

SIMULATION OF A SUPERSONIC STATOR VANE UNDER TWO-PHASE INLET CONDITIONS

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ABSTRACT

Two-phase expansion has previously been considered as a means to improve the performance of ORC systems for waste-heat recovery applications. However, ORC turbomachinery has almost exclusively been designed for operation with either superheated or saturated vapour, whilst during experimental testing turbine operation close the saturation region is generally avoided. However, for high temperature ORC systems, characterised by high molecular weight fluids, high expansion ratios and supersonic flows, it is postulated that a degree of wetness at the turbine inlet could be accommodated. This raises the question of whether an existing ORC turbine could be operated under two-phase inlet conditions, and whether two-phase expansion could be realised with existing ORC turbomachinery.

This work presents an investigation of the performance of a supersonic stator vane, initially designed for superheated expansion, under two-phase inlet conditions. The stator geometry evaluated is installed on the ORC test rig at Lappeenranta University of Technology and was designed for the expansion of MDM from turbine inlet conditions of 265 °C and 8 bar to a Mach number of 2.4. A one-dimensional isentropic analysis is first conducted under the assumption of homogeneous flow to predict the variation in pressure and velocity if the existing area distribution is mapped onto two-phase inlet conditions with the same inlet pressure. The results indicate that for inlet vapour qualities above 0.65 the two-phase region is confined to upstream of the throat. This investigation is followed by a two-dimensional CFD simulation of the stator vane with inlet vapour qualities of 0.85, 0.65 and 0.45. As an initial assumption the two-phase mixture is modelled as a homogeneous binary mixture, and non-equilibrium effects are neglected. The CFD results are consistent with the one-dimensional predictions, indicating that for vapour qualities of 0.85 and 0.65 the two-phase region is confined to upstream of the throat, whilst for a vapour quality of 0.45 the transition to superheated vapour occurs in very close proximity to the throat. In all cases, there is no significant shift in the velocity triangles at stator outlet, indicating similar rotor performance could be expected. Finally, the validity of the equilibrium assumption is discussed.

1 INTRODUCTION

As a means to improve the power output from organic Rankine cycle (ORC) systems for waste-heat recovery applications two-phase expansion has been widely considered. Whilst the thermodynamic benefits of two-phase expansion have been shown through thermodynamic simulations (Smith, 1993; Fischer, 2011), a major stumbling block has been the development of expander technologies suitable for two-phase expansion. Owing to the ability of volumetric expanders to accommodate two-phase inlet conditions, most research has focused on expansion machines such as scroll and screw expanders (Öhman and Lundqvist, 2013; Bianchi *et al.*, 2017). However, a major limitation of volumetric machines is their low built-in volume ratio which limits them to relatively low expansion ratios. This has led to one of the current authors proposing the combination of molecularly complex working fluids and single-stage radial-inflow turbines to achieve two-phase expansion for waste-heat recovery application with heat-source temperatures ranging between 150 and 250 °C (White, 2021). Initial numerical investigations have suggested this system could improve power output by up to 30% compared to single-phase ORC systems and indicated that a transition from two-phase to superheated questions can be obtained within the stator

vane ensuring dry conditions at the inlet to the rotor. Contrary to this, the convention within existing ORC turbine design is to ensure that turbine operation occurs completely within the superheated region, and it is not uncommon for designers to apply a certain amount of superheat at the turbine inlet to ensure this. Moreover, during experimental testing operation close to the saturation region is generally avoided to ensure there is no potential damage to the turbine rotor owing to the potential presence of liquid droplets.

Taken together, these observations lead to two questions: (i) could existing ORC turbine designs operate with inlet conditions that are close to, or within, the saturation dome; and (ii) could existing single-phase ORC turbine designs, designed for moderate heat-source temperature applications, operate with two-phase inlet conditions and thus be considered as potential expansion machines for two-phase ORC systems? The aim of this paper is to investigate these questions for the first time by investigating the performance of an existing single-phase ORC turbine stator vane under two-phase inlet conditions using a combination of one-dimensional isentropic analysis and homogeneous equilibrium CFD simulations.

