Published for SISSA by Springer

Received: *April 29, 2020*
Accepted: *June 29, 2020*
Published: *July 30, 2020*

Matching scalar leptoquarks to the SMEFT at one loop

Valerio Gherardi,[a,b] **David Marzocca**[b] **and Elena Venturini**[c]

[a] *SISSA, Via Bonomea 265, Trieste 34136, Italy*
[b] *INFN, Sezione di Trieste, SISSA, Via Bonomea 265, Trieste 34136, Italy*
[c] *Technische Universität München, Physik-Department, James-Franck-Straße 1, Garching 85748, Germany*

E-mail: vgherard@sissa.it, david.marzocca@ts.infn.it, elena.venturini@tum.de

Abstract: In this paper we present the complete one-loop matching conditions, up to dimension-six operators of the Standard Model effective field theory, resulting by integrating out the two scalar leptoquarks $S_1 \sim (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$ and $S_3 \sim (\bar{\mathbf{3}}, \mathbf{3})_{\frac{1}{3}}$. This allows a phenomenological study of low-energy constraints on this model at one-loop accuracy, which will be the focus of a subsequent work. Furthermore, it provides a rich comparison for functional and computational methods for one-loop matching, that are being developed. As a corollary result, we derive a complete set of dimension-six operators independent under integration by parts, but not under equations of motions, called *Green's basis*, as well as the complete reduction formulae from this set to the Warsaw basis.

Keywords: Beyond Standard Model, Heavy Quark Physics

ArXiv ePrint: 2003.12525

https://doi.org/10.1007/JHEP07(2020)225

JHEP07(2020)225

Contents

1 Introduction **1**

2 The $S_1 + S_3$ model **2**

2.1 Tree-level SMEFT matching conditions 4

3 Complete one-loop SMEFT matching conditions **4**

3.1 Example 6

3.2 One-loop matching conditions in the Warsaw basis 8

3.2.1 Renormalizable terms 9

3.2.2 Purely bosonic 10

3.2.3 Two-fermion operators 11

3.2.4 Four-fermion operators 14

4 Conclusions **19**

A A Green's basis for the SMEFT **20**

B Reduction of Green's basis to Warsaw basis **24**

B.1 Renormalizable operators 24

B.2 Purely bosonic operators 25

B.3 Two-fermion operators 25

B.4 Four-fermion operators 30

C One-loop matching conditions in the Green's basis **36**

C.1 Renormalizable operators 36

C.2 Purely bosonic operators 36

C.3 Two-fermion operators 38

C.4 Four-fermion operators 42

1 Introduction

The Standard Model (SM) of Particle Physics still represents the best description of high-energy phenomena at our disposal. Nevertheless, several experimental and theoretical shortcomings require the presence of some New Physics (NP). The lack of a NP discovery at the LHC and the SM's impressive phenomenological success could suggest the existence of a large separation between the Electroweak (EW) and NP scales. If this is the case, the SM should really be understood as the leading (renormalizable) approximation of a low energy Effective Field Theory (EFT), known as SMEFT [1–3].

Considering the SM as an EFT is not only conceptually, but also practically advantageous, for it allows to study a whole class of SM extensions in a model independent way. In fact, in the SMEFT framework, the effects of heavy NP are fully encoded in the Wilson Coefficients (WCs) of non-renormalizable operators, in terms of which low-energy observables can be computed without any reference to the specific ultraviolet (UV) model. The WCs for a given concrete UV SM extension can then be obtained by *matching*.

In this work we present the complete one-loop matching conditions, up to dimension-six SMEFT operators, resulting by integrating out the two scalar leptoquarks (LQ) S_1 and S_3 [4] with all admissible baryon and lepton number conserving couplings. From the phenomenological point of view, such an effort is motivated by the recent interest received by the model in the context of the deviations from the SM observed in B-meson decays, for which it could provide a combined explanation [5–12]. The complete one-loop matching allows a thourough study of the model's phenomenology, which was indeed one of our initial goals, and will be reported in a separate contribution [13].

Aside from the phenomenological interest, this work represents one of the very few available examples of complete one-loop matching to the SMEFT. In [14, 15] the one-loop matching for bosonic SMEFT operators from integrating out sfermions in the MSSM is derived, refs. [16, 17] perform the complete one-loop matching for a singlet scalar (see also [18]), and [19] considers the SM with an additional light sterile neutrino and heavy fermions and a scalar singlet. The model considered here, with two coloured and weakly-charged states coupled to all SM particles with non-trivial flavour structures, represents a very rich example of such a matching. While functional [20–28] and automated [29, 30] methods for the task are currently under development, diagrammatic calculation is still the state of the art in this subject, and will represent an important cross-check when these more sophisticated methods will be available.

The matching conditions are obtained by equating EFT and UV theory one-light-particle irreducible (1LPI) off-shell Green's functions at the matching scale. This operation produces a set of operators which are independent under integration by parts (IBP), but possibly redundant under the SM renormalizable equations of motion (EOMs). A complete set of such operators has been called *Green's* basis in [16], and must then be suitably reduced to an operator basis for S-matrix elements by applying the SM EOMs (or field redefinitions). As a byproduct of our work, we identify a complete Green's basis of dimension-six SMEFT operators by extending the *Warsaw* basis [3], and obtain the fully general reduction equations expressing Warsaw basis WCs in terms of Green's basis ones.

The paper is organized as follows: in section 2 we introduce the $S_1 + S_3$ model; in section 3 we give the complete one-loop matching conditions in the Warsaw basis, which is the main result of this paper. We conclude in section 4. Several results are contained in the appendices: in appendix A we discuss in full generality the Green's basis for the SMEFT; in appendix B we provide the reduction equations from the Green's to the Warsaw basis, and in appendix C we give the complete one-loop matching condition for the leptoquark theory in the Green's basis.

In the supplementary material we provide the complete one-loop matching in the Green's basis, the general reduction equations from the Green's to the Warsaw basis, as well as usage examples.

2 The $S_1 + S_3$ model

The UV model under consideration is defined by the SM gauge group and field content, with the addition of two colored scalar *leptoquarks*

$$S_1 \sim (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \quad \text{and} \quad S_3 \sim (\bar{\mathbf{3}}, \mathbf{3})_{\frac{1}{3}}\,, \tag{2.1}$$

where in parenthesis we indicate the representation under $(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$. The combination of these two scalars has been considered in the literature as a possible simultaneous explanation of charged and neutral current B-anomalies [6–12]. For such a purpose, both leptoquarks need to have TeV scale masses and, consequently, negligibly small baryon number violating couplings. Enforcing baryon number conservation, the part of the Lagrangian involving $S_{1,3}$ is:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LQ}} = & |D_\mu S_1|^2 + |D_\mu S_3|^2 - M_1^2 |S_1|^2 - M_3^2 |S_3|^2 + \\
& + \left((\lambda^{1L})_{i\alpha} \bar{q}_i^c \epsilon \ell_\alpha + (\lambda^{1R})_{i\alpha} \bar{u}_i^c e_\alpha\right) S_1 + (\lambda^{3L})_{i\alpha} \bar{q}_i^c \epsilon \sigma^I \ell_\alpha S_3^I + \text{h.c.} + \\
& - \lambda_{H1} |H|^2 |S_1|^2 - \lambda_{H3} |H|^2 |S_3^I|^2 - \left(\lambda_{H13} (H^\dagger \sigma^I H) S_3^{I\dagger} S_1 + \text{h.c.}\right) + \\
& - \lambda_{\epsilon H3} i \epsilon^{IJK} (H^\dagger \sigma^I H) S_3^{J\dagger} S_3^K - V_S(S_1, S_3),
\end{aligned}
\tag{2.2}
$$

where $\epsilon = i\sigma_2$, $\lambda_{H1}, \lambda_{H3}, \lambda_{\epsilon H3} \in \mathbb{R}$, $(\lambda^{1L})_{i\alpha}, (\lambda^{1R})_{i\alpha}, (\lambda^{3L})_{i\alpha}, \lambda_{H13} \in \mathbb{C}$, and the LQ self-interactions are described by

$$
\begin{aligned}
V_S(S_1, S_3) = & \frac{c_1}{2} (S_1^\dagger S_1)^2 + \\
& + c_{13}^{(1)} (S_1^\dagger S_1)(S_3^\dagger S_3) + c_{13}^{(8)} (S_1^\dagger T^A S_1)(S_3^\dagger T^A S_3) + \\
& + \frac{c_3^{(1)}}{2} (S_3^\dagger S_3)(S_3^\dagger S_3) + \frac{c_3^{(3)}}{2} (S_3^{I\dagger} \epsilon^{IJK} S_3^J)(S_3^{L\dagger} \epsilon^{LMK} S_3^M) + \\
& + \frac{c_3^{(5)}}{2} \left[\frac{(S_3^{I\dagger} S_3^J)(S_3^{I\dagger} S_3^J) + (S_3^{I\dagger} S_3^J)(S_3^{J\dagger} S_3^I)}{2} - \frac{1}{3} (S_3^\dagger S_3)(S_3^\dagger S_3) \right].
\end{aligned}
\tag{2.3}
$$

We denote SM quark and lepton fields by q_i, u_i, d_i, ℓ_α, and e_α. We adopt latin letters (i, j, k, …) for quark flavor indices and greek letters (α, β, γ, …) for lepton flavor indices. We work in the down-quark and charged-lepton mass eigenstate basis, where

$$
q_i = \begin{pmatrix} V_{ji}^* u_L^j \\ d_L^i \end{pmatrix}, \qquad \ell_\alpha = \begin{pmatrix} \nu_L^\alpha \\ e_L^\alpha \end{pmatrix}, \tag{2.4}
$$

and V is the CKM matrix. The Higgs field is denoted by H and its hypercharge is normalized to $Y_H = \frac{1}{2}$. The covariant derivative of a generic field $\Phi \sim (\rho_3^\Phi, \rho_2^\Phi)_{Y_\Phi}$ is defined by

$$
D_\mu \Phi = \left(\partial_\mu - i g' Y_\Phi B_\mu - i g (t_2^\Phi)^I W_\mu^I - i g_s (t_3^\Phi)^A G_\mu^A\right) \Phi, \tag{2.5}
$$

and the corresponding field strengths read

$$
\begin{aligned}
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g \epsilon^{IJK} W_\mu^J W_\nu^K, \\
G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s f^{ABC} G_\mu^B G_\nu^C.
\end{aligned}
\tag{2.6}
$$

The SM Yukawa lagrangian is defined by

$$
\mathcal{L}_{\mathrm{Yuk}} = -(y_E)_{\alpha\beta} \bar{\ell}_\alpha e_\beta H - (y_U)_{ij} \bar{q}_i u_j \widetilde{H} - (y_D)_{ij} \bar{q}_i d_j H + \text{h.c.}, \tag{2.7}
$$

where $\widetilde{H} = i\sigma_2 H^*$, and the Higgs potential reads

$$V_H = -m^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2. \tag{2.8}$$

Finally, for future convenience, we define the bi-lateral derivatives:

$$\begin{aligned} H^\dagger \overleftrightarrow{D}_\mu H &= H^\dagger (D_\mu H) - (D_\mu H)^\dagger H, \\ H^\dagger \overleftrightarrow{D}^I_\mu H &= H^\dagger \sigma^I (D_\mu H) - (D_\mu H)^\dagger \sigma^I H. \end{aligned} \tag{2.9}$$

2.1 Tree-level SMEFT matching conditions

Since the two extra scalar fields are, by assumption, heavier than the electroweak scale, for the purpose of low energy phenomenology, we can integrate them out and work instead with a non-renormalizable SMEFT lagrangian. This takes the form:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i C_i \mathcal{O}_i^{(6)} + \dots\,, \tag{2.10}$$

where $\mathcal{O}_i^{(6)}$ are dimension-six SMEFT operators, while the dots denote higher-dimension operators, which we neglect in the following. We adopt the Warsaw basis [3] for dimension-six operators. Separating the contributions arising at tree level from the one-loop generated ones, we can write

$$C_i = C_i^{(0)} + \frac{1}{(4\pi)^2} C_i^{(1)}\,. \tag{2.11}$$

At tree level, only a set of semi-leptonic operators is generated, with WCs:

$$\begin{aligned} [C_{lq}^{(1)}]^{(0)}_{\alpha\beta ij} &= \frac{\lambda^{1L*}_{i\alpha}\lambda^{1L}_{j\beta}}{4M_1^2} + \frac{3\lambda^{3L*}_{i\alpha}\lambda^{3L}_{j\beta}}{4M_3^2}\,, \\ [C_{lq}^{(3)}]^{(0)}_{\alpha\beta ij} &= -\frac{\lambda^{1L*}_{i\alpha}\lambda^{1L}_{j\beta}}{4M_1^2} + \frac{\lambda^{3L*}_{i\alpha}\lambda^{3L}_{j\beta}}{4M_3^2}\,, \\ [C_{lequ}^{(1)}]^{(0)}_{\alpha\beta ij} &= \frac{\lambda^{1R}_{j\beta}\lambda^{1L*}_{i\alpha}}{2M_1^2}\,, \\ [C_{lequ}^{(3)}]^{(0)}_{\alpha\beta ij} &= -\frac{\lambda^{1R}_{j\beta}\lambda^{1L*}_{i\alpha}}{8M_1^2}\,, \\ [C_{eu}]^{(0)}_{\alpha\beta ij} &= \frac{\lambda^{1R*}_{i\alpha}\lambda^{1R}_{j\beta}}{2M_1^2}. \end{aligned} \tag{2.12}$$

For the operator definitions, see tables 1, 2, 3 and 4.

3 Complete one-loop SMEFT matching conditions

We report in this section the complete one-loop SMEFT matching conditions for the S_1+S_3 model introduced in the previous section.

JHEP07(2020)225

The matching is performed diagrammatically, by equating 1LPI Green's functions in the UV and effective theory. As explained in the Introduction, this gives rise to a set of higher-dimensional effective operators in the Green's basis, which are then reduced to a minimal set of Warsaw basis operators by applying the SM equations of motion. This reduction is performed in full generality in appendix B, following the complete classification of Green's basis operators in appendix A. The one-loop matching to the Green's basis WCs for the $S_1 + S_3$ model is given in appendix C. Here we report the matching conditions in the Warsaw basis, which is the central result of the paper.

We performed our computations in a general R_ξ gauge, adopting the $\overline{\rm MS}$ subtraction scheme within Naive Dimensional Regularization (NDR). Whenever relevant, we explicitly checked the independence of matching conditions on the gauge fixing parameter ξ. The matching scale is μ_M, which in practical applications should be taken of order of the leptoquark masses $\mu_M \sim M_1, M_3$, but is otherwise arbitrary. For physics to be independent of μ_M, the resulting dependence of WCs from the matching scale should exactly correspond to their SMEFT renormalization group (RG) running [31–33], which we explicitly verified as a cross-check of our procedure. In practice, for all the operators that are not already generated at the tree-level, the explicit (logarithmic) scale dependence we obtain from the matching computation corresponds to the one from the SMEFT RG equations. For the operators listed in eq. (2.12), instead, one should also consider the running of the leptoquark couplings and masses with μ_M, schematically $[C_i]^{(0)}(\mu_M) \sim \lambda(\mu_M)\lambda(\mu_M)/M^2_{\rm LQ}(\mu_M)$. For instance, in the one-loop matching of these operators, there is a contribution from the quartic leptoquark couplings of eq. (2.3): the logarithmic scale dependence of such contribution cancels the one arising from the RG evolution of the LQ masses in the tree-level matching.

In contrast to the aforementioned gauge fixing and matching scale independence, matching conditions may (and do) explicitly depend on the definition of evanescent operators [34], i.e. operators which vanish in $d = 4$, but may be non-vanishing in $d \neq 4$. Two examples of evanescent Dirac structures relevant to us are:

$$\mathcal{E}_1 = P_L\gamma^\mu\gamma^\nu P_L \otimes P_L\gamma_\mu\gamma_\nu P_L - 4P_L \otimes P_L + P_L\sigma^{\mu\nu}P_L \otimes P_L\sigma_{\mu\nu}P_L, \tag{3.1}$$

$$\mathcal{E}_2 = P_L\gamma^\mu\gamma^\nu P_L \otimes P_R\gamma_\mu\gamma_\nu P_R - 4P_L \otimes P_R. \tag{3.2}$$

The NDR defining equations in $d = 4 - 2\epsilon$ dimensions, viz.

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \qquad \{\gamma^\mu, \gamma^5\} = 0, \qquad \mathrm{Tr}(\eta) = 4 - 2\epsilon \tag{3.3}$$

imply $\mathcal{E}_1 = -2\epsilon P_L \otimes P_L$, but do not univocally fix $\mathcal{E}_2$. Following the notation of ref. [35], we write:

$$\mathcal{E}_2 = 4a_{\rm ev}\epsilon P_L \otimes P_R + E^{(2)}_{LR}(a_{\rm ev}), \tag{3.4}$$

where the coefficient $a_{\rm ev}$ can be regarded as the definition of the evanescent operator $E^{(2)}_{LR}$ (e.g. for $a_{\rm ev} = -1/2$ one gets $E^{(2)}_{LR} = P_L\sigma^{\mu\nu}P_L \otimes P_R\sigma_{\mu\nu}P_R$, which vanishes in four dimensions, cf. ref. [35]).

In order to facilitate result comparisons, we report the matching conditions for general $a_{\rm ev}$ (the other scheme defining coefficients, $b_{\rm ev}$, $c_{\rm ev}$, etc., of ref. [35] do not enter in our one-loop computations). For practical calculations, ref. [36] recommends $a_{\rm ev} = b_{\rm ev} = \cdots = 1$, as in such scheme evanescent operators only affect two-loop anomalous dimensions.

We treat the Higgs mass term $m^2 H^\dagger H$ as an interaction (both in the SMEFT and UV theory) and work with a massless Higgs field propagator. By dimensional analysis, a diagram with internal Higgs lines and n insertions of m^2 is suppressed by a factor $(m^2/M^2)^n$ (where $M^2 = M^2_{1,3}$) relative to the same diagram with no insertions. Therefore, at dimension-six level, mass insertions can be relevant to the matching conditions for renormalizable operators (see below). However, in the present theory, one-loop diagrams with internal Higgs lines only give rise to dimension-six operators, so that m^2 does not contribute to the Green's basis matching conditions. It does, instead, contribute to the Warsaw basis matching conditions, where it makes its appearence through the Higgs EOM, see eq. (B.1).

As a further check, we have also recomputed the one-loop Green's basis WCs of pure-Higgs operators belonging to classes H^4D^2 and H^6 (see table 1) within the universal one-loop effective action (UOLEA) approach [21, 22, 26], and we find agreement with our diagrammatic results.

Integrating out the leptoquarks at one loop also generates contributions to SM renormalizable operators and, in particular, fermion kinetic terms. Such modifications can be undone by suitable field and SM coupling redefinitions, which however also introduce additional contributions to tree-level generated WCs.[1] In our case only fermion kinetic terms (i.e. wave-functions renormalizations) are relevant, as the tree-level WCs in eq. (2.12) do not depend on any SM coupling. The one-loop formulas below include the contributions due to fermion field renormalization.

3.1 Example

In this section we discuss in some details the matching of a specific Green's function, in order to illustrate some of the most relevant aspects of our computation.

Let us consider the off-shell Green's function $\mathcal{G} \equiv \langle e_\beta(p_1)\bar{e}_\alpha(p_2)H_b(q_1)H_a^\dagger(q_2)\rangle$, where all momenta are incoming and a, b are $\mathrm{SU}(2)_L$ indices. The matching conditions for this correlator are depicted diagrammatically in figure 1, where the left and right hand-side show the EFT and UV contributions, respectively. We briefly comment on the various steps of this computation.

We begin by listing the various contributions to $\mathcal{G}$, both in the SMEFT and the leptoquark model. The SMEFT operators which contribute at tree level to $\mathcal{G}$ are (cf. table 2 for the notation):

$$\begin{aligned}
[\mathcal{O}_{He}]_{\alpha\beta} &= (\bar{e}_\alpha\gamma^\mu e_\beta)(H^\dagger i\overleftrightarrow{D}_\mu H)\,,\\
[\mathcal{O}'_{He}]_{\alpha\beta} &= (\bar{e}_\alpha i\overleftrightarrow{\not{D}} e_\beta)(H^\dagger H)\,,\\
[\mathcal{O}''_{He}]_{\alpha\beta} &= (\bar{e}_\alpha\gamma^\mu e_\beta)\partial_\mu(H^\dagger H)\,.
\end{aligned} \tag{3.5}$$

[1] Since field redefinitions arise at one loop in our model, only tree-level WCs are affected. In general, any tree-level shift in SM couplings and wave-function renormalizations that could influence loop-generated coefficients should be taken into account, see e.g. [16].

Moreover, we must take into account a one-loop contribution from $\mathcal{O}_{eu}$, which is generated at the tree-level in our model according to eq. (2.12). Since this tree-level WC is fixed, the matching of $\mathcal{G}$ allows us to fix the coefficients of the operators in (3.5), see the left-hand side of figure 1. In the leptoquark model there are two diagrams contributing to $\mathcal{G}$, both mediated by S_1, shown in the right-hand side of figure 1: a box diagram proportional to (schematically) $y_U y_U^\dagger \lambda^{1R}\lambda^{1R\dagger}$, and a triangle diagram proportional to $\lambda_{H1}\lambda^{1R\dagger}\lambda^{1R}$.

By total momentum conservation, only three out of the four momenta p_1, p_2, q_1, q_2 are independent. Writing $(p_1, p_2, q_1, q_2) = (p-r,\, -p-r,\, q+r,\, -q+r)$, the tree-level contributions from the operators in eq. (3.5) read:

$$[\mathcal{G}^{\rm tree}_{\rm EFT}(\mu_M)]_{\alpha\beta} = 2\not{q}[G_{He}(\mu_M)]_{\alpha\beta} + 2\not{p}[G'_{He}(\mu_M)]_{\alpha\beta} - 2i\not{r}[G''_{He}(\mu_M)]_{\alpha\beta}, \tag{3.6}$$

where we drop here and below a global δ_{ab} factor, and we denote Green's basis WCs by G_i. The UV and EFT one-loop contributions are more easily computed when only one of the independent momenta p, q, r is non-vanishing, and yield respectively:

$$\left[\mathcal{G}^{\text{1-loop}}_{\rm UV}(\mu_M)\right]^{q=r=0}_{\alpha\beta} = -\not{p}\frac{N_c(\lambda^{1R\dagger}y_U^T y_U^* \lambda^{1R})_{\alpha\beta}}{(4\pi)^2 2M_1^2} + \not{p}\frac{N_c\lambda_{H1}(\lambda^{1R\dagger}\lambda^{1R})_{\alpha\beta}}{(4\pi)^2 2M_1^2}\,, \tag{3.7}$$

$$\left[\mathcal{G}^{\text{1-loop}}_{\rm UV}(\mu_M)\right]^{p=r=0}_{\alpha\beta} = -\not{q}\frac{N_c(\lambda^{1R\dagger}y_U^T y_U^* \lambda^{1R})_{\alpha\beta}}{(4\pi)^2 M_1^2}\log\frac{-q^2}{M_1^2}\,, \tag{3.8}$$

$$\left[\mathcal{G}^{\text{1-loop}}_{\rm UV}(\mu_M)\right]^{p=q=0}_{\alpha\beta} = 0\,, \tag{3.9}$$

and

$$\left[\mathcal{G}^{\text{1-loop}}_{\rm EFT}(\mu_M)\right]^{q=r=0}_{\alpha\beta} = 0\,, \tag{3.10}$$

$$\left[\mathcal{G}^{\text{1-loop}}_{\rm EFT}(\mu_M)\right]^{p=r=0}_{\alpha\beta} = \not{q}\frac{N_c(\lambda^{1R\dagger}y_U^T y_U^* \lambda^{1R})_{\alpha\beta}}{(4\pi)^2 M_1^2}\left(1+\log\frac{\mu_M^2}{-q^2}\right)\,, \tag{3.11}$$

$$\left[\mathcal{G}^{\text{1-loop}}_{\rm EFT}(\mu_M)\right]^{p=q=0}_{\alpha\beta} = 0\,, \tag{3.12}$$

where we employed the tree-level value of $[C_{eu}]^{(0)}_{\alpha\beta}$ given in eq. (2.12). Notice that the EFT computation presents an ultraviolet divergence, which we regulate in the $\overline{\rm MS}$ scheme at renormalization scale μ_M. On the other hand, on the basis of renormalizability, the UV contribution must be (and is) finite. Finally, both EFT and UV diagrams present an infrared divergence, corresponding to the $\log(-q^2)$ terms in eqs. (3.8) and (3.11). The agreement of these two terms, which is guaranteed by the EFT construction, provides a further check of validity of the computation.

Requiring $\mathcal{G}_{\rm EFT}(\mu_M) = \mathcal{G}_{\rm UV}(\mu_M)$, we finally obtain the matching conditions:

$$\begin{aligned}
[G_{He}(\mu_M)]_{\alpha\beta} &= -\frac{N_c(\lambda^{1R\dagger}y_U^T y_U^* \lambda^{1R})_{\alpha\beta}}{32\pi^2 M_1^2}\left(1+\log\frac{\mu_M^2}{M_1^2}\right)\,,\\
[G'_{He}(\mu_M)]_{\alpha\beta} &= -\frac{N_c(\lambda^{1R\dagger}y_U^T y_U^* \lambda^{1R})_{\alpha\beta}}{64\pi^2 M_1^2} + \frac{N_c\lambda_{H1}(\lambda^{1R\dagger}\lambda^{1R})_{\alpha\beta}}{64\pi^2 M_1^2}\,,\\
[G''_{He}(\mu_M)]_{\alpha\beta} &= 0\,.
\end{aligned} \tag{3.13}$$

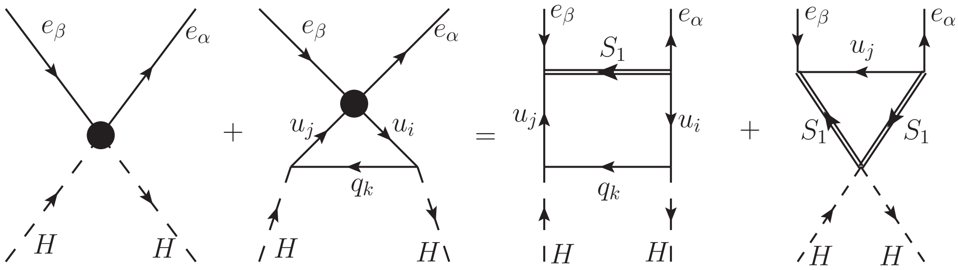

Figure 1. Diagrams for the matching of the $\langle \bar{e} e H^\dagger H \rangle$ Green function.

