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Market Design for Energy Communities

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Abstract

Energy systems worldwide are undergoing an accelerating transition from centralized and fossil-fueled power plants to distributed and renewable energy sources. While conventional power plants are controllable in their output, renewable energy sources are subject to the stochastic availability of their primary energy resource. As a result, new coordination schemes for the distributed energy production and consumption must be developed to reach the goal of a fully decarbonized energy system. One approach envisions spatially closely located end-customers who own roof-top photovoltaic systems, electrical heat pumps, and energy storage systems as participants in local energy markets. A local market participation of so-called prosumers would enable a decentralized coordination of the distributed energy supply and demand in a fully liberalized and market-oriented manner. In this line, the European Commission proposed energy communities as a novel regulatory framework that allows prosumers to engage in energy trading with their neighbors. As a barrier towards implementation, the participation in energy communities directly exposes prosumers to the uncertainty inherent to the stochastic power generation by renewable energy sources. This induces volatile and uncertain local market-clearing prices for energy, and thus volatility in the energy cost of prosumers. Consequently, the success of energy communities highly depends on the willingness of prosumers to accept stochasticity in their energy costs.

To this end, the main objective of this thesis is to propose local market design options for energy communities that accommodate uncertainty. First, this thesis develops local market design alternatives that enable an access economy for distributed energy resources within energy communities. An access economy enables the flexibility utilization of a distributed energy resource in the interest of multiple prosumers and not only in favor of the resource-owner herself. Second, this thesis incorporates both risk and ambiguity aversion against an uncertain future event into the decision-making problem of prosumers. A risk- or ambiguity-averse trading decision secures a prosumer against an uncertain future event, but highly affects the local market-clearing outcome for her competitors. Therefore, this thesis additionally proposes the exchange of financial products among community members to prevent the unintended externality of highly risk-averse decision making on prosumers with a low risk aversion. Methodologically, this thesis leverages game-theoretical models as analytical tools to investigate the interaction between individual decision makers in a market environment. This allows a rigorous and tractable evaluation of local market design alternatives in the presence of uncertainty.

The key findings of this thesis are as follows: An access economy for distributed energy resources enhances energy communities by substantially reducing the energy cost volatility for the majority of prosumers. Risk-averse decision making also yields a reduction in the energy cost volatility. In contrast, ambiguity-averse decision making does not necessarily reduce the energy cost volatility. However, it efficiently secures a local market participant against incomplete information on the probability distribution function describing an uncertain future event. A conservative attitude towards uncertainty increases the own expected energy cost, and simultaneously affects the energy cost of competitors. Nonetheless, suitable and well defined financial products traded among community members efficiently neutralize heterogeneous risk preferences. The findings presented in this thesis can inform and guide policy-makers as well as prosumers in implementing energy communities accommodating uncertainty as well as individual attitudes towards uncertainty.

Zusammenfassung

Energiesysteme weltweit erleben eine drastische Transformation von konventionellen Kraftwerken hin zu dezentralen und erneuerbaren Energien. Konventionelle Kraftwerke sind steuerbar in ihrer Produktion. Im Gegensatz dazu unterliegen erneuerbare Energien der stochastischen Verfügbarkeit ihrer Primärenergiequelle. Infolgedessen müssen neue Koordinationsmechanismen für die dezentrale Energieerzeugung und Verbrauch entwickelt werden, um das Ziel eines vollständig dekarbonisierten Energiesystems zu erreichen. Ein Ansatz betrachtet nahe beieinander liegende Endkundinnen und Endkunden, die beispielsweise Photovoltaikanlagen, elektrische Wärmepumpen und Energiespeichersysteme besitzen, als Teilnehmerinnen und Teilnehmer an lokalen Energiemärkten. Eine Marktteilnahme von sogenannten Prosumentinnen und Prosumenten würde eine verteilte Koordination des dezentralen Angebots und der dezentralen Nachfrage in einer liberalisierten und marktorientierten Weise ermöglichen. Daher hat die Europäische Kommission Energiegemeinschaften als neuartigen regulatorischen Rahmen vorgeschlagen. Dieser ermöglicht es Prosumentinnen und Prosumenten lokal Energie zu handeln. Ein Hindernis für die Umsetzung von Energiegemeinschaften besteht jedoch darin, dass die Energiegemeinschaftsmitglieder direkt der Unsicherheit einhergehend mit der erneuerbaren Energieerzeugung ausgesetzt sind. Volatile Energiekosten für Gemeinschaftsmitglieder sind die Folge. Daher hängt der Erfolg von Energiegemeinschaften in hohem Maße von der Bereitschaft ab Unsicherheit in den Energiekosten zu akzeptieren.

Das Hauptziel dieser Dissertation besteht darin, lokale Marktgestaltungsalternativen unter Berücksichtigung der individuellen Handelsstrategien unter Unsicherheiten vorzuschlagen. Unter anderem entwickelt diese Dissertation eine Zugangswirtschaft für verteilte Energieressourcen innerhalb von Energiegemeinschaften. Eine Zugangswirtschaft ermöglicht die flexible Nutzung einer dezentralen Energieressource im Interesse mehrerer Mitglieder. Des Weiteren werden in dieser Dissertation sowohl die Risiko- als auch die Ambiguitätsaversion gegenüber einem unsicheren zukünftigen Ereignis in die Handelsstrategie der Prosumentinnen und Prosumenten mit einbezogen. Sowohl eine risiko- als auch eine ambiguitätsaverse Handelsstrategie schützt vor einem unsicheren zukünftigen Ereignis, beeinflusst jedoch das lokale Marktergebnis für die Wettbewerberinnen und Wettbewerber. Daher untersucht diese Dissertation zusätzlich Finanzprodukte, die konservative Unsicherheitspräferenzen der Individuen ausgleichen. Diese Dissertation verwendet spieltheoretische Modelle. Diese ermöglichen die präzise Abbildung der Interaktion zwischen individuellen Entscheidungsträgerinnen und Entscheidungsträgern innerhalb einer Marktumgebung.

Die wichtigsten Ergebnisse dieser Dissertation sind folgende: Eine Zugangswirtschaft für verteilte Energieressourcen bereichert Energiegemeinschaften, indem sie die Energiekostenvolatilität für die Mehrheit der Mitglieder erheblich reduziert. Eine risikoaverse Handelsstrategie reduziert ebenfalls die Energiekostenvolatilität. Dies trifft für eine ambiguitätsaverse Handelsstrategie nicht unbedingt zu. Sie schützt jedoch vor unvollständigen Informationen bezüglich der Wahrscheinlichkeitsverteilung eines unsicheren Ereignisses. Eine konservative Einstellung gegenüber Unsicherheiten erhöht die eigenen erwarteten Energiekosten und beeinflusst gleichzeitig die Energiekosten der Wettbewerberinnen und Wettbewerber. Präzise definierte Finanzprodukte neutralisieren jedoch heterogene Risikopräferenzen. Die in dieser Dissertation vorgestellten Ergebnisse können sowohl politischen Entscheidungsträgerinnen und Entscheidungsträgern als auch Prosumentinnen und Prosumenten bei der Implementierung von Energiegemeinschaften in der Gegenwart von Unsicherheiten leiten.

Preface

This thesis was prepared at the Munich School of Engineering of the Technical University of Munich in partial fulfillment of the requirements for acquiring the degree Doctor of Engineering.

This dissertation summarizes the work carried out by the author during his Ph.D. project. It started on September, 1st 2017 and was completed on December, 20th 2020. From September 2017 until August 2018, he was hired by the Siemens AG as an external Ph.D. candidate within the group Energy System Modeling at the Corporate Technology. From December 2018 until December 2020, he was hired by the Technical University of Munich as a research assistant at the Chair of Renewable and Sustainable Energy Systems at the Department of Electrical and Computer Engineering.

This thesis is composed of five chapters that summarize the three attached scientific papers, two of which have been peer-reviewed and published. The remaining paper has been submitted and is presently being given full consideration for publication in a peer-reviewed journal.

Niklas Peter René Erich Vespermann
December 2020

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Danke,
Niklas

Munich, Germany, December 2020

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Chapter 1

Introduction

1.1 Towards a fully decarbonized and liberalized energy sector

Concerns about global warming motivate societies worldwide to transform their energy systems from fossil fuels to renewable energy sources [1]. The deployment of these renewable energy sources induces a change in the primary resource necessary to cover the energy demand, as well as where and how this energy is produced. While in the past, centralized conventional power plants supplied the majority of the energy demand, the future energy system will be characterized by a substantial number of distributed energy resources, such as photovoltaic systems, wind turbines, biogas power plants, and electrical heat pumps. Historically, conventional units were controllable in their power generation, and thereby adapted their output to any change in the electricity demand [2]. On the contrary, photovoltaic systems and wind turbines are subject to the stochastic and intermittent availability of the solar radiation and the wind speed, and thus are less controllable. As a result, new coordination schemes for the distributed energy supply and demand must be developed to unlock flexibility in the energy consumption and foster its adaption to the uncertain and volatile power generation [3].

One approach for a new coordination scheme of the distributed energy supply and demand envisions end-customers of electrical services as active participants in local energy markets [4–7]. In particular, the development of information and communication technologies in the last 20 years today empowers pro-active customers, the so-called *prosumers*, to fully control and time-shift their energy demand [8, 9]. A prosumer may even provide her[1] renewable power generation from a roof-top photovoltaic system to others at times when the energy is not needed for her own purposes [10–13]. To this end, the European Commission proposed, as illustrated in Figure 1.1, *energy communities* as a novel regulatory framework that makes energy trading among neighbors possible [14, 15]. Energy communities would enable a decentralized coordination of the energy supply and demand in a fully liberalized and market-oriented manner. Thereby, prosumers could actively decide on their response to the flexibility needs of their surrounding energy system. This distinguishes energy communities from other coordination schemes, such as the operation of distributed energy resources by an aggregator [16–19] or a distribution system operator [20–23].

[1]Throughout this thesis the pronouns “she” and “her”, and all variations thereof, refer to all types of players independent of the gender that may be observed in real-world applications.

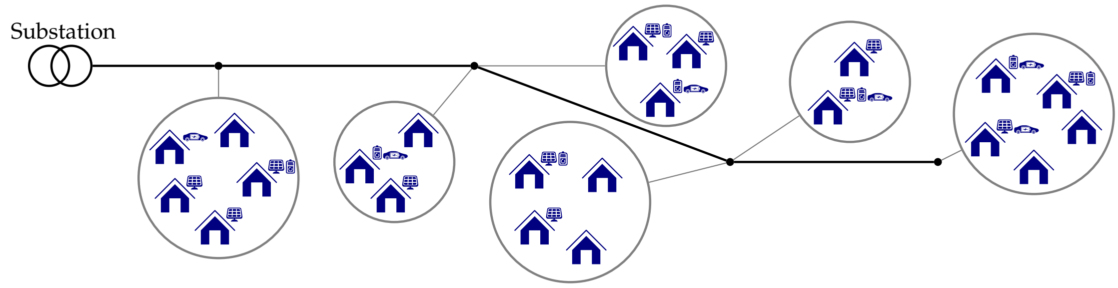

Figure 1.1 Energy communities: Associations of spatially closely located prosumers, the so-called energy communities, would enable a decentralized coordination of the distributed energy supply and demand in a fully liberalized and market-oriented manner.

A key challenge is that the participation in an energy community exposes prosumers to uncertainty. In particular, the power generation by local renewable energy sources at a future stage, although in the near future, is uncertain. This induces stochasticity in the local market-clearing prices for energy, and thus uncertainty in the energy cost of prosumers. The success of energy communities highly depends on the willingness of prosumers to accept uncertainty in their energy cost. A local market design that accommodates uncertainty would pave the way towards a fully decarbonized and liberalized energy sector.

To this end, this thesis studies local market design options for energy communities accommodating uncertainty as well as individual attitudes towards uncertainty. From a methodological perspective, this thesis leverages game-theoretical models as analytical tools to investigate the interaction among individual decision makers in a market environment [24]. These models enable a rigorous and tractable evaluation of local market design alternatives and the individual decision making under uncertainty. Thereby, findings presented in this thesis can guide prosumers and policy-makers implementing energy communities in the presence of uncertainty.

1.2 Local energy market design alternatives accommodating uncertainty

The stochastic and intermittent power generation by a renewable energy source causes an uncertain energy shortage or surplus for a resource owner at a future stage. In the case of a shortage, she may purchase energy in the local market to meet her energy demand. Given a surplus, she may sell energy in the local market, yielding additional revenue. However, the local market-clearing price for energy depends on the *community-wide* energy shortage or surplus. Additional energy imports from the distribution system may be necessary, causing the local-market clearing price to be equivalent to the marginal import cost. In contrast, exporting energy to the distribution system results in a local-market clearing price equal to the marginal export revenue [11]. These dependencies on the renewable power generation induce uncertain and volatile local market-clearing prices for energy, and thus energy *cost volatility* for community members.

Prosumers may seek to deploy distributed energy resources that can provide flexibility to compensate the uncertainty and volatility inherent to the renewable power generation. In this context, flexibility describes the potential to adequately respond to an uncertain future event with an alternative action than originally intended [25]. The deployment of distributed energy resources, such as energy storage systems or electrical heat pumps, can provide this flexibility [26]. However, within an energy community, one of those resources is most likely able to serve the flexibility needs of multiple prosumers. Therefore, an energy community should include an economic mechanism that allows the flexibility utilization of one distributed energy resource in favor of multiple community members [12, 27]. In any case, prosumers remain exposed to price uncertainty arising from their local market participation at times when their energy demand cannot be met by their own renewable power generation. Some prosumers may tolerate volatile and uncertain energy cost. However, other prosumers may be unwilling to accept stochasticity in their cost and prefer planning reliability as is the case today with a conventional energy supply contract. An individual attitude towards uncertainty is a natural behavior that can also be observed in other situations, e.g., trading on a stock exchange. However, the number of prosumers participating in a local energy market is quite limited, and thus the individual attitude towards uncertainty of one prosumer has significant potential to highly affect the local market-clearing outcome [28, 29]. The individual attitude towards uncertainty of local market participants should be explicitly respected in designing a local energy market for energy communities.

To this end, the objective to develop local market design alternatives that accommodate uncertainty leads to the following two main research questions: How to enable an access economy for distributed energy resources in energy communities? How to incorporate an individual attitude towards uncertainty into the decision-making problem of community members?

1.2.1 How to enable an access economy for distributed energy resources in energy communities?

An inadequate local market design may lead to excessive investments in distributed energy resources, such as energy storage systems [30]. These devices are necessary to efficiently compensate uncertain and volatile power generation [26]. In contrast, an *access economy* for distributed energy resources would allow for a joint flexibility utilization. An access economy describes a market-oriented access practice for goods or services, e.g., AirBnB for an accommodation or BlaBlaCar for a transportation service [31, 32]. This contrasts with an access practice outside a market environment, which is based on social norms, such as sharing within a family or a community library [33]. By integrating an access economy into a local market design, community members would have the possibility to pay for the access to the flexibility benefit of a distributed energy resource without direct ownership. On the contrary, a resource owner could recover her investment cost through an additional revenue stream enabled by the access economy. To this end, two questions arise: What is the best local market design for an energy community that enables an access economy for distributed energy resources? What are the implications of a local market comprising an access economy for policy-makers?

One possible direction that enables an access economy lies in the domain of *non-cooperative* market design options. In a non-cooperative market, each market participant is understood as a self-interest seeking player who maximizes her own utility [24]. Consequently, the local market should include an economic incentive for, e.g., an energy storage system owner to provide a share of the storage to other prosumers. Such an economic incentive could be achieved via *rights*, either physical or financial. The concept of storage rights is closely related to the idea of transmission rights [34, 35], which yield market participants an economic access to the interspatial energy arbitrage from the transmission system operation. In particular, a *physical storage right* enables the right holder to directly operate a share of the storage system, and thereby operational and economic access to energy storage [27, 36–38]. In contrast, a *financial storage right* only yields the right holder an economic access to energy storage through a claim on the profit realized by the intertemporal energy arbitrage. The storage system itself is understood as a communal asset and operated by a central entity in favor of all market participants [39, 40]. Both concepts could enable an access economy for distributed energy resources in energy communities. However, this would mean an additional market place for trading storage rights.

Another possible direction lies in the domain of *cooperative* market design options. In a cooperative market, prosumers negotiate a priori a cost allocation rule of the total operational community cost. The optimal output of all distributed energy resources is centrally determined according to a predefined objective [41]. The actual economic access to energy storage depends on the choice of the community cost allocation rule. A simple, but intuitive, cost allocation rule, e.g., based on the peak demand of each prosumer, does not necessarily result in a cost allocation that satisfies all community members [10]. More complex cost allocation rules, such as the *Shapley value* [42], describing the average marginal cost contribution of each prosumer, or the *nucleolus* [43], which minimizes the dissatisfaction of each prosumer, may suffer from a high computational burden.

1.2.2 How to incorporate an individual attitude towards uncertainty into the decision-making problem of community members?

An access economy for distributed energy resources can partly compensate uncertainty and volatility in the energy cost of prosumers. However, a holistic understanding of prosumers' decision making under uncertainty is still needed to design an efficient and reliable local energy market. In general, an uncertainty-aware player forecasts the probability distribution function of an uncertain event. Based on her probabilistic forecast, she chooses a market participation strategy that maximizes her uncertainty-dependent utility in *expectation* [44]. In practice, a player who is exposed to uncertainty may be concerned about costly uncertainty realizations, even though they are unlikely. This concern becomes highly relevant in the case that a market is characterized by a small number of participants. In this case, the decision of one player strongly influences the market-clearing outcome for her competitors. This motivates the following questions: How to incorporate an individual attitude towards uncertainty into the decision-making problem of prosumers? What are the economic implications arising from heterogeneous uncertainty attitudes?

One form of an attitude towards uncertainty is described by the notion of *risk aversion*. Given a probabilistic forecast, a risk-averse prosumer considers, in her market participation strategy, an increased value for the probability of a costly uncertainty realization and a decreased weight for an advantageous event [45, 46]. The risk-adjustment of probabilities can be achieved by incorporating a risk-measure function [47, 48] into the objective of each prosumer. Thereby, a prosumer maximizes her uncertainty-dependent utility in risk-averse expectation. Risk-averse decision making yields a reduction in the cost volatility, but leads to a higher cost in expectation. In addition, it affects the energy cost of other prosumers owing to their linkage via the local energy market [28, 29]. However, prosumers are heterogeneously risk-averse and some players may be willing to accept a greater cost volatility than others, while enjoying a lower energy cost in expectation. One possibility to neutralize heterogeneous risk preferences is *risk trading* [46, 49–51]. Risk trading describes the exchange of financial products to transfer the cost under a specific uncertainty realization from a highly risk-averse player to a market participant with a low risk aversion. For this purpose, the probability distribution function of an uncertain event must be exactly known to define a financial product for each uncertainty realization. In practice, this knowledge is not necessarily available a priori.

Missing information on the true probability distribution function not only prevents a market completeness for risk [52], but also induces *ambiguity aversion* by individuals [53]. In particular, a prosumer may be averse against empirical data on which her probabilistic forecast is based [54]. An ambiguity-averse player considers a family of probability distribution functions, the so-called *ambiguity set*, close to her empirical probabilistic forecast. Given the aversion against the available data, the player chooses a market participation strategy in expectation of the worst-case probability distribution function from her ambiguity set [55]. Thereby, she maximizes her uncertainty-dependent utility in ambiguity-averse expectation. This attitude towards uncertainty can be incorporated into the decision-making problem of prosumers through the concept of distributionally robust optimization [56]. The individual confidence in empirical data may be explicitly adjusted through the size of the ambiguity set. This set can be built based on, e.g., a Wasserstein probability distance metric [57].

1.3 Thesis contributions to local energy market design options

The main objective of this thesis is to propose local market design alternatives for energy communities that accommodate individual decision making under uncertainty. For this purpose, non-cooperative and cooperative game-theoretical models are developed that (*i*) enable an access economy for distributed energy resources in energy communities and (*ii*) incorporate an individual attitude towards uncertainty into the decision-making problem of each prosumer. This thesis analyzes the proposed local market design alternatives in terms of the local market efficiency, the expected cost and its volatility for prosumers and the community as a whole, and in terms of the computational time needed to numerically obtain a local market-clearing solution. Thereby, this thesis can assist policy-makers and prosumers in implementing energy communities in the presence of uncertainty. In the following, the main conceptual and methodological contributions of this thesis are summarized, which are represented by Publication **[A]**–**[C]**.

1.3.1 Conceptual contributions

From a conceptual perspective, this thesis proposes an access economy for distributed energy resources in energy communities based on the example of energy storage systems. In particular, Publication **[A]** develops an access economy to the benefit of energy storage based on both physical and financial storage rights. These rights are traded within a local *forward* market, i.e., a marketplace that clears well in advance to real-time. At the forward market stage, storage-owning prosumers may actively decide on the share of storage capacity offered to others or withheld for her own interests. In real-time, physical storage right holders directly dispatch their share of storage systems in local *spot* markets, i.e., real-time markets. Given financial storage rights, all storage systems are dispatched by a central entity in favor of the whole community. Financial storage right holders yield an economic access to energy storage through a claim on the intertemporal energy arbitrage. In addition, Publication **[A]** develops an access economy based on a cooperative market design in which all distributed energy resources, including storage systems, are dispatched in real-time by a central entity. Publication **[A]** focuses on the Shapley value and the nucleolus as two different community cost allocation rules determining the economic access to energy storage for prosumers. Ultimately, Publication **[A]** is the first work that provides a comprehensive comparison and discussion on how an access economy for energy communities could be enabled and which market properties are ensured by each local market design alternative.

Beyond an access economy, this thesis incorporates individual attitudes towards uncertainty into the decision-making problem of prosumers. In particular, Publication **[B]** addresses the notion of risk aversion in energy communities. Risk-averse local market participants trade energy in a local forward market followed by a local spot market. To prevent conservative decision making by risk-averse individuals, Publication **[B]** proposes risk trading via *Arrow-Debreu securities* [52]. This allows the transfer of energy cost under specific uncertainty realizations from highly risk-averse prosumers to community members with a low risk aversion. Depending on the availability of Arrow-Debreu securities, the local market may be fully or partially incomplete for risk, i.e., none or only some cost realizations can be transferred, or complete for risk, i.e., all cost realizations could be transferred. Publication **[B]** is the first work that offers a thorough discussion of the economic implications of heterogeneously risk-averse prosumers who engage in energy trading in a fully or partially incomplete, or complete, local market for risk. One necessary assumption for complete risk trading via Arrow-Debreu securities is that the probability distribution function of an uncertain event is exactly known. Since this is unlikely in practice, Publication **[C]** assesses the impact of energy trading in a local spot market among heterogeneously ambiguity-averse players. Each ambiguity-averse local market participant observes her empirical data describing an uncertain event. Based on her confidence in these data, she forms an ambiguity set comprising probability distribution functions close to the empirical probability distribution function. Given the problem complexity, publication **[C]** proposes the introduction of a local forward market that determines an energy production and consumption schedule as well as a participation plan for responding to any community-wide energy imbalances. Eventually, Publication **[C]** provides, for the first time, a discussion on ambiguity aversion in the context of local markets.

1.3.2 Methodological contributions and application results

From a methodological perspective on an access economy for distributed energy resources in energy communities, Publication **[A]** proposes two generalized mathematical formulations of non-cooperative local market design alternatives that enable an access economy via physical and financial storage rights, respectively. These formulations allow, for the first time, a *partial* access to energy storage systems, while the optimal level of access provision from the storage owner's perspective is determined endogenously. Furthermore, this work analytically investigates the proposed non-cooperative market design alternatives in terms of the existence and the uniqueness of a local market-clearing solution. Based on game-theoretical methods, Publication **[A]** shows that for each proposed non-cooperative local market design, a unique market equilibrium exists. The equivalence of a market equilibrium problem including physical storage rights to a social planner problem was proven previously [27]. However, Publication **[A]** is the first work showing that the local market equilibrium problem comprising financial storage rights can be efficiently solved by two sequential optimization problems. Beyond non-cooperative market design options, Publication **[A]** assesses an access economy based on cooperative market design options. In particular, this work applies the Shapley value as well as the nucleolus and interprets the resulting community cost allocation in terms of an economic access for prosumers to energy storage. Numerically, Publication **[A]** demonstrates that the non-cooperative market design alternatives scale well as the community size increases in terms of community members. In contrast, the cooperative market design alternatives scale poorly, and thus are more suitable for small energy communities. Independent of the local market design in place, an access economy for energy storage enhances energy communities by reducing the energy cost volatility for the majority of prosumers. At the same time, the operational cost of the community as a whole remains unchanged. Consequently, the preferred local market design enabling an access economy for distributed energy resources highly depends on the community member preferences. This finding highlights the importance of assessing individual risk and ambiguity preferences in energy communities.

In this regard, Publication **[B]** and Publication **[C]** incorporate risk and ambiguity aversion into the decision-making problem of local market participants, respectively. Publication **[B]** applies the well-known conditional value-at-risk [58] as a coherent risk-measure function [48] to each local market participant. In addition, Publication **[B]** considers risk trading via Arrow-Debreu securities. The collection of all risk-averse decision-making problems gives rise to a risk-averse Nash game with risk trading. Depending on the availability of Arrow-Debreu securities, the local market may range from a fully incomplete to a complete market for risk. Publication **[B]** studies game-theoretical properties and shows that for each case, a local market equilibrium exists. However, this work shows that in any case, multiple Nash equilibria may be found. This prevents a distinct conclusion for policy-makers and prosumers on the local market-clearing outcomes. Lastly, Publication **[B]** discusses the equivalence of the risk-averse Nash game to a social planner problem. For heterogeneous risk aversion, this can be confirmed only given a complete market for risk. Beyond risk aversion, Publication **[C]** incorporates ambiguity aversion into the decision-making problem of local market participants based on the concept of *distributionally robust optimization* and Wasserstein ambiguity sets. For the purpose of complete Wasserstein ambiguity sets, Publication **[C]** considers the uncertain event to follow a continuous probability distribution function. This contrasts with a finite and discrete probability distribution function previously considered in this thesis. The collection of all distributionally robust decision-making problems motivates a distributionally robust game [59–61]. This game is intractable owing to the continuous probability distribution function. However, Publication **[C]** provides a tractable problem reformulation based on chance constraints [62] and linear decision rules [63]. Publication **[C]** shows, for the first time, the existence of an equivalent social planner problem to the tractable reformulation, although players may possess asymmetric information regarding uncertainty [64]. The solution of the equivalent social planner problem is unique. This implies the existence of a unique local market-clearing solution.

Numerical findings in Publication **[B]** indicate that risk trading efficiently protects prosumers with a low risk aversion from highly risk-averse decision making by competitors. A significant community cost saving can be realized in the case that prosumers engage in risk trading and sufficient Arrow-Debreu securities are available for trading. Given the case of insufficient information about the uncertain event, and thus heterogeneous ambiguity aversion, Publication **[C]** identifies that local market participants with a comparatively low utility from energy consumption are highly exposed to the ambiguity aversion of rivals.

1.4 Thesis structure

The remainder of this Ph.D. thesis is structured as follows: Chapter 2 discusses the advent of energy communities, provides preliminary assumptions on the local market design, and introduces the local market structure applied throughout this thesis. Furthermore, a game-theoretical model, which represents the local market-clearing problem of an energy community, is introduced. Chapter 3 proposes local market design alternatives that enable an access economy for distributed energy resources based on the example of energy storage systems. Given the finding that the preferred choice of a local market design highly depends on the individual preference of each community member towards her energy cost volatility, Chapter 4 incorporates risk aversion into the decision-making problem of prosumers. In addition, this chapter discusses how risk trading among community members may neutralize heterogeneous risk aversion. Moreover, Chapter 4 discusses ambiguity aversion against empirical data describing an uncertainty event. For this purpose, it assumes that the uncertain event follows a continuous probability distribution function, as opposed to a finite and discrete probability distribution function considered previously in this thesis. Lastly, Chapter 5 concludes by summarizing the main contributions and proposing potential future research directions arising from this thesis.

1.5 List of publications

The relevant publications summarized in this thesis are listed in the following:

- **[A]** Niklas Vespermann, Thomas Hamacher, and Jalal Kazempour: "Access economy for storage in energy communities", in *IEEE Transactions on Power Systems*, forthcoming, 2020, DOI: 10.1109/TPWRS.2020.3033999.
- **[B]** Niklas Vespermann, Thomas Hamacher, and Jalal Kazempour: "Risk trading in energy communities", in *IEEE Transactions on Smart Grid*, forthcoming, 2020, DOI: 10.1109/TSG.2020.3030319.
- **[C]** Niklas Vespermann, Thomas Hamacher, and Jalal Kazempour: "On ambiguity-averse market equilibrium", submitted to *Optimization Letters*, (under review, second round), 2022.

Another publication has been prepared during the course of the Ph.D. studies, but omitted from this thesis since it is not directly related to the primary objective.

- **[D]** Niklas Vespermann, Matthias Huber, Simon Paulus, Michael Metzger, and Thomas Hamacher: "The impact of network tariffs on PV investment decisions by consumers", in *15th International Conference on the European Energy Market (EEM)* 2018, DOI: 10.1109/EEM.2018.8469944.

Chapter 2

Fundamentals of Energy Communities

This chapter provides preliminaries that are fundamental to the following chapters. Section 2.1 discusses the advent of energy communities. Section 2.2 outlines the local market structure considered throughout this thesis. Section 2.3 introduces preliminary assumptions on the local market design. Based on the local market structure and assumptions in place, Section 2.4 provides a game-theoretical model representing the local market of an energy community. Lastly, Section 2.5 shows the solution existence and uniqueness for the underlying local market-clearing problem.

2.1 The advent of energy communities

Pro-active customers, the so-called prosumers, are able to control and time-shift their energy consumption [8, 9]. Thereby, prosumers embody the potential to contribute to the necessary power system flexibility to reach the goal of a fully decarbonized energy system. A coordination of the distributed energy supply and demand would allow the further integration of distributed energy resources, such as photovoltaic systems and electrical heat pumps. At the same time, a coordination could prevent overloads in the distribution system without additional network upgrades [65]. However, a major challenge remains: How to achieve the coordination among the distributed energy production and consumption?

One approach envisions prosumers as participants in local energy markets [4–7]. Those markets could coordinate energy supply and demand in a fully liberalized and market-oriented manner. However, efficient regulations that enable the local energy trading are still lacking. The European Commission responded to this shortcoming through the renewable energy directive [14, Art. 21] and the directive on common rules for the internal market for electricity [15, Art. 16]. More specifically, the member states of the European Union have been asked to provide prosumers with a fair market access and to make energy trading among neighbors possible. At the same time, prosumers should not be exposed to disproportional levies and surcharges. In this line, energy communities are proposed as a novel regulatory framework that allows prosumers to directly engage in energy trading [15, Art. 16]. The successful implementation of energy communities would enable prosumers to actively decide on their response to the flexibility needs of their neighbors or the distribution system. This characteristic distinguishes the concept of an energy community from other coordination schemes, such as an operational access to distributed energy

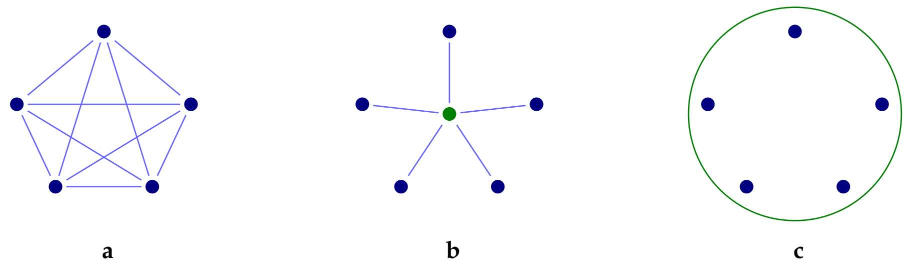

Figure 2.1 Local energy market paradigms: Figure 2.1a shows a peer-to-peer market structure. Prosumers, marked by blue dots, negotiate bilateral contracts. Figure 2.1b depicts an auction-based local market design. A community manager, marked by the green dot, clears the local market, and thereby determines energy exchanges between prosumers. Lastly, Figure 2.1c illustrates a coalitional local market design. A community manager, indicated by the green circle, centrally determines all energy production, consumption, and exchange activities inside the community.

resources by an aggregator [16–19] or by the distribution system operator [20–23].

Today, there is a range of pilot projects demonstrating energy communities. For example, "The Energy Collective" project is implemented in Bofællesskabet Svalin, Denmark, and proposes a consumer-centric local electricity market [66]. The "EnergyLab Nordhavn" project is implemented in the Nordhavn neighborhood of Copenhagen, Denmark, and focuses besides others on a market-based coordination of the local integrated heat and electricity system [67]. The "Pebbles" project is implemented in Wildpoldsried, Germany, and assesses the power system integration of a local energy market as a virtual power plant [68]. All these projects allow prosumers to directly engage in energy trading with their neighbors through a local market. However, the market frameworks in place highly differ in how they enable local energy trading.

2.2 A local market for energy communities

In general, local market design options for energy communities can be distinguished by three organizational paradigms, as depicted in Figure 2.1 [4, 6]. Within a *peer-to-peer* market, as shown in Figure 2.1a, prosumers negotiate fully decentralized bilateral contracts regarding local energy exchanges [5, 69, 70]. Figure 2.1b illustrates an *auction-based* local market design. Prosumers submit energy production and consumption bids to the local market. A community manager clears the local market, and thereby determines the energy exchanges between prosumers [11, 13, 71]. In both cases, prosumers are understood as self-interest seeking players who actively decide on their energy production, consumption, and trades. Therefore, these paradigms lie in the domain of *non-cooperative* market design options. In contrast, Figure 2.1c depicts a *coalitional* local market design. A community manager centrally determines all energy production, consumption, and exchange activities inside the energy community. This takes place according to a predefined objective, e.g., the minimization of the total operational community cost. Prosumers are inactive during the operational stage [10, 12, 72]. Post operation, the total community cost is systematically redistributed among community members. This paradigm lies in the domain of *cooperative* market design options.

Peer-to-peer local market design options enable fully decentralized energy trades, and thereby the highest degree of a market liberalization [6]. However, significant computational challenges and communication complexity issues exist, preventing a robust system operation [73]. In contrast, coalitional local market design options achieve a robust system operation by a central entity [12, 13]. However, since local market participants are inactive during the operational stage, this paradigm yields the lowest degree of a market liberalization among the three alternatives. This thesis mainly focuses on auction-based local market design options. An auction-based local market enables an active market participation by prosumers, while ensuring a comparatively low computational and communicational burden [11, 13]. Note that Chapter 3 also discusses coalitional local market design options in the context of an access economy for distributed energy resources in energy communities. However, peer-to-peer local market design options are not in the scope of this thesis and left fully aside for future research.

Following the concept of an auction-based local market design, Figure 2.2 depicts the local market structure considered throughout this thesis. The energy community comprises three types of players, namely prosumers, a spatial arbitrageur, and a price-setter. Prosumers have a price-inelastic energy demand and potentially own a roof-top photovoltaic system with an uncertain power generation. All prosumers minimize their energy cost by meeting their price-inelastic energy demand through the uncertain photovoltaic power generation. In the case of a shortage, prosumers can purchase energy in the local market. In the case of a photovoltaic power generation surplus, energy can be sold in the local market. Given a community-wide energy shortage or surplus, the spatial arbitrageur imports and exports energy from and to the distribution system, respectively. Lastly, the price-setter, who is a fictitious player [74], reveals local market-clearing prices evolving under free trade and perfect competition. In detail, the price-setter reveals market-clearing prices, which minimize the cost for buyers and maximize the revenue for sellers. In addition, such market-clearing prices implicitly ensure the balance between the total energy supply and demand inside the community. In a perfectly competitive market environment, the spatial arbitrageur and the price-setter can be institutionally interpreted as a community manager who fulfills both tasks, ensuring liquidity and revealing local market-clearing prices [75].

2.3 Preliminary assumptions on the local market design

In the following, assumptions on the local market design are introduced. These assumptions significantly simplify the reality, but allow the formulation of tractable game-theoretical models, and thereby enable general insights on local market design options in the presence of uncertainty. In particular, this thesis assumes that all players participate in the local market as price-takers. This implies that they bid according to their true cost and preferences in a perfectly competitive local market. In addition, all players have a perfect foresight of future events. Given the small size of an energy community, internal network constraints are neglected. However, constraints observed by the spatial arbitrageur on the energy import and export are explicitly modeled. Owing to the close spatial distance between prosumers, their photovoltaic power generation profiles are assumed to be identical for all photovoltaic systems in the community. The photovoltaic power generation is the only source of uncertainty. However, it causes the local market-clearing price to be uncertain from the perspective of prosumers and the spatial arbitrageur. Consequently, their local

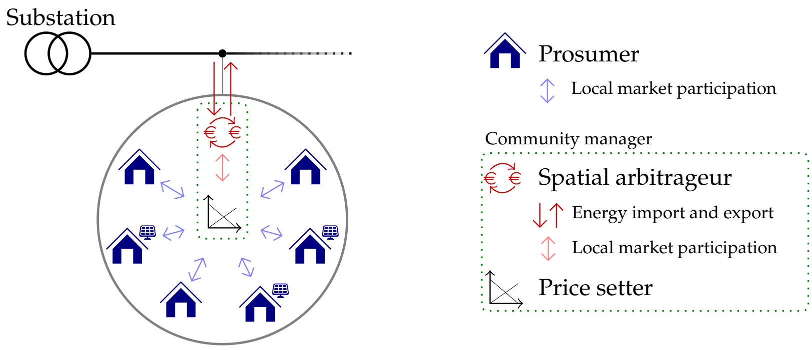

Figure 2.2 The local market structure: The energy community comprises three types of players. Prosumers trade energy in the local market. A spatial arbitrageur ensures liquidity. A price-setter reveals local market-clearing prices. Given a perfectly competitive local market environment, a combination of the spatial arbitrageur and the price-setter can be institutionally interpreted as a community manager.

market participation strategy is *uncertainty dependent*. However, from the price-setters' perspective, the market participation of prosumers and the spatial arbitrageur is uncertain. Therefore, her choice for a local market-clearing price is also uncertainty dependent. Throughout this thesis, the renewable power generation as the only uncertain parameter as well as uncertainty-dependent decision variables of players are indicated by a tilde $(\tilde{\cdot})$.

For simplicity, this chapter assumes that players trade energy in a single local *spot market*, and thus observe the uncertainty realization in real-time and simultaneously decide on their local market participation strategy. However, in Chapter 3 and Chapter 4 the local spot market is preceded by a local *forward market*, i.e., a marketplace that clears well in advance to real-time. In addition, Chapter 3 considers multiple sequentially clearing spot markets, i.e., one spot market for each time step, to account for energy storage.

At this stage, it is assumed that all players are risk- and ambiguity-neutral, and possess identical information about the uncertain renewable power generation. The uncertainty is modeled by the set Ω comprised of a finite number of discrete scenarios ω. Each scenario describes one specific uncertainty realization of the renewable power generation. Chapter 4 relaxes the assumption of risk- and ambiguity-neutral decision making, and additionally accounts for a potentially incomplete market for risk and ambiguity, respectively. Under ambiguity aversion, Chapter 4 also relaxes the assumption of finite and discrete scenarios and considers the uncertain renewable power generation to follow a continuous probability distribution function.

2.4 A Nash equilibrium problem describing the local energy market

This section introduces mathematically the decision-making problem of prosumers, the spatial arbitrageur, and the price-setter. At this stage, the potential uncertainty realizations of the renewable power generation are independent from each other. Therefore, each player solves *one* optimization problem *per* uncertainty realization $\omega \in \Omega$.

Each prosumer $n \in \mathcal{N}$ minimizes her uncertainty-dependent local spot market energy cost $J_{n\omega}$.

The prosumer n observes a, from her perspective, uncertain spot market energy price $\tilde{\lambda}_\omega$ and determines her uncertainty-dependent energy trade $\tilde{p}_{n\omega}$ by solving the following optimization problem:

$$\Bigg\{\underset{\tilde{p}_{n\omega}}{\text{Min}}\ J_{n\omega} := \tilde{\lambda}_\omega \tilde{p}_{n\omega} + c(\tilde{p}_{n\omega}) \tag{2.1a}$$

$$\text{s.t. } \tilde{p}_{n\omega} + \tilde{S}_{n\omega} - D_n = 0\colon \phi_{n\omega}, \tag{2.1b}$$

$$\tilde{p}_{n\omega} \in \mathcal{P}_n := \{\tilde{p}_{n\omega} \mid -P_n \leq \tilde{p}_{n\omega} \leq P_n\colon \underline{\chi}^{\tilde{p}}_{n\omega}, \overline{\chi}^{\tilde{p}}_{n\omega}\} \Bigg\},\ \forall n \in \mathcal{N}, \forall \omega \in \Omega. \tag{2.1c}$$

Under each uncertainty realization ω, each prosumer n minimizes her spot market energy cost by buying energy, i.e., non-negative values of $\tilde{p}_{n\omega}$, and selling energy, i.e., non-positive values of $\tilde{p}_{n\omega}$, according to the uncertain local spot market energy price $\tilde{\lambda}_\omega$ (2.1a). In addition, the objective function (2.1a) is endowed with a quadratic regularizer $c(\tilde{p}_{n\omega}) = \frac{1}{2}\beta\tilde{p}^2_{n\omega}$, wherein β is a small positive constant, e.g., 10^{-3}. A sufficiently small value for β does not alter the aggregated cost of all prosumers in comparison to $\beta = 0$. However, this regularizer ensures mathematically the market-clearing solution uniqueness [76]. In addition, identical prosumers incur the same energy cost. The energy trade $\tilde{p}_{n\omega}$ of prosumer n under each uncertainty realization ω is motivated by the need to ensure the balance (2.1c) between her uncertain photovoltaic power generation $\tilde{S}_{n\omega}$ and her price-inelastic energy demand D_n. The trading decision $\tilde{p}_{n\omega}$ lies within a closed, convex, and compact set $\mathcal{P}_n$. This set imposes a sufficiently large lower and upper bound P_n on $\tilde{p}_{n\omega}$, such that the energy trade $\tilde{p}_{n\omega}$ remains unconstrained. Symbols following a colon denote the dual variables of the respective constraints.

The spatial arbitrageur minimizes her uncertainty-dependent spot market energy import and export cost J^{ar}_ω. For a given, from her perspective, uncertain spot market energy price $\tilde{\lambda}_\omega$, the spatial arbitrageur determines her uncertainty-dependent energy trade $\tilde{p}^{\text{ar}}_\omega$ as follows:

$$\Bigg\{\underset{\tilde{p}^{\text{ar}}_\omega}{\text{Min}}\ J^{\text{ar}}_\omega := (C - \tilde{\lambda}_\omega)\tilde{p}^{\text{ar}}_\omega \tag{2.2a}$$

$$\text{s.t. } \tilde{p}^{\text{ar}}_\omega \in \mathcal{P}^{\text{ar}} := \{\tilde{p}^{\text{ar}}_\omega \mid -P^{\text{ar}} \leq \tilde{p}^{\text{ar}}_\omega \leq P^{\text{ar}}\colon \underline{\chi}^{\tilde{p}^{\text{ar}}}_\omega, \overline{\chi}^{\tilde{p}^{\text{ar}}}_\omega\} \Bigg\},\ \forall \omega \in \Omega. \tag{2.2b}$$

Under each uncertainty realization ω, the spatial arbitrageur imports energy, i.e., non-negative values of $\tilde{p}^{\text{ar}}_\omega$, and exports energy, i.e., non-positive values of $\tilde{p}^{\text{ar}}_\omega$, at the fixed cost C. For energy imports and exports she receives and pays, respectively, the uncertain spot market energy price $\tilde{\lambda}_\omega$ (2.2a). The trading decision $\tilde{p}^{\text{ar}}_\omega$ lies within a closed, compact, and convex set $\mathcal{P}^{\text{ar}}$, which imposes a lower and upper bound P^{ar} on $\tilde{p}^{\text{ar}}_\omega$ (2.2b). These bounds describe potential network constraints between the community and the distribution system.

Lastly, the price-setter minimizes and maximizes the uncertainty-dependent local spot market energy cost and revenue for buyers and sellers J^{ps}_ω, respectively. She observes, from her perspective, uncertain spot market energy trades $\tilde{p}_{n\omega}$ and $\tilde{p}^{\text{ar}}_\omega$, and determines an uncertainty-dependent spot market energy price $\tilde{\lambda}_\omega$ by solving the following optimization problem:

$$\Bigg\{\underset{\tilde{\lambda}_\omega}{\text{Min}}\ J^{\text{ps}}_\omega := \tilde{\lambda}_\omega\Big(\sum_{n\in\mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_\omega\Big), \tag{2.3a}$$

$$\text{s.t. } \tilde{\lambda}_\omega \in \mathcal{L} := \{\tilde{\lambda}_\omega \mid -\Lambda \leq \tilde{\lambda}_\omega \leq \Lambda\colon \underline{\chi}^{\tilde{\lambda}}_\omega, \overline{\chi}^{\tilde{\lambda}}_\omega\} \Bigg\},\ \forall \omega \in \Omega. \tag{2.3b}$$

Under each uncertainty realization ω, the price-setter chooses the spot market energy price $\tilde{\lambda}_\omega$, such that the cost for energy buyers, i.e., non-negative values of $\tilde{p}_{n\omega}$ and non-positive values of $\tilde{p}^{\text{ar}}_\omega$, is minimized and the revenue for energy sellers, i.e., non-positive values of $\tilde{p}_{n\omega}$ and non-negative values of $\tilde{p}^{\text{ar}}_\omega$, is maximized (2.3a). The spot market energy price $\tilde{\lambda}_\omega$ lies within a closed, compact, and convex set $\mathcal{L}$. This set imposes a sufficiently large lower and upper bound Λ on $\tilde{\lambda}_\omega$, such that the spot market energy price remains unconstrained (2.3b).

