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TIME-VARIANT RELIABILITY-BASED STRUCTURAL OPTIMIZATION USING SORM

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Structural optimization under time-invariante reliability constraints is sufficiently well known. The same problem under time-dependent loads and resistances has not yet found satisfying solutions. Recently, a new attempt has been made where structural reliability is determined by the outcrossing approach in the context of first-order reliability methodology (FORM). In the paper an algorithm is designed with which outcrossing rates determined by asymptotic second-order reliability methods (SORM) can be used as constraints in structural optimization. The method is developed for two different types of stationary load models, rectangular wave renewal processes and Gaussian processes, respectively. An example application demonstrates the new methodology.

Keywords: Reliability-oriented structural optimization; time-invariant and time-variant structural reliability; random processes; outcrossing rates; one-level optimization

1. INTRODUCTION

Structural optimization with reliability constraints or optimal reliability-oriented structural design has been the subject of research for many years. Many different approaches have been studied all assuming that the constraints on reliability are time-independent (for example, see [11, 15, 18, 19] for typical approaches). The first contribution known to the authors which deals with time-variant aspects is due to Rosenblueth/Mendoza [22]. The main concepts developed in this fundamental paper

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were refined by Hasofer [12] and later again by Rosenblueth [23]. Unfortunately, the reliability models used in those studies were rather simple.

Recently, a new attempt has been made in [17] to formulate the time-variant case using standard structural reliability methodology. Reliabilities are determined by the outcrossing approach for stationary cases in the context of first-order reliability methodology (FORM). Two types of load models, rectangular wave renewal processes and Gaussian processes, respectively, are considered. When using FORM in time-invariant reliability it is essential that a "most likely failure point" exists in the so-called standard space. The same point is also the "critical" point in time-variant reliability indicating the "point of maximum local outcrossing rate". In this paper an extension to second-order reliability methods (SORM) is proposed also using a formulation in the standard space but additionally iterating for improved second-order crossing rates. A similar proposal has been made in [9] for time-invariant reliability constraints.

2. TIME-INVARIANT COMPONENT FAILURE PROBABILITY

Let $\mathbf{X} = (X_1, \ldots, X_n)^T$ denote a n-dimensional vector of random variables with distribution function $F_{\mathbf{x}}(\mathbf{x})$. A d-dimensional vector of cost parameters $\mathbf{p}$ can involve deterministic parameters but also parameters of the distribution function $F_{\mathbf{x}}(\mathbf{x})$. $G(\mathbf{x}, \mathbf{p})$ is a state (performance) function or failure function. $G(\mathbf{x}, \mathbf{p}) > 0$ denotes the safe state and $G(\mathbf{x}, \mathbf{p}) \leq 0$ the failure state. $G(\mathbf{x}, \mathbf{p}) = 0$ will also be denoted by failure surface. It is assumed that the probability density $f_{\mathbf{x}}(\mathbf{x})$ exists and the probability distribution function is continuously differentiable. The failure probability then is

$$P_f(\mathbf{p}) = \int_{G(\mathbf{x}, \mathbf{p}) \leq 0} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}. \qquad (1)$$

For large dimension n of the random variables and complex state function an exact evaluation by numerical integration can require considerable computational effort. Therefore, some special methods have been devised which can do the integration efficiently. A probability distribution transformation

$$\mathbf{T} : \mathbb{R}^n \rightarrow \mathbb{R}^n$$
$$\mathbf{T} : \mathbf{X} \longmapsto \mathbf{U}$$

always exists which maps an arbitrary n-dimensional random vector $\mathbf{X} = (X_1, \ldots, X_n)^T$ into an independent standard normal vector $\mathbf{U} = (U_1, \ldots, U_n)^T$ (Hohenbichler/Rackwitz [14], Der Kiureghian/Liu [8], Winterstein/Bjerager [29]). With a (new) state function $G(\mathbf{x}, \mathbf{p}) = g(\mathbf{T}(\mathbf{x}), \mathbf{p}) = g(\mathbf{u}, \mathbf{p})$ and the failure domain $\mathcal{F}_{\mathbf{p}} = \{\mathbf{u} : g(\mathbf{u}, \mathbf{p}) \leq 0\} \subset \mathbb{R}^n$, we then have

$$P_f(\mathbf{p}) = \int_{g(\mathbf{u}, \mathbf{p}) \leq 0} \varphi_{\mathbf{U}}(\mathbf{u}) d\mathbf{u} = \int_{\mathcal{F}_{\mathbf{p}}} P_{\mathbf{U}}(d\mathbf{u}), \qquad (2)$$

where $\varphi_{\mathbf{U}}(\mathbf{u})$ is the standard normal density and $P_{\mathbf{U}}(\cdot)$ is the standard normal distribution law.

If $g(\mathbf{u}, \mathbf{p}) \approx \boldsymbol{\alpha}^T \mathbf{u} + \beta_{\mathbf{p}}$ with $\beta_{\mathbf{p}} = -\boldsymbol{\alpha}^T \mathbf{u}^*$ and where $\mathbf{u}^*$ is the solution of the following optimization problem

$$(\beta P) \quad \textbf{minimize} \quad \|\mathbf{u}\|$$
$$\text{subject to} \quad g(\mathbf{u}, \mathbf{p}) \leq 0,$$

there is (Hasofer/Lind [13] and Rackwitz/Fiessler [20])

$$P_f(\mathbf{p}) \approx \Phi(-\beta_{\mathbf{p}}).$$

$\Phi(\cdot)$ is the standard normal integral. This approximation method is called first-order reliability method (FORM). The solution point $\mathbf{u}^*$ of the optimization problem (βP), the so called design point, maximum likely failure point or β-point, defines the first-order reliability index

$$\beta_{\mathbf{p}} = \|\mathbf{u}^*\|. \qquad (3)$$

$\boldsymbol{\alpha}$ is the vector of direction cosines of the solution point. Reference to the parameter vector $\mathbf{p}$ is omitted here and in the following whenever this is possible without loosing clarity.