As a cross-check, we observe that the μ_M dependence of $[G_{He}(\mu_M)]_{\alpha\beta}$ corresponds to the SMEFT RG running of C_{He} due to C_{eu} [32],

$$(4\pi)^2 \mu \frac{d[C_{He}]_{\alpha\beta}}{d\mu} = -2N_c [C_{eu}]_{\alpha\beta ij} (y_U^T y_U^*)_{ij} \,, \tag{3.14}$$

once eq. (2.12) is taken into account.

3.2 One-loop matching conditions in the Warsaw basis

In the following we report the complete one-loop matching conditions of the $S_1 + S_3$ model to dimension-six SMEFT operators in the Warsaw basis. Definitions of the operators can be found in tables 1, 2, 3 and 4, while the $C_i^{(1)}$ coefficients are defined as in eq. (2.11). For convenience, we make the following definitions:

$$L_{1,3} \equiv \ln\left(\frac{\mu_M^2}{M_{1,3}^2}\right) \,, \tag{3.15}$$

$$\begin{aligned} h(M_1, M_3) &\equiv \frac{M_1^4 - M_3^4 - 2M_1^2 M_3^2 \log\frac{M_1^2}{M_3^2}}{(M_1^2 - M_3^2)^3} \,, \\ &\approx \frac{1}{3M_1^2}\left(1 - \delta m + \frac{7}{10}\delta m^2 + \mathcal{O}(\delta m^3)\right) \qquad (\delta m \equiv M_3/M_1 - 1) \,, \end{aligned} \tag{3.16}$$

$$n(M_1, M_3) \equiv \frac{M_1^2 - M_3^2 + M_3^2 \log\frac{M_3^2}{M_1^2}}{(M_3^2 - M_1^2)^2} \approx -\frac{1}{2M_1^2}\left(1 - \frac{2}{3}\delta m + \mathcal{O}(\delta m^2)\right) \,, \tag{3.17}$$

$$\begin{aligned} &\Lambda_q^{(n)} \equiv \lambda^{nL*}\lambda^{nLT}, \qquad &&\Lambda_q^{(31)} \equiv \lambda^{3L*}\lambda^{1LT}, \qquad &&\Lambda_u \equiv \lambda^{1R*}\lambda^{1RT}, \\ &\Lambda_\ell^{(n)} \equiv \lambda^{nL\dagger}\lambda^{nL}, \qquad &&\Lambda_\ell^{(31)} \equiv \lambda^{3L\dagger}\lambda^{1L}, \qquad &&\Lambda_e \equiv \lambda^{1R\dagger}\lambda^{1R}, \end{aligned} \tag{3.18}$$

$$X_{1U}^{1L} \equiv \lambda^{1L\dagger} y_U^* \lambda^{1R}, \qquad X_{1E}^{1L} \equiv (\lambda^{1R} y_E^\dagger \lambda^{1L\dagger})^T,$$

$$X_{1U}^{3L} \equiv \lambda^{3L\dagger} y_U^* \lambda^{1R}, \qquad X_{1E}^{3L} \equiv (\lambda^{1R} y_E^\dagger \lambda^{3L\dagger})^T,$$

$$X_{2F}^{nL} \equiv \lambda^{nL\dagger} y_F^* y_F^T \lambda^{nL}, \qquad X_{2U}^{1R} \equiv \lambda^{1R\dagger} y_U^T y_U^* \lambda^{1R}, \tag{3.19}$$

$$X_{2E}^{nL} \equiv (\lambda^{nL} y_E y_E^\dagger \lambda^{nL\dagger})^T, \qquad X_{2E}^{1R} \equiv (\lambda^{1R} y_E^\dagger y_E \lambda^{1R\dagger})^T,$$

$$X_{3U}^{1L} \equiv \lambda^{1L\dagger} y_U^* y_U^T y_U^* \lambda^{1R}, \qquad X_{3E}^{1L} \equiv (\lambda^{1R} y_E^\dagger y_E y_E^\dagger \lambda^{1L\dagger})^T,$$

where $n = 1, 3$ and $F = U, D$, and the superscript T stands for transpose. The fermion wave function renormalizations, are given by $(Z_\psi)_{ij} = \delta_{ij} + \frac{1}{(4\pi)^2}(\delta Z_\psi)_{ij}$, where

$$(\delta Z_\ell)_{\alpha\beta} = \frac{N_c}{2}\left[\left(\frac{1}{2}+L_1\right)(\Lambda_\ell^{(1)})_{\alpha\beta} + 3\left(\frac{1}{2}+L_3\right)(\Lambda_\ell^{(3)})_{\alpha\beta}\right], \tag{3.20}$$

$$(\delta Z_e)_{\alpha\beta} = \frac{N_c}{2}\left(\frac{1}{2}+L_1\right)(\Lambda_e)_{\alpha\beta}, \tag{3.21}$$

$$(\delta Z_q)_{ij} = \frac{1}{2}\left[\left(\frac{1}{2}+L_1\right)(\Lambda_q^{(1)})_{ij} + 3\left(\frac{1}{2}+L_3\right)(\Lambda_q^{(3)})_{ij}\right], \tag{3.22}$$

$$(\delta Z_u)_{ij} = \frac{1}{2}\left(\frac{1}{2}+L_1\right)(\Lambda_u)_{ij}, \tag{3.23}$$

$$(\delta Z_d)_{ij} = 0. \tag{3.24}$$

3.2.1 Renormalizable terms

$$\begin{aligned}(4\pi)^2(\delta y_E)_{\alpha\beta} = &-\frac{1}{2}\left[(\delta Z_\ell)_{\alpha\gamma}\delta_{\delta\beta} + \delta_{\alpha\gamma}(\delta Z_e)_{\delta\beta}\right](y_E)_{\gamma\delta} + \\ &- N_c\left[1 + L_1 + \frac{1}{2}\left(\frac{1}{2}+L_1\right)\frac{m^2}{M_1^2}\right](X_{1U}^{1L})_{\alpha\beta}\,,\end{aligned} \tag{3.25}$$

$$\begin{aligned}(4\pi)^2(\delta y_U)_{ij} = &-\frac{1}{2}\left[(\delta Z_q)_{ik}\delta_{jl} + \delta_{ik}(\delta Z_u)_{lj}\right](y_U)_{kl} + \\ &-\left[1 + L_1 + \frac{1}{2}\left(\frac{1}{2}+L_1\right)\frac{m^2}{M_1^2}\right](X_{1E}^{1L})_{ij}\,,\end{aligned} \tag{3.26}$$

$$(4\pi)^2(\delta y_D)_{ij} = 0\,, \tag{3.27}$$

$$\begin{aligned}(4\pi)^2\delta\lambda = &-N_c\left[\lambda_{H1}^2 L_1 + (3\lambda_{H3}^2 + 2\lambda_{\epsilon H3}^2)L_3 + 2|\lambda_{H13}|^2\left(1 + \frac{M_3^2 L_3 - M_1^2 L_1}{M_3^2 - M_1^2}\right)\right] + \\ &-\frac{N_c}{15} g^4 m^2 \frac{1}{M_3^2}\,,\end{aligned} \tag{3.28}$$

$$(4\pi)^2\delta m^2 = N_c\left[\lambda_{H1}(1+L_1)M_1^2 + 3\lambda_{H3}(1+L_3)M_3^2 - 2m^4\left(\frac{\lambda_{\epsilon H3}^2}{3M_3^2} + |\lambda_{H13}|^2 h(M_1, M_3)\right)\right]. \tag{3.29}$$

3.2.2 Purely bosonic

X^3.

$$C_{3G}^{(1)} = \frac{1}{360} g_s^3 \left(\frac{3}{M_3^2} + \frac{1}{M_1^2} \right), \tag{3.30}$$

$$C_{3W}^{(1)} = \frac{N_c}{90} g^3 \frac{1}{M_3^2}, \tag{3.31}$$

$$C_{3\widetilde{G}}^{(1)} = C_{3\widetilde{W}}^{(1)} = 0. \tag{3.32}$$

$X^2 H^2$.

$$C_{HG}^{(1)} = \frac{g_s^2}{12} \left(\frac{3\lambda_{H3}}{M_3^2} + \frac{\lambda_{H1}}{M_1^2} \right), \tag{3.33}$$

$$C_{HW}^{(1)} = \frac{N_c}{3} g^2 \frac{\lambda_{H3}}{M_3^2}, \tag{3.34}$$

$$C_{HB}^{(1)} = \frac{N_c}{6} g'^2 \left(3\frac{\lambda_{H3} Y_{S_3}^2}{M_3^2} + \frac{\lambda_{H1} Y_{S_1}^2}{M_1^2} \right), \tag{3.35}$$

$$C_{HWB}^{(1)} = -N_c \frac{g g' Y_{S_3} \lambda_{\epsilon H3}}{3M_3^2}, \tag{3.36}$$

$$C_{H\widetilde{G}}^{(1)} = C_{H\widetilde{W}}^{(1)} = C_{H\widetilde{B}}^{(1)} = C_{H\widetilde{W}B}^{(1)} = 0. \tag{3.37}$$

$H^4 D^2$.

$$\begin{aligned} C_{H\Box}^{(1)} = &-N_c \left(\frac{1}{40} g^4 + \frac{1}{20} g'^4 Y_H^2 Y_{S_3}^2 + \frac{3\lambda_{H3}^2 - 2\lambda_{\epsilon H3}^2}{12} \right) \frac{1}{M_3^2} - N_c \left(\frac{1}{60} g'^4 Y_H^2 Y_{S_1}^2 + \frac{\lambda_{H1}^2}{12} \right) \frac{1}{M_1^2} + \\ &+ \frac{N_c}{2} |\lambda_{H13}|^2 h(M_1, M_3), \end{aligned} \tag{3.38}$$

$$C_{HD}^{(1)} = -\frac{N_c}{15} g'^4 Y_H^2 \left(\frac{3Y_{S_3}^2}{M_3^2} + \frac{Y_{S_1}^2}{M_1^2} \right) - 2N_c \left(\frac{\lambda_{\epsilon H3}^2}{3M_3^2} + |\lambda_{H13}|^2 h(M_1, M_3) \right). \tag{3.39}$$

H^6.

$$\begin{aligned} C_H^{(1)} = &-N_c \left(\frac{1}{30} g^4 \lambda + \frac{1}{2} \lambda_{H3}^3 + \lambda_{H3} \lambda_{\epsilon H3}^2 \right) \frac{1}{M_3^2} - N_c \frac{\lambda_{H1}^3}{6M_1^2} + 2N_c \lambda \left(\frac{\lambda_{\epsilon H3}^2}{3M_3^2} + |\lambda_{H13}|^2 h(M_1, M_3) \right) \\ &+ \frac{N_c |\lambda_{H13}|^2}{M_1^2 - M_3^2} \left(\lambda_{H3} - \lambda_{H1} + \frac{\ln\left(\frac{M_1^2}{M_3^2}\right)}{M_1^2 - M_3^2} (\lambda_{H1} M_3^2 - \lambda_{H3} M_1^2) \right). \end{aligned} \tag{3.40}$$

3.2.3 Two-fermion operators

$\psi^2 XH$.

$$[C_{uG}]^{(1)}_{ij} = -\frac{1}{24} g_s \left(3\frac{(\Lambda_q^{(3)} y_U)_{ij}}{M_3^2} + \frac{(\Lambda_q^{(1)} y_U)_{ij}}{M_1^2} \right) - \frac{1}{24} g_s \frac{(y_U \Lambda_u)_{ij}}{M_1^2} - \frac{1}{4} g_s \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{3.41}$$

$$[C_{uW}]^{(1)}_{ij} = -\frac{1}{24} g \left(3\frac{(\Lambda_q^{(3)} y_U)_{ij}}{M_3^2} - \frac{(\Lambda_q^{(1)} y_U)_{ij}}{M_1^2} \right) - \frac{1}{8} g \left(\frac{1}{2} + L_1 \right) \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{3.42}$$

$$\begin{aligned}[C_{uB}]^{(1)}_{ij} = &-\frac{1}{24} g' (Y_q + 3Y_\ell) \left(3\frac{(\Lambda_q^{(3)} y_U)_{ij}}{M_3^2} + \frac{(\Lambda_q^{(1)} y_U)_{ij}}{M_1^2} \right) - \frac{1}{24} g' (Y_u - 3Y_e) \frac{(y_U \Lambda_u)_{ij}}{M_1^2} + \\ &+ \frac{1}{4} g' \left[(Y_l + Y_e) L_1 + \frac{1}{2} Y_l + \frac{3}{2} Y_e - Y_u \right] \frac{(X^{1L}_{1E})_{ij}}{M_1^2},\end{aligned} \tag{3.43}$$

$$[C_{dG}]^{(1)}_{ij} = -\frac{1}{24} g_s \left(3\frac{(\Lambda_q^{(3)} y_D)_{ij}}{M_3^2} + \frac{(\Lambda_q^{(1)} y_D)_{ij}}{M_1^2} \right), \tag{3.44}$$

$$[C_{dW}]^{(1)}_{ij} = -\frac{1}{24} g \left(3\frac{(\Lambda_q^{(3)} y_D)_{ij}}{M_3^2} - \frac{(\Lambda_q^{(1)} y_D)_{ij}}{M_1^2} \right), \tag{3.45}$$

$$[C_{dB}]^{(1)}_{ij} = -\frac{1}{24} g' (Y_q + 3Y_\ell) \left(3\frac{(\Lambda_q^{(3)} y_D)_{ij}}{M_3^2} + \frac{(\Lambda_q^{(1)} y_D)_{ij}}{M_1^2} \right), \tag{3.46}$$

$$[C_{eW}]^{(1)}_{\alpha\beta} = -\frac{N_c}{24} g \left(3\frac{(\Lambda_\ell^{(3)} y_E)_{\alpha\beta}}{M_3^2} - \frac{(\Lambda_\ell^{(1)} y_E)_{\alpha\beta}}{M_1^2} \right) - \frac{N_c}{8} g \left(\frac{1}{2} + L_1 \right) \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}, \tag{3.47}$$

$$\begin{aligned}[C_{eB}]^{(1)}_{\alpha\beta} = &-\frac{N_c}{24} g' (Y_\ell + 3Y_q) \left(3\frac{(\Lambda_\ell^{(3)} y_E)_{\alpha\beta}}{M_3^2} + \frac{(\Lambda_\ell^{(1)} y_E)_{\alpha\beta}}{M_1^2} \right) - \frac{N_c}{24} g' (Y_e - 3Y_u) \frac{(y_E \Lambda_e)_{\alpha\beta}}{M_1^2} + \\ &+ \frac{N_c}{4} g' \left[(Y_q + Y_u) L_1 + \frac{1}{2} Y_q + \frac{3}{2} Y_u - Y_e \right] \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}.\end{aligned} \tag{3.48}$$

$\psi^2 H^2 D$.

$$\begin{aligned}[C^{(1)}_{Hq}]^{(1)}_{ij} = &-\frac{N_c}{30} g'^4 Y_H Y_q \delta_{ij} \left(\frac{3Y_{S_3}^2}{M_3^2} + \frac{Y_{S_1}^2}{M_1^2} \right) - \frac{1}{24} \frac{(y_U \Lambda_u y_U^\dagger)_{ij}}{M_1^2} + \\ &+ \frac{1}{3} g'^2 Y_H \left\{ 3 \left(\frac{1}{6} Y_q + Y_\ell L_3 \right) \frac{(\Lambda_q^{(3)})_{ij}}{M_3^2} + \left(\frac{1}{6} Y_q + Y_\ell L_1 \right) \frac{(\Lambda_q^{(1)})_{ij}}{M_1^2} \right\} + \\ &- \frac{1}{4} \left(3(1 + L_3) \frac{(X^{3L}_{2E})_{ij}}{M_3^2} + (1 + L_1) \frac{(X^{1L}_{2E})_{ij}}{M_1^2} \right),\end{aligned} \tag{3.49}$$

JHEP07(2020)225

$$[C_{Hq}^{(3)}]_{ij}^{(1)} = -\frac{N_c}{60} g^4 \delta_{ij} \frac{1}{M_3^2} + \frac{1}{24} \frac{(y_U \Lambda_u y_U^\dagger)_{ij}}{M_1^2} + \\ + \frac{1}{12} g^2 \left(\left(\frac{1}{2} + L_3 \right) \frac{(\Lambda_q^{(3)})_{ij}}{M_3^2} - \left(-\frac{1}{6} + L_1 \right) \frac{(\Lambda_q^{(1)})_{ij}}{M_1^2} \right) + \\ - \frac{1}{4} \left((1+L_3) \frac{(X_{2E}^{3L})_{ij}}{M_3^2} - (1+L_1) \frac{(X_{2E}^{1L})_{ij}}{M_1^2} \right), \tag{3.50}$$

$$[C_{Hu}]_{ij}^{(1)} = -\frac{N_c}{30} g'^4 Y_H Y_u \delta_{ij} \left(\frac{3Y_{S_3}^2}{M_3^2} + \frac{Y_{S_1}^2}{M_1^2} \right) + \frac{1}{12} \left(3 \frac{(y_U^\dagger \Lambda_q^{(3)} y_U)_{ij}}{M_3^2} + \frac{(y_U^\dagger \Lambda_q^{(1)} y_U)_{ij}}{M_1^2} \right) + \\ + \frac{1}{3} g'^2 Y_H \left(\frac{8Y_e - Y_{S_1}}{6} + Y_e L_1 \right) \frac{(\Lambda_u)_{ij}}{M_1^2} + \frac{1}{2} (1+L_1) \frac{(X_{2E}^{1R})_{ij}}{M_1^2}, \tag{3.51}$$

$$[C_{Hd}]_{ij}^{(1)} = -\frac{N_c}{30} g'^4 Y_H Y_d \delta_{ij} \left(\frac{3Y_{S_3}^2}{M_3^2} + \frac{Y_{S_1}^2}{M_1^2} \right) + \frac{1}{12} \left(3 \frac{(y_D^\dagger \Lambda_q^{(3)} y_D)_{ij}}{M_3^2} + \frac{(y_D^\dagger \Lambda_q^{(1)} y_D)_{ij}}{M_1^2} \right), \tag{3.52}$$

$$[C_{Hud}]_{ij}^{(1)} = \frac{1}{12} \left(3 \frac{(y_U^\dagger \Lambda_q^{(3)} y_D)_{ij}}{M_3^2} + \frac{(y_U^\dagger \Lambda_q^{(1)} y_D)_{ij}}{M_1^2} \right), \tag{3.53}$$

$$[C_{H\ell}^{(1)}]_{\alpha\beta}^{(1)} = -\frac{N_c}{30} g'^4 Y_H Y_\ell \delta_{\alpha\beta} \left(\frac{3Y_{S_3}^2}{M_3^2} + \frac{Y_{S_1}^2}{M_1^2} \right) + \frac{N_c}{24} \frac{(y_E \Lambda_e y_E^\dagger)_{\alpha\beta}}{M_1^2} + \\ + \frac{N_c}{3} g'^2 Y_H \left(3 \left(\frac{1}{6} Y_\ell + Y_q L_3 \right) \frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2} + \left(\frac{1}{6} Y_\ell + Y_q L_1 \right) \frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2} \right) + \\ - \frac{N_c}{4} \left\{ 3(1+L_3) \frac{-(X_{2U}^{3L})_{\alpha\beta} + (X_{2D}^{3L})_{\alpha\beta}}{M_3^2} + (1+L_1) \frac{-(X_{2U}^{1L})_{\alpha\beta} + (X_{2D}^{1L})_{\alpha\beta}}{M_1^2} \right\}, \tag{3.54}$$

$$[C_{H\ell}^{(3)}]_{\alpha\beta}^{(1)} = -\frac{N_c}{60} g^4 \delta_{\alpha\beta} \frac{1}{M_3^2} + \frac{N_c}{24} \frac{(y_E \Lambda_e y_E^\dagger)_{\alpha\beta}}{M_1^2} + \\ + \frac{N_c}{12} g^2 \left(\left(\frac{1}{2} + L_3 \right) \frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2} - \left(-\frac{1}{6} + L_1 \right) \frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2} \right) + \\ - \frac{N_c}{4} \left\{ (1+L_3) \frac{(X_{2U}^{3L})_{\alpha\beta} + (X_{2D}^{3L})_{\alpha\beta}}{M_3^2} - (1+L_1) \frac{(X_{2U}^{1L})_{\alpha\beta} + (X_{2D}^{1L})_{\alpha\beta}}{M_1^2} \right\}, \tag{3.55}$$

$$[C_{He}]_{\alpha\beta}^{(1)} = -\frac{N_c}{30} g'^4 Y_H Y_e \delta_{\alpha\beta} \left(\frac{3Y_{S_3}^2}{M_3^2} + \frac{Y_{S_1}^2}{M_1^2} \right) + \frac{N_c}{12} \left(3 \frac{(y_E^\dagger \Lambda_\ell^{(3)} y_E)_{\alpha\beta}}{M_3^2} + \frac{(y_E^\dagger \Lambda_\ell^{(1)} y_E)_{\alpha\beta}}{M_1^2} \right) + \\ + \frac{N_c}{3} g'^2 Y_H \left(\frac{8Y_u - Y_{S_1}}{6} + Y_u L_1 \right) \frac{(\Lambda_e)_{\alpha\beta}}{M_1^2} - \frac{N_c}{2} (1+L_1) \frac{(X_{2U}^{1R})_{\alpha\beta}}{M_1^2}. \tag{3.56}$$

$\psi^2 H^3$.

$$[C_{uH}]^{(1)}_{ij} = -\frac{N_c}{60}g^4(y_U)_{ij}\frac{1}{M_3^2}+N_c(y_U)_{ij}\left(\frac{\lambda^2_{\epsilon H3}}{3M_1^2}+|\lambda_{H13}|^2h(M_1,M_3)\right)+$$
$$+\frac{1}{12}\left(3\frac{(y_Uy_U^\dagger\Lambda_q^{(3)}y_U)_{ij}}{M_3^2}+\frac{(y_Uy_U^\dagger\Lambda_q^{(1)}y_U)_{ij}}{M_1^2}\right)+\frac{1}{12}\frac{(y_U\Lambda_uy_U^\dagger y_U)_{ij}}{M_1^2}+$$
$$-\frac{\lambda}{2}\left(L_1+\frac{1}{2}\right)\frac{(X^{1L}_{1E})_{ij}}{M_1^2}+\frac{1}{2}\frac{(y_UX^{1L\dagger}_{1E}y_U)_{ij}}{M_1^2}+$$
$$-\frac{1}{4}\left(\frac{(X^{3L}_{2E}y_U)_{ij}+(3\lambda_{H3}-2\lambda_{\epsilon H3})(\Lambda_q^{(3)}y_U)_{ij}}{M_3^2}+\frac{(X^{1L}_{2E}y_U)_{ij}+\lambda_{H1}(\Lambda_q^{(1)}y_U)_{ij}}{M_1^2}\right)+$$
$$-\frac{1}{2}\left[\left(\lambda^*_{H13}\Lambda_q^{(31)}y_U\right)_{ij}n(M_1,M_3)+\left(\lambda_{H13}\Lambda_q^{(31)\dagger}y_U\right)_{ij}n(M_3,M_1)\right]+$$
$$+\frac{1}{4}\frac{(y_UX^{1R}_{2E})_{ij}+\lambda_{H1}(y_U\Lambda_u)_{ij}}{M_1^2}+\frac{(1+L_1)(X^{1L}_{3E})_{ij}-\lambda_{H1}(X^{1L}_{1E})_{ij}}{M_1^2}-\frac{\lambda^*_{H13}(X^{3L}_{1E})_{ij}\log\frac{M_1^2}{M_3^2}}{M_1^2-M_3^2},\tag{3.57}$$

$$[C_{dH}]^{(1)}_{ij} = -\frac{N_c}{60}g^4(y_D)_{ij}\frac{1}{M_3^2}+N_c(y_D)_{ij}\left(\frac{\lambda^2_{\epsilon H3}}{3M_1^2}+|\lambda_{H13}|^2h(M_1,M_3)\right)+$$
$$+\frac{1}{12}\left(3\frac{(y_Dy_D^\dagger\Lambda_q^{(3)}y_D)_{ij}}{M_3^2}+\frac{(y_Dy_D^\dagger\Lambda_q^{(1)}y_D)_{ij}}{M_1^2}\right)+$$
$$-\frac{1}{4}\left(\frac{2(X^{3L}_{2E}y_D)_{ij}+(3\lambda_{H3}+2\lambda_{\epsilon H3})(\Lambda_q^{(3)}y_D)_{ij}}{M_3^2}+\frac{\lambda_{H1}(\Lambda_q^{(1)}y_D)_{ij}}{M_1^2}\right)+$$
$$+\frac{1}{2}\left[\left(\lambda^*_{H13}\Lambda_q^{(31)}y_D\right)_{ij}n(M_1,M_3)+\left(\lambda_{H13}\Lambda_q^{(31)\dagger}y_D\right)_{ij}n(M_3,M_1)\right],\tag{3.58}$$

$$[C_{eH}]^{(1)}_{\alpha\beta} = -\frac{N_c}{60}g^4(y_E)_{\alpha\beta}\frac{1}{M_3^2}+N_c(y_E)_{\alpha\beta}\left(\frac{\lambda^2_{\epsilon H3}}{3M_1^2}+|\lambda_{H13}|^2h(M_1,M_3)\right)+$$
$$+\frac{N_c}{12}\left(3\frac{(y_Ey_E^\dagger\Lambda_\ell^{(3)}y_E)_{\alpha\beta}}{M_3^2}+\frac{(y_Ey_E^\dagger\Lambda_\ell^{(1)}y_E)_{\alpha\beta}}{M_1^2}\right)+\frac{N_c}{12}\frac{(y_E\Lambda_ey_E^\dagger y_E)_{\alpha\beta}}{M_1^2}+$$
$$-\frac{N_c}{2}\left(\frac{1}{2}+L_1\right)\lambda\frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}+\frac{N_c}{2}\frac{(y_EX^{1L\dagger}_{1U}y_E)_{\alpha\beta}}{M_1^2}+$$
$$-\frac{N_c}{4}\frac{(X^{3L}_{2U}y_E)_{\alpha\beta}+2(X^{3L}_{2D}y_E)_{\alpha\beta}+(3\lambda_{H3}+2\lambda_{\epsilon H3})(\Lambda_\ell^{(3)}y_E)_{\alpha\beta}}{M_3^2}+$$
$$-\frac{N_c}{4}\frac{(X^{1L}_{2U}y_E)_{\alpha\beta}+\lambda_{H1}(\Lambda_\ell^{(1)}y_E)_{\alpha\beta}}{M_1^2}-\frac{N_c}{4}\frac{(y_EX^{1R}_{2U})_{\alpha\beta}+\lambda_{H1}(y_E\Lambda_e)_{\alpha\beta}}{M_1^2}+$$
$$-\frac{N_c}{2}\left[\left(\lambda^*_{H13}\Lambda_\ell^{(31)}y_E\right)_{\alpha\beta}n(M_1,M_3)+\left(\lambda_{H13}\Lambda_\ell^{(31)\dagger}y_E\right)_{\alpha\beta}n(M_3,M_1)\right]+$$
$$+N_c\frac{(1+L_1)(X^{1L}_{3U})_{\alpha\beta}-\lambda_{H1}(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}-N_c\frac{\lambda^*_{H13}(X^{3L}_{1U})_{\alpha\beta}\log\frac{M_1^2}{M_3^2}}{M_1^2-M_3^2}.\tag{3.59}$$