The collection of the decision-making problems (2.1)–(2.3) states a set of *Nash equilibrium problems*, i.e., one Nash equilibrium problem per uncertainty realization $\omega \in \Omega$. For each uncertainty realization ω, the corresponding *Nash game* is given by $\Gamma_\omega(\mathcal{Z}, K_\omega, \{J_{i\omega}\}_{\forall i \in \mathcal{Z}})$. The symbol $\mathcal{Z}$ states the set of all players, while $J_{i\omega}$ denotes their respective objective function. Furthermore, $K_\omega = (K_{n_1\omega} \times \cdots \times K_{N\omega} \times K^{\text{ar}}_\omega \times K^{\text{ps}}_\omega)$ defines the strategy set of the Nash game, in which $K_{n_1\omega}$ and $K_{N\omega}$ are the strategy sets of the first and last prosumer, respectively. The symbol K^{ar}_ω denotes the strategy set of the spatial arbitrageur, while K^{ps}_ω refers to the strategy set of the price-setter.

In order to determine a market-clearing solution, the so-called *Nash equilibrium point*, all decision-making problems within the Nash game have to be solved simultaneously. A Nash equilibrium point is found if no player deviates unilaterally from her decision given that no other player changes her market participation strategy [24]. One possible approach determines the Nash equilibrium point based on an equivalent formulation of the Nash equilibrium problem as a mixed complementarity problem [75]. A mixed complementarity problem computes the point where the Karush-Kuhn-Tucker optimality conditions of all optimization problems are simultaneously fulfilled. For this purpose, the Karush-Kuhn-Tucker conditions of (2.1) write as follows:

$$\frac{\partial \mathcal{L}_{n\omega}}{\partial \tilde{p}_{n\omega}} = \tilde{\lambda}_\omega + \beta \tilde{p}_{n\omega} + \phi_{n\omega} - \underline{\chi}^{\tilde{p}}_{n\omega} + \overline{\chi}^{\tilde{p}}_{n\omega} = 0, \qquad \forall n, \omega, \tag{2.4a}$$

$$\tilde{p}_{n\omega} + \tilde{S}_{n\omega} - D_n = 0, \qquad \forall n, \omega, \tag{2.4b}$$

$$0 \le \tilde{p}_{n\omega} + P_n \perp \underline{\chi}^{\tilde{p}}_{n\omega} \ge 0,\ 0 \le P_n - \tilde{p}_{n\omega} \perp \overline{\chi}^{\tilde{p}}_{n\omega} \ge 0, \qquad \forall n, \omega, \tag{2.4c}$$

in which $\mathcal{L}_{n\omega}$ denotes the Lagrangian function of (2.1). Furthermore, the Karush-Kuhn-Tucker conditions of (2.2) are given by the following set of equations:

$$\frac{\partial \mathcal{L}^{\text{ar}}}{\partial \tilde{p}_\omega} = C - \tilde{\lambda}_\omega - \underline{\chi}^{\tilde{p}}_\omega + \overline{\chi}^{\tilde{p}}_\omega = 0, \qquad \forall \omega, \tag{2.5a}$$

$$0 \le \tilde{p}^{\text{ar}}_\omega + P^{\text{ar}} \perp \underline{\chi}^{\tilde{p}^{\text{ar}}}_\omega \ge 0,\ 0 \le P^{\text{ar}} - \tilde{p}^{\text{ar}}_\omega \perp \overline{\chi}^{\tilde{p}^{\text{ar}}}_\omega \ge 0, \qquad \forall \omega. \tag{2.5b}$$

in which $\mathcal{L}^{\text{ar}}_\omega$ states the Lagrangian function of (2.2). Lastly, the Karush-Kuhn-Tucker conditions associated with (2.3) write as follows:

$$\frac{\partial \mathcal{L}^{\text{ps}}_\omega}{\partial \tilde{\lambda}_\omega} = \sum_{n \in \mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_\omega - \underline{\chi}^{\tilde{\lambda}}_\omega + \overline{\chi}^{\tilde{\lambda}}_\omega = 0, \qquad \forall \omega, \tag{2.6a}$$

$$0 \le \tilde{\lambda}_\omega - \Lambda \perp \underline{\chi}^{\tilde{\lambda}} \ge 0,\ 0 \le \Lambda - \tilde{\lambda}_\omega \perp \overline{\chi}^{\tilde{\lambda}} \ge 0, \qquad \forall \omega. \tag{2.6b}$$

in which $\mathcal{L}^{\text{ps}}_\omega$ refers to the Lagrangian function of (2.3). It is worth noting that for inactive price constraints, i.e., a sufficiently large set $\mathcal{L}$ in (2.3b), the dual variables $\underline{\chi}^{\tilde{\lambda}}$ and $\overline{\chi}^{\tilde{\lambda}}$ in (2.6a) become zero. Thereby, the local spot market-clearing price revealed by the price-setter implicitly ensures the balance between the total energy supply and demand inside the community.

2.5 On the existence of a unique Nash equilibrium point

In economic analysis, two properties of a Nash equilibrium point are of major interest: its existence as well as its uniqueness. Only that way, policy-makers and analysts are able to derive a distinct conclusion from a game-theoretical model. To assess the existence and uniqueness of a Nash equilibrium point, this thesis considers Nash games in their equivalent form of Variational Inequality problems [77]. In detail, the Nash game $\Gamma_\omega(\mathcal{Z}, K_\omega, \{J_{i\omega}\}_{\forall i\in\mathcal{Z}}),\ \forall\omega\in\Omega$, is equivalent to a Variational Inequality problem with the game map

$$F_\omega(x_\omega) = [\nabla_{\tilde{p}_{n_1\omega}} J_{n_1\omega}(\tilde{p}_{n_1\omega}), \cdots, \nabla_{\tilde{p}_{N\omega}} J_{N\omega}(\tilde{p}_{N\omega}), \nabla_{\tilde{p}^{\text{ar}}_\omega} J^{\text{ar}}_\omega(\tilde{p}^{\text{ar}}_\omega), \nabla_{\tilde{\lambda}_\omega} J^{\text{ps}}_\omega(\tilde{\lambda}_\omega)]^\top, \qquad \forall\omega\in\Omega,$$

in which $x_\omega = [\tilde{p}_{n_1\omega}, \cdots, \tilde{p}_{N\omega}, \tilde{p}^{\text{ar}}_\omega, \tilde{\lambda}_\omega],\ \forall\omega\in\Omega$, denotes the strategy vector of the game.

For the Nash game $\Gamma_\omega(\mathcal{Z}, K_\omega, \{J_{i\omega}\}_{\forall i\in\mathcal{Z}}),\ \forall\omega\in\Omega$, the strategy set K_ω is closed, compact, convex, and non-empty. The game map $F_\omega(x_\omega)$ is continuous, since all cost functions $J_{i\omega}$ are continuously differentiable. Therefore, a solution set to the Variational Inequality problem exists, implying the existence of a Nash equilibrium point [77].

The singleton nature of this Nash equilibrium point can be assessed by considering the Jacobian matrix of the game map $F_\omega(x_\omega)$ as

$$\nabla_{x_\omega} F_\omega(x_\omega) = \begin{pmatrix} \beta & \cdots & 0 & 0 & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & \beta & 0 & 1 \\ 0 & \cdots & 0 & \beta & -1 \\ 1 & \cdots & 1 & -1 & 0 \end{pmatrix}, \qquad \forall\omega\in\Omega.$$

The Jacobian matrix is symmetric, as highlighted by the red diagonal entries. Therefore, the corresponding Nash game is *integrable* [75]. This implies that an *equivalent* optimization problem solving the Variational Inequality problem exists; its objective function is given by

$$\left\{\begin{aligned} \theta(x) &= \int_0^1 F(x^0 + t(x - x^0))^\top (x - x^0) dt \\ &\overset{x^0\to 0}{=} \int_0^1 t \begin{pmatrix} \tilde{\lambda}_\omega + \beta\tilde{p}_{n_1\omega} \\ \vdots \\ \tilde{\lambda}_\omega + \beta\tilde{p}_{N\omega} \\ -\tilde{\lambda}_\omega + C \\ \tilde{p}_{n_1\omega} + \cdots + \tilde{p}_{N\omega} - \tilde{p}^{\text{ar}}_\omega \end{pmatrix}^\top \begin{pmatrix} \tilde{p}_{n_1\omega} \\ \vdots \\ \tilde{p}_{N\omega} \\ \tilde{p}^{\text{ar}}_\omega \\ \tilde{\lambda}_\omega \end{pmatrix} dt && \text{(2.7a)} \\ &= \int_0^1 [t(\sum_{n\in\mathcal{N}} (\tilde{\lambda}_\omega\tilde{p}_{n\omega} + \beta\tilde{p}^2_{n\omega}) - \tilde{\lambda}_\omega\tilde{p}^{\text{ar}}_\omega + \tilde{\lambda}_\omega(\sum_{n\in\mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_\omega)) + C\tilde{p}^{\text{ar}}_\omega] dt && \text{(2.7b)} \\ &= (\sum_{n\in\mathcal{N}} (\tilde{\lambda}_\omega\tilde{p}_{n\omega} + \beta\tilde{p}^2_{n\omega}) - \tilde{\lambda}_\omega\tilde{p}^{\text{ar}}_\omega + \tilde{\lambda}_\omega(\sum_{n\in\mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_\omega))\frac{1}{2}[t^2]_0^1 + C\tilde{p}^{\text{ar}}_\omega && \text{(2.7c)} \\ &= \frac{1}{2}(\sum_{n\in\mathcal{N}} (\tilde{\lambda}_\omega\tilde{p}_{n\omega} + \beta\tilde{p}^2_{n\omega}) - \tilde{\lambda}_\omega\tilde{p}^{\text{ar}}_\omega + \tilde{\lambda}_\omega(\sum_{n\in\mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_\omega)) + C\tilde{p}^{\text{ar}}_\omega && \text{(2.7d)} \\ &= \tilde{\lambda}_\omega(\sum_{n\in\mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_\omega) + \sum_{n\in\mathcal{N}} \frac{1}{2}\beta p^2_{n\omega} + C\tilde{p}^{\text{ar}}_\omega && \text{(2.7e)} \end{aligned}\right\}\forall\omega\in\Omega.$$

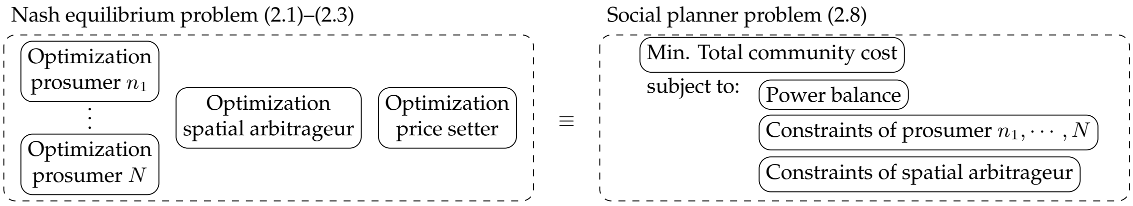

Figure 2.3 The equivalence of a Nash equilibrium to a social planner solution: The Nash equilibrium problem is mathematically equivalent to the social planner problem. This implies that any solution to the Nash equilibrium problem is also a solution to the social planner problem and vice versa.

This yields the following optimization problem:

$$\left\{ \begin{aligned} &\underset{\tilde{p}_{n\omega}, \tilde{p}^{\text{ar}}_{\omega}}{\text{Min}} \sum_{n \in \mathcal{N}} c(\tilde{p}_{n\omega}) + C \tilde{p}^{\text{ar}}_{\omega}, && \text{(2.8a)} \\ &\text{s.t. (2.1b), (2.1c)}, \ \forall n; \ \text{(2.2b)}, && \text{(2.8b)} \\ &\sum_{n \in \mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_{\omega} = 0 \colon \tilde{\lambda}_{\omega} \end{aligned} \right\}, \ \forall \omega \in \Omega, \quad \text{(2.8c)}$$

for which the Karush-Kuhn-Tucker optimality conditions are given by the following set of equations:

$$\frac{\partial \mathcal{L}^{\text{sp}}}{\partial \tilde{p}_{n\omega}} = \text{(2.4a)}, \ \text{(2.4b)–(2.4c)}, \qquad \forall n, \omega, \qquad \text{(2.9a)}$$

$$\frac{\partial \mathcal{L}^{\text{sp}}}{\partial \tilde{p}^{\text{ar}}_{\omega}} = \text{(2.5a)}, \ \text{(2.5b)}, \qquad \forall \omega, \qquad \text{(2.9b)}$$

$$\sum_{n \in \mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}^{\text{ar}}_{\omega} = 0, \qquad \forall \omega, \qquad \text{(2.9c)}$$

in which $\mathcal{L}^{\text{sp}}_{\omega}$ states the Lagrangian function of (2.8). A comparison of the Karush-Kuhn-Tucker conditions (2.9) to the Karush-Kuhn-Tucker conditions (2.4), (2.5), and (2.6) confirms, for inactive price constraints, the equivalence of the optimization problem (2.8) to the Nash equilibrium problem (2.1)–(2.3). The objective function (2.8a) is strongly convex and quadratic. Therefore, the solution of (2.8) is unique, implying a unique Nash equilibrium point for the Nash game $\Gamma_{\omega}(\mathcal{Z}, K_{\omega}, \{J_{i\omega}\}_{\forall i \in \mathcal{Z}}), \ \forall \omega \in \Omega$.

In fact, the optimization problem (2.8) can be interpreted as the problem of a social planner. As illustrated in Figure 2.3, the second welfare theorem states that the equivalence between perfectly competitive and complete trading and a welfare optimization "is a trivial reformulation of the usual one in welfare economics" [78, Chapter VIII]. However, note that an equivalent optimization problem does not exist for all Nash games proposed throughout this thesis.

Chapter 3

Towards an Access Economy for Energy Communities

Based on the game-theoretical fundamentals introduced in Chapter 2, this chapter proposes an access economy for distributed energy resources based on the example of energy storage systems. In particular, Section 3.1 discusses an access economy in the domain of non-cooperative market design options. The focus is on both physical and financial storage rights that are explicitly traded among community members. Section 3.2 proposes an access economy through a coalitional community operation which associates with a cooperative market design. The actual economic access to energy storage depends on the cost allocation rule in place. Lastly, Section 3.3 discusses the most representative numerical results of Publication **[A]**.

3.1 Non-cooperative access practices

In the domain of a non-cooperative market design, a merchant owner of an energy storage system usually participates in the market as an intertemporal arbitrageur. An intertemporal arbitrageur charges the storage during low-price periods and discharges it during high-price periods [79]. This results in a storage system operation exclusively in the interest of the storage owner herself. Other community members may attempt to install an energy storage system as well to compensate uncertainty and volatility in the renewable power generation. This would lead to excessive investments within energy communities since one storage system is most likely able to serve the flexibility needs of multiple prosumers. The following question arises: How to provide an economic incentive for prosumers to jointly operate an energy storage system, while each individual community member is a self-interest seeking local market participant?

The current practice of the transmission system operation in electricity markets in the United States of America offers a promising approach. In energy markets applying nodal pricing, each node of the transmission system has an individual energy price. The system operator purchases energy at one node, transports it via transmission lines, and sells the energy at a different node. Given that the transmission system is not congested and power losses are not taken into account, prices are equal at all nodes. In the case that the transmission system is congested, energy prices may differ among nodes. Consequently, the system operator collects a profit from the transportation, the so-called interspatial energy arbitrage or congestion rent [80]. However, customers of electrical services can purchase transmission rights, either physical or financial [34, 35, 81]. Transmission rights offer a win–win situation for both the transmission system operator and customers. On the one hand, the transmission system operator can recover her investment cost upfront without financially depending on the uncertain real-time operation. On the other hand, transmission rights guarantee right holders either a share of the transportation capacity of a specific transmission line or a claim on a share of the congestion rent collected by the transmission system operator. Thereby, right holders can secure themselves against uncertain prices at a future stage owing to a congested transmission system.

A similar practice can be applied to the energy storage system operation. In fact, a storage also transports energy, but in the time dimension. Similar to the case of nodal pricing, energy prices in time differ as well. Also similar to the transmission system operation, prices would be equivalent over time given a sufficiently large storage system. Based on these parallels between transmission system operation and storage system operation, a range of works proposes physical [27, 37] and financial storage rights [39, 40]. In this line, this thesis considers, as illustrated in Figure 3.1, physical and financial storage rights in the context of a local market for energy communities. Storage rights are traded among community members in a local forward market, i.e., a marketplace that clears well in advance to real-time. Thereby, a storage owner may recover her investment cost upfront without being exposed to uncertainty in the operational stage. The forward market for storage rights is followed by sequentially clearing spot markets for energy, i.e., one real-time market for each time step of interest. In real-time, physical storage right holders, as depicted in Figure 3.1a, directly dispatch their awarded share of energy storage systems, and thus yield through an operational access, an economic access to energy storage. In contrast, under financial storage rights, as shown in Figure 3.1b, no prosumer has an operational access to any storage system. The energy storage systems themselves are optimally dispatched in local spot markets by the community manager, who potentially collects a surplus from the intertemporal energy arbitrage. This surplus is redistributed post operation among financial storage right holders, who thereby yield an economic access to energy storage.

In the following, two game-theoretical models are developed: one for physical storage rights and another for financial storage rights. To account for energy storage over time, the assumption of a single local spot market, introduced in Section 2.3, is relaxed to multiple sequentially clearing local spot markets, i.e., one spot market for each hourly time step t of the planning horizon $\mathcal{T}$. In contrast to Chapter 2, this chapter considers an energy community manager who simultaneously fulfills the task of the spatial arbitrageur and the price-setter. Storage rights are traded once among prosumers in the local forward market, but are valid for all uncertainty realizations $\omega \in \Omega$ in all local spot markets $t \in \mathcal{T}$. This causes a coupling of the potential renewable power generation uncertainty realizations $\omega \in \Omega$, as opposed to the case in the Nash equilibrium problem (2.1)–(2.3) in Chapter 2. Therefore, each player solves the deterministic equivalent of a two-stage stochastic decision-making problem [82]. The first stage determines storage right trades in the local forward market, while the second stage explicitly accounts for all uncertainty realizations $\omega \in \Omega$ and potential recourses in local spot markets. This optimizes the objective of each player in *expectation*, as noted by $\mathbb{E}_{\Omega}[\cdot]$.

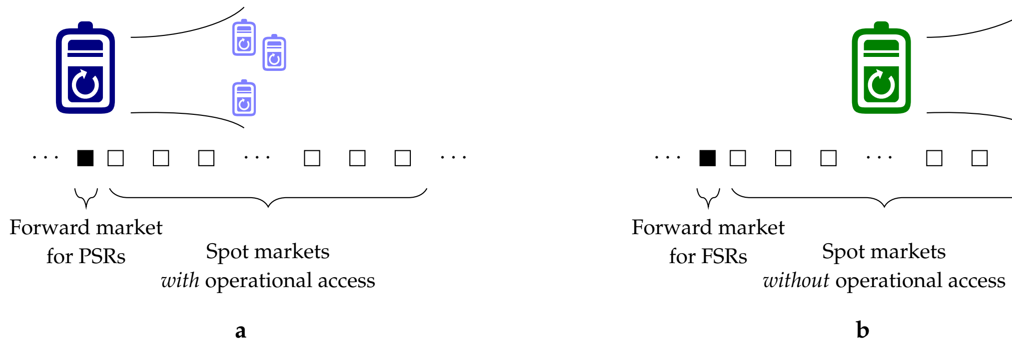

Figure 3.1 Access economy through physical and financial storage rights: Figure 3.1a depicts an access economy through physical storage rights (PSRs). These rights are traded in a local forward market. In local spot markets, right holders directly dispatch a share of the storage system. Similarly, as shown in Figure 3.1b, financial storage rights (FSRs) are also traded in a local forward market. However, in local spot markets, the storage system is operated by the community manger. The intertemporal energy arbitrage is redistributed post operation among storage right holders.

3.1.1 Trading physical storage rights for operational and economic access

Each prosumer $n \in \mathcal{N}$ minimizes and maximizes her local forward market physical storage right cost and revenue, as well as her uncertainty-dependent energy cost in all local spot markets $t \in \mathcal{T}$ in expectation. For given physical storage right prices μ_s^{PSR} and uncertain spot market energy prices $\tilde{\lambda}_{t\omega}$, she determines her local forward and uncertainty-dependent spot market participation strategy by solving the following optimization problem:

$$\Bigg\{ \underset{x^{\text{PSR}}_{s\in\Phi_n}, q^{\text{PSR}}_{ns}, \tilde{p}_{nt\omega}, \tilde{e}_{nst\omega}}{\text{Min}} \; -\sum_{s\in\Phi_n} \mu_s^{\text{PSR}} x_s^{\text{PSR}} + \sum_{s\in\mathcal{S}} \mu_s^{\text{PSR}} q_{ns}^{\text{PSR}} + \sum_{t\in\mathcal{T}} \mathbb{E}_\Omega \left[\tilde{\lambda}_{t\omega}\tilde{p}_{nt\omega} + c(\tilde{p}_{nt\omega})\right] \tag{3.1a}$$

$$\text{s.t. } x^{\text{PSR}}_{s\in\Phi_n} \in \mathcal{E}_{s\in\Phi_n},\; q^{\text{PSR}}_{ns} \in \mathcal{Q}, \qquad \forall s, \tag{3.1b}$$

$$\tilde{p}_{nt\omega} + \tilde{S}_{nt\omega} + \sum_{s\in\mathcal{S}} \tilde{e}_{nst\omega} - D_{nt} = 0, \qquad \forall t,\omega, \tag{3.1c}$$

$$\tilde{p}_{nt\omega} \in \mathcal{P}_n, \qquad \forall t,\omega, \tag{3.1d}$$

$$\tilde{e}_{nst\omega} \in \mathcal{E}_s(q^{\text{PSR}}_{ns}), \qquad \forall s,t,\omega \Bigg\},\; \forall n \in \mathcal{N}. \tag{3.1e}$$

Each prosumer n maximizes her forward market revenue by selling physical storage rights x_s^{PSR} according to physical storage right prices μ_s^{PSR} (3.1a). The prosumer n is able to sell storage rights x_s^{PSR} in the case that she owns a storage system $s \in \Phi_n$. The selling decision x_s^{PSR} is constrained by the maximum storage system capacity $\mathcal{E}_s$ (3.1b). At the same time, each prosumer n can buy physical storage rights q_{ns}^{PSR} from any storage system $s \in \mathcal{S}$ in the community. Therefore, the prosumer n also minimizes her forward market cost by buying physical storage rights q_{ns}^{PSR} according to physical storage right prices μ_s^{PSR}. The buying decision q_{ns}^{PSR} lies within a sufficiently large, closed, compact, and convex set $\mathcal{Q}$, such that the buying decision q_{ns}^{PSR} remains unconstrained (3.1b). Furthermore, the prosumer n minimizes her expected energy cost over all uncertainty realizations $\omega \in \Omega$ in all spot markets $t \in \mathcal{T}$. In each spot market t, she pays and receives for her spot market energy trade $\tilde{p}_{nt\omega}$ the uncertain spot market energy price $\tilde{\lambda}_{t\omega}$, respectively. Lastly, the objective function (3.1a) is endowed with a quadratic regularizer $c(\tilde{p}_{nt\omega}) = \frac{1}{2}\beta\tilde{p}^2_{nt\omega}$

associated with spot market energy trades $\tilde{p}_{nt\omega}$. Under each uncertainty realization ω and for each time step t, the prosumer n has to ensure her individual power balance (3.1c) between her uncertain renewable power generation $\tilde{S}_{nt\omega}$, her demand D_{nt}, and her spot market energy trade $\tilde{p}_{nt\omega}$. At the same time, she may have operational access to storage systems and choose the storage system operation $\tilde{e}_{nst\omega}$ in spot markets. The energy trade $\tilde{p}_{nt\omega}$ lies within a sufficiently large, closed, compact, and convex set $\mathcal{P}_n$ (3.1d). The operational access to storage systems $\tilde{e}_{nst\omega}$ depends on awarded physical storage rights q_{ns}^{PSR} in the forward market and the technical characteristic of the respective storage system s. The constraining set is given by $\mathcal{E}_s(q_{ns}^{\text{PSR}})$ (3.1e).

The energy community manager minimizes and maximizes the local forward market cost and revenue for physical storage right buyers and sellers, respectively. At the same time, she minimizes and maximizes the uncertainty-dependent local spot market cost and revenue for energy buyers and sellers in expectation, respectively. In local spot markets, she additionally fulfills the task of the spatial arbitrageur and minimizes the expected energy import and export cost. For given forward market trading decisions x_s^{PSR} and q_{ns}^{PSR} as well as uncertain spot market trading decisions $\tilde{p}_{nt\omega}$, she determines forward market prices for physical storage rights μ_s^{PSR}, uncertainty-dependent spot market energy prices $\tilde{\lambda}_{t\omega}$, and uncertainty-dependent energy imports and exports $\tilde{p}_{t\omega}^{\text{ar}}$ as follows:

$$\underset{\mu_s^{\text{PSR}},\tilde{\lambda}_{t\omega},\tilde{p}_{t\omega}^{\text{ar}}}{\text{Min}} \sum_{s\in\mathcal{S}} \mu_s^{\text{PSR}}(-x_s^{\text{PSR}} + \sum_{n\in\mathcal{N}} q_{ns}^{\text{PSR}}) + \sum_{t\in\mathcal{T}} \mathbb{E}_{\Omega}\big[C\tilde{p}_{t\omega}^{\text{ar}} + \tilde{\lambda}_{t\omega}(\sum_{n\in\mathcal{N}} \tilde{p}_{nt\omega} - \tilde{p}_{t\omega}^{\text{ar}})\big] \tag{3.2a}$$

$$\text{s.t. } \mu_s^{\text{PSR}} \in \mathcal{M}, \qquad \forall s, \tag{3.2b}$$

$$\tilde{\lambda}_{t\omega} \in \mathcal{L}, \qquad \forall t,\omega, \tag{3.2c}$$

$$\tilde{p}_{t\omega}^{\text{ar}} \in \mathcal{P}^{\text{ar}}, \qquad \forall t,\omega. \tag{3.2d}$$

The community manager reveals forward market prices for physical storage rights μ_s^{PSR}, such that the forward market revenue for storage right sellers x_s^{PSR} is maximized and the forward market cost for storage right buyers q_{ns}^{PSR} is minimized (3.2a). These prices μ_s^{PSR} lie within a sufficiently large, closed, compact, and convex set $\mathcal{M}$ (3.2b). At the same time, the community manager accounts for all uncertainty realizations $\omega \in \Omega$ in real-time and reveals spot market energy prices $\tilde{\lambda}_{t\omega}$, one for each time step $t \in \mathcal{T}$. These prices minimize and maximize the expected spot market cost and revenue of energy buyers and sellers, respectively. Spot market energy prices $\tilde{\lambda}_{t\omega}$ lie within a sufficiently large, closed, convex, and compact set $\mathcal{L}$ (3.2c). Lastly, the community manager determines for each time step t the spot market energy import and export $\tilde{p}_{t\omega}^{\text{ar}}$ according to the fixed cost C. At the same time, she receives and pays the spot market energy price $\tilde{\lambda}_{t\omega}$, respectively. Energy import and export decisions $\tilde{p}_{t\omega}^{\text{ar}}$ lie within a closed, convex, and compact set $\mathcal{P}^{\text{ar}}$ (3.2d).

3.1.2 Trading financial storage rights for economic access

Following the concept of financial storage rights, prosumers do not have operational access to storage systems. However, financial storage rights yield prosumers a claim on a share of the merchandising surplus from the storage system operation. Each prosumer $n \in \mathcal{N}$ minimizes and maximizes her local forward market financial storage right cost and revenue, as well as her uncertainty-dependent energy cost in all local spot markets $t \in \mathcal{T}$ in expectation. In addition, each prosumer accounts for her expected revenue in local spot markets according to her awarded financial storage rights. For given financial storage right prices μ_s^{FSR}, uncertain spot market energy prices $\tilde{\lambda}_{t\omega}$, and uncertain financial storage right values $\tilde{\gamma}^{\text{FSR}}_{st\omega}$ [39], she chooses her local forward and uncertainty-dependent spot market participation strategy as follows:

$$\left\{ \underset{x^{\text{FSR}}_{s\in\Phi_n}, q^{\text{FSR}}_{ns}, \tilde{p}_{nt\omega}}{\text{Min}} \quad -\sum_{s\in\Phi_n} \mu^{\text{FSR}}_s x^{\text{FSR}}_s + \sum_{s\in\mathcal{S}} \mu^{\text{FSR}}_s q^{\text{FSR}}_{ns} + \sum_{t\in\mathcal{T}} \mathbb{E}_\Omega \left[\tilde{\lambda}_{t\omega} \tilde{p}_{nt\omega} + c(\tilde{p}_{nt\omega}) - \sum_{s\in\mathcal{S}} \tilde{\gamma}^{\text{FSR}}_{st\omega} q^{\text{FSR}}_{ns} \right] \right. \tag{3.3a}$$

$$\text{s.t. } x^{\text{FSR}}_{s\in\Phi_n} \in \mathcal{E}_{s\in\Phi_n},\ q^{\text{FSR}}_{ns} \in \mathcal{Q}, \qquad \forall s, \tag{3.3b}$$

$$\tilde{p}_{nt\omega} + \tilde{S}_{nt\omega} - D_{nt} = 0, \qquad \forall \omega, t, \tag{3.3c}$$

$$\left. \tilde{p}_{nt\omega} \in \mathcal{P}_n, \qquad \forall \omega, t \right\},\ \forall n \in \mathcal{N}. \tag{3.3d}$$

Similar to (3.2a), each prosumer n minimizes her forward market cost and maximizes her forward market revenue by buying q^{FSR}_{ns} and selling financial storage rights x^{FSR}_s according to financial storage right prices μ^{FSR}_s (3.3a). Storage right trades are constrained in (3.3b). At the same time, the prosumer n minimizes her expected energy cost in all spot markets $t \in \mathcal{T}$ from spot market energy trades $\tilde{p}_{nt\omega}$ according to uncertain spot market energy prices $\tilde{\lambda}_{t\omega}$. Again, the objective function (3.3a) is endowed with a quadratic regularizer $c(\tilde{p}_{nt\omega}) = \frac{1}{2}\beta\tilde{p}^2_{nt\omega}$ associated with spot market energy trades $\tilde{p}_{nt\omega}$. In contrast to (3.2a), the prosumer n additionally accounts for her expected revenue from her awarded financial storage rights q^{FSR}_{ns} in the forward market and uncertain financial storage right values $\tilde{\gamma}^{\text{FSR}}_{st\omega}$ in spot markets. Under each uncertainty realization ω and for each time step t, the prosumer n has to ensure her individual power balance (3.3c) without operational access to any storage system. Again, spot market energy trades $\tilde{p}_{nt\omega}$ lie within a sufficiently large, closed, compact, and convex set $\mathcal{P}_n$ (3.3d).

Similar to the case of physical storage rights, the energy community manager minimizes and maximizes the cost and revenue for financial storage right buyers and sellers in the local forward. At the same time, she minimizes and maximizes the uncertainty-dependent local spot market energy cost and revenue for buyers and sellers in expectation. In addition, she minimizes the uncertainty-dependent energy import and export cost as well as the uncertainty-dependent intertemporal energy arbitrage from the storage operation in local spot markets, also in expectation. For given forward market trading decisions x^{FSR}_s and q^{FSR}_{ns}, as well as uncertain spot market trading decisions $\tilde{p}_{nt\omega}$, the community manager chooses financial storage right prices μ^{FSR}_s, uncertainty-dependent spot market energy prices $\tilde{\lambda}_{t\omega}$, and uncertainty-dependent energy imports and exports $\tilde{p}^{\text{ar}}_{t\omega}$. Moreover, she determines the uncertainty-dependent operation $\tilde{e}_{st\omega}$ off all storage systems within the community. For this purpose, she solves the following optimization problem:

$$\underset{\mu^{\text{FSR}}_s, \tilde{\lambda}_{t\omega}, \tilde{p}^{\text{ar}}_{t\omega}, \tilde{e}_{st\omega}}{\text{Min}} \sum_{s\in\mathcal{S}} \mu^{\text{FSR}}_s \Big(-x^{\text{FSR}}_s + \sum_{n\in\mathcal{N}} q^{\text{FSR}}_{ns}\Big) + \sum_{t\in\mathcal{T}} \mathbb{E}_\Omega \Big[C\tilde{p}^{\text{ar}}_{t\omega} + \tilde{\lambda}_{t\omega} \Big(\sum_{n\in\mathcal{N}} \tilde{p}_{nt\omega} - \tilde{p}^{\text{ar}}_{t\omega} + \sum_{s\in\mathcal{S}} \tilde{e}_{st\omega} \Big) \Big] \tag{3.4a}$$

$$\text{s.t. } \mu^{\text{FSR}}_s \in \mathcal{M}, \qquad \forall s, \tag{3.4b}$$

$$\tilde{\lambda}_{t\omega} \in \mathcal{L}, \qquad \forall t, \omega, \tag{3.4c}$$

$$\tilde{p}^{\text{ar}}_{t\omega} \in \mathcal{P}^{\text{ar}}, \qquad \forall t, \omega, \tag{3.4d}$$

$$\tilde{e}_{st\omega} \in \mathcal{E}_s : \gamma^{\text{FSR}}_{st\omega}, \qquad \forall s, t, \omega. \tag{3.4e}$$

Similar to (3.2), the community manager determines financial storage right prices μ^{FSR}_s and spot market energy prices $\tilde{\lambda}_{t\omega}$, such that in the forward and all spot markets, the cost for buyers is minimized and the revenue for sellers is maximized (3.4a). She also fulfills the task of the spatial arbitrageur and identifies spot market energy imports and exports $\tilde{p}^{\text{ar}}_{t\omega}$ according to the fixed cost C

and spot market energy prices $\tilde{\lambda}_{t\omega}$. Financial storage right prices and spot market energy prices lie within a sufficiently large, closed, compact, and convex set $\mathcal{M}$ (3.4b) and $\mathcal{L}$ (3.4c), respectively. Spot market energy imports and exports $\tilde{p}^{\text{ar}}_{t\omega}$ lie within the closed, compact, and convex set $\mathcal{P}^{\text{ar}}$ (3.4d). In contrast to (3.2), the community manager determines the operation of all storage systems $s \in \mathcal{S}$ in sport markets. Therefore, she additionally accounts for the expected intertemporal energy arbitrage from the operation $\tilde{e}_{st\omega}$ of all storage systems according to spot market energy prices $\tilde{\lambda}_{t\omega}$. The storage system operation $\tilde{e}_{st\omega}$ is constrained by the technical characteristic $\mathcal{E}_s$ of the respective storage system s (3.4e). Note that the dual variable $\gamma^{\text{FSR}}_{st\omega}$ associated with the storage system capacity constraint in (3.4e) describes the financial storage right value of the storage system s at time step t under uncertainty realization ω. The sum of all financial storage right values $\gamma^{\text{FSR}}_{st\omega}$ for one specific storage system s represents the total intertemporal energy arbitrage realized by its operation in spot markets under the uncertainty realization ω [39].

Publication **[A]** provides the proof that for both Nash equilibrium problems, either with physical or financial storage rights, a unique market-clearing outcome exists. Moreover, this publication shows the equivalence between the proposed Nash equilibrium problem with physical storage rights and a single social planner problem. For the Nash equilibrium problem with financial storage rights, this is not the case. However, Publication **[A]** proposes two optimization problems that have to be solved sequentially. This solving approach enables the assessment of the local market-clearing solution for the Nash equilibrium problem including financial storage rights.

3.2 Cooperative access practices

Another access practice to distributed energy resources, such as energy storage systems, is based on a cooperative market design [24, 41]. Within a cooperative market design, as illustrated in Figure 3.2, community members negotiate well in advance to real-time a community cost allocation rule. Local spot markets, which determined the dispatch of all distributed energy resources in the presence of a non-cooperative market design, are replaced by a centralized operation through the energy community manager. Consequently, no prosumer has an operational access to storage systems. However, post operation, the total community cost, including the benefit of storage system operation, is redistributed among community members according to the cost allocation rule agreed upon by prosumers. Thereby, the cost allocation rule in place determines the degree of economic access individuals have to energy storage.

For the sake of an access economy through a cooperative market design, this section assumes that all prosumers reveal their true cost, preferences, and information about their distributed energy resources. Note that the negotiation process regarding the community cost allocation rule is not within the scope of this thesis and left aside for future research. The coalitional operating problem solved by the energy community manager determines the community operation for each uncertainty realization $\omega \in \Omega$, separately. Therefore, the community manager solves a deterministic optimization problem structurally identical to the social planner problem (2.8) in Chapter 2.

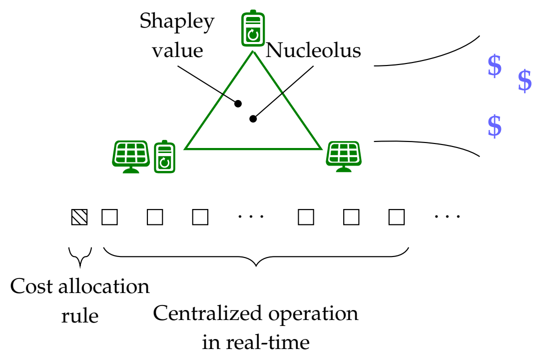

Figure 3.2 Access economy through a coalitional community operation: Community members agree a priori on a cost allocation rule, e.g., the Shapley value or the nucleolus. In real-time, all distributed energy resources within the community are dispatched by the community manager. Post operation, the total community cost is redistributed among all community members.

3.2.1 The coalitional operating problem

Given the true cost, preferences, and information by prosumers, the energy community manager minimizes the total community cost under each uncertainty realization $\omega \in \Omega$. She determines the optimal operation of all distributed energy resources by solving the following social planner problem:

$$\Big\{ \underset{\tilde{p}_{nt\omega},\tilde{p}^{\text{ar}}_{t\omega},\tilde{e}_{st\omega},}{\text{Min}} \sum_{t\in\mathcal{T}} \Big(C\tilde{p}^{\text{ar}}_{t\omega} + \sum_{n\in\mathcal{N}} c(\tilde{p}_{nt\omega}) \Big) \tag{3.5a}$$

$$\text{s.t. } \tilde{p}_{nt\omega} + \tilde{S}_{nt\omega} + \sum_{s\in\Phi_n} \tilde{e}_{st\omega} - D_{nt} = 0, \quad \forall n,t, \tag{3.5b}$$

$$\tilde{p}_{nt\omega} \in \mathcal{P}_n, \quad \forall n,t, \tag{3.5c}$$

$$\tilde{e}_{st\omega} \in \mathcal{E}_s, \quad \forall s,t, \tag{3.5d}$$

$$\tilde{p}^{\text{ar}}_{t\omega} \in \mathcal{P}^{\text{ar}}, \quad \forall t, \tag{3.5e}$$

$$\sum_{n\in\mathcal{N}} \tilde{p}_{nt\omega} - \tilde{p}^{\text{ar}}_{t\omega} = 0, \quad \forall t \Big\},\ \forall\omega. \tag{3.5f}$$

Under each uncertainty realization ω, the energy community manager minimizes over all time steps $t \in \mathcal{T}$ the cost from the energy import and export $\tilde{p}^{\text{ar}}_{t\omega}$ according to the fixed cost C (3.5a). Moreover, she accounts for all quadratic regularizers $c(\tilde{p}_{nt\omega}) = \frac{1}{2}\beta\tilde{p}^2_{nt\omega}$ associated with energy exchanges among prosumers. The community manager ensures that for each time step t, the power balance (3.5b) of each prosumer $n \in \mathcal{N}$ is fulfilled. In addition, she accounts for the constraining set $\mathcal{P}_n$ on energy exchanges among prosumers (3.5c), as well as the constraints $\mathcal{E}_s$ on the operation $\tilde{e}_{st\omega}$ of each storage system $s \in \mathcal{S}$ (3.5d). Moreover, she respects the constraining set $\mathcal{P}^{\text{ar}}$ on energy exchanges $\tilde{p}^{\text{ar}}_{t\omega}$ with the distribution system (3.5e). Lastly, the equality constraint (3.5f) defines, for each time step t, the balance between the total energy supply and demand inside the community. The operational point determined by the community manager is unique since the objective function (3.5a) is strongly convex and all decision variables lie within closed, convex, and compact sets.

3.2.2 On the cost allocation

The total community cost, comprising the cost and revenue from the storage system operation, is redistributed among prosumers. Simple, but intuitive, cost allocation rules do not necessarily result in a situation that pleases all prosumers [83]. Therefore, all possible coalitions $\mathcal{S} \subseteq \mathcal{N}$, i.e., all possible combinations of prosumers given by $\mathcal{S} \in 2^N$, must be considered [12]. Only in this way can a satisfactory financial burden assigned to each prosumer. However, the exponential growth of the coalition set $\mathcal{S} \in 2^N$ indicates the computational burden as the community size increases in terms of community members.

For each coalition $\mathcal{S}$ and each uncertainty realization $\omega \in \Omega$, a cost saving function from cooperating is given by

$$v_\omega(\mathcal{S}) = \sum_{n \in \mathcal{S}} C_{n\omega} - C_{\mathcal{S}\omega}, \qquad \forall \omega, \tag{3.6}$$

in which $C_{n\omega}$ is the cost for each prosumer $n \in \mathcal{S}$ if she operates her distributed energy resources individually [13]. The symbol $C_{\mathcal{S}\omega}$ refers to the cost for the coalition $\mathcal{S}$ if prosumers operate their distributed energy resources jointly. Based on the cost saving function, a coalitional game is given by the pair $(\mathcal{N}, v_\omega(\mathcal{S}))$. One important characteristic of a coalitional game is super-additivity. This means that the value of a union of two selected coalitions, i.e., $\mathcal{S} \in 2^N$ and $\mathcal{V} \in 2^N$ with $\mathcal{S} \cap \mathcal{V} = \varnothing$, is not less than the sum of their separate values, i.e., $v_\omega(\mathcal{S}) + v_\omega(\mathcal{V}) \leq v_\omega(\mathcal{S} \cup \mathcal{V})$ [12, 13]. This implies that a cooperation among all prosumers, the so-called grand coalition $\mathcal{S} = \mathcal{N}$, is most beneficial.

However, super-additivity implies neither that all players benefit individually, nor that a cooperation among all prosumers is stable. Incentives for prosumers may exist to leave the grand coalition $\mathcal{N}$. Therefore, three important properties have to be considered, namely *efficiency*, *individual rationality*, and *stability* [13]. A cost allocation rule is efficient if it redistributes the whole value of the grand coalition $\mathcal{N}$, i.e., $\sum_{n \in \mathcal{N}} x_{n\omega} = v_\omega(\mathcal{N})$, in which $x_{n\omega}$ is the value assigned to each prosumer $n \in \mathcal{N}$. Individual rationality is given if each prosumer benefits from the grand coalition at least as much as she would gain individually, i.e., $x_{n\omega} \geq v_\omega(n)$. Lastly, a coalition is stable if the cost allocation lies within the *core* C^*_ω [12], given by

$$C^*_\omega = \Big\{ \{x_{n\omega}\}_{\forall n \in \mathcal{N}} \mid \sum_{n \in \mathcal{N}} x_{n\omega} = v_\omega(\mathcal{N});\ v_\omega(\mathcal{S}) - \sum_{n \in \mathcal{S}} x_{n\omega} \geq 0, \forall n \in \mathcal{N} \Big\}, \qquad \forall \omega. \tag{3.7}$$

The equality constraint ensures an efficient cost allocation, while the inequality constraint guarantees the stability of the grand coalition.

In Publication **[A]**, two cost allocation rules that enable an access economy for energy storage are considered more in detail, namely the Shapley value [84] and the nucleolus [43]. The Shapley value describes the average marginal contribution of each prosumer to the total community cost [10, 42, 83]. However, since the Shapley value does not guarantee a stable coalition comprising all community members, Publication **[A]** studies the nucleolus [12] as well. The nucleolus proposes a cost allocation that minimizes the dissatisfaction of each individual community member, and thereby guarantees a stable energy community.

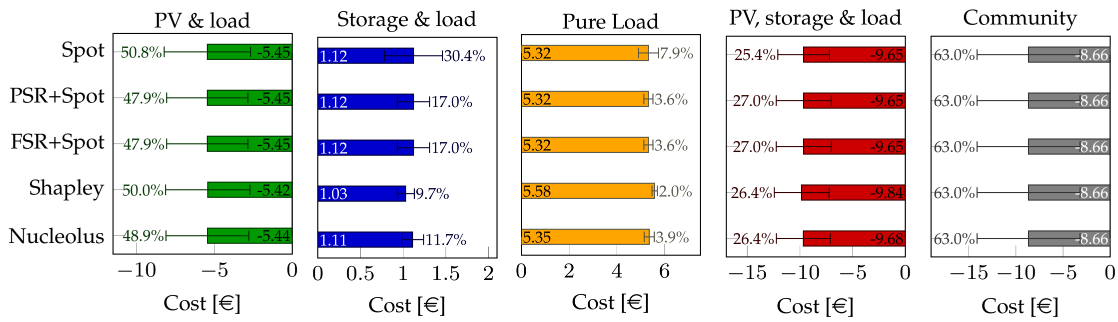

Figure 3.3 Expected cost and its volatility in the presence of an access economy: Numbers within color bars refer to the expected cost in €. Numbers next to lines show the standard deviation as a percentage of the expected cost. Note that spot refers to the benchmark case of sequentially clearing spot markets without an access economy for energy storage systems. This figure is adapted from Publication **[A]**.

