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# Optimization and the life quality index as an alternative for the development of new design regulations

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**Structures should be optimal with respect to the economic investment, the benefits derived from their existence, the expected consequences in case of failure and the degree of protection to human life and limb. This paper presents the implications of a new optimization strategy. A renewal model for the sequence of structural failures is used to define the objective function for optimizing the design of structures. The Life Quality Index, which is a compound social indicator, is included in the optimization for the efficiency of the measures to save human lives. This criterion balances quality–adjusted life years saved against the associated cost for society. These concepts are then applied to the seismic design of structures. The results show that safety standards used in current practice in earthquake engineering should be reviewed in the light of optimization of resources and saving human lives. They also show the importance of different socio–economic characteristics in the definition of risk acceptability.**

In any country, resources should be allocated to all aspects of life that ensure the well–being of its citizens in proportion to their potential to ''buy'' life quality years. Therefore, a rational and efficient investment in structural safety requires the appropriate assignment of resources in accordance with a cost effective strategy. In structural terms, building structures ought to be designed and constructed for safety standards that reflect the interests and needs of the society. Current format of codes of practice define minimum requirements to ensure safety. The safety level that is inherent in any code or standard is supposed to represent a value judgment of the society but is incomplete as long as it does not recognize the characteristics of the investment from the perspective of the public interest. Infrastructure development should be optimal with respect to the investment in any facility, the benefits derived from its existence, the expected consequences in case of failure and the degree of protection to human life and limb[1].

At present, structural design is based on rational and widely accepted mechanical models despite there are still many sources of uncertainty both inherently random and epistemic in nature. Structural loads and strengths are unpredictable, databases are limited and modeling of performance limit states cannot be carried out accurately[2]. Although these aspects are common to all countries, the consequences of safety–decisions for a society in terms of development and quality of life have been kept aside widely.

This paper discusses some key issues relevant for the future development of new design specifications and presents a strategy for deciding on the optimum structural design criteria for taking up a project considering both direct economic and human life losses, within a given social context. The proposal is illustrated with two cases: (1) a formal strategy for the definition of the ALARP region and (2) the definition of the design acceleration values for a seismic area.

## ACCEPTABILITY OF RISK

The definition of acceptable level of risk has always been a key issue in structural design. In reliability terms, this is related to the decision on whether the probability of a limit state violation is acceptable or not. However, the decision about acceptance has to include also an assessment of the consequences of failure and the context within which an unfavorable event might happen. Among the most common criteria for making decisions about risk acceptance are: the comparison between the calculated probability of structural

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failure with other risks in society, the definition of the ALARP region, the calibration at past and present practice and the cost–benefit analysis.

Defining acceptable risk by comparing death rates of different activities within a society may be misleading. Acceptability of risks varies with age, sex, socio-economic conditions, level of education, cultural background, available information, media influence, physiological aspects, and so forth. Special attention should be given to the differentiation between individual and collective risks. An individual acts with respect to his/her needs, preferences and lifestyle. Thus, risk acceptance depends on the degree to which the risk is incurred voluntarily. Collective (public) risk is of concern for the government, or the operator of a technical facility, who acts on behalf of society as a whole and is not concerned with the individual's safety. The ALARP approach defines a region of acceptable values of probability of failure in a plot of the occurrence probability of adverse events versus their consequences. Although this approach might be appealing, there are significant difficulties in its interpretation, openness of decision processes, morality of actions and comparability between facilities[3]. Calibration of acceptable levels of risk at past and present practice has also been used for defining target reliabilities. It is tacitly assumed that this practice is optimal although this is not at all obvious. Rackwitz[4] argues that despite its development based on trial and error, it cannot give totally wrong numbers because of their long history. But analysis shows that there is great variation of reliability levels. Finally, a reliability oriented cost–benefit analysis considers that technical facilities should be economically optimal. But then, the question of the economical value of human life or more precisely, the question how to reduce the risk to human life, cannot be avoided. This approach has been recently updated by including the Life Quality Index (LQI) as proposed by Nathwani et al.[5], leading to the conclusion that risk acceptability from the public perspective is essentially a matter of efficient investment into life saving measures. This approach will be discussed in detail later in this paper.

Acceptable risks for structural engineering, measured in terms of annual probability, have shown to be below the level for common chronic disease ($10^{-3}$/yr) but somewhat above the "de minimis" risk threshold, around $10^{-7}$/yr[6], where individuals and society are indifferent to the risk[2]. Since this range is extremely wide, a strategy for selecting appropriate target reliabilities and risk acceptance criteria becomes a key element for making decisions on structural safety. The optimization strategy presented in the following sections is suggested as a dependable and effective approach for defining the criteria for optimum structural design.

## CODE DEVELOPMENT

In the second half of the nineteen century, the theory of elasticity became essential part of the design engineering practice. The design philosophy which evolved out of the application of elasticity theory is called "allowable stress" design. In reliability terms, the "factor of safety" is the crucial issue and its value has changed throughout the years. For instance, Galambos[7] presents data that shows that the factor of safety has decreased from 2 in 1890 to 1.67 in 1963 for mild steel structures. Early empirical design procedures based on a simple global safety coefficient were progressively replaced by "limit state design" approaches. This design method appeared initially in the Soviet Union in 1940s and in USA and Europe after the 60's. This gave a differentiated appraisal of the reliability in terms of limit state functions. Currently, the limit state design philosophy, also called partial safety factor method, has been implemented in most of the national design specifications.

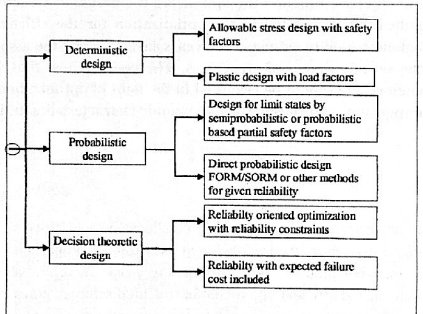


FIG. 1. DESIGN STRATEGIES BASED ON DIFFERENT RELIABILITY MODELS.

