the second–order probability of the second event. The numbers given in figures 6 to 9 correspond to this calculation scheme. A third calculation method corresponds to the asymptotic theory outlined in [10] for the handling of complex events including equalities. The numerical results for this method and the former method as presented in figure 10 showed unsystematic differences. Finally, the results were checked with the importance sampling scheme described in [11] and which are also given in figure 10. It is seen that in most of the examples the results according to [10] are relatively close to the importance sampling updates.

The discrepancies between the four calculation methods require some comments. Of course, the results according to [10] should deserve more credibility. As mentioned, crude FORM appears to be inadequate which must be attributed to the relative large curvatures of the failure surfaces. Also, the abovementioned second approximation appears to be applicable only in cases where a more sophisticated calculation method has validated its suitability. In the examples the ordering of the total path probability did not change for the various methods, however. The third method can generally be recommended because the numerical effort for an importance sampling update increases roughly by three orders of magnitude in order to reduce the coefficient of variation of the probability estimate below 10 %. The third method should yield sufficiently accurate results also in degenerate cases where the second method must fail. It should be noted that the dimension of the vector uncertain variables is 10 or 13 in the examples which clearly discriminates other methods such as numerical integration or elementary Monte Carlo methods.

6. Further Discussion

The formulations above are presented with reference to a SDOF–Daniels system which may be considered as the mechanically and probabilistically simplest redundant system. It is mechanically simple because the loads are equally shared by the remaining components. In this sense it is also the most favorable system among all other mechanical systems from

a reliability point of view. It is probabilistically simple because the complicated ordering conditions for the resistances in eq. (3.2) can be replaced an order statistics consideration if certain probabilistic dependence structures for the resistances hold.

However, the formulations also carry over to more complex systems with some straightforward modifications. Arbitrary mechanical systems can be chosen but, for reasons which cannot be discussed here, the system properties such as masses, dampings and stiffnesses must be assumed functionally dependent. The mechanics become especially simple if a modal decomposition is possible and are further facilitated if the excitation can be approximated by Gaussian white noise. The response vector is obtained as the superposition of the responses in the various vibration modes. Probabilistically, the events $\mathcal{A}_k$ in eq. (3.2) need to be assessed as in eq. (3.5). The probability computations analogous to eq. (3.23) involve a number of additional intersecting events which principally is not a serious difficulty when applying the methods in [9,10]. Some numerical experience shows that only a few and mostly only one of those restrictions become active in the most likely failure region of the probability integral; a finding which also appeared in [7] for the static case. From a numerical point of view it is now necessary to use one of the branch and bound methods in order to identify the most critical paths to failure since there usually is a large number of them (see again [7] for further details). Simultaneously, the probabilities $P(\mathcal{Q}_k)$ must be determined as in [7]. Due to the usually less favorable stress redistribution regime in general systems those systems provide comparatively less extra reliability by redundancy especially for small system damping.

From a practical point of view some general conclusions can already be drawn although the numerical experience still is limited and some open questions remain. Quite generally, the extra reliability provided by redundancy of structural systems with brittle components is rather limited unless the degree of redundancy is very large. If the dynamic effects during stress redistribution must be considered most structures behave like weakest–link systems because the failure of the weakest component triggers progressive failure of the others. As this statement rests on the assumption of componental brittleness one has to clarify this notion. Many structural materials are, in fact, not really brittle. The other extreme of ideal plasticity is also not met frequently. Mostly, one observes a load deformation characteristic as shown in figure 11. But even if the materials behave rather ductile as in figure 11, structural components such as bars or beams in frame structures frequently behave almost brittle on a structural scale because the overall deformation is the sum of quasi–elastic deformations integrated over a large portion of the component and the quasi–plastic deformation concentrating only in small regions of the component. Thus, the notion of brittleness as relevant for reliability considerations must be defined on a structural scale. A component behaves brittle as long as the quasi–plastic deformations remain small as compared with the total quasi–elastic deformations in the

component. With this in mind a large category of structures belongs to the class of structures discussed in this paper. The exact borderline between brittle and non–brittle componental behavior still needs to be defined by appropriate criteria.