3.3 Assessing the potential of an access economy

To shed light on the proposed local market design alternatives, Publication **[A]** considers, among other numerical analyses, the expected cost and its volatility for four representative prosumers as well as for the community as a whole. A non-cooperative local market design without an access economy for energy storage serves as a benchmark for evaluation. The first prosumer owns a pohotovoltaic system, but has no storage system. The second prosumer owns a storage system, but has no renewable power generation. The third prosumer is a pure consumer owning neither a phtvoltaic, nor a storage system. Lastly, the fourth prosumer owns both a phtvoltaic and a storage system.

Figure 3.3 shows the expected cost and its standard deviation for each prosumer and for the community as a whole under the non-cooperative as well as the cooperative market design alternatives. An important observation in Figure 3.3 is that under the three non-cooperative market design alternatives, with or without storage rights, the expected cost of each prosumer remains unchanged. Under the two cooperative market design alternatives, the expected cost of each prosumer slightly changes, and thus depends on the technologies owned by the respective prosumer.

The cost distribution each prosumer yields over uncertainty realizations $\omega \in \Omega$ strongly depends on the local market design in place. This cost volatility decreases for the majority of prosumers in the presence of an access economy for energy storage. In particular, the first prosumer reduces her cost volatility through storage rights, and thereby more efficiently utilizes her uncertain photovoltaic power generation. However, the renewable power generation yields no advantage in the cooperative market design alternatives. Renewable power generation is rewarded only by a slight reduction in the cost volatility, while the expected revenue decreases as well. The second prosumer remarkably reduces her cost volatility by selling storage rights, and thereby being less exposed to uncertain local spot market-clearing prices. In the cooperative market design alternatives, the energy storage system provides a great contribution that is rewarded by a reduction in the expected energy cost as well as a significant decrease in the cost volatility. The third prosumer also reduces her cost volatility through storage rights, and thereby compensates uncertain local spot market energy prices. However, in the cooperative market design alternatives she yields higher expected cost, but a decrease in her energy cost volatility. Lastly, the fourth prosumer, who owns a photovoltaic and a storage system, yields an increased revenue volatility by selling storage rights since she cannot fully utilize her storage system to compensate her own uncertain power generation. In the cooperative market design alternatives, the renewable power generation and the potential of energy storage are rewarded by an increase in the expected revenue, although the revenue volatility increases as well.

Even though individual costs change in terms of the expected value and its standard deviation, the expected total community cost and its volatility remain unchanged for all proposed local market design alternatives. This implies that the proposed local market design alternatives do not affect the operational strategy of the community as a whole. However, they distinguish in the cost allocation among prosumers, i.e., who pays whom which amount of money.

To this end, Publication **[A]** provides additional insights into an energy community with 16 prosumers. The computational time necessary for solving the proposed local market design alternatives highlights the limitation of cooperative market design options as the community size increases in terms of prosumers. From a policy perspective, Publication **[A]** provides a comprehensive discussion on the benefits and drawbacks of each local market design. Publication **[A]** concludes that the preferred choice from the perspective of prosumers for an access economy either via storage rights or a coalitional operation depends on three aspects: the technologies each prosumer owns, the individual preference towards cost volatility, and the market design properties, which are of utmost attraction to community members.

Chapter 4

On Risk and Ambiguity Preferences of Energy Community Members

A key insight from the research on an access economy for distributed energy resources in Chapter 3 was that the preferred choice for a local market design highly depends on the individual preferences of community members. In particular, the volatility that prosumers are willing to accept in their energy cost. Therefore, this chapter addresses risk and ambiguity preferences of individual local market participants. Section 4.1 incorporates the *risk aversion* against costly uncertainty realizations into the decision-making problem of prosumers and proposes risk trading among community members to neutralize heterogeneous risk aversion. Section 4.2 relaxes the assumption of an exactly known probability distribution function and incorporates *ambiguity aversion* against the probabilistic forecast into the decision-making problem of local market participants. For this purpose, Section 4.2 considers the uncertain renewable power generation to follow continuous probability distribution function, as opposed to finite and discrete uncertainty realizations considered previously. Lastly, Section 4.3 provides the most representative numerical results of Publication **[B]** and Publication **[C]**.

4.1 Incorporating risk aversion into decision-making problems

Decision making solely in expectation of a given probability distribution function does not pay special attention to extreme events. However, a player may be concerned about costly uncertainty realizations at a future stage, even though they are unlikely. This preference is referred to as risk aversion. As shown in Figure 4.1, a risk-averse player observes a probability distribution function and explicitly assigns a higher value to the probability of a costly uncertainty realization and a lower weight to an advantageous uncertainty realization [45, 49]. Thereby, she derives a *risk-adjusted* probability distribution function describing the uncertain event. Based on this function, she chooses a market participation strategy in risk-averse expectation. Thereby, a risk-averse player reduces her cost volatility, but simultaneously increases her expected cost compared to a risk-neutral decision making. Note that in practice local market participants are heterogeneously risk-averse. This means that players have an individual risk-averse expectation of their energy cost, even though their cost may be equivalent given a risk-neutral expectation.

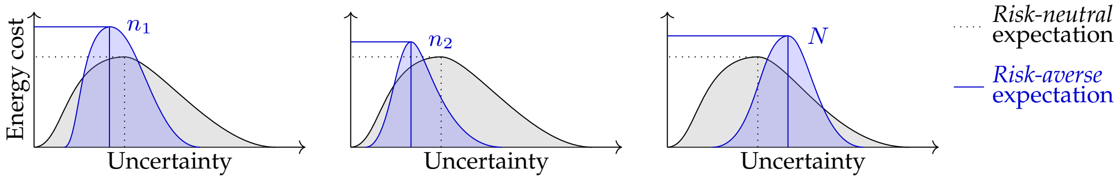

Figure 4.1 Risk-averse decision making: A risk-averse player assigns a higher weight to costly events, and thereby considers a risk-adjusted probability distribution function in her decision-making problem. In practice, local market participants are heterogeneously risk-averse. Consequently, the players n_1, n_2 to N have an individual risk-averse expectation of their energy cost, even though their cost may be equivalent given a risk-neutral expectation.

4.1.1 A coherent risk measure function and risk trading

To assess the impact of heterogeneous risk aversion in energy communities, this thesis incorporates a coherent risk-measure function $\rho(\cdot)$ into the decision-making problem of prosumers [47, 48]. In detail, this thesis applies the dual representation of the conditional value-at-risk [58] given by

$$\rho(Z_\omega) = \max_{\pi_\omega^\rho} \sum_{\omega \in \Omega} \pi_\omega^\rho Z_\omega \tag{4.1a}$$

$$\text{s.t.} \sum_{\omega \in \Omega} \pi_\omega^\rho = 1, \tag{4.1b}$$

$$0 \leq \pi_\omega^\rho \leq \frac{1}{\alpha} \pi_\omega^\Theta, \qquad \forall \omega. \tag{4.1c}$$

The dual representation of a coherent risk-measure function $\rho(\cdot)$ states a constrained optimization problem. For the conditional value-at-risk, the objective function (4.1a) maximizes an uncertainty-dependent cost Z_ω through the choice of risk-adjusted probabilities π_ω^ρ [45]. The equality constraint (4.1b) ensures that the sum of all risk-adjusted probabilities is equal to one. The lower bound of the inequality constraint (4.1c) ensures the non-negativity of risk-adjusted probabilities π_ω^ρ. The upper bound of (4.1c) allows an increased weight of uncertainty realizations $\omega \in \Omega$ according to physical probabilities π_ω^Θ, i.e., the real world observations [46], and the parameter $\alpha \in (0, 1]$. This parameter indicates the percentile of the conditional value-at-risk, and thereby the risk aversion of the respective player.

Under the assumption of homogeneous risk aversion [85], all market participants have identical risk attitudes. However, this assumption would lead to an overestimation of the local market potential since participants are in practice heterogeneously risk-averse. Owing to their interaction on a local energy market, the risk-averse decision making of one prosumer causes not only the own expected energy cost to increase, but also affects the cost of her competitors. However, some prosumers may be willing to accept a greater cost volatility, while enjoying a lower energy cost in expectation. One possibility to neutralize heterogeneous risk aversion, and thereby to prevent the unintended externality of highly risk-averse decision making on prosumers with a low risk aversion, is *risk trading*. Risk trading describes the exchange of a financial product, e.g., an *Arrow-Debreu security*, to transfer the cost under a specific uncertainty realization from a highly risk-averse player to a player with a low risk aversion [52]. For each uncertainty realization $\omega \in \Omega$, an Arrow-Debreu security is an unconstrained contract between a security seller and a security buyer. In a forward market, i.e., a marketplace that clears well in advance to real-time, the seller receives a market-driven price, the so-called *risk price* μ_ω^{AD}. In return, the buyer receives a payment of 1 in real-time given the specific uncertainty realization ω.

The following section proposes a risk-averse Nash equilibrium problem with risk trading. In this game, prosumers and a spatial arbitrageur trade energy in a local forward market which determines a tentative energy production and consumption schedule. In addition, Arrow-Debreu securities are available for risk trading. The local forward market is followed by a single local spot market in which the tentative schedule is adjusted according to the respective uncertainty realization of the renewable power generation. Furthermore, local market participants yield a revenue or a cost according to their awarded Arrow-Debreu securities. The assumptions stated in Section 2.3 are relaxed in the sense that prosumers and the spatial arbitrageur are potentially risk-averse. Depending on the availability of Arrow-Debreu securities, the local market may be either fully or partially incomplete for risk, or complete for risk. This means that Arrow-Debreu securities are either available for none, for a subset, or for all uncertainty realizations $\omega \in \Omega$. Similar to the decision-making problems (3.1)–(3.4) in Chapter 3, the uncertainty realizations $\omega \in \Omega$ of the renewable power generation are coupled owing to the local forward market trading decision for energy and Arrow-Debreu securities. Therefore, each local market participant solves the deterministic counterpart of a two-stage stochastic optimization problem [82]. The first stage associates with energy and Arrow-Debreu security trades in the local forward market. The second stage explicitly accounts in potentially risk-averse expectation for each uncertainty realization and respective recourses of players in the local spot market.

4.1.2 Heterogeneous risk aversion: Risk trading among community members

Each risk-averse prosumer $n \in \mathcal{N}$ minimizes her local forward and her uncertainty-dependent local spot market energy cost in risk-averse expectation. At the same time, she accounts for her local forward market cost and revenue from Arrow-Debreu security trades, as well as the revenue and cost from awarded and sold Arrow-Debreu securities under each uncertainty realization $\omega \in \Omega$ in the local spot market. For a given forward market energy price λ, forward market risk prices μ_ω^{AD}, and an uncertain spot market energy price $\tilde{\lambda}_\omega$, she chooses her forward and uncertainty-dependent spot market participation strategy by solving the following optimization problem:

$$\Big\{ \underset{p_n, a_{n\omega}, \tilde{p}_{n\omega}}{\mathrm{Min}} \ \lambda p_n + c(p_n) + \sum_{\omega \in \Omega} \mu_\omega^{\mathrm{AD}} a_{n\omega} + \rho_n \Big(\tilde{\lambda}_\omega \tilde{p}_{n\omega} + c(\tilde{p}_{n\omega}) - a_{n\omega} \Big) \tag{4.2a}$$

$$\text{s.t. } p_n + \tilde{p}_{n\omega} + \tilde{S}_{n\omega} - D_n = 0, \qquad \forall \omega, \tag{4.2b}$$

$$\{p_n, \tilde{p}_{n\omega}\} \in \mathcal{P}_n, \qquad \forall \omega \Big\}, \ \forall n \in \mathcal{N}. \tag{4.2c}$$

Each prosumer n minimizes her energy cost from the forward market energy trade p_n according to the forward market energy price λ (4.2a). In addition, she accounts for a quadratic regularizer $c(p_n) = \frac{1}{2}\beta p_n^2$ associated with the forward market energy trade p_n. Furthermore, the prosumer n incurs a forward market cost or earns a forward market revenue associated with her Arrow-Debreu security trades $a_{n\omega}$ according to forward market risk prices μ_ω^{AD}. A positive value for $a_{n\omega}$ implies that prosumer n buys securities for the uncertainty realization ω, while a negative value for $a_{n\omega}$ indicates that prosumer n sells securities. The uncertainty-dependent spot market cost is endowed with the coherent risk measure function $\rho_n(\cdot)$. The spot market cost consists of the spot market energy trade $\tilde{p}_{n\omega}$ according to the uncertain spot market energy price $\tilde{\lambda}_\omega$, a quadratic regularizer $c(\tilde{p}_{n\omega}) = \frac{1}{2}\beta\tilde{p}_{n\omega}^2$ associated with the spot market energy trade $\tilde{p}_{n\omega}$, and a cost or revenue from Arrow-Debreu securities $a_{n\omega}$ awarded in the forward market. Under each uncertainty realization ω, the prosumer n has to ensure her individual power balance (4.2b) between the forward market energy trade p_n, the uncertainty-dependent spot market energy trade $\tilde{p}_{n\omega}$, her demand D_n, and the uncertain renewable power generation $\tilde{S}_{n\omega}$. Both the forward p_n and the spot market trading decision $\tilde{p}_{n\omega}$ lie within a sufficiently large, closed, compact, and convex set $\mathcal{P}_n$ (4.2c).

The risk-averse spatial arbitrageur also participates in risk trading. She minimizes her local forward and uncertainty-dependent local spot market cost from energy import and export in risk-averse expectation. In addition, she accounts for her forward market cost and revenue from Arrow-Debreu security trades as well as the revenue and cost from awarded Arrow-Debreu securities under each uncertainty realization ω in the local spot market. For a given forward market energy price λ, forward market risk prices μ_ω^{AD}, and an uncertain spot market energy price $\tilde{\lambda}_\omega$, she chooses her forward and uncertainty-dependent spot market participation strategy as follows:

$$\underset{p^{\text{ar}}, b_\omega, \tilde{p}_\omega^{\text{ar}}}{\text{Min}} \; (C - \lambda)p^{\text{ar}} + \sum_{\omega \in \Omega} \mu_\omega^{\text{AD}} b_\omega + \rho^{\text{ar}}\Big((C' - \tilde{\lambda}_\omega)\tilde{p}_\omega^{\text{ar}} - b_\omega\Big) \tag{4.3a}$$

$$\text{s.t. } \{p^{\text{ar}}, \tilde{p}_\omega^{\text{ar}}\} \in \mathcal{P}^{\text{ar}}, \qquad \forall \omega. \tag{4.3b}$$

The spatial arbitrageur minimizes her energy cost in the forward market from the energy import and export p^{ar} according to the fixed cost C (4.3a). She receives and pays for the forward market energy import and export the forward market energy price λ, respectively. Furthermore, the spatial arbitrageur incurs a forward market cost or earns a forward market revenue associated with Arrow-Debreu security trades b_ω according to forward market risk prices μ_ω^{AD}. A positive value for b_ω indicates a demand, while a negative value for b_ω states a supply of securities. Similar to risk-averse prosumers, the uncertainty-dependent spot market cost is endowed with the risk-measure function $\rho^{\text{ar}}(\cdot)$. The spot market cost consists of the spot market energy import and export $\tilde{p}_\omega^{\text{ar}}$ according to the fixed cost C' and the uncertain spot market energy price $\tilde{\lambda}_\omega$, and a cost or revenue from Arrow-Debreu securities b_ω awarded in the forward market. The forward p^{ar} and the spot market trading decision $\tilde{p}_\omega^{\text{ar}}$ lie within the closed, compact, and convex set $\mathcal{P}^{\text{ar}}$ (4.3b).

Lastly, the price-setter minimizes and maximizes the forward market cost and revenue for energy and Arrow-Debreu security buyers and sellers, respectively. In addition, she also minimizes and maximizes the uncertainty-dependent spot market cost and revenue for energy buyers and sellers in risk-averse expectation. For given forward market p_n, p^{ar}, $a_{n\omega}$, and b_ω, as well as uncertain spot market trading decisions $\tilde{p}_{n\omega}$ and $\tilde{p}_\omega^{\text{ar}}$, she reveals a forward market energy price λ, forward market risk prices μ_ω^{AD}, and an uncertainty-dependent spot market energy price $\tilde{\lambda}_\omega$ by solving the following optimization problem:

$$\underset{\lambda, \mu_\omega^{\text{AD}}, \tilde{\lambda}_\omega}{\text{Min}} \; \lambda(\sum_{n \in \mathcal{N}} p_n - p^{\text{ar}}) + \sum_{\omega \in \Omega} \mu_\omega^{\text{AD}}(\sum_{n \in \mathcal{N}} a_{n\omega} + b_\omega) + \rho^{\cap}\Big(\tilde{\lambda}_\omega(\sum_{n \in \mathcal{N}} \tilde{p}_{n\omega} - \tilde{p}_\omega^{\text{ar}})\Big) \tag{4.4a}$$

$$\text{s.t. } \mu_\omega^{\text{AD}} \in \mathcal{A}, \qquad \forall \omega, \tag{4.4b}$$

$$\{\lambda, \tilde{\lambda}_\omega\} \in \mathcal{L}, \qquad \forall \omega. \tag{4.4c}$$

The price-setter chooses a forward market energy price λ, forward market risk prices $\mu_{\omega}^{\mathrm{AD}}$, and an uncertainty-dependent spot market energy price $\tilde{\lambda}_{\omega}$, such that the cost for buyers is minimized and the revenue for sellers is maximized (4.4a). Furthermore, the price-setter applies a coherent risk measure function $\rho^{\cap}(\cdot)$ to the uncertainty-dependent spot market cost and revenue. However, the "risk aversion" of the price-setter results from the lowest common risk aversion of all community members [49]. Forward market risk prices $\mu_{\omega}^{\mathrm{AD}}$ lie within a sufficiently large, closed, compact, and convex set $\mathcal{A}$ (4.4b). Similarly, the forward λ and uncertainty-dependent spot market energy price $\tilde{\lambda}_{\omega}$ lie within a sufficiently large, closed, compact, and convex set $\mathcal{L}$ (4.4c). Thereby, prices remain unconstrained.

The risk-averse Nash equilibrium problem (4.2)–(4.4) provides a generalized formulation of a local energy market comprising a forward and a spot market. Depending on the parameterization of the risk-averse Nash equilibrium problem, it is able to describe several setups including risk-neutral as well as homogeneous or heterogeneous risk-averse local market participants. Moreover, the risk-averse Nash equilibrium problem may range from a fully incomplete to a complete local market for risk. This depends on the availability of a Arrow-Debreu security for each uncertainty realization $\omega \in \Omega$. Publication **[B]** shows that for any setup, a local market-clearing solution exists. However, the solution is not necessarily unique. Lastly, Publication **[B]** discusses the equivalence of the risk-averse Nash equilibrium problem with risk trading to a risk-averse social planner problem. For heterogeneous risk aversion, this can be confirmed only given a complete market for risk.

4.2 Incorporating ambiguity aversion into decision-making problems

Risk trading via Arrow-Debreu securities depends on the exact knowledge of the probability distribution function. Only in that way can a security be defined for each uncertainty realization $\omega \in \Omega$. So far, it was assumed that the true probability distribution function is exactly known and publicly available for all local market participants. However, this is unlikely the case in practice. This leaves the local market incomplete for risk and additionally induces ambiguity aversion by individuals against the probabilistic forecast [53, 54]. As illustrated in Figure 4.2, an ambiguity-averse player estimates, e.g., based on empirical data, a probability distribution function describing an uncertain event. Depending on her confidence in her data, she builds a family of potential probability distribution functions, the so-called ambiguity set, around the estimated one. Given the ambiguity set, she chooses her market participation strategy in ambiguity-averse expectation, i.e., based on the *worst-case* probability distribution function from her ambiguity set [55, 57]. Thereby, an ambiguity-averse player secures herself against incomplete information about an uncertain event, but simultaneously increases her expected cost in comparison to ambiguity-neutral expectation. Similar to the case of risk aversion, local market participants are heterogeneously ambiguity-averse. This means that players have an individual ambiguity-averse expectation of their energy cost, even though their cost may be equivalent in ambiguity-neutral expectation. Also similar to the case of heterogeneous risk aversion, an ambiguity-averse decision making by one player not only increases her own expected energy cost, but also affects the energy cost of her competitors. However, a suitable market product that completes the market for ambiguity, and thereby neutralizes the externality from highly ambiguity-averse decision making on players with a low ambiguity aversion, is not in the scope of this thesis and left aside for future research.

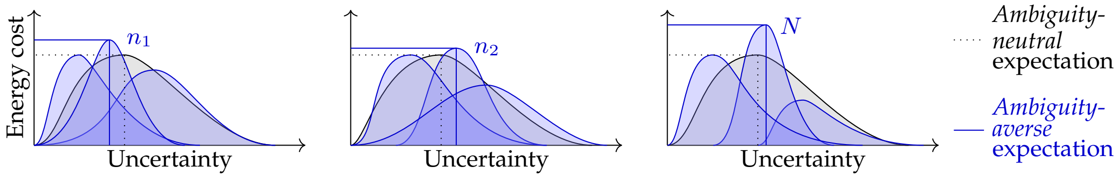

Figure 4.2 Ambiguity-averse decision making: An ambiguity-averse player estimates a probability distribution function describing an uncertain event. Depending on her confidence in the underlying empirical data, she considers an ambiguity set and chooses her market participation strategy in ambiguity-averse expectation. In practice local market participants are heterogeneously ambiguity-averse. Consequently, the local market participants n_1, n_2 to N have an individual ambiguity-averse expectation of their energy cost, even though their cost may be equivalent in ambiguity-neutral expectation.

4.2.1 Distributionally robust optimization with a Wasserstein ambiguity set

This thesis incorporates ambiguity aversion into the decision-making problem of local market participants through a distributionally robust optimization problem [56] given by

$$\underset{z}{\text{Min}} \max_{F \in \mathcal{D}} \mathbb{E}_F[g(z, \xi)], \tag{4.5}$$

in which $g(z, \xi)$ denotes an uncertainty-dependent cost function. An ambiguity-averse player makes a decision z in expectation $\mathbb{E}_F[\cdot]$ of her uncertainty-dependent cost $g(z, \xi)$. At the same time, she considers the uncertain parameter ξ, e.g., the renewable power generation, to follow the worst-case probability distribution function F from her ambiguity set $\mathcal{D}$. This thesis considers an endogenously built ambiguity set via a Wasserstein probability distance metric [57]. In detail, the ambiguity set $\mathcal{D}$ comprises all probability distribution functions F in the neighborhood of an empirical probability distribution function $\widehat{F}_i$, in which $i \in \mathcal{I}$ denotes the set of empirical data. The distance between a probability distribution function F and the empirical one $\widehat{F}_i$ is given by a Wasserstein distance $\Delta(\cdot)$ as

$$\Delta(F, \widehat{F}_i) = \min_{\Pi} \int \Big(\sum_{i \in \mathcal{I}} |\xi - \widehat{\xi}_i|^p \Big)^{\frac{1}{p}} \Pi(d\xi, d\widehat{\xi}_i), \tag{4.6a}$$

in which Π is a joint probability distribution function describing both the uncertain parameter ξ and the empirical data $\widehat{\xi}_i$. The symbol p denotes an arbitrary norm on the difference between the uncertain parameter ξ and the empirical data $\widehat{\xi}_i$. The marginals of the joint probability distribution function are given by F and $\widehat{F}_i$. Thereby, a Wasserstein ambiguity set [57] is defined as

$$\mathcal{D} = \big\{ F \in \mathcal{M}(\Xi) \colon \Delta(F, \widehat{F}_i) \leq \rho \big\}. \tag{4.6b}$$

The space of all probability distribution functions $\mathcal{M}$ is constrained by a support. The support is defined as $\Xi = \{\xi \in \mathbb{R} \colon \underline{H} \leq \xi \leq \overline{H}\}$, in which the lower $\underline{H}$ and upper bound $\overline{H}$ constrain the uncertain parameter ξ. Thereby, the worst-case probability distribution function takes realistic values, e.g., the minimum and maximum power generation of a renewable energy source. The non-negative parameter ρ constrains the distance between the probability distribution functions inside the ambiguity set and the empirical probability distribution function $\widehat{F}_i$. Therefore, ρ can be interpreted as the confidence in the available set of empirical data.

Based on these concepts, the next section proposes an ambiguity-averse Nash equilibrium problem. In this game, prosumers and the spatial arbitrageur trade energy in a single local spot market. In contrast to previous Nash equilibrium problems, the renewable power generation is considered to be a separate local market participant and the only source of uncertainty. Moreover, this section explicitly accounts for the utility of prosumers from energy consumption. The assumptions stated in Section 2.3 are relaxed in the sense that prosumers and the spatial arbitrageur may be heterogeneously ambiguity-averse. As a result, the local market may be incomplete for ambiguity. For the sake of complete Wasserstein ambiguity sets, i.e., ambiguity sets comprising *all* potential probability distribution functions in the neighborhood of an empirical one, this section assumes that the renewable power generation $\tilde{S}(\xi)$ depends on an uncertain event ξ. The uncertain event ξ, e.g, the solar radiation, follows a continuous probability distribution function, as opposed to the case of finite and discrete uncertainty realizations $\omega \in \Omega$ in the previous Nash equilibrium problems. Each ambiguity-averse market participant solves a distributionally robust optimization problem to determine her local spot market utility at a future stage in ambiguity-averse expectation. Note that for the sake of coherency with previous chapters, the following ambiguity-averse Nash equilibrium problem is slightly changed from the formulation used in Publication **[C]**.

4.2.2 Heterogeneous ambiguity aversion: Energy trading under continuous probability distribution functions

The renewable power generation $\tilde{S}(\xi)$, depending on the uncertain event ξ, has no trading decision and receives under any uncertainty realization of ξ the uncertain local spot market energy price $\tilde{\lambda}(\xi)$ according to

$$\tilde{\lambda}(\xi)\tilde{S}(\xi). \tag{4.7}$$

Each prosumer $n \in \mathcal{N}$ maximizes her uncertainty-dependent local spot market utility from energy consumption in ambiguity-averse expectation. For a given uncertain local spot market energy price $\tilde{\lambda}(\xi)$, each prosumer n determines her uncertainty-dependent spot market participation strategy by solving the following optimization problem:

$$\Big\{ \underset{\tilde{p}_n(\xi)}{\text{Min}} \; \max_{F_n \in \mathcal{D}_n} \; \mathbb{E}_{F_n}\left[(\tilde{\lambda}(\xi) - U_n)\tilde{p}_n(\xi) + c(\tilde{p}_n(\xi))\right] \tag{4.8a}$$

$$\text{s.t. } \tilde{p}_n(\xi) \in \mathcal{P}_n \Big\}, \; \forall n \in \mathcal{N}. \tag{4.8b}$$

Each prosumer n minimizes her uncertainty-dependent negative spot market utility in ambiguity-averse expectation (4.8a). The spot market utility consists of the spot market energy trade $\tilde{p}_n(\xi)$ according to the uncertain spot market energy price $\tilde{\lambda}(\xi)$. This yields the prosumer n the utility U_n. In addition, the spot market utility consists of a quadratic regularizer $c(\tilde{p}_n(\xi)) = \frac{1}{2}\beta\tilde{p}_n(\xi)^2$ associated with the spot market energy trade $\tilde{p}_n(\xi)$. At the same time, each prosumer n expects the uncertain parameter ξ to follow the worst-case probability distribution function F_n from her ambiguity set $\mathcal{D}_n$. Lastly, the spot market energy trade $\tilde{p}_n(\xi)$ lies within the closed, compact, and convex set $\mathcal{P}_n$ (4.8b).

The spatial arbitrageur minimizes her uncertainty-dependent local spot market cost from energy import and export in ambiguity-averse expectation. For a given uncertain local spot market energy price $\tilde{\lambda}(\xi)$, the spatial arbitrageur chooses her uncertainty-dependent spot market participation strategy as follows:

$$\underset{\tilde{p}^{\mathrm{ar}}(\xi)}{\mathrm{Min}} \max_{F^{\mathrm{ar}} \in \mathcal{D}^{\mathrm{ar}}} \mathbb{E}_{F^{\mathrm{ar}}}\big[(C - \tilde{\lambda}(\xi))\tilde{p}^{\mathrm{ar}}(\xi)\big] \tag{4.9a}$$

$$\text{s.t. } \tilde{p}^{\mathrm{ar}}(\xi) \in \mathcal{P}^{\mathrm{ar}}. \tag{4.9b}$$

The spatial arbitrageur minimizes her uncertainty-dependent spot market cost by importing and exporting energy $\tilde{p}^{\mathrm{ar}}(\xi)$ according to the fixed cost C (4.9a). She receives and pays the uncertain spot market energy price $\tilde{\lambda}(\xi)$, respectively. The spatial arbitrageur expects the uncertain parameter ξ to follow the worst-case probability distribution function F^{ar} from her ambiguity set $\mathcal{D}^{\mathrm{ar}}$. Her spot market energy trade $\tilde{p}^{\mathrm{ar}}(\xi)$ lies within a closed, compact, and convex set $\mathcal{P}^{\mathrm{ar}}$ (4.9b).

Lastly, the price-setter minimizes and maximizes the uncertainty-dependent spot market cost and revenue for energy buyers and sellers, respectively. For given uncertain spot market energy trades $\tilde{p}_n(\xi)$ and $\tilde{p}^{\mathrm{ar}}(\xi)$, the price-setter determines the uncertainty-dependent spot market energy price $\tilde{\lambda}(\xi)$ by solving the following optimization problem:

$$\underset{\tilde{\lambda}(\xi)}{\mathrm{Min}} \; \tilde{\lambda}(\xi)\Big(\sum_{n \in \mathcal{N}} \tilde{p}_n(\xi) - \tilde{p}^{\mathrm{ar}}(\xi) - \tilde{S}(\xi)\Big). \tag{4.10a}$$

$$\tilde{\lambda}(\xi) \in \Lambda. \tag{4.10b}$$

Under each uncertainty realization of ξ, the price-setter chooses an uncertainty-dependent spot market energy price $\tilde{\lambda}(\xi)$, such that the cost for energy buyers is minimized and the revenue for energy sellers is maximized (4.10a). The uncertainty-dependent spot market energy price $\tilde{\lambda}(\xi)$ lies within a closed, compact, and convex set Λ (4.10b).

The ambiguity-averse Nash equilibrium problem (4.7)–(4.10) provides a generalized problem formulation. Depending on the characteristics of the Wasserstein ambiguity sets, players may be ambiguity-neutral, homogeneously ambiguity-averse, or heterogeneously ambiguity-averse. Heterogeneous ambiguity aversion may arise owing to an individual confidence in empirical data or even asymmetric uncertainty information. The proposed ambiguity-averse Nash equilibrium problem is intractable owing to variables depending on the continuous uncertainty realization of ξ. This leads to an infinite-dimensional problem structure. Therefore, Publication **[C]** applies linear decision rules [63, 86] to approximate the stochastic variables. Chance constraints are used as an operator to translate infinite dimensional constraints into a single dimension [62]. These reformulations result in a tractable distributionally robust game. Publication **[C]** interprets this game as a local market design in which energy is traded in a local forward market. At the same time, the forward market determines a market-driven contribution plan for all community members to jointly compensate any community imbalances owing to the uncertainty realization of ξ in real-time. Publication **[C]** proves that, for the tractable game, a unique market-clearing solution exists. Moreover, Publication **[C]** shows the equivalence of the tractable distributionally robust game with Wasserstein ambiguity sets to a social planner problem, although players may possess asymmetric uncertainty information [64].

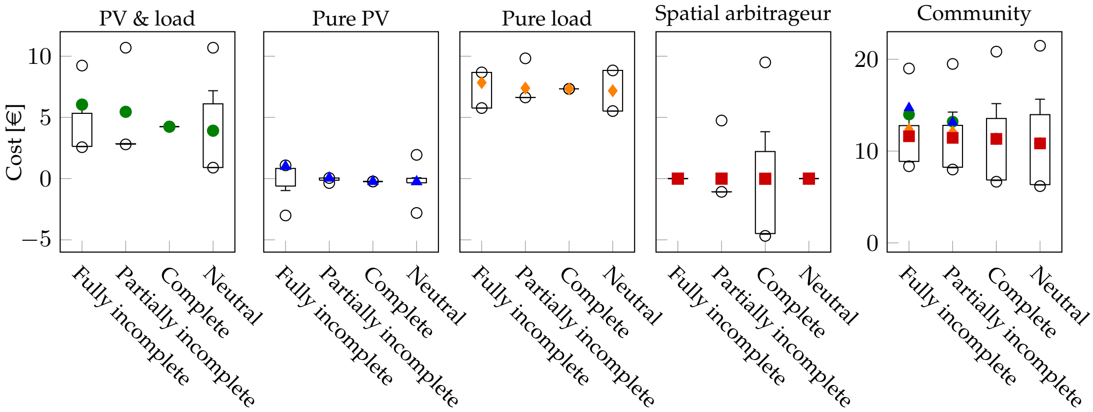

Figure 4.3 Towards a complete market for risk: The cost distribution for market participants and for the community as a whole depends on the market completeness for risk. Colored squares indicate cost from the perspective of each community member in risk-averse expectation. The cost distribution over uncertainty realizations is given by boxes, horizontal lines, and circles. Boxes show the cost between the second and the third quartile. Horizontal lines indicate the 5th and 95th quantile. Circles highlight outliers beyond the 5th and the 95th quantiles. This figure is adapted from Publication **[B]**.

4.3 The impact of individual risk and ambiguity preferences on competitors

To assess the impact of heterogeneous risk and ambiguity aversion, this section provides the most representative numerical results of Publication **[B]** and Publication **[C]**. In particular, Publication **[B]** numerically assesses heterogeneous risk aversion in energy communities. To this end, three risk-averse setups are considered, namely a fully incomplete, a partially incomplete, and a complete local market for risk. A risk-neutral setup serves as the benchmark for comparison. The energy community is comprised of three prosumers. The first prosumer has a constant demand, owns a photovoltaic system, and is moderately risk-averse. The second prosumer solely owns a photovoltaic system, while being highly risk-averse. The third prosumer solely has a constant demand and a low risk aversion. Lastly, the spatial arbitrageur is the least risk-averse local market participant.

The potential of risk trading is illustrated in Figure 4.3. Moving from a fully incomplete market to a complete market for risk, the cost distribution over uncertainty realizations $\omega \in \Omega$ for prosumers decreases. Given a complete market for risk, prosumers fully erase their cost volatility. However, the spatial arbitrageur, who is the least risk-averse local market participant in this example, absorbs the cost volatility of prosumers. Her cost volatility increases, although it remains unchanged in expectation. Looking at the total community cost, it can be observed that moving from a fully incomplete market to a complete market for risk reduces the expected community cost. Furthermore, owing to risk trading, the expected community cost from the perspective of each player converges towards a common expectation. However, the community cost volatility increases as the local market becomes complete for risk.

This finding emphasizes that risk-trading complements energy communities in the case that

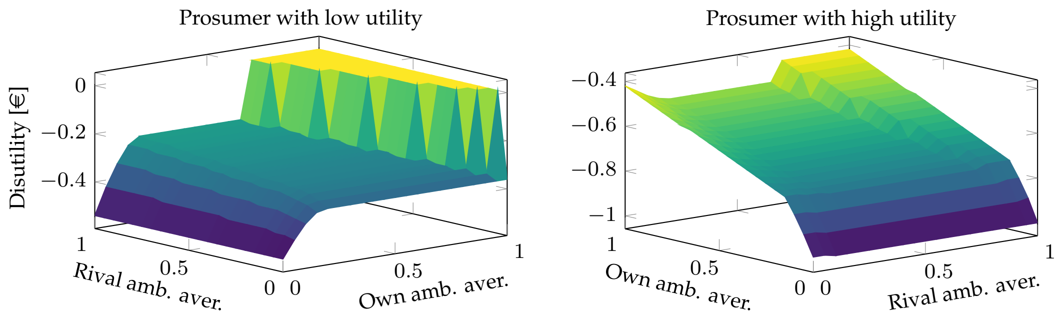

Figure 4.4 The impact of ambiguity aversion on competitors: The expected disutility, i.e., the negative utility, of each prosumer depends not only on her own ambiguity aversion (amb. aver.), but also on the ambiguity aversion of her competitor. Given highly ambiguity-averse decision making by both prosumers, the prosumer with a low consumption utility does not yield any utility. In contrast, the prosumer with a high consumption utility still enjoys a utility from energy consumption. This implies that a prosumer with a comparatively low consumption utility is highly exposed to the ambiguity aversion her competitors. This figure is adapted from Publication **[C]**.

community members are heterogeneously risk-averse. However, risk-trading highly depends on the availability of financial products to be traded. Therefore, Publication **[B]** provides additional insights into Arrow-Debreu security trades and how the market completeness for risk affects the market participation strategy of individuals. This work shows that risk trading efficiently protects community members with a low risk aversion from conservative decisions made by highly risk-averse prosumers.

Given incomplete information on the probability distribution function of an uncertain event, Publication **[C]** assesses the impact of an heterogeneous ambiguity aversion on competitors based on an illustrative example comprised of two prosumers only. The first prosumer has a low utility from energy consumption, while the second prosumer yields a comparatively higher utility from consumption.

Figure 4.4 shows the expected disutility, i.e., the negative utility, of both prosumers depending on their respective ambiguity aversion. As the ambiguity aversion of each prosumer increases, their expected disutility increases as well. However, the expected disutility of one prosumer also depends on the ambiguity aversion of her competitor. In particular, the disutility reduction for the prosumer with a low consumption utility significantly depends on the ambiguity aversion of the prosumer with a high consumption utility. In the case that both prosumers are highly ambiguity-averse, the prosumer with a low consumption utility yields a utility of zero. In contrast, the prosumer with high consumption utility is significantly less exposed to the ambiguity aversion of the rival prosumer. Even in the case that both prosumers are highly ambiguity-averse, she still yields a utility. This implies that a local market participant with a low consumption utility is highly exposed to the rival ambiguity aversion.

Chapter 5

Conclusion

This thesis addressed the challenges of coordinating the distributed energy production and consumption activities in a fully liberalized and market-oriented manner. To this end, a novel regulatory framework of spatially close located prosumers, the so-called energy communities, were considered. Energy communities enable prosumers to directly engage in energy trading with their neighbors via local energy markets. However, community members are directly exposed to the uncertainty inherent to the stochastic power generation by renewable energy sources, and thus to uncertain local market-clearing prices for energy. Therefore, this thesis explicitly addressed local market design options accommodating uncertainty as well as individual attitudes towards uncertainty. First, this thesis developed non-cooperative and cooperative game-theoretical models that enable an access economy for distributed energy resources in energy communities based on the example of energy storage system. This allows the flexibility utilization of an storage system in the interest of multiple community members. Second, this thesis incorporated risk and ambiguity aversion into the decision-making problem of prosumers. In addition, this thesis proposed the exchange of financial products to neutralize heterogeneous risk aversion, and thereby the unintended consequences of highly risk-averse decision making on prosumers with a low risk aversion. Based on these game-theoretical models, the proposed local market design alternatives were analyzed in terms of the market efficiency, the expected cost and its volatility for prosumers and the community as a whole, as well as the computational time needed to numerically solve the local market-clearing problems. This chapter summarizes in Section 5.1 the main contributions and findings. Based on these insights, Section 5.2 concludes with future research directions.

5.1 Summary

To enable an access economy for energy storage systems, this thesis proposed local market design alternatives based on non-cooperative and cooperative game-theoretical frameworks. For the given framework, it has been proven that for each non-cooperative market design, a unique market-clearing solution exists. Moreover, this thesis proposed an efficient solving approach for each Nash equilibrium problem based on its equivalence to a social planner problem. The key insight from this research is that an access economy for energy storage systems enhances energy communities by reducing the cost volatility for the majority of prosumers. It was shown that the total community cost is independent of the local market design. In contrast, the cost volatility observed by each prosumer significantly depends on the local market design in place. This implies that the proposed market design alternatives do not affect the operational strategy of the community as a whole, but distinguish in the cost allocation among community members, i.e., who pays whom which amount of money. Consequently, the market design choice depends on the preferences of community members. In particular, it depends on both the technologies each prosumer owns as well as the magnitude of the energy cost volatility that a prosumer is willing to accept. It also depends on the market design properties, which are of utmost attraction to community members. Lastly, from a computational perspective, the size of an energy community is a key factor in determining a suitable and tractable market design. From a policy perspective, all proposed market mechanisms ensure that identical prosumers incur the same cost. This implies that none of the local market design alternatives discriminate any market participant.

Given the importance of the cost volatility that prosumers are willing to accept, this thesis incorporated individual attitudes towards uncertainty into the decision-making problem of local market participants. The concepts of interest were risk and ambiguity aversion. Focusing on risk aversion, this thesis applied the conditional value-at-risk as the risk measure function to each individual community member. In addition, Arrow-Debreu securities were proposed as financial products to be traded among local market participants. Based on these concepts, this thesis proposed a generalized game-theoretical formulation of a local energy market in which the risk aversion of local market participants can be freely chosen. Moreover, the local energy market itself may be fully or partially incomplete, or complete for risk. It was shown that a local market-clearing solution exists for any degree of market completeness for risk. However, the solution is not necessarily unique in any of these cases. Lastly, this thesis discussed the equivalence of the risk-averse Nash game with risk trading to a social planner problem. Under heterogeneous risk aversion, this can be confirmed only given a complete market for risk.

The concept of complete risk trading via Arrow-Debreu securities necessitates that the probability distribution function of an uncertain event is exactly known. Only in that way can a security be defined for each uncertainty realization. However, this knowledge availability is unlikely in practice, which leaves the local market incomplete for risk and simultaneously induces ambiguity aversion against the probabilistic forecast. Therefore, this thesis addressed the notion of heterogeneously ambiguity-averse local market participants. Ambiguity aversion was incorporated into the decision-making problem of each prosumer through the concept of distributionally robust optimization and Wasserstein ambiguity sets. For the purpose of complete Wasserstein ambiguity sets, the assumption of finite and discrete uncertainty realizations, applied previously in this thesis, was relaxed to a continuous probability distribution function describing an uncertain event. This thesis proposed a generalized and tractable formulation of a distributionally robust game. Furthermore, it has been proven that for this game an equivalent social planner problem exists with a unique solution. This implies the existence of a unique local market-clearing outcome.

This thesis emphasized, based on numerical results, that risk trading complements energy communities when prosumers are heterogeneously risk-averse. More specifically, risk trading protects prosumers with a low risk aversion from conservative decisions made by highly risk-averse local market participants. Given that sufficient Arrow-Debreu securities are available for trading, a significant community cost saving can be realized. Eventually, all community members benefit individually from risk-trading. However, this thesis also discussed that, in the case of incomplete information on the probability distribution function of an uncertain event, local market participants may be ambiguity averse. Numerical results showed that a prosumer with a comparatively low energy consumption utility is highly exposed to the ambiguity aversion of her competitors.

5.2 Perspectives for future research

The insights gained from the research projects summarized in this thesis open up a range of potential future research directions related to local market design options for energy communities exposed to uncertainty. The research on an access economy for energy communities laid out the fundamental concepts for the design of a local market. While this thesis addressed an access economy for energy storage systems, a promising future research direction is the generalization of the concept of physical and financial storage rights to other distributed energy resources. In particular, distributed energy resources, such as electrical heat pumps, thermal energy storages, or electric vehicles, could be aggregated as a virtual storage system for which rights are available for trading. The problem formulation in this thesis can serve as a starting point. However, the current literature lacks a suitable aggregation approach of multiple distributed energy resources with uncertain availability.

Given an access economy in energy communities, one can hypothesize that a community member with operational access to distributed energy resources may have a strong incentive to manipulate the local energy market for her own benefit. In particular, in the case that only a few community members have operational access to distributed energy resources. The strategic behavior of individuals should be addressed, such that a local market design for an energy community is resilient against such threats. A starting point may be the application of the well-known Stackelberg game through a hierarchical problem formulation [87–89] in the presence of physical storage rights. Thereby, one can assess a situation in which a strategic prosumer anticipates the response of her competitors. A holistic understanding of market vulnerabilities with respect to strategic behavior by individuals would ensure the local market efficiency and incentive compatibility.

Along the research stream of incorporating individual attitudes towards uncertainty into the decision-making problem of prosumers, the key driver for conservative attitudes are the unlikely, but costly, uncertainty realizations. A central research question lies in analyzing options that reduce the energy cost volatility for prosumers. In general, the deployment of energy storage systems levelizes energy prices over time. The charging in low-price periods causes prices to increase, while the discharging in high-price periods drives energy prices down. Eventually, one can hypothesize that a sufficiently large energy storage system or a sufficient number of storage systems fully erase any volatility in local energy prices observed by prosumers. This would also erase the energy cost volatility. The consideration of storage systems in the presence of heterogeneous risk aversion and risk trading is of interest. In particular, research on a tipping-point is promising from which onward risk trading via Arrow-Debreu securities becomes obsolete owing to energy storage.