A classification of existing design alternatives in terms of safety and reliability is shown in Figure 1 and has been presented elsewhere as a hierarchy of structural reliability measures. This hierarchy consists of four levels: (1) code leve methods; (2) "second moment methods"; (3) "exact methods"; and "decision methods"[3].

Level 1 is a non-probabilistic generalized version of the traditional safety and load factor formats. Level 2 procedures deal with nominal probabilities based on normal distributions and simple forms of the limit state function[3]. In level 3 (i.e., the direct probabilistic design) the FORM/SORM is commonly used. In the time–invariant first order reliability method, the most difficult task is the identification of β–point(s) or "design point(s)", which is associated to a safety index. However, for 90% of all applications, the first order reliability analysis fulfils all practical needs and its accuracy is usually more than sufficient. The time–invariant second order reliability method (SORM) describes better the form (i.e. curvature) of the limit state function. It enhances the result by fitting a parabola, in the β–point, to the true failure surface. It is even an asymptotically exact method, i.e., when the reliability index β becomes large (or failure probabilities become small). Time–variant reliability methods have also been developed covering a large spectrum of applications and mostly based on FORM/SORM principles. Rackwitz[8,9] states that "FORM/SORM methodology is well established and, apart from some technical aspects, little can be added and very little can be removed". Finally, in level 4, the decision theoretic design addresses the issue that structures should be economically optimal. Thus, the design has to be performed to minimize the expected cost or maximize the expected benefit.

Essentially the same development and the same theoretical basis underlies the modern limit states design codes of USA, Canada, Europe and many other countries. The reliability level is measured through a set of partial safety factors that have been reduced to a minimum. Explicit probabilities of satisfying structural behavior have been inferred from deterministically looking codes of practice. The calibration of partial safety factors found in modern codes of practice have shown that there is a great variation between different structural members, materials and design procedures in terms of probabilities[10]. Although the code format and the implied comparative levels of reliability may vary somewhat from code to code, the reliability theory has proved a common ground through which codes of different jurisdictions can be compared[7].

## FUTURE OF DESIGN REGULATIONS

The ultimate objective of any structure is to be fit for its purpose and economically feasible. Fitness for purpose includes safety, which means that it will not fail during its intended life in any manner that would kill or harm its users or cause severe economic loss[7]. Safety in a broader sense includes not only the design, but also quality assurance procedures, avoidance of human error, etc. The function of a code of practice is to regulate design so that the resulting artefact is safe, serviceable and economical. A design code is a common standard against which all structures of the same type are to be measured; thus, it should be a minimum standard.

There exists the perception within the engineering community that current design specifications lead to structures that are not or not always optimal. This is mainly caused by (1) the need for design requirements which are general enough to cover a significant number of cases; and (2) the use of deterministic (or quasi-deterministic) approach (i.e. semi-probabilistic partial safety factor method) which manages in a cursory manner the uncertainty of the variables and models.

The debate within the engineering community regarding the need for a new code format is growing fast. The new directions for design specifications have to consider the following aspects: (1) structures should be optimal in terms of capital investment and the saving of life years; (2) the probabilistic nature of the design variables need to be included within the design models and processes; and (3) the socio–economic characteristics of the area where the structure is built should be taken into account.

Designing, erecting, maintaining and replacing structural facilities is a decision problem, where the maximum expected benefit is sought and the reliability requirements are fulfilled simultaneously at the decision point. This implies that there must be a balance between the economic benefit and the potential consequences, in terms of casualties (i.e. life years lost)[10]. Most existing approaches to risk of human life attempt to quantify life through a monetary value despite all ethical and moral considerations. However, modern approaches address the pro–blem by considering the cost of saving life years, or more precisely, the cost to reduce the risk to life[10]. Later in this paper, a social indicator, which is applicable for the investment, in many aspects of life (i.e. health, road safety, structural safety), in the public interest will be discussed.

As a contribution to the development of a new design paradigm, the following sections will be discussed: (1) the one level structural optimisation of cost and reliability, as proposed by Rackwitz[10] (2) a rational model to account for the cost of saving lives based on the recently proposed Life Quality Index; and (3) two illustrative applications of these concepts to the definition of the ALARP region and the determination of the design acceleration for a seismic area.

## OPTIMIZATION OF STRUCTURAL FACILITIES

A reliability–based optimization process consists of defining the optimum value of the vector parameter $p$ for which the building is financially feasible. The vector parameter $p$ stands for any measure capable to control the risk of failure. For instance, the dimension of the structural elements, the reinforcement, the quality assurance program during construction, the maintenance program during service and so forth. The general objective function for maximization can be expressed as:

$$Z(p) = B - C(p) - D(p) \qquad (1)$$

where $B$ is the benefit derived from the structure assumed independent of the vector parameter $p$; $C(p)$ is the cost of design and construction, and $D(p)$ the expected failure cost. Since the structure will eventually fail after sometime, the optimization has to be performed at the decision point (i.e. $t = 0$). Therefore, all cost need to be discounted. In this paper, a continuous discounting function will be considered (i.e., $\delta(t) = \exp[-\gamma t]$). In addition, several replacement strategies were defined: (1) the facility is given up after service or failure or (2) the facility is systematically replaced after failure. Further, it is possible to distinguish between structures that fail upon completion or never and structures that fail at a random point in time.

TABLE 1
OBJECTIVE FUNCTION FOR VARIOUS REPLACEMENT STRATEGIES

| Replacement strategy | Objective function |
|---|---|
| Failure upon completion due to time invariant loads | $Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H)\frac{P_f(p)}{1 - P_f(p)}$ |
| – Systematic reconstruction Random failure in time – Given up after completion | $Z(p) = \frac{b}{\gamma + \lambda(p)} - C(p) - H\frac{\lambda(p)}{\gamma + \lambda(p)}$ |
| Systematic reconstruction Random failure in time due to random disturbances | $Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H)\frac{\lambda P_f(p)}{\gamma + \lambda P_f(p)}$ |

Assuming a constant benefit (i.e., $b = \beta C_0$), the objective function for all cases is presented in Table 1, where $H$ is the cost of failure, $\gamma$ is the annual discount rate corrected for in/deflation averaged over sufficiently long periods and $\lambda(p)$ the rate of failure for stationary Poissonian failure processes. The random failure in time with systematic reconstruction can be applied to earthquakes in which the seismic events follow a Poissonian process with occurrence rate $\lambda$ and failures can occur independently with probability $P_f(p)$[11]. It is emphasized that the values of $b$ and $\gamma$ play a very important role since they are the main financial parameters considered in the optimization. A detailed discussion on participation and importance of the interest and benefit rates can be found in reference 10.