Breitung established in [3] the following asymptotic result, called second-order reliability method (SORM). For $\beta_{\mathbf{p}} \to \infty$ there is

$$P_f(\mathbf{p}) \sim \Phi(-\beta_{\mathbf{p}}) \cdot \prod_{i=1}^{n-1} (1 - \beta_{\mathbf{p}} \kappa_i)^{-1/2}, \tag{4}$$

where κ_i are the main curvatures of the state function in the solution point.

If $\mathbf{u}^*$ is an optimal point for (βP), the β-point is a Kuhn-Tucker-point. In [15] the following theorem is proved.

THEOREM 1 (β-Point-Theorem in Time-Invariant Case) *If u^*, with $u^* \neq 0$, is the solution point of optimization problem (βP), then the following two statements hold for each p*:

(*a*) $g(u^*, p) = 0$,
(*b*) $u^{*T} \nabla_u g(u^*, p) + \|u^*\| \, \|\nabla_u g(u^*, p)\| = 0$. ■

3. THE OUTCROSSING APPROACH FOR TIME-VARIANT RELIABILITY

3.1. Basic Formulations

Let T be the random time between exists into the failure domain. Then, the time-dependent failure probability is

$$P_f(t, \mathbf{p}) = \mathcal{P}(T \leq t|\mathbf{p}),$$

where $[0, t]$ is the considered time interval. The distribution function of the first passage time T must be known. The first passage time is the time where the component enters the failure domain for the first time given that the component was in the safe state at time $\tau = 0$. Exact first passage time distributions are known for only few types of processes which generally are of little practical interest in structural reliability. However it is frequently possible to determine outcrossing rates and to use bounds or asymptotic results for first passage time. Therefor, all subsequent derivations for the determination of the probability of first passage failure are based on the so-called outcrossing approach. It will

even be demonstrated that for cost optimal design it is the outcrossing rate itself and not time-dependent failure probabilities. The outcrossing approach rests on a few assumptions which are fulfilled in most practical cases.

As in time-invariant component reliability there exists a state function depending on random vectors and on random process variables. More specifically, for the purpose of versatile modelling we distinguish between three types of variables

- $\mathbf{R}$ is a $n_{\mathbf{R}}$-dimensional random vector as in time-invariant reliability. This vector is used to model resistance variables and its most important characteristic is that it is non-ergodic.
- $\mathbf{Q}$ is a $n_{\mathbf{Q}}$-dimensional vector of stationary and ergodic sequences. It is used to model long term variations in time, *e.g.*, traffic or sea states. These variables determine the fluctuating parameters of the random process variables described next.
- $\mathbf{S}$ is a $n_{\mathbf{s}}$-dimensional vector of sufficiently mixing random process variables whose parameters can depend on $\mathbf{Q}$ and/or $\mathbf{R}$.

The state function $G(\mathbf{r}, \mathbf{q}, \mathbf{s}(t), t, \mathbf{p}) = \bar{g}(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}(t)}, t, \mathbf{p})$ can contain time as a parameter, too, and must be twice differentiable. The safe state of the structure is defined for $G(\mathbf{r}, \mathbf{q}, \mathbf{s}(t), t, \mathbf{p}) > 0$, the state for $G(\mathbf{r}, \mathbf{q}, \mathbf{s}(t), t, \mathbf{p}) = 0$ and the failure state for $G(\mathbf{r}, \mathbf{q}, \mathbf{s}(t), t, \mathbf{p}) \leq 0$, respectively.

The rate of outcrossings into the failure domain conditional on $\mathbf{r}, \mathbf{q}$ and $\mathbf{p}$ is defined as

$$\begin{aligned} &\nu^+(\mathcal{F}, \tau|\mathbf{r}, \mathbf{q}, \mathbf{p}) \\ &\quad = \lim_{\Delta \downarrow 0} \frac{1}{\Delta} \mathcal{P}(\{G(\mathbf{r}, \mathbf{q}, \mathbf{S}(\tau), \tau, \mathbf{p}) > 0\} \\ &\qquad\qquad \cap \{G(\mathbf{r}, \mathbf{q}, \mathbf{S}(\tau + \Delta), \tau + \Delta, \mathbf{p}) \leq 0\}), \end{aligned}$$

where $\mathcal{F} = \{(\mathbf{S}(t), t) : G(\mathbf{r}, \mathbf{q}, \mathbf{S}(t), t, \mathbf{p}) \leq 0\}$ denotes the failure domain conditional on $\mathbf{r}, \mathbf{q}$ and $\mathbf{p}$. The rate of outcrossings only exists in cases where the limiting operation can be performed. It exists for regular point processes of crossings. Note, the time-dependent outcrossing rates are additive for a regular process. The number of crossings in the time interval $[t_1, t_2]$ is denoted by $N^+(t_1, t_2)$. If the random process depends on random variables $\mathbf{r}, \mathbf{q}$ and on deterministic cost parameters $N^+(t_1, t_2)|\mathbf{r}, \mathbf{q}, \mathbf{p}$ denotes the number of crossings conditional on $\mathbf{r}, \mathbf{q}$ and $\mathbf{p}$. The mean number of crossings $N^+(t_1, t_2)$ in the time interval $[t_1, t_2]$ conditional on $\mathbf{r}, \mathbf{q}$ and $\mathbf{p}$ can then be determined from

$$E(N^+(t_1, t_2)|\mathbf{r}, \mathbf{q}, \mathbf{p}) = \int_{t_1}^{t_2} \nu^+(\mathcal{F}, \tau|\mathbf{r}, \mathbf{q}, \mathbf{p})d\tau.$$