An Arrow-Debreu security is a highly stylized financial product, which can be traded only for predefined uncertainty realizations. This implies that the probability distribution function of an uncertain event must be exactly known in order to complete the local market for risk. However, as discussed in this thesis, the true probability distribution function of an uncertain event, such as the renewable power generation, is unknown. Therefore, the theoretical approach proposed in this thesis will, in practice, leave the local market of an energy community incomplete for risk. Research on a less stylized and more practical financial product is highly promising. This thesis laid out a game-theoretical framework based on continuous probability distribution functions. In this model, the true probability distribution function does not need to be exactly known. The proposed distributionally robust game could be extended by adding a risk-measure function, e.g., the conditional value-at-risk, to the objective function of each individual local market participant. The resulting problem structure should be studied in detail. Thereby, one may gain insights on how to define a suitable financial product that completes the local market for both risk and ambiguity.

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Collection of Relevant Publications

[A] Niklas Vespermann, Thomas Hamacher, and Jalal Kazempour: "Access economy for storage in energy communities", in *IEEE Transactions on Power Systems*, early access, 2020, DOI: 10.1109/TPWRS.2020.3033999.

[B] Niklas Vespermann, Thomas Hamacher, and Jalal Kazempour: "Risk trading in energy communities", in *IEEE Transactions on Smart Grid*, early access, 2020, DOI: 10.1109/TSG.2020.3030319.

[C] Niklas Vespermann, Thomas Hamacher, and Jalal Kazempour: "On ambiguity-averse market equilibrium", submitted to *Optimization Letters*, (under review, second round), 2022.

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Access Economy for Storage in Energy Communities

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***Abstract*—We address the market design issue for a local energy community, comprising prosumers, consumers, photovoltaic, and energy storage systems, all connected as a community to a distribution grid. Our work explores different market design options based on cooperative and non-cooperative game-theoretic models that enable an economic access to the benefits of energy storage for prosumers without a direct ownership of a storage system. We compare market outcomes in terms of the community cost as well as the individual cost. We pay special attention to potential uncertainties, and investigate financial instruments that allow storage systems to be utilized by multiple prosumers. In particular, we explore a case where a prosumer that owns a storage system provides rights, either physical or financial, rather than participating in the local market as an arbitrageur. Moreover, we consider a cooperative market design where energy community members agree on the Shapley value or the nucleolus as a community cost allocation rule. Our results show that an access economy for energy storage systems enhances energy communities by reducing the cost volatility for most prosumers, while the expected operational cost of the community as a whole remains unchanged.**

***Index Terms*—Access economy, energy community, game theory, local energy market, storage right, uncertainty**

I. Introduction

A. Motivation and Aim

In the last 20 years a significant increase in the installment of roof-top solar photovoltaic (PV) systems in residential areas has been observed [1]. This trend is mainly driven by the aim of decarbonizing the energy sector. Support schemes, such as fixed feed-in tariffs, are the driving force for the increased deployment of PV systems. However, as governments start to cut back support schemes, new regulatory frameworks and business models must be developed. This it the only way investments in renewable energy sources by proactive end-customers, the so-called *prosumers*[1], remain economical and beneficial for energy systems [2], [3].

One widely discussed approach is the notion of *energy communities*, which allows prosumers to maximize local usage of PV power generation via local energy exchanges, exempted from network and other surcharges [4]. However, the intermittent and volatile injection by PV systems necessitates the deployment of energy storage systems to alleviate renewable energy curtailment and efficiently deal with the fluctuating power generation [5]. Nevertheless, it is more likely that within an energy community one energy storage system is capable to serve the needs of multiple prosumers.

To this end, we are interested in an *access economy*[2] for energy storage systems within energy communities, i.e., enabling prosumers to pay for the access to benefits of an energy storage system without its direct ownership [9]. In this regard, we assess different paradigms of an access economy for energy storage: On the one hand, we explore the case where a prosumer who owns an energy storage system provides storage rights, either physical or financial, to other community members rather than participating as an intertemporal arbitrageur in a local energy market. On the other hand, we analyze the case where community members agree on a community cost[3] allocation rule, while the storage system is dispatched by a system operator.

A local energy market design that enables an access economy for energy storage systems in energy communities gives rise to two fundamental questions: First, what do potential market designs look like? And second, what are the regulatory implications of an access economy for energy storage?

To answer these questions, we consider an energy community that comprises multiple prosumers as well as an energy community manager, who acts as a non-profit oriented system operator. We represent uncertain PV power generation by a finite set of discrete scenarios. We first start with a benchmark case of a non-cooperative market design. In this case, every prosumer who owns an energy storage system participates as an intertemporal arbitrageur in local spot markets, where the only product to be traded is energy. We continue with the case, where each prosumer who owns a storage system offers *physical storage rights* (PSRs) in a local forward market, which clears once in advance to spot markets. The energy storage system is then scheduled in spot markets according to the underlying objective of a PSR holder. Similarly, we consider the case of *financial storage rights* (FSRs). However, under this paradigm the storage system is optimally dispatched from a social perspective by the energy community manager. We move beyond non-cooperative market designs and consider the case of a cooperative market design, where energy community members agree on a community cost allocation rule among all prosumers, while energy storage as well as PV systems are optimally utilized for the whole system. We

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[1]In this work, each prosumer is a power consumer with a fixed load, who may also locally produce renewable power by her own PV system. A prosumer may also own an energy storage system.

[2]We understand the access economy as an access practice to a good or a service guided by market norms, e.g., AirBnB or BlaBlaCar, in contrast to an access practice enabled outside the market and guided by social norms, e.g., intrafamily sharing, food sharing, or the community library [6]–[8].

[3]Community cost refers to the total operational cost of the community.

study implications of different cost allocation rules such as the well-known Shapley value and the nucleolus.

B. Literature Review and Contributions

The research on local energy markets has gained significant interest in the last five years. Generally, three different organizational paradigms of a local energy market are proposed [10], [11], which are (*i*) a peer-to-peer market structure that allows prosumers to directly engage in bilateral contracts [12]–[14], (*ii*) a more structured local market with a community manager, who administers trading activities inside the energy community [15]–[17][4], and (*iii*) a highly organized paradigm, in which a group of prosumers are centrally managed according to a predefined objective [19]–[21]. In this work, we build non-cooperative market designs upon the paradigm (*ii*) with a community manager and study an access economy for energy storage via storage rights.[5] For cooperative market designs we follow paradigm (*iii*) where all devices are optimally dispatched in favor of the whole community by the community manager as the central entity.

The concept of storage rights, either physical or financial, lies in the domain of non-cooperative market designs and is highly related to the idea of transmission rights [22], [23]. In fact, the storage owner offers storage rights to other market participants in order to, e.g., recover investment costs upfront or not be involved in the market operation. In the paradigm of PSRs, a storage right holder directly dispatches a part of the energy storage system, and thereby yields operational as well as economic access to energy storage [24]–[27]. In contrast, under FSRs the storage system is understood as a communal asset and optimally dispatched by the system operator, who potentially collects a merchandising surplus due to intertemporal energy arbitrage. This surplus is redistributed *a posteriori* among storage right holders, which again, yield economic access to energy storage [28], [29].

Another approach that allows economic access to energy storage lies in the domain of cooperative market designs. Here, the challenge is to define a cost allocation rule which results in a *stable* energy community in the sense of a lasting cooperation of community members. Simple but intuitive cost allocation rules, e.g., per capita or based on the peak demand, fail this requirement [19]. Therefore, a more complex cost allocation rule is studied in [30], which shows how the Shapley value can be applied for an energy community. However, since the Shapley value does not always result in a stable cost allocation, [20] analyzes the nucleolus as an alternative. Given a stable cost allocation rule, [31] moves beyond [20], and [30] and additionally considers the investment decision in energy storage systems, while the cost associated to the investment and the operation are shared among prosumers. All these works conclude that deploying storage systems next to renewable energy sources within an energy community leads to significant cost savings. However, in practice this economic benefit depends highly on the billing mechanism observed by individuals as discussed in [2], which studies the impact of policy designs on local exchanges of an excess PV power generation between community members. Both [2] and [3] conclude that a proper policy design is necessary to encourage an economically efficient operation of distributed energy resources.

In contrast to [9], which studies trading of unused stored energy among firms, we focus on prosumers, which yield an economic access to energy storage via various local market designs. In that direction, [26] has studied the case of an aggregator, who invests in an energy storage system and provides access to individual market participants. While [26] considers a single entity providing access to her storage system, [27] has studied the case of a non-cooperative local market design, where multiple storage-owning prosumers offer PSRs. However, both [26] and [27] leave a comparison to FSRs to the side, for which—to the best of our knowledge—no work exists with a focus on local markets. In this paper, we propose a mathematical formulation, which can be efficiently solved and readily applied to questions beyond local energy markets. In contrast to [25], [27]–[29], we provide a generalized problem formulation for both PSR and FSR forward markets where the storage-owning prosumer actively decides on the *share* of storage capacity, which is offered to others or withheld for her own interests. This allows a *partial access* to storage, such that the optimal level of access provision from the storage owner's perspective is endogenously determined. To the best of our knowledge, this is the first work in the research community proposing such a generalized model. Lastly, [20] and [30] consider a cooperative market design for energy communities although they derive no conclusion about the economic access to energy storage for individuals.

In summary, this paper extends existing concepts that enable an access economy for storage and offers a thorough study built upon game-theoretical approaches, ranging from non-cooperative to cooperative setups, on local energy communities with a focus on regulatory implications.

C. Main Findings and Paper Organization

Our main findings are as follows: In a non-cooperative market design every prosumer's *expected* cost remains unchanged, irrespective of the availability or the absence of storage rights, either physical or financial. However, most prosumers benefit from an access economy for energy storage through the availability of storage rights by reducing their cost *volatility*. Cooperative market designs provide a rigorous framework for implementation although the Shapley value suffers from stability and might leave incentives for some prosumers to split off from the energy community. In contrast, the nucleolus gives rise to a stable community cost allocation. However, its calculation is highly intensive. We find that non-cooperative market designs scale well in terms of community

[4]Note that hybrid forms combining a full peer-to-peer local market with a local market administrator are envisioned as well [18].

[5]Under some assumptions, e.g., perfect competition, complete markets, rational decision-making, and interaction with existing markets, both paradigms (*i*) and (*ii*) provide the same market outcomes [15]. Therefore, our conclusions on the potential of storage rights can be applied to a peer-to-peer market as well. However, the mathematical problem formulation will be different.

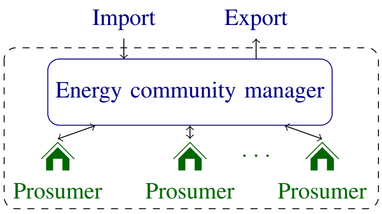
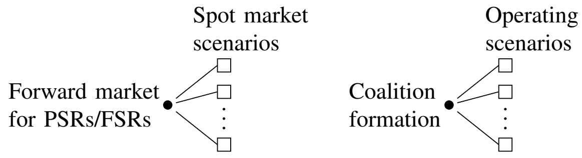


Fig. 1: Structure of a local energy community.



(a) (b)

Fig. 2: Elements of non-cooperative market designs are depicted in Figure 2a. Such elements for cooperative market designs are presented in Figure 2b.

members, while cooperative market designs do not, and thus are more suitable for small energy communities.

The rest of this paper is organized as follows. In Section II we introduce the energy community structure and conceptually explain the non-cooperative as well as cooperative market organization. Based on our assumptions, in Section III we mathematically introduce individual decision-making problems and define non-cooperative and cooperative games. Section IV provides numerical results. Lastly, we conclude our work in Section V and provide a discussion on our proposed local market designs. Proofs and solving approaches are available in Appendices A and B, while the source code can be found in [32].

II. Preliminaries

A. Energy Community Structure

We understand a local energy community as an aggregation of a few prosumers, which are spatially located very close to each other and physically connected to the distribution system as a single entity [10]. Within such an energy community we consider two types of players as illustrated in Figure 1: On the one hand, prosumers attempt to minimize their energy cost from meeting their demand by an uncertain and volatile PV power generation, energy exchange, and the utilization of an energy storage system. On the other hand, an energy community manager administers the energy community as a non-profit oriented entity. Her task is to ensure liquidity within the energy community by importing and exporting electricity from and to a retailer[6] for energy. Moreover, the energy community manager reveals local market-clearing prices evolving under free trade and perfect competition when considering a non-cooperative market design[7], and operates all devices within the energy community in the presence of a cooperative market design. As pointed out in Section I-B, we assume all prosumers buy/sell energy through the community manager, and do not exchange energy bilaterally in a peer-to-peer fashion.

B. Non-Cooperative Market Design

Figure 2a illustrates elements of non-cooperative market designs. We start by a benchmark design of sequentially clearing local *spot* markets. In this benchmark, the only market product to be traded is energy. Each prosumer considers the entire optimization horizon and individually operates her devices according to her underlying objective and the realization of the uncertain PV power generation. Prosumers pay/are paid based on local spot market-clearing prices for exchanged energy.

We extend this non-cooperative market design and additionally consider a local *forward* market for PSRs, which clears once in advance to local spot markets.[8] A storage-owning prosumer sells PSRs in the forward market, and thereby yields a revenue upfront to spot markets. In spot markets, PSR holders directly dispatch the awarded share of an energy storage system to minimize their individual energy cost.

Finally, we consider a local forward market, where a storage-owning prosumer offers FSRs to other energy community members. Unlike the case for PSRs, energy storage systems are dispatched in spot markets by the energy community manager, which thereby yields a merchandising surplus from intertemporal energy arbitrage. This surplus is redistributed among FSR holders according to the realized value of FSRs in local spot markets, i.e., the marginal contribution of a single FSR to the merchandising surplus incurred by the energy community manager.

It is worth noting that in contrast to [26], community members separately bid—depending on their needs—to get economic access to charging, discharging, and reservoir devices of an energy storage system. Both PSRs and FSRs decompose in charging, discharging, and energy capacity rights [25], [28]. This formulation enables an additional degree of freedom with respect to the economic access practice of storage rights, since it is not necessarily the case that prosumers need a fixed ration of charging, discharging, and energy rights.

C. Cooperative Market Design

Figure 2b depicts the cooperative market design, where we consider as the community cost allocation rule (*i*), the Shapley value, and (*ii*), the nucleolus. Prosumers anticipate the expected cost from joining a coalition given the cost allocation rule energy community members agree on before the operational stage. Note that we do not endogenously consider this negotiation process, but rather assume that the decision for either the Shapley value or the nucleolus is made upfront to the optimization horizon. During operation, all devices are optimally dispatched for the energy community as a whole by the energy community manager. Based on the realization of the uncertain PV power generation, the community cost is systematically distributed among energy community members according to the predefined cost alloca-

[6]A retailer is a self-interested profit-seeking entity which buys great energy volumes at the wholesale market and sells energy to many small customers.

[7]In a perfectly competitive market environment the community manager simultaneously fulfills tasks of a spatial arbitrageur and a price setter [33].

[8]Storage rights, either physical or financial, are valid for the entire optimization horizon, which can—without loss of generality—span the period of a day, a month or a year.

tion rule [17], [20], [30].

D. Additional Assumptions

We assume all prosumers as well as the energy community manager participate as price-takers in perfectly competitive local forward and spot markets, and bid according to their true cost/utility.[9] While each price-taker prosumer minimizes her own energy cost, the community manager fulfills her tasks in the interest of the community as a whole. Furthermore, given the small size of an energy community we consider a single-node local system, and thus neglect network constraints *within* the energy community. However, we consider the capacity limitation of the single network that connects the whole community to the distribution system. All players are risk-neutral and have symmetric information on the uncertain PV power generation, which is the only source of the uncertainty, although other sources of the uncertainty can be incorporated in the same manner. We represent the uncertain PV power generation by a set of discrete and a finite number of scenarios $\omega \in \Omega$. To avoid pricing non-convexities and in line with [9], [25], and [27], binary variables indicating the status of storage systems are not considered.[10] Lastly, we consider hourly time steps in the spot market.

III. Mathematical Formulation

In Section III-A we formulate non-cooperative games among players, while Section III-B provides the mathematical formulation of cooperative games. The operator $\mathbb{E}[\cdot]$ refers to the expected value given the uncertain PV power generation. Symbols followed by a colon denote dual variables of the respective constraints.

A. Non-Cooperative Market Design

For non-cooperative games, we introduce one optimization problem per prosumer as well as one optimization problem for the community manager. All these optimization problems have to be solved *simultaneously*, yielding a pure Nash equilibrium problem, whose solution is a Nash equilibrium point. Aligned with findings of seminal works such as [38] and [39], we will show that each proposed Nash equilibrium problem—assuming no market failures—can be equivalently solved by a single optimization problem, or sequential ones, maximizing the social welfare.

In the following, we start with a Nash equilibrium problem with local spot markets only, and extend it later by adding a forward market for *(i)* PSRs, and *(ii)* FSRs.

1) Spot Markets: Here, we develop a non-cooperative market design with spot markets only, and thus no storage rights to be traded in the forward market. The optimal operation and energy trades are determined in sequential spot markets, one market per hour, wherein devices are exclusively dispatched by respective prosumers. Since there is no forward market affecting spot market trades under all scenarios, prosumers make their spot decisions for each scenario ω independently from other scenarios. Therefore, the decision-making problem of each prosumer is deterministic, although her decision variables are indexed by ω and subject to the PV power generation realized under the underlying scenario.

Let us consider a set of time periods $t \in \mathcal{T}$, a set of prosumers $n \in \mathcal{N}$, and a set of energy storage systems indexed by $s \in \mathcal{S}$. The set Φ_n indicates storage systems belonging to prosumer n. Under each scenario ω of local spot market realizations, prosumer n minimizes her energy cost for the given spot market prices $\lambda_{t\omega}$ as

$$\left\{ \underset{\Xi_n^{\mathrm{S}}}{\text{Min}} \sum_t \left(\underbrace{\lambda_{t\omega} p_{nt\omega}}_{\text{Energy cost}} + \underbrace{\frac{1}{2}\beta p_{nt\omega}^2}_{\text{Regularizer}} \right) \right. \tag{1a}$$

subject to

$$p_{nt\omega} + S_{nt\omega} + \sum_{s\in\Phi_n} (p^{\downarrow}_{st\omega} - p^{\uparrow}_{st\omega}) - D_{nt} = 0, \ \forall t, \tag{1b}$$

$$-\overline{P}_n \le p_{nt\omega} \le \overline{P}_n, \ \forall t, \tag{1c}$$

$$e_{st\omega} = e_{s(t-1)\omega} + \eta^{\uparrow}_s p^{\uparrow}_{st\omega} - \eta^{\downarrow}_s p^{\downarrow}_{st\omega}, \forall s\in\Phi_n, 1 < t < |\mathcal{T}|, \tag{1d}$$

$$e_{st\omega} = E^{\text{ini}}_s, \ \forall s \in \Phi_n, t = 1, t = |\mathcal{T}|, \tag{1e}$$

$$0 \le p^{\uparrow}_{st\omega} \le \overline{P}^{\uparrow}_s, \ : \underline{\gamma}^{\uparrow}_{st\omega}, \overline{\gamma}^{\uparrow}_{st\omega}, \ \forall s \in \Phi_n, t, \tag{1f}$$

$$0 \le p^{\downarrow}_{st\omega} \le \overline{P}^{\downarrow}_s, \ : \underline{\gamma}^{\downarrow}_{st\omega}, \overline{\gamma}^{\downarrow}_{st\omega}, \ \forall s \in \Phi_n, t, \tag{1g}$$

$$\left. 0 \le e_{st\omega} \le \overline{E}_s : \underline{\gamma}^{\text{e}}_{st\omega}, \overline{\gamma}^{\text{e}}_{st\omega}, \ \forall s\in\Phi_n, t \right\} \forall n\in\mathcal{N}, \omega\in\Omega, \tag{1h}$$

where the set of variables Ξ_n^{S} includes $p_{nt\omega}$ as well as $\{p^{\uparrow}_{st\omega}, p^{\downarrow}_{st\omega}, e_{st\omega}\}$ $\forall s \in \Phi_n$. Each prosumer n pays/is paid based on the local spot market-clearing price $\lambda_{t\omega}$ for exchanged energy $p_{nt\omega}$ at time period t under scenario ω. Note that market-clearing prices $\lambda_{t\omega}$ are treated as parameters within (1), but they are variables in the Nash equilibrium problem (1)–(2). Note that problem (2) refers to the energy community manager's problem, which we provide later in this section. A positive value of $p_{nt\omega}$ indicates a demand, while a negative value states a supply of energy. The first term in (1a) represents energy cost. Moreover, without loss of generality, we consider a quadratic regularizer in (1a), where β is a small positive constant, e.g., 10^{-3}, to ensure a strictly convex objective function.[11] Institutionally, this regularizer can be interpreted as a transaction cost arising from energy trades.

Each prosumer n has to ensure her individual power balance (1b). The inelastic deterministic demand D_{nt} has to be met by traded energy $p_{nt\omega}$, PV power generation $S_{nt\omega}$, power charged $p^{\uparrow}_{st\omega}$, and discharged $p^{\downarrow}_{st\omega}$ from the energy storage system $s \in \Phi_n$ at each time step t and scenario ω. Note that $S_{nt\omega}$ is the sole uncertain parameter of the problem. Constraint (1c) restricts energy trades by the parameter $\overline{P}_n$.[12]

[9]Relevant works that consider an energy storage system exhibiting market power as a price-maker player are available in [34]–[36].

[10]Note that under some conditions, which hold in our model, this does not result in simultaneous charging and discharging of energy storage systems [37, Prop. 2].

[11]A value $\beta = 0$ makes the objective function linear and multiple solutions may exist in terms of prosumers' cost, although the community cost will be the same across all solutions. Very small values for β, e.g., 10^{-3}, do not alter the community cost. However, we achieve a unique solution for the Nash equilibrium problem, where identical players incur the same cost [40].

[12]We introduce theoretical bounds $\overline{P}_n$ on energy exchanges to achieve closed and compact decision sets for players. This is necessary for conclusions on existence of a game solution [41]. However, we select sufficiently large values for $\overline{P}_n$, and check *a posteriori* that (1c) is always inactive.

Constraints (1d)–(1h) define the operational region of the energy storage system s. The state of charge variable $e_{st\omega}$ is given by (1d), where parameters $\eta_s^{\uparrow}$ and $\eta_s^{\downarrow}$ denote charging and discharging efficiencies, respectively. Without loss of generality, (1e) enforces the energy stored $e_{st\omega}$ in the last time period, i.e., $t = |\mathcal{T}|$, to be equal to the initial state of charge E_s^{ini}. The energy stored for the beginning of the next time horizon will be zero if (1e) is relaxed.[13] Finally, (1f)–(1h) restrict the level of energy charged, discharged, and stored by capacity limits $\overline{P}_s^{\uparrow}, \overline{P}_s^{\downarrow}$, and $\overline{E}_s$, respectively. We use dual variables associated to (1f)–(1h) later when we introduce a forward market for FSRs.

Given trading decisions $p_{nt\omega}$ of prosumers, the community manager clears local spot markets under each scenario ω by minimizing the operational cost of the community as

$$\left\{ \underset{\Theta^{\text{S}}}{\text{Min}} \sum_t \Big[\underbrace{C_t p^{\text{i}}_{t\omega} - R_t p^{\text{e}}_{t\omega}}_{\text{Community cost}} + \underbrace{\lambda_{t\omega} \big(\sum_{n\in\mathcal{N}} p_{nt\omega} - p^{\text{i}}_{t\omega} + p^{\text{e}}_{t\omega} \big)}_{\text{Buyers' spot market cost / Sellers' spot market revenue}} \Big] \right. \quad (2a)$$

subject to

$$0 \le p^{\text{i}}_{t\omega} \le \overline{P}^{\text{i}}, \ \forall t, \quad (2b)$$

$$0 \le p^{\text{e}}_{t\omega} \le \overline{P}^{\text{e}}, \ \forall t, \quad (2c)$$

$$\left. -\overline{\Lambda} \le \lambda_{t\omega} \le \overline{\Lambda}, \ \forall t \right\} \forall \omega \in \Omega, \quad (2d)$$

where the variable set is $\Theta^{\text{S}} = \{p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}, \lambda_{t\omega}\}$. Note that market-clearing prices $\lambda_{t\omega}$ are variables in (2), and therefore, also in the Nash equilibrium problem (1)–(2), although they are parameters in (1). In the objective function (2a), the energy community manager minimizes the cost incurred by purchasing power $p^{\text{i}}_{t\omega}$ and the revenue obtained by selling power $p^{\text{e}}_{t\omega}$ from and to a retailer for energy. She imports energy at the cost C_t and exports at the price R_t.[14] Thereby, the energy community manager ensures liquidity of local spot markets. Moreover, the community manager is in charge of setting local spot market-clearing prices $\lambda_{t\omega}$ by minimizing the cost of energy buyers while maximizing the revenue of energy sellers. Note that from the community manager's perspective, demands are $p_{nt\omega} \ge 0$ as well as the energy export $p^{\text{e}}_{t\omega}$. In contrast, the sources of supply are $p_{nt\omega} \le 0$ as well as the energy import $p^{\text{i}}_{t\omega}$.

Energy imports and exports are constrained in (2b)–(2c) by upper bounds $\overline{P}^{\text{i}}$ and $\overline{P}^{\text{e}}$, corresponding to the potential capacity limitation of the network between the whole community and the distribution system. Local spot market-clearing prices are constrained in (2d) by $\overline{\Lambda}$.[15] It is worth noting that the energy community manager chooses local market-clearing prices such that energy supply and demand are perfectly balanced.[16]

Definition 1. Based on (1) and (2), we define the non-cooperative Nash game $\Gamma^{\text{spot}}(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$, where $\mathcal{Z}$ is the set of all players and J_i their respective objective function. $K = (K_1 \times \cdots \times K_{\mathcal{N}} \times K_{\text{m}})$ denotes the strategy set of the game, where K_n is the strategy set of prosumer $n \in \mathcal{N}$ and K_{m} the strategy set of the energy community manager.

Proposition 1. Due to strictly convex objective functions and closed convex decision sets for all players, a unique Nash equilibrium for the non-cooperative Nash game $\Gamma^{\text{spot}}(\cdot)$ exists.

Proof 1. We provide the proof in Appendix A-A. ■

Remark 1. We solve the non-cooperative Nash game $\Gamma^{\text{spot}}(\cdot)$ using its equivalent formulation as a single optimization problem, provided in Appendix A-A.

2) PSR Forward Market and Spot Markets: We extend the previous local market design and consider the case wherein a storage-owning prosumer offers PSRs to other prosumers. Recall that storage rights to be traded in the forward market are valid for the entire optimization horizon and scenarios, and thus are not indexed by t and ω. This results in a two-stage stochastic program where local forward market trades for PSRs are here-and-now decisions, while local spot market trades are wait-and-see decisions until the realization of the uncertain PV power generation is observed. Unlike (1), where each prosumer optimizes her local spot market trades for each scenario separately, here each prosumer minimizes her *expected* cost, accounting for all scenarios at once.

For given PSR prices, i.e., a charging right price $\mu_s^{P\uparrow}$, a discharging right price $\mu_s^{P\downarrow}$, and an energy capacity right price μ_s^{Pe}, as well as hourly spot market prices $\lambda_{t\omega}$, each prosumer n minimizes her expected energy cost by buying and selling PSRs in the local forward market and trading energy in local spot markets. This problem writes as

$$\left\{ \underset{\Xi_n^{\text{P}}}{\text{Min}} \sum_{s\in\Phi_n} \big(\underbrace{\mu_s^{\text{P}\uparrow} x_s^{\text{P}\uparrow} + \mu_s^{\text{P}\downarrow} x_s^{\text{P}\downarrow} + \mu_s^{\text{Pe}} x_s^{\text{Pe}}}_{\text{Revenue in forward market for PSR sellers}} \big) \right.$$

$$+ \sum_s \Big[\underbrace{\mu_s^{\text{P}\uparrow} q_{ns}^{\text{P}\uparrow} + \mu_s^{\text{P}\downarrow} q_{ns}^{\text{P}\downarrow} + \mu_s^{\text{Pe}} q_{ns}^{\text{Pe}}}_{\text{Cost in forward market for PSR buyers}} + \underbrace{\frac{1}{2}\beta ({q_{ns}^{\text{P}\uparrow}}^2 + {q_{ns}^{\text{P}\downarrow}}^2 + {q_{ns}^{\text{Pe}}}^2)}_{\text{Regularizer}} \Big]$$

$$+ \sum_t \mathbb{E}_\omega \Big[\underbrace{\lambda_{t\omega} p_{nt\omega}}_{\text{Cost in spot markets}} + \underbrace{\frac{1}{2}\beta p_{nt\omega}^2}_{\text{Regularizer}} \Big] \quad (3a)$$

subject to

$$-\overline{P}_s^{\uparrow} \le x_s^{\text{P}\uparrow} \le 0, \ \forall s \in \Phi_n, \quad (3b)$$

$$-\overline{P}_s^{\downarrow} \le x_s^{\text{P}\downarrow} \le 0, \ \forall s \in \Phi_n, \quad (3c)$$

$$-\overline{E}_s \le x_s^{\text{Pe}} \le 0, \ \forall s \in \Phi_n, \quad (3d)$$

$$0 \le q_{ns}^{\text{P}\uparrow}, q_{ns}^{\text{P}\downarrow}, q_{ns}^{\text{Pe}} \le \overline{Q}^{\text{P}}, \ \forall s, \quad (3e)$$

[13] Alternatively, [42] includes a reward term in the objective function associated with the foreseen *value* of the stored energy in the beginning of the next time horizon. However, it might be challenging to estimate such a value.

[14] For $R_t \le C_t$, energy import appears if there is an energy shortage within the community, while export takes place if there is an energy excess.

[15] Again, by theoretical bounds we achieve compact and closed decision sets to mathematically prove the existence of the Nash equilibrium solution.

[16] This can be mathematically shown by taking the derivative of the Lagrangian function of (2) with respect to $\lambda_{t\omega}$. It writes as $\sum_{n\in\mathcal{N}} (p_{nt\omega} - p^{\text{i}}_{t\omega} + p^{\text{e}}_{t\omega}) - \underline{\chi}_{t\omega} + \overline{\chi}_{t\omega} = 0, \forall t, \omega$, where $\underline{\chi}_{t\omega}$ and $\overline{\chi}_{t\omega}$ are dual variables corresponding to lower and upper bounds of (2d), respectively. For inactive price constraints yielding $\underline{\chi}_{t\omega} = \overline{\chi}_{t\omega} = 0$, the total supply equals to the total demand at any time step t and in every scenario ω. However, active price constraints may lead to "economic curtailments" and market inefficiencies as discussed in [43].

$$p_{nt\omega} + S_{nt\omega} + \sum_{s}(p^{\downarrow}_{nst\omega} - p^{\uparrow}_{nst\omega}) - D_{nt} = 0,\ \forall t, \omega, \quad (3f)$$

$$\text{(1c)},\ \forall t, \omega, \quad (3g)$$

$$e_{nst\omega} = e_{ns(t-1)\omega} + \eta^{\uparrow}_{s} p^{\uparrow}_{nst\omega} - \eta^{\downarrow}_{s} p^{\downarrow}_{nst\omega}, \forall s, 1 < t < |\mathcal{T}|, \omega, \quad (3h)$$

$$e_{nst\omega} = \frac{q^{\text{Pe}}_{ns}}{\overline{E}_s} E^{\text{ini}}_{s},\ \forall s, t = 1, t = |\mathcal{T}|, \omega, \quad (3i)$$

$$0 \leq p^{\uparrow}_{nst\omega} \leq q^{\text{P}\uparrow}_{ns},\ \forall s, t, \omega, \quad (3j)$$

$$0 \leq p^{\downarrow}_{nst\omega} \leq q^{\text{P}\downarrow}_{ns},\ \forall s, t, \omega, \quad (3k)$$

$$0 \leq e_{nst\omega} \leq q^{\text{Pe}}_{ns},\ \forall s, t, \omega \Bigg\}\ \forall n \in \mathcal{N}, \quad (3l)$$

where $\Xi^{\text{P}}_n = \{x^{\text{P}\uparrow}_{s\in\Phi_n}, x^{\text{P}\downarrow}_{s\in\Phi_n}, x^{\text{Pe}}_{s\in\Phi_n}, q^{\text{P}\uparrow}_{ns}, q^{\text{P}\downarrow}_{ns}, q^{\text{Pe}}_{ns}, p_{nt\omega}, p^{\uparrow}_{nst\omega}, p^{\downarrow}_{nst\omega}, e_{nst\omega}\}$. The first two lines of the objective function (3a) maximize the revenue and minimize the cost of prosumer n in the local forward market obtained from buying and selling PSRs, which consist of charging, discharging, and energy rights. In addition, the last line of (3a) minimizes her expected cost in spot markets incurred by energy trades. For the sake of a general formulation, we consider that each prosumer n is able to sell PSRs in the case that she owns a storage system $s \in \Phi_n$, and at the same time, she can buy PSRs from any storage system s in the community belonging to herself or other prosumers. This allows modeling a prosumer, who can *partially* provide other prosumers with access to her own storage system, and uses the rest of the energy storage system for her own benefit. For this purpose, we differentiate between physical charging, discharging, and capacity right quantities of storage system $s \in \Phi_n$ to be *sold* by prosumer n, i.e., non-positive variables $x^{\text{P}\uparrow}_{s}, x^{\text{P}\downarrow}_{s}$, and x^{Pe}_{s}, and those PSR quantities to be *bought* by prosumer n from any storage system s belonging to herself or others, i.e., non-negative variables $q^{\text{P}\uparrow}_{ns}, q^{\text{P}\downarrow}_{ns}$, and q^{Pe}_{ns}. PSR prices $\mu^{\text{P}\uparrow}_{s}, \mu^{\text{P}\downarrow}_{s}$, and μ^{Pe}_{s} as well as spot market prices $\lambda_{t\omega}$ are parameters within (3), while they are variables within the Nash equilibrium problem (3)–(4). We provide the problem (4) of the energy community manager later in this section. Similar to (1a), we consider quadratic regularizers, weighted by a small positive factor β in (3a) for both PSR and energy quantities.

Constraints (3b)–(3d) restrict the amount of PSR quantities to be sold. Constraints (3e) impose a theoretical upper bound $\overline{Q}^{\text{P}}$ to PSR quantities to be bought. Based on awarded PSRs, prosumer n dispatches the storage system s in local spot markets through (3f)–(3l). Storage-related variables are indexed by n. For example, $e_{nst\omega}$ denotes the stored energy belonging to prosumer n in storage system s at time t under scenario ω, although the energy storage system s itself may not belong to that prosumer. Moreover, each prosumer has access and also restores the initial state of charge E^{ini}_{s} according to her awarded capacity rights (3i). It is worth noting that for operational access to a storage system a prosumer needs to award all types of physical rights, i.e., charging right $q^{\text{P}\uparrow}_{ns}$, discharging right $q^{\text{P}\downarrow}_{ns}$, and energy right q^{Pe}_{ns}. Otherwise, charging $p^{\uparrow}_{nst\omega}$ or discharging $p^{\downarrow}_{nst\omega}$ cannot take place since equality constraint (3h) or (3i) would be violated. Eventually, the set of constraints (3h)–(3l) ensures that individual operational strategies of an energy storage system according to the objective of PSR holders are in line with the physical operational region of the storage system itself.

Given local forward market decisions of prosumers, i.e., $x^{\text{P}\uparrow}_{s}$, $x^{\text{P}\downarrow}_{s}$, x^{Pe}_{s}, $q^{\text{P}\uparrow}_{ns}$, $q^{\text{P}\downarrow}_{ns}$, q^{Pe}_{ns} and their trading decisions $p_{nt\omega}$ in local spot markets, the energy community manager minimizes the operational cost of the community as

$$\underset{\Theta^{\text{P}}}{\text{Min}} \sum_{s} \underbrace{\Big[\mu^{\text{P}\uparrow}_{s}(x^{\text{P}\uparrow}_{s} + \sum_{n\in\mathcal{N}} q^{\text{P}\uparrow}_{ns}) + \mu^{\text{P}\downarrow}_{s}(x^{\text{P}\downarrow}_{s} + \sum_{n\in\mathcal{N}} q^{\text{P}\downarrow}_{ns}) + \mu^{\text{Pe}}_{s}(x^{\text{Pe}}_{s} + \sum_{n\in\mathcal{N}} q^{\text{Pe}}_{ns})\Big]}_{\text{Buyers' forward market cost / Sellers' forward market revenue}}$$
$$+ \sum_{t} \mathbb{E}_{\omega}\Big[C_t p^{\text{i}}_{t\omega} - R_t p^{\text{e}}_{t\omega} + \lambda_{t\omega}\big(\sum_{n\in\mathcal{N}} p_{nt\omega} - p^{\text{i}}_{t\omega} + p^{\text{e}}_{t\omega}\big)\Big] \quad (4a)$$

subject to

$$\text{(2b)–(2d)},\ \forall t, \omega, \quad (4b)$$

$$-\overline{M}^{\text{P}} \leq \mu^{\text{P}\uparrow}_{s}, \mu^{\text{P}\downarrow}_{s}, \mu^{\text{Pe}}_{s} \leq \overline{M}^{\text{P}},\ \forall s, \quad (4c)$$

where the variable set is $\Theta^{\text{P}} = \{\mu^{\text{P}\uparrow}_{s}, \mu^{\text{P}\downarrow}_{s}, \mu^{\text{Pe}}_{s}, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}, \lambda_{t\omega}\}$. In contrast to (2a), the objective function (4a) additionally maximizes the revenue for PSR sellers and minimizes the cost for PSR buyers by revealing forward market-clearing prices $\mu^{\text{P}\uparrow}_{s}, \mu^{\text{P}\downarrow}_{s}$, and μ^{Pe}_{s} based on the expectation about spot market scenarios. Such prices are constrained by a theoretical bound $\overline{M}^{\text{P}}$ in (4c). Similar to (2), the problem (4) ensures the power balance in the community as well as the balance between sold and bought rights, provided that price constraints are not binding.[17]

Definition 2. Based on (3) and (4) we define the non-cooperative Nash game $\Gamma^{\text{PSR}}(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$, where $\mathcal{Z}$ is the set of all players and J_i their respective objective function. $K = (K_1 \times \cdots \times K_{\mathcal{N}} \times K_{\text{m}})$ denotes the strategy set of the game, where K_n is the strategy set of prosumer $n \in \mathcal{N}$ and K_{m} the strategy set of the energy community manager.

Proposition 2. Similarly as for $\Gamma^{\text{spot}}(\cdot)$, a unique Nash equilibrium for the non-cooperative Nash game $\Gamma^{\text{PSR}}(\cdot)$ exists.

Proof 2. The proof is available in Appendix A-B. ■

Remark 2. We solve $\Gamma^{\text{PSR}}(\cdot)$ in its equivalent formulation as a single optimization problem, provided in Appendix A-B.

3) FSR Forward Market and Spot Markets: We follow the concept introduced by Taylor in [28], defining *FSR values* in terms of dual variables associated with physical capacity and energy constraints of an energy storage system.

For given FSR prices $\mu^{\text{F}\uparrow}_{s}, \mu^{\text{F}\downarrow}_{s}, \mu^{\text{Fe}}_{s}$, spot market prices $\lambda_{t\omega}$ as well as FSR values $\overline{\gamma}^{\uparrow}_{st\omega}, \overline{\gamma}^{\downarrow}_{st\omega}, \overline{\gamma}^{\text{e}}_{st\omega}$ each prosumer n minimizes her expected cost from FSR trades in the local forward market and energy exchanges in spot markets as

$$\Bigg\{\underset{\Xi^{\text{F}}_{n}}{\text{Min}} \sum_{s\in\Phi_n} \big(\underbrace{\mu^{\text{F}\uparrow}_{s} x^{\text{F}\uparrow}_{s} + \mu^{\text{F}\downarrow}_{s} x^{\text{F}\downarrow}_{s} + \mu^{\text{Fe}}_{s} x^{\text{Fe}}_{s}}_{\text{Revenue in forward market for FSR sellers}} \big)$$

[17]For example, by taking the derivative of the Lagrangian function associated to (4) with respect to $\mu^{\text{P}\uparrow}_{s}$ and assuming price constraints (4c) are inactive, one can conclude that $x^{\text{P}\uparrow}_{s} + \sum_{n\in\mathcal{N}} q^{\text{P}\uparrow}_{ns} = 0,\ \forall s$.

$$+\sum_s \Big[\underbrace{\mu_s^{\mathrm{F}\uparrow} q_{ns}^{\mathrm{F}\uparrow} + \mu_s^{\mathrm{F}\downarrow} q_{ns}^{\mathrm{F}\downarrow} + \mu_s^{\mathrm{Fe}} q_{ns}^{\mathrm{Fe}}}_{\text{Cost in forward market for FSR buyers}} + \underbrace{\frac{1}{2}\beta ({q_{ns}^{\mathrm{F}\uparrow}}^2 + {q_{ns}^{\mathrm{F}\downarrow}}^2 + {q_{ns}^{\mathrm{Fe}}}^2)}_{\text{Regularizer}} \Big]$$

$$+ \sum_t \mathbb{E}_\omega \Big[\underbrace{\lambda_{t\omega} p_{nt\omega}}_{\text{Cost in spot markets}} + \underbrace{\frac{1}{2}\beta p_{nt\omega}^2}_{\text{Regularizer}}$$

$$- \sum_s (\underbrace{\overline{\gamma}_{st\omega}^{\uparrow} q_{ns}^{\mathrm{F}\uparrow} + \overline{\gamma}_{st\omega}^{\downarrow} q_{ns}^{\mathrm{F}\downarrow} + \overline{\gamma}_{st\omega}^{\mathrm{e}} q_{ns}^{\mathrm{Fe}}}_{\text{Revenue in spot markets from FSR values}}) \Big] \tag{5a}$$

subject to

$$-\overline{P}_s^{\uparrow} \leq x_s^{\mathrm{P}\uparrow} \leq 0, \ \forall s \in \Phi_n, \tag{5b}$$

$$-\overline{P}_s^{\downarrow} \leq x_s^{\mathrm{P}\downarrow} \leq 0, \ \forall s \in \Phi_n, \tag{5c}$$

$$-\overline{E}_s \leq x_s^{\mathrm{Pe}} \leq 0, \ \forall s \in \Phi_n, \tag{5d}$$

$$0 \leq q_{ns}^{\mathrm{F}\uparrow}, q_{ns}^{\mathrm{F}\downarrow}, q_{ns}^{\mathrm{Fe}} \leq \overline{Q}^{\mathrm{F}}, \ \forall s, \tag{5e}$$

$$p_{nt\omega} + S_{nt\omega} - D_{nt} = 0, \ \forall t, \omega, \tag{5f}$$

$$\left.(1\mathrm{c}), \ \forall t, \omega \ \right\} \forall n \in \mathcal{N}, \tag{5g}$$

where $\Xi_n^{\mathrm{F}} = \{x_{s\in\Phi_n}^{\mathrm{F}\uparrow}, x_{s\in\Phi_n}^{\mathrm{F}\downarrow}, x_{s\in\Phi_n}^{\mathrm{Fe}}, q_{ns}^{\mathrm{F}\uparrow}, q_{ns}^{\mathrm{F}\downarrow}, q_{ns}^{\mathrm{Fe}}, p_{nt\omega}\}$. Similar to (3a), the objective function (5a) maximizes the revenue and minimizes the cost of prosumer n from FSR trades in the local forward market as well as her expected cost from energy trades in local spot markets. Also similar to (3), we differentiate between financial charging, discharging, and capacity right quantities to be sold by a prosumer in the case that she owns a storage system $s \in \Phi_n$, i.e., non-positive variables $x_s^{\mathrm{F}\uparrow}, x_s^{\mathrm{F}\downarrow}$, and x_s^{Fe}, and those FSR quantities to be bought by a prosumer from any storage system s, i.e., non-negative variables $q_{ns}^{\mathrm{F}\uparrow}, q_{ns}^{\mathrm{F}\downarrow}$, and q_{ns}^{Fe}. These amounts are constrained by (5b)–(5e). FSR prices $\mu_s^{\mathrm{F}\uparrow}, \mu_s^{\mathrm{F}\downarrow}, \mu_s^{\mathrm{Fe}}$, FSR values $\overline{\gamma}_{st\omega}^{\uparrow}, \overline{\gamma}_{st\omega}^{\downarrow}, \overline{\gamma}_{st\omega}^{\mathrm{e}}$ as well as spot market prices $\lambda_{t\omega}$ are parameters within (5), while they are variables within the Nash equilibrium problem (5)–(6). We provide problem (6) later in this section.

Problem (5) with FSRs is different in two ways compared to problem (3) with PSRs. First, each prosumer additionally accounts in (5a) for the expected revenue from awarded FSRs $q_{ns}^{\mathrm{F}\uparrow}, q_{ns}^{\mathrm{F}\downarrow}, q_{ns}^{\mathrm{Fe}}$ according to realized FSR values $\overline{\gamma}_{st\omega}^{\uparrow}, \overline{\gamma}_{st\omega}^{\downarrow}, \overline{\gamma}_{st\omega}^{\mathrm{e}}$ in local spot markets. And second, each prosumer has to ensure her individual power balance in (5f) *without* an operational access to an energy storage system.