2 ONE-DIMENSIONAL ISENTROPIC ANALYSIS

2.1 One-dimensional isentropic modelling

A one-dimensional isentropic analysis is conducted to investigate the variation in pressure, density and velocity if the area distribution for an existing ORC stator vane is mapped onto two-phase inlet conditions. Under the homogeneous thermal equilibrium assumption, the behavior of a two-phase mixture is analogous to a single-phase fluid and thus for the isentropic expansion of either a single- or two-phase fluid the pressure, velocity, density and area are related through the one-dimensional steady-state flow equations that consist of a single mass, momentum and energy equation. Considering the expansion process defined in Tab. 1 and shown by the black line in Fig. 1, which refers to the design point for a supersonic stator currently installed on the ORC test rig at Lappeenranta University of Technology (LUT) (Uusitalo et al., 2020), the required area distribution for the design point can be obtained by solving the one-dimensional isentropic flow equations for a range of pressures ranging between $p_{0,in}$ and p_{out} . This area distribution is mapped onto two-phase inlet conditions with the same inlet pressure, but with varying inlet vapour quality, q_0 (see Fig. 1). This is done by first determining the choked mass-flow rate for the two-phase inlet condition, which is found from the density and speed of sound at the throat (*i.e.*, ρ^* and a^*) and the throat area. Once the mass-flow rate is known the flow equations are solved numerically at each point of the known area distribution to determine the corresponding pressure, density and velocity.

Table 1: Design point operating conditions.

Working fluid	MDM	
Stagnation temperature	$T_{0,in}$	265 °C
Stagnation pressure	$p_{0,in}$	8.0 bar
Outlet static pressure	p_{out}	0.4 bar
Outlet Mach number	M_{out}	2.4 -

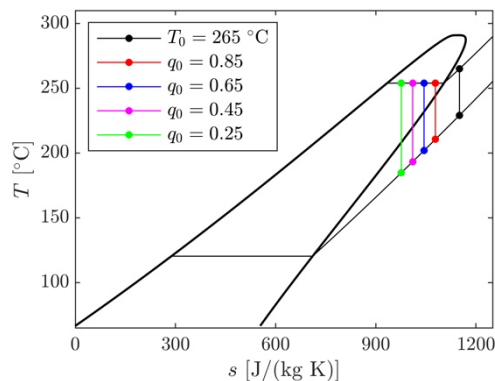


Figure 1: Expansion of MDM from a stagnation pressure of 8 bar to 0.4 bar with varying inlet conditions.

Within the superheated region fluid properties are obtained directly from the multi-parameter equation of state for MDM (Colonna *et al.*, 2008). Within the two-phase region properties such as enthalpy and specific volume can be computed by conserving entropy and using the mass-fraction weighted sum of the saturated properties at a defined pressure. The speed of sound for a homogeneous two-phase mixture undergoing phase change is computed using the model proposed by Brennen (2005), which neglects surface tension and assumes that heat transfer between the two phases occurs instantaneously. This model is consistent with the homogeneous thermal equilibrium assumption as it correctly accounts for the effect

of mass exchange between the two phases. This ensures the sonic line is correctly placed at the throat but does lead to a discontinuity at the transition between the two-phase and superheated regions.

2.2 One-dimensional isentropic results

The results from the isentropic analysis are reported in Fig. 2, which report the variation in velocity, pressure, Mach number and vapour quality as expansion proceeds from left to right. The markers indicate the transition from two-phase to superheated conditions, and the discontinuity in Mach number is consistent with the discontinuity in the speed of sound. In general, as q_0 is reduced the velocity at each nozzle location is reduced, whilst static pressure is increased. However, despite differences in the local pressure, the overall pressure ratio of the nozzle is conserved, although the velocity and hence Mach number at the nozzle outlet reduces with reducing q_0 . Considering the transition from two-phase to superheated conditions, it is observed that expansion from all inlet vapour qualities achieve superheated conditions at the nozzle outlet, and the transition occurs further along the length of the nozzle as vapour quality is reduced. More interestingly, the results reveal that for inlet vapour qualities above approximately 0.65 this transition occurs upstream of the stator throat. This provides an initial suggestion that existing supersonic stator designs could accommodate two-phase inlet qualities with relatively high vapour qualities whilst still operating with superheated conditions within the diverging section of the nozzle.