Schall *et al.*, showed in [24] that an upper bound for the first passage probability is

$$P_f(t_1, t_2, \mathbf{p}) \leq P_f(t_1, \mathbf{p}) + E_{\mathbf{R}}(E_{\mathbf{Q}}(E(N^+(t_1, t_2)|\mathbf{R}, \mathbf{Q}, \mathbf{p}))). \quad (5)$$

In most practical cases $P_f(t_1)$ is negligible small. It will be computed as in time-invariant reliability. If it is guaranteed that the processes start from the origin, then it is $P_f(t_1) = 0$, of course. An asymptotic result can also be derived (Cramer/Leadbetter [7] and Breitung [5]) which in the stationary case is

$$P_f(t_1, t_2, \mathbf{p}) \sim 1 - E_{\mathbf{R}}(\exp(-E_{\mathbf{Q}}(\nu^+(\mathcal{F})|\mathbf{R}, \mathbf{Q}, \mathbf{p})(t_2 - t_1)). \quad (6)$$

This is an exponential distribution implying that the failure process is a stationary Poisson process.

3.2. Outcrossing Rates for Rectangular Wave Renewal Vector Processes

In [6] Breitung/Rackwitz have proven that in the stationary case the outcrossing rate can be calculated as the product of the jump rate λ_i and the probability that a component of the rectangular wave jumps from the safe domain $\bar{\mathcal{F}}$ into the failure domain $\mathcal{F}$, summed up over all n_s components of the rectangular wave renewal process. Ignoring for the moment all **R**- and **Q**-variables the mean outcrossing rate is

$$\begin{aligned}\nu^+(\mathcal{F}) &= \sum_{i=1}^{n_s} \lambda_i \cdot \mathcal{P}(\{\mathbf{S}_i^- \in \bar{\mathcal{F}}\} \cap \{\mathbf{S}_i^+ \in \mathcal{F}\}) \\ &= \sum_{i=1}^{n_s} \lambda_i \cdot (\mathcal{P}(S_i^+ \in \mathcal{F}) - \mathcal{P}(\{\mathbf{S}_i^- \in \mathcal{F}\} \cap \{\mathbf{S}_i^+ \in \mathcal{F}\})),\end{aligned}$$

where $\mathbf{S}_i^-$ is the vector of jumping components just before a jump of the ith component $\mathbf{S}_i^-$ and $\mathbf{S}_i^+$ after a jump. It is assumed that at a jump the component $\mathbf{S}_i$ changes its position from a random value to a new random value.

Then, the outcrossing rate can be given in the simple case of linear failure surfaces $\partial\mathcal{F} = \alpha_s^T\mathbf{u}^s + \beta_\mathbf{p} = 0$ as

$$\begin{aligned}\nu^+(\mathcal{F}(\mathbf{p})) &= \sum_{i=1}^{n_s} \lambda_i \cdot (\Phi(-\beta_\mathbf{p}) - \Phi_2(-\beta_\mathbf{p}, -\beta_\mathbf{p}; \rho_i)) \\ &= \Phi(-\beta_\mathbf{p}) \cdot \sum_{i=1}^{n_s} \lambda_i \cdot \left(1 - \frac{\Phi_2(-\beta_\mathbf{p}, -\beta_\mathbf{p}; \rho_i)}{\Phi(-\beta_\mathbf{p})}\right) \\ &= \Phi(-\beta_\mathbf{p}) \cdot \sum_{i=1}^{n_s} \lambda_i',\end{aligned} \quad (7)$$

with $\Phi_2(\cdot, \cdot; \cdot)$ the two-dimensional normal integral. Note that the reliability index $\beta_\mathbf{p}$ is determined in the entire **R-Q-S**-space, *i.e.*, $\beta_\mathbf{p} = \|\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})^T\|$. The correlation coefficient of the two state variables before 0 and after a jump equals

$$\rho_i = 1 - \left(\frac{\frac{\partial g}{\partial u_i^s}(\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})}{\|\nabla_{\mathbf{u}^s} g(\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})\|}\right)^2, \quad i = 1, \ldots, n_s$$

Formally, the last factor can be interpreted as a first-order correction to the jump rates then denoted by $\boldsymbol{\lambda}'$. It is defined by

$$\lambda_i' = \lambda_i\left(1 - \frac{\Phi_2(-\beta_\mathbf{p}, -\beta_\mathbf{p}; \rho_i)}{\Phi(-\beta_\mathbf{p})}\right), \quad i = 1, \ldots, n_s$$

and can be interpreted in the same way as in the linear case. It depends on all random and on cost variables in the iteration points.