Given prosumers' trading decisions in the FSR forward market, i.e., $x_s^{\mathrm{F}\uparrow}, x_s^{\mathrm{F}\downarrow}, x_s^{\mathrm{Fe}}, q_{ns}^{\mathrm{F}\uparrow}, q_{ns}^{\mathrm{F}\downarrow}$, and q_{ns}^{Fe}, as well as their energy exchanges in spot markets $p_{nt\omega}$, the community manager minimizes the operational cost of the community as

$$\underset{\Theta^{\mathrm{F}}}{\mathrm{Min}}$$

$$\sum_s \underbrace{\Big[\mu_s^{\mathrm{F}\uparrow}(x_s^{\mathrm{F}\uparrow} + \sum_{n\in\mathcal{N}} q_{ns}^{\mathrm{F}\uparrow}) + \mu_s^{\mathrm{F}\downarrow}(x_s^{\mathrm{F}\downarrow} + \sum_{n\in\mathcal{N}} q_{ns}^{\mathrm{F}\downarrow}) + \mu_s^{\mathrm{Fe}}(x_s^{\mathrm{Fe}} + \sum_{n\in\mathcal{N}} q_{ns}^{\mathrm{Fe}})\Big]}_{\text{Buyers' forward market cost / Sellers' forward market revenue}}$$

$$+ \sum_t \mathbb{E}_\omega \Big[C_t p_{t\omega}^{\mathrm{i}} - R_t p_{t\omega}^{\mathrm{e}}$$

$$+ \underbrace{\lambda_{t\omega} \big(\sum_{n\in\mathcal{N}} p_{nt\omega} - p_{t\omega}^{\mathrm{i}} + p_{t\omega}^{\mathrm{e}} + \sum_s (p_{st\omega}^{\downarrow} - p_{st\omega}^{\uparrow}) \big) \Big]}_{\text{Buyers' spot market cost / Sellers' spot market revenue}} \tag{6a}$$

subject to

$$(1\mathrm{d})\text{–}(1\mathrm{h}), \ \forall s, t, \omega, \tag{6b}$$

$$(2\mathrm{b})\text{–}(2\mathrm{d}), \ \forall t, \omega, \tag{6c}$$

$$-\overline{M}^{\mathrm{F}} \leq \mu_s^{\mathrm{F}\uparrow}, \mu_s^{\mathrm{F}\downarrow}, \mu_s^{\mathrm{Fe}} \leq \overline{M}^{\mathrm{F}}, \ \forall s, \tag{6d}$$

where the set of primal variables is $\Theta^{\mathrm{F}} = \{\mu_s^{\mathrm{F}\uparrow}, \mu_s^{\mathrm{F}\downarrow}, \mu_s^{\mathrm{Fe}}, p_{t\omega}^{\mathrm{i}}, p_{t\omega}^{\mathrm{e}}, \lambda_{t\omega}, p_{st\omega}^{\uparrow}, p_{st\omega}^{\downarrow}, e_{st\omega}\}$. Moreover, $\overline{\gamma}_{st\omega}^{\uparrow}, \overline{\gamma}_{st\omega}^{\downarrow}$, and $\overline{\gamma}_{st\omega}^{\mathrm{e}}$ are dual variables of each storage system associated with its charging, discharging and reservoir capacity constraints (1f)–(1h) within (6b), referring to as "FSR values" [28].

The main difference of (6) compared to the problem (4) with PSRs is that the community manager operates *all* energy storage systems. In contrast to (4a), the third line of (6a) determines the charging and discharging levels of storage systems. Therefore, the energy community manager enforces the import/export and price constraints (6c)–(6d), and also the operational region of all energy storage systems within the energy community (6b).

Definition 3. Based on (5) and (6) we define the non-cooperative Nash game $\Gamma^{\mathrm{FSR}}(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$, where $\mathcal{Z}$ is the set of all players and J_i their respective objective function. $K = (K_1 \times \cdots \times K_{\mathcal{N}} \times K_{\mathrm{m}})$ denotes the strategy set of the game, where K_n is the strategy set of prosumer $n \in \mathcal{N}$ and K_{m} the strategy set of the energy community manager.

Proposition 3. Again, a unique Nash equilibrium for the non-cooperative Nash game $\Gamma^{\mathrm{FSR}}(\cdot)$ exists.

Proof 3. The proof is stated in Appendix A-C. ■

Remark 3. In contrast to $\Gamma^{\mathrm{spot}}(\cdot)$ and $\Gamma^{\mathrm{PSR}}(\cdot)$, the non-cooperative Nash game $\Gamma^{\mathrm{FSR}}(\cdot)$ cannot be solved as a single convex optimization problem due to products of FSR volumes and values in (5a). However, $\Gamma^{\mathrm{FSR}}(\cdot)$ can be either solved as a mixed complementarity problem [33] or by two sequential optimization problems as formulated in Appendix A-C.

B. Cooperative Market Design

In this section, we pay attention to cooperative market designs for energy communities, where prosumers agree on a community cost allocation rule. The energy community manager determines the optimal dispatch of all devices for each scenario of uncertain PV power generation ω as

$$\Big\{ \underset{\Phi}{\mathrm{Min}} \sum_t \big(C_t p_{t\omega}^{\mathrm{i}} - R_t p_{t\omega}^{\mathrm{e}} + \sum_{n\in\mathcal{N}} \frac{1}{2}\beta p_{nt\omega}^2 \big) \tag{7a}$$

subject to

$$(1\mathrm{b})\text{–}(1\mathrm{c}), \ \forall n, t, \tag{7b}$$

$$(1\mathrm{d})\text{–}(1\mathrm{h}), \ \forall s, t, \tag{7c}$$

$$(2\mathrm{b})\text{–}(2\mathrm{c}), \ \forall t, \tag{7d}$$

$$\left.\sum_{n\in\mathcal{N}} p_{nt\omega} - p_{t\omega}^{\mathrm{i}} + p_{t\omega}^{\mathrm{e}} = 0 \colon \lambda_{t\omega}, \ \forall t \ \right\} \forall \omega, \tag{7e}$$

where the variable set is $\Phi = \{p_{nt\omega}, p^{\uparrow}_{st\omega}, p^{\downarrow}_{st\omega}, e_{st\omega}, p^{\mathrm{i}}_{t\omega}, p^{\mathrm{e}}_{t\omega}\}$. The objective function (7a) minimizes the community cost from importing and exporting energy. Constraints (7b) and (7c) define the operational region for each prosumer and energy storage systems. Constraint (7d) describes bounds for energy imports and exports. Finally, (7e) enforces the power balance of the energy community as a whole.

To derive the financial burden for each prosumer, we first have to consider all coalitions $\mathcal{S} \subseteq \mathcal{N}$, i.e., all possible combinations of prosumers given by $\mathcal{S} \in 2^{\mathcal{N}}$.[18] The case where all prosumers cooperate with each other is called the grand coalition $\mathcal{S} = \mathcal{N}$. For each coalition $\mathcal{S}$ we define a cost saving function

$$v(\mathcal{S}) = \sum_{n \in \mathcal{S}} C_n - C_{\mathcal{S}}, \tag{8}$$

where C_n is the cost for each prosumer $n \in \mathcal{S}$ if she operates her devices individually, and $C_{\mathcal{S}}$ is the cost for coalition $\mathcal{S}$ if prosumers operate their devices jointly.

A coalitional game is now given by the pair $(\mathcal{N}, v)$. One important characteristic of our coalitional game is super-additivity. This means that the value of a union of two selected coalitions, i.e., $\mathcal{S} \in 2^{\mathcal{N}}$ and $\mathcal{V} \in 2^{\mathcal{N}}$ with $\mathcal{S} \cap \mathcal{V} = \varnothing$, is not less than the sum of their separate values, i.e., $v(\mathcal{S}) + v(\mathcal{V}) \leq v(\mathcal{S} \cup \mathcal{V})$ [17]. Given that the importing cost C_t is higher than the exporting price R_t, our coalitional game is super-additive [44].[19] Thus, a cooperation among all prosumers, i.e., the grand coalition $\mathcal{N}$, is most beneficial. However, super-additivity does not imply that all players benefit individually, or that a cooperation among all prosumers is stable.[20]

Therefore, we have to consider three important properties, that have to be fulfilled by a cost allocation rule, namely *efficiency*, *individual rationality*, and *stability*. A cost allocation rule is efficient if it redistributes the whole value, i.e., the cost saving, of the grand coalition $\mathcal{N}$, i.e., $\sum_{n \in \mathcal{N}} x_n = v(\mathcal{N})$, where $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ is the payoff vector and x_n the payment to player $n \in \mathcal{N}$. Individual rationality is given if each player benefits from the grand coalition at least as much as she would gain individually, i.e., $x_n \geq v(\{n\})$. Finally, a coalition is stable if the cost allocation lies within the *core* $\mathbf{C}$ [20], given by

$$\mathbf{C} = \Big\{\mathbf{x} | \sum_{n \in \mathcal{N}} x_n = v(\mathcal{N}), v(\mathcal{S}) - \sum_{n \in \mathcal{S}} x_n \geq 0, \forall n \in \mathcal{N}\Big\}, \tag{9}$$

where the equality constraint ensures an efficient cost allocation, while the inequality constraint guarantees the stability of the grand coalition.

In the following we consider two cost allocation rules, namely the *Shapley value* [45] and the *nucleolus* [46].

1) Shapley Value: The Shapley value [45] states the average marginal contribution of each prosumer $n \in \mathcal{S}$ to coalitions $\mathcal{S} \in 2^{\mathcal{N}}$ as

$$\phi_n(v) = \sum_{\mathcal{S} \in 2^{\mathcal{N}}, \forall n \in \mathcal{S}} \frac{(|\mathcal{S}| - 1)!(N - |\mathcal{S}|)!}{N!} \big[v(\mathcal{S}) - v(\mathcal{S} \backslash \{n\})\big], \tag{10}$$

where N is the total number of prosumers. Note that the Shapley value is not necessarily a stable cost allocation rule [20].

2) Nucleolus: The nucleolus minimizes the excesses of all coalitions, and thereby the dissatisfaction of prosumers. In contrast to the Shapley value the nucleolus yields a stable coalition among all prosumers [46]. We provide the mathematical program to calculate the nucleolus in Appendix B.

[18] The exponential growth of the power set indicates the computational challenge as the size of the energy community increases in terms of energy community members.

[19] Given $C_t \geq R_t$, the total community cost is a concave function in dependence of the number of energy community members.

[20] In this context, stable means that no incentive for prosumers to leave the grand coalition exists.

TABLE I: Properties of non-cooperative and cooperative local market designs.

	Spot	PSR+Spot	FSR+Spot	Shapley	Nucleolus
Operational access	✗	✓	✗	✗	✗
Economic access	✗	✓	✓	✓	✓
Market efficiency	✓*	✓*	✓*	✓	✓
Individual rationality	✓	✓	✓	✓	✓
Revenue adequacy	✓	✓	✓	✓	✓
Incentive compatibility	✓*	✓*	✓*	✓**	✓**
Stability	✓	✓	✓	✗	✓
Computational time	Low	Low	Low	High	High

* Given the assumption of a perfectly competitive market environment.
** Given that prosumers reveal their true costs/utilities.

C. Properties of Local Market Designs

We summarize properties of non-cooperative and cooperative market designs in Table I. Except for the market design with spot markets only, all other local market designs enable an access economy for energy storage, although they differ highly in their approach: On the one hand, the concept of a forward market for PSRs enables an *operational access* for individuals, who yield *economic access* via a direct utilization of an energy storage system. On the other hand, a forward market for FSRs as well as cooperative market designs leave the operation of energy storage systems exclusively to the energy community manager, while prosumers benefit via a redistribution of the generated value from intertemporal energy arbitrage.

Table I also provides the list of desirable economic market properties that are satisfied or jeopardized under different market designs. The first property is *market efficiency*, implying that market outcomes align with the social welfare maximization. This property is given by definition for cooperative market designs. However, for non-cooperative market designs this only holds under the assumption of a perfectly competitive market environment. The second desirable property is *individual rationality*, implying that no prosumer is left with a negative utility. The third property is *revenue adequacy*, meaning that the energy community manager has no financial deficit. Both properties are fulfilled by all market designs. Lastly, the fourth desirable property is *incentive compatibility*, i.e., all prosumers bid according to their true costs/utilities. This property is only ensured under perfect competition.

All local market designs ensure *stability* in the sense that there is no incentive for prosumers to leave the energy community, except for the Shapley value, for which this cannot be generally guaranteed [17]. Finally, we note that non-cooperative market designs computationally scale well

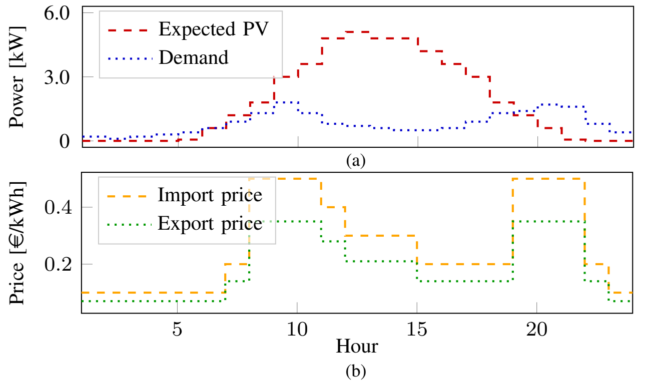


Fig. 3: Input data: Plot 3a shows the expected PV power generation and the demand for each prosumer. Plot 3b depicts prices for importing and exporting electricity.

in terms of energy community members, while cooperative market designs do not due to the exponential increase in possible coalitions as we show later in Section IV-B.

IV. Numerical Study

We apply all game-theoretic models introduced in Section III to numerically assess different alternatives of an economic access to storage systems in energy communities. In the following we explain the method used to generate PV power generation scenarios in Section IV-A. In Section IV-B we discuss scalability issues and report the computational time as the size of an energy community increases in terms of the number of prosumers. We then present an illustrative example in Section IV-C, where we consider four representative prosumers to shed light on the individual and the community cost under different local market designs. A sensitivity analysis in terms of the uncertainty is presented in Section IV-D, where we gradually increase the variance of the underlying probability distribution. Finally, in Section IV-E we consider an energy community with 16 prosumers and multiple energy storage systems, which differ in their round-trip efficiency.

Throughout these analyses, we consider a time horizon of 24 hours. The local forward market for trading storage rights is cleared once before the given time horizon, while local spot markets are cleared for every hour. As input data, Figure 3a shows the expected PV power generation as well as the daily demand profile of each prosumer. In the illustrative example we assume that the demand profiles of all prosumers are identical, while in Section IV-E we slightly change the demand of each added prosumer by up to 20%. Again as input data, Figure 3b provides the hourly importing and exporting prices for electricity observed by the energy community manager.

A. Modeling Uncertainty

The uncertain nature of the PV power generation $S_{nt\omega} \forall n, t$ is modeled via a set of scenarios $\omega \in \Omega$ that accounts for an intertemporal structure of the PV power availability [47]. To obtain this set of scenarios over the optimization horizon $\mathcal{T}$, we first generate a random vector $\boldsymbol{X} = (X_1, X_2, ..., X_{\mathcal{T}})^\top$, following a multivariate Gaussian distribution $\boldsymbol{X}^\omega \sim \mathcal{N}(\mu_0, \Sigma)$

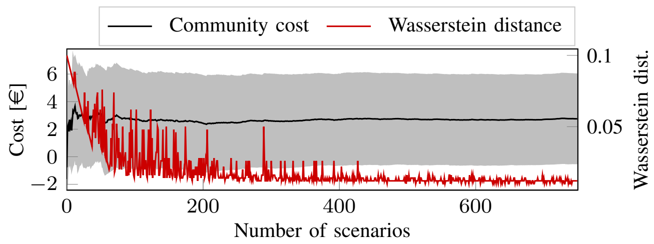


Fig. 4: Market design with spot markets only: Expected community cost marked in black and its standard deviation illustrated by the shaded area are shown as a function of the number of scenarios. The red curve shows the Wasserstein distance between each set of scenarios and its predecessor.

with $\boldsymbol{X}^\omega$ being the ω^{th} realization of the random vector. The vector of mean values μ_0 is a $1 \times \mathcal{T}$ zero vector. The covariance matrix Σ is a symmetric $\mathcal{T} \times \mathcal{T}$ matrix, whose diagonal elements are equal to 1, and whose off-diagonal elements are calculated in the same way suggested in [48], i.e.,

$$\Sigma = \exp\left(\frac{-|t - t'|}{\nu}\right), \; 0 \leq t, t' \leq \mathcal{T}, \tag{11}$$

where the range parameter ν controls the strength of correlation of random variables across time steps.

Applying the inverse probit function $\Phi(\cdot)$ and the predictive cumulative distribution function $\hat{F}_t(\cdot)$ on each time step t of the ω^{th} realization of the random vector $\boldsymbol{X}^\omega$, one normalized PV power generation trajectory $s_{t\omega} \in [0, 1]$ per scenario ω spanning the optimization horizon $\mathcal{T}$ is obtained through the following transformation:

$$s_{t\omega} = \hat{F}_t^{-1} \left(\Phi \left(\boldsymbol{X}^\omega\right)\right), \; \forall t, \forall \omega. \tag{12}$$

The probability density function $\hat{f}_t$ is modeled by a Beta distribution $B(\mu_t, \sigma^2)$. We normalize the trajectory μ_t shown in Figure 3a and consider this path as the conditional expectation of the stochastic process describing the PV power generation. Moreover, we set the variance to $\sigma^2 = 0.025$.

By applying this method, an arbitrary number of PV power generation scenarios around the expected trajectory can be sampled. However, in order to appropriately select the number of scenarios characterizing the PV power generation uncertainty, we provide in Figure 4 the evolution of the expected community cost and its standard deviation for the market design with only spot markets as the number of scenarios increases. It also shows the Wasserstein distance between each set of scenarios and its predecessor. Accordingly, we find out that from 500 scenarios onward changes hardly ever take place. Therefore, we choose 500 scenarios to properly represent the underlying probability distribution of PV power generation. Note that all scenarios are equiprobable.

B. Computational Aspects

One important property for a local energy market design is its computational scalability, especially in cases with a comparatively high number of community members. We have solved all optimization problems with Gurobi Optimizer 9.0.1 under Python 3.7.6 using a 314 GB-RAM computer with 72 cores, each clocking at 2.3 GHz. In the case of cooperative

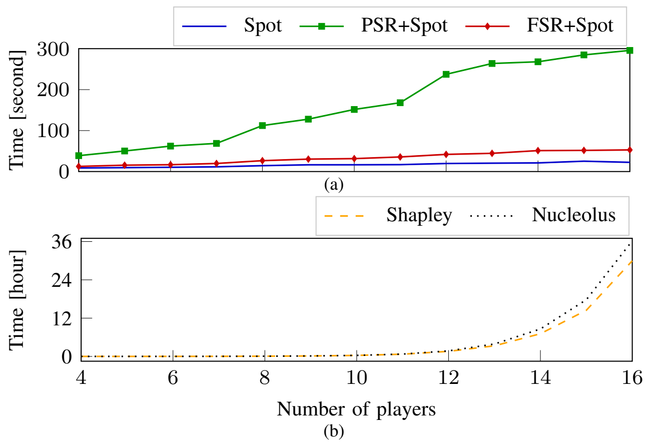


Fig. 5: Computational time as a function of the number of prosumers in the community. The plot 5a corresponds to non-cooperative market design alternatives, while the plot 5b associates with the cooperative ones.

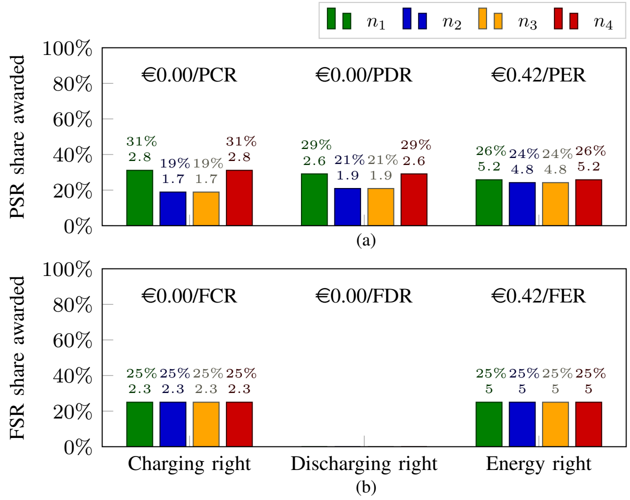


Fig. 6: Market-clearing prices and awarded quantities of physical charging rights (PCR), discharging rights (PDR), and energy rights (PER), as well as those of financial charging rights (FCR), discharging rights (FDR), and energy rights (FER) in percentage of the total available energy storage capacity in the community. Numbers below percentage values indicate the absolute value of tota PSR/FSR quantities bought by prosumers n_1 to n_4.

games, we have parallelized the computation of coalitions across 72 cores. Recall that all source codes are available in [32].

Figure 5 shows the computational time as the number of prosumers gradually increases from 4 to 16. The non-cooperative market design with a forward market for FSRs scales nicely with the number of community members, as the computational time remains within seconds, see Figure 5a. Unlike the case of FSRs, the computational time for the case of PSRs increases up to 5 minutes, since we compute the dispatch of many small energy storage systems as a result of providing operational access to PSR holders. For cooperative market designs shown in Figure 5b, we observe that the computational time increases exponentially up to 36 hours. This effect is mainly driven by the exponential increase in the number of possible coalitions of community members by $2^{\mathcal{N}}$. Calculating the nucleolus is slightly more time consuming than the Shapley value, although the difference is negligible compared to the determination of solutions for all possible coalitions in the first place.

C. Illustrative Example with Four Prosumers

Let us consider a community with four prosumers, namely n_1, n_2, n_3, and n_4. Prosumer n_1 owns a PV system, but has no storage. Prosumer n_2 owns a storage system, but has no renewable power generation. Prosumer n_3 is a pure consumer owning neither a PV nor a storage system. Finally, prosumer n_4 owns a PV and a storage system. The storage systems belonging to prosumer n_2 and n_4 are of 10-kWh energy capacity, 4.5-kW charging and discharging power capacity, and have a round-trip efficiency of 0.9. The initial state of charge E_s^{ini} of both energy storage systems is equal to zero. The PV system of prosumer n_4 is identical to that of n_1. The daily demand profile is assumed to be identical for all prosumers as shown in Figure 3a.

In our non-cooperative market designs the access economy is enabled through either PSRs or FSRs. However, these two paradigms fundamentally follow different approaches. While PSRs enable economic access via operational access for each right holder, FSRs enable economic access through a claim to the intertemporal energy arbitrage earned by the energy community manager, where right values are determined based on the limiting components of an energy storage system. Therefore, PSR and FSR quantities awarded by prosumers differ as shown in Figure 6. According to this figure, charging and discharging right prices for both PSRs and FSRs are zero, while energy rights have a price of €0.42/right.

The market outcomes of the forward market for PSRs are shown in Figure 6a. Prosumers n_1 and n_4 award a slightly greater share of physical charging, discharging, and energy rights compared to prosumers n_2 and n_3, who award equal shares. Thereby, prosumers n_1 and n_4 reduce spot market activities and optimally utilize their PV systems for meeting their own demand. Note that prosumers have to award all types of rights, i.e., physical charging, discharging, and energy rights, to get operational, and thereby economic access to energy storage systems.

Figure 6b illustrates the forward market outcomes for FSRs. Here, no prosumer awards financial discharging rights, since the value of those rights in spot markets is zero. However, financial charging and energy rights have positive values, from which all prosumers award equal shares.

Figure 7 shows market outcomes for non-cooperative market designs with respect to the expected cost/revenue of energy community members incurred/earned in local forward and spot markets as well as the community cost as a whole.

In detail, Figure 7a corresponds to a market design with spot markets only. According to this figure, on the one hand prosumer n_1 incurs a cost in spot markets from meeting demand in time periods when PV power is not sufficiently available. On the other hand, she receives a revenue from selling energy when her PV power generation exceeds her own demand. Prosumer n_2, who owns a storage system, incurs a cost in spot markets from supplying her demand and charging

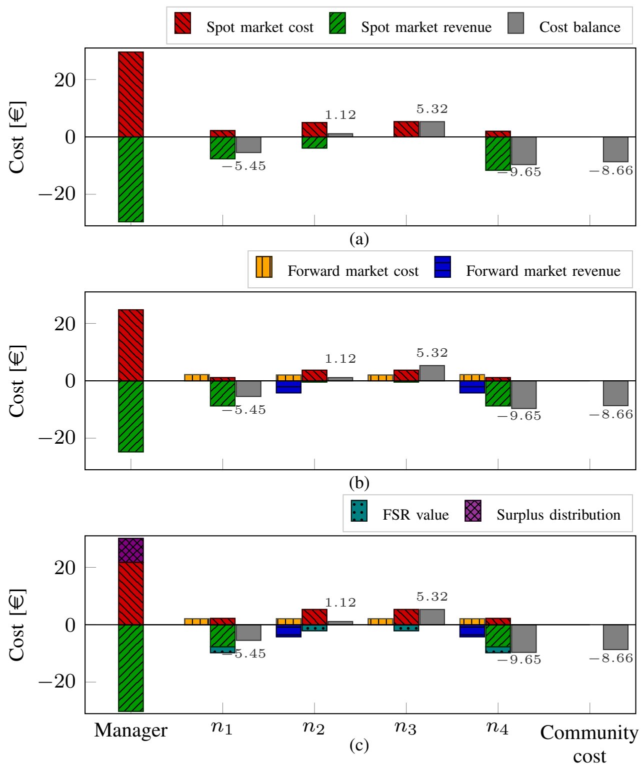


Fig. 7: Expected cost/revenue of community members and the community cost under non-cooperative market designs. Plot 7a corresponds to a market design with spot markets only. Plot 7b considers a forward market for trading PSRs, while plot 7c takes a forward market for trading FSRs into account.

the energy storage system. Moreover, she earns a revenue by discharging the storage system and selling her energy excess in local spot markets. Prosumer n_3 solely incurs a cost from meeting her demand. Finally, prosumer n_4, who owns a PV and a storage system, incurs a cost from charging the storage system while earning a revenue from discharging the storage and selling the excess power generation from the PV system. By operating both PV and storage systems, prosumer n_4 earns the highest profit among all prosumers. The energy community manager has a cost due to importing and a revenue for exporting energy, although her financial balance is zero.

Figure 7b associates with a market design including a local forward market for trading PSRs. Prosumers n_2 and n_4 as storage-owning community members earn a revenue in the forward market by selling PSRs. All prosumers, including n_2 and n_4, incur some cost in the forward market by buying PSRs. This shows that prosumers n_2 and n_4 provide a *partial access* to other prosumers, and operate a part of the storage system in local spot markets for their own benefit. By awarding PSRs, the amount of energy exchanges in spot markets, and thus cost for prosumers in those markets, decrease as, e.g., prosumer n_1 now has access to a storage system, and more efficiently utilizes her own PV system for meeting her own demand.

Figure 7c corresponds to the market design with a local forward market for trading FSRs. In contrast to the case with PSRs, the community manager collects a merchandising surplus from scheduling all energy storage systems in spot markets. This surplus is redistributed *a posteriori* among FSR holders according to the resulting FSR values and awarded shares.

An important observation made in Figure 7 is that under all three non-cooperative market designs, with or without rights, the overall *expected cost* for each community member incurred in local forward and spot markets is unchanged. In expectation, prosumer n_1 and prosumer n_4 earn €5.45 and €9.65, while prosumer n_2 and prosumer n_3 pay €1.12 and €5.32, respectively. The community manager is budget-balanced, and the expected community revenue is €8.66. However, each community member yields a cost distribution across the 500 scenarios of uncertain PV power generation, which depends on the local market design in place. This difference is illustrated in Figure 8, which shows the expected cost and its standard deviation for each prosumer under non-cooperative and cooperative market designs.

According to Figure 8, prosumer n_2, who owns a storage system, remarkably reduces her *cost volatility* by selling storage rights. Her cost standard deviation decreases from 30.4% to 17.0%. The reason for such a reduced cost volatility is that by selling rights, prosumer n_2 participates less in spot markets with volatile prices induced by the uncertain PV power generation. Moreover, under cooperative market designs her cost volatility reduces even further to 9.7% under the Shapley value. The cost/revenue volatility for prosumers n_1 and n_3 also reduces compared to a non-cooperative market design without an access economy for energy storage. For the case of either PSRs or FSRs, both prosumers n_1 and n_3 are sole right buyers. Here, prosumer n_1 experiences a slight revenue volatility reduction from 50.8% to 47.9%, since her uncertain PV power generation predominates her relatively small amount of awarded storage rights. Under cooperative market designs prosumer n_1 earns an expected revenue slightly lower than that under non-cooperative market designs. However, the revenue volatility reduces as well. Prosumer n_3, who is a pure load, observes a slight increase in her expected cost under cooperative market designs, although her cost volatility remarkably decreases by up to 70% when considering the Shapley value as the cost allocation rule. Lastly, prosumer n_4, who owns a PV and a storage system, experiences an increased revenue volatility by 6% due to selling storage rights in non-cooperative market designs. Her cost volatility is the lowest under a non-cooperative market design without an access economy for energy storage, since it is the only case wherein she fully schedules her storage to tackle the PV power generation uncertainty. When she offers storage rights to others or participates in a cooperative market design, less storage capacity is available for her individual needs, and thus her revenue volatility increases. However, her expected revenue slightly increases under cooperative market designs by 2%.

It is worth noting that although we observe changes in individual costs in terms of the expected value and the standard deviation, Figures 7 and 8 indicate that the expected community cost as a whole and its volatility remain unchanged across all proposed local market designs.

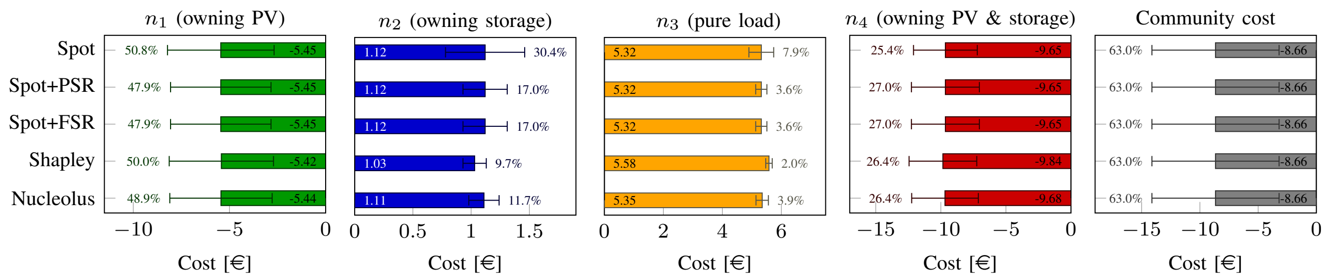


Fig. 8: Cost of prosumers n_1 to n_4: Color bars present the expected cost. Black lines show the cost ranging from the expected value minus the standard deviation to the expected value plus the standard deviation. Numbers within color bars and next to lines show the expected cost in € and its standard deviation as a percentage of the expected cost, respectively.

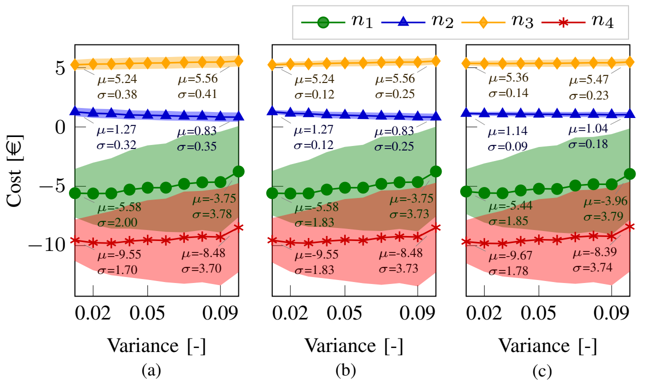


Fig. 9: Plot 9a shows the expected cost μ and its standard deviation σ, highlighted by shaded areas, for each prosumer under sole spot markets. Plot 9b depicts the case for a FSR forward and spot markets. Plot 9c graphs the cooperative market outcomes with the nucleolus as the cost allocation rule.

D. Impact of the Uncertainty

We assess the impact of the uncertainty by gradually increasing the variance σ^2 of the underlying probability distribution from 0.01 to 0.1 as shown in Figure 9. Generally, as the variance of the uncertain PV power generation increases the cost/revenue volatility for all prosumers increases too. However, while prosumer n_2, who owns a storage system, observes a decrease in her expected cost, prosumer n_3, who is a pure load, experiences an increase in her expected cost. Similarly, prosumers n_1 and n_4, which both own a PV system, incur a reduction in their expected revenue as the variance increases. These trends hold true for all proposed local market designs. Finally, we note that on the one hand the revenue volatility for prosumers n_2 and n_4, who both own PV systems, increases the most under the cooperative market design with the nucleolus as the cost allocation rule. On the other hand, the increase in the cost volatility for prosumers n_1 and n_3 is the lowest under this local market design.

E. Energy Community with 16 Prosumers

In this section we keep our initial four prosumers unchanged and add twelve new prosumers, resulting in an energy community with 16 prosumers in total. Six out of 16 prosumers own a PV system each, two prosumers own a storage system each, six prosumers are pure demands, while each of the remaining two prosumers owns both PV and storage systems.

TABLE II: PSR and FSR prices for storage systems s_1 to s_4 in [€/right]. Equal right prices for all storage systems are highlighted in blue, while unequal prices are shown in red.

	s_1	s_2	s_3	s_4
Physical charging right	0.020	0.020	0.020	0.020
Physical discharging right	0.027	0.027	0.027	0.027
Physical energy right	0.513	0.513	0.461	0.461
Financial charging right	0.018	0.018	0.018	0.018
Financial discharging right	0.027	0.027	0.027	0.027
Financial energy right	0.514	0.514	0.463	0.463

Each new prosumer has the same load pattern as depicted in Figure 3a, although the whole demand profile is randomly increased/decreased by up to 20%. The two newly added storage systems, namely s_3 and s_4, have a round-trip efficiency of 0.8, which is slightly lower than that of two existing storage systems s_1 and s_2 with a round-trip efficiency of 0.9. Finally, we consider for all energy storage systems an initial state of charge E_s^{ini} equal to zero.

Figure 10 shows the expected cost of each prosumer under different local market designs, where we group prosumers by technologies they own. A general trend that we observe is similar to our previous observations made in Figure 8. The expected cost for individual prosumers under non-cooperative market designs remains unchanged. PV-owning prosumers benefit from cooperative market designs by an increase in their expected profit, while the storage-owning prosumers and pure loads experience a slight increase in their expected cost.

Here, we draw attention to FSR and PSR prices as given in Table II. Right prices for existing storage systems s_1 and s_2 are slightly higher than those for newly added storage systems s_3 and s_4. The reason for lower right prices obtained for s_3 and s_4 is their comparatively lower efficiency with respect to s_1 and s_2. This holds true for both PSRs and FSRs. In addition, physical charging right prices are slightly higher than financial ones, while the opposite holds true for energy right prices.

V. Conclusion

This work provides and analyzes various local market design alternatives for local energy communities, ranging from non-cooperative to cooperative game-theoretic setups. In particular, this work explores market design alternatives that enable an access economy for energy storage systems. We

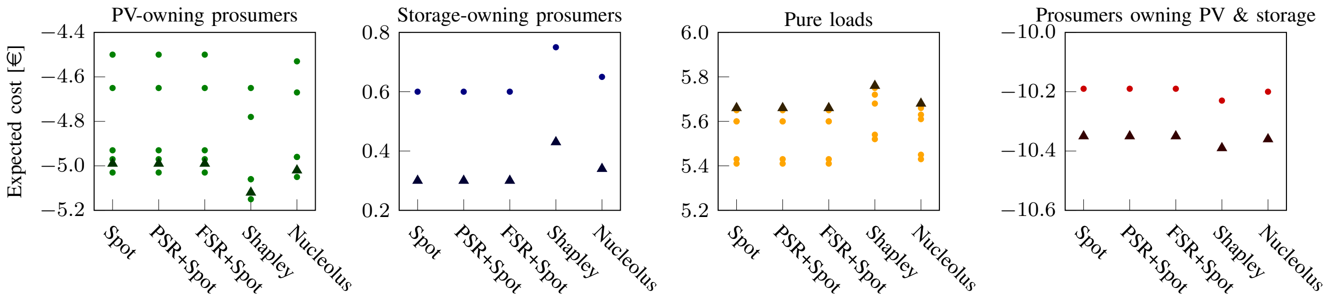


Fig. 10: The expected cost of prosumers under different non-cooperative and cooperative market designs. Prosumers are grouped by the technology, where we consider 6 PV-owning prosumers, 2 storage-owning prosumers, 6 pure loads, and 2 prosumers owning both PV and storage systems. Triangles show the expected cost of the 4 initial prosumers, while dots depict the expected cost of the 12 new prosumers.

observe that an access economy for such devices enhances energy communities by reducing the cost/revenue volatility of most prosumers. Each market design alternative ensures some desirable market properties, while jeopardizing others. Therefore, there is not a specific market design that outperforms other alternatives. This work does not derive an absolute conclusion on which of the proposed market designs is more appealing. Rather, we provide a framework which allows market regulators and policy makers to evaluate the implications of each market design for energy communities. The choice of a market design depends on the preference of energy community members, in particular the magnitude of cost/revenue volatility they are willing to accept. In addition, it depends on which market properties are most attractive to members of a given energy community. Finally, the size of an energy community is a key factor in determining a market design, because the computational time significantly increases in the case of cooperative market designs as the number of prosumers grows.

One key finding of our analyses is that the expected community cost as well as the cost volatility of the whole community across scenarios representing uncertainty are independent of the local market design. Consequently, proposed local market designs do not affect the operational strategy of PV and energy storage systems, energy exchanges among prosumers as well as energy imports, and exports to and from the energy community. However, what distinguishes various market designs is the *cost allocation* among energy community members, i.e., who pays to whom which amount of money. In the case of non-cooperative market designs, the expected cost of each prosumer is unchanged over all non-cooperative market designs, although the cost distribution across scenarios is dependent on the market design in hand. The cost volatility reduces when either PSRs or FSRs are available. In cooperative market designs the expected costs of individuals and the standard deviation of those costs change, depending on technologies belonging to prosumers as well as the number of energy community members. Consequently, prosumers' preferred choice for either a non-cooperative market design with storage rights or a cooperative market design depends on three aspects, namely the technologies they own, their individual preference on whether they are indifferent of the cost volatility or not, as well as the size of the energy community.

From a policy perspective we note that all proposed local market designs enable an access economy for storage systems. In all market design alternatives explored in this paper, we find that identical prosumers incur the same cost, which implies that no market participant will be discriminated. However, we hypothesize that energy storage owners may have a strong incentive for manipulating markets, especially if there are only a few players in the community owning storage. Therefore, a market mechanism should be resilient against potential market power to be exercised by storage owners. Note that exercising market power by storage owners is not a concern in the non-cooperative market design with FSRs, as well as cooperative market designs. The reason for this is that in such market design alternatives energy storage systems are operated not by their owners, but by the energy community manager in favor of the whole community.

As a potential area of research, it would be of interest to take into account risk-averse prosumers, and also consider their potential strategic behaviors. In addition, the computational issue of cooperative market designs needs to be tackled by developing a computational scheme for all coalitions based on, e.g., an approximation for some coalitions given results for other coalitions.

Appendix A
Proofs and Solving Approaches

The proofs are based on equivalent forms of Variational Inequality (VI) problems and strict monotonicity of players' preferences [41].

A. *Proof of Proposition 1*

We state problem $\Gamma^{\text{spot}}(\cdot)$ of finding a Nash equilibrium as a VI(F, K) with the game map

$$F(z) = [\nabla_1 J_1(z_1, z_{-1}), \cdots, \nabla_m J^m(z_m, z_{-m})]^\top,$$

where $z = [p_{nt\omega}, p^{\uparrow}_{st\omega}, p^{\downarrow}_{st\omega}, e_{st\omega}, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}, \lambda_{t\omega}]$ denotes the strategy vector. For game $\Gamma^{\text{spot}}(\cdot)$ the strategy set K_i is compact, convex, and non-empty. Moreover, game map $F(z)$ is continuous, since all cost functions $J_{i \in \mathcal{Z}}$ are continuously differentiable. Thus, a solution set SOL(K, F) exists.

To show the singleton nature of the solution set SOL(K, F) we consider the Jacobian matrix of $F(z)$ as

$$\nabla z F(z)=\begin{pmatrix} \beta & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & \cdots & -1 & 1 & 0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\0\\0\\0\end{matrix}}\right\}\text{(1a)} \\ \\ \left.\vphantom{\begin{matrix}0\\0\\0\end{matrix}}\right\}\text{(2a)} \end{matrix}$$

The above Jacobian matrix is symmetric, highlighted by the red diagonal entries, meaning that the corresponding game is *integrable* [33]. This implies that an equivalent optimization problem solving the VI(F, K) exists, whose objective function is given by

$$\theta(z) = \int_0^1 F(z^0 + t(z - z^0))^\top (z - z^0) dt$$
$$= \sum_{t \in \mathcal{T}} \big(C_t p^{\text{i}}_{t\omega} - R_t p^{\text{e}}_{t\omega} + \sum_{n \in \mathcal{N}} \frac{1}{2} \beta p^2_{nt\omega}\big). \quad (13)$$

The non-cooperative Nash game $\Gamma^{\text{spot}}(\cdot)$ can be solved as

$$\{\underset{\Phi^{\text{spot}}}{\text{Min}}\ (13) \quad (14a)$$
$$\text{subject to}$$
$$(1c)\text{–}(1h),\ \forall n, t, \quad (14b)$$
$$(2b)\text{–}(2c),\ \forall t, \quad (14c)$$
$$(7e),\ \forall t\ \},\ \forall \omega, \quad (14d)$$

where $\Phi^{\text{spot}} = \{p_{nt\omega}, p^{\uparrow}_{st\omega}, p^{\downarrow}_{st\omega}, e_{st\omega}, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}\}$. We derive the cost incurred by prosumers and the energy community manager by evaluating objective functions (1a) and (2a) at the solution of (14). Note that the objective function (13) is quadratic and convex. This confirms that a unique Nash equilibrium point for the Nash game $\Gamma^{\text{spot}}(\cdot)$ exists. ∎

B. Proof of Proposition 2

Following Appendix A-A we state $\Gamma^{\text{PSR}}(\cdot)$ as VI(F, K) with

$$F(z) = [\nabla_1 J_1(z_1, z_{-1}), \cdots, \nabla_m J^m(z_m, z_{-m})]^\top,$$

where z=$[x^{\text{P}\uparrow}_s, x^{\text{F}\downarrow}_s, x^{\text{Pe}}_s, q^{\text{P}\uparrow}_{ns}, q^{\text{P}\downarrow}_{ns}, q^{\text{Pe}}_{ns}, p_{nt\omega}, p^{\uparrow}_{nst\omega}, p^{\downarrow}_{nst\omega}, e_{nst\omega}, \mu^{\text{P}\uparrow}_s, \mu^{\text{P}\downarrow}_s, \mu^{\text{Pe}}_s, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}, \lambda_{t\omega}]$ denotes the strategy vector. The corresponding Jacobian matrix writes as

$$\nabla z F(z)=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \pi_\omega\beta & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \pi_\omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -\pi_\omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \pi_\omega \\
0 & 0 & 0 & 0 & 0 & 0 & \pi_\omega & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -\pi_\omega & \pi_\omega & 0
\end{pmatrix} \begin{matrix} \text{(3a)} \\ \\ \text{(4a)} \end{matrix}$$

The above Jacobian matrix is symmetric. This shows there exists an equivalent optimization problem, whose objective function is

$$\theta(z) = \int_0^1 F(z^0 + t(z - z^0))^\top (z - z^0) dt$$
$$= \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}} \frac{1}{2} \beta ({q^{\text{P}\uparrow}_{ns}}^2 + {q^{\text{P}\downarrow}_{ns}}^2 + {q^{\text{Pe}}_{ns}}^2)$$
$$+ \sum_{t \in \mathcal{T}} \mathbb{E}_\omega \big[(C_t p^{\text{i}}_{t\omega} - R_t p^{\text{e}}_{t\omega} + \sum_{n \in \mathcal{N}} \frac{1}{2} \beta p^2_{nt\omega}). \quad (15)$$

Accordingly, the non-cooperative Nash game $\Gamma^{\text{PSR}}(\cdot)$ can be solved using an equivalent optimization as

$$\underset{\Phi^{\text{PSR}}}{\text{Min}}\ (15) \quad (16a)$$
$$\text{subject to}$$
$$(3b)\text{–}(3l),\ \forall n, t, \omega \quad (16b)$$
$$(4b),\ \forall t, \omega \quad (16c)$$
$$(7e),\ \forall t, \omega, \quad (16d)$$

where $\Phi^{\text{PSR}} = \{x^{\text{P}\uparrow}_s, x^{\text{P}\downarrow}_s, x^{\text{Pe}}_s, q^{\text{P}\uparrow}_{ns}, q^{\text{P}\downarrow}_{ns}, q^{\text{Pe}}_{ns}, p_{nt\omega}, p^{\uparrow}_{nst\omega}, p^{\downarrow}_{nst\omega}, e_{nst\omega}, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}\}$. We derive the cost incurred by prosumers and the energy community manager by evaluating objective functions (3a) and (4a) at the solution of (16). Again, since the objective function (15) is quadratic and convex, a unique Nash equilibrium point exists. ∎

C. Proof of Proposition 3

Finally, we state problem $\Gamma^{\text{FSR}}(\cdot)$ of finding a Nash equilibrium as a VI(F, K) with the game map

$$F(z) = [\nabla_1 J_1(z_1, z_{-1}), \cdots, \nabla_m J^m(z_m, z_{-m})]^\top,$$

where $z = [x^{\text{F}\uparrow}_s, x^{\text{F}\downarrow}_s, x^{\text{Fe}}_s, q^{\text{F}\uparrow}_{ns}, q^{\text{F}\downarrow}_{ns}, q^{\text{Fe}}_{ns}, p_{nt\omega}, \mu^{\text{F}\uparrow}_s, \mu^{\text{F}\downarrow}_s, \mu^{\text{Fe}}_s, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}, \lambda_{t\omega}, p^{\uparrow}_{st\omega}, p^{\downarrow}_{st\omega}, e_{st\omega}]^\top$ denotes the strategy vector. We consider the Jacobian matrix of $F(z)$ as

$$\nabla z F(z)=\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \beta & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \pi_\omega\beta & \cdots & 0 & 0 & 0 & 0 & 0 & \pi_\omega & 0 & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -\pi_\omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \pi_\omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \pi_\omega & \cdots & 0 & 0 & 0 & -\pi_\omega & \pi_\omega & 0 & \pi_\omega & -\pi_\omega & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \pi_\omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -\pi_\omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \begin{matrix} \text{(5a)} \\ \\ \text{(6a)} \end{matrix}$$

Since the above Jacobian matrix is symmetric, an equivalent optimization problem solving the VI(F, K) exists, whose objective function is

$$\theta(z) = \int_0^1 F(z^0 + t(z - z^0))^\top (z - z^0) dt$$
$$= \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}} \frac{1}{2} \beta ({q^{\text{F}\uparrow}_{ns}}^2 + {q^{\text{F}\downarrow}_{ns}}^2 + {q^{\text{Fe}}_{ns}}^2) + \sum_{t \in \mathcal{T}} \mathbb{E}_\omega \big[C_t p^{\text{i}}_{t\omega} - R_t p^{\text{e}}_{t\omega}$$
$$+ \sum_{n \in \mathcal{N}} \big(\frac{1}{2} \beta p^2_{nt\omega} - \sum_{s \in \mathcal{S}} (\overline{\gamma}^{\uparrow}_{st\omega} q^{\text{F}\uparrow}_{sn} + \overline{\gamma}^{\downarrow}_{st\omega} q^{\text{F}\downarrow}_{ns} + \overline{\gamma}^{\text{e}}_{st\omega} q^{\text{Fe}}_{ns})\big) \big] \quad (17)$$

The objective function (17) comprises bi-linear terms as products of FSRs awarded by prosumers and FSR values realized from the storage operation. However, we can decompose (17) and solve the non-cooperative Nash game $\Gamma^{\text{FSR}}(\cdot)$ by two sequential optimization problems. The first optimization problem (18) minimizes the total system cost, and determines energy exchanges and the optimal dispatch of all devices for each spot market scenario ω as

$$\underset{\Phi^{\text{FSR}_1}}{\text{Min}} \sum_{t\in\mathcal{T}} \mathbb{E}_\omega\big[C_t p^{\text{i}}_{t\omega} - R_t p^{\text{e}}_{t\omega} + \sum_{n\in\mathcal{N}} \frac{1}{2}\beta p^2_{nt\omega}\big] \quad (18a)$$

subject to

$$(2b)\text{–}(2c),\ \forall t,\omega, \quad (18b)$$

$$(5f)\text{–}(5g),\ \forall n,t,\omega, \quad (18c)$$

$$(6b),\ \forall s,t,\omega, \quad (18d)$$

$$\sum_{n\in\mathcal{N}} p_{nt\omega} - p^{\text{i}}_{t\omega} + p^{\text{e}}_{t\omega} + \sum_{s\in\mathcal{S}} (p^{\uparrow}_{st\omega} - p^{\downarrow}_{st\omega}) = 0 : \lambda_{t\omega}, \forall t,\omega, \quad (18e)$$

where the set of decision variables includes $\Phi^{\text{FSR}_1} = \{p_{nt\omega}, p^{\text{i}}_{t\omega}, p^{\text{e}}_{t\omega}, p^{\uparrow}_{t\omega}, p^{\downarrow}_{t\omega}, e_{t\omega}\}$.