On the other hand the assumptions underlying method 1 probably have been discriminated unduly. A load deformation characteristic as in fig. 11, in fact, implies substantial energy absorption during the rupture process which can be modeled by a large damping relevant for the dynamics of the system just in the transient phase of redistribution. Although the details of the dynamics of redistribution which would be capable to take account of such phenomena are not yet fully understood, the authors are inclined to presume that the dynamic effects of redistribution can be neglected in many cases. This leads to the tentative recommendation to apply method 2 in applications.

7. Summary and Conclusions

The reliability of redundant, dynamic structural systems with brittle components and loaded by Gaussian white noise is investigated. The mechanics of stress redistribution after failure of one or more components is formulated and the upcrossing rates are calculated mechanically exact and under several simplifying assumptions which can be considered as approximating limiting cases. The necessary, multidimensional probability integrations are performed using well–known first– or second–order reliability methods including importance sampling updating.

The proposed methods are illustrated at a simple three–component Daniels system. The effect of the magnitude of damping, the manner in which damping changes during system degradation and the direction of loading are studied. Particular emphasis is given to the methodology of the non–trivial probability calculations.

The following conclusions under the assumption of perfect brittleness of the components can be drawn:

- The dynamic effects during stress redistribution after componental failure must only be taken into account for small system damping.
- In this case the consideration of the maximum possible dynamic impulse yields conservative and fairly accurate reliability estimates.
- For sufficiently large system damping the dynamics of stress redistribution can be neglected.

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Appendix: Auxiliary functions in eq. (2.3)

The index m is dropped in this section for simplicity. Thus, ζ_m, $\omega_{o,m}$ and $\omega_{d,m} = \omega_{o,m}\sqrt{1-\zeta_m^2}$ are denoted by ζ, ω_o and ω_d, respectively. Similarly a_m, b_m,.... are referred to as a, b,... .

Case 1 $\zeta < 1$

$$a(\tau) = e^{-\zeta\omega_0\tau}\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\tau + \cos\omega_d\tau\right]$$

$$b(\tau) = \frac{1}{\omega_d}e^{-\zeta\omega_0\tau}\sin\omega_d\tau$$

$$\dot{a}(\tau) = -\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0\tau}\sin\omega_d\tau$$

$$\dot{b}(\tau) = e^{-\zeta\omega_0\tau}\left[\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d\tau + \cos\omega_d\tau\right]$$

$$\int_0^\tau b(u)du = \frac{1}{\omega_0^2}[1-a(\tau)]$$

$$\int_0^\tau \dot{b}(u)du = b(\tau)$$

$$\int_0^\tau b(u)^2du = \frac{1}{4\zeta\omega_0^3}\{1-e^{-2\zeta\omega_0\tau}[1+\frac{2\zeta^2}{1-\zeta^2}(\sin\omega_d\tau)^2 + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin 2\omega_d\tau]\}$$

$$\int_0^\tau b(u)\dot{b}(u)du = \frac{1}{2\omega_d^2}e^{-2\zeta\omega_0\tau}(\sin\omega_d\tau)^2$$

$$\int_0^\tau \dot{b}(u)^2du = \frac{1}{4\zeta\omega_0}\{1-e^{-2\zeta\omega_1\tau}[1+\frac{2\zeta^2}{1-\zeta^2}(\sin\omega_d\tau)^2 - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin 2\omega_d\tau]\}$$

Case 2 $\zeta = 1$

$$a(\tau) = e^{-\omega_0\tau}$$

$$b(\tau) = \tau e^{-\omega_0\tau}$$

$$\dot{a}(\tau) = -\omega_0 e^{-\omega_0\tau}$$

$$\dot{b}(\tau) = (1-\omega_0\tau)e^{-\omega_0\tau}$$

$$\int_0^\tau b(u)du = \frac{1}{\omega_0^2} - (\frac{\tau}{\omega_0} + \frac{1}{\omega_0^2})e^{-\omega_0\tau}$$