If the failure surface is approximated by a linear surface in the β-point, then the probability of having jumps into the failure domain depends on $\beta_\mathbf{p}$ and the probability of having jumps from the failure domain into the failure domain can be studied in terms of the general parameters $\beta_\mathbf{p}$ and ρ_i. The probability of jumps from the failure domain into the failure domain can frequently be neglected and therefore

$$\nu^+_{\text{FORM}}(\mathcal{F}(\mathbf{p})) \leq \Phi(-\beta_\mathbf{p}) \cdot \sum_{i=1}^{n_s} \lambda_i. \quad (8)$$

In [5] Breitung proved that in the stationary case the outcrossing rate can even be approximated by

$$\nu^{+}_{\mathrm{SORM}}(\mathcal{F}(\mathbf{p})) = \Phi(-\beta_{\mathbf{p}}) \cdot \prod_{j=1}^{n_s-1} (1-\beta_{\mathbf{p}}\kappa_j)^{-1/2} \cdot \sum_{i=1}^{n_s} \lambda_i. \tag{9}$$

where κ_j are the main curvatures of the state function in the solution point with respect to the jump variables S_i. This result is asymptotically exact ($\beta_{\mathbf{p}} \to \infty$). It can further be improved for non-asymptotic cases by taking

$$\nu^{+}(\mathcal{F}(\mathbf{p})) \sim \Phi(-\beta_{\mathbf{p}}) \cdot \prod_{j=1}^{n-1} (1-\beta_{\mathbf{p}}\kappa_j)^{-1/2} \cdot \sum_{i=1}^{n_s} \lambda_i \cdot \left(1 - \frac{\Phi_2(-\beta_{\mathbf{p}}, -\beta_{\mathbf{p}}; \rho_i)}{\Phi(-\beta_{\mathbf{p}}) \cdot \prod_{j=1}^{n_s-1}(1-\beta_{\mathbf{p}}\kappa_j)^{-1/2}}\right)$$
$$= \Phi(-\beta_{\mathbf{p}}) \cdot \prod_{j=1}^{n-1} (1-\beta_{\mathbf{p}}\kappa_j)^{-1/2} \cdot \sum_{i=1}^{n_s} \lambda_i''.$$

3.3. Outcrossing Rates for Gaussian Processes

The determination of outcrossing rates for Gaussian processes is well known by Rice's formula [7, 21] and extensions to vector processes exist. For differentiable Gaussian vector processes $\mathbf{S}(t)$ given by the mean vector $m_s(t)$ and symmetric, positive definite matrix

$$C_{\mathbf{S}}(t_1, t_2) = \{\sigma_{ij}(t_1, t_2); \quad i, j = 1, \ldots, n\}$$

of covariance functions the crossing rates are computed according to the generalization of Rice's formula put forward by Belyaev [1]. It is useful to introduce some transformations. After standardization by

$$V_i(t) = \frac{\mathbf{S}_i(t) - m_i(t)}{\sigma_{ii}(t)} \qquad i = 1, \ldots, n$$

and subsequent diagonalization of the correlation coefficient matrix of $V(t)$ by $V(t) = A(t)\mathbf{u}_s(t)$ where $A(t)$ is a triangular matrix, possibly

depending on time, and $\mathbf{u}_s(t)$ a vector of independent standard normal processes such that $V(t)$ and $A(t)\mathbf{u}_s(t)$ have the same cross-correlation coefficient matrix for any time t and where the coefficients in $A(t_1, t_2)$ are by Choleski's decomposition scheme

$$A(t_1,t_2) = \begin{pmatrix} a_{11}(t_1,t_2) = \rho_{\mathbf{u}_s,11}(t_1,t_2) & \\ a_{i1}(t_1,t_2) = \rho_{\mathbf{u}_s,i1}(t_1,t_2); & 2 \le i \le n \\ a_{ii}(t_1,t_2) = \left(\rho_{\mathbf{u}_s,ii}(t_1,t_2) - \sum_{k=1}^{i-1} a_{ik}^2(t_1,t_2)\right)^{1/2}; & 2 \le i \le n \\ a_{ij}(t_1,t_2) = \frac{1}{a_{jj}(t_1,t_2)} \left(\rho_{\mathbf{u}_s,ij}(t_1,t_2) - \sum_{k=1}^{j-1} a_{ik}(t_1,t_2) a_{jk}(t_1,t_2)\right); & 1 < j < i \le n \end{pmatrix}$$

the matrix of autocorrelation functions $R(t_1, t_2)$ is a diagonal matrix. Also, the cross-correlation matrix of the derivative processes $\ddot{R}(t_1, t_2)$ obtained by twice differentiating the auto-correlation matrix is a diagonal matrix. The matrix of the correlations between processes and derivatives $\dot{R}(t_1, t_2)$, in general, is not of diagonal form and is skew-symmetric, *i.e.*, $\dot{R}^T(t_1, t_2) = -\dot{R}(t_1, t_2)$. For stationary vector processes the cross-correlations do not depend on time and the two time arguments can be replaced by $\tau = t_2 - t_1$. Then, Belyaev's formula [1] is

$$\nu^{+}(\mathcal{F}, \tau) = \int_{\partial\mathcal{F}(\tau)} E(-\boldsymbol{\alpha}_s^T(\mathbf{s}, \tau)\dot{\mathbf{S}}(\tau) | \mathbf{S}(\tau) = \mathbf{s}) \cdot \varphi_n(\mathbf{s}) ds_{\partial\mathcal{F}}, \tag{10}$$

where $\partial\mathcal{F}(\tau) = \{(\mathbf{s}(\tau), \tau) : G(\mathbf{s}(\tau), \tau) \le 0\}$, $\boldsymbol{\alpha}_s(\mathbf{s}) = \nabla G(\mathbf{s}) / \|\nabla G(s)\|$ the surface normal and $ds_{\partial\mathcal{F}}$ means surface integration.