JHEP07(2020)225

3.2.4 Four-fermion operators

Four-quark.[2]

$$[C_{qq}^{(1)}]_{ijkl}^{(1)} = -\frac{1}{240}g_s^4\left(\frac{1}{2}\delta_{il}\delta_{kj}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)-\frac{N_c}{60}g'^4Y_q^2\delta_{ij}\delta_{kl}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+$$
$$+\frac{1}{72}g_s^2\left(\frac{1}{2}\delta_{kj}\delta_{im}\delta_{ln}+\frac{1}{2}\delta_{il}\delta_{km}\delta_{jn}-\frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn}-\frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}\right)\left(3\frac{(\Lambda_q^{(3)})_{mn}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{mn}}{M_1^2}\right)+$$
$$+\frac{1}{6}g'^2Y_q\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)\left\{3\left(\frac{1}{6}Y_q+Y_\ell L_3\right)\frac{(\Lambda_q^{(3)})_{mn}}{M_3^2}+\left(\frac{1}{6}Y_q+Y_\ell L_1\right)\frac{(\Lambda_q^{(1)})_{mn}}{M_1^2}\right\}+$$
$$-\frac{1}{16}\left(9\frac{(\Lambda_q^{(3)})_{il}(\Lambda_q^{(3)})_{kj}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{il}(\Lambda_q^{(1)})_{kj}}{M_1^2}+3\frac{\log\frac{M_3^2}{M_1^2}\left[(\Lambda_q^{31})_{il}(\Lambda_q^{31\dagger})_{kj}+(\Lambda_q^{31\dagger})_{il}(\Lambda_q^{31})_{kj}\right]}{M_3^2-M_1^2}\right), \tag{3.60}$$

$$[C_{qq}^{(3)}]_{ijkl}^{(1)} = -\frac{1}{480}g_s^4\delta_{il}\delta_{kj}\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)-\frac{N_c}{120}g^4\delta_{ij}\delta_{kl}\frac{1}{M_3^2}+$$
$$+\frac{1}{144}g_s^2\left(\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)\left(3\frac{(\Lambda_q^{(3)})_{mn}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{mn}}{M_1^2}\right)+$$
$$+\frac{1}{24}g^2\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)\left(\left(\frac{1}{2}+L_3\right)\frac{(\Lambda_q^{(3)})_{mn}}{M_3^2}-\left(-\frac{1}{6}+L_1\right)\frac{(\Lambda_q^{(1)})_{mn}}{M_1^2}\right)+$$
$$-\frac{1}{16}\left(\frac{(\Lambda_q^{(3)})_{il}(\Lambda_q^{(3)})_{kj}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{il}(\Lambda_q^{(1)})_{kj}}{M_1^2}-\frac{\log\frac{M_3^2}{M_1^2}\left[(\Lambda_q^{31})_{il}(\Lambda_q^{31\dagger})_{kj}+(\Lambda_q^{31\dagger})_{il}(\Lambda_q^{31})_{kj}\right]}{M_3^2-M_1^2}\right), \tag{3.61}$$

$$[C_{uu}]_{ijkl}^{(1)} = -\frac{g_s^4}{240}\left(\delta_{il}\delta_{kj}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)-\frac{N_c}{60}g'^4Y_u^2\delta_{ij}\delta_{kl}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+$$
$$+\frac{1}{6}g'^2Y_u\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)\left(\frac{8Y_e-Y_{S_1}}{6}+Y_eL_1\right)\frac{(\Lambda_u)_{mn}}{M_1^2}+$$
$$+\frac{1}{72}g_s^2\left(-\frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn}-\frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}+\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)\frac{(\Lambda_u)_{mn}}{M_1^2}+$$
$$-\frac{1}{8}\frac{(\Lambda_u)_{il}(\Lambda_u)_{kj}}{M_1^2}, \tag{3.62}$$

$$[C_{dd}]_{ijkl}^{(1)} = -\frac{g_s^4}{240}\left(\delta_{il}\delta_{kj}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)-\frac{N_c}{60}g'^4Y_d^2\delta_{ij}\delta_{kl}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right), \tag{3.63}$$

$$[C_{ud}^{(1)}]_{ijkl}^{(1)} = -\frac{N_c}{30}g'^4Y_dY_d\delta_{ij}\delta_{kl}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+$$
$$+\frac{1}{3}g'^2Y_d\left(\frac{8Y_e-Y_{S_1}}{6}+Y_eL_1\right)\frac{\delta_{kl}(\Lambda_u)_{ij}}{M_1^2}, \tag{3.64}$$

[2]Box diagram contributions to four-quark operators from S_1 and S_3, taken separately, have also been computed in [37]. We find agreement except for $C_{qq}^{(1)}$ and $C_{qq}^{(3)}$, where we found an inconsistency in [37]. We thank the authors for clarifications about this point.

$$[C_{ud}^{(8)}]_{ijkl}^{(1)} = -\frac{g_s^4}{60}\delta_{ij}\delta_{kl}\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)+\frac{1}{18}g_s^2\frac{\delta_{kl}(\Lambda_u)_{ij}}{M_1^2}, \tag{3.65}$$

$$\begin{aligned}[C_{qu}^{(1)}]_{ijkl}^{(1)} = &-\frac{N_c}{30}g'^4 Y_q Y_u \delta_{ij}\delta_{kl}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\ &+\frac{1}{3}g'^2 Y_u\left\{3\left(\frac{1}{6}Y_q+Y_\ell L_3\right)\frac{\delta_{kl}(\Lambda_q^{(3)})_{ij}}{M_3^2}+\left(\frac{1}{6}Y_q+Y_\ell L_1\right)\frac{\delta_{kl}(\Lambda_q^{(1)})_{ij}}{M_1^2}\right\}+\\ &+\frac{1}{3}g'^2 Y_q\left(\frac{8Y_e-Y_{S_1}}{6}+Y_e L_1\right)\frac{\delta_{ij}(\Lambda_u)_{kl}}{M_1^2}+\\ &+\frac{1}{12}\left(\frac{1}{2}+L_1\right)\frac{(y_U^\dagger)_{kj}(X_{1E}^{1L})_{il}+(y_U)_{il}(X_{1E}^{1L\dagger})_{kj}}{M_1^2}-\frac{1}{12}\frac{(\Lambda_q^{(1)})_{ij}(\Lambda_u)_{kl}}{M_1^2},\end{aligned} \tag{3.66}$$

$$\begin{aligned}[C_{qu}^{(8)}]_{ijkl}^{(1)} = &-\frac{1}{60}g_s^4\delta_{ij}\delta_{kl}\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)+\\ &+\frac{1}{18}g_s^2\left(3\frac{(\Lambda_q^{(3)})_{ij}\delta_{kl}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{ij}\delta_{kl}}{M_1^2}\right)+\frac{1}{18}g_s^2\frac{\delta_{ij}(\Lambda_u)_{kl}}{M_1^2}+\\ &+\frac{1}{2}\left(\frac{1}{2}+L_1\right)\frac{(X_{1E}^{1L})_{il}(y_U^\dagger)_{kj}+(y_U)_{il}(X_{1E}^{1L\dagger})_{kj}}{M_1^2}-\frac{1}{2}\frac{(\Lambda_q^{(1)})_{ij}(\Lambda_u)_{kl}}{M_1^2},\end{aligned} \tag{3.67}$$

$$\begin{aligned}[C_{qd}^{(1)}]_{ijkl}^{(1)} = &-\frac{N_c}{30}g'^4 Y_q Y_d \delta_{ij}\delta_{kl}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\ &+\frac{1}{3}g'^2 Y_d\left\{3\left(\frac{1}{6}Y_q+Y_\ell L_3\right)\frac{\delta_{kl}(\Lambda_q^{(3)})_{ij}}{M_3^2}+\left(\frac{1}{6}Y_q+Y_\ell L_1\right)\frac{\delta_{kl}(\Lambda_q^{(1)})_{ij}}{M_1^2}\right\},\end{aligned} \tag{3.68}$$

$$[C_{qd}^{(8)}]_{ijkl}^{(1)} = -\frac{1}{60}g_s^4\delta_{ij}\delta_{kl}\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right)+\frac{1}{18}g_s^2\left(3\frac{(\Lambda_q^{(3)})_{ij}\delta_{kl}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{ij}\delta_{kl}}{M_1^2}\right), \tag{3.69}$$

$$[C_{quqd}^{(1)}]_{ijkl}^{(1)} = -\frac{1}{2}\left(L_1+\frac{1}{2}\right)\frac{(X_{1E}^{1L})_{ij}(y_D)_{kl}}{M_1^2}, \tag{3.70}$$

$$[C_{quqd}^{(8)}]_{ijkl}^{(1)} = 0. \tag{3.71}$$

Four-lepton.

$$\begin{aligned}[C_{\ell\ell}]_{\alpha\beta\gamma\delta}^{(1)} = &-\frac{N_c}{120}g^4(2\delta_{\alpha\delta}\delta_{\gamma\beta}-\delta_{\alpha\beta}\delta_{\gamma\delta})\frac{1}{M_3^2}-\frac{N_c}{60}g'^4 Y_\ell^2\delta_{\alpha\beta}\delta_{\gamma\delta}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\ &+\frac{N_c}{24}g^2\left(-\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma}+2\delta_{\gamma\beta}\delta_{\alpha\rho}\delta_{\delta\sigma}-\delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma}+2\delta_{\alpha\delta}\delta_{\gamma\rho}\delta_{\beta\sigma}\right)\times\\ &\times\left(\left(L_3+\frac{1}{2}\right)\frac{(\Lambda_\ell^{(3)})_{\rho\sigma}}{M_3^2}-\left(L_1-\frac{1}{6}\right)\frac{(\Lambda_\ell^{(1)})_{\rho\sigma}}{M_1^2}\right)+\\ &+\frac{N_c}{6}g'^2 Y_\ell\left(\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma}+\delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma}\right)\times\\ &\times\left[3\left(\frac{1}{6}Y_\ell+Y_q L_3\right)\frac{(\Lambda_\ell^{(3)})_{\rho\sigma}}{M_3^2}+\left(\frac{1}{6}Y_\ell+Y_q L_1\right)\frac{(\Lambda_\ell^{(1)})_{\rho\sigma}}{M_1^2}\right]+\end{aligned}$$

$$
-\frac{N_c}{8}\left\{\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}(\Lambda_\ell^{(3)})_{\gamma\delta}}{M_3^2}+4\frac{(\Lambda_\ell^{(3)})_{\alpha\delta}(\Lambda_\ell^{(3)})_{\gamma\beta}}{M_3^2}+\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_\ell^{(1)})_{\gamma\delta}}{M_1^2}+\right.
$$

$$
-\frac{\log\frac{M_3^2}{M_1^2}}{M_3^2-M_1^2}\left[(\Lambda_\ell^{(31)})_{\alpha\beta}(\Lambda_\ell^{(31)\dagger})_{\gamma\delta}+(\Lambda_\ell^{(31)\dagger})_{\alpha\beta}(\Lambda_\ell^{(31)})_{\gamma\delta}+\right.
$$

$$
\left.\left.-2(\Lambda_\ell^{(31)})_{\alpha\delta}(\Lambda_\ell^{(31)\dagger})_{\gamma\beta}-2(\Lambda_\ell^{(31)\dagger})_{\alpha\delta}(\Lambda_\ell^{(31)})_{\gamma\beta}\right]\right\}, \tag{3.72}
$$

$$
[C_{ee}]^{(1)}_{\alpha\beta\gamma\delta}=-\frac{N_c}{60}g'^4Y_e^2\delta_{\alpha\beta}\delta_{\gamma\delta}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+
$$

$$
+\frac{N_c}{6}g'^2Y_\ell\left(\frac{8Y_u-Y_{S_1}}{6}+Y_uL_1\right)(\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma}+\delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma})\frac{(\Lambda_e)_{\rho\sigma}}{M_1^2}+
$$

$$
-\frac{N_c}{8}\frac{(\Lambda_e)_{\alpha\beta}(\Lambda_e)_{\gamma\delta}}{M_1^2}, \tag{3.73}
$$

$$
[C_{\ell e}]^{(1)}_{\alpha\beta\gamma\delta}=-\frac{N_c}{30}g'^4Y_\ell Y_e\delta_{\alpha\beta}\delta_{\gamma\delta}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+
$$

$$
+\frac{N_c}{3}g'^2Y_e\left(3\left(\frac{1}{6}Y_\ell+Y_qL_3\right)\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}\delta_{\gamma\delta}}{M_3^2}+\left(\frac{1}{6}Y_\ell+Y_qL_1\right)\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}\delta_{\gamma\delta}}{M_1^2}\right)+
$$

$$
+\frac{N_c}{3}g'^2Y_\ell\left(\frac{8Y_u-Y_{S_1}}{6}+Y_uL_1\right)\frac{\delta_{\alpha\beta}(\Lambda_e)_{\gamma\delta}}{M_1^2}+
$$

$$
+\frac{N_c}{4}\left(\frac{1}{2}+L_1\right)\frac{(X_{1U}^{1L})_{\alpha\delta}(y_E^\dagger)_{\gamma\beta}+(X_{1U}^{1L\dagger})_{\gamma\beta}(y_E)_{\alpha\delta}}{M_1^2}-\frac{N_c}{4}\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_e)_{\gamma\delta}}{M_1^2}. \tag{3.74}
$$

Semileptonic.

$$
[C_{\ell q}^{(1)}]^{(1)}_{\alpha\beta ij}=\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)\left[g_s^2\frac{N_c^2-1}{2N_c}+g'^2(Y_q-Y_\ell)^2\right]\left(\frac{3\lambda_{i\alpha}^{3L*}\lambda_{j\beta}^{3L}}{M_3^2}+\frac{\lambda_{i\alpha}^{1L*}\lambda_{j\beta}^{1L}}{M_1^2}\right)+ \tag{3.75}
$$

$$
-\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)g^2\,3\left(\frac{\lambda_{i\alpha}^{3L*}\lambda_{j\beta}^{3L}}{M_3^2}+\frac{\lambda_{i\alpha}^{1L*}\lambda_{j\beta}^{1L}}{M_1^2}\right)+ \tag{3.76}
$$

$$
-\left[\frac{1}{2}(\delta Z_\ell)_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{lj}+\frac{1}{2}\delta_{\alpha\gamma}(\delta Z_\ell)_{\delta\beta}\delta_{ik}\delta_{lj}+\right.
$$

$$
\left.+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}(\delta Z_q)_{ik}\delta_{jl}+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}(\delta Z_q)_{lj}\right][C_{\ell q}^{(1)}]^{(0)}_{\gamma\delta kl}-\frac{N_c}{30}g'^4Y_\ell Y_q\delta_{\alpha\beta}\delta_{ij}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+
$$

$$
+\frac{1}{3}g'^2Y_\ell\delta_{\alpha\beta}\left\{3\left(\frac{1}{6}Y_q+Y_\ell L_3\right)\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}+\left(\frac{1}{6}Y_q+Y_\ell L_1\right)\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right\}+
$$

$$
+\frac{N_c}{3}g'^2Y_q\delta_{ij}\left(3\left(\frac{1}{6}Y_\ell+Y_qL_3\right)\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2}+\left(\frac{1}{6}Y_\ell+Y_qL_1\right)\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2}\right)+
$$

$$
\begin{aligned}
&-\frac{1}{4}\left(3\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}(\Lambda_q^{(3)})_{ij}}{M_3^2}+\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_q^{(1)})_{ij}}{M_1^2}\right)+\\
&+\left(c_1(1+L_1)+\frac{9}{4}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}\right)\frac{\lambda^{1L\dagger}_{\alpha i}\lambda^{1L}_{j\beta}}{M_1^2}+\\
&+\left(\frac{9}{4}(1+L_1)c_{13}^{(1)}\frac{M_1^2}{M_3^2}+\frac{3}{2}(1+L_3)\left[5c_3^{(1)}-c_3^{(3)}+\left(2+\frac{N_c^2-1}{2N_c}\right)c_3^{(5)}\right]\right)\frac{\lambda^{3L\dagger}_{\alpha i}\lambda^{3L}_{j\beta}}{M_3^2},
\end{aligned}
\tag{3.77}
$$

$$
[C_{\ell q}^{(3)}]^{(1)}_{\alpha\beta ij}=\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)\left[g_s^2\frac{N_c^2-1}{2N_c}+g'^2(Y_q-Y_\ell)^2\right]\left(\frac{\lambda^{3L*}_{i\alpha}\lambda^{3L}_{j\beta}}{M_3^2}-\frac{\lambda^{1L*}_{i\alpha}\lambda^{1L}_{j\beta}}{M_1^2}\right)+
\tag{3.78}
$$

$$
-\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)g^2\left(-\frac{3\lambda^{3L*}_{i\alpha}\lambda^{3L}_{j\beta}}{M_3^2}+\frac{\lambda^{1L*}_{i\alpha}\lambda^{1L}_{j\beta}}{M_1^2}\right)+
\tag{3.79}
$$

$$
\begin{aligned}
&-\left[\frac{1}{2}(\delta Z_\ell)_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{lj}+\frac{1}{2}\delta_{\alpha\gamma}(\delta Z_\ell)_{\delta\beta}\delta_{ik}\delta_{lj}+\right.\\
&\left.+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}(\delta Z_q)_{ik}\delta_{jl}+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}(\delta Z_q)_{lj}\right][C_{\ell q}^{(3)}]^{(0)}_{\gamma\delta kl}-\frac{N_c}{60}g^4\delta_{\alpha\beta}\delta_{ij}\frac{1}{M_3^2}+\\
&+\frac{1}{12}g^2\delta_{\alpha\beta}\left(\left(\frac{1}{2}+L_3\right)\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}-\left(-\frac{1}{6}+L_1\right)\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right)+\\
&+\frac{N_c}{12}g^2\delta_{ij}\left(\left(\frac{1}{2}+L_3\right)\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2}-\left(-\frac{1}{6}+L_1\right)\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2}\right)+\\
&-\frac{1}{4}\left(2\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}(\Lambda_q^{(3)})_{ij}}{M_3^2}+\frac{\log\frac{M_3^2}{M_1^2}\left[(\Lambda_\ell^{(31)})_{\alpha\beta}(\Lambda_q^{(31)})_{ij}+(\Lambda_\ell^{(31)\dagger})_{\alpha\beta}(\Lambda_q^{(31)\dagger})_{ij}\right]}{M_3^2-M_1^2}\right)+\\
&-\left(c_1(1+L_1)+\frac{9}{4}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}\right)\frac{\lambda^{1L\dagger}_{\alpha i}\lambda^{1L}_{j\beta}}{M_1^2}+\\
&+\left(\frac{3}{4}(1+L_1)c_{13}^{(1)}\frac{M_1^2}{M_3^2}+\frac{1}{2}(1+L_3)\left[5c_3^{(1)}-c_3^{(3)}+\left(2+\frac{N_c^2-1}{2N_c}\right)c_3^{(5)}\right]\right)\frac{\lambda^{3L\dagger}_{\alpha i}\lambda^{3L}_{j\beta}}{M_3^2},
\end{aligned}
\tag{3.80}
$$

$$
[C_{eu}]^{(1)}_{\alpha\beta ij}=\frac{1}{2}\left(\frac{1}{2}+a_{\rm ev}\right)\left[g_s^2\frac{N_c^2-1}{2N_c}+g'^2(Y_u-Y_e)^2\right]\frac{\lambda^{1R*}_{i\alpha}\lambda^{1R}_{j\beta}}{M_1^2}+
\tag{3.81}
$$

$$
\begin{aligned}
&-\left[\frac{1}{2}(\delta Z_e)_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{lj}+\frac{1}{2}\delta_{\alpha\gamma}(\delta Z_e)_{\delta\beta}\delta_{ik}\delta_{lj}+\right.\\
&\left.+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}(\delta Z_u)_{ik}\delta_{jl}+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}(\delta Z_u)_{lj}\right][C_{eu}]^{(0)}_{\gamma\delta kl}+\\
&-\frac{N_c}{30}g'^4Y_eY_u\delta_{\alpha\beta}\delta_{ij}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\
&+\frac{1}{3}g'^2Y_e\left(\frac{8Y_e-Y_{S_1}}{6}+Y_eL_1\right)\frac{\delta_{\alpha\beta}(\Lambda_u)_{ij}}{M_1^2}+\\
&+\frac{N_c}{3}g'^2Y_u\left(\frac{8Y_u-Y_{S_1}}{6}+Y_uL_1\right)\frac{\delta_{ij}(\Lambda_e)_{\alpha\beta}}{M_1^2}+\\
&-\frac{1}{4}\frac{(\Lambda_e)_{\alpha\beta}(\Lambda_u)_{ij}}{M_1^2}+\left(2c_1(1+L_1)+\frac{9}{2}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}\right)\frac{\lambda^{1R\dagger}_{\alpha i}\lambda^{1R}_{j\beta}}{M_1^2},
\end{aligned}
\tag{3.82}
$$