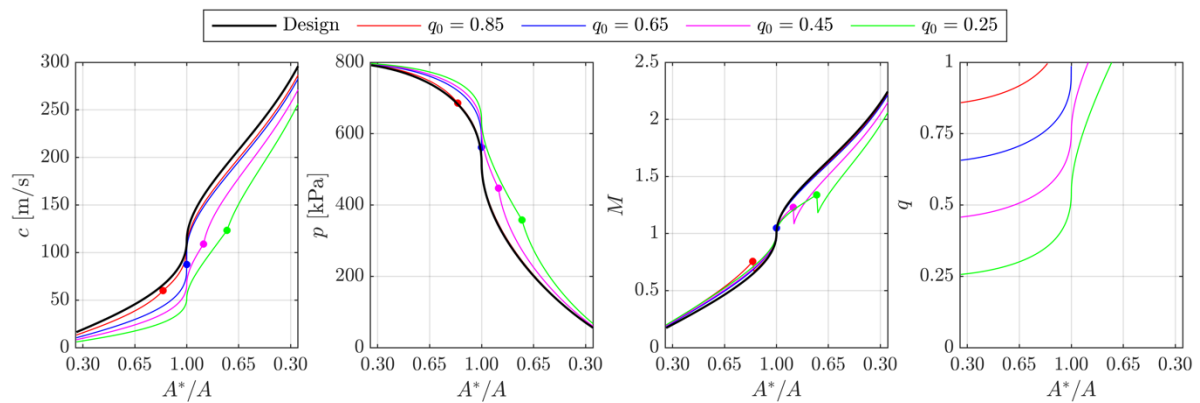


Figure 2: Variation in velocity c , pressure p , Mach number M , and vapour quality q , for an isentropic expansion of MDM from different stagnation vapour qualities (q_0) within a nozzle sized for the design point. The expansion process occurs from left to right with the throat being positioned at $A^*/A = 1$. The circular markers indicate the transition from two-phase to superheated conditions.

To investigate this further, the isentropic analysis has been extended to investigate the vapour quality at the throat q^* following the two-phase expansion from any location within the saturation dome. The results from this analysis are reported in Fig. 3. The left-hand plot reports the results on a T - s diagram with the outer black line indicating the saturation dome and the red line corresponding to the conditions where $q^* = 1$. The right-hand plot correlates to this red line and reports the minimum inlet vapour quality $q_{0,\min}$ that results in a full dry expansion downstream of the throat at different inlet stagnation pressures. These results indicate that as the inlet pressure is increased, lower inlet vapour qualities could be accommodated. For pressures between 4 and 12 bar the minimum vapour quality ranges between 0.74 and 0.57 respectively. These pressures correspond to approximately 30% and 85% of the critical pressure and could be considered the range of inlet pressures for which a subcritical ORC system might operate with MDM.

3 HOMOGENEOUS EQUILIBRIUM CFD SIMULATION

3.1 CFD modelling

Computational-fluid dynamic (CFD) simulations have been setup to expand on the one-dimensional isentropic analysis. The simulations are quasi- two-dimensional viscous simulations conducted using ANSYS CFX (version 18). The modelling strategy is similar to one previously applied to preliminary two-phase stator designs (White, 2021). Consistent with the assumptions made in Section 2, the two-phase fluid modelled by specifying a homogeneous binary mixture constructed from look-up tables generated