In standard space the second order outcrossing rate in the stationary case is (Breitung, [4])

$$\nu^{+}_{\mathrm{SORM}}(\mathcal{F}(\mathbf{p})) = \frac{\varphi(\beta_{\mathbf{p}})}{\sqrt{2\pi}} \cdot \left(\frac{1}{|\det(H)|}\right)^{1/2} \cdot \omega_0, \tag{11}$$

where $\beta_{\mathbf{p}} = \|(\mathbf{u}^{\mathbf{r}*}, \mathbf{u}^{\mathbf{q}*}, \mathbf{u}^{\mathbf{s}*})\|$ is the reliability index and the variance of the velocity process normal to the failure surface in the critical point is

$$\omega_0^2 = \frac{\boldsymbol{\alpha}_s^T(\mathbf{s})}{\|\boldsymbol{\alpha}_s(\mathbf{s})\|} \cdot \left(\ddot{\mathbf{R}} + \dot{\mathbf{R}}^T \frac{G_{\mathbf{S}}}{\|\nabla_{\mathbf{u}^s} g(\mathbf{u}^s | \mathbf{u}^r, \mathbf{u}^q, \mathbf{p})\|} \dot{\mathbf{R}}\right) \cdot \frac{\boldsymbol{\alpha}_s(\mathbf{s})}{\|\boldsymbol{\alpha}_s(\mathbf{s})\|},$$

where it is noted that the second term in parenthesis is a generally rather small second order correction. G_s is the matrix of second derivatives of $g(\mathbf{u}^s|\mathbf{u}^r, \mathbf{u}^q, \mathbf{p}) = 0$ at $(\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})^T$ with respect to the S-variables. Formally, ω_0 depends on all random vectors and on cost variables $\mathbf{p}$ in the all iteration points. H is a matrix of the form

$$H = I - \frac{\beta_\mathbf{p}}{\|\nabla_{\mathbf{u}^{r,q,s}} g(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s, \mathbf{p})\|} \cdot G_{\mathbf{R},\mathbf{Q},\mathbf{S}},$$

with I the identity matrix and the matrix $G_{\mathbf{R},\mathbf{Q},\mathbf{S}}$ collecting the second derivatives of $g(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s, \mathbf{p}) = 0$ at $(\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})^T$ with respect to $(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s)$. This result is asymptotically exact $(\beta_\mathbf{p} \to \infty)$.

The corresponding FORM results are nicely compact (Veneziano *et al.* [28]). If the failure surface is linear

$$\omega_0^2 = \frac{\boldsymbol{\alpha}_s^T(\mathbf{s})}{\|\boldsymbol{\alpha}_s(\mathbf{s})\|} \cdot \ddot{\mathbf{R}} \cdot \frac{\boldsymbol{\alpha}_s(\mathbf{s})}{\|\boldsymbol{\alpha}_s(\mathbf{s})\|} \tag{12}$$

remains and det $(H) = 1$ so that

$$\nu^+_{\text{FORM}}(\mathcal{F}(\mathbf{p})) = \frac{\varphi(\beta_\mathbf{p})}{\sqrt{2\pi}} \cdot \omega_0. \tag{13}$$

Note that this expression does not involve the matrix $\dot{\mathbf{R}}(t_1, t_2)$.

3.4. Optimality Conditions in the Stationary, Time-variant Case

It is seen that also for time-variant reliability the β-point plays an important role. Now it is no more the most likely failure point but the point of maximum local outcrossing rate.

The β-Point-Problem in time-variant and stationary case has the following form:

$$(\beta P\text{-}TV\text{-}ST) \quad \textbf{minimize} \quad \|(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s)^T\|$$
$$\text{subject to} \quad g(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s, \mathbf{p}) \le 0.$$

The state function $G(\mathbf{r}, \mathbf{q}, \mathbf{s}, \mathbf{p}) = G(\mathbf{T}(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s), \mathbf{p}) = g(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s, \mathbf{p})$ does not contain time as a parameter and must be twice differentiable. The solution point $\mathbf{u}^* = (\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})^T$ of the time-variant and stationary

problem $(\beta P\text{-}TV\text{-}ST)$ defines the so called design point or β-point. The first-order reliability index $\beta_\mathbf{p}$ defines analogous

$$\beta_\mathbf{p} = \|\mathbf{u}^*\| = \|(\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})^T\|.$$

The following important corollary expresses sufficient conditions for the optimality of the solution $\mathbf{u}^*$. The proof is based on the proof of the time-invariant β-Point-Theorem and is an extension of the time-invariant proof, mentioned in [15].

COROLLARY 2 (β-Point-Conditions in Time-Variant and Stationary Case) *If* $u^* = (u^{r*}, u^{q*}, u^{s*})^T$, *with* $u^* \neq 0$, *is the solution point of optimization problem* $(\beta P\text{-}TV\text{-}ST)$, *then the following two statements hold for each* p:

(*a*) $g(u^{r*}, u^{q*}, u^{s*}, p) = 0$,

(*b*) $(u^{r*}, u^{q*}, u^{s*}, p)^T \nabla_u g(u^{r*}, u^{q*}, u^{s*}, p)$
$+ \|(u^{r*}, u^{q*}, u^{s*}, p)^T\| \; \|\nabla_u g(u^{r*}, u^{q*}, u^{s*}, p)\| = 0.$

Note that the gradient of the state function surface is defined in the entire R-Q-S-space, *i.e.*, $\nabla_\mathbf{u}(\cdot) = \nabla_{\mathbf{u}^r,\mathbf{u}^q,\mathbf{u}^s}(\cdot)$, and similarly the norm of the optimal vector $\mathbf{u}^* = (\mathbf{u}^{r*}, \mathbf{u}^{q*}, \mathbf{u}^{s*})^T$.

