3D Acoustic-Elastic Simulations for Tsunami-Genesis

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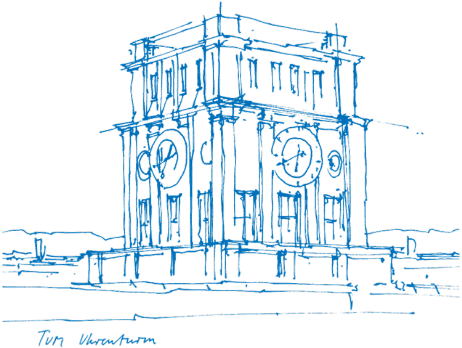

Acknowledgments

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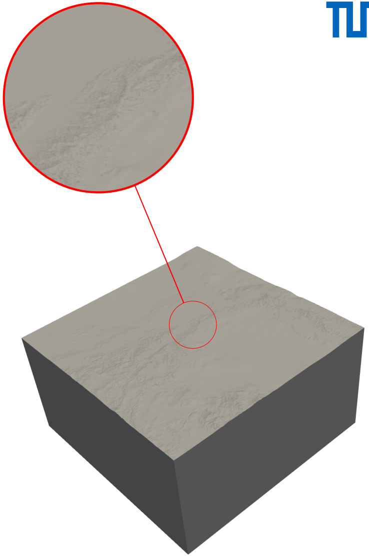
SeisSol

What

- Seismic wave propagation in (an)**isotropic elastic** material
- **Acoustic** wave propagation
- Viscoelastic wave propagation
- **Off-fault plasticity**
- **Dynamic earthquake rupture**

How

- Numerics: ADER-DG
- Unstructured tetrahedral meshes with local time-stepping
- Optimized Hybrid MPI + OpenMP Parallelization



Palu mesh, 22 mio. elements

Available (**open-source**) at `https://github.com/SeisSol/SeisSol/`.

The ADER-DG Approach[1]

Solve linear hyperbolic equations of the form

$$\frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{Q}\frac{\partial \boldsymbol{q}}{\partial x} + \boldsymbol{B}\frac{\partial \boldsymbol{q}}{\partial y} + \boldsymbol{C}\frac{\partial \boldsymbol{q}}{\partial z} = 0 \tag{1}$$

with $\boldsymbol{q}$ vector of variables, $\boldsymbol{x} = (x, y, z)$ position, t time, $\boldsymbol{A}(\boldsymbol{x})$, $\boldsymbol{B}(\boldsymbol{x})$, $\boldsymbol{C}(\boldsymbol{x})$ flux matrices.
Discontinuous Galerkin (DG) divides domain into disjoint elements, approximates solutions by piecewise-polynomials.
Elements are connected by solving the Riemann problem.
ADER-Approach uses element-local Taylor expansion for time integration instead of Runge-Kutta procedures.
Advantages: One-step scheme, arbitrary order in time and space

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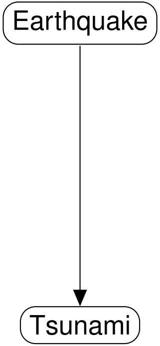
[1] V. A. Titarev and E. F. Toro. “ADER: Arbitrary High Order Godunov Approach”. In: *Journal of Scientific Computing* 17.1 (Dec. 2002).

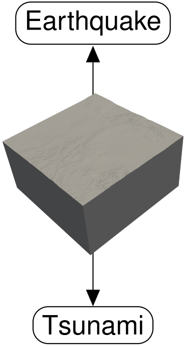
Earthquake-Tsunami Coupling Workflows

Two-step Coupling

- Separate simulations
- 3D Earthquake simulation produces output
- Used to drive **2D** tsunami simulation



Fully Coupled

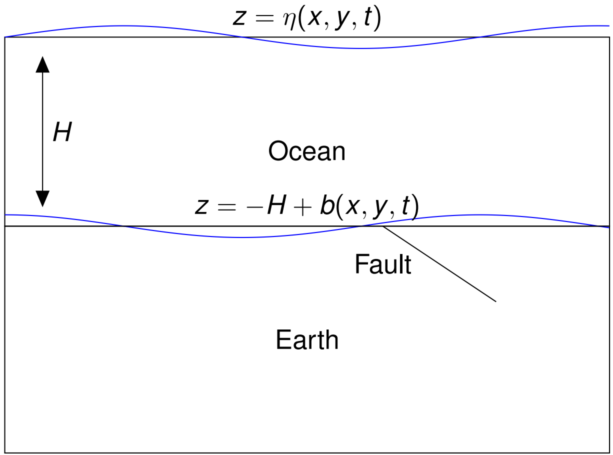


- One **3D** simulation

	Static Initial Condition	**Time-dependent Seafloor Velocity**	**Time-dependent Sea Surface Velocity**	**Fully-Coupled**
Model	1. From earthquake model: record final seafloor displacement. 2. In tsunami model: Use as initial condition	1. From earthquake model: record time-dependent **sea floor** velocity 2. In tsunami model: use as forcing term	1. From earthquake model: record time-dependent **sea surface** velocity 2. In tsunami model: use as forcing term	**Simultaneously** solve earthquake dynamics and ocean response (including gravity)
Model valid when	• In shallow water limit • Acoustic waves are not generated • Tsunami waves do not propagate over source duration	• In shallow water limit • Acoustic waves are not generated	• In shallow water limit (if non-dispersive shallow water solver)	Valid in all cases