Given optimal FSR values $(\overline{\gamma}^{\uparrow\star}_{st\omega}, \overline{\gamma}^{\downarrow\star}_{st\omega}, \overline{\gamma}^{\text{e}\star}_{st\omega})$, we derive the FSR trading decisions. The second optimization problem (19) minimizes the expected negative revenue from awarding FSR as

$$\underset{\Phi^{\text{FSR}_2}}{\text{Min}} \sum_{n\in\mathcal{N}} \sum_{s\in\mathcal{S}} \Big(\frac{1}{2}\beta({q^{\text{F}\uparrow}_{ns}}^2 + {q^{\text{F}\downarrow}_{ns}}^2 + {q^{\text{Fe}}_{ns}}^2) - \sum_{t\in\mathcal{T}} \mathbb{E}_\omega\big[\overline{\gamma}^{\uparrow\star}_{st\omega} q^{\text{F}\uparrow}_{ns} + \overline{\gamma}^{\downarrow\star}_{st\omega} q^{\text{F}\downarrow}_{ns} + \overline{\gamma}^{\text{e}\star}_{st\omega} q^{\text{Fe}}_{ns}\big]\Big) \quad (19a)$$

subject to

$$(5b)\text{–}(5d),\ \forall s \in \Phi_n, \quad (19b)$$

$$(5e),\ \forall n, s, \quad (19c)$$

$$x^{\text{F}\uparrow}_s + \sum_{n\in\mathcal{N}} q^{\text{F}\uparrow}_{ns} = 0 : \mu^{\text{F}\uparrow}_s,\ \forall s, \quad (19d)$$

$$x^{\text{F}\downarrow}_s + \sum_{n\in\mathcal{N}} q^{\text{F}\downarrow}_{ns} = 0 : \mu^{\text{F}\downarrow}_s,\ \forall s, \quad (19e)$$

$$x^{\text{Fe}}_s + \sum_{n\in\mathcal{N}} q^{\text{Fe}}_{ns} = 0 : \mu^{\text{Fe}}_s,\ \forall s, \quad (19f)$$

where the set of decision variables is $\Phi^{\text{FSR}_2} = \{x^{\text{F}\uparrow}_s, x^{\text{F}\downarrow}_s, x^{\text{Fe}}_s, q^{\text{F}\uparrow}_{ns}, q^{\text{F}\downarrow}_{ns}, q^{\text{Fe}}_{ns}\}$.

Given FSR $(x^{\text{F}\uparrow}_s, x^{\text{F}\downarrow}_s, x^{\text{Fe}}_s, q^{\text{F}\uparrow}_{ns}, q^{\text{F}\downarrow}_{ns}, q^{\text{Fe}}_{ns})$ and spot market $p_{nt\omega}$ trading decisions, the local forward $(\mu^{\text{F}\uparrow}_s, \mu^{\text{F}\downarrow}_s, \mu^{\text{Fe}}_s)$ and spot $\lambda_{t\omega}$ market-clearing prices as well as FSR values $(\overline{\gamma}^{\uparrow}_{st\omega}, \overline{\gamma}^{\downarrow}_{st\omega}, \overline{\gamma}^{\text{e}}_{st\omega})$, we derive each prosumer's and the energy community manager's cost by evaluating objective functions (5a) and (6a) at these values. Note that objective functions (18a) and (19a) are strongly convex. Thus, a unique optimal solution exists, implying that there is a unique Nash equilibrium solution to the Nash game $\Gamma^{\text{FSR}}(\cdot)$. ■

Appendix B
Calculating the Nucleolus

According to [20] and [46], we solve an optimization problem to determine the nucleolus as

$$\epsilon_1 = \min_{\epsilon,\pi_n} \epsilon \quad (20a)$$

subject to

$$\sum_{n\in\mathcal{N}} \pi_n - v(\mathcal{N}) = 0, \quad (20b)$$

$$0 \le \pi_n,\ \forall n, \quad (20c)$$

$$v(\mathcal{S}) - \sum_{n\in\mathcal{N}} \pi_n \le \epsilon,\ \forall \mathcal{S} \notin \{\mathcal{N}, \emptyset\}. \quad (20d)$$

Constraint (20b) ensures the efficient allocation, while (20d) ensures that the excess of any coalition is lower than or equal to the maximum excess ϵ_1. Based on (20) we define Θ as the set of coalitions for which (20d) is binding.

For $l > 1$ we determine the maximum excess over all coalitions that are not binding in the previous iteration as

$$\epsilon_l = \min_{\epsilon,\pi_n} \epsilon \quad (21a)$$

subject to

$$(20b)\text{–}(20c), \quad (21b)$$

$$v(\mathcal{S}) - \sum_{n\in\mathcal{N}} \pi_n \le \epsilon_k,\ \forall \mathcal{S} \in \Theta_k, \forall k \in [1, l-1], \quad (21c)$$

$$v(\mathcal{S}) - \sum_{n\in\mathcal{N}} \pi_n \le \epsilon,\ \forall \mathcal{S} \notin \{\Theta_k, \mathcal{N}, \emptyset\}, \forall k \in [1, l-1]. \quad (21d)$$

Constraints in (21b) ensure the efficient allocation. Constraint (21c) ensures that excesses binding in the previous iteration are binding, while (21d) sets the maximum excess ϵ for all remaining coalitions.

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Risk Trading in Energy Communities

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***Abstract*—Local energy communities are proposed as a regulatory framework to enable the market participation of end-consumers. However, volatile local market-clearing prices, and consequently, volatile cost give rise to local market participants with generally heterogeneous risk attitudes. To prevent the increased operational cost of communities due to conservative trading decisions in the forward stage, e.g., a day-ahead market, we propose risk trading in energy communities via financial hedging products, the so-called Arrow-Debreu securities. The conditional value-at-risk serves as our risk measure for players to study different degrees of market completeness for risk. We define a risk-averse Nash game with risk trading and solve the Nash equilibrium problem for an incomplete market for risk as a mixed complementarity problem. We show that such a Nash equilibrium problem reduces to a single optimization problem if the market is complete for risk. Numerical findings indicate that a significant community cost saving can be realized when players engage in risk trading and sufficient financial hedging products are available. Moreover, risk trading efficiently protects less risk-averse players from highly risk-averse decision-making of rival players.**

***Index Terms*—Arrow-Debreu security, conditional value-at-risk, energy community, market completeness for risk, mixed complementarity problem, risk trading, two-stage stochastic Nash equilibrium problem**

I. Introduction

A. Motivation and Aim

The development of distributed energy resources and the progress in information and communication technologies empower end-consumers to engage in energy trading. Local energy communities are proposed as a regulatory framework that allows the market participation of these proactive consumers, the so-called prosumers [1]–[3]. Energy trading within a local energy community enables its members to efficiently utilize distributed energy resources, such as roof-top photovoltaic (PV) systems, battery storage units, and thermal-electric appliances. In this way, community members are expected to reduce their energy cost [4].

However, the uncertainty inherent to the intermittent injection of small-scale renewable energy sources such as PV systems causes increased price variability within the energy community. As a result, the volatility of energy cost for local energy market participants, the so-called *players*, increases.

Generally, energy markets are organized in a temporally sequential manner to ensure a cost-efficient matching of supply and demand given the physical requirements of the technical system, such as the instantaneous balance of electricity production and consumption. In this context, players make decisions in a *forward* market, e.g., a day-ahead market, before the realization of an uncertain event. The potential realizations of the uncertainty in a *spot* market, e.g., a real-time market, induce volatile prices, i.e., the spot market price might be comparatively higher or lower than the forward market price depending on the realization of the uncertain event. This price volatility leads to a cost volatility for players, giving rise to *risk-averse* preferences.[1] At the stage before uncertainty realization, i.e., the forward market, a risk-averse player tends to make conservative decisions such that the volatility of her overall cost is reduced in both markets. This risk-averse decision-making comes with the cost of an increased disbenefit, i.e., a higher cost or a lower profit for individual players as well as a higher operational cost for the whole energy community.

Under the common assumption of *homogeneous* risk aversion [6], all players have identical risk attitudes. However, in practice we rather observe *heterogeneous* risk aversion, where individual players have different risk preferences, such that some players are willing to accept a greater cost volatility induced by an uncertain event, e.g., the power generation of renewable energy sources, and thereby, a greater risk than others. Consequently, if less risk-averse players could take over the cost volatility of highly risk-averse players, these highly risk-averse players are able to make less conservative forward market decisions while ensuring a low cost volatility. As a result, the total cost of the energy community as well as disbenefits of risk-averse players resulting from conservative forward market decisions decrease.

To this purpose, we propose *risk trading* within local energy communities as a vehicle for the cost volatility transfer. Risk trading describes the exchange of financial hedging products in a market to reduce the cost volatility by a different mean than conservative forward market decisions. However, this transfer depends highly on the availability of financial hedging products. In this context, local energy markets might range from a *fully incomplete market for risk*, where no financial hedging products are available, to a *complete market for risk* [7], in which *all* potential realizations of the uncertainty can be hedged. Both degrees of market completeness for risk constitute extremes, while intermediate cases are those in which risk trading is possible only for a part of potential

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[1] Risk aversion is a natural attitude of decision-makers, which can be observed not only in long-term planning decision-making problems but also in short-term operational ones. However, since operational decisions are made more often, e.g., on an hourly basis, decision-makers are able to adjust their risk preferences over time. They can even learn how to play in dynamic games with risky payoffs [5].

realizations of the uncertainty.

The concept of risk trading gives rise to two important market-driven questions: First, how to define and implement a financial hedging product for energy communities to transfer the cost volatility from highly risk-averse to less risk-averse players? Second, does a potentially unique market equilibrium under heterogeneous risk aversion and risk trading exist?

To answer these questions, we consider a two-stage stochastic decision-making process for heterogeneously risk-averse energy community members, who engage in risk trading. The first stage corresponds to trading decisions in a local forward market for energy and financial hedging products. The second stage represents trading decisions in a local spot market, where the uncertain PV power generation is characterized through a set of discrete scenarios. We apply the well-known Conditional Value-at-Risk (CVaR) [8] as the risk measure for players, and then define a risk-averse non-cooperative Nash game with risk trading based on the collection of all players' decision-making problems. Next, we solve the resulting two-stage stochastic Nash equilibrium problem as a mixed complementarity problem [9]. However, for the special case of a complete market for risk, we solve a single two-stage stochastic optimization problem, which is *equivalent* to the Nash equilibrium problem. We eventually evaluate different degrees of market completeness for risk in terms of individual costs and total community cost.

B. Literature Review

Neglecting risk aversion causes a systematic overestimation of benefits as shown in [10], where heterogeneously risk-averse players are considered for a generation expansion planning problem. Reference [11] discusses the effect of different risk preferences on market-clearing outcomes. Risk-averse stochastic producers avoid the forward market, which causes the system cost and market-clearing prices to increase. However, if risk trading among players is available, the cost volatility can be shifted through risk trading to less risk-averse players as discussed in [12]. Thereby, even highly risk-averse players make moderate forward market decisions, since the volatile cost is hedged. Thus, market-clearing prices and the system cost decrease. Reference [13] even highlights the potential benefits of risk trading in the case of ill-designed electricity markets.

From a methodological perspective, [14] and [15] link heterogeneous risk aversion and complete as well as incomplete markets for risk by stating the problem as a risk-averse Nash equilibrium problem with risk trading. If the market is incomplete for risk, multiple equilibria may exist [16], which are differently stable and may have different system cost as discussed in [17]. However, if the market is complete for risk, [14], [15], and [18] show how a risk-averse social planner solution can be interpreted as a perfectly competitive risk-averse Nash equilibrium, where sufficient hedging products are available.

The principles for market clearing and risk trading in an energy community are not different than those in other electricity markets, e.g., the transmission-level wholesale markets. However, since the number of players in local energy markets is quite limited, their heterogeneous risk preferences potentially have a great impact on market outcomes. In this context of heterogeneous risk aversion in energy communities the recent work [19] addresses the impact of risk trading on the notion of fairness among community members. Moreover, [20] studies heterogeneous risk-averse prosumers, where risk trading is based on a distributed implementation of financial products. A risk-averse equilibrium is computed given a generalized potential game structure.

However, a great challenge remains in defining an applicable financial product that enables risk trading. Here, [21] studies a local energy market on the distribution system level, i.e., a market organization layer above energy communities, where financial hedging rights are proposed to reduce the price volatility due to distribution network constraints. Moreover, [22] considers the simultaneous trading of energy and the uncertain part of power generation by PV systems. Note that financial products proposed by [21] and [22] leave the market partially incomplete for risk [23].

C. Contributions and Paper Organization

To the best of our knowledge, there are only two works in the existing literature that account for heterogeneously risk-averse players in the context of local energy communities. While [19] focuses on the notion of fairness, our work places strong emphasis on game-theoretical analyses and properties. In addition, [20] studies the risk-averse equilibrium based on a generalized Nash equilibrium problem, while we provide a formulation which yields a Nash equilibrium problem, and therefore rigorous conclusions on the solution existence and uniqueness. Moreover, [19] and [20] neglect a thorough analysis of different degrees of market completeness for risk.

Methodologically, we move beyond [14] and [15] and mathematically prove that no equivalent optimization problem necessarily exists in the case that the market is incomplete for risk, while the degree of incompleteness—either fully or partially—is arbitrary. Moreover, in contrast to [14], [15], and [18], we show that although for the complete case an equivalent optimization problem exists, multiple Nash equilibria might still be found.

From the application perspective, we offer a thorough study on local energy communities with a focus on risk-averse prosumers, who engage in risk trading. Based on our numerical results, we identify that prosumers with a deterministic demand avoid the uncertain spot market, while prosumers with stochastic generation tend to postpone trading decisions to the spot market, where the realization of the uncertain power generation is observed. Moreover, by risk trading a significant community cost saving can be realized, while all players yield reduced disbenefits.

The remainder of this paper is organized as follows. In Section II we introduce the structure of the energy community and describe methods for representing risk aversion and risk trading. We present risk-neutral optimization problems and start by defining a risk-neutral Nash game in Section III. In Section IV we extend risk-neutral optimization problems by adding the CVaR as a risk measure for players as well as risk trading among energy community members and define the risk-averse Nash game with risk trading. In Section V we present and discuss numerical results. We conclude our work in Section VI. Mathematical proofs of propositions are provided in Appendix A. Appendix B includes the resulting mixed complementarity problem. Lastly, Appendix C provides the formulation for a risk-averse social planner optimization problem whose solution is equivalent to a risk-averse Nash game in the case that the market is complete for risk.

II. Preliminaries

A. Energy Communities

We understand a local energy community as an aggregation of *a few* prosumers, which are spatially located very close to each other and physically connected to the distribution system as a single entity. For example, energy communities were recently demonstrated through “The Energy Collective” project [24] implemented in Bofællesskabet Svalin, Denmark, the “EnergyLab Nordhavn” project [25] implemented in the Nordhavn neighborhood of Copenhagen, Denmark, as well as the “pebbles” project [26] implemented in Wildpoldsried, Germany. These projects enable prosumers to directly engage in energy trading with their neighbors via a local energy market within the energy community. This is one key aspect that the European Commission asks its member states for by the renewable energy directive, Article 21 [27].

Local markets for energy communities should be differentiated from any other market schemes on the distribution system level that may geographically cover a whole feeder or even a suburb. Distribution-level markets might be designed for trading energy [28] or flexibility [29], taking into account power losses and technical constraints such as limits on nodal voltage magnitudes and apparent power flow of lines. We treat local markets for energy communities as one market-organization layer below any distribution-level market, and rather see the energy community as a whole as a potential market participant in such distribution-level markets. However, we leave the explicit consideration for future research.

B. Definitions

In this study we consider a local forward as well as a local spot market, and represent the probability distribution of an uncertain realization in the spot market by a finite number of discrete scenarios $\omega \in \Omega$. Each scenario embodies a collection of the PV power generation for all prosumers under that scenario. Prior to the uncertainty realization, the local forward market determines the energy production and consumption schedule for each community member for, e.g., the next day, as well as a local forward market-clearing price. In real time when the uncertainty is realized, any deviations from the local forward market schedule are balanced in the local spot market, providing a local spot market-clearing price.

In the local forward market, risk-neutral players make decisions based on *physical probabilities* π_ω^Θ of uncertain realizations, i.e., empirical real-world observations [15]. Risk-averse players, however, observe physical probabilities and

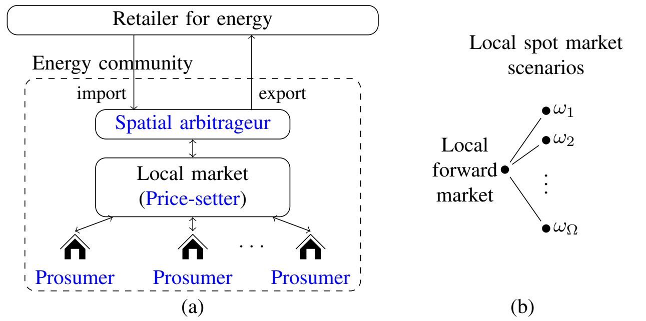


Fig. 1: Plot 1a shows the structure of the energy community, where explicitly considered players are marked in blue. Plot 1b depicts the two-stage stochastic decision-making process of energy community members.

increase/decrease the weight of some scenarios to derive *risk-adjusted probabilities* π_ω^ρ [10], which describe their individual risk preferences on uncertain realizations. In fact, a risk-averse player treats risk-adjusted probabilities as her individual decision variables, such that she considers comparatively higher weights for scenarios under which she incurs a high cost, or achieves a low profit. Similarly, she considers comparatively lower weights for scenarios, resulting in a low cost, or a high profit. Note that the sum of risk-adjusted probabilities should still be equal to one. By decision-making based on resulting risk-adjusted probabilities, the risk-averse player reduces her cost volatility at the cost of increased disbenefits.

To outweigh conservative forward market decisions and to reach a consensus among energy community members on risk preferences, we consider *risk trading* via a financial market product, the so-called *Arrow-Debreu security* [30].[2] For each scenario $\omega \in \Omega$, an Arrow-Debreu security is an unconstrained contract between a security buyer and a seller in the forward market. Both the buyer and seller are local energy community members. The buyer pays to the seller in the forward market based on a market-driven price, the so-called *risk price* μ_ω, whose value is lower than or equal to 1. In return, the buyer receives from the seller a payment of 1 in the spot market *if* scenario ω realizes. For example, if a prosumer buys 100 securities from another prosumer for a given scenario ω, she should pay €$100\mu_\omega$ but will be paid back €100 if that scenario occurs. We will show later in Section V how risk-adjusted probabilities converge towards a consensus on risk preferences if risk trading is possible.

C. Local Market Structure

Within the energy community we explicitly consider three types of players as illustrated in Figure 1a, namely prosumers, a spatial arbitrageur and a price setter. Moreover, Figure 1b schematically depicts the underlying two-stage stochastic decision-making process of each player.

Prosumers, who potentially have an inelastic demand and own PV systems with an uncertain power generation, buy and sell energy as well as Arrow-Debreu securities in the

[2]While the definition of Arrow-Debreu securities in [30] is based on a discretization of an uncertain event, recent efforts have been made to consider risk trading for a continuous probability distribution of an uncertain event [31].

local forward market. At the same time, they anticipate all possible local spot market realizations $\omega \in \Omega$, i.e., the spot market-clearing price and the uncertain PV power generation in scenario ω, as well as their respective response in terms of energy trades and the cost/revenue from buying/selling Arrow-Debreu securities for each scenario ω.

In the case of a shortage or a surplus of energy the spatial arbitrageur imports and exports energy from and to the energy community. Since an energy community supplies and demands fairly small energy volumes, a direct access for the spatial arbitrageur to the wholesale market is rather unlikely. Therefore, we consider a retailer[3], who serves as an intermediary for the energy community. The spatial arbitrageur decides on energy imports and exports as well as her Arrow-Debreu security trades in the local forward market. She also takes into account all possible realizations of local spot market-clearing prices as well as her optimal recourse in terms of energy imports and exports and the cost/revenue from Arrow-Debreu securities for each scenario ω.

Finally, a price setter, who is a fictitious player [32], reveals local market-clearing prices evolving under free trade and perfect competition. In detail, the price setter reveals a local forward market-clearing price for energy and prices for Arrow-Debreu securities given trading decisions by prosumers and energy imports/exports by the spatial arbitrageur. At the same time, she anticipates the recourse by prosumers due to the uncertain PV power generation as well as the optimal response by the spatial arbitrageur and reveals a spot market-clearing price for each scenario $\omega \in \Omega$.[4]

D. Towards a Complete Market for Risk

We study four different cases as summarized in Table I. In a case wherein all players are risk-neutral, a Nash equilibrium is determined by an optimization problem. The fully incomplete case constitutes a setting where prosumers and the spatial arbitrageur are heterogeneously risk-averse and no financial hedging products are available. We relax the constraint on the availability of hedging products and consider a partially incomplete market for risk, where some realizations of the uncertain PV power generation can be hedged. For these two cases of an incomplete market for risk, we rely on solving the Nash equilibrium problem as a mixed complementarity problem along with its challenges [16], [17], such as potential multiplicity, instability, and computational burden. However, if the market is complete for risk, i.e., all realizations of uncertain PV power generation can be hedged, the Nash equilibrium problem reduces to an equivalent optimization problem [18]. According to [15], expected disbenefits decrease as we move from a fully incomplete to a complete market for risk, as illustrated in the last column of Table I.

[3]A retailer is a self-interested profit-seeking entity which buys great energy volumes at the wholesale market and sells energy to many small customers.

[4]In a perfectly competitive market environment the spatial arbitrageur and the price setter can be interpreted as a community manager, who simultaneously fulfills both tasks, ensuring liquidity and revealing prices [9].

TABLE I: Problem overview with respect to the degree of risk trading.

Risk attitude	Risk trading	Problem type	Expected disbenefits
Neutral	-	Optimization*	⋆
Averse	Fully incomplete	Equilibrium+	⋆⋆⋆⋆
Averse	Partially incomplete	Equilibrium+	⋆⋆⋆
Averse	Complete	Optimization*	⋆⋆

*Equivalent Nash equilibrium problem exists.
+Equivalent optimization problem does not necessarily exist.

E. Overview of the Conditional Value-at-Risk

The seminal work [33] defines coherent risk measures $\rho(\cdot)$ of an uncertain disbenefit function Z_ω. We will apply the dual representation of a coherent risk measure defined by [34] as

$$\rho(Z_\omega) = \max_{\pi^\rho_\omega \in \mathcal{D}} \sum_{\omega \in \Omega} \pi^\rho_\omega Z_\omega, \tag{1a}$$

where the risk set $\mathcal{D}$ defines the feasible region for risk-adjusted probabilities π^ρ_ω. For the CVaR as a coherent risk measure in particular, the risk set $\mathcal{D}^{\text{CVaR}}$ [10] is given by

$$\mathcal{D}^{\text{CVaR}} = \big\{\pi^\rho_\omega : \sum_{\omega \in \Omega} \pi^\rho_\omega = 1,\ 0 \le \pi^\rho_\omega \le \frac{1}{\alpha}\pi^\Theta_\omega\big\}, \tag{1b}$$

where the equality constraint ensures that the sum of risk-adjusted probabilities is still equal to one. The lower bound of the inequality constraint ensures that all risk-adjusted probabilities π^ρ_ω are non-negative. The upper bound allows increasing the weight of some scenarios according to physical probabilities π^Θ_ω and the risk aversion parameter $\alpha \in (0, 1]$ indicating the percentile of the CVaR measure. For the special case $\alpha = 1$, risk-adjusted probabilities are equal to physical probabilities, which represents a risk-neutral attitude.

In fact, (1a) with the risk set (1b) states a constrained optimization problem that determines risk-adjusted probabilities according to the risk aversion expressed by the CVaR. This optimization problem is an equivalent representation of the CVaR metric expressed by the well-known linear programming problem [8] as

$$\rho^{\text{CVaR}}(Z_\omega) = \min_\zeta \big\{\zeta + \frac{1}{\alpha} \sum_{\omega \in \Omega} \pi^\Theta_\omega \big(Z_\omega - \zeta\big)^+\big\}, \tag{1c}$$

which states the weighted mean deviation from the α^{th} quantile, where ζ denotes the value-at-risk. However, implementing the CVaR by (1c) causes the objective function to be non-smooth, i.e., non-continuously differentiable, owing to the positively defined term in $(\cdot)^+$, and thus, limits our possibilities for deriving optimality conditions to draw conclusions on the uniqueness of the equilibrium point in game-theoretic models.

Remark 1. In Section IV we combine the dual representation of a coherent risk measure (1a) with the risk set of the CVaR (1b) when introducing the generic framework of a risk-averse Nash equilibrium problem with risk trading. This allows us to use a framework that relies on analyzing the resulting game-theoretic models in their equivalent forms of Variational Inequality (VI) problems [35]. This VI representation enables us to draw conclusions on the existence and uniqueness of a game solution. For the case of a complete market for risk, we apply the CVaR metric (1c) in the equivalent optimization problem of the Nash equilibrium problem. This allows us to efficiently solve the equivalent optimization problem and derive numerical values of trades and prices.

F. Assumptions

We assume that all players are price takers, resulting in a perfectly competitive local energy market. The PV power generation is the only source of uncertainty, though other sources of uncertainty can be incorporated in the same manner. Owing to the low spatial distance, we assume that all PV systems have the same power generation profile and players possess the same information on scenarios as well as identical beliefs on physical probabilities π^{Θ}_{ω}.[5] Furthermore, owing to the size of the energy community and the fact that spatial distances and trade volumes are relatively small, we neglect network constraints *within* the energy community. However, network constraints *between* the energy community and the distribution system, which are observed by the spatial arbitrageur are of interest and are modeled in this work. Results change in a way that a network congestion and consequently limited energy imports or exports affect local forward and spot market-clearing prices. Moreover, we consider the local market clearing for a single hour only, since we do not consider any technology with time-coupling constraints, e.g., energy storage units. A problem extension to consider multiple time steps is mathematically straightforward, though it complicates the solution interpretation on how risk trading affects local energy market outcomes. Lastly, we consider two trading floors, e.g., day-ahead and real-time, only, and exclude additional floors, such as intraday markets. The possibility of peer-to-peer trading among prosumers within the community is also discarded.

In the next two sections we introduce the mathematical formulation of each player's problem. We start by describing a risk-neutral energy community in Section III. In Section IV, we extend the risk-neutral problem formulation to a setting with risk-averse players and risk trading.

Notation: We use a tilde, i.e., $\tilde{(\cdot)}$, for those symbols associated with the local spot market. Symbols followed by a colon denote dual variables of respective constraints. We use these dual variables when we derive Karush-Kuhn-Tucker (KKT) conditions of optimization problems in Appendix B.

III. Market Clearing with Risk-Neutral Players

In a risk-neutral setting, each prosumer $n \in \mathcal{P}$ minimizes her expected energy cost J_n of meeting the demand as

$$\Big\{ \underset{p_n, \tilde{p}_{n\omega}}{\text{Min}}\, J_n = \underbrace{\lambda p_n}_{\substack{\text{Forward market}\\ \text{cost}}} + \underbrace{\frac{1}{2}\beta p_n^2}_{\text{Regularizer}} + \sum_{\omega \in \Omega} \pi^{\Theta}_{\omega} \big(\underbrace{\tilde{\lambda}_{\omega} \tilde{p}_{n\omega}}_{\substack{\text{Spot market}\\ \text{cost}}} + \underbrace{\frac{1}{2}\beta \tilde{p}_{n\omega}^2}_{\text{Regularizer}} \big) \quad \text{(2a)}$$

$$\text{s.t. } p_n + \tilde{p}_{n\omega} + \tilde{S}_{n\omega} - D_n = 0 \; : \tilde{\phi}_{n\omega}, \; \forall \omega, \quad \text{(2b)}$$

$$-\overline{P}_n \le p_n \le \overline{P}_n : \underline{\chi}^{\mathrm{p}}_n, \overline{\chi}^{\mathrm{p}}_n, \quad \text{(2c)}$$

$$-\overline{P}_n \le \tilde{p}_{n\omega} \le \overline{P}_n : \underline{\chi}^{\tilde{\mathrm{p}}}_{n\omega}, \overline{\chi}^{\tilde{\mathrm{p}}}_{n\omega}, \; \forall \omega \Big\} \; \forall n. \quad \text{(2d)}$$

[5]A relevant model accounting for asymmetric information about scenarios, but without modeling risk aversion, is available in [36].

The first term in the objective function (2a) states the cost incurred from power trades in the local forward market. Positive values of p_n indicate a demand while negative values state a supply. Each prosumer n pays/is paid based on the local forward market-clearing price λ for her energy exchange p_n. The second term in (2a) states a regularizer [37] for forward market trades, where β is a small positive constant[6], e.g., 10^{-3}. Institutionally, this regularizer can be interpreted as a transaction cost arising from trades. The third and fourth terms of (2a) refer to the expected cost and the regularizer in the local spot market, weighted by physical probabilities π^{Θ}_{ω}. The prosumer n pays/is paid based on the local spot market-clearing price $\tilde{\lambda}_{\omega}$ for her power exchange $\tilde{p}_{n\omega}$ in scenario ω. Note that market-clearing prices λ and $\tilde{\lambda}_{\omega}$ are parameters within (2), while they are variables in the Nash equilibrium problem, i.e., the collection of all prosumers' $n \in \mathcal{P}$, the spatial arbitrageur's, and the price setter's optimization problems, which are solved simultaneously.

For each scenario ω, the prosumer n has to ensure the satisfaction of her individual power balance, enforced by (2b). Her demand D_n has to be met by the power exchange p_n in the forward market, $\tilde{p}_{n\omega}$ in the spot market, and her PV power generation $\tilde{S}_{n\omega}$, which is a scenario-dependent parameter. Finally, (2c) and (2d) restrict power exchanges within the energy community by parameters $\overline{P}_n$. We introduce theoretical bounds $\overline{P}_n$ on power exchanges to achieve a closed and compact decision set for all players [12]. This will be necessary later for proving the existence of the game solution [35]. However, we select sufficiently large values for $\overline{P}_n$, and check the equilibrium solution *a posteriori* to ensure (2c) and (2d) are always inactive.

Furthermore, the spatial arbitrageur $\mathcal{P}^{ar}$ minimizes her expected cost J^{ar} from trading energy between the energy community and a retailer for energy as

$$\underset{p^{\mathrm{i}}, p^{\mathrm{e}}, \tilde{p}^{\mathrm{i}}_{\omega}, \tilde{p}^{\mathrm{e}}_{\omega}}{\text{Min}} \; J^{ar} = \underbrace{(C^{\mathrm{i}} - \lambda) p^{\mathrm{i}}}_{\text{Forward market import cost}} - \underbrace{(C^{\mathrm{e}} - \lambda) p^{\mathrm{e}}}_{\text{Forward market export cost}}$$

$$+ \sum_{\omega \in \Omega} \pi^{\Theta}_{\omega} \big[\underbrace{(\tilde{C}^{\mathrm{i}} - \tilde{\lambda}_{\omega}) \tilde{p}^{\mathrm{i}}_{\omega}}_{\text{Spot market import cost}} - \underbrace{(\tilde{C}^{\mathrm{e}} - \tilde{\lambda}_{\omega}) \tilde{p}^{\mathrm{e}}_{\omega}}_{\text{Spot market export cost}} \big] \quad \text{(3a)}$$

$$\text{s.t. } 0 \le p^{\mathrm{i}} \le \overline{P}^{\mathrm{i}} : \underline{\chi}^{\mathrm{p^i}}, \overline{\chi}^{\mathrm{p^i}}, \quad \text{(3b)}$$

$$0 \le p^{\mathrm{e}} \le \overline{P}^{\mathrm{e}} : \underline{\chi}^{\mathrm{p^e}}, \overline{\chi}^{\mathrm{p^e}}, \quad \text{(3c)}$$

$$0 \le \tilde{p}^{\mathrm{i}}_{\omega} \le \overline{P}^{\mathrm{i}} : \underline{\chi}^{\mathrm{\tilde{p}^i}}_{\omega}, \overline{\chi}^{\mathrm{\tilde{p}^i}}_{\omega}, \; \forall \omega, \quad \text{(3d)}$$

$$0 \le \tilde{p}^{\mathrm{e}}_{\omega} \le \overline{P}^{\mathrm{e}} : \underline{\chi}^{\mathrm{\tilde{p}^e}}_{\omega}, \overline{\chi}^{\mathrm{\tilde{p}^e}}_{\omega}, \; \forall \omega. \quad \text{(3e)}$$

The first term of the objective function (3a) corresponds to the cost in the forward market from importing power p^{i} at the fixed price C^{i} while being paid at the local forward market-clearing price λ. Similarly, the second term represents the cost from exporting energy p^{e} at the forward market-clearing price λ, while receiving the fixed exporting price C^{e}. The risk-

[6]Very small values for β do not alter the total operational cost of the energy community in comparison to $\beta = 0$. However, $\beta = 0$ yields linear objective functions, and may give rise to multiple trading solutions for players [38]. We introduce regularizers to ensure strictly monotone objective functions. This allows us to draw conclusions on the uniqueness of the solution. In fact, a very small value for the regularizer ensures identical cost for players, who have identical risk preferences, production, and consumption profiles.

neutral spatial arbitrageur weights local spot market scenarios according to physical probabilities π_ω^Θ. The third and fourth terms of (3a) resemble the cost from energy arbitrage between the energy community and the retailer under each scenario of the spot market, where $\tilde{C}^{\mathrm{i}}$ and $\tilde{C}^{\mathrm{e}}$ are fixed importing and exporting prices, respectively. Constraints (3b)–(3e) set bounds for importing $\overline{P}^{\mathrm{i}}$ and exporting $\overline{P}^{\mathrm{e}}$ power to and from the energy community.

Finally, for given values of $p_n, p^{\mathrm{i}}, p^{\mathrm{e}}, \tilde{p}_{n\omega}, \tilde{p}_\omega^{\mathrm{i}}$, and $\tilde{p}_\omega^{\mathrm{e}}$, the price-setter $\mathcal{P}^{ps}$ derives forward and spot market-clearing prices, i.e., λ and $\tilde{\lambda}_\omega$, as

$$\underset{\lambda,\tilde{\lambda}_\omega}{\text{Min}}\ J^{ps} = \underbrace{\lambda\big(\sum_{n\in N} p_n - p^{\mathrm{i}} + p^{\mathrm{e}}\big)}_{\text{Buyers' forward market cost / sellers' forward market revenue}} + \sum_{\omega\in\Omega} \pi_\omega^\Theta \big[\underbrace{\tilde{\lambda}_\omega\big(\sum_{n\in N}\tilde{p}_{n\omega} - \tilde{p}_\omega^{\mathrm{i}} + \tilde{p}_\omega^{\mathrm{e}}\big)}_{\text{Buyers' spot market cost / sellers' spot market revenue}}\big] \tag{4a}$$

$$\text{s.t. } -\overline{\Lambda} \le \lambda \le \overline{\Lambda} : \underline{\chi}^\lambda, \overline{\chi}^\lambda, \tag{4b}$$

$$-\overline{\Lambda} \le \tilde{\lambda}_\omega \le \overline{\Lambda} : \underline{\chi}_\omega^{\tilde{\lambda}}, \overline{\chi}_\omega^{\tilde{\lambda}},\ \forall\omega. \tag{4c}$$

The first line of the objective function (4a) minimizes the cost for energy buyers and maximizes the revenue for energy sellers in the local forward market. By sellers, we refer to producers and energy imports. Similarly, by buyers, we refer to consumers and energy exports. The second line minimizes/maximizes the expected cost/revenue in the local spot market, where each scenario is weighted by the physical probability π_ω^Θ. Constraints (4b) and (4c) set the lower and upper bounds on market-clearing prices[7]. Note that in the case that (4b) and (4c) are inactive, the KKT conditions of (4) enforce power balance conditions within the energy community in both forward and spot markets.

Definition 1. Given optimization problems (2)–(4), we define $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$ as the risk-neutral non-cooperative Nash game, where $\mathcal{Z} = (\mathcal{P} \cup \mathcal{P}^{ar} \cup \mathcal{P}^{ps})$ is the set of all players. $K = (K_1 \times \cdots \times K_{\mathcal{P}} \times K^{ar} \times K^{ps})$ denotes the strategy set of the game, where K_i is the strategy set of player $i \in \mathcal{Z}$.

Proposition 1. For the risk-neutral non-cooperative Nash game $\Gamma(\cdot)$ an equivalent optimization problem exists. In addition, the Nash equilibrium solution is unique.

Proof 1. We provide the proof in Appendix A-A. ■

IV. Market Clearing with Risk-Averse Players and Risk Trading

We extend optimization problems of risk-neutral prosumers $n \in \mathcal{P}$ and the risk-neutral spatial arbitrageur $\mathcal{P}^{ar}$ by adding the coherent risk measure function $\rho(\cdot)$ as stated in (1a) over local spot market scenarios. The risk set of each player $\mathcal{D}^{\mathrm{CVaR}}$ is built upon the CVaR measure as defined

[7] Again, we theoretically consider a price floor and a price cap to achieve a compact and closed decision set for the price-setter, and thereby, to mathematically prove the solution existence. In our numerical study, we will consider a very large value for parameter $\overline{\Lambda}$ to ensure bounds are inactive. We refer the interested reader to [39], addressing how an active price cap may cause market inefficiency.

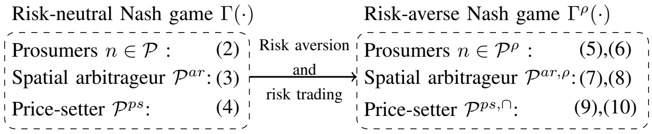


Fig. 2: By adding risk aversion and risk trading to the risk-neutral Nash game, each player simultaneously solves two optimization problems within the risk-averse Nash equilibrium problem with risk trading.

in (1b). Moreover, we introduce risk trading among prosumers and the spatial arbitrageur. Finally, we extend the optimization problem of the price-setter $\mathcal{P}^{ps}$ such that she clears the market and respects prosumers' and the spatial arbitrageur's risk preferences when revealing local market-clearing prices. Figure 2 depicts the affiliation of optimization problems moving from the risk-neutral Nash game $\Gamma(\cdot)$ with one optimization problem per player to the risk-averse Nash game $\Gamma^\rho(\cdot)$ with risk trading, where we consider two optimization problems per player.

We first start with risk-averse prosumers $n \in \mathcal{P}^\rho$. Each prosumer is able to hedge the risk induced by the uncertain PV power generation in scenario ω by trading Arrow-Debreu securities $a_{n\omega}$ in the local forward market. We develop two optimization problems related to each risk-averse prosumer. These two problems are solved simultaneously within the Nash equilibrium problem. The first problem of prosumer n minimizes her risk-adjusted expected cost $J_n^{\rho_1}$ by determining her trading decisions p_n, $\tilde{p}_{n\omega}$, and $a_{n\omega}$ while risk-adjusted probabilities $\pi_{n\omega}^\rho$ are given. The second one exhibits the dual representation (1a) of the CVaR as risk measure function $\rho^{\mathrm{CVaR}}(\cdot)$ and determines risk-adjusted probabilities, while her trading decisions are treated as fixed parameters. The first problem writes as

$$\Big\{\underset{p_n,\tilde{p}_{n\omega},a_{n\omega}}{\text{Min}}\ J_n^{\rho_1} = \lambda p_n + \frac{1}{2}\beta p_n^2 + \underbrace{\sum_{\omega\in\Omega}\mu_\omega a_{n\omega}}_{\text{Forward market hedging cost/revenue}} + \sum_{\omega\in\Omega}\pi_{n\omega}^\rho\big(\tilde{\lambda}_\omega \tilde{p}_{n\omega} + \frac{1}{2}\beta\tilde{p}_{n\omega}^2 - \underbrace{a_{n\omega}}_{\text{Hedging cost/revenue in }\omega}\big) \tag{5a}$$

$$\text{s.t. (2b)–(2d)}\Big\}\ \forall n. \tag{5b}$$

Compared to (2a) in the risk-neutral setting, the objective function (5a) includes risk-adjusted probabilities $\pi_{n\omega}^\rho$ instead of physical ones. It also comprises one additional free variable per scenario, i.e., Arrow-Debreu security $a_{n\omega}$, and two additional cost/revenue components related to such securities. The first line of (5a) includes $\sum_{\omega\in\Omega}\mu_\omega a_{n\omega}$, which refers to the total cost/revenue of prosumer n over scenarios in the local forward market, obtained by trading securities $a_{n\omega}$ at the risk price μ_ω. A positive value for $a_{n\omega}$ implies that prosumer n buys securities in the forward market for scenario ω, and thereby lowers her associated risk under that scenario. In contrast, a negative value for $a_{n\omega}$ means that prosumer n sells securities in the forward market, and therefore is willing to accept a higher risk under scenario ω. Note that similar to λ and $\tilde{\lambda}_\omega$, the risk price μ_ω is a parameter in (5), while it is a variable within the Nash equilibrium problem. Note also that $a_{n\omega}$ and μ_ω are Nash equilibrium variables in the forward stage, though they are indexed by scenario ω. If scenario ω occurs in the local spot market, the seller/buyer of security $a_{n\omega}$ pays/is paid at price 1, as given in the second line of (5a).