$$\int_0^\tau \dot{b}(u)du = \tau e^{-\omega_0\tau}$$

$$\int_0^\tau b(u^2)du = \frac{1}{4\omega_0^3} - e^{-2\omega_0\tau}(\frac{1}{4\omega_0^3} + \frac{\tau}{2\omega_0^2} + \frac{\tau^2}{2\omega_0})$$

$$\int_0^\tau b(u)\dot{b}(u)du = \frac{1}{2}\tau^2 e^{-2\omega_0\tau}$$

$$\int_0^\tau \dot{b}(u)^2du = \frac{1}{4\omega_0} - e^{-2\omega_0\tau}(\frac{1}{4\omega_0} - \frac{\tau}{2} + \frac{\tau^2}{2}\omega_0)$$

Case 3 $\zeta > 1$

$$a(\tau) = \frac{1}{2\omega_0\sqrt{\zeta^2-1}}(\omega_{20}e^{\omega_{10}\tau} - \omega_{10}e^{\omega_{20}\tau})$$

$$b(\tau) = \frac{1}{2\omega_0\sqrt{\zeta^2-1}}(-e^{\omega_{10}\tau} + e^{\omega_{20}\tau})$$

$$\dot{a}(\tau) = \frac{\omega_{10}\ \ \omega_{20}}{2\omega_0\sqrt{\zeta^2-1}}(e^{\omega_{10}\tau} - e^{\omega_{20}\tau})$$

$$\dot{b}(\tau) = \frac{1}{2\omega_0\sqrt{\zeta^2-1}}(-\omega_{10}e^{\omega_{10}\tau} + \omega_{20}e^{\omega_{20}\tau})$$

$$\int_0^\tau b(\omega)du = \frac{1}{2\omega_0\sqrt{\zeta^2-1}}[\frac{1}{\omega_{10}}(1-e^{\omega_{10}\tau}) - \frac{1}{\omega_{20}}(1-e^{\omega_{20}\tau})]$$

$$\int_0^\tau \dot{b}(\omega)du = \frac{1}{2\omega_0\sqrt{\zeta^2-1}}[e^{\omega_{20}\tau} - e^{\omega_{10}\tau}]$$

$$\int_0^\tau b(u)^2du = \frac{1}{4\omega_0^2(\zeta^2-1)}[\frac{e^{2\omega_{10}\tau}-1}{2\omega_{10}} + \frac{e^{2\omega_{20}\tau}-1}{2\omega_{20}} - 2\frac{e^{(\omega_{10}+\ \omega_{20})\tau}-1}{\omega_{10}\ +\ \omega_{20}}]$$

$$\int_0^\tau b(u)\dot{b}(u)du = \frac{1}{8\omega_0^2(\zeta^2-1)}(e^{\omega_{10}\tau} - e^{\omega_{20}\tau})^2$$

$$\int_0^\tau \dot{b}(u)^2du = \frac{1}{4\omega_0^2(\zeta^2-1)}[\frac{1}{2}\omega_{10}(e^{2\omega_{10}\tau}-1) +$$

$$+\frac{1}{2}\omega_{20}(e^{2\omega_{20}\tau}-1) - \frac{2\omega_{10}\ \ \omega_{20}}{\omega_{10}\ +\ \omega_{20}}(e^{(\omega_{10}+\ \omega_{20})\tau}-1)]$$