The merit of the objective function for random failures in time with systematic reconstruction (e.g., seismic design case), is that it does not depend on a specific lifetime of the structure, which is a random variable very difficult to quantify and usually much longer than values specified by codes of practice. The solution is based on failure intensities and not on time dependant failure probabilities. It is neither necessary to define arbitrary reference times of intended use nor it is necessary to compute first passage time distributions. The same targets, in terms of failure rates, can be set for temporary structures and monumental buildings, given the same marginal cost for reliability and failure consequences[8]. Thus, by using the third equation in Table 1 there is no need to perform the optimization in terms of the expected total cost over a time period (i.e. design life for a new facility, or remaining life for a retrofitted facility), usually called Lifecycle Cost Design criteria; but refer the analysis to yearly rates of occurrence of the event and annual failure probabilities.

## COST OF SAVING HUMAN LIVES (LIFE QUALITY INDEX)

### General aspects

There has been always a great amount of discussion about the cost of human life; but despite the moral and ethical considerations, economic values are still assigned, mainly by insurance companies. For instance, FEMA[12] reports suggest that, for the United States, the cost of injury can be taken as US$1,000/person and US$10,000/person for minor–and serious injury respectively. According to the same source, the life saving cost has been assessed at US$1,700,000. In general terms, fatality and injury losses can be evaluated using one of two approaches, namely, human capital approach and willingness–to–pay approach[13]. Decisions on the development of design and construction regulations are not related to individual but collective risks. Recently, the approaches to the question of risk to human life do not consider the monetary value but the cost to save life. Rackwitz[10] argues that only the public in democratic societies is authorized to decide on the human value in cost–benefit analysis, via, for instance, regulations based on the national constitution. Thus, a society can enhance life quality and safety in the long run if the stakes are efficiently invested into risk control.

### Life Quality Index (LQI)

Lind[14] argues that a measure of tolerable risk should be based on human values and be expressed in human terms. Many widely used social and economic indicators have been developed for international organizations as an attempt to measure and compare the "quality" of life and development of different societies. Basic social indicators are statistical time series, such as life expectancy or Gross Domestic Product (GDP), while compound social indicators are functions of this data for specific purposes, such as reflecting "how well a nation serves the well being of its citizens" (eg., Human Development Index HDI). Lind[14] argues further that any social index that is a differentiable function of life expectancy and GDP per person, implies a tolerable and, simultaneously, affordable risk value.

Although the management of public risks has several ethical, psychological and political dimensions, the core of the overall management of risk is a problem of allocation of economic resources to serve the public good. Risk management is the purchase of extra life expectancy. Thus, "the cost of life–saving is not so many dollars; rather the cost of a dollar is so much life"[14]. The cost of human life can be



taken into account by the Life Quality Index (LQI), which is a compound indicator of the well being of a society. It has been defined[5] as:

$$L = f(g)\,h(t) \qquad (2)$$

where the function $f(g)$ represents the intensity while the factor $h(t)$ represents the duration of the enjoyment of life. The LQI assumes that $f(g)$ and $h(t)$ are independent functions. The parameter $g$ is the individual mean contribution to the GDP and $t$ the time for enjoyment of life whose quality is measured by $g$. The GDP is roughly the sum of all incomes created by labour and capital (stored labour) in a country during a year. It is a measure for the productivity of a society. Life expectancy at birth (mean time to death), $e$, is the area under the survivor curve, which is function of the age. It is proposed as a measure of safety and the GDP per person, as a surrogate measure of the quality of life. Therefore, the GDP is somehow a measure of the possibility to buy additional life years through better medical care, improved safety in road traffic, structural safety, etc. On the whole, the LQI is a cost/efficiency–based criterion expressed in terms of a marginal utility that does not depend on absolute values of life expectancy at birth or the gross national product. It is based on considerations about the potential loss of life and does not deal with risks to any particular group, nor does it deals with risks to any identifiable person.

The LQI is based on the assumption that GDP per capita (i.e., $g$) is proportional to working time $w$, which is the fraction of $e$ devoted to economic activities in order to raise $g$. It has been observed that the value of $w$ varies between 10% for developed countries and 20% for undeveloped countries for the labour force in a country. Thus, if the time spent in economic activities is $we$ $(0 < w < 1)$, then, the time for enjoyment of life is $t = (1 - w)e$. It is, therefore, reasonable to assume that individuals maximize their income with respect to the time they spend in earning it, that is,

$$\frac{dL}{dw} = 0 \qquad (3)$$

After some mathematical manipulations, the LQI can be approximated as[5,10]:

$$L = g^w e^{(1-w)} (1-w)^{(1-w)} \approx g^w e^{(1-w)} \qquad (4)$$

An activity, regulation, project or undertaking changing life expectancy and involving cost is reasonable if[1,4]:

$$\frac{dg}{de} \geq -\frac{g}{e}\,\frac{(1-w)}{w} \qquad (5)$$

or,

$$\frac{dg}{g} + \frac{(1-w)}{w}\,\frac{de}{e} \geq 0 \qquad (6)$$

The equality in equation (6) corresponds to "optimal" investments into life saving. The ">" means that the investments into life saving are inefficient and "<" means that are not admissible. Equation (6) indicates what is both necessary and also affordable to a society for life saving undertakings. The beauty of the LQI is its simplicity despite there are values that cannot be accounted directly such as the cultural heritage, the extinction of rare species or religious beliefs.