The second optimization problem corresponding to each risk-averse prosumer $n \in \mathcal{P}^\rho$ resembles the risk measure function $\rho^{\text{CVaR}}(\cdot)$, takes her scenario-indexed trading decisions $\tilde{p}_{n\omega}$ and $a_{n\omega}$ into account as parameters, and endogenously determines her risk-adjusted probabilities $\pi^\rho_{n\omega}$ as

$$\Big\{\underset{\pi^\rho_{n\omega}}{\text{Min}}\; J^{\rho_2}_n = -\sum_{\omega\in\Omega} \pi^\rho_{n\omega}\big(\tilde{\lambda}_\omega \tilde{p}_{n\omega} + \frac{1}{2}\beta\tilde{p}^2_{n\omega} - a_{n\omega}\big) \quad (6a)$$

$$\text{s.t.}\; \sum_{\omega\in\Omega}\pi^\rho_{n\omega} - 1 = 0 : \phi^\rho_n, \quad (6b)$$

$$0 \le \pi^\rho_{n\omega} \le \frac{1}{\alpha_n}\pi^\Theta_\omega : \underline{\chi}^\rho_{n\omega}, \overline{\chi}^\rho_{n\omega},\; \forall\omega\Big\}\; \forall n, \quad (6c)$$

where the objective function (6a) minimizes the negative expected cost in the local spot market by optimally choosing values for risk-adjusted probabilities $\pi^\rho_{n\omega}$. Note that according to the definition of a coherent risk measure in (1a), the objective function (6a) maximizes the disbenefit for prosumer n in the local spot market by increasing the weight of the worst scenarios. Constraints (6b) and (6c) set the bounds of risk-adjusted probabilities $\pi^\rho_{n\omega}$ according to the risk set $\mathcal{D}^{\text{CVaR}}_n$ of each risk-averse prosumer n.

Similarly, as for risk-averse prosumers we consider two optimization problems for the risk-averse spatial arbitrageur $\mathcal{P}^{ar}$, who is also able to participate in risk trading. In fact, she minimizes her risk-adjusted expected cost J^{ar,ρ_1} as

$$\underset{p^{\text{i}},p^{\text{e}},\tilde{p}^{\text{i}}_\omega,\tilde{p}^{\text{e}}_\omega,b_\omega}{\text{Min}}\; J^{ar,\rho_1} = \big(C^{\text{i}} - \lambda\big)p^{\text{i}} - \big(C^{\text{e}} - \lambda\big)p^{\text{e}} + \underbrace{\sum_{\omega\in\Omega}\mu_\omega b_\omega}_{\text{Forward market hedging cost/revenue}}$$

$$+ \sum_{\omega\in\Omega}\pi^{ar}_\omega\big[\big(\tilde{C}^{\text{i}} - \tilde{\lambda}_\omega\big)\tilde{p}^{\text{i}}_\omega - \big(\tilde{C}^{\text{e}} - \tilde{\lambda}_\omega\big)\tilde{p}^{\text{e}}_\omega - \underbrace{b_\omega}_{\text{Hedging cost/revenue in }\omega}\big] \quad (7a)$$

$$\text{s.t. (3b)–(3e).} \quad (7b)$$

Compared to (3a) in the risk-neutral model, the first line of (7a) includes the total cost/revenue for the spatial arbitrageur from trading Arrow-Debreu securities b_ω at the risk price μ_ω in the local forward market. The second line refers to her expected cost in the local spot market weighted by risk-adjusted probabilities π^{ar}_ω, including the hedging cost/revenue under each scenario ω.

Similar to (6), the second optimization problem of the spatial arbitrageur within the Nash equilibrium problem determines her risk-adjusted probabilities π^{ar}_ω, while treating her scenario-indexed trading decisions $\tilde{p}^{\text{i}}_\omega$, $\tilde{p}^{\text{e}}_\omega$, and b_ω as parameters. This problem writes as

$$\underset{\pi^{ar}_\omega}{\text{Min}}\; J^{ar,\rho_2} = -\sum_{\omega\in\Omega}\pi^{ar}_\omega\big[\big(\tilde{C}^{\text{i}} - \tilde{\lambda}_\omega\big)\tilde{p}^{\text{i}}_\omega - \big(\tilde{C}^{\text{e}} - \tilde{\lambda}_\omega\big)\tilde{p}^{\text{e}}_\omega - b_\omega\big] \quad (8a)$$

$$\text{s.t.}\; \sum_{\omega\in\Omega}\pi^{ar}_\omega - 1 = 0 : \phi^{ar}, \quad (8b)$$

$$0 \le \pi^{ar}_\omega \le \frac{1}{\alpha^{ar}}\pi^\Theta_\omega : \underline{\chi}^{ar}_\omega, \overline{\chi}^{ar}_\omega,\; \forall\omega. \quad (8c)$$

The objective function (8a) minimizes the negative expected local spot market cost J^{ar,ρ_2} by choosing π^{ar}_ω. Constraints (8b) and (8c) ensure bounds for risk-adjusted probabilities π^{ar}_ω according to the risk set $\mathcal{D}^{\text{CVaR},ar}$ of the risk-averse spatial arbitrageur.

Finally, the price-setter $\mathcal{P}^{ps,\cap}$ also considers two optimization problems within the Nash equilibrium problem. In the first optimization problem, she minimizes the risk-adjusted expected cost for energy/security buyers, maximizes the risk-adjusted expected revenue for energy/security sellers, and determines market-clearing prices λ, $\tilde{\lambda}_\omega$ and μ_ω as

$$\underset{\lambda,\tilde{\lambda}_\omega,\mu_\omega}{\text{Min}}\; J^{ps,\cap_1} = \lambda\big(\sum_{n\in N}p_n - p^{\text{i}} + p^{\text{e}}\big) + \sum_{\omega\in\Omega}\mu_\omega\big(\sum_{n\in N}a_{n\omega} + b_\omega\big)$$

$$+ \sum_{\omega\in\Omega}\pi^\cap_\omega\big[\tilde{\lambda}_\omega\big(\sum_{n\in N}\tilde{p}_{n\omega} - \tilde{p}^{\text{i}}_\omega + \tilde{p}^{\text{e}}_\omega\big)\big] \quad (9a)$$

$$\text{s.t. (4b)–(4c),} \quad (9b)$$

$$-\overline{M} \le \mu_\omega \le \overline{M} : \underline{\chi}^\mu_\omega, \overline{\chi}^\mu_\omega,\; \forall\omega. \quad (9c)$$

For given values of Arrow-Debreu securities $a_{n\omega}$ traded by prosumers $n \in \mathcal{P}^\rho$ as well as securities b_ω traded by the spatial arbitrageur for scenarios $\omega \in \Omega$, the price-setter chooses risk prices μ_ω, such that the cost for buyers is minimized and the revenue for sellers is maximized. Moreover, she weights cost in the local spot market according to system-wide risk-adjusted probabilities $\pi^\cap_\omega$, which are considered as parameters in (9). Constraint (9c) imposes theoretical lower and upper bounds for risk prices μ_ω.

In the second optimization problem, the price-setter determines the system-wide risk-adjusted probabilities $\pi^\cap_\omega$ given scenario-indexed trading decisions $\tilde{p}_{n\omega}$, $\tilde{p}^{\text{i}}_\omega$, and $\tilde{p}^{\text{e}}_\omega$, as well as risk sets of prosumers $\mathcal{D}^{\text{CVaR}}_n$ and the spatial arbitrageur $\mathcal{D}^{\text{CVaR},ar}$ as

$$\underset{\pi^\cap_\omega}{\text{Min}}\; J^{ps,\cap_2} = -\sum_{\omega\in\Omega}\pi^\cap_\omega\big[\tilde{\lambda}_\omega\big(\sum_{n\in N}\tilde{p}_{n\omega} - \tilde{p}^{\text{i}}_\omega + \tilde{p}^{\text{e}}_\omega\big)\big] \quad (10a)$$

$$\text{s.t.}\; \sum_{\omega\in\Omega}\pi^\cap_\omega - 1 = 0 : \phi^\cap, \quad (10b)$$

$$0 \le \pi^\cap_\omega \le \frac{1}{\alpha^\cap}\pi^\Theta_\omega : \underline{\chi}^\cap_\omega, \overline{\chi}^\cap_\omega,\; \forall\omega. \quad (10c)$$

Aligned with the definition of a coherent risk measure in (1a), the objective function (10a) maximizes disbenefits. In other words, it minimizes the expected energy revenue of sellers and maximizes the energy cost of buyers in the local spot market, aiming at determining system-wide risk-adjusted probabilities $\pi^\cap_\omega$. Constraints (10b) and (10c) ensure bounds for system-wide risk-adjusted probabilities $\pi^\cap_\omega$, given the risk set $\mathcal{D}^{\text{CVaR},\cap}$. The risk set $\mathcal{D}^{\text{CVaR},\cap}$ is formed based on the intersection of risk sets of prosumers and the spatial arbitrageur. In fact, this risk set corresponds to the risk set of *the least* risk-averse player as described in [14]. This implies that $\alpha^\cap$ in (10c) is equal to $\min\{\alpha_1, \cdots, \alpha_N, \alpha^{ar}\}$.

Definition 2. Given (5)–(10), we define $\Gamma^\rho(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$ as the risk-averse non-cooperative Nash game, where $\mathcal{Z} = (\mathcal{P}^\rho \cup \mathcal{P}^{ar,\rho} \cup \mathcal{P}^{ps,\cap})$ is the set of all players. $K = (K_1 \times \cdots \times K_{\mathcal{P}^{\rho_{1,2}}} \times K^{ar,\rho_{1,2}} \times K^{ps,\cap_{1,2}})$ denotes the strategy set of the game, where K_i is the strategy set of player $i \in \mathcal{Z}$.

Remark 2. The risk-averse Nash game $\Gamma^\rho(\cdot)$ with risk trading provides a generalized representation. By adjusting the risk aversion parameter of players, i.e., $\alpha_1, \cdots, \alpha_{\mathcal{P}^\rho}, \alpha^{ar}$, and

TABLE II: A summary of problems to be solved.

	Problem
Fully incomplete	Mixed complementarity problem (15)–(20)
Partially incomplete	Mixed complementarity problem (15)–(20)
Complete	Optimization (21), followed by optimization (22)
Neutral	Optimization (12)

the availability of Arrow-Debreu securities, $\Gamma^{\rho}(\cdot)$ is able to describe several game setups, including both risk-neutral and risk-averse Nash games. The risk-averse setups range from a fully incomplete market for risk, where no Arrow-Debreu security is available for trading, to a complete market for risk, where Arrow-Debreu securities are available for trading in *all* scenarios. The intermediate setups represent partially incomplete markets for risk, where Arrow-Debreu securities are only available for trading in a subset of scenarios.

Proposition 2. If the risk-averse non-cooperative Nash game $\Gamma^{\rho}(\cdot)$ is either fully or partially incomplete for risk, then no equivalent optimization problem necessarily exists. In addition, multiple Nash equilibria may exist.

Proof 2. We provide the proof in Appendix A-B. ■

Proposition 3. If the risk-averse non-cooperative Nash game $\Gamma^{\rho}(\cdot)$ is complete for risk, then an equivalent optimization problem exists, where all players reach a consensus on risk-adjusted probabilities due to unconstrained risk trading. This is in line with the findings of [14], [15], and [18]. However, multiple Nash equilibria may still exist, since objective functions are not strongly convex in $a_{n\omega}$ and b_{ω}.

Proof 3. We provide the proof in Appendix A-C. ■

Remark 3. We reformulate the risk-averse Nash game $\Gamma^{\rho}(\cdot)$ with risk trading as a mixed complementarity problem [9] by concatenating KKT conditions from all optimization problems within $\Gamma^{\rho}(\cdot)$, i.e., optimization problems (5)–(10). Appendix B provides the full formulation of the resulting mixed complementarity problem. This mixed complementarity problem contains bilinear terms due to products of risk-adjusted probabilities and trading decisions, resulting in a mixed non-linear complementarity problem. However, it can be solved using PATH [40] or other mixed complementarity problem solvers.

Remark 4. The risk-averse Nash game $\Gamma^{\rho}(\cdot)$ with a complete market for risk can be solved in the same way as stated in Remark 3. However, Proposition 3 shows that an equivalent optimization problem for such a problem exists, which is a risk-averse social planner problem endowed with the risk measure function $\rho^{\cap}(\cdot)$ and the risk set $\mathcal{D}^{\text{CVaR},\cap}$. We provide such an optimization problem related to our risk-averse Nash game $\Gamma^{\rho}(\cdot)$ with a complete market for risk in Appendix C. By solving this convex optimization we avoid computational issues, coming along with solvers for mixed complementarity problems as highlighted in [17].

According to Remark 3 and Remark 4, we solve optimization problems for the risk-neutral Nash game $\Gamma(\cdot)$ and for the risk-averse Nash game $\Gamma^{\rho}(\cdot)$ when the market is complete for risk. In contrast, we solve mixed complementarity problems associated with $\Gamma^{\rho}(\cdot)$ when the market is partially or fully incomplete for risk. We provide a summary of problems to be solved in Table II.

TABLE III: Average computational time in seconds.

	Fully incomp.	Partially incomp.	Complete	Neutral
6 Scenarios	2.1	1.8	0.4	0.2
500 Scenarios	437.1	891.0	5.1	4.0

V. Numerical Study and Discussion

We apply models presented in Sections III and IV to analyze the impact of heterogeneous risk aversion within an energy community. We start with an illustrative example with 6 scenarios to gain insights into risk trading. In the second part, we turn our attention to a problem with 500 scenarios, where we shed light on the payment flows among energy community members in the local forward and spot markets. Moreover, we gradually increase the risk aversion of one prosumer to highlight the implication for rival players.

Throughout this section, we consider three players, namely n_1, n_2, and n_3, in addition to the spatial arbitrageur ar. Player n_1 is a prosumer who owns a stochastic PV system with a mean power generation of 6 kW while having a deterministic demand of 10 kW. Player n_2 owns a stochastic PV system only, with a mean power generation of 6 kW. Finally, player n_3 is an inelastic demand, with a deterministic load of 10 kW.

We consider the spatial arbitrageur to be the least risk-averse player with $\alpha^{ar} = 0.9$. Player n_1 is moderately risk-averse with $\alpha_1 = 0.5$. Player n_2 is highly risk-averse with $\alpha_2 = 0.3$, while player n_3 is slightly risk-averse with $\alpha_3 = 0.7$. The fixed importing price C^{i} in the local forward market for the spatial arbitrageur is €0.5/kW, while she receives a price C^{e} of €0.25/kW for exporting electricity in the forward market. In the local spot market, the fixed price for importing $\tilde{C}^{\text{i}}$ is €0.75/kW, while the exporting price $\tilde{C}^{\text{e}}$ is €0.125/kW. This incentivizes the energy community to optimally settle in the forward market, since costs are lower and revenues are higher. The regularizer is set to $\beta = 0.01$. The lower and upper bounds for power trades and clearing prices—if unequal to zero—are chosen such that those bounds are never binding[8]. Lastly, for a partially incomplete market for risk, Arrow-Debreu securities are available for a third of the scenarios only.

A. Computational Issues

We use Gurobi Optimizer 9.0.1 under Python 3.7.6 to solve optimization problems, and PATH [40] under GAMS 24.6 to solve mixed complementarity problems. All these problems

[8]Note that upper bounds on imports and exports for the spatial arbitrageur represent the network capacity constraint between the energy community and the distribution system. These bounds, if active, alter local market-clearing prices. In particular, if upper bounds for energy imports and exports are binding and the energy community exhibits an energy surplus, local market-clearing prices decrease, while in the case of an energy shortage local market-clearing prices increase. Therefore, the network congestion changes the cost/revenue of players n_1 to n_3, while the spatial arbitrageur earns a profit from energy arbitrage. This profit comes from the price difference between the local market-clearing price and the buying/selling price of the retailer.

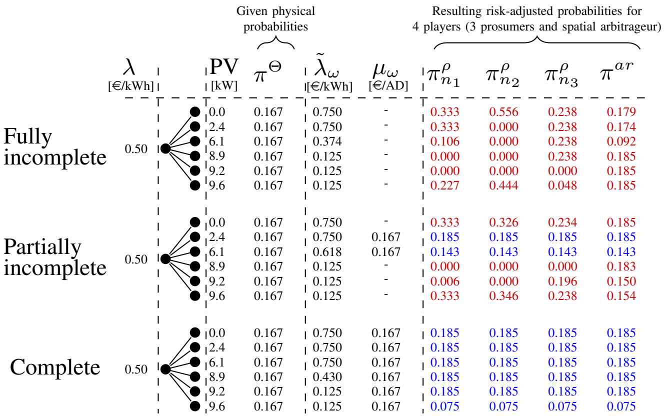


Fig. 3: Forward λ, spot $\tilde{\lambda}_{\omega}$, and Arrow-Debreu (AD) security μ_{ω} market-clearing prices as well as risk-adjusted probabilities per player resulting from the stochastic PV power generation, which is reported per player, under different degrees of market completeness for risk, i.e., fully incomplete, partially incomplete, and complete. Equally weighted scenarios among four players are indicated in blue, while differently weighted scenarios are indicated in red.

are solved on a 8 GB-RAM computer clocking at 2.40 GHz. Note that all source codes are available in our online companion [41].

Optimization problems in general and mixed complementarity problems with at least one risk-neutral player scale well as the number of scenarios increases. Table III provides the computational time required to solve the underlying problem. For the case of 6 scenarios the computational time remains within seconds, while for the case of 500 scenarios the computational time needed by the PATH solver significantly increases. We observe computational challenges in the PATH solver when the problem size increases with respect to the number of scenarios. For such a case, an alternative solution algorithm is to use a decomposition approach, e.g., an alternating direction method of multipliers, to compute a risk-averse Nash equilibrium [42], though we leave it for future research.

We emphasize that the proposed Nash equilibrium problem does not serve as a tool for clearing a local energy market, where computational time significantly matters, but rather provides a framework for analyzing the implications of heterogeneous risk aversion and risk trading within local energy communities.

B. Illustrative Example with 6 Scenarios

Figure 3 presents the PV power generation $\tilde{S}_{n\omega}$ as scenario-dependent input data, as well as optimization/Nash equilibrium problem outcomes for different degrees of market completeness for risk, including forward λ and spot $\tilde{\lambda}_{\omega}$ market-clearing prices, risk prices μ_{ω}, and risk-adjusted probabilities of different players, i.e., $\pi^{\rho}_{n\omega}$ and π^{ar}_{ω}. In the fully incomplete case, risk-adjusted probabilities vary highly among players. The most risk-averse player n_2 weights the scenario with zero PV power generation the most with $\pi^{\rho}_{n_2\omega_1} = 0.556$, since no revenues are achieved. Interestingly, the weight of the scenario with the highest PV power generation is increased since this event causes the local spot market-clearing price to reduce, which also induces lower revenues. A similar behavior, but less extreme, is observed for player n_1. In contrast, player n_3, who is a sole consumer, increases weights for scenarios with high spot market-clearing prices while accordingly reducing weights of scenarios with low spot market-clearing prices. As we move towards the complete case, for scenarios where Arrow-Debreu securities are available, players reach a consensus on risk-adjusted probabilities, marked in the partially incomplete and complete cases in blue.

TABLE IV: Power schedules in the local forward market [kW].

	n_1	n_2	n_3	Spatial arbitrageur
Fully incomplete	3.72	-2.27	5.34	6.79
Partially incomplete	2.91	-2.15	4.89	5.65
Complete	2.00	-2.99	4.86	3.87
Neutral	1.67	-3.33	4.68	3.03

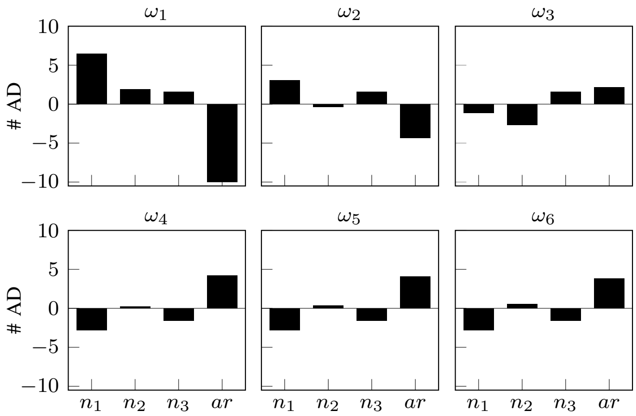


Fig. 4: Arrow-Debreu (AD) security trades among the three players n_1 to n_3 and the spatial arbitrageur (ar) for all scenarios ω_1 to ω_6 in the forward stage of a complete local energy market for risk.

Based on risk-adjusted probabilities given in Figure 3, players adjust their forward market trades, as listed in Table IV. If the market is fully incomplete for risk, player n_2, who owns a stochastic PV system, reduces her local forward market trades and postpones trading decisions to be made in the local spot market when the realization of PV power generation is observed. In contrast, player n_3, who has a deterministic load, prefers meeting the majority of her demand in the local forward market, and thereby avoids the price volatility over local spot market scenarios. Player n_1 has a stochastic PV power generation as well as a deterministic load. We observe that she meets her demand in the local forward market while postponing decisions regarding PV power generation to the local spot market. The spatial arbitrageur imports/exports to/from the energy community according to trading strategies of players n_1 to n_3.

Risk trading outweighs heterogeneous risk aversion and shifts risk-adjusted forward trades towards observations in a risk-neutral setting. Figure 4 illustrates the Arrow-Debreu security trades in the forward stage for each scenario in a complete market for risk, which allow reaching a consensus on risk preferences. All three players n_1 to n_3 buy Arrow-Debreu securities for scenario ω_1 with the lowest PV power generation, while the least risk-averse player, who in our case is the spatial arbitrageur, is the only Arrow-Debreu security

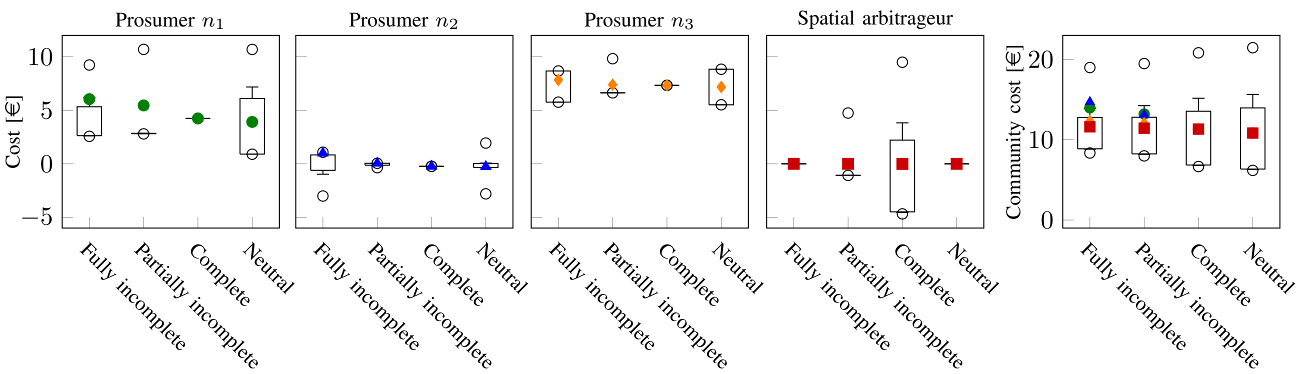


Fig. 5: Cost distribution over scenarios ω_1 to ω_6 for players as well as for the community as a whole in dependence of the degree of market completeness for risk. Colored markers refer to the risk-adjusted expected cost from each player's perspective. The distribution of the cost over scenarios is given by boxes, horizontal lines, and circles. Each box highlights the cost between the second and the third quartile. Horizontal lines show the 5th and 95th quantiles. Finally, circles indicate outliers below the 5th and beyond the 95th quantiles.

seller. However, for the following scenarios, we observe the opposite trend. In most cases, risk-averse players n_1 to n_3 emerge as Arrow-Debreu security sellers, while the spatial arbitrageur appears as a Arrow-Debreu security buyer. Therefore, players n_1 to n_3 erase their cost volatility, while the spatial arbitrageur absorbs it. Thus, unconstrained risk trading leads to a consensus on risk-adjusted probabilities corresponding to the risk preference of the least risk-averse player, i.e., the spatial arbitrageur.

The effect of risk trading on the cost distribution as well as the distribution of the total community cost is shown in Figure 5. Moving from a fully incomplete market to a complete market for risk, the cost distribution for players reduces to the point where in the complete case costs in all scenarios are identical. However, since the spatial arbitrageur absorbs the cost volatility of players n_1 to n_3, her cost volatility increases. Nevertheless, her expected cost remains unchanged. If all players are risk-neutral, they realize the lowest disbenefit. Similarly, looking at the total community cost we observe that moving from a fully incomplete market to a complete market for risk, the total expected community cost reduces by 5%, while the community cost volatility increases. This cost reduction as a result of completing the market for risk confirms findings by [15] as discussed in Section II-D. Moreover, owing to risk trading, the expected community cost from each player's perspective converges towards a common belief, indicating a consensus on risk preferences.

C. Assessing Risk Aversion with 500 Scenarios

In this section we assume the spatial arbitrageur to be risk-neutral, i.e., $\alpha^{ar} = 1$, because we expect the spatial arbitrageur to be a necessary automated service for the energy community in a similar way as the price setter clears the local market. Moreover, this assumption allows us to solve mixed complementarity problems with the PATH solver under GAMS without any computational issues as the number of scenarios increases.

Figure 6 illustrates the evolution of the expected community cost and its standard deviation in the risk-neutral Nash game

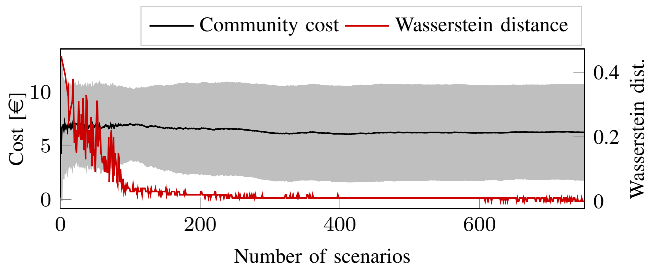
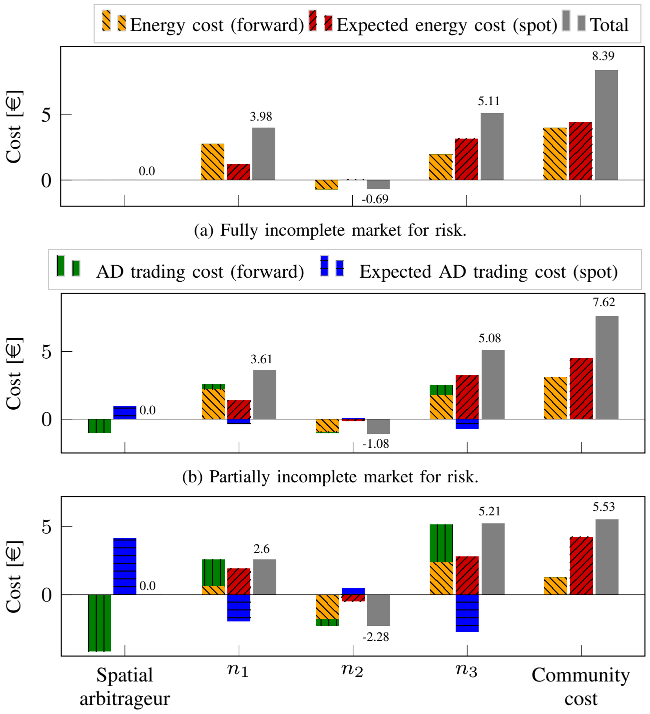


Fig. 6: Expected community cost and standard deviation highlighted by the shaded area in the risk-neutral Nash game. The Wasserstein distance shows the convergence against a good representation of the probability distribution as the number of scenarios increases.

with the number of scenarios. Moreover, we show the Wasserstein distance [43] between each set of sampled scenarios and its predecessor. Based on these numerical findings, we observe that from 500 scenarios onward changes arising from one additional scenario representing uncertain PV power generation hardly take place. Therefore, we choose 500 discrete scenarios to approximate the underlying continuous probability distribution of PV power generation.

Figure 7 presents the expected cost of all players as well as the total community cost. It provides details of costs specifically incurred in the local forward and spot markets by trading energy and Arrow-Debreu securities. Figure 7a corresponds to a fully incomplete market for risk, where players n_1 to n_3 incur the highest cost among all cases. This causes the expected community cost with 8.39€ to be comparatively high too.

As risk trading is possible for one-third of 500 scenarios, Figure 7b illustrates how players n_1 to n_3 reduce their energy trades in the local forward market and increase more profitable activities in the local spot market, though subject to the PV power generation uncertainty. This uncertainty is partially hedged by awarded Arrow-Debreu securities. Note that the cost for all players n_1 to n_3 as well as the community cost decrease to some extent in comparison to those costs in the fully incomplete case. In particular, the expected community cost decreases from 8.39€ in the fully incomplete case to 7.62€

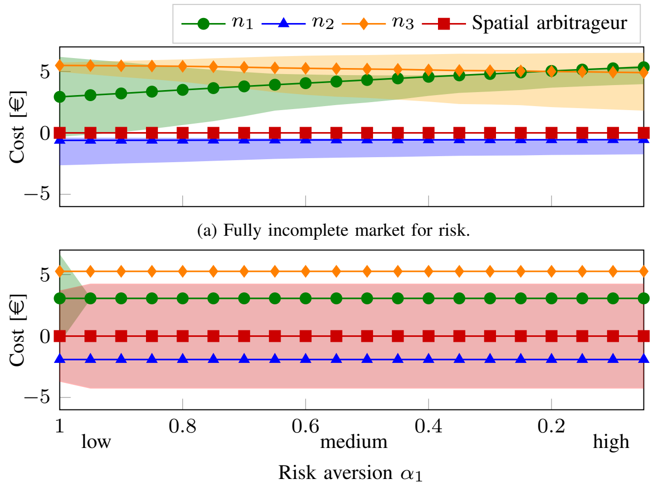


(a) Fully incomplete market for risk.

(b) Partially incomplete market for risk.

(c) Complete market for risk.

Fig. 7: Expected cost of players n_1, n_2, n_3 and the spatial arbitrageur over 500 scenarios incurred by trading energy and Arrow-Debreu (AD) securities in local forward and spot markets. Plot 7a corresponds to a fully incomplete market for risk, plot 7b associates with a partially incomplete market for risk, and lastly, plot 7c refers to a complete market for risk.



(a) Fully incomplete market for risk.

(b) Complete market for risk.

Fig. 8: Cost under an increasing risk aversion of player n_1. Plot 8a graphs observations for a fully incomplete market for risk, while plot 8b illustrates the situation for a complete market for risk. Marked lines refer to the expected cost while shaded areas highlight the cost standard deviation.

in the partially incomplete case, implying a 9.18% saving in the community cost.

Figure 7c shows results for a case wherein the local energy market is complete for risk, i.e., risk trading is possible for all 500 scenarios. The three players n_1 to n_3 as well as the community as a whole incur the lowest cost among all three cases. In particular, the expected community cost drops to 5.53€, i.e., a 34.09% saving in the community cost compared to a fully incomplete market for risk. Moreover, this plot shows that energy trades in the local forward market have been reduced even further, while the engagement of players n_1 to n_3 in the local spot market has been increased. It is worth noting that the expected cost of each player in the local spot market is fully compensated by Arrow-Debreu securities. This causes a significant cost volatility for the spatial arbitrager, who is the greatest security seller in this case study.

Finally, we note that the spatial arbitrageur yields a zero cost in expectation in all three cases. In particular, since constraints on energy imports and exports are never binding, the local forward and spot market-clearing prices are equal to importing and exporting prices. Moreover, the revenue of the spatial arbitrageur from selling Arrow-Debreu securities in the local forward market are fully balanced with her expected cost in the local spot market for compensating the security buyers.

In the following, we gradually increase the risk aversion of one of the players, e.g., player n_1, and investigate impacts on her rivals. Figure 8a shows such an effect on the expected cost as well as the cost standard deviation. For the fully incomplete case the expected cost for player n_1 increases significantly as her risk aversion α_1 increases. However, owing to her risk-adjusted forward market position the cost standard deviation decreases. Moreover, we observe that the risk aversion of player n_1 impacts the expected cost and the cost standard deviation of players n_2 and n_3. Player n_3 incurs comparatively lower expected cost, while her cost standard deviation increases. The reason for this is that player n_1 trades her stochastic PV power generation in the local spot market, and thereby lowers the local spot market-clearing price from which player n_3 profits.

As noted previously, if the market is complete for risk, risk-averse players n_1 to n_3 fully erase their cost volatility, while the spatial arbitrageur absorbs all the cost volatility as graphed in Figure 8b. Thus, the standard deviation of players n_1 to n_3 is zero, while the spatial arbitrageur experiences a remarkable cost volatility, though with an expected value of zero.

VI. Conclusion and Future Work

This work proposes risk trading within energy communities to outweigh market inefficiencies arising from heterogeneously risk-averse community members. We have formulated a two-stage stochastic Nash equilibrium problem and show different solution approaches depending on the degree of market completeness for risk. Risk trading complements local energy markets within energy communities when players have heterogeneous risk preferences. Risk trading protects players with slight risk aversion from conservative decisions made by highly risk-averse players. As a result, a significant system cost saving can be realized, while disbenefits for all players are reduce.

This work opens a wide range of research questions to be addressed in the future. The role of an energy storage system should be considered, since energy arbitrage over time can reduce the volatility of local market-clearing prices, and therefore the cost volatility for players by a different mean than risk trading. In addition, the small number of energy community members gives rise to potential strategic behavior by some local market participants. This aspect should not be neglected in designing a suitable risk trading product. If the spatial size of an energy community increases, network constraints within the community and resulting power losses have to be respected too. In addition, it is of interest to include the possibility of peer-to-peer energy trading among prosumers within the energy community. Another interesting research direction is to explore risk trading among retailers/aggregators on a distribution system level, i.e., one level above energy communities. Furthermore, we note that Arrow-Debreu security is a highly stylized financial product, which can only be traded for predefined scenarios. Therefore, the probability distribution of an uncertain event must be certainly known to complete the market for risk. Research on a less stylized product is of interest for determining an optimal trade-off between market completeness for risk and applicability of a financial product. Lastly, from a computational perspective, research on different solution algorithms for Nash equilibrium problems, e.g., based on various decomposition algorithms, is promising.

Appendix A
Proofs

Proofs are based on equivalent forms of Variational Inequality (VI) problems and strict monotonicity of players' preferences [35].

A. Proof of Proposition 1

We state the problem $\Gamma(\cdot)$ of finding a Nash equilibrium as a $\text{VI}(F, K)$ with the game map $F(z) = [\nabla_1 J_1(z_1, z_{-1}), \cdots, \nabla_{ar} J^{ar}(z_{ar}, z_{-ar}), \nabla_\lambda J^\lambda(z_\lambda, z_{-\lambda})]^\top$, where $z = [p_1, \tilde{p}_{1\omega}, \cdots, p^{\text{i}}, p^{\text{e}}, \tilde{p}^{\text{i}}_\omega, \tilde{p}^{\text{e}}_\omega, \lambda, \tilde{\lambda}_\omega]$ denotes the strategy vector. For the game $\Gamma(\cdot)$ the strategy set K_i is compact, convex, and non-empty. Moreover, the game map $F(z)$ is continuous, since all cost functions $J_{i \in \mathcal{Z}}$ are continuously differentiable. Therefore, a solution set $\text{SOL}(K, F)$ exists.

To show the singleton nature of the solution set $\text{SOL}(K, F)$ we derive the Jacobian matrix of $F(z)$ as

$$\nabla z F(z) = \begin{pmatrix} \beta & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \pi^\Theta_\omega \beta & \cdots & 0 & 0 & 0 & 0 & 0 & \pi^\Theta_\omega \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -\pi^\Theta_\omega \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \pi^\Theta_\omega \\ 1 & 0 & \cdots & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & \pi^\Theta_\omega & \cdots & 0 & 0 & -\pi^\Theta_\omega & \pi^\Theta_\omega & 0 & 0 \end{pmatrix} \begin{matrix} J_n \\ \\ \\ \\ J^{ar} \\ \\ \\ J^{ps} \\ \\ \end{matrix}.$$

The Jacobian matrix above is symmetric indicated by the blue diagonal entries, meaning that the corresponding game is *integrable* [9]. This implies that an equivalent optimization problem solving the $\text{VI}(F, K)$ exists, whose objective function is given by

$$\theta(z) = \int_0^1 F(z^0 + t(z - z^0))^\top (z - z^0) dt =$$

$$C^{\text{i}} p^{\text{i}} - C^{\text{e}} p^{\text{e}} + \sum_{n \in N} \frac{1}{2} \beta p_n^2$$
$$+ \sum_{\omega \in \Omega} \pi^\Theta_\omega (\tilde{C}^{\text{i}} \tilde{p}^{\text{i}}_\omega - \tilde{C}^{\text{e}} \tilde{p}^{\text{e}}_\omega + \sum_{n \in N} \frac{1}{2} \beta \tilde{p}^2_{n\omega}), \tag{11}$$

which motivates the optimization problem

$$\underset{p_n, p^{\text{i}}, p^{\text{e}}, \tilde{p}_{n\omega}, \tilde{p}^{\text{i}}_\omega, \tilde{p}^{\text{e}}_\omega}{\text{Min.}} \quad (11) \tag{12a}$$
$$\text{s.t. (2b), (2c), (3b), (3d).} \tag{12b}$$

The objective function of the resulting optimization problem is convex and quadratic. This confirms that a unique Nash equilibrium point for the risk-neutral Nash game $\Gamma(\cdot)$ exists. ■

B. Proof of Proposition 2

Following Appendix A-A we state $\Gamma^\rho(\cdot)$ as $\text{VI}^\rho(F, K)$ with $F(z) = [\nabla_1 J_1^{\rho_1}(z_1, z_{-1}), \nabla_1 J_1^{\rho_2}(z_1, z_{-1}), \cdots, \nabla_{ar} J^{ar,\rho_1}(z_{ar}, z_{-ar}), \nabla_{ar} J^{ar,\rho_2}(z_{ar}, z_{-ar}), \nabla_{ps} J^{ps,\cap_1}(z_{ps}, z_{-ps}), \nabla_{ps} J^{ps,\cap_2}(z_{ps}, z_{-ps})]^\top$, where $z = [p_1, \tilde{p}_{1\omega}, a_{1\omega}, \pi^\rho_{1\omega}, \cdots, p^{\text{i}}, p^{\text{e}}, \tilde{p}^{\text{i}}_\omega, \tilde{p}^{\text{e}}_\omega, b_\omega, \pi^{ar}_\omega, \mu_\omega, \lambda, \tilde{\lambda}_\omega, \pi^\cap_\omega]$ denotes the strategy vector of the game. Moreover, to apply tools from VI we assume constraints on Arrow-Debreu security trades, although they are never binding. We write the Jacobian matrix as

$$\nabla z F(z) = \begin{pmatrix} \beta & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \beta \pi^\rho_{n\omega} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi^\rho_{n\omega} & 0 \\ 0 & 0 & 0 & -1 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_n & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \tilde{C}^{\text{i}} & 0 & 0 & -\pi^{ar}_\omega & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -\tilde{C}^{\text{e}} & 0 & 0 & \pi^{ar}_\omega & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\tilde{C}^{\text{i}} & \tilde{C}^{\text{e}} & 1 & q^{ar} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \pi^\cap_\omega & 0 & 0 & \cdots & 0 & 0 & -\pi^\cap_\omega & \pi^\cap_\omega & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q^\cap \end{pmatrix} \begin{matrix} \\ J^{\rho_1}_n \\ \\ J^{\rho_2}_n \\ \\ \\ \\ \\ J^{ar,\rho_1} \\ \\ J^{ar,\rho_2} \\ \\ J^{ps,\cap_1} \\ \\ J^{ps,\cap_2} \end{matrix},$$

where $q_n = -\frac{1}{\pi^\rho_{n\omega}}(\tilde{\lambda}_\omega \tilde{p}_{n\omega} - \frac{1}{2}\beta \tilde{p}^2_{n\omega})$, $q^{ar} = \frac{1}{\pi^{ar}_\omega} \tilde{\lambda}_\omega (\tilde{p}^{\text{i}}_\omega - \tilde{p}^{\text{e}}_\omega)$, $q^\cap = -\frac{1}{\pi^\cap_\omega}(\frac{1}{2}\beta \tilde{p}^2_{n\omega} + \tilde{C}^{\text{i}} \tilde{p}^{\text{i}}_\omega - \tilde{C}^{\text{e}} \tilde{p}^{\text{e}}_\omega)$. Since the market is incomplete for risk, players potentially apply different risk-adjusted probabilities. Therefore, the Jacobian matrix of $\Gamma^\rho(\cdot)$ is asymmetric as highlighted by the red entries. Thus, an equivalent optimization problem does not necessarily exist. ■

C. Proof of Proposition 3

If the market is complete for risk, all players apply identical risk-adjusted probabilities, i.e., $\pi^\rho_{n\omega} = \pi^{ar}_\omega = \pi^\cap_\omega$. This observation is based on the equivalence by the zero-gradient conditions with respect to Arrow-Debreu securities among all players for the case of unconstrained risk trading as given in Appendix B. Thus, the Jacobian matrix of $\Gamma^\rho(\cdot)$ is symmetric, with a skew symmetric inner part. An equivalent optimization problem solving $\text{VI}^\rho(F, K)$ exists. Its objective is given by

$$\theta(z) = \int_0^1 F(z^0 + t(z - z^0))^\top (z - z^0) dt =$$

$$C^{\mathrm{i}}p^{\mathrm{i}} - C^{\mathrm{e}}p^{\mathrm{e}} + \sum_{n\in N}\frac{1}{2}\beta p_n^2 + \sum_{\omega\in\Omega}\pi_\omega^{\cap}\big(\tilde{C}^{\mathrm{i}}\tilde{p}^{\mathrm{i}}_\omega - \tilde{C}^{\mathrm{e}}\tilde{p}^{\mathrm{e}}_\omega + \sum_{n\in N}\frac{1}{2}\beta\tilde{p}^2_{n\omega}\big), \quad (13)$$

where $\pi_\omega^{\cap} \in \mathcal{D}^{\mathrm{CVaR},\cap}$. This gives rise to

$$\underset{p_n,p^{\mathrm{i}},p^{\mathrm{e}},\tilde{p}_{n\omega},\tilde{p}^{\mathrm{i}}_\omega,\tilde{p}^{\mathrm{e}}_\omega}{\text{Min.}} \quad (13) \quad (14a)$$

$$\text{s.t. (2b), (2c), (3b), (3d).} \quad (14b)$$

However, we cannot derive a conclusion on the solution uniqueness, since Arrow-Debreu security trades are not explicitly stated in the optimization problem. Therefore, the objective function (13) is not strongly convex in $a_{n\omega}$ and b_ω. ■

Appendix B
Mixed Complementarity Problem

We reformulate the Nash equilibrium problem as a mixed non-linear complementarity problem, which solves the risk-averse Nash game $\Gamma^\rho(\cdot)$ with risk trading. In the following, we provide this problem by concatenating the KKT conditions associated with prosumers' optimization problems (5)–(6), the spatial arbitrageur's optimization problems (7)–(8), and the price setter's optimization problems (9)–(10). Note that $\mathcal{L}$ refers to the Lagrangian function of the underlying optimization problem.

