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DISCUSSION

DISCUSSION TO: HARBITZ, A., AN EFFICIENT SAMPLING METHOD TO PROBABILITY OF FAILURE CALCULATION

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It is, in fact, true that the Monte-Carlo method as a tool for failure probability calculations has been left aside for some time because of the rapid progress of other reliability methods, in particular of the first- and second-order reliability methods (FORM and SORM). The author should be congratulated for taking up the subject again. Yet, it appears that he does not make full use of the concept of importance sampling as an extension of FORM/SORM, which has also been investigated by Shinozuka [1] and Melchers [2] to some extent.

All studies so far recommended the Monte-Carlo method with importance sampling as a substitute for FORM and SORM. But this is a somewhat misleading argument since the domain of importance, i.e. the β-point, must just be found by FORM/SORM algorithms [3]. This is usually the computationally involved part of the analysis. There is, in fact, little point in using Monte-Carlo integration as a substitute for the simple FORM result or even a SORM result which is known to be asymptotically exact. The most efficient way to use that technique is by estimating the error of FORM/SORM results in the sense of Hohenbichler [4]. With the notation of the paper one may write:

$$P_f = P(A) + (P(Q) - P(A)) = P(A)(1 + R)$$

$$= P(A)\left(1 + \int [I_R(\boldsymbol{x})] f(\boldsymbol{x})\, d\boldsymbol{x}\right)$$

$$= P(A)\left(1 + \int [I_R(\boldsymbol{x})] \frac{f(\boldsymbol{x})}{h(\boldsymbol{x})} h(\boldsymbol{x})\, d\boldsymbol{x}\right)$$

where, in generalizing the subject of the paper, $Q = \bigcup \bigcap Q_{ij}$ is the true failure domain, $A = \bigcup \bigcap A_{ij}$ its approximation according to FORM or SORM [3], $f(\boldsymbol{x}) = (2\pi)^{n/2} \exp(-\|\boldsymbol{x}\|^2/2)$ and $h(\boldsymbol{x})$ the importance sampling function. Then, an estimate of R is [5]:

$$R = \frac{1}{N} \sum_{j=1}^{N} [I_R(\boldsymbol{x})] \frac{f(\boldsymbol{x}_j)}{h(\boldsymbol{x}_j)}$$

The function $I_R(\boldsymbol{x})$ takes on the values:

$$I_R(\boldsymbol{x}) = \begin{cases} +1: & (\boldsymbol{X} \in Q) \cap (\boldsymbol{X} \notin A) \\ 0: & \{(\boldsymbol{X} \in Q) \cap (\boldsymbol{X} \in A)\} \cup \{(\boldsymbol{X} \notin Q) \cap (\boldsymbol{X} \notin A)\} \\ -1: & (\boldsymbol{X} \notin Q) \cap (\boldsymbol{X} \in A) \end{cases}$$

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The primary problem is to choose $h(x)$ in an optimal manner. It is essential to locate as many points as possible in the domain where the function I_R takes on non-zero values. This suggests to use more directly the information gained by the β-point(s) instead of "blindly" choosing the directions of sample points. In view of the general failure domain under consideration a component k in the overall series system (union) most first be selected from a discrete probability distribution function with probability masses, e.g. from:

$$p_k = P(K=k) = P(A_k)/P(\cup A_r) \text{ with } A_r = \bigcap_s A_{rs}$$

Following Melchers [2] or the arguments in Ref. [3] that only the vicinity of the β-points contributes substantially to the failure probability and, for the same reason, the suggestion in the paper to sample only in a domain for which $\|x_k^*\| \leqslant \|x\| \leqslant \|x_k^*\| + 3$, with x_k^* the local β-point determined from $\beta_k = \min(\|x\|)$ for $x \in Q_k$, the importance sampling function becomes:

$$h_k(x) = \frac{(2\pi)^{-n/2} \exp(-\|x - x_k^*\|/2)}{\bar{\Gamma}_n(\beta_k^2) - \bar{\Gamma}_n((\beta_k + 3)^2)}$$

$\bar{\Gamma}_n(\cdot)$ is the complementary chi-square distribution. The proposed strategy to sample "important" points, of course, can be improved once more specific information is available for the problem at hand. The special case treated in Ref. [4] is just one such fortunate example. Experience with the computer program PROBAN [6] where the above checking procedure has been implemented showed that a reliable error estimate of SORM probability estimates increases the numerical effort by roughly a factor of ten.

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