JHEP07(2020)225

$$
\begin{aligned}
[C_{ed}]^{(1)}_{\alpha\beta ij} =& -\frac{N_c}{30} g'^4 Y_e Y_d \delta_{\alpha\beta}\delta_{ij}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\
&+\frac{N_c}{3} g'^2 Y_d \left(\frac{8Y_u - Y_{S_1}}{6}+Y_u L_1\right)\frac{\delta_{ij}(\Lambda_e)_{\alpha\beta}}{M_1^2},
\end{aligned} \tag{3.83}
$$

$$
\begin{aligned}
[C_{qe}]^{(1)}_{ij\alpha\beta} =& -\frac{N_c}{30} g'^4 Y_q Y_e \delta_{ij}\delta_{\alpha\beta}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\
&+\frac{1}{3} g'^2 Y_e \left\{3\left(\frac{1}{6}Y_q+Y_\ell L_3\right)\frac{\delta_{kl}(\Lambda_q^{(3)})_{ij}}{M_3^2}+\left(\frac{1}{6}Y_q+Y_\ell L_1\right)\frac{(\Lambda_q^{(1)})_{ij}\delta_{\alpha\beta}}{M_1^2}\right\}+\\
&+\frac{N_c}{3} g'^2 Y_q \left(\frac{8Y_u - Y_{S_1}}{6}+Y_u L_1\right)\frac{\delta_{ij}(\Lambda_e)_{\alpha\beta}}{M_1^2}+\\
&-\frac{1}{4}\frac{(\Lambda_q^{(1)})_{ij}(\Lambda_e)_{\alpha\beta}}{M_1^2}-\frac{3}{4}(1+L_3)\frac{(\lambda^{3L*}y_E^*)_{i\alpha}(\lambda^{3L}y_E)_{j\beta}}{M_3^2}+\\
&-\frac{1}{4}(1+L_1)\frac{(\lambda^{1L*}y_E^* - y_U\lambda^{1R*})_{i\alpha}(\lambda^{1L}y_E - y_U^*\lambda^{1R})_{j\beta}}{M_1^2},
\end{aligned} \tag{3.84}
$$

$$
\begin{aligned}
[C_{\ell u}]^{(1)}_{\alpha\beta ij} =& -\frac{N_c}{30} g'^4 Y_\ell Y_u \delta_{ij}\delta_{\alpha\beta}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\
&+\frac{1}{3} g'^2 Y_\ell \left(\frac{8Y_e - Y_{S_1}}{6}+Y_e L_1\right)\frac{\delta_{\alpha\beta}(\Lambda_u)_{ij}}{M_1^2}+\\
&+\frac{N_c}{3} g'^2 Y_u \left(3\left(\frac{1}{6}Y_\ell+Y_q L_3\right)\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}\delta_{ij}}{M_3^2}+\left(\frac{1}{6}Y_\ell+Y_q L_1\right)\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}\delta_{ij}}{M_1^2}\right)+\\
&-\frac{1}{4}\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_u)_{ij}}{M_1^2}-\frac{3}{4}(1+L_3)\frac{(\lambda^{3L\dagger}y_U^*)_{\alpha i}(\lambda^{3L\,T}y_U)_{\beta j}}{M_3^2}+\\
&-\frac{1}{4}(1+L_1)\frac{(\lambda^{1L\,\dagger}y_U^* - y_E\lambda^{1R\dagger})_{\alpha i}(\lambda^{1L\,T}y_U - y_E^*\lambda^{1RT})_{\beta j}}{M_1^2},
\end{aligned} \tag{3.85}
$$

$$
\begin{aligned}
[C_{\ell d}]^{(1)}_{\alpha\beta ij} =& -\frac{N_c}{30} g'^4 Y_\ell Y_d \delta_{ij}\delta_{\alpha\beta}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right)+\\
&+\frac{N_c}{3} g'^2 Y_d \left(3\left(\frac{1}{6}Y_\ell+Y_q L_3\right)\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}\delta_{ij}}{M_3^2}+\left(\frac{1}{6}Y_\ell+Y_q L_1\right)\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}\delta_{ij}}{M_1^2}\right)+\\
&-\frac{1}{4}\left(3(1+L_3)\frac{(\lambda^{3L\dagger}y_D^*)_{\alpha i}(\lambda^{3L\,T}y_D)_{\beta j}}{M_3^2}+(1+L_1)\frac{(\lambda^{1L\,\dagger}y_D^*)_{\alpha i}(\lambda^{1L\,T}y_D)_{\beta j}}{M_1^2}\right),
\end{aligned} \tag{3.86}
$$

$$
\begin{aligned}
[C_{\ell edq}]^{(1)}_{\alpha\beta ij} =& -\frac{N_c}{2}\left(\frac{1}{2}+L_1\right)\frac{(y_D^\dagger)_{ij}(X_{1U}^{1L})_{\alpha\beta}}{M_1^2}+\\
&-\frac{1}{2}\left(-3(1+L_3)\frac{(\lambda^{3L\,\dagger}y_D^*)_{\alpha i}(\lambda^{3L}y_E)_{j\beta}}{M_3^2}+(1+L_1)\frac{(\lambda^{1L\,\dagger}y_D^*)_{\alpha i}(\lambda^{1L}y_E)_{j\beta}}{M_1^2}\right)+\\
&+\frac{1}{2}(1+L_1)\frac{(\lambda^{1L\,\dagger}y_D^*)_{\alpha i}(y_U^*\lambda^{1R})_{j\beta}}{M_1^2},
\end{aligned} \tag{3.87}
$$

$$
\begin{aligned}
[C^{(1)}_{\ell equ}]^{(1)}_{\alpha\beta ij} =& -\left[\frac{1}{2}(\delta Z_\ell)_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{lj}+\frac{1}{2}\delta_{\alpha\gamma}(\delta Z_e)_{\delta\beta}\delta_{ik}\delta_{lj}+\right.\\
&\left.+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}(\delta Z_q)_{ik}\delta_{jl}+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}(\delta Z_u)_{lj}\right][C^{(1)}_{\ell equ}]^{(0)}_{\gamma\delta kl}+
\end{aligned}
$$

$$
+\frac{1}{2}\left(L_1+\frac{1}{2}\right)\frac{(y_E)_{\alpha\beta}(X_{1E}^{1L})_{ij}}{M_1^2}+\frac{N_c}{2}\left(\frac{1}{2}+L_1\right)\frac{(y_U)_{ij}(X_{1U}^{1L})_{\alpha\beta}}{M_1^2}+
$$

$$
+\frac{\lambda_{\alpha i}^{1L\dagger}\lambda_{j\beta}^{1R}}{M_1^2}\left\{2(1+L_1)c_1+\frac{9}{2}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}+\right.
$$

$$
\left.-\frac{3}{2}\left(\frac{3}{2}+L_1\right)\left[(Y_q-Y_\ell)(Y_u-Y_e)g'^2+\frac{N_c^2-1}{2N_c}g_s^2\right]\right\}, \tag{3.88}
$$

$$
[C_{\ell equ}^{(3)}]_{\alpha\beta ij}^{(1)} = -\left[\frac{1}{2}(\delta Z_\ell)_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}\delta_{lj}+\frac{1}{2}\delta_{\alpha\gamma}(\delta Z_e)_{\delta\beta}\delta_{ik}\delta_{lj}\right.
$$

$$
\left.+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}(\delta Z_q)_{ik}\delta_{jl}+\frac{1}{2}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ik}(\delta Z_u)_{lj}\right][C_{\ell equ}^{(3)}]_{\gamma\delta kl}^{(0)}+
$$

$$
+\frac{\lambda_{\alpha i}^{1L\dagger}\lambda_{j\beta}^{1R}}{M_1^2}\left\{-\frac{1}{4}(1+L_1)c_1-\frac{9}{8}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}+\right.
$$

$$
\left.-\frac{1}{8}\left(\frac{3}{2}+L_1\right)\left[(Y_q-Y_\ell)(Y_u-Y_e)g'^2+\frac{N_c^2-1}{2N_c}g_s^2\right]\right\}. \tag{3.89}
$$

4 Conclusions

In this work we have presented the complete one-loop matching conditions, up to dimension-six SMEFT operators, for the $S_1 + S_3$ leptoquark model. This is one of the few available examples of a complete one-loop matching onto the SMEFT, and is substantially richer than previous ones due to the presence of two heavy fields charged under the SM gauge groups, coupled to SM fermions with a non-trivial flavour structure, and with potential couplings with the Higgs boson as well as themselves. The matching was performed diagrammatically, by direct comparison of full theory and EFT 1LPI off-shell Green's functions, and can serve as a cross-check for functional or computer methods devoted to the same task.

As a by-product of this work, we have extended the Warsaw basis of dimension-six SMEFT operators, to a full Green's basis, where only integration by parts (without SM EOMs) are used to reduce the number of independent operators. This set provides an operator basis for off-shell 1PI Green's functions. We have provided the complete reduction equations from Green's to Warsaw basis, which we believe to be of general interest for any kind of SMEFT matching or computation beyond the leading order. All relevant information related to the Green's basis is contained in the appendices.

The model studied in the present paper has been known for a while to provide a good candidate combined explanation of neutral- and charged-current B-physics anomalies. The tools developed in the present paper allows a thorough and complete study of the model's phenomenology, which we will explore in a separate contribution.

Acknowledgments

We thank D. Sutherland and A. Crivellin for useful discussions. DM acknowledges support by the INFN grant SESAMO, MIUR grant PRIN_2017L5W2PT. D.M is also partially supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme, grant agreement 833280 (FLAY). EV has been partially supported by the DFG Cluster of Excellence 2094 ORIGINS, the Collaborative Research Center SFB1258 and the BMBF grant 05H18WOCA1 and thanks the Munich Institute for Astro- and Particle Physics (MIAPP) for hospitality.

A A Green's basis for the SMEFT

In this appendix we present a basis of dimension-six IBP independent SMEFT operators, i.e. the Green's basis, extending the Warsaw basis of IBP and EOM independent operators. The operator basis is given in tables 1 (bosonic operators), 2 (single fermionic current operators), and 3, 4 (four fermion operators), in which Warsaw basis operators are highlighted in blue; we count 132 independent operators for a single generation of SM fermions, including baryon number violating ones. The following discussion is mainly a re-adaptation of the line of reasoning of ref. [3] (to which we refer the reader for further clarification), with the important exception that we are not allowed to use of EOMs. The strategy is to simply examine all possible Lorentz-invariant combinations of gauge field strengths, covariant derivatives, standard model fermions and the Higgs field, denoted X, D, ψ and H respectively.

In tables 1, 2, 3 and 4, we list all Green's basis operators (Warsaw basis [3] operators in blue color).

Bosonic operators.

$\boldsymbol{X^3}$. All independent operators are contained in Warsaw basis.

$\boldsymbol{X^2D^2}$. If we allow X to be possibly dual, there is no need to consider contractions involving the ϵ tensor. Thus, the indices of the two derivatives must either be contracted (a) between themselves, (b) with the indices of a single tensor or (c) with the indices of the two different tensors. In the case (b), antisymmetry of X and $[D_\mu, D_\nu] \sim X_{\mu\nu}$ brings us to X^3 class. In the case (a), we first note that taking both tensors to be dual is equivalent to considering no dual tensor. For the other two possibilities, we can take all derivatives to act on a single tensor and use Bianchi identities for the Yang-Mills tensors to obtain (here Y is possibly dual, but X is not):

$$Y^{\mu\nu} D^\rho D_\rho X_{\mu\nu} = -Y^{\mu\nu} D^\rho (D_\mu X_{\nu\rho} + D_\nu X_{\rho\mu}),$$

which is equivalent to case (c). We are thus left with case (c) where, due to Bianchi identities $D^\mu \widetilde{F}_{\mu\nu} = 0$, there exist only the following three possibilities:

$$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2\,, \qquad \mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W_{\mu\nu})^2\,, \qquad \mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G_{\mu\nu})^2\,. \tag{A.1}$$

$\boldsymbol{X^2H^2}$. All independent operators are contained in Warsaw basis.

$\boldsymbol{XD^4}$. Because of its antisymmetry, the indices of X must be contracted with two derivatives. This brings us to class X^2D^2 (see above).

XH^2D^2. By hypercharge conservation, these operators must involve one field H and one conjugate $H^\dagger$. Using the IBP freedom, we can assume the two derivatives to act either on H or X. Moreover, since the indices of X must be contracted with the two derivatives, the latters get antisymmetrized. Thus, if the two derivatives both act on X or H, we are moved to X^2H^2 class (see above). The remaining possibilities are two operators which, modulo a total divergence and X^2H^2 class operators, can be taken as:

$$\begin{aligned} \mathcal{O}_{BDH} &= \partial^\nu B_{\mu\nu}(H^\dagger i\overleftrightarrow{D}^\mu H)\,, \\ \mathcal{O}_{WDH} &= (D^\nu W_{\mu\nu})^I(H^\dagger i\overleftrightarrow{D}^{\mu I} H)\,. \end{aligned} \tag{A.2}$$

Another possibility sometimes used in the literature is to use instead the operators $\mathcal{O}_{HHB} = i(D_\mu H)^\dagger(D_\nu H)B^{\mu\nu}$ and $\mathcal{O}_{HHW} = i(D_\mu H)^\dagger\sigma^I(D_\nu H)W^{I\,\mu\nu}$, related via

$$\begin{aligned} \mathcal{O}_{BDH} &= 2\mathcal{O}_{HHB} + g'Y_H\mathcal{O}_{HB} + \frac{g}{2}\mathcal{O}_{HWB}\,, \\ \mathcal{O}_{WDH} &= 2\mathcal{O}_{HHW} + g'Y_H\mathcal{O}_{HWB} + \frac{g}{2}\mathcal{O}_{HW}\,. \end{aligned} \tag{A.3}$$

H^2D^4. The covariant derivatives must be contracted between themselves, either through the metric $\eta_{\mu\nu}$ or the volume form ϵ. Contracting them through the ϵ tensor moves us to H^2X^2 class. Hence, modulo total divergencies, the only operator in this class is:

$$\mathcal{O}_{DH} = (D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)\,. \tag{A.4}$$

H^4D^2. Because of hypercharge conservation, these operators involve exactly two fields H and two conjugate fields H^*. Moreover, the two derivatives must be contracted together. We must thus form Lorentz and $\mathrm{SU}(2)_L$ singlets by choosing four fields out of the four independent scalars:

$$H,\ H^*,\ D_\mu D^\mu H,\ D_\mu D^\mu H^*,$$

and the two independent vectors

$$D_\mu H, D^\mu H^*,$$

and, of course, by taking exactly two derivatives. We explore all possible field contents:

a) $D_\mu H, D^\mu H, H^*, H^*$. The two H^*'s must necessarily form a triplet, so that we have a single $\mathrm{SU}(2)_L$ contraction. Given the field content, such operator is clearly non-hermitian.

b) $D_\mu H^*, D^\mu H^*, H, H$. The two H's must necessarily form a triplet, so that we have a single $\mathrm{SU}(2)_L$ contraction. This is the hermitian conjugate of the previous case (a).

c) $D_\mu H, H, D^\mu H^*, H^*$. If H and H^* are contracted in a singlet (triplet), so must be $D_\mu H$ and $D^\mu H^*$, so that we have two independent $\mathrm{SU}(2)_L$ contractions, which are readily verified to be hermitian.

d) $D_\mu D^\mu H, H, H^*, H^*$. By integration by parts, we can reduce this to cases (a, b, c) above.

e) $H, H, H^*, D_\mu D^\mu H^*$. By integration by parts, we can reduce this to cases (a, b, c) above.

Thus we have, modulo divergencies, four independent hermitian singlets. We can complement the two Warsaw basis ones as follows:

$$\begin{aligned}
\mathcal{O}_{H\Box} &= (H^\dagger H)\Box(H^\dagger H)\,,\\
\mathcal{O}_{HD} &= (H^\dagger D_\mu H)^\dagger(H^\dagger D^\mu H)\,,\\
\mathcal{O}'_{HD} &= (H^\dagger H)(D_\mu H)^\dagger(D^\mu H)\,,\\
\mathcal{O}''_{HD} &= (H^\dagger H)D_\mu\left(H^\dagger i\overleftrightarrow{D}^\mu H\right)\,.
\end{aligned}\tag{A.5}$$

Other operators sometimes encountered in the literature are:

$$\begin{aligned}
\mathcal{O}_T^{(1)} &= \frac{1}{2}(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger\overleftrightarrow{D}^\mu H) = -2\mathcal{O}_{HD} - \frac{1}{2}\mathcal{O}_{H\Box}\,,\\
\mathcal{O}_T^{(3)} &= \frac{1}{2}(H^\dagger\overleftrightarrow{D}_\mu^I H)(H^\dagger\overleftrightarrow{D}^{\mu I} H) = -2\mathcal{O}'_{HD} - \frac{1}{2}\mathcal{O}_{H\Box}\,.
\end{aligned}\tag{A.6}$$

H^6. This operator is contained in Warsaw basis.

Two-fermion operators. Before considering the various cases, let us make a preliminary observation: these operators are obtained by contracting a scalar, vector or tensor current built out of two fermionic fields, with a corresponding current obtained from bosonic fields; it is easy to see that the only scalar (tensor) two-fermion currents allowed are $\overline{q}(\sigma_{\mu\nu})u$, $\overline{q}(\sigma_{\mu\nu})d$, $\overline{\ell}(\sigma_{\mu\nu})e$ and their conjugates, while the only vector currents allowed are $\overline{\psi}\gamma^\mu\psi$, with $\psi = q,\, u,\, d,\, \ell,\, e$, and $\overline{u}\gamma^\mu d$ with its conjugate. This is so because these operators conserve both lepton and baryon numbers. In fact, since B changes by integer units only, clearly $\Delta B = 0$. Moreover, since $\Delta(B-L) = 0$ at dimension-six level [38], this also implies $\Delta L = 0$.

$\Psi^2 D^3$. By hypercharge invariance, from the list of allowed two-fermion currents above, we can only pick $J^\mu = \overline{\psi}\gamma^\mu\psi$, with $\psi = q,\, u,\, d,\, \ell,\, e$. If we contract J^μ with the three covariant derivatives through an ϵ tensor, we move to $\psi^2 DX$ class (see below). Thus, at least two derivatives must be contracted with each other. Taking all derivatives to act on ψ, the only remaining possibility is:

$$\overline{\psi}i\not{D}D^2\psi = \frac{1}{2}\overline{\psi}\left\{i\not{D}, D^2\right\}\psi + (\psi^2 DX).$$

We are thus left with the five hermitian operators:

$$\mathcal{O}_{\psi D} = \frac{i}{2}\overline{\psi}\left\{D_\mu D^\mu, \not{D}\right\}\psi\,, \qquad \psi = q,\, u,\, d,\, \ell,\, e\,. \tag{A.7}$$

$\Psi^2 XD$. By hypercharge invariance, the two fermions must pair in a vector current $J^\mu = \overline{\psi}\gamma^\mu\psi$, with $\psi = q,\, u,\, d,\, \ell,\, e$. If we allow X to be dual, we do not need to consider contractions through the ϵ tensor, and the Lorentz structure is completely specified by $J^\mu X_{\mu\nu}D^\nu$. Using the IBP freedom, we can arrange that D^μ never acts on $\overline{\psi}$. We are left with the possibilities summarized by:

$$\begin{aligned}
\mathcal{O}_{X\psi} &= (\overline{\psi}\, t^a_{X\psi}\gamma^\mu\psi)D^\nu X^a_{\mu\nu}, &\quad \psi &\in \{q,\, u,\, d,\, \ell,\, e\}\,, &\quad X &\in \{G,\, W,\, B\}\,,\\
\mathcal{O}'_{X\psi} &= (\overline{\psi}\, t^a_{X\psi}\gamma^\mu iD^\nu\psi)X^a_{\mu\nu}, &\quad \psi &\in \{q,\, u,\, d,\, \ell,\, e\}\,, &\quad X &\in \left\{G,\, \widetilde{G},\, W,\, \widetilde{W},\, B,\, \widetilde{B}\right\}.
\end{aligned}\tag{A.8}$$

Here $t_{B\psi} \equiv 1$, while $t_{W\psi}$ and $t_{G\psi}$ are the SU(2) and SU(3) generators in the representation of ψ (possibly zero). These three combinations are independent, since:

- The Feynman rules of the pure derivative parts of $\mathcal{O}^{(\prime)}_{X\psi}$ vanish at some special kinematical configuration. These configurations are distinct for $\mathcal{O}_{X\psi}$ and $\mathcal{O}'_{X\psi}$.
- $\mathcal{O}'_{X\psi}$ is CP-odd for dual X, and CP-even otherwise.

Notice that $\mathcal{O}_{X\psi}$ vanishes for dual X, by Bianchi identities.

$\Psi^2 D^2 H$. By gauge invariance, the allowed field contents are those appearing in the Yukawa lagrangian (i.e. $\bar{q}u\widetilde{H}$, $\bar{q}dH$ and $\bar{\ell}eH$, plus hermitian conjugates). Let us take, for the sake of clarity, $\bar{\ell}eH$. Integrating by parts, we can make the two derivatives act either on e or H. If they both act on e or H, combining $\bar{\ell}$ and e into a tensor current moves us to class $\psi^2 XH$ (see below). We must then combine $\bar{\ell}$ and e into a scalar current, and we are left with:

$$\begin{aligned} \mathcal{O}_{eHD1} &= (\bar{\ell}e)D_\mu D^\mu H, \\ \mathcal{O}_{eHD3} &= (\bar{\ell}D_\mu D^\mu e)H. \end{aligned} \tag{A.9}$$

If, instead, one derivative acts on e and the other on H, we get other two possibilities:

$$\begin{aligned} \mathcal{O}_{eHD2} &= (\bar{\ell}i\sigma_{\mu\nu}D^\mu e)D^\nu H, \\ \mathcal{O}_{eHD4} &= (\bar{\ell}D_\mu e)D^\mu H. \end{aligned} \tag{A.10}$$

$\Psi^2 XH$. All independent operators are contained in Warsaw basis.

$\Psi^2 DH^2$. By Lorentz invariance, the two fermions must combine into a vector current, that is $J^\mu = \bar{\psi}\gamma^\mu\psi$, with $\psi = q, u, d, \ell, e$, or $J^\mu = \bar{u}\gamma^\mu d$ together with its conjugate. Moreover, currents involving the two SU(2) doublets q or ℓ, can either form an SU(2) singlet or triplet, to be coupled to the corresponding Higgs singlet or triplet current (cf. eqs. (2.9)). Finally, since the external fields have three independent momenta, for a given current $J^\mu = \bar{\psi}_1\gamma^\mu\psi_2$, we can form at most three independent Lorentz singlets, which we conveniently choose as:

$$\begin{aligned} \mathcal{O} &= [\bar{\psi}_1\gamma^\mu\psi_2]\cdot[H^\dagger i\overleftrightarrow{D}_\mu H] \\ \mathcal{O}' &= [\bar{\psi}_1 i\overleftrightarrow{D}_\mu\gamma^\mu\psi_2]\cdot[H^\dagger H] \\ \mathcal{O}'' &= [\bar{\psi}_1\gamma^\mu\psi_2]\cdot[D_\mu(H^\dagger H)] \end{aligned} \tag{A.11}$$

(the objects in square brackets are either SU(2) singlets or triplets, when possible). This results in the operators listed in the $\psi^2 DH^2$ box of table 2 (notice that for the non-hermitian $\bar{u}d$ current, the operators $\mathcal{O}'_{Hud}$ and $\mathcal{O}''_{Hud}$ vanish identically, as $(\widetilde{H}^\dagger H) = 0$).

$\Psi^2 H^3$. All independent operators are contained in Warsaw basis.

Four-fermion operators. All independent operators are contained in Warsaw basis.

B Reduction of Green's basis to Warsaw basis

We give in this appendix the reduction equations from the Green's basis to the Warsaw basis, which can be obtained by applying the SM EOMs to the operator basis derived in the previous appendix. The SM EOM are:

$$
\begin{aligned}
(D_\mu D^\mu H)^a &= m^2 H^a - \lambda(H^\dagger H)H^a - \bar{e}y_E^\dagger \ell^a + i(\sigma_2)^{ab}\bar{q}^b y_U u - \bar{d}y_D^\dagger q^a\,,\\
(D^\nu G_{\mu\nu})^A &= g_s(\bar{q}_i\gamma_\mu T^A q_i + \bar{u}_i\gamma_\mu T^A u_i + \bar{d}_i\gamma_\mu T^A d_i)\,,\\
(D^\nu W_{\mu\nu})^I &= \frac{g}{2}(H^\dagger i \overleftrightarrow{D}_\mu^I H + \bar{\ell}_\alpha\gamma_\mu\sigma^I\ell_\alpha + \bar{q}_i\gamma_\mu\sigma^I q_i)\,,\\
(\partial^\nu B_{\mu\nu}) &= g'\left(Y_H H^\dagger i\overleftrightarrow{D}_\mu H + \sum_f Y_f \bar{f}\gamma_\mu f\right),\\
i\not{D}\ell &= y_E e H\,,\\
i\not{D}e &= y_E^\dagger H^\dagger \ell\,,\\
i\not{D}q &= y_U u\widetilde{H} + y_D d H\,,\\
i\not{D}u &= y_U^\dagger \widetilde{H}^\dagger q\,,\\
i\not{D}d &= y_D^\dagger H^\dagger q\,.
\end{aligned}
\tag{B.1}
$$

Schematically, the change of basis formulae are given in the form $C_i = \sum_j a_{ij}\, G_j$, where C_i and G_j are the Warsaw and Green's basis WCs, respectively, and a_{ij} is a function of SM couplings. All quantities are understood to be evaluated at the same scale.

B.1 Renormalizable operators

$$
Z_\Phi = Z_\Phi^{\mathrm{G}} \qquad \Phi = H,\, q,\, u,\, d,\, \ell,\, e,\, G,\, W,\, B
$$

$$
\delta\lambda = \delta\lambda^{\mathrm{G}} - g^2 m^2 G_{2W} + 4\lambda m^2 G_{DH} + 4 g m^2 G_{WDH}, \tag{B.2}
$$

$$
\delta m^2 = (\delta m^2)^{\mathrm{G}} + m^4 G_{DH} - m^4 G'_{HD}, \tag{B.3}
$$

$$
(\delta y_E)_{\alpha\beta} = (\delta y_E^{\mathrm{G}})_{\alpha\beta} + m^2 G_{DH}(y_E)_{\alpha\beta} - m^2[G_{eHD1}]_{\alpha\beta} - \frac{1}{2}m^2[G_{eHD2}]_{\alpha\beta} + \frac{1}{2}m^2[G_{eHD4}]_{\alpha\beta}, \tag{B.4}
$$

$$
(\delta y_U)_{ij} = (\delta y_U^{\mathrm{G}})_{ij} + m^2 G_{DH}(y_U)_{ij} - m^2[G_{uHD1}]_{ij} - \frac{1}{2}m^2[G_{uHD2}]_{ij} + \frac{1}{2}m^2[G_{uHD4}]_{ij}, \tag{B.5}
$$

$$
(\delta y_D)_{ij} = (\delta y_D^{\mathrm{G}})_{ij} + m^2 G_{DH}(y_D)_{ij} - m^2[G_{dHD1}]_{ij} - \frac{1}{2}m^2[G_{dHD2}]_{ij} + \frac{1}{2}m^2[G_{dHD4}]_{ij}. \tag{B.6}
$$

B.2 Purely bosonic operators

X^3.

$$\begin{aligned} C_{3G} &= G_{3G}, \\ C_{3\widetilde{G}} &= G_{3\widetilde{G}}, \\ C_{3W} &= G_{3W}, \\ C_{3\widetilde{W}} &= G_{3\widetilde{W}}. \end{aligned} \tag{B.7}$$

X^2H^2.

$$\begin{aligned} C_{HG} &= G_{HG}, \\ C_{H\widetilde{G}} &= G_{H\widetilde{G}}, \\ C_{HW} &= G_{HW}, \\ C_{H\widetilde{W}} &= G_{H\widetilde{W}}, \\ C_{HB} &= G_{HB}, \\ C_{H\widetilde{B}} &= G_{H\widetilde{B}}, \\ C_{HWB} &= G_{HWB}, \\ C_{H\widetilde{W}B} &= G_{H\widetilde{W}B}. \end{aligned} \tag{B.8}$$

H^4D^2.

$$C_{H\Box} = -\frac{3}{8}g^2 G_{2W} - \frac{1}{2}g'^2 Y_H^2 G_{2B} + \frac{3}{2}g G_{WDH} + g' Y_H G_{BDH} + G_{H\Box} + \frac{1}{2}G'_{HD}, \tag{B.9}$$

$$C_{HD} = -2g'^2 Y_H^2 G_{2B} + 4g' Y_H G_{BDH} + G_{HD}. \tag{B.10}$$

H^6.

$$C_H = -\frac{1}{2}g^2\lambda G_{2W} + 2g\lambda G_{WDH} + \lambda^2 G_{DH} + \lambda G'_{HD} + G_H. \tag{B.11}$$

B.3 Two-fermion operators

$\psi^2 XH$.