in which

$$\omega_{10} = -\omega_0(\zeta + \sqrt{\zeta^2-1})$$

$$\omega_{20} = -\omega_0(\zeta - \sqrt{\zeta^2-1})$$

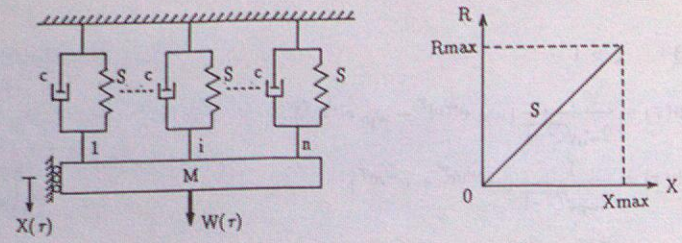


Figure 1: Daniels system

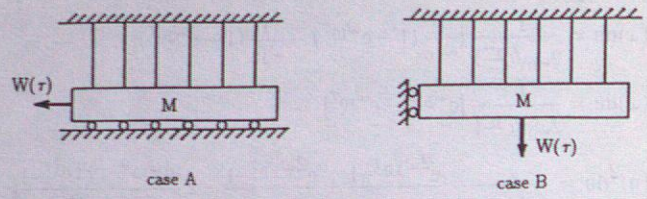
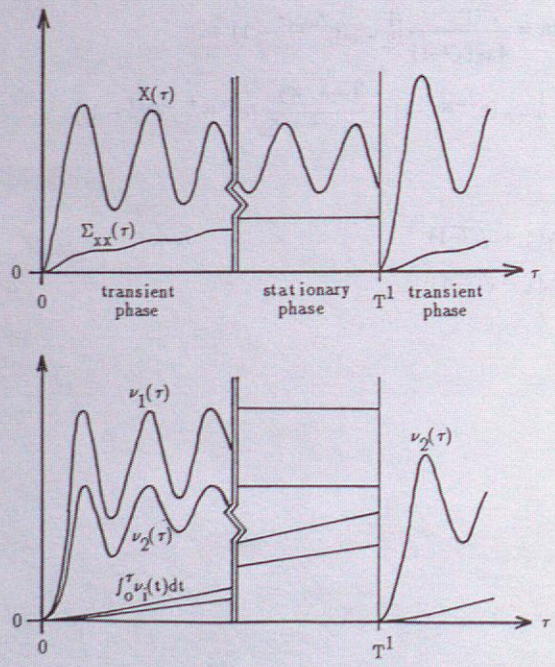