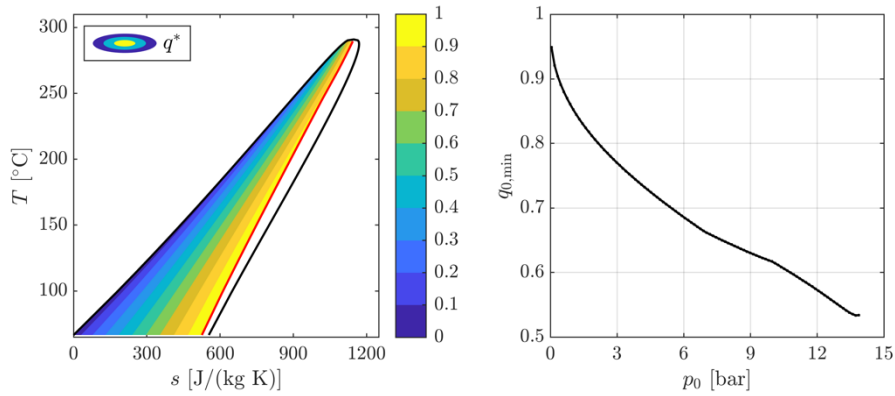


Figure 3: Vapour qualities related to the two-phase expansion of MDM. Left: throat vapour quality (q^*) for isentropic expansion from within the saturation dome; right: minimum stagnation vapour quality ($q_{0,min}$) that results in dry expansion downstream of the throat at different stagnation pressures (p_0).

for the liquid and vapour phases using a multi-parameter equation of state for MDM (Colonna *et al.*, 2008). Each look-up table ranges between 1 kPa and 1200 kPa and 320 K and 600 K with 500 steps for both temperature and pressure. The use of a homogeneous binary mixture allows the use of CFX, which has been used to simulate supersonic single-phase ORC turbines (Harinck *et al.*, 2013; De Servi *et al.*, 2019), to be extended to two-phase operating conditions whilst still solving a single set of flow equations for the two-phase mixture. The inlet boundary condition is defined using the inlet stagnation conditions and inlet vapour quality, the outlet boundary condition is defined using an average static pressure, and a periodic boundary is applied. The $k-\omega$ SST turbulence model is used, and the non-dimensional height of the first element near the wall is set to $y^+ \approx 1$ to avoid the use of wall functions. The mesh is constructed using ANSYS TurboGrid, which doesn't permit meshes with a single element in the spanwise direction. As such, the quasi- two-dimensional domain is defined using two elements in the spanwise direction and symmetry is applied. A grid independence study was conducted at the design point, using the boundary conditions defined in Tab. 1. Three meshes were created with 52,000, 104,000 and 196,000 elements, with 26,000, 52,000 and 98,000 elements in the blade-to-blade region respectively. The grid independence study results are reported in Fig. 4, which shows the mesh with 104,000 elements agrees with the finest mesh to within 0.5%. The grid convergence index (Celik *et al.*, 2008) for the mass-flow rate and stator loss coefficient ζ are 0.30% and 0.02% respectively. The Mach number contours are reported in Fig. 5.

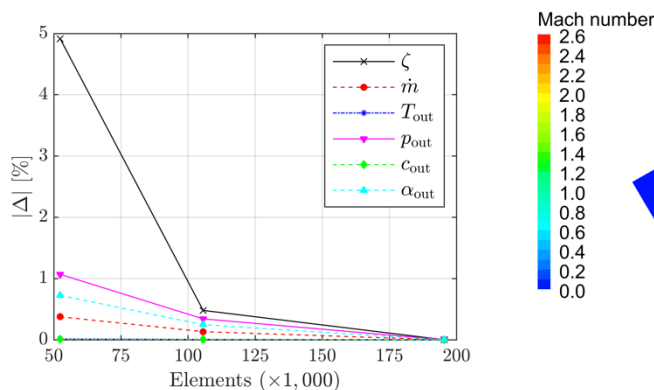


Figure 4: Grid independence study results.

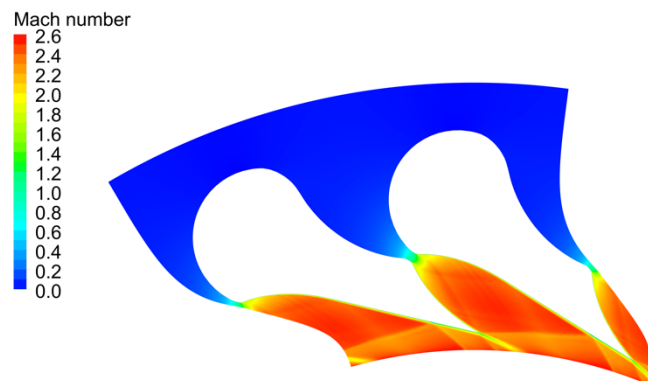


Figure 5: Design point Mach number contours.