4. RELIABILITY-ORIENTED STRUCTURAL OPTIMIZATION

4.1. Objective Functions

Structural optimization can be performed with respect to reliability or with respect to other criteria such as weight or cost. In the following we will consider only optimization with respect to weight or cost with or without a reliability constraint. If cost optimization is chosen the question arises whether to include expected failure cost or not. Simple cost optimization subject to a reliability constraint, no doubt, is a much simpler task than optimization of cost including expected failure cost, possibly subject to a reliability constraint. Rosenblueth and Hashofer [12, 22] proposed the following objective function

$$C_{\text{total}}(\mathbf{p}) = C(\mathbf{p}) + D(\mathbf{p}), \tag{14}$$

in which $C(\mathbf{p})$ are the building cost depending on cost; parameters $\mathbf{p}$. The expected failure cost are denoted by $D(\mathbf{p})$. All quantities will be measured in monetary units. In [22] Rosenblueth/Mendoza distinguished between two reconstruction policies. On the one hand, they considered structures which are designed to fulfill only "one mission", *i.e.*, structures which will be abandoned after the mission or after failure. On the other hand, they considered structures which are systematically rebuilt after failure. For most structures this policy should be adopted and this is the case which is of interest herein. They further considered Poissonian failure processes, *i.e.*, processes with outcrossing rate $\nu^+(\mathcal{F}(\mathbf{p}))$ and fulfilling the conditions for Eq. (6). The reconstruction times will be assumed to be negligible short as compared to the inter-arrival times of failure. All quantities are expected values which have to be capitalized down to the decision point at $t = 0$. The (continuous) capitalization or discount function is

$$d(t) = \exp[-\gamma t],$$

with γ the discount rate. For a yearly discount rate γ', there is $\gamma = \ln(1+\gamma')$.

By considering systematic reconstruction and, thus, infinitely many "renewals" (see also Rosenblueth/Mendoza, [22]) the expected cost of failure including immediate reconstruction can be determined as

$$D(\mathbf{p}) = (C(\mathbf{p}) + H(\mathbf{p}))\frac{\nu^+(\mathcal{F}(\mathbf{p}))}{\gamma},$$

a result also found elsewhere in the literature. Note that the parameter time, *i.e.*, some usually unknown time of use or of reference time of the structure, has disappeared and the failure cost (or the failure rate) are formally increased by a factor of $1/\gamma$. If $\nu^+(\mathcal{F}(\mathbf{p}))$ depends on some uncertain parameter vector $\mathbf{R}$ we have finally

$$C_{\text{total}}(\mathbf{p}) = C(\mathbf{p}) + (C(\mathbf{p}) + H(\mathbf{p}))\frac{E_{\mathbf{R}}(\nu^+(\mathcal{F}(\mathbf{p})))}{\gamma}, \quad (15)$$

in noting that in the presence of ergodic sequences $\mathbf{Q}$, the Poissonian nature of the failure process is maintained. More discussion of this basic result can be found in [17]. It should be mentioned that the case of one mission is slightly more complicated but can be simplified to Eq. (15) in good approximation.

4.2. Cost Optimization with Time-variant FORM-Reliability Constraints

As in [15] for time-invariant componential optimization, the so-called one-level optimization scheme is applied here [17]. The necessary first-order optimality condition for design points from Corollary 2 are inserted into the cost optimization problem.

$$\begin{array}{ll}
\textbf{minimize} & C_{\text{total}}(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}}, \mathbf{p}) \\
\text{subject to} & \textbf{\textit{constraint for reliability}} \\
 & g(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}}, \mathbf{p}) = 0 \\
 & (\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}})^T \nabla_{\mathbf{u}} g(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}}, \mathbf{p}) \\
 & \quad + \|(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}})^T\| \, \|\nabla_{\mathbf{u}} g(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}}, \mathbf{p})\| = 0 \\
 & \textit{constraints on cost vector} \\
 & \textit{simple bounds for random and cost vector}, \quad \textbf{(CRP-TV)}
\end{array}$$

The reliability constraint in **(CRP-TV)** is specified by a failure rate constraint

$$E_{\mathbf{R}}(E_{\mathbf{Q}}(\nu^+(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}}, \mathbf{p}))) \leq \nu^+_{\text{maximum}}, \quad (16)$$

where ν^+_{maximum} is the maximum allowable outcrossing rate, given by the user. Note, that in this formulation the outcrossing rate $\nu^+(\cdot, \cdot, \cdot, \cdot)$ depends on the optimization variables $(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}})^T$ and $\mathbf{p}$.

Using the total cost including a discount or actualization aspect of building and failure cost as in Eq. (15) leads in a first-order formulation to:

$$\textbf{minimize} \quad C(\mathbf{p}) + (C(\mathbf{p}) + H(\mathbf{p}))\frac{E_R[\nu^+_{\text{FORM}}(\mathbf{u}^{\mathbf{r}}, \mathbf{u}^{\mathbf{q}}, \mathbf{u}^{\mathbf{s}}, \mathbf{p})]}{\gamma}$$