Figure 2: Loading directions



mean and covariance of responce,
Figure 3: mean upcrossing rate and its integral

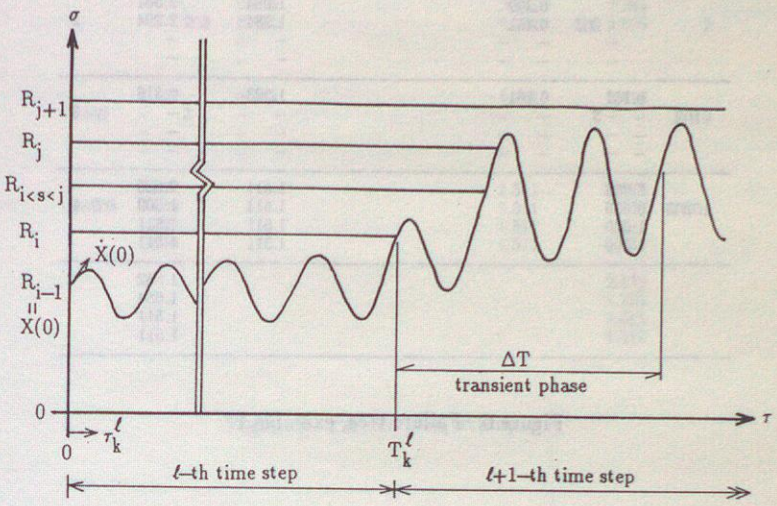


Figure 4: Sample path with level crossings

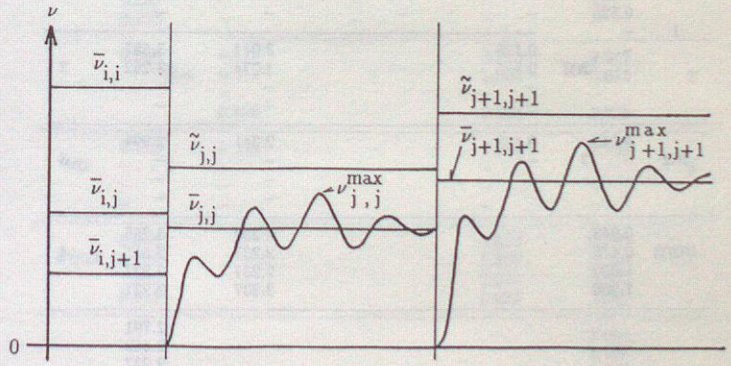


Figure 5: Upcrossing rates used in methods 1 to 4

	V_k^1	V_k^2	V_k^3	$\beta(\Sigma T_k^\ell)$	β_k	k
		0.574	0.574	1.998	2.222	
	2	– 3	–	–	–	1
	0.783	–	–	–	–	
	0.461	–	–	–	–	
1	–					
	–	0.209		1.984	2.581	
	2∩3	0.461		1.984	2.294	2
		–		–	–	
		–		–	–	
	0.161	0.161		1.523	2.316	
1∩2	– 3	–		–	–	3
	–	–		–	–	
	–	–		–	–	
	0.056			1.511	2.680	
1∩2∩3	0.539			1.511	1.809	4
	1.000			1.511	1.511	
	1.000			1.511	1.511	
					1.852	
					1.684	
					1.511	
					1.511	

Figure 6: Failure tree, example 1

	V_k^1	V_k^2	V_k^3	$\beta(\Sigma T_k^\ell)$	β_k	k
		0.635	0.635	3.154	3.284	
	2	– 3	–	–	–	1
	0.814	–	–	–	–	
	0.525	–	–	–	–	
1	–					
	–	0.178		3.074	3.557	
	2∩3	0.525		3.074	3.262	2
		–		–	–	
		–		–	–	
	0.143	0.143		2.341	2.994	
1∩2	– 3	–		–	–	3
	–	–		–	–	
	–	–		–	–	
	0.043			2.237	3.266	
1∩2∩3	0.475			2.237	2.511	4
	1.000			2.237	2.237	
	1.000			2.237	2.237	
					2.791	
					2.480	
					2.237	
					2.237	

Figure 7: Failure tree, example 2

	V_k^1	V_k^2	V_k^3	$\beta(\Sigma T_k^\ell)$	β_k	k
		0.602	0.602	2.508	2.683	
	2	– 3	–	–	–	1
	0.781	–	–	–	–	
	0.517	–	–	–	–	
1	–					
	–	0.179		2.361	2.941	
	2∩3	0.517		2.361	2.596	2
		–		–	–	
		–		–	–	
	0.162	0.162		1.791	2.516	
1∩2	– 3	–		–	–	3
	–	–		–	–	
	–	–		–	–	
	0.057			1.511	2.678	
1∩2∩3	0.483			1.511	1.858	4
	1.000			1.511	1.511	
	1.000			1.511	1.511	
					2.172	
					1.796	
					1.511	
					1.511	

Figure 8: Failure tree, example 3

	V_k^1	V_k^2	V_k^3	$\beta(\Sigma T_k^\ell)$	β_k	k
		0.635	0.635	2.826	2.975	
	2	– 3	–	–	–	1
	0.802	–	–	–	–	
	0.512	–	–	–	–	
1	–					
	0.486	0.178		2.803	3.317	
	2∩3	0.512		2.803	3.013	2
		–		–	–	
		0.486		2.803	3.029	
	0.150	0.150		2.104	2.788	
1∩2	– 3	–		–	–	3
	–	–		–	–	
	–	–		–	–	
	0.048			2.077	3.120	
1∩2∩3	0.488			2.077	2.357	4
	1.000			2.077	2.077	
	0.514			2.077	2.337	
					2.544	
					2.307	
					2.077	
					2.292	

Figure 9: Failure tree, example 4

Ex. No. \ method	1	2	3	4
1	2.013	2.222	2.428	2.375
2	3.006	3.284	3.487	3.409
3	2.316	2.683	2.853	2.749
4	2.689	2.975	3.242	3.229

method 1: Crude FORM
method 2: Simplified SORM
method 3: SORM
method 4: FORM updated by importance sampling

Figure 10: β by different calculation methods (k = 1)

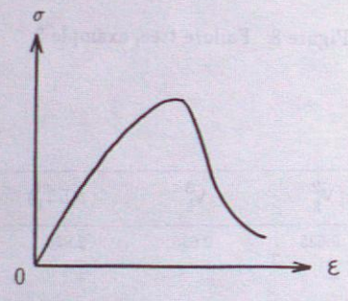


Figure 11: σ–ε relationship of non–brittle element

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