Modified from L. S. Abrahams et al. “Comparison of techniques for coupled earthquake and tsunami modeling”. In: *AGU Fall Meeting Abstracts* (2020)

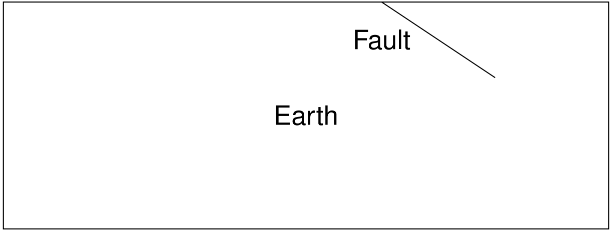
Two-Way Elastic-Acoustic Coupling



With:

- $\eta(x, y, t)$ sea surface height
- H height of the ocean
- $b(x, y, t)$ time dependent bathymetry
- Figures in 2D for illustration, all simulations are **3D**

Earth Model



$$\frac{\partial \sigma_{ij}}{\partial t} - \lambda \delta_{ij} u_k \frac{\partial}{\partial x_k} - \mu \left(\frac{\partial}{\partial x_j} u_i + \frac{\partial}{\partial x_i} u_j \right) = 0, \qquad \rho \frac{\partial}{\partial t} u_i - \frac{\partial}{\partial x_j} \sigma_{ij} = 0 \tag{2}$$

δ_{ij} Kronecker delta, summation implied

- Isotropic elastic medium
- Velocity-stress formulation
- u, v, w velocities
- $\boldsymbol{\sigma}$ stress tensor
- ρ density, (μ, λ) Lame parameters
- Dynamic rupture earthquake source (e.g. rate & state friction)

Ocean Model[2]

Modeled as **linear acoustic** medium, $\boldsymbol{q} = (u, v, w, p)$
Treated as special case of elastic wave equation with $\mu = 0$.
Pressure p sum of background pressure p_0 (in **hydrostatic equilibrium**) and perturbation p'.

$$\begin{aligned} p &= p_0 + p'(x, y, z) \\ p_0 &= p_a + \rho g(-z) \end{aligned} \tag{3}$$

with atmospheric pressure p_a and $g = 9.81\,\mathrm{m/s^2}$.
Pressure at some point is:

$$p(x, y, z, t) = p_a + \rho g(-z) + p'(x, y, z, t) - \rho g u_z(x, y, z, t) \tag{4}$$

with z-displacements u_z.

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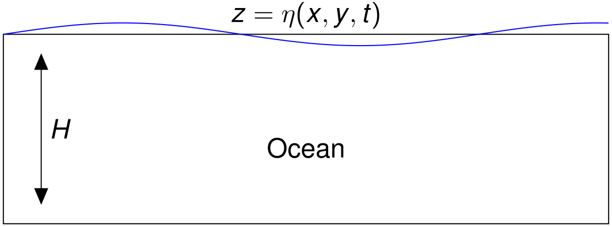
[2] G. C. Lotto and E. M. Dunham. “High-order finite difference modeling of tsunami generation in a compressible ocean from offshore earthquakes”. In: *Computational Geosciences* 19.2 (2015).

Ocean Model: Free Surface



Physical free surface boundary condition at sea surface height η:

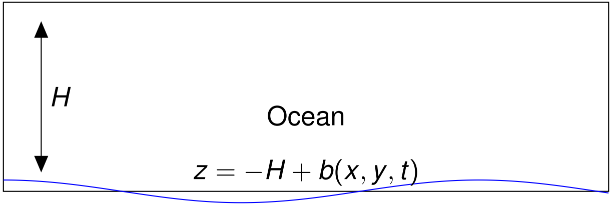
$$p(x, y, \eta) = 0 \tag{5}$$

Too expensive, instead use **linearization** and **hydrostatic assumption**:

$$\begin{gathered} p(x, y, z = 0) = -\rho g \eta(x, y) \\ \frac{\partial \eta}{\partial t} = u \end{gathered} \tag{6}$$

Important to use *u* at boundary (solution of Riemann problem), otherwise unstable!

Ocean Model: Sea floor



For sloping $b(x, t)$ we have:

$$b(x, y, t) \approx u_z(x, y, t) - \frac{\mathrm{d}H}{\mathrm{d}x} u_x(x, y, t) - \frac{\mathrm{d}H}{\mathrm{d}y} u_y(x, y, t), \tag{7}$$

Our method contains typical Tanioka & Satake[3] correction!