3.2 CFD results

The CFD simulations were completed for the LUT stator vane across the same pressure defined in Tab. 1, but with inlet vapour qualities of 0.85, 0.65 and 0.45. The performance of the stator, assessed by the averaged properties at the outlet of the domain, is reported in Tab. 2. The rotor-inlet Mach triangles are also constructed using the averaged velocities, assuming the design rotor rotational speed, and these are reported in Fig. 6. Overall, these results are consistent with the one-dimensional analysis, with velocity and Mach number reducing as the inlet vapour quality is reduced. The results also reveal that there is not

a significant variation in the rotor-inlet flow angle, and hence in the Mach triangle; although the deviation does increase as the vapour quality is reduced, which could be attributed to a shift in the phase boundary, as discussed later. Ultimately, the consistency in the Mach triangles suggests that rotor performance may not be significantly affected by the introduction of two-phase inlet conditions.

Table 2: Averaged properties at stator outlet obtained from the CFD simulation.

	Design	q_0			
		0.85	0.65	0.45	
T_{out}	499.1	480.6	471.9	463.3	K
p_{out}	26.96	26.96	26.76	27.19	kPa
c_{out}	332.4	323.6	318.5	312.1	m/s
α_{out}	63.77	63.55	63.86	62.53	°
M_{out}	2.51	2.50	2.48	2.45	-
\dot{m}^*	0.212	0.216	0.213	0.227	kg/s

* linear scaling for full three-dimensional stator

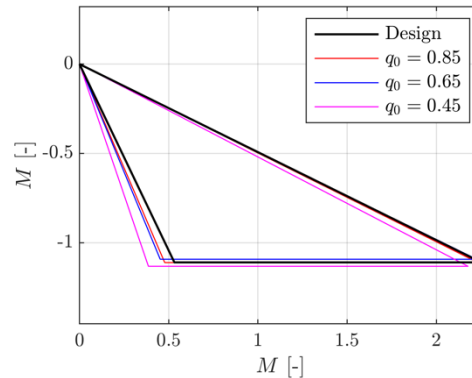


Figure 6: Stator outlet Mach number triangles based on averaged velocities.

The streamwise variations in the vapour quality, degree of superheat and normalised entropy are reported in Fig. 7. In all cases, full vapourisation is achieved within the stator vane, such that the fluid leaves the stator in a superheated state. Unsurprisingly, the lower the inlet vapour quality, the longer the vapourisation process takes and the lower the outlet superheat. The plots showing superheat and entropy confirm that the homogeneous binary mixture consistently models the fluid behaviour during the transition between the two-phase and superheated regions. The location of the transition from two-phase to superheated flows is examined in Fig. 8, which reports the vapour quality contours within the converging-diverging portion of the stator. At an inlet vapour quality of 0.85 the transition occurs upstream of the throat, at a vapour quality of 0.65 the transition occurs in close proximity to the throat, whilst at an inlet vapour quality of 0.45 the transition occurs during the initial expansion section of the diverging nozzle.

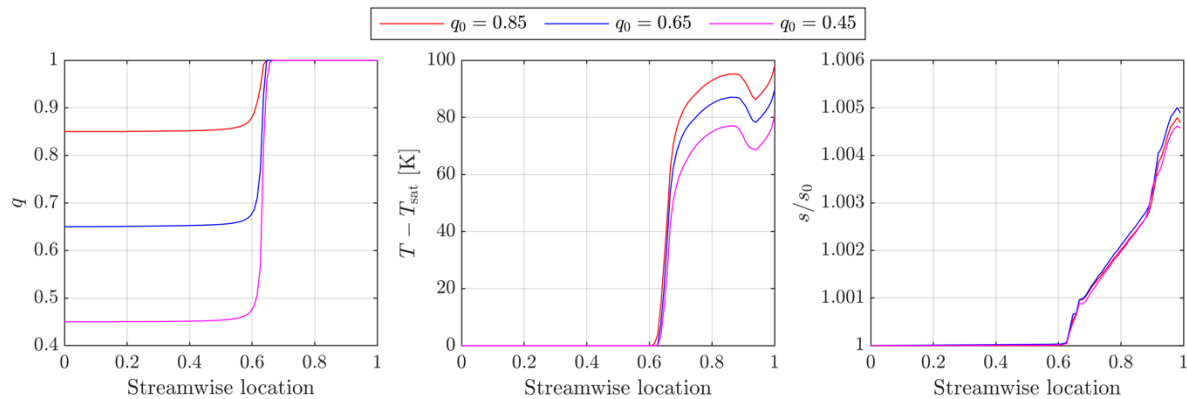


Figure 7: Variation in circumferentially averaged vapour quality (left), degree of superheat (centre) and normalised entropy (right) with streamwise location for different inlet vapour qualities.