subject to

$$
\begin{aligned}
&E_{\mathbf{R}}(E_{\mathbf{Q}}(\nu^{+}_{\mathrm{FORM}}(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s},\mathbf{p}))) \le \nu^{+}_{\mathrm{maximum}}\\
&g(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s},\mathbf{p}) = 0\\
&(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{T}\nabla_{u}g(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s},\mathbf{p})\\
&\quad + \|(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{T}\|\|\nabla_{u}g(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s},\mathbf{p})\| = 0\\
&K(\mathbf{p}) \overset{=}{\geq} \mathbf{0}_{m'+m}\\
&((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{l},\mathbf{p}^{l}) \le ((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s}),\mathbf{p})\\
&(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s},\mathbf{p}) \le ((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{u},\mathbf{p}^{u}), \qquad \textbf{(CRP-TV)}
\end{aligned}
$$

where $K(\cdot)$ denotes a constraint vector of m' equality constraints and m inequality constraints of structural performance requirements on the design vector. The parameters $((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{l},\mathbf{p}^{l})$, $((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{u},\mathbf{p}^{u})$ are simple lower and upper bounds for the standard normal vector $(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})$ and the cost parameter is $\mathbf{p}$. Obviously, the objective function as well as the reliability constraint (16) are determined only to first order. For the outcrossing rate formula (8) and/or (13) can be used. It is recognized that this formulation rests on a formulation of the reliability problem in standard space. It resembles very much formulations for cost optimization with time-invariant reliability constraints [15]. The computational task is best solved by one of the well-known SQP-algorithms.

5. RELIABILITY-BASED COST OPTIMIZATION WITH TIME-VARIANT SORM-CONSTRAINTS

The presented one-level approach for cost- and reliability-oriented structural optimization is based on a first-order approximation for the respective reliability problem. It does not generally fulfill the reliability constraint (16). An equivalent SORM-formulation is not possible because the SORM-correction factors in (9) and (11) are valid only in the $\beta_{\mathbf{p}}$-point. Even if a formulation could be found third-order derivatives of the state function would be necessary which may limit the numerical feasibility of such an algorithm. Therefore, the one-level

optimization is at least to be improved with respect to a reliability constraint evaluated according to SORM.

An obvious and rather effective method to fulfill SORM-based reliability constraints (failure rates) is additional iteration. The modified optimization problem (**MCRP-TV**) is used in the following algorithm to solve the general cost optimization problem with a SORM constraint.

Algorithm for (MCRP-TV)

Step 0: Evaluate: $((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{0},\mathbf{p}^{0})^{*}$ as solution of (**CRP-TV**)

Evaluate: $\nu^{0}_{\mathrm{FORM}} = E_{\mathbf{R}}(E_{\mathbf{Q}}(\nu^{+}_{\mathrm{FORM}}((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{0},\mathbf{p}^{0})))$

Set: $k = 1$

Step 1: Evaluate: $\nu^{k-1}_{\mathrm{SORM}} = E_{\mathbf{R}}(E_{\mathbf{Q}}(\nu^{+}_{\mathrm{SORM}}((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k-1},\mathbf{p}^{k-1})))$

Step 2: If $\nu^{k-1}_{\mathrm{SORM}} \le \nu^{+}_{\mathrm{maximum}}$ **STOP!**

Step 3: Evaluate: $((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k})^{*}$ as solution of $(\mathbf{MCRP\text{-}TV})^{k}$

Evaluate: $\nu^{k}_{\mathrm{FORM}} = E_{\mathbf{R}}(E_{\mathbf{Q}}(\nu^{k}_{\mathrm{FORM}}((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k-1},\mathbf{p}^{k-1})))$

Set: $k = k+1$

Step 4: Goto Step 1!

$((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k})^{*}$ Solution of the Optimization Problem $(\mathbf{MCRP\text{-}TV})^{k}$:

minimize $C(\mathbf{p}) + (C(\mathbf{p}) + H(\mathbf{p}))\dfrac{E_{R}[\nu^{+}_{\mathrm{FORM}}((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k})]}{\gamma}$

subject to:

$$
\begin{aligned}
&E_{\mathbf{R}}(E_{\mathbf{Q}}(\nu^{+}_{\mathrm{FORM}}((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k}))) \le \frac{(\nu^{k-1}_{\mathrm{FORM}})^{2}}{\nu^{k-1}_{\mathrm{SORM}}}\\
&g((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k}) = 0\\
&((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k})^{T}\nabla_{u}g(\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k})\\
&\quad + \|((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k})^{T}\|\,\|\nabla_{u}g((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k})\| = 0\\
&H(\mathbf{p}^{k}) \overset{=}{\geq} \mathbf{0}_{m'+m}\\
&((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{l},\mathbf{p}^{l}) \le ((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{k},\mathbf{p}^{k}) \le ((\mathbf{u}^{r},\mathbf{u}^{q},\mathbf{u}^{s})^{u},\mathbf{p}^{u})
\end{aligned}
$$

In words: In (**CRP-TV**) the first-order reliability constraint is necessarily fulfilled. If the second-order reliability constraint is also fulfilled the algorithm stops. If it is not fulfilled a new sharper auxiliary reliability constraint must be set up. It appears natural to multiply the original FORM-constraint by a factor smaller than unity. Its magnitude, for example, can be chosen to be the ratio of the FORM and the SORM rate. Other similar choices are possible. This auxiliary reliability constraint is, if necessary, sharpened consecutively, *i.e.*:

$$\nu^k_{\text{FORM}} \frac{\nu^k_{\text{FORM}}}{\nu^k_{\text{SORM}}} \leq \nu^{k-1}_{\text{FORM}} \frac{\nu^{k-1}_{\text{FORM}}}{\nu^{k-1}_{\text{SORM}}} \leq \nu^+_{\text{maximum}}; \quad k = 0, 1, 2, \ldots$$

Clearly, any improvement of the FORM result, for example, by importance sampling, can be used instead of SORM.