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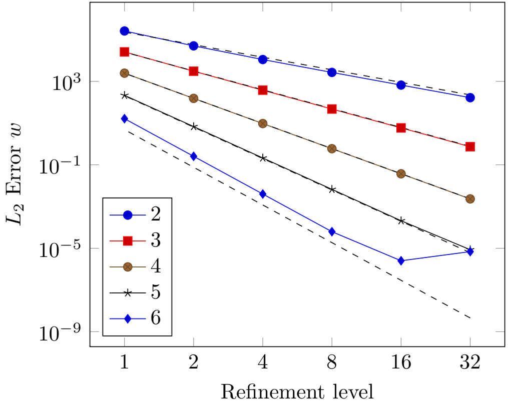
[3] Y. Tanioka and K. Satake. “Tsunami generation by horizontal displacement of ocean bottom”. In: *Geophysical research letters* 23.8 (1996).

Convergence[4]



Model verification scenario:

- Acoustic only model
- Simple water tank
- Corresponds to acoustic wave mode in ocean
- Boundary conditions: Modified free surface top, bottom rigid, other free surface
- Simulation time 2.562 s, one wave period
- Number of elements from 600 to ca. 19.7 million elements

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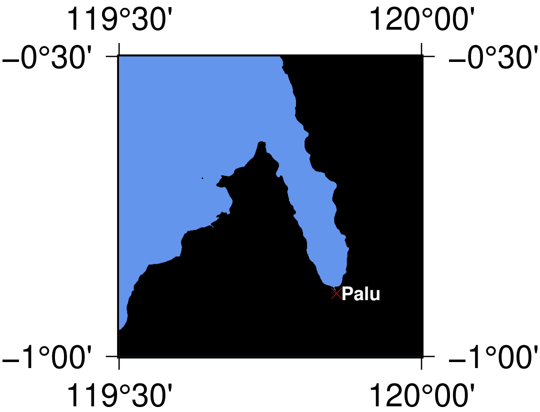
[4] L. S. Abrahams et al. “Verification of a 3D Fully-Coupled Earthquake and Tsunami Model”. In: *AGU Fall Meeting Abstracts* 43 (Dec. 2019).

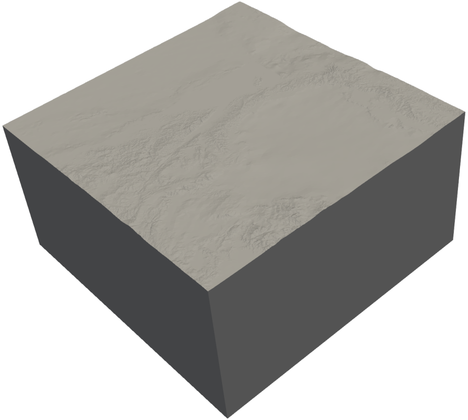
Example: Palu, Sulawesi September 2018

- $M_w 7.5$ strike-slip earthquake
- Propagation at supershear speed crossing narrow Palu bay
- Followed by unexpectedly localized tsunami
- Complicated geometry (bath-tub like bay, very shallow water)



Our setup

- Added water layer to existing earthquake model[5].
- Fully coupled model (including plasticity, dynamic earthquake rupture)
- Resulting mesh roughly **22 million elements**, used order 6
- Simulation time: 150 s
- Took 6h28min on **256 nodes** of SuperMUC-NG (2 Intel Skylake Xeon Platinium 8174 CPUs)



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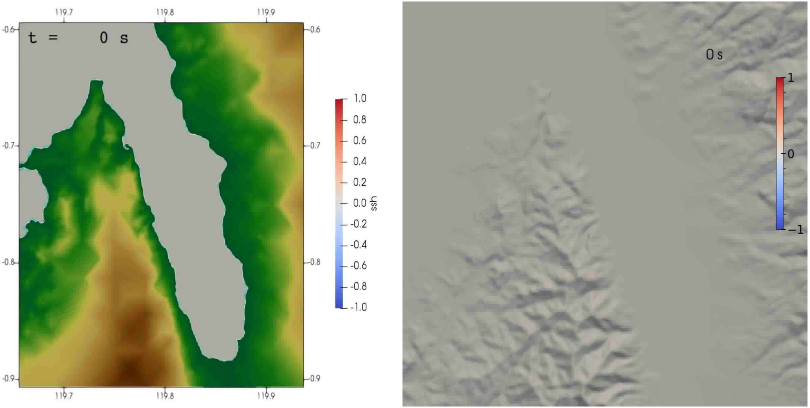
[5] T. Ulrich et al. “Coupled, Physics-based Modelling Reveals Earthquake Displacements are Critical to the 2018 Palu, Sulawesi Tsunami”. In: *Pure and Applied Geophysics* (2019)

Results



Left: Two-step coupling, **right: fully-coupled**

Conclusion

- Fully coupled elastic-acoustic simulations capture more effects than typical two-step strategies
- Linearization of free surface boundary conditions efficient way of tracking sea surface height
- Our method achieves high-order convergence
- Results for Palu scenario are very promising
- Further work on numerics, performance and scenarios