At this stage it is worth commenting on the potential effect the transition point will have on nozzle performance. Considering the design case, alongside the $q_0 = 0.85$ and $q_0 = 0.65$ cases, the conditions at the throat are dry which means the expansion process within the diverging section is fully superheated and the amount of flow turning can be assessed using the Prandtl-Meyer function. Since this is an important parameter affecting supersonic nozzle design, and that fixing the amount of flow turning at the design Mach number of 2.4 and considering expansion from $q_0 = 0.85$ and $q_0 = 0.65$ leads to small variations in the Mach number (*i.e.*, 2.36 and 2.34 respectively), it can be rationalised that similar nozzle performance could be expected for these cases. For $q_0 = 0.45$, where the transition occurs downstream of the throat, two-phase inlet conditions could have a more significant influence on stator performance.

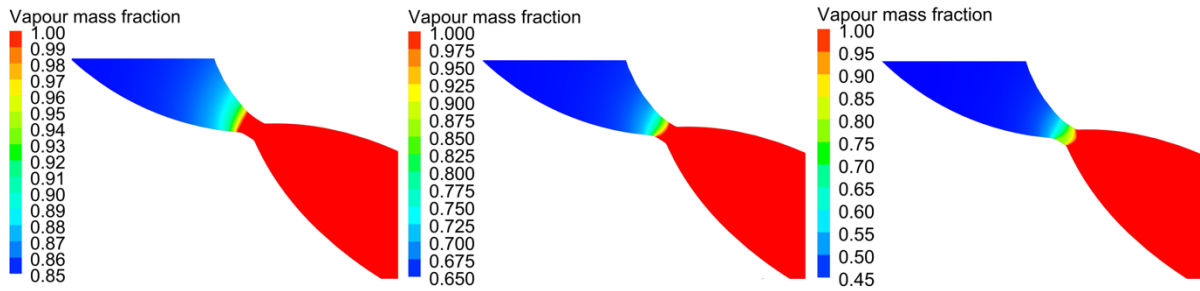


Figure 8: Vapour quality contours within the converging-diverging portion of the stator vane for different inlet vapour qualities: left: $q_0 = 0.85$; centre: $q_0 = 0.65$; right: $q_0 = 0.45$.

The Mach number contours for the three two-phase cases are reported in Fig. 9. The circumferential variation in the properties at the domain outlet are reported in Fig. 10 where $\theta = 0^\circ$ is defined at the bottom-left corner of the contour plots and an increase in θ is consistent with rotation in the anti-clockwise direction. The findings from Figs. 9 and 10 are consistent with the previous discussion, with the flow field for the $q_0 = 0.85$ and $q_0 = 0.65$ cases being very similar the design point, albeit with a slight reduction in the Mach number. This supports the suggestion that the rotor inlet flow conditions are unaltered by the introduction of two-phase inlet conditions with vapour qualities above 0.65 and thus similar performance for the whole turbine stage could be expected. For the $q_0 = 0.45$ case a more significant variation in the flow field is observed, consistent off-design operation. Firstly, there appears to be a slight shift in the shock at $\theta \approx 5^\circ$ that originates from the reflection of the trailing-edge shock on the suction side of the stator vane. There is also a strengthening of the weak shock that occurs at $\theta \approx 13^\circ$ which appears to arise due the complex flow field resulting from the interaction of an oblique shock pattern within the diverging portion of the nozzle and the trailing-edge shock. This causes a reduction in the flow angle and an increase in static pressure at that location. Having said this, this deviation is relatively small compared to the overall variations observed in the circumferential direction. Further investigations considering the whole turbine stage would be required to confirm the extent to which this may affect turbine operation.