6. NUMERICAL EXAMPLE

The above algorithm is illustrated at a practical example already used in [17]. The minimization of total cost under time-variant constraints is carried out by the non-linear optimization algorithm NLPQL by Schittkowski [25]. All necessary functions values and gradients are taken numerically.

The numerical example is a steel column and is investigated in detail with FORM in [17]. The vector of cost parameter is defined by $\mathbf{p} = (\mu_b, \mu_d, \mu_h)$:

Variable	*Symbol*	*Unit*	*Bounds*
Mean of Flange Breadth	μ_b	mm	$(200, 400)$
Mean of Flange Thickness	μ_d	mm	$(10, 30)$
Mean of Height of Steel Profile	μ_h	mm	$(100, 500)$

The steel column has a constant length of 7500 [mm].

The function of total cost $C_{\text{total}}(\mathbf{u}^r, \mathbf{u}^q, \mathbf{u}^s, \mathbf{p})$ includes failure cost of

$$H = 50\,000[\text{CU}] \quad (\text{CU} = \text{currency unit})$$

discounted continuously with rate $\gamma = 5\%$ per year and with:

$$C(\mathbf{p}) = (\mu_b \mu_d + 5[\text{mm}] \cdot \mu_h) \cdot [\text{CU/mm}^2]$$

The independent uncertian vector $\mathbf{X} = ((F_s, P_1, B, D, H, F_0, E), (P_2, P_3))$ and its stochastic characteristics are given by:

Variable	*Symbol*	*Distribution*	*Type*	*Mean/st. dev.*	*Unit*	*Jump rate*
Yield Stress	F_s	LogN	R	400/35	MPa	–
Flange Breadth	B	LogN	R	$\mu_b/3$	mm	–
Flange Thickness	D	LogN	R	$\mu_d/2$	mm	–
Height of Profile	H	LogN	R	$\mu_h/5$	mm	–
Initial Deflection	F_0	N	R	30/10	mm	–
Youngs Modulus	E	Weibull	R	21000/4200	MPa	–
Dead Weight Load	P_1	N	R	500000/50000	N	–
Variable Load	P_2	Gumbel	S	600000/90000	N	0.1 [1/year]
Variable Load	P_3	Gumbel	S	600000/90000	N	10 [1/year]

The state function in terms of the random vector **X**, the parameter (μ_b, μ_d, μ_h) and auxiliary functions $\mathcal{A}_s, \mathcal{M}_s, \mathcal{M}_i, \mathcal{E}_b, \mathcal{P} = P_1 + P_2 + P_3$ is defined by:

$$G(\mathbf{x}, \mathbf{p}) = F_s - \mathcal{P}\left(\frac{1}{\mathcal{A}_s} + \frac{F_0}{\mathcal{M}_s} \cdot \frac{\mathcal{E}_b}{\mathcal{E}_b - \mathcal{P}}\right).$$

where

$$\mathcal{A}_s = 2BD, \quad \text{(area of section)}$$

$$\mathcal{M}_s = BDH, \quad \text{(modulus of section)}$$

$$\mathcal{M}_i = \frac{1}{2}BDH^2, \quad \text{(moment of inertia)}$$

$$\mathcal{E}_b = \frac{\pi^2 E \mathcal{M}_i}{s^2}, \quad \text{(Euler buckling load)}$$

The admissible failure rate is $\nu^+_{\text{maximum}} = 10^{-4}$/year. No other constraints on cost or design parameters are imposed in this example.

The results for optimization with the **(MCRP-TV)**-algorithm are:

(MCRP-TV)-Algorithm	***Iteration*: 1**	***Iteration*: 2**	***Iteration*: 3**
Auxiliary Failure Rate Constraint	$\mathbf{1.000 \cdot 10^{-4}}$	$\mathbf{0.476 \cdot 10^{-5}}$	$\mathbf{0.217 \cdot 10^{-4}}$
Optimal total cost	**4697.60 CU**	**4732.98 CU**	**4789.65 CU**
Optimal design vector $\mathbf{p}^*$	(200, 20.44, 100)	(200, 20.90, 100)	(200, 21.37, 100)
Iterations in Local Optimization	19	20	18
State Function Calls in Optimization	3371	3557	3171
FORM-Failure rate at optimum	$0.999 \cdot 10^{-4}$	$0.476 \cdot 10^{-4}$	$0.217 \cdot 10^{-4}$
SORM-Failure-rate at optimum	$2.102 \cdot 10^{-4}$	$1.044 \cdot 10^{-4}$	$0.523 \cdot 10^{-4}$

For the parameters selected the FORM result already fulfills the overall failure rate constraint. The corresponding SORM result is too large. It requires two more iterations according to the (**MCRP-TV**)-algorithm in order to meet the overall failure rate constraint, which, then is no more active.

7. SUMMARY AND CONCLUSION

Based on an optimization scheme for structural components developed for time-invariant reliability constraints the theory is generalized to time-variant reliability constraints within the context of FORM. Reliabilities are computed by the outcrossing approach for rectangular wave renewal processes and for Gaussian vector processes, at present for non-intermittent, stationary processes only. The optimization scheme is a one-level scheme which requires second order derivatives of the structural state function. The objective function takes into account capitalized failure cost and systematic reconstruction. Reliability constraints must be given in terms of failure rates instead of failure probabilities for given reference periods. A suitable scheme for an updating algorithm with respect to reliability constraints according to SORM is proposed. Other updating schemes than SORM, for example, by importance sampling can be used as well. It is found, that the numerical effort, even without SORM updates, is relatively high.

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