The effect of life saving can be estimated through the cost of averting a fatality in terms of gain in life expectancy $\Delta e$. The cost of the corresponding safety measure is expressed as a reduction $\Delta g$ of the GDP. Rackwitz[10] defines the Implied Cost of Averting a Fatality (ICAF) as the cost, $\Delta C = -\Delta g$, per year to extend a person life by $\Delta e$, that is,

$$\Delta C = -\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{1-(1/w)}\right] \qquad (7)$$

Since $\Delta C$ is a yearly cost (undiscounted) and the ICAF is what has to be spent for safety related investments into technical projects at the decision point $t = 0$, then

$$ICAF = |\Delta g|\,e \qquad (8)$$

The ICAF is number which society should be willing to pay for saving life according to its ethical principles and to what society can afford (Table 2). Note that the ICAF cannot be taken as the value of human life. It enters into optimization as a fictitious number at the decision point and is independent of interest or benefit rates.

The application of equation 5 can be performed if the gain (or loss) in the life expectancy is measured by changes in the mortality rate of a group. The detailed calculations of the survival curve can be found in references 5 and 10. If some life saving operation reduces mortality uniformly, the impact of a reduction of risk can be measured in terms of a small change $dM$ of crude mortality $M$ (i.e., number of deaths due to all causes in group per year/group size). Nathwani et. al.[5] relate the quantity $de/e$ empirically to changes in $dM$ by:

$$\frac{de}{e} \approx -C_{FM}\,dM \approx -C_{F\delta}\,\frac{dM}{M} \qquad (9)$$

with, for instance, $C_{FM} \approx 19$ ($C_{F\delta} = 0.13$) for Canada with $M = 0.0073$ in 1990. The value of $C_{F\delta}$ varies from 0 to 1 depending upon the age structure of the population and the life expectancy of the group. It is a measure of the shape (convexity) of the survival curve where $C_{F\delta} \approx 0$ corresponds to the case where all people die at the same age and $C_{F\delta} \approx 1$ to the case of constant mortality at all ages.

TABLE 2
SOCIAL INDICATORS FOR SELECTED COUNTRIES[10]

| Country | g (US$) (GDP/ capita) | $e$ (Years) | w | *ICAF* | $C_{F\delta}$ | $G_F$ |
|---|---|---|---|---|---|---|
| Canada | 27,330 | 79.72 | 0.125 | $2.1\times 10^6$ | 0.13 | $3.4\times 10^6$ |
| USA | 34,260 | 77.70 | 0.125 | $2.6\times 10^6$ | 0.15 | $4.7\times 10^6$ |
| Germany | 25,010 | 77.71 | 0.125 | $1.9\times 10^6$ | 0.13 | $2.2\times 10^6$ |
| France | 24,470 | 78.73 | 0.125 | $1.9\times 10^6$ | 0.14 | $2.6\times 10^6$ |
| Brazil | 7,320 | 67.59 | 0.15 | $4.8\times 10^5$ | 0.21 | $9.3\times 10^5$ |
| Colombia | 5,890 | 70.61 | 0.15 | $4.0\times 10^5$ | 0.21 | $1.3\times 10^6$ |
| México | 8,810 | 72.68 | 0.15 | $6.2\times 10^5$ | 0.18 | $1.8\times 10^6$ |
| Ecuador | 2,900 | 70.61 | 0.15 | $2.0\times 10^6$ | 0.20 | $6.0\times 10^5$ |
| China | 3,940 | 70.63 | 0.18 | $2.6\times 10^5$ | 0.19 | $5.1\times 10^5$ |
| India | 2,390 | 63.53 | 0.15 | $1.5\times 10^5$ | 0.31 | $3.7\times 10^5$ |
| Indonesia | 2,840 | 65.60 | 0.15 | $1.8\times 10^5$ | 0.26 | $6.6\times 10^5$ |
| Japan | 26,460 | 80.74 | 0.15 | $2.1\times 10^6$ | 0.13 | $2.3\times 10^6$ |
| Egypt | 3,690 | 66.58 | 0.15 | $2.4\times 10^5$ | 0.23 | $6.2\times 10^5$ |
| Kenya | 1,010 | 52.39 | 0.15 | $5.2\times 10^4$ | 0.48 | $1.9\times 10^5$ |
| Nigeria | 790 | 47.36 | 0.18 | $3.7\times 10^4$ | 0.52 | $1.3\times 10^5$ |
| South Africa | 9,180 | 55.40 | 0.15 | $5.0\times 10^5$ | 0.43 | $1.4\times 10^6$ |

### Applications of the LQI to Technical Facilities

The important aspect of the LQI is that it can be used as a rational strategy for investing into life saving projects. Implicit in the LQI is the idea that safety is not an isolated measure, but it is context–dependant. In fact, safety is highly related to the resources available to afford it. If it is assumed that the life risk in and from any facility is uniformly distributed over the age and sexes of those affected, the total cost of safety related regulation per member of the group and year is:

$$dg = -dC(p) = -\frac{1}{N}\sum_{i=1}^{n} dC_i(p) \qquad (10)$$

where $n$ is the total number of objects under discussion, each with incremental cost $dC_i$ and $N$ is the group size. If $N_F$ is the total mean number of fatalities avoided per year by the regulation, then,

$$-\frac{dC(p)}{g}+\frac{(1-w)}{w}(-C_{FM}\,dM)=\frac{dC(p)}{g}+\frac{(1-w)}{w}\left(-C_{F\delta}\frac{dM}{M}\right)\geq 0 \qquad (11)$$

If the relationship $de/e \approx C_{FM}\,dM \approx C_{F\delta}(dM/M)$ is included for the change in quality adjusted life expectancy and $dM$ is replaced by the failure rate $dv_f^+$

$$\frac{-dC(p)}{dv_f^+}\geq -kC_{FM}\,g\frac{(1-w)}{w}=-k\frac{C_{F\delta}}{M}g\frac{(1-w)}{w}=-kG_F \qquad (12)$$

where $dM = k\,dv_f^+$, with $k < 1$. The constant $k$ relates the changes in mortality to the changes in the failure rate. It can be interpreted as a person's probability of actually being killed in case of failure. $G_F$ is slightly larger than *ICAF* for developed countries (see table 2) but can be up to five times larger for less developed countries with very low crude mortality. The *ICAF* depends explicitly on life expectancy, the constant $G_F$ only indirectly via $C_{Fd}/M$. While the *ICAF* is relatively insensitive to the exact value of $w$, the constant $G_F$ is inversely proportional to $w$ in first approximation. If w is not related only to the labour force in a country but to the whole population, the value of $w$ as given in Table 2 must be divided roughly by 2 and $G_F$ as given in Table 2 must be multiplied by about 2. If more than one endangered person (i.e. $N_F$ persons) are involved, it is useful to introduce another constant $K_F = k\,G_F\,N_F$. This constant must be determined carefully through risk analysis for any specific family of structural facilities and is probably the most difficult task when applying equation (12).