$$\begin{aligned} [C_{uG}]_{ij} = {} & \frac{1}{4}g_s (y_U)_{lj}[G_{qD}]_{il} + \frac{1}{4}g_s (y_U)_{il}[G_{uD}]_{lj} + \\ & - \frac{i}{4}(y_U)_{lj}[G'_{Gq}]_{il} - \frac{1}{4}(y_U)_{lj}[G'_{\widetilde{G}q}]_{il} + \frac{i}{4}(y_U)_{il}[G'_{Gu}]_{lj} - \frac{1}{4}(y_U)_{il}[G'_{\widetilde{G}u}]_{lj} + \\ & + \frac{1}{2}g_s \, [G_{uHD3}]_{ij} + \\ & + [G_{uG}]_{ij}, \end{aligned} \tag{B.12}$$

$$\begin{aligned} [C_{uW}]_{ij} = {} & \frac{1}{8}g (y_U)_{lj}[G_{qD}]_{il} + \\ & - \frac{i}{4}(y_U)_{lj}[G'_{Wq}]_{il} - \frac{1}{4}(y_U)_{lj}[G'_{\widetilde{W}q}]_{il} + \end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{8} g \, [G_{uHD2}]_{ij} + \frac{1}{8} g \, [G_{uHD4}]_{ij} + \\
&+ [G_{uW}]_{ij}, \qquad \text{(B.13)}
\end{aligned}
$$

$$
\begin{aligned}
[C_{uB}]_{ij} = &\frac{1}{4} g' Y_q (y_U)_{lj} [G_{qD}]_{il} + \frac{1}{4} g' Y_u (y_U)_{il} [G_{uD}]_{lj} + \\
&- \frac{i}{4} (y_U)_{lj} [G'_{Bq}]_{il} - \frac{1}{4} (y_U)_{lj} [G'_{\widetilde{B}q}]_{il} + \frac{i}{4} (y_U)_{il} [G'_{Bu}]_{lj} - \frac{1}{4} (y_U)_{il} [G'_{\widetilde{B}u}]_{lj} + \\
&+ \frac{1}{4} g' Y_H \, [G_{uHD2}]_{ij} + \frac{1}{2} g' Y_u \, [G_{uHD3}]_{ij} - \frac{1}{4} g' Y_H \, [G_{uHD4}]_{ij} + \\
&+ [G_{uB}]_{ij}, \qquad \text{(B.14)}
\end{aligned}
$$

$$
\begin{aligned}
[C_{dG}]_{ij} = &+ \frac{1}{4} g_s (y_D)_{lj} [G_{qD}]_{il} + \frac{1}{4} g_s (y_D)_{il} [G_{dD}]_{lj} + \\
&- \frac{i}{4} (y_D)_{lj} [G'_{Gq}]_{il} - \frac{1}{4} (y_D)_{lj} [G'_{\widetilde{G}q}]_{il} + \frac{i}{4} (y_D)_{il} [G'_{Gd}]_{lj} - \frac{1}{4} (y_D)_{il} [G'_{\widetilde{G}d}]_{lj} + \\
&+ \frac{1}{2} g_s \, [G_{dHD3}]_{ij} + \\
&+ [G_{dG}]_{ij}, \qquad \text{(B.15)}
\end{aligned}
$$

$$
\begin{aligned}
[C_{dW}]_{ij} = &+ \frac{1}{8} g (y_D)_{lj} [G_{qD}]_{il} + \\
&- \frac{i}{4} (y_D)_{lj} [G'_{Wq}]_{il} - \frac{1}{4} (y_D)_{lj} [G'_{\widetilde{W}q}]_{il} + \\
&- \frac{1}{8} g \, [G_{dHD2}]_{ij} + \frac{1}{8} g \, [G_{dHD4}]_{ij} + \\
&+ [G_{dW}]_{ij}, \qquad \text{(B.16)}
\end{aligned}
$$

$$
\begin{aligned}
[C_{dB}]_{ij} = &+ \frac{1}{4} g' Y_q (y_D)_{lj} [G_{qD}]_{il} + \frac{1}{4} g' Y_d (y_D)_{il} [G_{dD}]_{lj} + \\
&- \frac{i}{4} (y_D)_{lj} [G'_{Bq}]_{il} - \frac{1}{4} (y_D)_{lj} [G'_{\widetilde{B}q}]_{il} + \frac{i}{4} (y_D)_{il} [G'_{Bd}]_{lj} - \frac{1}{4} (y_D)_{il} [G'_{\widetilde{B}d}]_{lj} + \\
&- \frac{1}{4} g' Y_H \, [G_{dHD2}]_{ij} + \frac{1}{2} g' Y_d \, [G_{dHD3}]_{ij} + \frac{1}{4} g' Y_H \, [G_{dHD4}]_{ij} + \\
&+ [G_{dB}]_{ij}, \qquad \text{(B.17)}
\end{aligned}
$$

$$
\begin{aligned}
[C_{eW}]_{\alpha\beta} = &+ \frac{1}{8} g (y_E)_{\delta\beta} [G_{\ell D}]_{\alpha\delta} + \\
&- \frac{i}{4} (y_E)_{\delta\beta} [G'_{W\ell}]_{\alpha\delta} - \frac{1}{4} (y_E)_{\delta\beta} [G'_{\widetilde{W}\ell}]_{\alpha\delta} +
\end{aligned}
$$

$$- \frac{1}{8} g [G_{eHD2}]_{\alpha\beta} + \frac{1}{8} g [G_{eHD4}]_{\alpha\beta} +$$

$$+ [G_{eW}]_{\alpha\beta}, \tag{B.18}$$

$$[C_{eB}]_{\alpha\beta} = \frac{1}{4} g' Y_\ell (y_E)_{\delta\beta} [G_{\ell D}]_{\alpha\delta} + \frac{1}{4} g' Y_e (y_E)_{\alpha\delta} [G_{eD}]_{\delta\beta} +$$

$$- \frac{i}{4} (y_E)_{\delta\beta} [G'_{B\ell}]_{\alpha\delta} - \frac{1}{4} (y_E)_{\delta\beta} [G'_{\widetilde{B}\ell}]_{\alpha\delta} + \frac{i}{4} (y_E)_{\alpha\delta} [G'_{Be}]_{\delta\beta} - \frac{1}{4} (y_E)_{\alpha\delta} [G'_{\widetilde{B}e}]_{\delta\beta} +$$

$$- \frac{1}{4} g' Y_H [G_{eHD2}]_{\alpha\beta} + \frac{1}{2} g' Y_e [G_{eHD3}]_{\alpha\beta} + \frac{1}{4} g' Y_H [G_{eHD4}]_{\alpha\beta} +$$

$$+ [G_{eB}]_{\alpha\beta}. \tag{B.19}$$

$\boldsymbol{\psi^2 H^2 D}$.

$$[C^{(1)}_{Hq}]_{ij} = -g'^2 Y_H Y_q \delta_{ij} G_{2B} + g' Y_q \delta_{ij} G_{BDH} +$$

$$+ \frac{1}{4} (y_U)_{ik} (y_U^\dagger)_{lj} [G_{uD}]_{kl} - \frac{1}{4} (y_D)_{ik} (y_D^\dagger)_{lj} [G_{dD}]_{kl} +$$

$$+ g' Y_H [G_{Bq}]_{ij} - \frac{1}{2} g' Y_H [G'_{\widetilde{B}q}]_{ij} +$$

$$+ \frac{1}{8} (y_U^\dagger)_{lj} [G_{uHD2}]_{il} + \frac{1}{8} (y_U)_{il} [G_{uHD2}]^*_{jl} +$$

$$- \frac{1}{4} (y_U^\dagger)_{lj} [G_{uHD3}]_{il} - \frac{1}{4} (y_U)_{il} [G_{uHD3}]^*_{jl} +$$

$$+ \frac{1}{8} (y_U^\dagger)_{lj} [G_{uHD4}]_{il} + \frac{1}{8} (y_U)_{il} [G_{uHD4}]^*_{jl} +$$

$$- \frac{1}{8} (y_D^\dagger)_{lj} [G_{dHD2}]_{il} - \frac{1}{8} (y_D^\dagger)_{il} [G_{dHD2}]^*_{jl} +$$

$$+ \frac{1}{4} (y_D^\dagger)_{lj} [G_{dHD3}]_{il} + \frac{1}{4} (y_D)_{il} [G_{dHD3}]^*_{jl} +$$

$$- \frac{1}{8} (y_D^\dagger)_{lj} [G_{dHD4}]_{il} - \frac{1}{8} (y_D)_{il} [G_{dHD4}]^*_{jl} +$$

$$+ [G^{(1)}_{Hq}]_{ij}, \tag{B.20}$$

$$[C^{(3)}_{Hq}]_{ij} = -\frac{1}{4} g^2 \delta_{ij} G_{2W} + \frac{1}{2} g \delta_{ij} G_{WDH} +$$

$$- \frac{1}{4} (y_U)_{ik} (y_U^\dagger)_{lj} [G_{uD}]_{kl} - \frac{1}{4} (y_D)_{ik} (y_D^\dagger)_{lj} [G_{dD}]_{kl} +$$

$$+ \frac{1}{2} g [G_{Wq}]_{ij} - \frac{1}{4} g [G'_{\widetilde{W}q}]_{ij} +$$

$$- \frac{1}{8} (y_U^\dagger)_{lj} [G_{uHD2}]_{il} - \frac{1}{8} (y_U)_{il} [G_{uHD2}]^*_{jl} +$$

$$+ \frac{1}{4} (y_U^\dagger)_{lj} [G_{uHD3}]_{il} + \frac{1}{4} (y_U)_{il} [G_{uHD3}]^*_{jl} +$$

$$- \frac{1}{8} (y_U^\dagger)_{lj} [G_{uHD4}]_{il} - \frac{1}{8} (y_U)_{il} [G_{uHD4}]^*_{jl} +$$

$$
\begin{aligned}
&-\frac{1}{8}(y_D^\dagger)_{lj}\,[G_{dHD2}]_{il}-\frac{1}{8}(y_D^\dagger)_{il}\,[G_{dHD2}]^*_{jl}+\\
&+\frac{1}{4}(y_D^\dagger)_{lj}\,[G_{dHD3}]_{il}+\frac{1}{4}(y_D)_{il}\,[G_{dHD3}]^*_{jl}+\\
&-\frac{1}{8}(y_D^\dagger)_{lj}\,[G_{dHD4}]_{il}-\frac{1}{8}(y_D)_{il}\,[G_{dHD4}]^*_{jl}+\\
&+[G^{(3)}_{Hq}]_{ij}, \qquad\qquad (B.21)
\end{aligned}
$$

$$
\begin{aligned}
[C_{Hu}]_{ij}=&-g'^2Y_HY_u\delta_{ij}G_{2B}+g'Y_u\delta_{ij}G_{BDH}+\\
&-\frac{1}{2}(y_U^\dagger)_{ik}(y_U)_{lj}[G_{qD}]_{kl}+\\
&+g'Y_H[G_{Bu}]_{ij}-\frac{1}{2}g'Y_H[G'_{\widetilde{B}u}]_{ij}+\\
&-\frac{1}{4}(y_U^\dagger)_{il}\,[G_{uHD2}]_{lj}-\frac{1}{4}(y_U)_{lj}\,[G_{uHD2}]^*_{li}+\\
&+\frac{1}{4}(y_U^\dagger)_{il}\,[G_{uHD4}]_{lj}+\frac{1}{4}(y_U)_{lj}\,[G_{uHD4}]^*_{li}+\\
&+[G_{Hu}]_{ij}, \qquad\qquad (B.22)
\end{aligned}
$$

$$
\begin{aligned}
[C_{Hd}]_{ij}=&-g'^2Y_HY_d\delta_{ij}G_{2B}+g'Y_d\delta_{ij}G_{BDH}+\\
&-\frac{1}{2}(y_D^\dagger)_{ik}(y_D)_{lj}[G_{qD}]_{kl}+\\
&+g'Y_H[G_{Bd}]_{ij}-\frac{1}{2}g'Y_H[G'_{\widetilde{B}d}]_{ij}+\\
&+\frac{1}{4}(y_D^\dagger)_{il}\,[G_{dHD2}]_{lj}+\frac{1}{4}(y_D)_{lj}\,[G_{dHD2}]^*_{li}+\\
&-\frac{1}{4}(y_D^\dagger)_{il}\,[G_{dHD4}]_{lj}-\frac{1}{4}(y_D)_{lj}\,[G_{dHD4}]^*_{li}+\\
&+[G_{Hd}]_{ij}, \qquad\qquad (B.23)
\end{aligned}
$$

$$
\begin{aligned}
[C_{Hud}]_{ij}=&-\frac{1}{2}(y_U^\dagger)_{ik}(y_D)_{lj}[G_{qD}]_{kl}+\\
&-\frac{1}{2}(y_D)_{lj}\,[G_{uHD2}]^*_{li}+\frac{1}{2}(y_D)_{lj}\,[G_{uHD4}]^*_{li}+\\
&+\frac{1}{2}(y_U^\dagger)_{il}\,[G_{dHD2}]_{lj}-\frac{1}{2}(y_U^\dagger)_{il}\,[G_{dHD4}]_{lj}+\\
&+[G_{Hud}]_{ij}, \qquad\qquad (B.24)
\end{aligned}
$$

$$
\begin{aligned}
[C^{(1)}_{H\ell}]_{\alpha\beta}=&-g'^2Y_HY_\ell\delta_{\alpha\beta}G_{2B}+g'Y_\ell\delta_{\alpha\beta}G_{BDH}+\\
&-\frac{1}{4}(y_E)_{\alpha\gamma}(y_E^\dagger)_{\delta\beta}[G_{eD}]_{\gamma\delta}+\\
&+g'Y_H[G_{B\ell}]_{\alpha\beta}-\frac{1}{2}g'Y_H[G'_{\widetilde{B}\ell}]_{\alpha\beta}+\\
&-\frac{1}{8}(y_E^\dagger)_{\delta\beta}[G_{eHD2}]_{\alpha\delta}-\frac{1}{8}(y_E)_{\alpha\delta}[G_{eHD2}]^*_{\beta\delta}+
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{4}(y_E^\dagger)_{\delta\beta}[G_{eHD3}]_{\alpha\delta}+\frac{1}{4}(y_E)_{\alpha\delta}[G_{eHD3}]^*_{\beta\delta}+\\
&-\frac{1}{8}(y_E^\dagger)_{\delta\beta}[G_{eHD4}]_{\alpha\delta}-\frac{1}{8}(y_E)_{\alpha\delta}[G_{eHD4}]^*_{\beta\delta}+\\
&+[G^{(1)}_{H\ell}]_{\alpha\beta},
\end{aligned}
\tag{B.25}
$$

$$
\begin{aligned}
[C^{(3)}_{H\ell}]_{\alpha\beta}=&-\frac{1}{4}g^2\delta_{\alpha\beta}G_{2W}+\frac{1}{2}g\delta_{\alpha\beta}G_{WDH}+\\
&-\frac{1}{4}(y_E)_{\alpha\gamma}(y_E^\dagger)_{\delta\beta}[G_{eD}]_{\gamma\delta}+\\
&+\frac{g}{2}[G_{W\ell}]_{\alpha\beta}-\frac{g}{4}[G'_{\widetilde{W}\ell}]_{\alpha\beta}+\\
&-\frac{1}{8}(y_E^\dagger)_{\delta\beta}[G_{eHD2}]_{\alpha\delta}-\frac{1}{8}(y_E)_{\alpha\delta}[G_{eHD2}]^*_{\beta\delta}+\\
&+\frac{1}{4}(y_E^\dagger)_{\delta\beta}[G_{eHD3}]_{\alpha\delta}+\frac{1}{4}(y_E)_{\alpha\delta}[G_{eHD3}]^*_{\beta\delta}+\\
&-\frac{1}{8}(y_E^\dagger)_{\delta\beta}[G_{eHD4}]_{\alpha\delta}-\frac{1}{8}(y_E)_{\alpha\delta}[G_{eHD4}]^*_{\beta\delta}+\\
&+[G^{(3)}_{H\ell}]_{\alpha\beta},
\end{aligned}
\tag{B.26}
$$

$$
\begin{aligned}
[C_{He}]_{\alpha\beta}=&-g'^2Y_HY_e\delta_{\alpha\beta}G_{2B}+g'Y_e\delta_{\alpha\beta}G_{BDH}+\\
&-\frac{1}{2}(y_E^\dagger)_{\alpha\gamma}(y_E)_{\delta\beta}[G_{\ell D}]_{\gamma\delta}+\\
&+g'Y_H[G_{Be}]_{\alpha\beta}-\frac{1}{2}g'Y_H[G'_{\widetilde{B}e}]_{\alpha\beta}+\\
&+\frac{1}{4}(y_E^\dagger)_{\alpha\delta}[G_{eHD2}]_{\delta\beta}+\frac{1}{4}(y_E)_{\delta\beta}[G_{eHD2}]^*_{\delta\alpha}+\\
&-\frac{1}{4}(y_E^\dagger)_{\alpha\delta}[G_{eHD4}]_{\delta\beta}-\frac{1}{4}(y_E)_{\delta\beta}[G_{eHD4}]^*_{\delta\alpha}+\\
&+[G_{He}]_{\alpha\beta}.
\end{aligned}
\tag{B.27}
$$

ψ^2H^3.

$$
\begin{aligned}
[C_{uH}]_{ij}=&-\frac{1}{4}g^2(y_U)_{ij}G_{2W}+g(y_U)_{ij}G_{WDH}+\lambda(y_U)_{ij}G_{DH}+\frac{1}{2}(y_U)_{ij}G'_{HD}-i(y_U)_{ij}G''_{HD}+\\
&-\frac{1}{2}(y_U)_{lj}(y_Uy_U^\dagger)_{ik}[G_{qD}]_{kl}-\frac{1}{2}(y_U)_{ik}(y_U^\dagger y_U)_{lj}[G_{uD}]_{kl}+\\
&-\lambda[G_{uHD1}]_{ij}+\\
&+\frac{1}{2}\left(\frac{1}{2}(y_U^\dagger y_U)_{lj}\delta_{ik}+\frac{1}{2}(y_Uy_U^\dagger)_{ik}\delta_{lj}-\lambda\delta_{ik}\delta_{lj}\right)[G_{uHD2}]_{kl}-\frac{1}{2}(y_U)_{il}(y_U)_{kj}\,[G_{uHD2}]^*_{kl}\\
&-\frac{1}{2}(y_U^\dagger y_U)_{lj}\delta_{ik}\,[G_{uHD3}]_{kl}-\frac{1}{2}(y_U)_{il}(y_U)_{kj}\,[G_{uHD3}]^*_{kl}+\\
&+\frac{1}{2}\left(\frac{1}{2}(y_U^\dagger y_U)_{lj}\delta_{ik}-\frac{1}{2}(y_Uy_U^\dagger)_{ik}\delta_{lj}+\lambda\delta_{ik}\delta_{lj}\right)[G_{uHD4}]_{kl}+\\
&+(y_U)_{lj}[G'^{(1)}_{Hq}]_{il}+i(y_U)_{lj}[G''^{(1)}_{Hq}]_{il}-(y_U)_{lj}[G'^{(3)}_{Hq}]_{il}-i(y_U)_{lj}[G''^{(3)}_{Hq}]_{il}+
\end{aligned}
$$

$$+(y_U)_{ik}[G'_{Hu}]_{kj} - i(y_U)_{ik}[G''_{Hu}]_{kj} +$$

$$+[G_{uH}]_{ij}, \tag{B.28}$$

$$\begin{aligned}
[C_{dH}]_{ij} = &-\frac{1}{4}g^2(y_D)_{ij}G_{2W} + g(y_D)_{ij}G_{WDH} + \lambda(y_D)_{ij}G_{DH} + \frac{1}{2}(y_D)_{ij}G'_{HD} + i(y_D)_{ij}G''_{HD} + \\
&-\frac{1}{2}(y_D)_{lj}(y_D y_D^\dagger)_{ik}[G_{qD}]_{kl} - \frac{1}{2}(y_D)_{ik}(y_D^\dagger y_D)_{lj}[G_{dD}]_{kl} + \\
&-\lambda\,[G_{dHD1}]_{ij} + \\
&+\frac{1}{2}\left(\frac{1}{2}\delta_{ik}(y_D^\dagger y_D)_{lj} + \frac{1}{2}(y_D y_D^\dagger)_{ik}\delta_{lj} - \lambda\delta_{ik}\delta_{lj}\right)[G_{dHD2}]_{kl} - \frac{1}{2}(y_D)_{il}(y_D)_{kj}\,[G_{dHD2}]^*_{kl} + \\
&-\frac{1}{2}(y_D^\dagger y_D)_{lj}\,[G_{dHD3}]_{il} - \frac{1}{2}(y_D)_{il}(y_D)_{kj}\,[G_{dHD3}]^*_{kl} + \\
&+\frac{1}{2}\left(\frac{1}{2}\delta_{ik}(y_D^\dagger y_D)_{lj} - \frac{1}{2}(y_D y_D^\dagger)_{ik}\delta_{lj} + \lambda\delta_{ik}\delta_{lj}\right)[G_{dHD4}]_{kl} + \\
&+(y_D)_{lj}[G'^{(1)}_{Hq}]_{il} + i(y_D)_{lj}[G''^{(1)}_{Hq}]_{il} + (y_D)_{lj}[G'^{(3)}_{Hq}]_{il} + i(y_D)_{lj}[G''^{(3)}_{Hq}]_{il} + \\
&+(y_D)_{ik}[G'_{Hd}]_{kj} - i(y_D)_{ik}[G''_{Hd}]_{kj} + \\
&+[G_{dH}]_{ij},
\end{aligned} \tag{B.29}$$

$$\begin{aligned}
[C_{eH}]_{\alpha\beta} = &-\frac{1}{4}g^2(y_E)_{\alpha\beta}G_{2W} + g(y_E)_{\alpha\beta}G_{WDH} + \lambda(y_E)_{\alpha\beta}G_{DH} + \frac{1}{2}(y_E)_{\alpha\beta}G'_{HD} + i(y_E)_{\alpha\beta}G''_{HD} + \\
&-\frac{1}{2}(y_E)_{\delta\beta}(y_E y_E^\dagger)_{\alpha\gamma}[G_{\ell D}]_{\gamma\delta} - \frac{1}{2}(y_E)_{\alpha\gamma}(y_E^\dagger y_E)_{\delta\beta}[G_{eD}]_{\gamma\delta} + \\
&-\lambda[G_{eHD1}]_{\alpha\beta} + \\
&+\frac{1}{2}\left(\frac{1}{2}\delta_{\alpha\gamma}(y_E^\dagger y_E)_{\delta\beta} + \frac{1}{2}(y_E y_E^\dagger)_{\alpha\gamma}\delta_{\delta\beta} - \lambda\delta_{\alpha\gamma}\delta_{\delta\beta}\right)[G_{eHD2}]_{\gamma\delta} - \frac{1}{2}(y_E)_{\alpha\delta}(y_E)_{\gamma\beta}[G_{eHD2}]^*_{\gamma\delta} + \\
&-\frac{1}{2}(y_E^\dagger y_E)_{\delta\beta}[G_{eHD3}]_{\alpha\delta} - \frac{1}{2}(y_E)_{\alpha\delta}(y_E)_{\gamma\beta}[G_{eHD3}]^*_{\gamma\delta} + \\
&+\frac{1}{2}\left(\frac{1}{2}\delta_{\alpha\gamma}(y_E^\dagger y_E)_{\delta\beta} - \frac{1}{2}(y_E y_E^\dagger)_{\alpha\gamma}\delta_{\delta\beta} + \lambda\delta_{\alpha\gamma}\delta_{\delta\beta}\right)[G_{eHD4}]_{\gamma\delta} + \\
&+(y_E)_{\delta\beta}[G'^{(1)}_{H\ell}]_{\alpha\delta} + i(y_E)_{\delta\beta}[G''^{(1)}_{H\ell}]_{\alpha\delta} + (y_E)_{\delta\beta}[G'^{(3)}_{H\ell}]_{\alpha\delta} + i(y_E)_{\delta\beta}[G''^{(3)}_{H\ell}]_{\alpha\delta} + \\
&+(y_E)_{\alpha\gamma}[G'_{He}]_{\gamma\beta} - i(y_E)_{\alpha\gamma}[G''_{He}]_{\gamma\beta} + \\
&+[G_{eH}]_{\alpha\beta}.
\end{aligned} \tag{B.30}$$

B.4 Four-fermion operators

Four-quark.