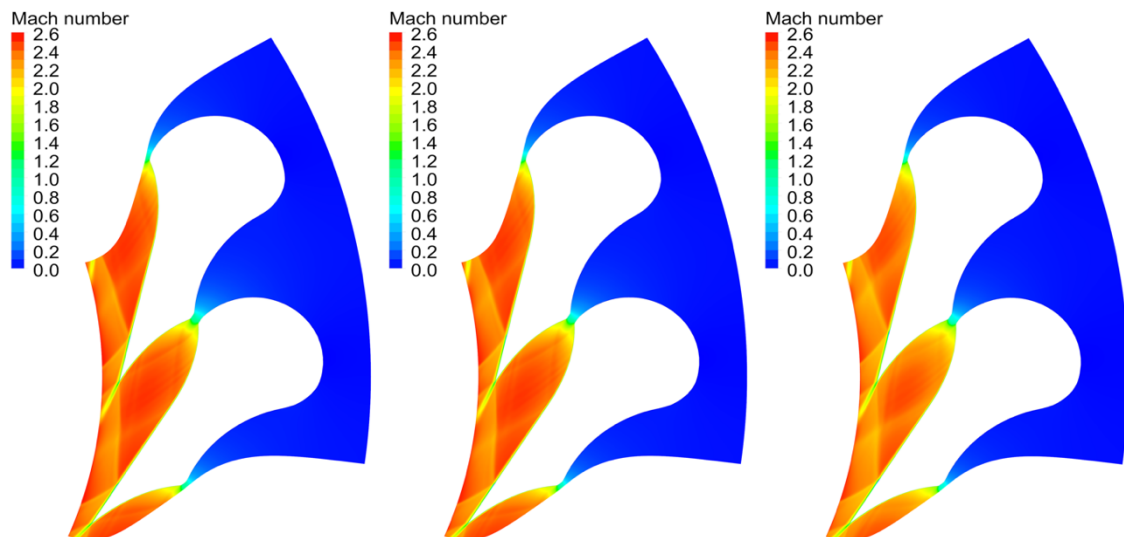


Figure 9: Mach number contours for different inlet vapour qualities: left: $q_0 = 0.85$; centre: $q_0 = 0.65$; right: $q_0 = 0.45$. Speed of sound is defined using the ANSYS built-in function which calculates the speed of sound based on the volume-weighted sum of the acoustic impedances of the saturated components.

4 COMMENTS ON NON-EQUILIBRIUM EFFECTS

The analysis presented has assumed thermal and mechanical equilibrium between the liquid and vapour phases, which corresponds to both phases being at the same velocity ($c_L = c_V$), pressure ($p_L = p_V = p_{\text{sat}}$) and temperature ($T_L = T_V = T_{\text{sat}}$). This assumption is valid if the two phases are well mixed, the size of the disperse particles is sufficiently small, and the effects of surface tension are neglected. Moreover, the

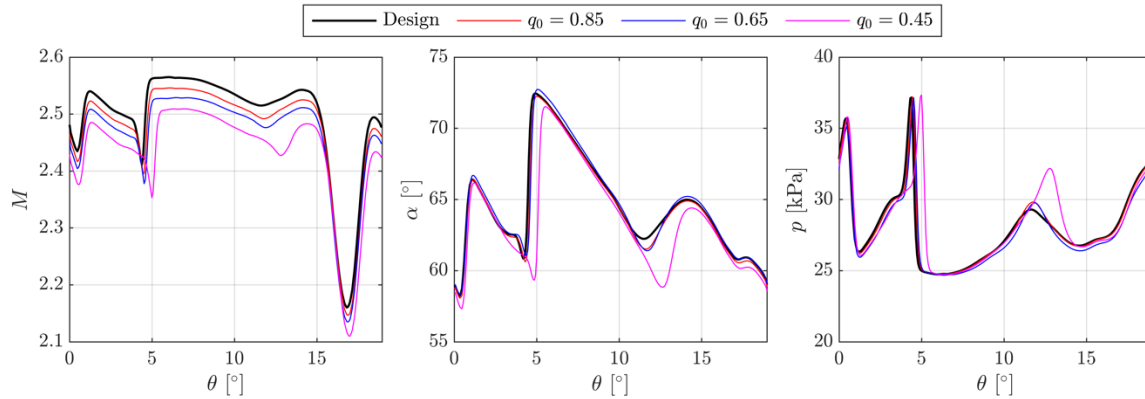


Figure 10: Circumferential variation in Mach number (left), flow angle (centre) and static pressure (right) at the stator outlet predicted from the CFD simulations.