Equation 12 is an indicator of what is affordable to a society for life saving undertakings. $dC/dv_f^+$ depends on the sensitivity of both $dC$ and $dv_f^+$ with respect to the design parameter $p$. It follows that there is not a unique "optimal" failure probability and, hence, no unique admissible failure probability or failure rate should be specified in codes. Specification of a single value for the admissible failure probability (or failure rate) is but a gross simplification and, in most cases sub-optimal.

## APPLICATIONS TO RELEVANT ASPECTS OF DESIGN SPECIFICATIONS

In the following sections, two examples which illustrate the possible application of optimization and the LQI in practice will be presented. These examples are intended as an approximation to a new design in practice. More examples can be found in reference 10.

### Definition of the acceptable (ALARP) region

The ALARP region defines acceptable values of probability of failure in a plot of the occurrence probability of adverse events versus their consequences. The main criticism to the ALARP region is that the definition of the bounds has ethical and moral implications that are not widely accepted. However, the ALARP region can be defined as a side result of an optimization process. For the case of Poissonian failure events with rate $\lambda(p)$, the criterion of economically reasonable structures with systematic reconstruction is:

$$\frac{b}{\gamma}-C(p)-(C(p)+H_M+H_F)\frac{\lambda(p)}{\gamma}\geq 0 \qquad (13)$$

Then, at the optimum (i.e., $p = p^*$)

$$\lambda(p^*)\leq\frac{\left(\beta\frac{C_0}{C(p^*)}-\gamma\right)}{1+(H_M+H_F)/C(p^*)} \qquad (14)$$



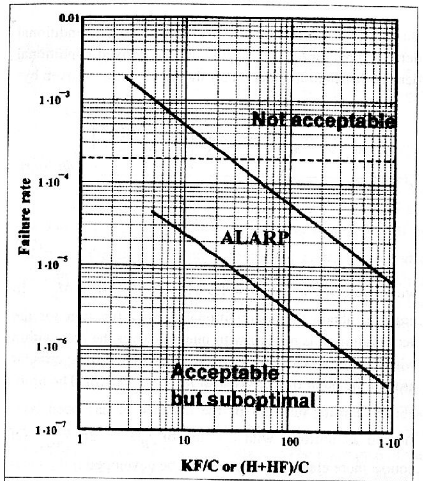


FIG.2 EXAMPLE OF THE ALARP REGION

It can be observed in equation 14 that the failure rate must be smaller than the difference of benefit (multiplied by $C_0/C(p^*) \approx 1$) and interest rate divided by a factor which is between 1 and 10 or more under practical conditions. This defines the lower bound of the ALARP-region.

According to Rackwitz[10] optimal structures are always safer than the LQI-criterion would require. This allows to set another optimization task:

$$Z'(p) = C(p) + K_F\frac{1}{\lambda(p)} \qquad (15)$$

which is equivalent to the requirement that the LQI has to be maximized. Therefore, the upper bound for the ALARP region can be defined as

$$\lambda(p)_{Lim}\leq\left(\frac{Z'(p_{Lim})}{C(p_{Lim})}-1\right)\frac{1}{K_F/C(p_{Lim})} \qquad (16)$$

The values of $p^*$ and $p_{Lim}$ have to be found for each combination of $C(p)$ and $K_F$ or $H_M + H_F$ respectively.

An example of the ALARP region for $\beta = 0.045$ and $\gamma = 0.02$ is shown in Figure 2. The bounds depend on benefit and interest rates and on the specific stochastic problem at hand. The interpretation of the upper bound is as usual but $Z(p_{Lim})$ is negative throughout. The lower bound corresponds to the optimal solution. The way in which the ALARP region is computed shows that the region can change for different contexts. It is emphasised that the ALARP region, defined as before, has quite a different interpretation as compared to its traditional meaning.

### Design acceleration for earthquake design

For earthquake design, the definition of the expected ground motion acceleration is required. If the optimization model and the LQI are considered, the design acceleration will vary as a function of the socioeconomic context. The results shown in this section are the result of a more detailed study[15].

### Optimization model

The model assumes that if the structure fails, it will be reconstructed and taken up to the reliability standards which it had before the event so that consecutive failure events are independent. If the cost of reconstruction is taken into account in the analysis and losses after the earthquake are divided into direct economic losses and fatalities, all depending on a local measure for the size of an earthquake, equation 3 (in Table 1) can be rewritten as function of the vector parameter $p$ and the peak ground acceleration $a$, as:

$$Z(p)=\frac{b}{\gamma}-C(p)-(C_R(a,p)+H_o+H_M(a)+H_F(a))\frac{P_f(p\mid a)\lambda}{\gamma+P_f(p\mid a)\lambda} \qquad (17)$$

where $C_R(a, p)$ is the cost of retrofitting the structure, $H_o$ is the cost of loss of opportunity, $H_M$ corresponds to the direct economic losses and $H_F(a)$ to the losses due to human fatalities. Note that the probability of failure $P_f(p \mid a)$ is related to both the occurrence of the earthquake and the characteristics of the structure measured through the parameter $p$. Thus, the objective function for the optimization, in terms of the vector parameter p only, can be computed taking the expectation with respect to the uncertain accelerations (i.e., a):

$$Z(p)=\frac{b}{\gamma}-C(p)-E_A\left[(C_R(a,p)+H_o+H_M(a)+H_F(a))\frac{P_f(p\mid a)\lambda}{\lambda+P_f(p\mid a)\lambda}\right] \qquad (18)$$