$$\begin{aligned}
[C^{(1)}_{qq}]_{ijkl} = &-\frac{1}{4}g_s^2\left(\frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)G_{2G} - \frac{1}{2}g'^2Y_q^2\delta_{ij}\delta_{kl}G_{2B} + \\
&+\frac{1}{4}g_s\left(\frac{1}{2}\delta_{kj}\delta_{im}\delta_{ln} + \frac{1}{2}\delta_{il}\delta_{km}\delta_{jn} - \frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn} - \frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}\right)[G_{Gq}]_{mn} + \\
&-\frac{1}{8}g_s\left(\frac{1}{2}\delta_{kj}\delta_{im}\delta_{ln} + \frac{1}{2}\delta_{il}\delta_{km}\delta_{jn} - \frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn} - \frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}\right)[G'_{\widetilde{G}q}]_{mn} +
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}g'Y_q\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G_{Bq}]_{mn}+\\
&-\frac{1}{4}g'Y_q\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G'_{\widetilde{B}q}]_{mn}+\\
&+[G^{(1)}_{qq}]_{ijkl},
\end{aligned}
\tag{B.31}
$$

$$
\begin{aligned}
[C^{(3)}_{qq}]_{ijkl}=&-\frac{1}{8}g_s^2\delta_{il}\delta_{kj}G_{2G}-\frac{1}{8}g^2\delta_{ij}\delta_{kl}G_{2W}+\\
&+\frac{1}{8}g_s\left(\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)[G_{Gq}]_{mn}+\\
&-\frac{1}{16}g_s\left(\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)[G'_{\widetilde{G}q}]_{mn}+\\
&+\frac{1}{4}g\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G_{Wq}]_{mn}+\\
&-\frac{1}{8}g\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G'_{\widetilde{W}q}]_{mn}+\\
&+[G^{(3)}_{qq}]_{ijkl},
\end{aligned}
\tag{B.32}
$$

$$
\begin{aligned}
[C_{uu}]_{ijkl}=&-\frac{1}{4}g_s^2\left(\delta_{il}\delta_{kj}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)G_{2G}-\frac{1}{2}g'^2Y_u^2\delta_{ij}\delta_{kl}G_{2B}+\\
&+\frac{1}{4}g_s\left(-\frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn}-\frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}+\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)[G_{Gu}]_{mn}+\\
&-\frac{1}{8}g_s\left(-\frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn}-\frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}+\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)[G'_{\widetilde{G}u}]_{mn}+\\
&+\frac{1}{2}g'Y_u\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G_{Bu}]_{mn}+\\
&-\frac{1}{4}g'Y_u\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G'_{\widetilde{B}u}]_{mn}+\\
&+[G_{uu}]_{ijkl},
\end{aligned}
\tag{B.33}
$$

$$
\begin{aligned}
[C_{dd}]_{ijkl}=&-\frac{1}{4}g_s^2\left(\delta_{il}\delta_{kj}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)G_{2G}-\frac{1}{2}g'^2Y_d^2\delta_{ij}\delta_{kl}G_{2B}+\\
&+\frac{1}{4}g_s\left(-\frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn}-\frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}+\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)[G_{Gd}]_{mn}+\\
&-\frac{1}{8}g_s\left(-\frac{1}{3}\delta_{kl}\delta_{im}\delta_{jn}-\frac{1}{3}\delta_{ij}\delta_{km}\delta_{ln}+\delta_{kj}\delta_{im}\delta_{ln}+\delta_{il}\delta_{km}\delta_{jn}\right)[G'_{\widetilde{G}d}]_{mn}+\\
&+\frac{1}{2}g'Y_d\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G_{Bd}]_{mn}+\\
&-\frac{1}{4}g'Y_d\left(\delta_{kl}\delta_{im}\delta_{jn}+\delta_{ij}\delta_{km}\delta_{ln}\right)[G'_{\widetilde{B}d}]_{mn}+\\
&+[G_{dd}]_{ijkl},
\end{aligned}
\tag{B.34}
$$

$$
\begin{aligned}
[C_{ud}^{(1)}]_{ijkl} = & -g'^2 Y_u Y_d \delta_{ij}\delta_{kl} G_{2B} + \\
& + g' Y_d \delta_{kl} [G_{Bu}]_{ij} - \frac{1}{2} g' Y_d \delta_{kl} [G'_{\widetilde{B}u}]_{ij} + \\
& + g' Y_u \delta_{ij} [G_{Bd}]_{kl} - \frac{1}{2} g' Y_u \delta_{ij} [G'_{\widetilde{B}d}]_{kl} + \\
& + [G_{ud}^{(1)}]_{ijkl},
\end{aligned} \tag{B.35}
$$

$$
\begin{aligned}
[C_{ud}^{(8)}]_{ijkl} = & -g_s^2 \delta_{ij}\delta_{kl} G_{2G} + \\
& + g_s \delta_{kl} [G_{Gu}]_{ij} - \frac{1}{2} g_s \delta_{kl} [G'_{\widetilde{G}u}]_{ij} + g_s \delta_{ij} [G_{Gd}]_{kl} - \frac{1}{2} g_s \delta_{ij} [G'_{\widetilde{G}d}]_{kl} + \\
& + [G_{ud}^{(8)}]_{ijkl},
\end{aligned} \tag{B.36}
$$

$$
\begin{aligned}
[C_{qu}^{(1)}]_{ijkl} = & -g'^2 Y_q Y_u \delta_{ij}\delta_{kl} G_{2B} - \frac{1}{6} (y_U)_{il} (y_U^\dagger)_{kj} G_{DH} + \\
& + g' Y_u \delta_{kl} [G_{Bq}]_{ij} - \frac{1}{2} g' Y_u \delta_{kl} [G'_{\widetilde{B}q}]_{ij} + \\
& + g' Y_q \delta_{ij} [G_{Bu}]_{kl} - \frac{1}{2} g' Y_q \delta_{ij} [G'_{\widetilde{B}u}]_{kl} + \\
& + \frac{1}{6} (y_U^\dagger)_{kj} [G_{uHD1}]_{il} + \frac{1}{6} (y_U)_{il} [G_{uHD1}]^*_{jk} + \\
& + \frac{1}{12} (y_U^\dagger)_{kj} [G_{uHD2}]_{il} + \frac{1}{12} (y_U)_{il} [G_{uHD2}]^*_{jk} + \\
& - \frac{1}{12} (y_U^\dagger)_{kj} [G_{uHD4}]_{il} - \frac{1}{12} (y_U)_{il} [G_{uHD4}]^*_{jk} \\
& + [G_{qu}^{(1)}]_{ijkl},
\end{aligned} \tag{B.37}
$$

$$
\begin{aligned}
[C_{qu}^{(8)}]_{ijkl} = & -g_s^2 \delta_{ij}\delta_{kl} G_{2G} - (y_U)_{il} (y_U^\dagger)_{kj} G_{DH} + \\
& + g_s \delta_{kl} [G_{Gq}]_{ij} - \frac{1}{2} g_s \delta_{kl} [G'_{\widetilde{G}q}]_{ij} + g_s \delta_{ij} [G_{Gu}]_{kl} - \frac{1}{2} g_s \delta_{ij} [G'_{\widetilde{G}u}]_{kl} + \\
& + (y_U^\dagger)_{kj} [G_{uHD1}]_{il} + (y_U)_{il} [G_{uHD1}]^*_{jk} + \\
& + \frac{1}{2} (y_U^\dagger)_{kj} [G_{uHD2}]_{il} + \frac{1}{2} (y_U)_{il} [G_{uHD2}]^*_{jk} + \\
& - \frac{1}{2} (y_U^\dagger)_{kj} [G_{uHD4}]_{il} - \frac{1}{2} (y_U)_{il} [G_{uHD4}]^*_{jk} + \\
& + [G_{qu}^{(8)}]_{ijkl},
\end{aligned} \tag{B.38}
$$

$$
\begin{aligned}
[C_{qd}^{(1)}]_{ijkl} = & -g'^2 Y_q Y_d \delta_{ij}\delta_{kl} G_{2B} - \frac{1}{6} (y_D)_{il} (y_D^\dagger)_{kj} G_{DH} + \\
& + g' Y_d \delta_{kl} [G_{Bq}]_{ij} - \frac{1}{2} g' Y_d \delta_{kl} [G'_{\widetilde{B}q}]_{ij} +
\end{aligned}
$$

$$+ g'Y_q\delta_{ij}[G_{Bd}]_{kl} - \frac{1}{2}g'Y_q\delta_{ij}[G'_{\widetilde{B}d}]_{kl}+$$

$$+ \frac{1}{6}(y_D^\dagger)_{kj}\,[G_{dHD1}]_{il} + \frac{1}{6}(y_D)_{il}\,[G_{dHD1}]^*_{jk} +$$

$$+ \frac{1}{12}(y_D^\dagger)_{kj}\,[G_{dHD2}]_{il} + \frac{1}{12}(y_D)_{il}\,[G_{dHD2}]^*_{jk} +$$

$$- \frac{1}{12}(y_D^\dagger)_{kj}\,[G_{dHD4}]_{il} - \frac{1}{12}(y_D)_{il}\,[G_{dHD4}]^*_{jk} +$$

$$+ [G^{(1)}_{qd}]_{ijkl}, \tag{B.39}$$

$$[C^{(8)}_{qd}]_{ijkl} = -g_s^2\delta_{ij}\delta_{kl}G_{2G} - (y_D)_{il}(y_D^\dagger)_{kj}G_{DH}+$$

$$+ g_s\delta_{kl}[G_{Gq}]_{ij} - \frac{1}{2}g_s\delta_{kl}[G'_{\widetilde{G}q}]_{ij} + g_s\delta_{ij}[G_{Gd}]_{kl} - \frac{1}{2}g_s\delta_{ij}[G'_{\widetilde{G}d}]_{kl}+$$

$$+ (y_D^\dagger)_{kj}\,[G_{dHD1}]_{il} + (y_D)_{il}\,[G_{dHD1}]^*_{jk} +$$

$$+ \frac{1}{2}(y_D^\dagger)_{kj}\,[G_{dHD2}]_{il} + \frac{1}{2}(y_D)_{il}\,[G_{dHD2}]^*_{jk} +$$

$$- \frac{1}{2}(y_D^\dagger)_{kj}\,[G_{dHD4}]_{il} - \frac{1}{2}(y_D)_{il}\,[G_{dHD4}]^*_{jk} +$$

$$+ [G^{(8)}_{qd}]_{ijkl}, \tag{B.40}$$

$$[C^{(1)}_{quqd}]_{ijkl} = (y_U)_{ij}(y_D)_{kl}G_{DH}+$$

$$- (y_D)_{kl}[G_{uHD1}]_{ij} - \frac{1}{2}(y_D)_{kl}[G_{uHD2}]_{ij} + \frac{1}{2}(y_D)_{kl}[G_{uHD4}]_{ij}+$$

$$- (y_U)_{ij}[G_{dHD1}]_{kl} - \frac{1}{2}(y_U)_{ij}[G_{dHD2}]_{kl} + \frac{1}{2}(y_U)_{ij}[G_{dHD4}]_{kl}+$$

$$+ [G^{(1)}_{quqd}]_{ijkl}, \tag{B.41}$$

$$[C^{(8)}_{quqd}]_{ijkl} = [G^{(8)}_{quqd}]_{ijkl}. \tag{B.42}$$

Four-lepton.

$$[C_{\ell\ell}]_{\alpha\beta\gamma\delta} = -\frac{1}{8}g^2(2\delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\beta}\delta_{\gamma\delta})G_{2W} - \frac{1}{2}g'^2Y_\ell^2\delta_{\alpha\beta}\delta_{\gamma\delta}G_{2B}+$$

$$+ \frac{1}{4}g\left(-\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma} + 2\delta_{\gamma\beta}\delta_{\alpha\rho}\delta_{\delta\sigma} - \delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma} + 2\delta_{\alpha\delta}\delta_{\gamma\rho}\delta_{\beta\sigma}\right)[G_{W\ell}]_{\rho\sigma}+$$

$$- \frac{1}{8}g\left(-\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma} + 2\delta_{\gamma\beta}\delta_{\alpha\rho}\delta_{\delta\sigma} - \delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma} + 2\delta_{\alpha\delta}\delta_{\gamma\rho}\delta_{\beta\sigma}\right)[G'_{\widetilde{W}\ell}]_{\rho\sigma}+$$

$$+ \frac{1}{2}g'Y_\ell\left(\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma} + \delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma}\right)[G_{B\ell}]_{\rho\sigma}+$$

$$- \frac{1}{4}g'Y_\ell\left(\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma} + \delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma}\right)[G'_{\widetilde{B}\ell}]_{\rho\sigma}+$$

$$+ [G_{\ell\ell}]_{\alpha\beta\gamma\delta}, \tag{B.43}$$

JHEP07(2020)225

$$
\begin{aligned}
[C_{ee}]_{\alpha\beta\gamma\delta} = &-\frac{g'^2}{2} Y_e^2 \delta_{\alpha\beta}\delta_{\gamma\delta} G_{2B} + \\
&+ \frac{1}{2} g' Y_e \left(\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma} + \delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma}\right) [G_{Be}]_{\rho\sigma} + \\
&- \frac{1}{4} g' Y_e \left(\delta_{\gamma\delta}\delta_{\alpha\rho}\delta_{\beta\sigma} + \delta_{\alpha\beta}\delta_{\gamma\rho}\delta_{\delta\sigma}\right) [G'_{\widetilde{B}e}]_{\rho\sigma} + \\
&+ [G_{ee}]_{\alpha\beta\gamma\delta},
\end{aligned} \tag{B.44}
$$

$$
\begin{aligned}
[C_{\ell e}]_{\alpha\beta\gamma\delta} = &-g'^2 Y_\ell Y_e \delta_{\alpha\beta}\delta_{\gamma\delta} G_{2B} - \frac{1}{2}(y_E)_{\alpha\delta}(y_E^\dagger)_{\gamma\beta} G_{DH} + \\
&+ g' Y_e \delta_{\gamma\delta} \left([G_{B\ell}]_{\alpha\beta} - \frac{1}{2}[G'_{\widetilde{B}\ell}]_{\alpha\beta} \right) + \\
&+ g' Y_\ell \delta_{\alpha\beta} \left([G_{Be}]_{\gamma\delta} - \frac{1}{2}[G'_{\widetilde{B}e}]_{\gamma\delta} \right) + \\
&+ \frac{1}{2}(y_E^\dagger)_{\gamma\beta}[G_{eHD1}]_{\alpha\delta} + \frac{1}{2}(y_E)_{\alpha\delta}[G_{eHD1}]^*_{\beta\gamma} + \\
&+ \frac{1}{4}(y_E^\dagger)_{\gamma\beta}[G_{eHD2}]_{\alpha\delta} + \frac{1}{4}(y_E)_{\alpha\delta}[G_{eHD2}]^*_{\beta\gamma} + \\
&- \frac{1}{4}(y_E^\dagger)_{\gamma\beta}[G_{eHD4}]_{\alpha\delta} - \frac{1}{4}(y_E)_{\alpha\delta}[G_{eHD4}]^*_{\beta\gamma} + \\
&+ [G_{\ell e}]_{\alpha\beta\gamma\delta}.
\end{aligned} \tag{B.45}
$$

Semileptonic.

$$
\begin{aligned}
[C^{(1)}_{\ell q}]_{\alpha\beta ij} = &-g'^2 Y_\ell Y_q \delta_{\alpha\beta}\delta_{ij} G_{2B} + \\
&+ g' Y_\ell \delta_{\alpha\beta}[G_{Bq}]_{ij} - \frac{1}{2} g' Y_\ell \delta_{\alpha\beta}[G'_{\widetilde{B}q}]_{ij} + \\
&+ g' Y_q \delta_{ij}[G_{B\ell}]_{\alpha\beta} - \frac{1}{2} g' Y_q \delta_{ij}[G'_{\widetilde{B}\ell}]_{\alpha\beta} + \\
&+ [G^{(1)}_{\ell q}]_{\alpha\beta ij},
\end{aligned} \tag{B.46}
$$

$$
\begin{aligned}
[C^{(3)}_{\ell q}]_{\alpha\beta ij} = &-\frac{1}{4} g^2 \delta_{\alpha\beta}\delta_{ij} G_{2W} + \\
&+ \frac{1}{2} g \delta_{\alpha\beta}[G_{Wq}]_{ij} - \frac{1}{4} g \delta_{\alpha\beta}[G'_{\widetilde{W}q}]_{ij} + \\
&+ \frac{1}{2} g \delta_{ij}[G_{W\ell}]_{\alpha\beta} - \frac{1}{4} g \delta_{ij}[G'_{\widetilde{W}\ell}]_{\alpha\beta} + \\
&+ [G^{(3)}_{\ell q}]_{\alpha\beta ij},
\end{aligned} \tag{B.47}
$$

$$
\begin{aligned}
[C_{eu}]_{\alpha\beta ij} = &-g'^2 Y_e Y_u \delta_{\alpha\beta}\delta_{ij} G_{2B} + \\
&+ g' Y_e \delta_{\alpha\beta}[G_{Bu}]_{ij} - \frac{1}{2} g' Y_e \delta_{\alpha\beta}[G'_{\widetilde{B}u}]_{ij} + \\
&+ g' Y_u \delta_{ij}[G_{Be}]_{\alpha\beta} - \frac{1}{2} g' Y_u \delta_{ij}[G'_{\widetilde{B}e}]_{\alpha\beta} + \\
&+ [G_{eu}]_{\alpha\beta ij},
\end{aligned} \tag{B.48}
$$

$$
\begin{aligned}
[C_{ed}]_{\alpha\beta ij} = &-g'^2 Y_e Y_d \delta_{\alpha\beta}\delta_{ij} G_{2B}+\\
&+ g' Y_e \delta_{\alpha\beta}[G_{Bd}]_{ij} - \frac{1}{2} g' Y_e \delta_{\alpha\beta}[G'_{\widetilde{B}d}]_{ij}+\\
&+ g' Y_d \delta_{ij}[G_{Be}]_{\alpha\beta} - \frac{1}{2} g' Y_d \delta_{ij}[G'_{\widetilde{B}e}]_{\alpha\beta}+\\
&+ [G_{ed}]_{\alpha\beta ij},
\end{aligned}
\tag{B.49}
$$

$$
\begin{aligned}
[C_{qe}]_{ij\alpha\beta} = &-g'^2 Y_q Y_e \delta_{ij}\delta_{\alpha\beta} G_{2B}+\\
&+ g' Y_e \delta_{\alpha\beta}[G_{Bq}]_{ij} - \frac{1}{2} g' Y_e \delta_{\alpha\beta}[G'_{\widetilde{B}q}]_{ij}+\\
&+ g' Y_q \delta_{ij}[G_{Be}]_{\alpha\beta} - \frac{1}{2} g' Y_q \delta_{ij}[G'_{\widetilde{B}e}]_{\alpha\beta}+\\
&+ [G_{qe}]_{ij\alpha\beta},
\end{aligned}
\tag{B.50}
$$

$$
\begin{aligned}
[C_{\ell u}]_{\alpha\beta ij} = &-g'^2 Y_\ell Y_u \delta_{\alpha\beta}\delta_{ij} G_{2B}+\\
&+ g' Y_\ell \delta_{\alpha\beta}[G_{Bu}]_{ij} - \frac{1}{2} g' Y_\ell \delta_{\alpha\beta}[G'_{\widetilde{B}u}]_{ij}+\\
&+ g' Y_u \delta_{ij}[G_{B\ell}]_{\alpha\beta} - \frac{1}{2} g' Y_u \delta_{ij}[G'_{\widetilde{B}\ell}]_{\alpha\beta}+\\
&+ [G_{\ell u}]_{\alpha\beta ij},
\end{aligned}
\tag{B.51}
$$

$$
\begin{aligned}
[C_{\ell d}]_{\alpha\beta ij} = &-g'^2 Y_\ell Y_d \delta_{\alpha\beta}\delta_{ij} G_{2B}+\\
&+ g' Y_\ell \delta_{\alpha\beta}[G_{Bd}]_{ij} - \frac{1}{2} g' Y_\ell \delta_{\alpha\beta}[G'_{\widetilde{B}d}]_{ij}+\\
&+ g' Y_d \delta_{ij}[G_{B\ell}]_{\alpha\beta} - \frac{1}{2} g' Y_d \delta_{ij}[G'_{\widetilde{B}\ell}]_{\alpha\beta}+\\
&+ [G_{\ell d}]_{\alpha\beta ij},
\end{aligned}
\tag{B.52}
$$

$$
\begin{aligned}
[C^{(1)}_{\ell edq}]_{\alpha\beta ij} = &(y_E)_{\alpha\beta}(y_D^\dagger)_{ij} G_{DH}+\\
&- (y_E)_{\alpha\beta}[G_{dHD1}]^*_{ji} - \frac{1}{2}(y_E)_{\alpha\beta}[G_{dHD2}]^*_{ji} + \frac{1}{2}(y_E)_{\alpha\beta}[G_{dHD4}]^*_{ji}+\\
&- (y_D^\dagger)_{ij}[G_{eHD1}]_{\alpha\beta} - \frac{1}{2}(y_D^\dagger)_{ij}[G_{eHD2}]_{\alpha\beta} + \frac{1}{2}(y_D^\dagger)_{ij}[G_{eHD4}]_{\alpha\beta}+\\
&+ [G^{(1)}_{\ell edq}]_{\alpha\beta ij},
\end{aligned}
\tag{B.53}
$$

$$
\begin{aligned}
[C^{(1)}_{\ell equ}]_{\alpha\beta ij} = &-(y_E)_{\alpha\beta}(y_U)_{ij} G_{DH}+\\
&+ (y_E)_{\alpha\beta}[G_{uHD1}]_{ij} + \frac{1}{2}(y_E)_{\alpha\beta}[G_{uHD2}]_{ij} - \frac{1}{2}(y_E)_{\alpha\beta}[G_{uHD4}]_{ij}+\\
&+ (y_U)_{ij}[G_{eHD1}]_{\alpha\beta} + \frac{1}{2}(y_U)_{ij}[G_{eHD2}]_{\alpha\beta} - \frac{1}{2}(y_U)_{ij}[G_{eHD4}]_{\alpha\beta}+\\
&+ [G^{(1)}_{\ell equ}]_{\alpha\beta ij},
\end{aligned}
\tag{B.54}
$$

$$
[C^{(3)}_{\ell equ}]_{\alpha\beta ij} = [G^{(3)}_{\ell equ}]_{\alpha\beta ij}. \tag{B.55}
$$

***B*-violating.**

$$[C_{duq}]_{ijk\alpha} = [G_{duq}]_{ijk\alpha}, \tag{B.56}$$

$$[C_{qqu}]_{ijk\alpha} = [G_{qqu}]_{ijk\alpha}, \tag{B.57}$$

$$[C_{qqq}]_{ijk\alpha} = [G_{qqq}]_{ijk\alpha}, \tag{B.58}$$

$$[C_{duu}]_{ijk\alpha} = [G_{duu}]_{ijk\alpha}. \tag{B.59}$$

C One-loop matching conditions in the Green's basis

We report in this section the complete one-loop SMEFT matching contributions for the $S_1 + S_3$ model in the Green's basis. Green's basis WCs are denoted by G. We absorb the loop factor defining $G_i = \frac{1}{(4\pi)^2} G_i^{(1)}$.

C.1 Renormalizable operators

$$(\delta Z_\ell^{\rm G})_{\alpha\beta} = \frac{N_c}{2}\left[\left(\frac{1}{2} + L_1\right)(\Lambda_\ell^{(1)})_{\alpha\beta} + 3\left(\frac{1}{2} + L_3\right)(\Lambda_\ell^{(3)})_{\alpha\beta}\right], \tag{C.1}$$

$$(\delta Z_e^{\rm G})_{\alpha\beta} = \frac{N_c}{2}\left(\frac{1}{2} + L_1\right)(\Lambda_e)_{\alpha\beta}, \tag{C.2}$$

$$(\delta Z_q^{\rm G})_{ij} = \frac{1}{2}\left[\left(\frac{1}{2} + L_1\right)(\Lambda_q^{(1)})_{ij} + 3\left(\frac{1}{2} + L_3\right)(\Lambda_q^{(3)})_{ij}\right], \tag{C.3}$$

$$(\delta Z_u^{\rm G})_{ij} = \frac{1}{2}\left(\frac{1}{2} + L_1\right)(\Lambda_u)_{ij}, \tag{C.4}$$

$$(\delta Z_d^{\rm G})_{ij} = 0, \tag{C.5}$$

$$(\delta y_E^{\rm G})_{\alpha\beta} = -N_c(1 + L_1)(X_{1U}^{1L})_{\alpha\beta}, \tag{C.6}$$

$$(\delta y_U^{\rm G})_{ij} = -(1 + L_1)(X_{1E}^{1L})_{ij}, \tag{C.7}$$

$$(\delta y_D^{\rm G})_{ij} = 0, \tag{C.8}$$

$$\delta\lambda^{\rm G} = -N_c\left[\lambda_{H1}^2 L_1 + (3\lambda_{H3}^2 + 2\lambda_{\epsilon H3}^2)L_3 + 2|\lambda_{H13}|^2\left(1 + \frac{M_3^2 L_3 - M_1^2 L_1}{M_3^2 - M_1^2}\right)\right], \tag{C.9}$$

$$(\delta m^2)^{\rm G} = N_c\left[\lambda_{H1}(1 + L_1)M_1^2 + 3\lambda_{H3}(1 + L_3)M_3^2\right]. \tag{C.10}$$

C.2 Purely bosonic operators

***X*³.**

$$G_{3G}^{(1)} = \frac{g_s^3}{360}\left(\frac{3}{M_3^2} + \frac{1}{M_1^2}\right), \tag{C.11}$$

$$G_{3W}^{(1)} = \frac{g^3 N_c}{90 M_3^2}, \tag{C.12}$$

$$G_{3\widetilde{W}}^{(1)} = G_{3\widetilde{G}}^{(1)} = 0. \tag{C.13}$$

***X²D²*.**

$$G^{(1)}_{2G} = \frac{g_s^2}{60}\left(\frac{3}{M_3^2}+\frac{1}{M_1^2}\right), \tag{C.14}$$

$$G^{(1)}_{2W} = \frac{g^2 N_c}{15M_3^2}, \tag{C.15}$$

$$G^{(1)}_{2B} = \frac{g'^2 N_c}{30}\left(\frac{3Y_{S_3}^2}{M_3^2}+\frac{Y_{S_1}^2}{M_1^2}\right). \tag{C.16}$$

***X²H²*.**

$$G^{(1)}_{HG} = \frac{g_s^2}{12}\left(\frac{3\lambda_{H3}}{M_3^2}+\frac{\lambda_{H1}}{M_1^2}\right), \tag{C.17}$$

$$G^{(1)}_{HW} = \frac{g^2 N_c \lambda_{H3}}{3M_3^2}, \tag{C.18}$$

$$G^{(1)}_{HB} = \frac{g'^2 N_c}{6}\left(3\frac{\lambda_{H3}Y_{S_3}^2}{M_3^2}+\frac{\lambda_{H1}Y_{S_1}^2}{M_1^2}\right), \tag{C.19}$$

$$G^{(1)}_{HWB} = -N_c\frac{gg'Y_{S_3}\lambda_{\epsilon H3}}{3M_3^2}, \tag{C.20}$$

$$G^{(1)}_{H\widetilde{G}} = G^{(1)}_{H\widetilde{W}} = G^{(1)}_{H\widetilde{B}} = G^{(1)}_{H\widetilde{W}B} = 0. \tag{C.21}$$

***H²XD²*.**

$$G^{(1)}_{WDH} = G^{(1)}_{BDH} = 0. \tag{C.22}$$

***H²D⁴*.**

$$G^{(1)}_{DH} = 0. \tag{C.23}$$

***H⁴D²*.**

$$G^{(1)}_{H\Box} = -\frac{N_c}{12}\left(\frac{(3\lambda_{H3}^2+2\lambda_{\epsilon H3}^2)}{M_3^2}+\frac{\lambda_{H1}^2}{M_1^2}\right)-\frac{N_c}{2}\left|\lambda_{H13}\right|^2 h(M_1,M_3), \tag{C.24}$$

$$G^{(1)}_{HD} = -N_c\frac{2\lambda_{\epsilon H3}^2}{3M_3^2}-2N_c\left|\lambda_{H13}\right|^2 h(M_1,M_3), \tag{C.25}$$

$$G'^{(1)}_{HD} = +N_c\frac{2\lambda_{\epsilon H3}^2}{3M_3^2}+2N_c\left|\lambda_{H13}\right|^2 h(M_1,M_3), \tag{C.26}$$

$$G''^{(1)}_{HD} = 0. \tag{C.27}$$

***H⁶*.**

$$G^{(1)}_{H} = \frac{N_c}{6}\Bigg\{-\frac{3\lambda_{H3}^3+6\lambda_{\epsilon H3}^2\lambda_{H3}}{M_3^2}-\frac{\lambda_{H1}^3}{M_1^2}+ \\ +\frac{6\left|\lambda_{H13}\right|^2}{M_1^2-M_3^2}\left[\lambda_{H3}-\lambda_{H1}+\frac{\log\left(\frac{M_1^2}{M_3^2}\right)}{M_1^2-M_3^2}(\lambda_{H1}M_3^2-\lambda_{H3}M_1^2)\right]\Bigg\}. \tag{C.28}$$