thermal equilibrium assumption implies that heat transfer between the two phases occurs instantaneously. At high pressure and in close proximity to the critical point where surface tension effects are negligible and the difference in the density of the two phases is small, these assumptions are likely to be valid, as rationalised by for organic fluids (Nannan *et al.*, 2013) and for supercritical CO₂ compressors (Romei and Persico, 2021). For the moderate pressures and two-phase volumetric expansion ratios considered in this work (which are approximately 1.4, 2.3 and 3.8 for expansion to saturated vapour from inlet qualities of 0.85, 0.65 and 0.45) the validity of this assumption requires further investigation. It is also worth noting that the high speed of the flow will reduce the residence time of the droplets which will reduce the time available for the two-phases to exchange heat. Combined, these effects will likely lead to the transition from two-phase to dry conditions occurring further downstream than predicted by the homogeneous equilibrium models. As a first step, separate investigations into thermal and mechanical non-equilibrium effects could be conducted through the introduction of the homogeneous relaxation model (Downar-Zapolski *et al.*, 1996) or a slip model respectively. This should later be extended to consider a full non-equilibrium model that considers separate mass, energy and momentum equations for the two-phases with suitable correlations to determine mass, energy and momentum exchange terms.

5 CONCLUSIONS

This paper has explored the performance of a supersonic radial turbine stator vane, designed for single-phase expansion, under two-phase operating conditions. This has been conducted with a view to establishing whether existing designs could accommodate two-phase inlet conditions and whether this could open up the avenue to realising the potential thermodynamic benefits of two-phase expansion.

The results from a one-dimensional isentropic analysis assuming homogeneous equilibrium between the liquid and vapour phases has indicated that, for the expansion of MDM from an inlet stagnation pressure of 8 bar, inlet vapour qualities (q_0) above 0.65 lead to dry conditions at the stator throat. Inlet stagnation pressures of 4 and 12 bar correspond to minimum inlet vapour qualities of 0.74 and 0.57 respectively to achieve the same condition. These initial results are followed by CFD simulations of the stator vane at the design inlet stagnation pressure (*i.e.*, 8 bar) under the same homogeneous equilibrium assumption. The CFD results support the finding from the one-dimensional isentropic analysis, indicating that stator performance is unaffected by two-phase inlet conditions providing that the transition from two-phase to superheated conditions occurs upstream of the throat (*i.e.*, $q_0 \geq 0.65$). Subsequently, the flow conditions at the rotor inlet are unaffected, which indicates that similar performance of the full turbine stage could be expected. Reducing the vapour quality to 0.45 moves the wet-to-dry transition downstream of the throat, albeit still in close proximity to the throat, and introduces a slight shift in the stator outlet conditions.

These results provide the first indication that two-phase expansion could be possible using existing supersonic ORC turbines. Furthermore, they also provide ORC practitioners some confidence that admitting a small amount of liquid into a supersonic turbine during operation may not necessarily risk

damage to the turbine and may not have a significant effect on turbine performance. Nonetheless, these findings need further investigations to consider the potential role of non-equilibrium effects, which will likely lead to the transition from two-phase to superheated conditions occurring further downstream than predicted by homogeneous equilibrium models.

NOMENCLATURE

a	speed of sound	(m/s)	ρ	density	(kg/m ³)
A	area	(m ²)	ζ	stator loss coefficient	(-)
c	velocity	(m/s)			
M	Mach number	(-)	Subscript		
p	pressure	(Pa)	0	stagnation conditions	
q	vapour quality	(-)	in	inlet	
s	specific entropy	(J/(kg K))	min	minimum	
T	temperature	(°C)	out	outlet	
α	flow angle	(°)	*	critical throat properties	

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