### Probabilistic model of the ground motion

Earthquake hazard assessment focuses on defining the probability of exceedence of a particular ground motion parameter at a site, in a given period of time $T$. For convenience and in agreement with current practice, the peak ground acceleration as a function of earthquake magnitude $M$ and epicentral distance $R$ is taken. Attenuation laws relating peak ground acceleration with magnitude and epicentral distance have the following general form:

$$a = h(m,r) = b_1(r)e^{b_2 m} \qquad (19)$$

where $a$ is the acceleration at the site of interest, $b_2 = 0.573$, $m$ is the magnitude and $b_1(r)$ is a function of distance describing the energy dissipation. For the seismic conditions of California[16], which may also be applicable to the region considered later in this study, $b_1(r)$ is given as:

$$b_1(r) = \frac{9.81 * 0.0955}{\sqrt{r^2 + 7.3^2}} \exp(-0.00587r) \qquad (20)$$

The acceleration can be related directly to the magnitude, through the attenuation law, which has a density function limited by an upper value $M_u$. The data collected for the region of interest show that they are best fitted by an extreme value distribution type III (for maxima). The upper bound is defined by historical data of earthquake events and by the regional geological characteristics. In addition, only events with magnitudes which may cause significant damage are taken into account; i.e., $m \geq M_{min} = 4.0$. Therefore, the conditional probability density function for the magnitude will be given by:

$$f_M(m \mid m > M_{min}) = \frac{\frac{w}{M_u - u}\left(\frac{M_u - m}{M_u - u}\right)^{w-1} \exp\left[-\left(\frac{M_u - m}{M_u - u}\right)^{w}\right]}{1 - \exp\left[-\left(\frac{M_u - M_{min}}{M_u - u}\right)^{w}\right]};$$

$$m \leq M_u \qquad (21)$$

The values of $w$ and $u$ can be determined based on the available data of the seismicity of the area. The denominator of the equation corresponds to $1 - F_M(m)$, which is the probability of having an earthquake with magnitude higher than $M_{min}$. For a region characterized by a mean earthquake magnitude of 4.53 with a standard deviation of 0.5 ($w$ = 8.108 and $u$ = 4.349), the curve of probability of exceedence, with $M_u = 7.5$ and $M_{min} = 4.0$, is shown in Figure 3. The parameters of the distribution were computed by a maximum likelihood method. The yearly rate of occurrences of earthquakes has been determined as $\lambda$ = 2.9[1/year].

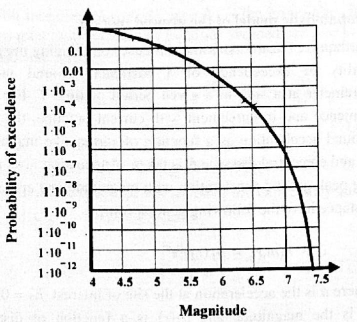


FIG.3 PROBABILITY OF EXCEEDENCE OF MAGNITUDES.

Based on the attenuation law considered and the conditional density function for the magnitude, the derived conditional density distribution function for the acceleration is given by:

$$f_A(a,r) = \frac{\frac{w}{M_u - u}\left(\frac{M_u - h^{-1}(a,r)}{M_u - u}\right)^{w-1} \exp\left[-\left(\frac{M_u - h^{-1}(a,r)}{M_u - u}\right)^{w}\right]}{1 - \exp\left[-\left(\frac{M_u - M_{min}}{M_u - u}\right)^{w}\right]} \frac{dh^{-1}(a,r)}{da} \qquad (22)$$

with $M_{min} < h^{-1}(a,r) = (1/b_2)\ln(a/(b_1(r)) < M_u$. In order to obtain the unconditional density function for the acceleration, it is necessary to integrate over the area within which the analysis is performed. It is assumed to be circular around the point of interest with $R_{max}$ = 200Km. The probability density function of the distance R has been considered as uniform with a value of $f_R(r) = 2r/R_{max}^2$. Of course more elaborated models can be developed if the location and earthquake pattern of the seismic sources are clearly identified. Assuming stochastic independence between magnitude and distance, the derived distribution for the acceleration, expressed in the general form presented in equation 6, can then be calculated as:

$$f_A(a) = \int_0^{R_{max}} f_A(a,r) f_R(r) dr \qquad (23)$$

The mean and the standard deviation for the acceleration without considering the uncertainty in the attenuation law, are 0.075 m/s$^2$ and 0.116 m/s$^2$ respectively; implying a coefficient of variation of 155%. The probability of exceedence can be computed as $1 - F_A(a)$ and is shown in Figure 4. According to the attenuation law, the maximum and minimum acceleration expected in site are 9.44 m/s$^2$ ($R$ = 0, $M = M_u$) and 0.014 m/s$^2$ ($R$ = 200 Km, $M$ = 4) respectively.

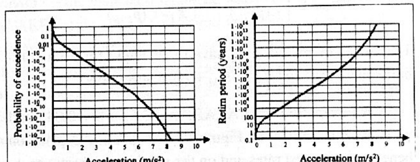


FIG.4 PROBABILITY OF EXCEEDENCE OF ACCELERATION

**Model of the probability of failure of the structural system**

The probabilistic model of the demand on the building structure subjected to a ground motion depends upon the probability distribution function of the acceleration and the



variation of the acceleration obtained from the response spectrum. The resistance depends upon the characteristics of the structures such as mass, stiffness, damping, and so forth. The simplified limit state function can be defined as:

$$g(R, S, A) = R - SA\varepsilon = 0 \qquad (24)$$

where $R$ is the resistance, $S$ is a variable that accounts for the variability of the response spectral acceleration of the system to the ground motion ($\mu_s = 1$, $\sigma_S = 0.6$), A will be taken as the peak ground acceleration at the base and $\varepsilon$ accounts for the uncertainty in the attenuation law which is in the order of 60% which was included as part of the spectral uncertainty. In practice, the design acceleration varies with the seismic construction requirements (e.g., detailing, ductility provided to the structure, minimum dimensions) for high, intermediate and low seismic regions, but for the purpose of this study, it is kept constant. The acceleration of the ground motion depends on the size of the earthquake and the epicentral distance. The resistance of the structure is a function of the vector parameter $p$, for which it has been designed and built to withstand the demand.