C.3 Two-fermion operators

$\psi^2 D^3$.

$$[G_{qD}]^{(1)}_{ij} = -\frac{1}{6}\left(3\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2} + \frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right), \tag{C.29}$$

$$[G_{uD}]^{(1)}_{ij} = -\frac{1}{6}\frac{(\Lambda_u)_{ij}}{M_1^2}, \tag{C.30}$$

$$[G_{dD}]^{(1)}_{ij} = 0, \tag{C.31}$$

$$[G_{\ell D}]^{(1)}_{\alpha\beta} = -\frac{N_c}{6}\left(3\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2} + \frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2}\right), \tag{C.32}$$

$$[G_{eD}]^{(1)}_{\alpha\beta} = -\frac{N_c}{6}\frac{(\Lambda_e)_{\alpha\beta}}{M_1^2}. \tag{C.33}$$

$\psi^2 X D$.

$$[G_{Gq}]^{(1)}_{ij} = \frac{1}{18}g_s\left(3\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right), \tag{C.34}$$

$$[G_{Wq}]^{(1)}_{ij} = \frac{1}{6}g\left(\left(L_3+\frac{5}{4}\right)\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}-\left(L_1+\frac{7}{12}\right)\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right), \tag{C.35}$$

$$[G'_{\widetilde{W}q}]^{(1)}_{ij} = \frac{1}{4}g\left(\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}-\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right), \tag{C.36}$$

$$[G_{Bq}]^{(1)}_{ij} = \frac{1}{3}g'\left\{3\left(\frac{7Y_\ell-2Y_{S_3}}{12}+Y_\ell L_3\right)\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}+\left(\frac{7Y_\ell-2Y_{S_1}}{12}+Y_\ell L_1\right)\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right\}, \tag{C.37}$$

$$[G'_{\widetilde{B}q}]^{(1)}_{ij} = \frac{1}{2}g'Y_\ell\left(3\frac{(\Lambda_q^{(3)})_{ij}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{ij}}{M_1^2}\right), \tag{C.38}$$

$$[G_{Gu}]^{(1)}_{ij} = \frac{1}{18}g_s\frac{(\Lambda_u)_{ij}}{M_1^2}, \tag{C.39}$$

$$[G_{Bu}]^{(1)}_{ij} = \frac{1}{3}g'\left(\frac{7Y_e-2Y_{S_1}}{12}+Y_e L_1\right)\frac{(\Lambda_u)_{ij}}{M_1^2}, \tag{C.40}$$

$$[G'_{\widetilde{B}u}]^{(1)}_{ij} = -\frac{1}{2}g'Y_e\frac{(\Lambda_u)_{ij}}{M_1^2}, \tag{C.41}$$

$$[G_{W\ell}]^{(1)}_{\alpha\beta} = \frac{N_c}{6}g\left(\left(L_3+\frac{5}{4}\right)\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2}-\left(L_1+\frac{7}{12}\right)\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2}\right), \tag{C.42}$$

$$[G'_{\widetilde{W}\ell}]^{(1)}_{\alpha\beta} = \frac{N_c}{4}g\left(\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}}{M_3^2}-\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}}{M_1^2}\right), \tag{C.43}$$

$$[G_{B\ell}]^{(1)}_{\alpha\beta} = \frac{N_c}{3} g' \left(3 \left(\frac{7Y_q - 2Y_{S_3}}{12} + Y_q L_3 \right) \frac{(\Lambda^{(3)}_\ell)_{\alpha\beta}}{M_3^2} + \left(\frac{7Y_q - 2Y_{S_1}}{12} + Y_q L_1 \right) \frac{(\Lambda^{(1)}_\ell)_{\alpha\beta}}{M_1^2} \right), \tag{C.44}$$

$$[G'_{\widetilde{B}\ell}]^{(1)}_{\alpha\beta} = \frac{N_c}{2} g' Y_q \left(3 \frac{(\Lambda^{(3)}_\ell)_{\alpha\beta}}{M_3^2} + \frac{(\Lambda^{(1)}_\ell)_{\alpha\beta}}{M_1^2} \right), \tag{C.45}$$

$$[G_{Be}]^{(1)}_{\alpha\beta} = \frac{N_c}{3} g' \left(\frac{7Y_u - 2Y_{S_1}}{12} + Y_u L_1 \right) \frac{(\Lambda_e)_{\alpha\beta}}{M_1^2}, \tag{C.46}$$

$$[G'_{\widetilde{B}e}]^{(1)}_{ij} = -\frac{N_c}{2} g' Y_u \frac{(\Lambda_e)_{\alpha\beta}}{M_1^2}, \tag{C.47}$$

$$\begin{aligned}
[G'_{Gq}]^{(1)}_{ij} &= [G'_{\widetilde{G}q}]^{(1)}_{ij} = [G'_{Wq}]^{(1)}_{ij} = [G'_{Bq}]^{(1)}_{ij} = 0, \\
[G'_{Gu}]^{(1)}_{ij} &= [G'_{\widetilde{G}u}]^{(1)}_{ij} = [G'_{Bu}]^{(1)}_{ij} = 0, \\
[G_{G(B)d}]^{(1)}_{ij} &= [G'_{G(B)d}]^{(1)}_{ij} = [G'_{\widetilde{G}(\widetilde{B})d}]^{(1)}_{ij} = 0, \\
[G'_{W\ell}]^{(1)}_{\alpha\beta} &= [G'_{B\ell}]^{(1)}_{\alpha\beta} = [G'_{Be}]^{(1)}_{\alpha\beta} = 0.
\end{aligned} \tag{C.48}$$

$\psi^2 H D^2$.

$$[G_{uHD1}]^{(1)}_{ij} = +\frac{1}{2} \left(L_1 + \frac{1}{2} \right) \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{C.49}$$

$$[G_{uHD2}]^{(1)}_{ij} = -\frac{1}{2} \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{C.50}$$

$$[G_{uHD3}]^{(1)}_{ij} = -\frac{1}{2} \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{C.51}$$

$$[G_{uHD4}]^{(1)}_{ij} = -\frac{1}{2} \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{C.52}$$

$$[G_{dHDn}]^{(1)}_{ij} = 0 \; (n = 1, 2, 3, 4), \tag{C.53}$$

$$[G_{eHD1}]^{(1)}_{\alpha\beta} = +\frac{N_c}{2} \left(L_1 + \frac{1}{2} \right) \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}, \tag{C.54}$$

$$[G_{eHD2}]^{(1)}_{\alpha\beta} = -\frac{N_c}{2} \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}, \tag{C.55}$$

$$[G_{eHD3}]^{(1)}_{\alpha\beta} = -\frac{N_c}{2} \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}, \tag{C.56}$$

$$[G_{eHD4}]^{(1)}_{\alpha\beta} = -\frac{N_c}{2} \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}. \tag{C.57}$$

$\psi^2 XH$.

$$[G_{uG}]^{(1)}_{ij} = 0, \tag{C.58}$$

$$[G_{uW}]^{(1)}_{ij} = -\frac{1}{8} g \left(L_1 + \frac{1}{2} \right) \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{C.59}$$

$$[G_{uB}]^{(1)}_{ij} = \frac{1}{4} g' \left[(Y_l + Y_e) L_1 + \frac{1}{2} Y_l + \frac{3}{2} Y_e \right] \frac{(X^{1L}_{1E})_{ij}}{M_1^2}, \tag{C.60}$$

$$[G_{dG}]^{(1)}_{ij} = [G_{dW}]^{(1)}_{ij} = [G_{dB}]^{(1)}_{ij} = 0, \tag{C.61}$$

$$[G_{eW}]^{(1)}_{\alpha\beta} = -\frac{N_c}{8} g \left(L_1 + \frac{1}{2} \right) \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}, \tag{C.62}$$

$$[G_{eB}]^{(1)}_{\alpha\beta} = \frac{N_c}{4} g' \left[(Y_q + Y_u) L_1 + \frac{1}{2} Y_q + \frac{3}{2} Y_u \right] \frac{(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}. \tag{C.63}$$

$\psi^2 DH^2$.

$$[G^{(1)}_{Hq}]^{(1)}_{ij} = -\frac{1}{4} \left(3(1+L_3) \frac{(X^{3L}_{2E})_{ij}}{M_3^2} + (1+L_1) \frac{(X^{1L}_{2E})_{ij}}{M_1^2} \right), \tag{C.64}$$

$$[G'^{(1)}_{Hq}]^{(1)}_{ij} = -\frac{1}{8} \left(3 \frac{(X^{3L}_{2E})_{ij} + 2\lambda_{H3} (\Lambda_q^{(3)})_{ij}}{M_3^2} + \frac{(X^{1L}_{2E})_{ij} + 2\lambda_{H1} (\Lambda_q^{(1)})_{ij}}{M_1^2} \right), \tag{C.65}$$

$$[G''^{(1)}_{Hq}]^{(1)}_{ij} = 0, \tag{C.66}$$

$$[G^{(3)}_{Hq}]^{(1)}_{ij} = -\frac{1}{4} \left((1+L_3) \frac{(X^{3L}_{2E})_{ij}}{M_3^2} - (1+L_1) \frac{(X^{1L}_{2E})_{ij}}{M_1^2} \right), \tag{C.67}$$

$$[G'^{(3)}_{Hq}]^{(1)}_{ij} = -\frac{1}{8} \left(\frac{(X^{3L}_{2E})_{ij} + 4\lambda_{\epsilon H3} (\Lambda_q^{(3)})_{ij}}{M_3^2} - \frac{(X^{1L}_{2E})_{ij}}{M_1^2} \right) + \\ + \frac{1}{4} \frac{\left[\lambda^*_{H13} \Lambda_q^{(31)} + \lambda_{H13} \Lambda_q^{(31)\dagger} \right]_{ij} \log \frac{M_3^2}{M_1^2}}{M_3^2 - M_1^2}, \tag{C.68}$$

$$[G''^{(3)}_{Hq}]^{(1)}_{ij} = -\frac{1}{2} \left[i\lambda^*_{H13} \Lambda_q^{(31)} - i\lambda_{H13} \Lambda_q^{(31)\dagger} \right]_{ij} \frac{M_1^2 - M_3^2 + \frac{1}{2}(M_1^2 + M_3^2) \log \frac{M_3^2}{M_1^2}}{(M_1^2 - M_3^2)^2}, \tag{C.69}$$

$$[G_{Hu}]^{(1)}_{ij} = \frac{1}{2} (1+L_1) \frac{(X^{1R}_{2E})_{ij}}{M_1^2}, \tag{C.70}$$

$$[G'_{Hu}]^{(1)}_{ij} = -\frac{1}{4} \frac{(X^{1R}_{2E})_{ij} + \lambda_{H1} (\Lambda_u)_{ij}}{M_1^2}, \tag{C.71}$$

$$[G''_{Hu}]^{(1)}_{ij} = 0\,, \tag{C.72}$$

$$[G_{Hd}]^{(1)}_{ij} = [G'_{Hd}]^{(1)}_{ij} = [G''_{Hd}]^{(1)}_{ij} = 0, \tag{C.73}$$

$$[G^{(1)}_{H\ell}]^{(1)}_{\alpha\beta} = -\frac{N_c}{4}\left\{3(1+L_3)\frac{-(X^{3L}_{2U})_{\alpha\beta}+(X^{3L}_{2D})_{\alpha\beta}}{M_3^2}+(1+L_1)\frac{-(X^{1L}_{2U})_{\alpha\beta}+(X^{1L}_{2D})_{\alpha\beta}}{M_1^2}\right\}, \tag{C.74}$$

$$[G'^{(1)}_{H\ell}]^{(1)}_{\alpha\beta} = -\frac{N_c}{8}\left\{3\frac{(X^{3L}_{2U})_{\alpha\beta}+(X^{3L}_{2D})_{\alpha\beta}+2\lambda_{H3}(\Lambda^{(3)}_\ell)_{\alpha\beta}}{M_3^2}+\right.$$

$$\left.+\frac{(X^{1L}_{2U})_{\alpha\beta}+(X^{1L}_{2D})_{\alpha\beta}+2\lambda_{H1}(\Lambda^{(1)}_\ell)_{\alpha\beta}}{M_1^2}\right\}, \tag{C.75}$$

$$[G''^{(1)}_{H\ell}]^{(1)}_{\alpha\beta} = 0, \tag{C.76}$$

$$[G^{(3)}_{H\ell}]^{(1)}_{\alpha\beta} = -\frac{N_c}{4}\left\{(1+L_3)\frac{(X^{3L}_{2U})_{\alpha\beta}+(X^{3L}_{2D})_{\alpha\beta}}{M_3^2}-(1+L_1)\frac{(X^{1L}_{2U})_{\alpha\beta}+(X^{1L}_{2D})_{\alpha\beta}}{M_1^2}\right\}, \tag{C.77}$$

$$[G'^{(3)}_{H\ell}]^{(1)}_{\alpha\beta} = +N_c\left\{-\frac{1}{8}\left(\frac{-(X^{3L}_{2U})_{\alpha\beta}+(X^{3L}_{2D})_{\alpha\beta}+4\lambda_{\epsilon H3}(\Lambda^{(3)}_\ell)_{\alpha\beta}}{M_3^2}-\frac{-(X^{1L}_{2U})_{\alpha\beta}+(X^{1L}_{2D})_{\alpha\beta}}{M_1^2}\right)+\right.$$

$$\left.-\frac{1}{4}\frac{\left[\lambda^*_{H13}\Lambda^{(31)}_\ell+\lambda_{H13}\Lambda^{(31)\dagger}_\ell\right]_{\alpha\beta}\log\frac{M_3^2}{M_1^2}}{M_3^2-M_1^2}\right\}, \tag{C.78}$$

$$[G''^{(3)}_{H\ell}]^{(1)}_{\alpha\beta} = \frac{N_c}{2}\left[i\lambda^*_{H13}\Lambda^{(31)}_\ell-i\lambda_{H13}\Lambda^{(31)\dagger}_\ell\right]_{\alpha\beta}\frac{M_1^2-M_3^2+\frac{1}{2}(M_1^2+M_3^2)\log\frac{M_3^2}{M_1^2}}{(M_1^2-M_3^2)^2}, \tag{C.79}$$

$$[G_{He}]^{(1)}_{\alpha\beta} = -\frac{N_c}{2}(1+L_1)\frac{(X^{1R}_{2U})_{\alpha\beta}}{M_1^2}, \tag{C.80}$$

$$[G'_{He}]^{(1)}_{\alpha\beta} = -\frac{N_c}{4}\frac{(X^{1R}_{2U})_{\alpha\beta}+\lambda_{H1}(\Lambda_e)_{\alpha\beta}}{M_1^2}, \tag{C.81}$$

$$[G''^{(3)}_{He}]^{(1)}_{\alpha\beta} = 0\,. \tag{C.82}$$

$\boldsymbol{\psi^2 H^3}$**.**

$$[G_{uH}]^{(1)}_{ij} = \frac{(1+L_1)(X^{1L}_{3E})_{ij}-\lambda_{H1}(X^{1L}_{1E})_{ij}}{M_1^2}-\frac{\lambda^*_{H13}(X^{3L}_{1E})_{ij}\log\frac{M_1^2}{M_3^2}}{M_1^2-M_3^2}, \tag{C.83}$$

$$[G_{dH}]^{(1)}_{ij} = 0, \tag{C.84}$$

$$[G_{eH}]^{(1)}_{\alpha\beta} = N_c\frac{(1+L_1)(X^{1L}_{3U})_{\alpha\beta}-\lambda_{H1}(X^{1L}_{1U})_{\alpha\beta}}{M_1^2}-N_c\frac{\lambda^*_{H13}(X^{3L}_{1U})_{\alpha\beta}\log\frac{M_1^2}{M_3^2}}{M_1^2-M_3^2}. \tag{C.85}$$

C.4 Four-fermion operators

Four-quark.

$$[G_{qq}^{(1)}]_{ijkl}^{(1)} = -\frac{1}{16}\left(9\frac{(\Lambda_q^{(3)})_{il}(\Lambda_q^{(3)})_{kj}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{il}(\Lambda_q^{(1)})_{kj}}{M_1^2}+3\frac{\log\frac{M_3^2}{M_1^2}\left[(\Lambda_q^{31})_{il}(\Lambda_q^{31\dagger})_{kj}+(\Lambda_q^{31\dagger})_{il}(\Lambda_q^{31})_{kj}\right]}{M_3^2-M_1^2}\right), \tag{C.86}$$

$$[G_{qq}^{(3)}]_{ijkl}^{(1)} = -\frac{1}{16}\left(\frac{(\Lambda_q^{(3)})_{il}(\Lambda_q^{(3)})_{kj}}{M_3^2}+\frac{(\Lambda_q^{(1)})_{il}(\Lambda_q^{(1)})_{kj}}{M_1^2}-\frac{\log\frac{M_3^2}{M_1^2}\left[(\Lambda_q^{31})_{il}(\Lambda_q^{31\dagger})_{kj}+(\Lambda_q^{31\dagger})_{il}(\Lambda_q^{31})_{kj}\right]}{M_3^2-M_1^2}\right), \tag{C.87}$$

$$[G_{uu}]_{ijkl}^{(1)} = -\frac{1}{8}\frac{(\Lambda_u)_{il}(\Lambda_u)_{kj}}{M_1^2}, \tag{C.88}$$

$$[G_{dd}]_{ijkl}^{(1)} = [G_{ud}^{(1)}]_{ijkl}^{(1)} = [G_{ud}^{(8)}]_{ijkl}^{(1)} = 0, \tag{C.89}$$

$$[G_{qu}^{(1)}]_{ijkl}^{(1)} = -\frac{1}{12M_1^2}(\Lambda_q^{(1)})_{ij}(\Lambda_u)_{kl}, \tag{C.90}$$

$$[G_{qu}^{(8)}]_{ijkl}^{(1)} = -\frac{1}{2M_1^2}(\Lambda_q^{(1)})_{ij}(\Lambda_u)_{kl}, \tag{C.91}$$

$$[G_{qd}^{(1)}]_{ijkl}^{(1)} = [G_{qd}^{(8)}]_{ijkl}^{(1)} = [G_{quqd}^{(1)}]_{ijkl}^{(1)} = [G_{quqd}^{(8)}]_{ijkl}^{(1)} = 0. \tag{C.92}$$

Four-lepton.

$$\begin{aligned}[G_{\ell\ell}]_{\alpha\beta\gamma\delta}^{(1)} = -\frac{N_c}{8}\Bigg\{&\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}(\Lambda_\ell^{(3)})_{\gamma\delta}}{M_3^2}+4\frac{(\Lambda_\ell^{(3)})_{\alpha\delta}(\Lambda_\ell^{(3)})_{\gamma\beta}}{M_3^2}+\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_\ell^{(1)})_{\gamma\delta}}{M_1^2}+\\ &-\frac{\log\frac{M_3^2}{M_1^2}}{M_3^2-M_1^2}\Big[(\Lambda_\ell^{(31)})_{\alpha\beta}(\Lambda_\ell^{(31)\dagger})_{\gamma\delta}+(\Lambda_\ell^{(31)\dagger})_{\alpha\beta}(\Lambda_\ell^{(31)})_{\gamma\delta}+\\ &-2(\Lambda_\ell^{(31)})_{\alpha\delta}(\Lambda_\ell^{(31)\dagger})_{\gamma\beta}-2(\Lambda_\ell^{(31)\dagger})_{\alpha\delta}(\Lambda_\ell^{(31)})_{\gamma\beta}\Big]\Bigg\},\end{aligned} \tag{C.93}$$

$$[G_{ee}]_{\alpha\beta\gamma\delta}^{(1)} = -\frac{N_c}{8}\frac{(\Lambda_e)_{\alpha\beta}(\Lambda_e)_{\gamma\delta}}{M_1^2}, \tag{C.94}$$

$$[G_{\ell e}]_{\alpha\beta\gamma\delta}^{(1)} = -\frac{N_c}{4}\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_e)_{\gamma\delta}}{M_1^2}. \tag{C.95}$$

Semileptonic.

$$\begin{aligned}[G_{\ell q}^{(1)}]_{\alpha\beta ij}^{(1)} = &\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)\left[g_s^2\frac{N_c^2-1}{2N_c}+g'^2(Y_q-Y_\ell)^2\right]\left(\frac{3\lambda_{i\alpha}^{3L*}\lambda_{j\beta}^{3L}}{M_3^2}+\frac{\lambda_{i\alpha}^{1L*}\lambda_{j\beta}^{1L}}{M_1^2}\right)+\\ &-\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)g^2\,3\left(\frac{\lambda_{i\alpha}^{3L*}\lambda_{j\beta}^{3L}}{M_3^2}+\frac{\lambda_{i\alpha}^{1L*}\lambda_{j\beta}^{1L}}{M_1^2}\right)+\\ &-\frac{1}{4}\left(3\frac{(\Lambda_\ell^{(3)})_{\alpha\beta}(\Lambda_q^{(3)})_{ij}}{M_3^2}+\frac{(\Lambda_\ell^{(1)})_{\alpha\beta}(\Lambda_q^{(1)})_{ij}}{M_1^2}\right)+\end{aligned}$$

$$+\left(c_1(1+L_1)+\frac{9}{4}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}\right)\frac{\lambda^{1L\dagger}_{\alpha i}\lambda^{1L}_{j\beta}}{M_1^2}+$$

$$+\left(\frac{9}{4}(1+L_1)c_{13}^{(1)}\frac{M_1^2}{M_3^2}+\frac{3}{2}(1+L_3)\left[5c_3^{(1)}-c_3^{(3)}+\left(2+\frac{N_c^2-1}{2N_c}\right)c_3^{(5)}\right]\right)\frac{\lambda^{3L\dagger}_{\alpha i}\lambda^{3L}_{j\beta}}{M_3^2}, \tag{C.96}$$

$$\begin{aligned}[G^{(3)}_{\ell q}]^{(1)}_{\alpha\beta ij} &= \frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)\left[g_s^2\frac{N_c^2-1}{2N_c}+g'^2(Y_q-Y_\ell)^2\right]\left(\frac{\lambda^{3L*}_{i\alpha}\lambda^{3L}_{j\beta}}{M_3^2}-\frac{\lambda^{1L*}_{i\alpha}\lambda^{1L}_{j\beta}}{M_1^2}\right)+\\
&-\frac{1}{4}\left(\frac{1}{2}+a_{\rm ev}\right)g^2\left(-\frac{3\lambda^{3L*}_{i\alpha}\lambda^{3L}_{j\beta}}{M_3^2}+\frac{\lambda^{1L*}_{i\alpha}\lambda^{1L}_{j\beta}}{M_1^2}\right)+\\
&-\frac{1}{4}\left(2\frac{(\Lambda^{(3)}_\ell)_{\alpha\beta}(\Lambda^{(3)}_q)_{ij}}{M_3^2}+\frac{\log\frac{M_3^2}{M_1^2}\left[(\Lambda^{(31)}_\ell)_{\alpha\beta}(\Lambda^{(31)}_q)_{ij}+(\Lambda^{(31)\dagger}_\ell)_{\alpha\beta}(\Lambda^{(31)\dagger}_q)_{ij}\right]}{M_3^2-M_1^2}\right)+\\
&-\left(c_1(1+L_1)+\frac{9}{4}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}\right)\frac{\lambda^{1L\dagger}_{\alpha i}\lambda^{1L}_{j\beta}}{M_1^2}+\\
&+\left(\frac{3}{4}(1+L_1)c_{13}^{(1)}\frac{M_1^2}{M_3^2}+\frac{1}{2}(1+L_3)\left[5c_3^{(1)}-c_3^{(3)}+\left(2+\frac{N_c^2-1}{2N_c}\right)c_3^{(5)}\right]\right)\frac{\lambda^{3L\dagger}_{\alpha i}\lambda^{3L}_{j\beta}}{M_3^2},\end{aligned} \tag{C.97}$$

$$\begin{aligned}[G_{eu}]^{(1)}_{\alpha\beta ij} &= \frac{1}{2}\left(\frac{1}{2}+a_{\rm ev}\right)\left[g_s^2\frac{N_c^2-1}{2N_c}+g'^2(Y_u-Y_e)^2\right]\frac{\lambda^{1R*}_{i\alpha}\lambda^{1R}_{j\beta}}{M_1^2}+\\
&-\frac{1}{4}\frac{(\Lambda_e)_{\alpha\beta}(\Lambda_u)_{ij}}{M_1^2}+\left(2c_1(1+L_1)+\frac{9}{2}(1+L_3)c_{13}^{(1)}\frac{M_3^2}{M_1^2}\right)\frac{\lambda^{1R\dagger}_{\alpha i}\lambda^{1R}_{j\beta}}{M_1^2},\end{aligned} \tag{C.98}$$

$$\begin{aligned}[G_{qe}]^{(1)}_{ij\alpha\beta} &= -\frac{1}{4}\frac{(\Lambda^{(1)}_q)_{ij}(\Lambda_e)_{\alpha\beta}}{M_1^2}-\frac{3}{4}(1+L_3)\frac{(\lambda^{3L*}y_E^*)_{i\alpha}(\lambda^{3L}y_E)_{j\beta}}{M_3^2}+\\
&-\frac{1}{4}(1+L_1)\frac{(\lambda^{1L*}y_E^*-y_U\lambda^{1R*})_{i\alpha}(\lambda^{1L}y_E-y_U^*\lambda^{1R})_{j\beta}}{M_1^2},\end{aligned} \tag{C.99}$$

$$\begin{aligned}[G_{\ell u}]^{(1)}_{\alpha\beta ij} &= -\frac{1}{4}\frac{(\Lambda^{(1)}_\ell)_{\alpha\beta}(\Lambda_u)_{ij}}{M_1^2}-\frac{3}{4}(1+L_3)\frac{(\lambda^{3L\dagger}y_U^*)_{\alpha i}(\lambda^{3L\,T}y_U)_{\beta j}}{M_3^2}+\\
&-\frac{1}{4}(1+L_1)\frac{(\lambda^{1L\,\dagger}y_U^*-y_E\lambda^{1R\dagger})_{\alpha i}(\lambda^{1L\,T}y_U-y_E^*\lambda^{1RT})_{\beta j}}{M_1^2},\end{aligned} \tag{C.100}$$

$$\begin{aligned}[G_{\ell d}]^{(1)}_{\alpha\beta ij} &= -\frac{1}{4}\left(3(1+L_3)\frac{(\lambda^{3L\dagger}y_D^*)_{\alpha i}(\lambda^{3L\,T}y_D)_{\beta j}}{M_3^2}+(1+L_1)\frac{(\lambda^{1L\,\dagger}y_D^*)_{\alpha i}(\lambda^{1L\,T}y_D)_{\beta j}}{M_1^2}\right),\\
[G_{\ell edq}]^{(1)}_{\alpha\beta ij} &= -\frac{1}{2}\left(-3(1+L_3)\frac{(\lambda^{3L\,\dagger}y_D^*)_{\alpha i}(\lambda^{3L}y_E)_{j\beta}}{M_3^2}+(1+L_1)\frac{(\lambda^{1L\,\dagger}y_D^*)_{\alpha i}(\lambda^{1L}y_E)_{j\beta}}{M_1^2}\right)+\\
&+\frac{1}{2}(1+L_1)\frac{(\lambda^{1L\,\dagger}y_D^*)_{\alpha i}(y_U^*\lambda^{1R})_{j\beta}}{M_1^2},\end{aligned} \tag{C.101}$$

$$[G^{(1)}_{\ell equ}]^{(1)}_{\alpha\beta ij} = \frac{\lambda^{1L\dagger}_{\alpha i}\lambda^{1R}_{j\beta}}{M_1^2}\left\{2c_1(1+L_1)+\frac{9}{2}(1+L_3)c^{(1)}_{13}\frac{M_3^2}{M_1^2}+\right.$$
$$\left.-\frac{3}{2}\left(\frac{3}{2}+L_1\right)\left[(Y_q-Y_\ell)(Y_u-Y_e)g'^2+\frac{N_c^2-1}{2N_c}g_s^2\right]\right\}, \tag{C.102}$$
$$[G^{(3)}_{\ell equ}]^{(1)}_{\alpha\beta ij} = \frac{\lambda^{1L\dagger}_{\alpha i}\lambda^{1R}_{j\beta}}{M_1^2}\left\{-\frac{1}{2}c_1(1+L_1)-\frac{9}{8}(1+L_3)c^{(1)}_{13}\frac{M_3^2}{M_1^2}+\right.$$
$$\left.-\frac{1}{8}\left(\frac{3}{2}+L_1\right)\left[(Y_q-Y_\ell)(Y_u-Y_e)g'^2+\frac{N_c^2-1}{2N_c}g_s^2\right]\right\}. \tag{C.103}$$

X^3		**X^2H^2**		**H^2D^4**	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G_{\mu\nu}^A G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}(H^\dagger H)$	**H^4D^2**	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G_{\rho\nu}^A)$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W_{\rho\nu}^I)$	$\mathcal{O}_{HWB}$	$W_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$	**H^6**	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}_{\mu\nu}^I B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu^I H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

Table 1. Bosonic operators in the Green's basis. Shaded ones are also included in Warsaw basis.