If both resistance and demand are modeled as lognormal distributions, the conditional probability of failure of the system can be expressed analytically as:

$$P_f(p \mid a) = \Phi\left[\frac{\ln\left(\frac{p}{sa}\sqrt{\frac{1 + V_S^2}{1 + V_R^2}}\right)}{\sqrt{\ln(1 + V_S^2)(1 + V_R^2)}}\right] \qquad (25)$$

The ratio $p/sa$ corresponds to the central safety factor, where $p$ is the mean value of the resistance. As mentioned, the variability of the attenuation law is included in the variability of the demand $V_s$, which was increased for the purpose of this study to 0.8. The conditional probability of failure of the system for different values of the vector parameter $p$ is presented in Figure 5. It can be observed that as the parameter $p$ increases, more strength is provided to the structure and therefore, the probability of failure

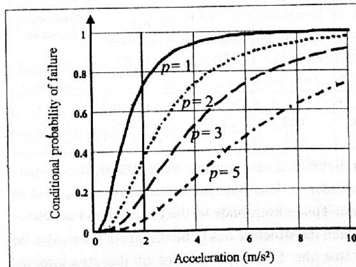


FIG.5 CONDITIONAL PROBABILITY OF FAILURE OF THE PROJECT

decreases for a given value of acceleration. It can also be observed that for $p$ = 1, and an acceleration value of 4 m/s$^2$, the probability of failure is close to 1. This corresponds to a MSK intensity of XI to XII, which means total destruction of the structure.

It is pointed out that equation 25 is valid only conditional on the acceleration, $a$. Therefore, expectation has to be taken to remove the condition. The unconditional probability of failure can be computed by integrating over the acceleration range ($a_{min}$ = 0.014 m/s$^2$ and $a_{max}$ = 9.44 m/s$^2$) as follows:

$$P_f(p) = \int_{a_{min}}^{a_{max}} P_f(a,p) f_A(a) da \qquad (26)$$

The unconditional probability only depends on the vector parameter $p$ and represents the failure probability of the structural system for a given capacity when subjected to acceleration with probability density as defined in equation 23. Because earthquakes follow a Poisson process, the failure rate is $v_f = \lambda P_f(p)$ and equation (3) in Table 1 applies.

**Calculation of Cost**

The definition of cost is fundamental for the optimization process. The cost values that are of interest for the optimization process are:

- Cost of construction
- Cost of retrofitting
- Cost of expected material losses
- Cost of human life losses

The construction cost of the structure consists of two parts; one that does not depend on the structural characteristics (non-structural elements) and the direct cost of the structure itself. The structural cost is a function of strength that is provided to withstand an earthquake, which is measured by a vector parameter $p$. The parameter $p$ might be, for instance, the mean horizontal shear force resistance of the structure. The following relationship has been considered for the cost of the building (Figure 6):

$$C(p) = (C_0 + C_1 p^{\delta}) \qquad (27)$$

where $C_0 = 1 \times 10^6$ is the construction cost that does not depend on the capacity characteristics, $C_1 = 1 \times 10^5$ is the construction cost that depends on the amount of resistance provided to the structure and $a$ is a constant that has been assumed to be $\delta$ = 1.1. Wen[17] estimated the value of power of cost function to be around 1.2.

The investment in retrofitting is related mainly to the structural system, although it may imply some repairs to non–structural elements. The cost of retrofitting depends on both the structural behaviour and the actual ground motion. The parameter $p$ defines the requirement, in structural terms to upgrade a structure to the reliability level of the current code of practice after it has been damaged; and the acceleration expected in site defines the expected structural damage. The retrofitting cost function has been defined as:

$$C_R(a,p) = (C_0 + C_2 p^{\delta} a^{\tau}) \quad (28)$$

where $C_2 = 8000$ is the cost of retrofitting the structure, $\tau$ is a constant that has been defined as 1.5 and $\delta$ is kept as 1.1. It has been assumed that the retrofitting cost cannot be higher than 60% of the total cost of the building; otherwise the building will not be repaired.

Although earthquake damage is usually measured in terms of the earthquake intensity (e.g., MMI, MSK) instead of the ground motion acceleration, in this paper, the analysis of cost will be made with reference to peak ground acceleration. Empirical relationships and tables were used as reference to relate damage observed and acceleration. The cost function of direct physical damage has been formulated as follows:

$$H_M(a) = C_M \eta^{a} \quad (29)$$

where $C_M = 1.5 \times 105$ represents the cost of material damage, $\eta$ is a constant taken as 1.25 and $a$ is the acceleration in m/s$^2$. For acceleration values of 2 m/s$^2$, which corresponds approximately to intensity IX (MSK scale), the cost of the damage will be around 25% of the total cost of the building; for $a$ = 5 m/s$^2$ (i.e. Intensity X, MSK) the cost of damage will be 45%; for acceleration values over 9 m/s$^2$ the cost of material losses can grow up to 110% of the total cost due to additional damages to the infrastructure that serves the building and the area (e.g., sewage, water or energy distribution systems). In addition to the direct cost of damage, the cost of loss of opportunity, $H_o$, was included as a constant depending upon the socio–economic climate: $H_o = \phi\, C_o$ but independent of acceleration.

The cost related to human life losses to be included in the optimization can be expressed as $H_F = ICAF$. It is a number, which society is willing to pay to save life years, i.e., investments in structural safety via codes of practice or the like. It enters into the optimization as a fictitious number at the decision point.

**Results of the optimization**

The characteristics of the ground motion used in the model correspond to data obtained from the US Geological Service (UGS) for a region with moderate to high seismic activity.

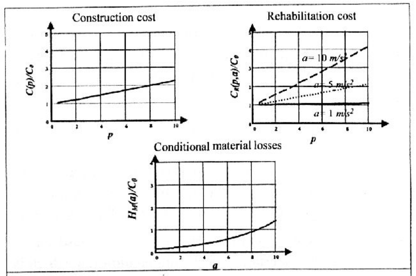


FIG.6 VARIATION OF COST OF CONSTRUCTION, RETROFITTING AND MATERIAL LOSSES

This study focuses on the implications of the LQI in the optimization process and the consequences in terms of structural safety for three different socio-economic contexts (Table 3).

TABLE 3
DATA FOR COMPARING DIFFERENT SOCIO–ECONOMIC LEVELS[18]

| Parameter | Socio–economic level | | |
|---|---|---|---|
| | **High** (Western Europe, USA, Japan) | **Moderate** (Latin America & Caribbean) | **Low** (Least developed countries) |
| $g$ | 23,500 | 6,500 | 1,500 |
| $e$ | 75 | 65 | 50 |
| $C_{F\delta}$ | 0.15 | 0.25 | 0.4 |
| $M$ | 0.01 | 0.0075 | 0.02 |
| $w$ | 0.125 | 0.15 | 0.18 |
| $H_0$ | 3.62$C_0$ | 1.0$C_0$ | 0.23$C_0$ |

Let us first consider a facility within which the maximum expected number of people killed in case of collapse is 10. The annual benefit is taken as $0.03C_0$ and the discount rate as 2% per year. Under these conditions, the values of the parameter $p$ and the corresponding design acceleration, obtained from the optimization, are presented in Table 4.

TABLE 4
DESIGN ACCELERATION VALUES FOR $p$–OPTIMUM FOR A LOW TO MODERATE SEISMICITY AREA

| Socio–economic level | Optimum–$p$ (Eq.5) | Acceleration (m/sec$^2$) | Failure probability of the structure | Return Period for the acceleration (years) |
|---|---|---|---|---|
| High | 2.3 | 2.07 | $2.30 \times 10^{-3}$ | 2569 |
| Moderate | 1.55 | 1.42 | $3.43 \times 10^{-3}$ | 381 |
| Low | 1.15 | 1.05 | $1.13 \times 10^{-2}$ | 135 |

A reliability analysis using FORM/SORM was performed in order to determine the acceleration associated to $p$–optimum. This corresponds to the peak ground acceleration for which the structure has to be designed. It can also be observed that the failure probability of the structure increases along with the socio–economic climate. For a scenario of ten fatalities, structural failure probabilities correspond to $\beta$ values between 2 and 3, which are common values in engineering practice for structural design of buildings structures. The corresponding return period for the peak ground acceleration, for all three socio-economic climates is shown in the last column of Table 4.



The results of the analysis for different number of fatalities and different socio-economic climates are summarized in Figure 7. It can be observed that in all cases, as the number of fatalities increases, there is also an increase in the required design acceleration. Also, the design acceleration changes depending upon the socio–economic condition of the population. This means that, as expected, a highly developed country should (and can) spend more in order to save life years than a developing country.

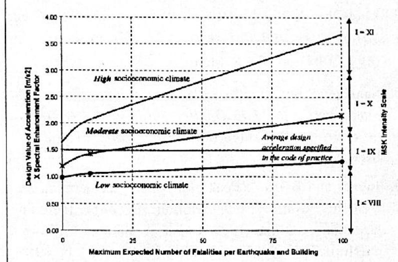


FIG.7 DESIGN ACCELERATION AS FUNCTION OF THE SOCIO-ECONOMIC LEVEL.

As the socio-economic characteristics of the society increase, the design acceleration and the associated return period also increase. On average, the return period specified by a code of practice is 475 years which corresponds to a design earthquake with a probability of being exceeded by 10% in fifty years. For $\lambda$ = 2.9, the earthquake specified by the code of practice has a design acceleration of 1.5 m/s$^2$ (Figure 7). Therefore, the results presented in Figure 7 suggest that design acceleration levels defined in current codes of practice are averaged values that in many cases are sub–optimal. They do not account for the number of fatalities nor for the socio–economic characteristics of the population. Therefore, the design criteria cannot be set in terms of the return period alone (e.g., 475 years), but it has also to consider construction and reconstruction costs, opportunity losses and the potential for human life saving.

If the objective function for the optimization does not include human life losses, the optimum $p$ values correspond to acceleration values of 1.63 m/s$^2$, 1.19 m/s$^2$ and 0.98 m/s$^2$ for high, moderate and low levels respectively. These values do not depend on the number of fatalities, nor on the socio–economic characteristics of the population. For this condition, the average design acceleration specified in the code of practice leads to overdesigns for moderate and low socio–economic climates and to underdesign of structures for highly developed countries. If human life losses are not considered, the difference in design acceleration between low and high social conditions is 1.65 m/s$^2$, which implies a huge difference in cost. Therefore, investments in structural safety should be in accordance with the economic development of a society.

For a given socio-economic climate, there is an important difference between the design acceleration with and without human life considerations. The difference between these two designs increases with the number of fatalities and also with the social context. For instance, for the high social climate and for the one hundred fatalities scenario, the difference of the required design acceleration with and without human losses is 2.05m/s$^2$; for moderate and low contexts this difference is 0.96 m/s$^2$ and 0.30 m/s$^2$ respectively. This implies that including the life saving criteria, as a factor in the optimization, in a highly developed society is much more expensive than for a low developed society.

## CONCLUSIONS

The structural design will move in the future to a new paradigm within which structures should be optimal in terms of capital investment and the saving of life years; the socio–economic characteristics of the area where the structure is built should be taken into account; and the probabilistic nature of the design variables need to be included in the design models and processes.

Since structures should be optimal in terms of the capital invested and the saving of life years, optimization techniques become essential for reliability–oriented optimal designs of technical facilities. Different objective functions have been proposed by Rackwitz (2000) in order to deal with different failure strategies. A compound social indicator of life expectancy, the Life Quality Index, is included as a criterion for the efficiency of the measures to save human lives. This criterion balances quality–adjusted life years saved against the associated cost for society.

Two examples were presented to illustrate the potential of the concepts discussed in the development of new regulations. First of all, the bounds for the ALARP region were calculated. Secondly, the acceleration for earthquake design of structures was computed for different social climates. The results show that countries with different socio-economic contexts should invest their resources in structural safety differently. Low developed countries should invest less in structural safety; and redirect the resources to other aspects such as education or health, which may prove to be more important for development. Selecting appropriate target reliabilities and risk acceptance criteria is paramount for making decisions on infrastructure development and might also define the speed at which this is achieved.

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