$\boldsymbol{\psi^2 D^3}$		$\boldsymbol{\psi^2 XD}$		$\boldsymbol{\psi^2 DH^2}$	
$\mathcal{O}_{qD}$	$\frac{i}{2}\overline{q}\left\{D_\mu D^\mu, \slashed{D}\right\} q$	$\mathcal{O}_{Gq}$	$(\overline{q}T^A\gamma^\mu q)D^\nu G^A_{\mu\nu}$	$\mathcal{O}^{(1)}_{Hq}$	$(\overline{q}\gamma^\mu q)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{uD}$	$\frac{i}{2}\overline{u}\left\{D_\mu D^\mu, \slashed{D}\right\} u$	$\mathcal{O}'_{Gq}$	$\frac{1}{2}(\overline{q}T^A\gamma^\mu i\overleftrightarrow{D}^\nu q)G^A_{\mu\nu}$	$\mathcal{O}'^{(1)}_{Hq}$	$(\overline{q}i\overleftrightarrow{\slashed{D}}q)(H^\dagger H)$
$\mathcal{O}_{dD}$	$\frac{i}{2}\overline{d}\left\{D_\mu D^\mu, \slashed{D}\right\} d$	$\mathcal{O}'_{\widetilde{G}q}$	$\frac{1}{2}(\overline{q}T^A\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{G}^A_{\mu\nu}$	$\mathcal{O}''^{(1)}_{Hq}$	$(\overline{q}\gamma^\mu q)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{\ell D}$	$\frac{i}{2}\overline{\ell}\left\{D_\mu D^\mu, \slashed{D}\right\} \ell$	$\mathcal{O}_{Wq}$	$(\overline{q}\sigma^I\gamma^\mu q)D^\nu W^I_{\mu\nu}$	$\mathcal{O}^{(3)}_{Hq}$	$(\overline{q}\sigma^I\gamma^\mu q)(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{eD}$	$\frac{i}{2}\overline{e}\left\{D_\mu D^\mu, \slashed{D}\right\} e$	$\mathcal{O}'_{Wq}$	$\frac{1}{2}(\overline{q}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu q)W^I_{\mu\nu}$	$\mathcal{O}'^{(3)}_{Hq}$	$(\overline{q}i\overleftrightarrow{\slashed{D}}^I q)(H^\dagger\sigma^I H)$
$\boldsymbol{\psi^2 HD^2}$ + **h.c.**		$\mathcal{O}'_{\widetilde{W}q}$	$\frac{1}{2}(\overline{q}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{W}^I_{\mu\nu}$	$\mathcal{O}''^{(3)}_{Hq}$	$(\overline{q}\sigma^I\gamma^\mu q)D_\mu(H^\dagger\sigma^I H)$
$\mathcal{O}_{uHD1}$	$(\overline{q}u)D_\mu D^\mu\widetilde{H}$	$\mathcal{O}_{Bq}$	$(\overline{q}\gamma^\mu q)\partial^\nu B_{\mu\nu}$	$\mathcal{O}_{Hu}$	$(\overline{u}\gamma^\mu u)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{uHD2}$	$(\overline{q}i\sigma_{\mu\nu}D^\mu u)D^\nu\widetilde{H}$	$\mathcal{O}'_{Bq}$	$\frac{1}{2}(\overline{q}\gamma^\mu i\overleftrightarrow{D}^\nu q)B_{\mu\nu}$	$\mathcal{O}'_{Hu}$	$(\overline{u}i\overleftrightarrow{\slashed{D}}u)(H^\dagger H)$
$\mathcal{O}_{uHD3}$	$(\overline{q}D_\mu D^\mu u)\widetilde{H}$	$\mathcal{O}'_{\widetilde{B}q}$	$\frac{1}{2}(\overline{q}\gamma^\mu i\overleftrightarrow{D}^\nu q)\widetilde{B}_{\mu\nu}$	$\mathcal{O}''_{Hu}$	$(\overline{u}\gamma^\mu u)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{uHD4}$	$(\overline{q}D_\mu u)D^\mu\widetilde{H}$	$\mathcal{O}_{Gu}$	$(\overline{u}T^A\gamma^\mu u)D^\nu G^A_{\mu\nu}$	$\mathcal{O}_{Hd}$	$(\overline{d}\gamma^\mu d)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{dHD1}$	$(\overline{q}d)D_\mu D^\mu H$	$\mathcal{O}'_{Gu}$	$\frac{1}{2}(\overline{u}T^A\gamma^\mu i\overleftrightarrow{D}^\nu u)G^A_{\mu\nu}$	$\mathcal{O}'_{Hd}$	$(\overline{d}i\overleftrightarrow{\slashed{D}}d)(H^\dagger H)$
$\mathcal{O}_{dHD2}$	$(\overline{q}i\sigma_{\mu\nu}D^\mu d)D^\nu H$	$\mathcal{O}'_{\widetilde{G}u}$	$\frac{1}{2}(\overline{u}T^A\gamma^\mu i\overleftrightarrow{D}^\nu u)\widetilde{G}^A_{\mu\nu}$	$\mathcal{O}''_{Hd}$	$(\overline{d}\gamma^\mu d)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{dHD3}$	$(\overline{q}D_\mu D^\mu d)H$	$\mathcal{O}_{Bu}$	$(\overline{u}\gamma^\mu u)\partial^\nu B_{\mu\nu}$	$\mathcal{O}_{Hud}$	$(\overline{u}\gamma^\mu d)(\widetilde{H}^\dagger iD_\mu H)$
$\mathcal{O}_{dHD4}$	$(\overline{q}D_\mu d)D^\mu H$	$\mathcal{O}'_{Bu}$	$\frac{1}{2}(\overline{u}\gamma^\mu i\overleftrightarrow{D}^\nu u)B_{\mu\nu}$	$\mathcal{O}^{(1)}_{H\ell}$	$(\overline{\ell}\gamma^\mu \ell)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{eHD1}$	$(\overline{\ell}e)D_\mu D^\mu H$	$\mathcal{O}'_{\widetilde{B}u}$	$\frac{1}{2}(\overline{u}\gamma^\mu i\overleftrightarrow{D}^\nu u)\widetilde{B}_{\mu\nu}$	$\mathcal{O}'^{(1)}_{H\ell}$	$(\overline{\ell}i\overleftrightarrow{\slashed{D}}\ell)(H^\dagger H)$
$\mathcal{O}_{eHD2}$	$(\overline{\ell}i\sigma_{\mu\nu}D^\mu e)D^\nu H$	$\mathcal{O}_{Gd}$	$(\overline{d}T^A\gamma^\mu d)D^\nu G^A_{\mu\nu}$	$\mathcal{O}''^{(1)}_{H\ell}$	$(\overline{\ell}\gamma^\mu \ell)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{eHD3}$	$(\overline{\ell}D_\mu D^\mu e)H$	$\mathcal{O}'_{Gd}$	$\frac{1}{2}(\overline{d}T^A\gamma^\mu i\overleftrightarrow{D}^\nu d)G^A_{\mu\nu}$	$\mathcal{O}^{(3)}_{H\ell}$	$(\overline{\ell}\sigma^I\gamma^\mu \ell)(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{eHD4}$	$(\overline{\ell}D_\mu e)D^\mu H$	$\mathcal{O}'_{\widetilde{G}d}$	$\frac{1}{2}(\overline{d}T^A\gamma^\mu i\overleftrightarrow{D}^\nu d)\widetilde{G}^A_{\mu\nu}$	$\mathcal{O}'^{(3)}_{H\ell}$	$(\overline{\ell}i\overleftrightarrow{\slashed{D}}^I \ell)(H^\dagger\sigma^I H)$
$\boldsymbol{\psi^2 XH}$ + **h.c.**		$\mathcal{O}_{Bd}$	$(\overline{d}\gamma^\mu d)\partial^\nu B_{\mu\nu}$	$\mathcal{O}''^{(3)}_{H\ell}$	$(\overline{\ell}\sigma^I\gamma^\mu \ell)D_\mu(H^\dagger\sigma^I H)$
$\mathcal{O}_{uG}$	$(\overline{q}T^A\sigma^{\mu\nu}u)\widetilde{H}G^A_{\mu\nu}$	$\mathcal{O}'_{Bd}$	$\frac{1}{2}(\overline{d}\gamma^\mu i\overleftrightarrow{D}^\nu d)B_{\mu\nu}$	$\mathcal{O}_{He}$	$(\overline{e}\gamma^\mu e)(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{uW}$	$(\overline{q}\sigma^{\mu\nu}u)\sigma^I\widetilde{H}W^I_{\mu\nu}$	$\mathcal{O}'_{\widetilde{B}d}$	$\frac{1}{2}(\overline{d}\gamma^\mu i\overleftrightarrow{D}^\nu d)\widetilde{B}_{\mu\nu}$	$\mathcal{O}'_{He}$	$(\overline{e}i\overleftrightarrow{\slashed{D}}e)(H^\dagger H)$
$\mathcal{O}_{uB}$	$(\overline{q}\sigma^{\mu\nu}u)\widetilde{H}B_{\mu\nu}$	$\mathcal{O}_{W\ell}$	$(\overline{\ell}\sigma^I\gamma^\mu \ell)D^\nu W^I_{\mu\nu}$	$\mathcal{O}''_{He}$	$(\overline{e}\gamma^\mu e)\partial_\mu(H^\dagger H)$
$\mathcal{O}_{dG}$	$(\overline{q}T^A\sigma^{\mu\nu}d)HG^A_{\mu\nu}$	$\mathcal{O}'_{W\ell}$	$\frac{1}{2}(\overline{\ell}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu \ell)W^I_{\mu\nu}$	$\boldsymbol{\psi^2 H^3}$ + **h.c.**	
$\mathcal{O}_{dW}$	$(\overline{q}\sigma^{\mu\nu}d)\sigma^I HW^I_{\mu\nu}$	$\mathcal{O}'_{\widetilde{W}\ell}$	$\frac{1}{2}(\overline{\ell}\sigma^I\gamma^\mu i\overleftrightarrow{D}^\nu \ell)\widetilde{W}^I_{\mu\nu}$	$\mathcal{O}_{uH}$	$(H^\dagger H)\overline{q}\widetilde{H}u$
$\mathcal{O}_{dB}$	$(\overline{q}\sigma^{\mu\nu}d)HB_{\mu\nu}$	$\mathcal{O}_{B\ell}$	$(\overline{\ell}\gamma^\mu \ell)\partial^\nu B_{\mu\nu}$	$\mathcal{O}_{dH}$	$(H^\dagger H)\overline{q}Hd$
$\mathcal{O}_{eW}$	$(\overline{\ell}\sigma^{\mu\nu}e)\sigma^I HW^I_{\mu\nu}$	$\mathcal{O}'_{B\ell}$	$\frac{1}{2}(\overline{\ell}\gamma^\mu i\overleftrightarrow{D}^\nu \ell)B_{\mu\nu}$	$\mathcal{O}_{eH}$	$(H^\dagger H)\overline{\ell}He$
$\mathcal{O}_{eB}$	$(\overline{\ell}\sigma^{\mu\nu}e)HB_{\mu\nu}$	$\mathcal{O}'_{\widetilde{B}\ell}$	$\frac{1}{2}(\overline{\ell}\gamma^\mu i\overleftrightarrow{D}^\nu \ell)\widetilde{B}_{\mu\nu}$		
		$\mathcal{O}_{Be}$	$(\overline{e}\gamma^\mu e)\partial^\nu B_{\mu\nu}$		
		$\mathcal{O}'_{Be}$	$\frac{1}{2}(\overline{e}\gamma^\mu i\overleftrightarrow{D}^\nu e)B_{\mu\nu}$		
		$\mathcal{O}'_{\widetilde{B}e}$	$\frac{1}{2}(\overline{e}\gamma^\mu i\overleftrightarrow{D}^\nu e)\widetilde{B}_{\mu\nu}$		

Table 2. Two-fermion operators in the Green's basis. Shaded ones are also included in Warsaw basis. Fermion family indices are omitted.

Four-quark		Four-lepton		Semileptonic	
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}\gamma^\mu q)(\bar{q}\gamma_\mu q)$	$\mathcal{O}_{\ell\ell}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{\ell}\gamma_\mu \ell)$	$\mathcal{O}_{\ell q}^{(1)}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{q}\gamma_\mu q)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}\gamma^\mu \sigma^I q)(\bar{q}\gamma_\mu \sigma^I q)$	$\mathcal{O}_{ee}$	$(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e)$	$\mathcal{O}_{\ell q}^{(3)}$	$(\bar{\ell}\gamma^\mu \sigma^I \ell)(\bar{q}\gamma_\mu \sigma^I q)$
$\mathcal{O}_{uu}$	$(\bar{u}\gamma^\mu u)(\bar{u}\gamma_\mu u)$	$\mathcal{O}_{\ell e}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{e}\gamma_\mu e)$	$\mathcal{O}_{eu}$	$(\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u)$
$\mathcal{O}_{dd}$	$(\bar{d}\gamma^\mu d)(\bar{d}\gamma_\mu d)$			$\mathcal{O}_{ed}$	$(\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d)$
$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}\gamma^\mu u)(\bar{d}\gamma_\mu d)$			$\mathcal{O}_{qe}$	$(\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e)$
$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}\gamma^\mu T^A u)(\bar{d}\gamma_\mu T^A d)$			$\mathcal{O}_{\ell u}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{u}\gamma_\mu u)$
$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}\gamma^\mu q)(\bar{u}\gamma_\mu u)$			$\mathcal{O}_{\ell d}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{d}\gamma_\mu d)$
$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}\gamma^\mu T^A q)(\bar{u}\gamma_\mu T^A u)$			$\mathcal{O}_{\ell edq}$	$(\bar{\ell}e)(\bar{d}q)$
$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}\gamma^\mu q)(\bar{d}\gamma_\mu d)$			$\mathcal{O}_{\ell equ}^{(1)}$	$(\bar{\ell}^r e)\epsilon_{rs}(\bar{q}^s u)$
$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}\gamma^\mu T^A q)(\bar{d}\gamma_\mu T^A d)$			$\mathcal{O}_{\ell equ}^{(3)}$	$(\bar{\ell}^r \sigma^{\mu\nu} e)\epsilon_{rs}(\bar{q}^s \sigma_{\mu\nu} u)$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}^r u)\epsilon_{rs}(\bar{q}^s d)$				
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}^r T^A u)\epsilon_{rs}(\bar{q}^s T^A d)$				

Table 3. Baryon and lepton number conserving four-fermion operators. All operators are included in Warsaw basis. Fermion family indices are omitted. Indices r, s, p, t, $\ldots$ denote the $\mathrm{SU}(2)_L$ fundamental representations.

B and L violating	
$\mathcal{O}_{duq}$	$\varepsilon_{abc}\epsilon_{rs}\left[(d^a)^T C u^b\right]\left[(q^{cr})^T C \ell^s\right]$
$\mathcal{O}_{qqu}$	$\varepsilon_{abc}\epsilon_{rs}\left[(q^{ar})^T C q^{bs}\right]\left[(u^c)^T C e\right]$
$\mathcal{O}_{qqq}$	$\varepsilon_{abc}\epsilon_{rs}\epsilon_{pt}\left[(q^{ar})^T C q^{bs}\right]\left[(q^{cp})^T C \ell^t\right]$
$\mathcal{O}_{duu}$	$\varepsilon_{abc}\left[(d^a)^T C u^b\right]\left[(u^c)^T C e\right]$

Table 4. Baryon and lepton number violating four-fermion operators. All operators are included in Warsaw basis. Fermion family indices are omitted. Indices r, s, p, t, $\ldots$ and a, b, c, $\ldots$ denote the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_c$ fundamental representations, respectively. C is the Dirac charge conjugation matrix.

References

[1] W. Buchmüller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation*, *Nucl. Phys. B* **268** (1986) 621 [INSPIRE].

[2] G.F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, *The Strongly-Interacting Light Higgs*, *JHEP* **06** (2007) 045 [hep-ph/0703164] [INSPIRE].

JHEP07(2020)225

[3] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, JHEP **10** (2010) 085 [arXiv:1008.4884] [INSPIRE].

[4] I. Doršner, S. Fajfer, A. Greljo, J.F. Kamenik and N. Košnik, *Physics of leptoquarks in precision experiments and at particle colliders*, Phys. Rept. **641** (2016) 1 [arXiv:1603.04993] [INSPIRE].

[5] M. Bauer and M. Neubert, *Minimal Leptoquark Explanation for the $R_{D^{(*)}}$, R_K, and $(g-2)_\mu$ Anomalies*, Phys. Rev. Lett. **116** (2016) 141802 [arXiv:1511.01900] [INSPIRE].

[6] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B-physics anomalies: a guide to combined explanations*, JHEP **11** (2017) 044 [arXiv:1706.07808] [INSPIRE].

[7] A. Crivellin, D. Müller and T. Ota, *Simultaneous explanation of $R(D^{(*)})$ and $b \to s\mu^+\mu^-$: the last scalar leptoquarks standing*, JHEP **09** (2017) 040 [arXiv:1703.09226] [INSPIRE].

[8] D. Marzocca, *Addressing the B-physics anomalies in a fundamental Composite Higgs Model*, JHEP **07** (2018) 121 [arXiv:1803.10972] [INSPIRE].

[9] P. Arnan, D. Becirevic, F. Mescia and O. Sumensari, *Probing low energy scalar leptoquarks by the leptonic W and Z couplings*, JHEP **02** (2019) 109 [arXiv:1901.06315] [INSPIRE].

[10] H. Yan, Y.-D. Yang and X.-B. Yuan, *Phenomenology of $b \to c\tau\bar{\nu}$ decays in a scalar leptoquark model*, Chin. Phys. C **43** (2019) 083105 [arXiv:1905.01795] [INSPIRE].

[11] I. Bigaran, J. Gargalionis and R.R. Volkas, *A near-minimal leptoquark model for reconciling flavour anomalies and generating radiative neutrino masses*, JHEP **10** (2019) 106 [arXiv:1906.01870] [INSPIRE].

[12] A. Crivellin, D. Müller and F. Saturnino, *Flavor Phenomenology of the Leptoquark Singlet-Triplet Model*, JHEP **06** (2020) 020 [arXiv:1912.04224] [INSPIRE].

[13] V. Gherardi, D. Marzocca and E. Venturini, *Low energy phenomenology of scalar leptoquarks at one-loop accuracy*, to appear (2020).

[14] R. Huo, *Effective Field Theory of Integrating out Sfermions in the MSSM: Complete One-Loop Analysis*, Phys. Rev. D **97** (2018) 075013 [arXiv:1509.05942] [INSPIRE].

[15] J.D. Wells and Z. Zhang, *Effective field theory approach to trans-TeV supersymmetry: covariant matching, Yukawa unification and Higgs couplings*, JHEP **05** (2018) 182 [arXiv:1711.04774] [INSPIRE].

[16] M. Jiang, N. Craig, Y.-Y. Li and D. Sutherland, *Complete One-Loop Matching for a Singlet Scalar in the Standard Model EFT*, JHEP **02** (2019) 031 [arXiv:1811.08878] [INSPIRE].

[17] U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini and A. Weiler, *Singlet night in Feynman-ville: one-loop matching of a real scalar*, JHEP **04** (2020) 164 [arXiv:2003.05936] [INSPIRE].

[18] M. Boggia, R. Gomez-Ambrosio and G. Passarino, *Low energy behaviour of standard model extensions*, JHEP **05** (2016) 162 [arXiv:1603.03660] [INSPIRE].

[19] M. Chala and A. Titov, *One-loop matching in the SMEFT extended with a sterile neutrino*, JHEP **05** (2020) 139 [arXiv:2001.07732] [INSPIRE].

[20] B. Henning, X. Lu and H. Murayama, *How to use the Standard Model effective field theory*, JHEP **01** (2016) 023 [arXiv:1412.1837] [INSPIRE].

[21] A. Drozd, J. Ellis, J. Quevillon and T. You, *The Universal One-Loop Effective Action*, JHEP **03** (2016) 180 [arXiv:1512.03003] [INSPIRE].

JHEP07(2020)225

[22] S.A.R. Ellis, J. Quevillon, T. You and Z. Zhang, *Mixed heavy-light matching in the Universal One-Loop Effective Action*, *Phys. Lett. B* **762** (2016) 166 [arXiv:1604.02445] [INSPIRE].

[23] B. Henning, X. Lu and H. Murayama, *One-loop Matching and Running with Covariant Derivative Expansion*, *JHEP* **01** (2018) 123 [arXiv:1604.01019] [INSPIRE].

[24] J. Fuentes-Martin, J. Portoles and P. Ruiz-Femenia, *Integrating out heavy particles with functional methods: a simplified framework*, *JHEP* **09** (2016) 156 [arXiv:1607.02142] [INSPIRE].

[25] Z. Zhang, *Covariant diagrams for one-loop matching*, *JHEP* **05** (2017) 152 [arXiv:1610.00710] [INSPIRE].

[26] S.A.R. Ellis, J. Quevillon, T. You and Z. Zhang, *Extending the Universal One-Loop Effective Action: Heavy-Light Coefficients*, *JHEP* **08** (2017) 054 [arXiv:1706.07765] [INSPIRE].

[27] M. Krämer, B. Summ and A. Voigt, *Completing the scalar and fermionic Universal One-Loop Effective Action*, *JHEP* **01** (2020) 079 [arXiv:1908.04798] [INSPIRE].

[28] T. Cohen, M. Freytsis and X. Lu, *Functional Methods for Heavy Quark Effective Theory*, *JHEP* **06** (2020) 164 [arXiv:1912.08814] [INSPIRE].

[29] F. del Aguila, Z. Kunszt and J. Santiago, *One-loop effective lagrangians after matching*, *Eur. Phys. J. C* **76** (2016) 244 [arXiv:1602.00126] [INSPIRE].

[30] I. Brivio et al., *Computing Tools for the SMEFT*, based on the contributions to *SMEFT-Tools 2019*, proceedings of the *1st Workshop on Tools for Low-Energy SMEFT Phenomenology*, Durham, United Kingdom, 12–14 June 2019, arXiv:1910.11003 [INSPIRE].

[31] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and lambda Dependence*, *JHEP* **10** (2013) 087 [arXiv:1308.2627] [INSPIRE].

[32] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, *JHEP* **01** (2014) 035 [arXiv:1310.4838] [INSPIRE].

[33] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159 [arXiv:1312.2014] [INSPIRE].

[34] S. Herrlich and U. Nierste, *Evanescent operators, scheme dependences and double insertions*, *Nucl. Phys. B* **455** (1995) 39 [hep-ph/9412375] [INSPIRE].

[35] W. Dekens and P. Stoffer, *Low-energy effective field theory below the electroweak scale: matching at one loop*, *JHEP* **10** (2019) 197 [arXiv:1908.05295] [INSPIRE].

[36] A. Buras, *Gauge Theories of Weak Decays*, Cambridge University Press (2020).

[37] C. Bobeth and A.J. Buras, *Leptoquarks meet ε'/ε and rare Kaon processes*, *JHEP* **02** (2018) 101 [arXiv:1712.01295] [INSPIRE].

[38] A. Kobach, *Baryon Number, Lepton Number, and Operator Dimension in the Standard Model*, *Phys. Lett. B* **758** (2016) 455 [arXiv:1604.05726] [INSPIRE].