The KKT conditions associated with (5) are as follows. Note that in order to obtain a closed and compact decision set, we consider theoretical lower and upper bounds on Arrow-Debreu security trades in the form of $-\overline{Y}_\omega \leq a_{n\omega} \leq \overline{Y}_\omega,\ \forall n,\omega$, whose dual variables are $\underline{\chi}^{\mathrm{a}}_{n\omega}$ and $\overline{\chi}^{\mathrm{a}}_{n\omega}$, respectively.

$$\frac{\partial\mathcal{L}}{\partial p_n} = \lambda + \beta p_n + \sum_{\omega\in\Omega}\tilde{\phi}_{n\omega} - \underline{\chi}^{\mathrm{p}}_n + \overline{\chi}^{\mathrm{p}}_n = 0,\ \forall n, \quad (15a)$$

$$\frac{\partial\mathcal{L}}{\partial \tilde{p}_{n\omega}} = \pi^\rho_{n\omega}\big(\tilde{\lambda}_\omega + \beta\tilde{p}_{n\omega}\big) + \tilde{\phi}_{n\omega} - \underline{\chi}^{\tilde{\mathrm{p}}}_{n\omega} + \overline{\chi}^{\tilde{\mathrm{p}}}_{n\omega} = 0,\ \forall n,\omega, \quad (15b)$$

$$\frac{\partial\mathcal{L}}{\partial a_{n\omega}} = \mu_\omega - \pi^\rho_{n\omega} - \underline{\chi}^{\mathrm{a}}_{n\omega} + \overline{\chi}^{\mathrm{a}}_{n\omega} = 0,\ \forall n,\omega, \quad (15c)$$

$$0 \leq p_n - \underline{P}_n \perp \underline{\chi}^{\mathrm{p}}_n \geq 0,\ \forall n, \quad (15d)$$

$$0 \leq \overline{P}_n - p_n \perp \overline{\chi}^{\mathrm{p}}_n \geq 0,\ \forall n, \quad (15e)$$

$$0 \leq \tilde{p}_{n\omega} - \underline{P}_n \perp \underline{\chi}^{\tilde{\mathrm{p}}}_{n\omega} \geq 0,\ \forall n,\omega, \quad (15f)$$

$$0 \leq \overline{P}_n - p^{\tilde{\mathrm{p}}}_{n\omega} \perp \overline{\chi}^{\tilde{\mathrm{p}}}_{n\omega} \geq 0,\ \forall n,\omega, \quad (15g)$$

$$0 \leq a_{n\omega} + \overline{Y}_\omega \perp \underline{\chi}^{\mathrm{a}}_{n\omega} \geq 0,\ \forall n,\omega, \quad (15h)$$

$$0 \leq \overline{Y}_\omega - a_{n\omega} \perp \overline{\chi}^{\mathrm{a}}_{n\omega} \geq 0,\ \forall n,\omega, \quad (15i)$$

$$p_n + \tilde{p}_{n\omega} + \tilde{S}_{n\omega} - D_n = 0,\ \forall n,\omega. \quad (15j)$$

The KKT conditions associated with (6) are

$$\frac{\partial\mathcal{L}}{\partial \pi^\rho_{n\omega}} = -\big(\tilde{\lambda}_\omega\tilde{p}_{n\omega} + \frac{1}{2}\beta\tilde{p}^2_{n\omega} - a_{n\omega}\big) + \phi^\rho_n - \underline{\chi}^\rho_{n\omega} + \overline{\chi}^\rho_{n\omega} = 0,\ \forall n,\omega, \quad (16a)$$

$$0 \leq \pi^\rho_{n\omega} \perp \underline{\chi}^\rho_{n\omega} \geq 0,\ \forall n,\omega, \quad (16b)$$

$$0 \leq \frac{1}{\alpha_n}\pi^\Theta_\omega - \pi_{n\omega} \perp \overline{\chi}^\rho_{n\omega} \geq 0,\ \forall n,\omega, \quad (16c)$$

$$\sum_{\omega\in\Omega}\pi^\rho_{n\omega} - 1 = 0,\ \forall n. \quad (16d)$$

The KKT conditions corresponding to (7) are as follows. Again, in order to achieve a closed and compact decision set, we consider theoretical lower and upper bounds on Arrow-Debreu security trades of the spatial arbitrageur in the form of $-\overline{Y}_\omega \leq b_\omega \leq \overline{Y}_\omega,\ \forall\omega$, whose dual variables are $\underline{\chi}^{\mathrm{b}}_\omega$ and $\overline{\chi}^{\mathrm{b}}_\omega$, respectively.

$$\frac{\partial\mathcal{L}}{\partial p^{\mathrm{i}}} = C^{\mathrm{i}} - \lambda - \underline{\chi}^{\mathrm{p^i}} + \overline{\chi}^{\mathrm{p^i}} = 0, \quad (17a)$$

$$\frac{\partial\mathcal{L}}{\partial p^{\mathrm{e}}} = -C^{\mathrm{e}} + \lambda - \underline{\chi}^{\mathrm{p^e}} + \overline{\chi}^{\mathrm{p^e}} = 0, \quad (17b)$$

$$\frac{\partial\mathcal{L}}{\partial \tilde{p}^{\mathrm{i}}_\omega} = \pi^{ar}_\omega\big(\tilde{C}^{\mathrm{i}} - \tilde{\lambda}_\omega\big) - \underline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{i}}}_\omega + \overline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{i}}}_\omega = 0,\ \forall\omega, \quad (17c)$$

$$\frac{\partial\mathcal{L}}{\partial \tilde{p}^{\mathrm{e}}_\omega} = -\pi^{ar}_\omega\big(\tilde{C}^{\mathrm{e}} - \tilde{\lambda}_\omega\big) - \underline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{e}}}_\omega + \overline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{e}}}_\omega = 0,\ \forall\omega, \quad (17d)$$

$$\frac{\partial\mathcal{L}}{\partial b_\omega} = \mu_\omega - \pi^{ar}_\omega - \underline{\chi}^{\mathrm{b}}_\omega + \overline{\chi}^{\mathrm{b}}_\omega = 0,\ \forall\omega, \quad (17e)$$

$$0 \leq p^{\mathrm{i}} \perp \underline{\chi}^{\mathrm{p^i}} \geq 0, \quad (17f)$$

$$0 \leq \overline{P}^{\mathrm{i}} - p^{\mathrm{i}} \perp \overline{\chi}^{\mathrm{p^i}} \geq 0, \quad (17g)$$

$$0 \leq p^{\mathrm{e}} \perp \underline{\chi}^{\mathrm{p^e}} \geq 0, \quad (17h)$$

$$0 \leq \overline{P}^{\mathrm{e}} - p^{\mathrm{e}} \perp \overline{\chi}^{\mathrm{p^e}} \geq 0, \quad (17i)$$

$$0 \leq \tilde{p}^{\mathrm{i}}_\omega \perp \underline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{i}}}_\omega \geq 0,\ \forall\omega, \quad (17j)$$

$$0 \leq \overline{P}^{\mathrm{i}} - \tilde{p}^{\mathrm{i}}_\omega \perp \overline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{i}}}_\omega \geq 0,\ \forall\omega, \quad (17k)$$

$$0 \leq \tilde{p}^{\mathrm{e}}_\omega \perp \underline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{e}}}_\omega \geq 0,\ \forall\omega, \quad (17l)$$

$$0 \leq \overline{P}^{\mathrm{e}} - \tilde{p}^{\mathrm{e}}_\omega \perp \overline{\chi}^{\tilde{\mathrm{p}}^{\mathrm{e}}}_\omega \geq 0,\ \forall\omega, \quad (17m)$$

$$0 \leq b_\omega + \overline{Y}_\omega \perp \underline{\chi}^{\mathrm{b}}_\omega \geq 0,\ \forall\omega, \quad (17n)$$

$$0 \leq \overline{Y}_\omega - b_\omega \perp \overline{\chi}^{\mathrm{b}}_\omega \geq 0,\ \forall\omega. \quad (17o)$$

The KKT conditions corresponding to (8) are

$$\frac{\partial\mathcal{L}}{\partial \pi^{ar}_\omega} = -\big[(\tilde{C}^{\mathrm{i}} - \tilde{\lambda}_\omega)\tilde{p}^{\mathrm{i}}_\omega - (\tilde{C}^{\mathrm{e}} - \tilde{\lambda}_\omega)\tilde{p}^{\mathrm{e}}_\omega - b_\omega\big] + \phi^{ar} - \underline{\chi}^{ar}_\omega + \overline{\chi}^{ar}_\omega = 0,\ \forall\omega, \quad (18a)$$

$$0 \leq \pi^{ar}_\omega \perp \underline{\chi}^{ar}_\omega \geq 0,\ \forall\omega, \quad (18b)$$

$$0 \leq \frac{1}{\alpha^{ar}}\pi^\Theta_\omega - \pi^{ar}_\omega \perp \overline{\chi}^{ar}_\omega \geq 0,\ \forall\omega, \quad (18c)$$

$$\sum_{\omega\in\Omega}\pi^{ar}_\omega - 1 = 0. \quad (18d)$$

The KKT conditions associated with (9) are

$$\frac{\partial\mathcal{L}}{\partial \lambda} = \sum_{n\in N}p_n - p^{\mathrm{i}} + p^{\mathrm{e}} - \underline{\chi}^\lambda + \overline{\chi}^\lambda = 0, \quad (19a)$$

$$\frac{\partial\mathcal{L}}{\partial \tilde{\lambda}_\omega} = \pi^{\cap}_\omega\big(\sum_{n\in N}\tilde{p}_{n\omega} - \tilde{p}^{\mathrm{i}}_\omega + \tilde{p}^{\mathrm{e}}_\omega\big) - \underline{\chi}^{\tilde{\lambda}}_\omega + \overline{\chi}^{\tilde{\lambda}}_\omega = 0,\ \forall\omega, \quad (19b)$$

$$\frac{\partial\mathcal{L}}{\partial \mu_\omega} = \sum_{n\in N}a_{n\omega} + b_\omega - \underline{\chi}^\mu_\omega + \overline{\chi}^\mu_\omega = 0,\ \forall\omega, \quad (19c)$$

$$0 \leq \overline{\Lambda} + \lambda \perp \underline{\chi}^\lambda \geq 0, \quad (19d)$$

$$0 \leq \overline{\Lambda} - \lambda \perp \overline{\chi}^\lambda \geq 0, \quad (19e)$$

$$0 \leq \overline{\Lambda} + \tilde{\lambda}_\omega \perp \underline{\chi}^{\tilde{\lambda}}_\omega \geq 0,\ \forall\omega, \quad (19f)$$

$$0 \leq \overline{\Lambda} - \tilde{\lambda}_\omega \perp \overline{\chi}^{\tilde{\lambda}}_\omega \geq 0,\ \forall\omega, \quad (19g)$$

$$0 \leq \overline{M} + \mu_\omega \perp \underline{\chi}^{\mu}_{\omega} \geq 0, \ \forall \omega, \tag{19h}$$

$$0 \leq \overline{M} - \mu_\omega \perp \overline{\chi}^{\mu}_{\omega} \geq 0, \ \forall \omega. \tag{19i}$$

Finally, the KKT conditions associated with (10) are

$$\frac{\partial \mathcal{L}}{\partial \pi^{\cap}_{\omega}} = -\tilde{\lambda}_\omega \big(\sum_{n \in N} \tilde{p}_{n\omega} - \tilde{p}^{\mathrm{i}}_{\omega} + \tilde{p}^{\mathrm{e}}_{\omega} \big) + \phi^{\cap} - \underline{\chi}^{\cap}_{\omega} + \overline{\chi}^{\cap}_{\omega} = 0, \ \forall \omega, \tag{20a}$$

$$0 \leq \pi^{\cap}_{\omega} \perp \underline{\chi}^{\cap}_{\omega} \geq 0, \ \forall \omega, \tag{20b}$$

$$0 \leq \frac{1}{\alpha^{\cap}} \pi^{\Theta}_{\omega} - \pi^{\cap}_{\omega} \perp \overline{\chi}^{\cap}_{\omega} \geq 0, \ \forall \omega, \tag{20c}$$

$$\sum_{\omega \in \Omega} \pi^{\cap}_{\omega} - 1 = 0. \tag{20d}$$

The resulting mixed complementarity problem is the collection of conditions (15)–(20).

Appendix C
Complete Market for Risk

We solve the risk-averse Nash game with a complete market for risk based on a risk-averse social planner problem, which minimizes the negative expected risk-adjusted system cost as

$$\underset{\Xi}{\text{Min}} \ \underbrace{C^{\mathrm{i}} p^{\mathrm{i}} - C^{\mathrm{e}} p^{\mathrm{e}} + \sum_{n \in N} \frac{1}{2} \beta p_n^2}_{\text{Community cost in the forward market}} \ \underbrace{- \phi^{\cap} + \frac{1}{\alpha^{\cap}} \sum_{\omega \in \Omega} \pi^{\Theta}_{\omega} \overline{\chi}^{\cap}_{\omega}}_{\text{CVaR metric (1c)}} \tag{21a}$$

$$\text{s.t. (2b)–(2c), (3b)–(3d),} \tag{21b}$$

$$\underbrace{\phi^{\cap} + \tilde{C}^{\mathrm{i}} \tilde{p}^{\mathrm{i}}_{\omega} - \tilde{C}^{\mathrm{e}} \tilde{p}^{\mathrm{e}}_{\omega} + \sum_{n \in N} \frac{1}{2} \beta \tilde{p}^2_{n\omega}}_{\text{Community cost in the spot market}} \leq \overline{\chi}^{\cap}_{\omega} : \mu_\omega, \ \forall \omega, \tag{21c}$$

$$0 \leq \overline{\chi}^{\cap}_{\omega}, \ \forall \omega, \tag{21d}$$

where $\Xi = \{p_n, p^{\mathrm{i}}, p^{\mathrm{e}}, \tilde{p}_{n\omega}, \tilde{p}^{\mathrm{i}}_{\omega}, \tilde{p}^{\mathrm{e}}_{\omega}, \phi^{\cap}, \overline{\chi}^{\cap}_{\omega}\}$. The objective function (21a) minimizes the total community cost in the forward market as well as the total expected community cost in the spot market, which are endowed with the CVaR metric introduced in (1c). The variable $\phi^{\cap}$ shows the value-at-risk, and $\overline{\chi}^{\cap}_{\omega}$ is a non-negative auxiliary variable. Constraints (21c) and (21d) ensure the non-negativity of CVaR-related variables, where the dual variable μ_ω of (21c) corresponds to system-wide risk-adjusted probabilities [10], and thus, risk prices.

Given the optimal values obtained for a risk-adjusted social plan $(p_n, p^{\mathrm{i}}, p^{\mathrm{e}}, \tilde{p}_{n\omega}, \tilde{p}^{\mathrm{i}}_{\omega}, \tilde{p}^{\mathrm{e}}_{\omega})$, as well as risk prices μ_ω, forward λ, and spot $\tilde{\lambda}_\omega$ market-clearing prices, we solve the following optimization problem to derive values for Arrow-Debreu securities traded, i.e., $a_{n\omega}$ and b_ω:

$$\underset{\Phi}{\text{Min}} \sum_{n \in N} \big(\sum_{\omega \in \Omega} \mu^*_\omega a_{n\omega} \underbrace{- \phi^{\rho}_n + \frac{1}{\alpha_n} \sum_{\omega \in \Omega} \pi^{\Theta}_{\omega} \overline{\chi}^{\rho}_{n\omega}}_{\text{Prosumer's CVaR metric (1c)}} \big) + \sum_{\omega \in \Omega} \mu^*_\omega b_\omega - \underbrace{\phi^{ar} + \frac{1}{\alpha^{ar}} \sum_{\omega \in \Omega} \pi^{\Theta}_{\omega} \overline{\chi}^{ar}_{\omega}}_{\text{Spatial arbitrageur's CVaR metric (1c)}} \tag{22a}$$

$$\text{s.t. } \phi^{\rho}_n + \underbrace{\tilde{\lambda}^*_\omega \tilde{p}^*_{n\omega} + \frac{1}{2} \beta \tilde{p}^{*2}_{n\omega} - a_{n\omega}}_{\text{Prosumer's spot market cost}} \leq \overline{\chi}^{\rho}_{n\omega}, \ \forall n, \omega, \tag{22b}$$

$$\phi^{ar} + \underbrace{\big(\tilde{C}^{\mathrm{i}} - \tilde{\lambda}^*_\omega\big) \tilde{p}^{\mathrm{i},*}_{\omega} - \big(\tilde{C}^{\mathrm{e}} - \tilde{\lambda}^*_\omega\big) \tilde{p}^{\mathrm{e},*}_{\omega} - b_\omega}_{\text{Spatial arbitrageur's spot market cost}} \leq \overline{\chi}^{ar}_{\omega}, \ \forall \omega, \tag{22c}$$

$$0 \leq \overline{\chi}^{\rho}_{n\omega}, \ \forall n, \omega, \tag{22d}$$

$$0 \leq \overline{\chi}^{ar}_{\omega}, \ \forall \omega, \tag{22e}$$

where $\Phi = \{a_{n\omega}, \phi^{\rho}_n, \overline{\chi}^{\rho}_{n\omega}, b_\omega, \phi^{ar}, \overline{\chi}^{ar}_{\omega}\}$. Parameters denoted by $(\cdot)^*$ correspond to values obtained from the risk-averse social planner problem (21). The first line of the objective function (22a) corresponds to Arrow-Debreu security trades by risk-averse prosumers, while the second line refers to the spatial arbitrageur's trades. Constraints (22b) and (22c) define the CVaR metric for each prosumer and the spatial arbitrageur, respectively. Lastly, (22d) and (22e) ensure the non-negativity of CVaR-related variables.

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On ambiguity-averse market equilibrium

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Abstract We develop a Nash equilibrium problem representing a perfectly competitive market wherein all players are subject to the same source of uncertainty with an unknown probability distribution. Each player—depending on her individual access to and confidence over empirical data—builds an ambiguity set containing a family of potential probability distributions describing the uncertain event. The ambiguity set of different players is not necessarily identical, yielding a market with potentially heterogeneous ambiguity aversion. Built upon recent developments in the field of Wasserstein distributionally robust chance-constrained optimization, each ambiguity-averse player maximizes her own expected payoff under the worst-case probability distribution within her ambiguity set. Using an affine policy and a conditional value-at-risk approximation of chance constraints, we define a tractable Nash game. We prove that under certain conditions a unique Nash equilibrium point exists, which coincides with the solution of a single optimization problem. Numerical results indicate that players with comparatively lower consumption utility are highly exposed to rival ambiguity aversion.

Keywords Distributionally robust equilibrium problem · Nash game · Wasserstein ambiguity set · Heterogeneous ambiguity aversion

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1 Introduction

This article considers a perfectly competitive market for a single commodity that clears well in advance of the realization of an uncertain event ξ. This event is a common source of uncertainty for all market players, namely $y_1, y_2, ..., Y$. These players are uncertainty-aware, and forecast the probability distribution $f(\xi)$ describing the uncertain event ξ. Based on the individual probabilistic forecast, each player solves a stochastic optimization problem to determine her optimal market participation strategy, aiming to maximize her expected payoff. The collection of individual optimization problems results in a stochastic Nash equilibrium problem, whose solution provides the market-clearing outcome.

1.1 Ambiguity aversion: definition and its heterogeneity

One extreme case in modeling the common source of uncertainty is to assume that the true probability distribution $f(\xi)$ is known and publicly available for all players. This case is illustrated in Figure 1(a). However, it is rather unlikely that this assumption holds true in reality.

Pursuing a more general case, we relax the assumption on the availability of the true probability distribution $f(\xi)$ and generate a family of potential distributions, the so-called *ambiguity set*. This case is depicted in Figure 1(b). In this case, the players are *ambiguity-averse* [1], [2], meaning that they endogenously determine the worst-case distribution in their ambiguity set, and optimize their market participation strategy problem against such a distribution.[1] Although this case offers a more general framework for modeling uncertainty compared to the extreme case in Figure 1(a), it is not the most general case as it assumes *homogeneous* ambiguity aversion, i.e., an identical ambiguity set for all players.[2]

The most general case, schematically depicted in Figure 1(c), is the one wherein every market player possesses her own private empirical data and builds her individual ambiguity set, which is not necessarily identical to that of other players. The rationale behind this case is that even if the empirical data are publicly available, market players may still differently build their individual ambiguity sets, reflecting their heterogeneous confidence in those empirical data. Hereafter, we call this case as the one with *heterogeneous* ambiguity aversion.

[1] Another potential generalization of the first case would be the case in which players possess different probability distribution functions and each one believes that her function is the true one. However, this would lead to a discussion on asymmetric information about an uncertain event [3], whereas this work focuses on ambiguity aversion against an uncertain event.

[2] Note that in this case the worst-case distribution of players, in contrast to their ambiguity set, is not necessarily identical.

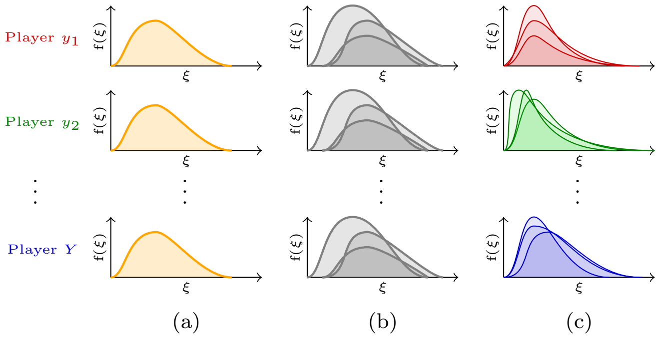

Fig. 1: Plot 1(a) shows the case in which all players know the true probability distribution. Plot 1(b) illustrates the case in which the true distribution is unknown and thus players consider an ambiguity set, although it is identical for all. Plot 1(c) refers to the case in which each player forms her own individual ambiguity set, resulting in heterogeneous ambiguity aversion.

1.2 Ambiguity aversion via distributionally robust chance-constrained optimization

We use a distributionally robust optimization approach [4]–[6] to include individual ambiguity sets within stochastic decision-making problems of players. This gives rise to a generalized formulation of a distributionally robust Nash equilibrium problem. We apply a Wasserstein probability distance metric to build individual ambiguity sets [7], [8]. Unlike the illustration in Figure 1, the ambiguity set of each player includes an infinite number of probability distributions that are sufficiently close to the empirical distribution. With this approach, each ambiguity-averse player maximizes her payoff in expectation with respect to the worst-case probability distribution in her ambiguity set.

The stochastic optimization problems of players may include their operational constraints. This is the case of market players in physical systems, e.g., energy or transportation systems. In the case the uncertain parameter appears in constraints, the resulting optimization problem will embody an infinite number of probabilistic constraints, since every constraint should be fulfilled for any realization drawn from the worst-case probability distribution. Aiming to achieve a tractable problem formulation, we enforce probabilistic constraints in the form of distributionally robust chance constraints [9], [10]. We decompose the uncertain event $L(\xi)$ into a deterministic forecast L and a stochastic component ξ, showing the uncertain forecast error. Additionally, we recast uncertainty-dependent decision variables using an affine policy [11]. By introducing a linear reformulation of distributionally robust objective functions [7] as well as applying the worst-case Conditional Value-at-Risk (CVaR) approximation of distributionally robust chance constraints [9], [12], we define a tractable convex Nash game. For this Nash game, we show—given a quadratic regularizer in the objective function of certain players as well as convex and compact strategy sets for all players—the existence of a unique

Nash equilibrium point. In addition, we provide the mathematical formulation of an equivalent single and convex optimization problem that can be efficiently solved.

1.3 State of the art, contributions, and paper organization

From a mathematical point of view, the work at hand generally lies in the domain of stochastic Nash games [13]–[16]. More precisely, this work models payoff functions by distributionally robust expected values and reformulates stochastic strategy sets through distributionally robust chance constraints, resulting in a distributionally robust Nash game [8], [10], [17]–[20]. The existing works on distributionally robust games can be divided into two research strands. The first one includes those works that build ambiguity sets using moments, e.g., mean and covariance, whose values are captured from the empirical data. Examples of such works are [10], [17], [18] and [19]. The research works within the second strand, e.g., [8], [19], [20], define ambiguity sets based on probabilistic distance metrics, e.g., Wasserstein metric. In both strands the possible existence of a Nash equilibrium point was proven [10], [19]. In addition, [8] and [10] show the equivalence of a distributionally robust chance-constrained Nash game to a single optimization problem.

From a conceptual perspective, our work investigates a market equilibrium given ambiguity-averse market players: The article at hand offers for the first time a comprehensive problem formulation of a market in which players may be ambiguity-averse and are subject to the same source of uncertainty. Depending on the parameterization of individual Wasserstein ambiguity sets, the proposed tractable Nash game is able to model various circumstances in which all players are (*i*) ambiguity-neutral, (*ii*) homogeneously ambiguity-averse, and (*iii*) heterogeneously ambiguity-averse owing to individual confidence in empirical data and/or access to private empirical data.

From a methodological perspective, differently to [20] that studies a generalized distributionally robust Nash equilibrium problem with coupling constraints, we consider a pure distributionally robust Nash equilibrium problem in which market players are only linked through their payoff functions. Their decision sets are independent of each other. Similar to [8] and [10] we are interested in providing an analytical proof for the existences of a Nash equilibrium point. While [8] and [10] address a general game-theoretic framework, this work relies on an affine policy, the worst-case CVaR approximation of distributionally robust chance constraints, and quadratic regularizers, and thereby, proves the existence and uniqueness of a Nash equilibrium point. Furthermore, we show that for the underlying Nash game built upon Wasserstein ambiguity sets, the Nash equilibrium point coincides with the solution of a single optimization problem that can be efficiently solved by commercial solvers. Our numerical results highlight that the realized utility of a market player with a comparatively low consumption utility highly depends on the degree of ambiguity aversion of the rival market players.

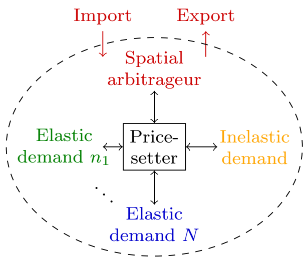

Fig. 2: Market structure with four types of players, namely a price-inelastic stochastic demand, a number of price-elastic demands, a spatial arbitrageur and a fictitious price-setter.

The remainder of this paper is laid out as follows. In Section 2 we introduce the distributionally robust Nash equilibrium problem. Section 3 provides the problem reformulation based on distributionally robust chance constraints and an affine policy. In Section 4 we provide a linear reformulation of distributionally robust objective functions as well as the worst-case CVaR constraints as approximation of distributionally robust chance constraints, and define a tractable Nash game. We discuss numerical results in Section 5. Section 6 concludes. The methodology for the linear reformulation of objective functions and the worst-case CVaR approximation of chance constraints as well as all mathematical proofs are available in four appendices. The source code is publicly available in [21].

2 Problem statement

We consider a perfectly competitive *local* market.[3] Four types of players exist, as illustrated in Figure 2. The first type of players is a single *price-inelastic demand*, representing the aggregation of all inelastic demands—these demands are willing to buy electricity an any price. This player is a pure stochastic load without a decision variable. The second type of players corresponds to a number of *price-elastic demands* $n \in \mathcal{N}$ indicated by $(\cdot)^{\mathrm{Ed}}$, who maximize their own consumption utility. The third type of players is a single *spatial arbitrageur* indicated by $(\cdot)^{\mathrm{Ar}}$, who maximizes her profit from importing and exporting the trading commodity between the local market and the outside, e.g., a wholesale market. Thereby, she ensures liquidity of the local market. The last player is a single *price-setter*, who is a fictitious player [22], indicated by $(\cdot)^{\mathrm{Ps}}$, who reveals social welfare maximizing prices.

An example of such a market is a local energy market inside an energy community, in which a number of spatially closely located households owning rooftop photovoltaic systems with uncertain power generation trade electricity [23], [24]. Such a local market may contribute to matching electric-

[3] The assumption of a perfectly competitive local market provides a benchmark estimation on the market impact of ambiguity aversion. In practice, market power in an imperfect competition can be an issue in a local market, although it is left aside to be addressed in future research.

ity supply and demand without stressing the surrounding infrastructure, e.g., high-voltage transmission and low-voltage distribution networks. In addition, a local energy market would allow the direct market participation of comparatively small entities, which usually do not have access to wholesale markets. However, the efficiency of such a local market significantly depends on the uncertainty and risk aversion of the market participants [25].

We model the ambiguity-averse decision-making problem of a given player through a distributionally robust optimization problem of the form

$$\underset{z}{\text{Min}} \max_{F \in \mathcal{D}} \mathbb{E}_F[g(z,\xi)], \tag{1}$$

where $g(z,\xi)$ is an uncertainty-dependent disutility function. In detail, the player in question makes the decision z in expectation $\mathbb{E}_F[\cdot]$ of her disutility $g(z,\xi)$, given the uncertain parameter ξ. This parameter follows the worst-case probability distribution F that is endogenously selected from the ambiguity set $\mathcal{D}$. Throughout this work we indicate parameters and variables depending on the uncertain event ξ by a tilde, i.e., $\widetilde{(\cdot)}$.

2.1 Distributionally robust Nash equilibrium problem

The consumption of the price-inelastic aggregated demand is the only source of uncertainty in this work, denoted by $\tilde{L}(\xi)$ including the one and only stochastic parameter ξ.[4] Given the market-clearing price $\tilde{\lambda}(\xi)$ under any realization of ξ, this demand pays

$$\tilde{\lambda}(\xi)\tilde{L}(\xi). \tag{2}$$

For the same given market-clearing price $\tilde{\lambda}(\xi)$, each price-elastic demand n minimizes her expected disutility as

$$\left\{ \underset{\tilde{d}_n(\xi)}{\text{Min}} \max_{F_n^{\text{Ed}} \in \mathcal{D}_n^{\text{Ed}}} \mathbb{E}_{F_n^{\text{Ed}}}\left[\tilde{\lambda}(\xi)\tilde{d}_n(\xi) - U_n\tilde{d}_n(\xi)\right] \right. \tag{3a}$$

$$\left. \text{s.t. } 0 \le \tilde{d}_n(\xi) \le \overline{D}_n \right\}, \ \forall n \in \mathcal{N}, \tag{3b}$$

where the variable $\tilde{d}_n(\xi)$ is her consumption, whose value is enforced by (3b) to lie between zero and the maximum consumption level $\overline{D}_n$. The parameter U_n in the objective function (3a) indicates the value of one unit of the trading commodity for demand n. Accordingly, $U_n\tilde{d}_n(\xi)$ gives the total value that demand n gains by consuming $\tilde{d}_n(\xi)$, whereas $\tilde{\lambda}(\xi)\tilde{d}_n(\xi)$ is the total payment of this price-elastic demand. This player builds the ambiguity set $\mathcal{D}_n^{\text{Ed}}$ and minimizes her expected disutility under the worst-case probability distribution F_n^{Ed}.

Similarly, the spatial arbitrageur minimizes her expected disutility as

$$\underset{\tilde{p}(\xi)}{\text{Min}} \max_{F^{\text{Ar}} \in \mathcal{D}^{\text{Ar}}} \mathbb{E}_{F^{\text{Ar}}}\left[C\tilde{p}(\xi) - \tilde{\lambda}(\xi)\tilde{p}(\xi)\right] \tag{4a}$$

[4] Later in Section 3.2 we decompose the uncertain price-inelastic demand $\tilde{L}(\xi)$ into a deterministic component L and a separate stochastic component ξ.

$$\text{s.t.} \ -\overline{P} \leq \tilde{p}(\xi) \leq \overline{P}, \tag{4b}$$

where the variable $\tilde{p}(\xi)$ denotes the amount of the trading commodity to be imported to—if $\tilde{p}(\xi) > 0$—or exported from—if $\tilde{p}(\xi) < 0$—the local market, both at an identical fixed cost C. This cost shows the price of the commodity outside the local market. If $\tilde{p}(\xi) > 0$, the arbitrageur buys the trading commodity outside the local market at price C and sells it back in the local market at price $\tilde{\lambda}(\xi)$. Similarly, if $\tilde{p}(\xi) < 0$, the arbitrageur buys the trading commodity from the local market at price $\tilde{\lambda}(\xi)$ and sells it back outside the local market at price C. The constraint (4b) sets the bound $\overline{P}$ on $\tilde{p}(\xi)$, indicating the potential capacity limit of the trade between the local and the outside market. One can hypothesize that the market-clearing price $\tilde{\lambda}(\xi)$ will be equal to C if this constraint is non-binding, otherwise it may take a different value. The arbitrageur builds the ambiguity set $\mathcal{D}^{\text{Ar}}$ and minimizes her expected disutility under the worst-case probability distribution F^{Ar}.

Finally, for given trading decisions $\tilde{d}_n(\xi)$ and $\tilde{p}(\xi)$ the price-setter determines the market-clearing price $\tilde{\lambda}(\xi)$ by maximizing the utility of all players as

$$\underset{\tilde{\lambda}(\xi)}{\text{Max}} \ \tilde{\lambda}(\xi)\Big(\tilde{p}(\xi) - \sum_{n\in\mathcal{N}} \tilde{d}_n(\xi) - \tilde{L}(\xi)\Big). \tag{5}$$

The price-setter chooses the price $\tilde{\lambda}(\xi)$ in (5) under any realization of ξ such that the cost for buyers is minimized and the revenue for sellers is maximized.

Recall that the price $\tilde{\lambda}(\xi)$ is given in the optimization problem (3) of each price-elastic demand and in the optimization problem (4) of the spatial arbitrageur. In contrast, the price $\tilde{\lambda}(\xi)$ is a variable in the optimization problem (5) of the price-setter, while variables in (3) and (4), i.e., $\tilde{d}_n(\xi)$ and $\tilde{p}(\xi)$, are given in (5). This makes these three problems interconnected, such that they should be solved at once.[5] The collection of optimization problems (3), (4) and (5) constitutes the distributionally robust Nash equilibrium problem.

2.2 Wasserstein ambiguity sets

This section explains how to build the ambiguity set $\mathcal{D}_n^{\text{Ed}}$ for each elastic demand n as well as the ambiguity set $\mathcal{D}^{\text{Ar}}$ for the spatial arbitrageur. The ambiguity set $\mathcal{D}_n^{\text{Ed}}$ comprises all probability distributions F_n^{Ed} in the neighborhood of a central empirical probability distribution $\widehat{F}_n^{\text{Ed}}$, for which $i \in \mathcal{I}_n^{\text{Ed}}$

[5] The reason for considering such a fictitious player, i.e., the price-setter, is that without it, all other players, i.e., price-inelastic aggregated demand, price-elastic demands, and spatial arbitrageur, will be linked via a common constraint, namely the demand-supply balance equality. It would result in a *generalized* Nash equilibrium problem with shared constraints, for which the proof of existence and uniqueness of a Nash equilibrium point is not necessarily straightforward. In contrast, the chosen problem structure comprising the fictitious price-setter yields a pure Nash equilibrium problem, for which the existence and uniqueness of a Nash equilibrium point can be proven in a straightforward manner. With this fictitious player, the strategy of each player still *implicitly* depends on the strategy of each other player through the price-setter's decision variable $\tilde{\lambda}(\xi)$.

denotes the set of empirical samples, e.g., historical observations, available to the respective elastic demand n. Following [7], we measure the distance between a distribution F_n^{Ed} and the empirical distribution $\widehat{F}_n^{\text{Ed}}$ based on the Wasserstein distance $\varDelta(\cdot,\cdot)$ as

$$\varDelta(F_n^{\text{Ed}}, \widehat{F}_n^{\text{Ed}}) = \min_{\varPi_n^{\text{Ed}}} \int \Big(\sum_{i \in \mathcal{I}_n^{\text{Ed}}} |\xi - \widehat{\xi}_{ni}^{\text{Ed}}|^p \Big)^{\frac{1}{p}} \varPi_n^{\text{Ed}}(d\xi, d\widehat{\xi}_{ni}^{\text{Ed}}), \ \forall n, \tag{6a}$$

in which $\varPi_n^{\text{Ed}}$ is a joint probability distribution of the uncertain parameter ξ and empirical data $\widehat{\xi}_{ni}^{\text{Ed}}$ with marginals F_n^{Ed} and $\widehat{F}_n^{\text{Ed}}$, respectively. The symbol p refers to an arbitrary norm[6] to be applied on the difference between the uncertain parameter ξ and empirical data $\widehat{\xi}_{ni}^{\text{Ed}}$.

Similarly, the spatial arbitrageur has access to her own individual empirical samples $i \in \mathcal{I}^{\text{Ar}}$, which are not necessarily identical to those of other players. We measure her Wasserstein distance $\varDelta(\cdot,\cdot)$ as

$$\varDelta(F^{\text{Ar}}, \widehat{F}^{\text{Ar}}) = \min_{\varPi^{\text{Ar}}} \int \Big(\sum_{i \in \mathcal{I}^{\text{Ar}}} |\xi - \widehat{\xi}_{i}^{\text{Ar}}|^p \Big)^{\frac{1}{p}} \varPi^{\text{Ar}}(d\xi, d\widehat{\xi}_{i}^{\text{Ar}}). \tag{6b}$$

We now define Wasserstein ambiguity sets $\mathcal{D}_n^{\text{Ed}}$ and $\mathcal{D}^{\text{Ar}}$ as

$$\mathcal{D}_n^{\text{Ed}} = \big\{ F_n^{\text{Ed}} \in \mathcal{M}(\varXi) \colon \varDelta(F_n^{\text{Ed}}, \widehat{F}_n^{\text{Ed}}) \le \rho_n^{\text{Ed}} \big\}, \ \forall n, \tag{6c}$$

$$\mathcal{D}^{\text{Ar}} = \big\{ F^{\text{Ar}} \in \mathcal{M}(\varXi) \colon \varDelta(F^{\text{Ar}}, \widehat{F}^{\text{Ar}}) \le \rho^{\text{Ar}} \big\}, \tag{6d}$$

in which the support $\varXi = \{\xi \in \mathbb{R} \colon \underline{H} \le \xi \le \overline{H}\}$ restricts the uncertain parameter ξ by a lower bound $\underline{H}$ and an upper bound $\overline{H}$, such that the worst-case probability distribution takes realistic values. We assume that all players have perfect and common information about the support. Lastly, the non-negative parameters ρ_n^{Ed} and ρ^{Ar} in (6c) and (6d), the so-called *Wasserstein radii*, limit the distance between probability distributions F_n^{Ed} and F^{Ar} within ambiguity sets and empirical probability distributions $\widehat{F}_n^{\text{Ed}}$ and $\widehat{F}^{\text{Ar}}$, respectively.

Figure 3 illustrates the implication of empirical probability distributions $\widehat{F}_n^{\text{Ed}}$ and $\widehat{F}^{\text{Ar}}$ as well as the choice of ρ_n^{Ed} and ρ^{Ar}, describing the confidence in those empirical distributions, and therefore the aversion against ambiguity in the empirical data [2], [26].[7]

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[6] We will apply later in Appendix A the infinity norm to derive a linear reformulation.

[7] In the case the intersection of ambiguity sets of different players is empty, the underlying Nash equilibrium problem might be infeasible. However, the feasibility can be restored by allowing involuntarily curtailment of the price-inelastic aggregated demand $\tilde{L}(\xi)$ while considering a significant cost (penalty) incurred by not fully supplying such a demand. This work has only focused on cases wherein the intersection of ambiguity sets is not empty, and leaves the potential issue of feasibility restoration for the future work.

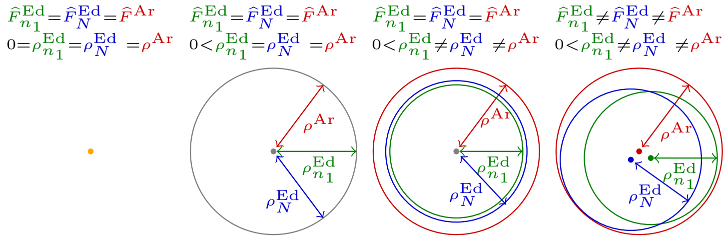


Fig. 3: The Wasserstein ambiguity sets $\mathcal{D}_n^{\text{Ed}}$ and $\mathcal{D}^{\text{Ar}}$ can represent four different circumstances. In the first case there is no ambiguity, and therefore all players consider a single and common probability distribution, see plot 3(a). In the second case there is homogeneous ambiguity aversion among all players, see plot 3(b). In the third case there is heterogeneous ambiguity aversion among players owing to their individual confidences in common empirical data, see plot 3(c-1). Finally, in the fourth case there is heterogeneous ambiguity aversion among players owing to not only their individual confidences but also their individual empirical data, see plot 3(c-2).

3 Towards computational tractability

The distributionally robust Nash equilibrium problem (3)–(5) is computationally intractable, since it optimizes over infinite-dimensional variables $\tilde{d}_n(\xi)$, $\tilde{p}(\xi)$, and $\tilde{\lambda}(\xi)$, subject to infinite-dimensional constraints (3b) and (4b). To achieve tractability, we apply some convex reformulations as illustrated in Figure 4. For the sake of clarity, this figure includes the inelastic demand, although there is no optimization problem for this player. In Section 3.1 we use distributionally robust chance-constrained programming [7] to cope with the infinite-dimensional nature of constraints (3b) and (4b). We then introduce an affine policy [11] in Section 3.2 to decompose uncertainty-dependent decision variables, and analytically derive the market-clearing price in Section 3.3. Based on a linear reformulation of distributionally robust objective functions [7] as well as the worst-case CVaR approximation of distributionally robust chance constraints [9], [12], [27] we define a tractable Nash game.[8]

3.1 Distributionally robust chance constraints

We consider a generic individual distributionally robust chance constraint of the form

$$\min_{F\in\mathcal{D}} \mathbb{P}_F[h(z,\xi)\leq 0]\geq 1-\epsilon, \tag{7}$$

[8] The methodology to linearly reformulate a distributionally robust objective function [7] as well as the worst-case CVaR approximation of a distributionally robust chance constraint [9], [12], [27] is available in Appendix A.

	Equilibrium problem	Chance constraints	Affine policy	Reformulation, approximation	Nash game
(Inelastic Demand:	(2)	⟶	(10b)	⟶	(16))
Elastic Demand $n \in \mathcal{N}$:	(3)	(8)	(11)	(18)	(18)
Spatial arbitrageur:	(4)	(9)	(12)	(20)	(20)
Price setter:	(5)	⟶	(13)	⟶	(21)

Fig. 4: By introducing distributionally robust chance constraints, applying an affine policy, and reformulating objective functions as well as approximating chance constraints, we derive a tractable Nash game corresponding to the distributionally robust Nash equilibrium problem (3)–(5).

where the decision z is made under the worst-case probability distribution F that is endogenously determined from the given ambiguity set $\mathcal{D}$. The probability $\mathbb{P}_F[\cdot]$ of the probabilistic constraint $h(z,\xi) \leq 0$ to be fulfilled is greater than or equal to $1-\epsilon$. Note that ϵ is a parameter to be tuned by the respective decision-maker, whose value lies between zero and one. Accordingly, we rewrite constraints (3b) as

$$\min_{F_n^{\mathrm{Ed}} \in \mathcal{D}_n^{\mathrm{Ed}}} \mathbb{P}_{F_n^{\mathrm{Ed}}}\big[0 \leq \tilde{d}_n(\xi)\big] \geq 1-\epsilon, \ \forall n, \tag{8a}$$

$$\min_{F_n^{\mathrm{Ed}} \in \mathcal{D}_n^{\mathrm{Ed}}} \mathbb{P}_{F_n^{\mathrm{Ed}}}\big[\tilde{d}_n(\xi) \leq \overline{D}_n\big] \geq 1-\epsilon, \ \forall n. \tag{8b}$$

Similarly, constraints (4b) are rewritten as

$$\min_{F^{\mathrm{Ar}} \in \mathcal{D}^{\mathrm{Ar}}} \mathbb{P}_{F^{\mathrm{Ar}}}\big[-\overline{P} \leq \tilde{p}(\xi)\big] \geq 1-\epsilon, \tag{9a}$$

$$\min_{F^{\mathrm{Ar}} \in \mathcal{D}^{\mathrm{Ar}}} \mathbb{P}_{F^{\mathrm{Ar}}}\big[\tilde{p}(\xi) \leq \overline{P}\big] \geq 1-\epsilon. \tag{9b}$$

Without loss of generality, we consider identical ϵ in all aforementioned chance constraints.[9]

3.2 Affine policy

We decompose the uncertain event, i.e., the consumption $\tilde{L}(\xi)$ of the aggregated inelastic demand, as

$$\tilde{L}(\xi) = L + \xi. \tag{10a}$$

The parameter L is the nominal, e.g., tentative, inelastic demand, which is independent of uncertainty. However, ξ is the uncertain deviation, either positive or negative, from L at a future stage. Substituting (10a) in (2) yields the consumption cost of the inelastic demand as

$$\tilde{\lambda}(\xi)(L+\xi). \tag{10b}$$

[9] Assigning different values for ϵ motivates a case where market players are heterogeneously risk averse against the violation risk of operational constraints. This risk aversion is beyond the scope of this work, and therefore we consider an identical value ϵ for all players, which can be interpreted as a case with homogeneously risk-averse players.

We apply an affine policy [11] to decisions made by price-elastic demands and the spatial arbitrageur. Accordingly, the probabilistic decision variables $\tilde{d}_n(\xi)$ and $\tilde{p}(\xi)$ are approximated by

$$\tilde{d}_n(\xi) = d_n - \alpha_n^{\mathrm{Ed}}\xi, \ \forall n, \tag{10c}$$
$$\tilde{p}(\xi) = p + \alpha^{\mathrm{Ar}}\xi, \tag{10d}$$

where variables d_n and p are nominal trades given the expected inelastic demand L. In addition, the free variables, i.e., either positive or negative, α_n^{Ed} and α^{Ar}, the so-called *participation factors*, are in per-unit and show the linear response of the price-elastic demand n and the spatial arbitrageur at a future stage to the uncertain deviation ξ, respectively. In other words, they indicate the contribution of the corresponding player to offset any supply–demand imbalance at the future stage, when the uncertainty ξ is realized. For example, consider a deviation $\xi > 0$, meaning that the realized consumption of the inelastic demand is more than the tentative one. According to (10c) and (10d), the price-elastic demand n and the spatial arbitrageur would respond to this deviation by decreased consumption—ensured by the minus in (10c)—and by additional imports—enforced by the plus in (10d)—, respectively.

By introducing distributionally robust chance constraints (8a) and (8b), and by applying the affine policy used in (10c), problem (3) of each price-elastic demand n reads

$$\Bigg\{ \underset{d_n, \alpha_n^{\mathrm{Ed}}}{\mathrm{Min}} \ \max_{F_n^{\mathrm{Ed}} \in \mathcal{D}_n^{\mathrm{Ed}}} \ \mathbb{E}_{F_n^{\mathrm{Ed}}} \big[\big(\tilde{\lambda}(\xi) - U_n\big)\big(d_n - \alpha_n^{\mathrm{Ed}}\xi\big)\big] \tag{11a}$$
$$\text{s.t.} \ \min_{F_n^{\mathrm{Ed}} \in \mathcal{D}_n^{\mathrm{Ed}}} \ \mathbb{P}_{F_n^{\mathrm{Ed}}} \big[0 \le (d_n - \alpha_n^{\mathrm{Ed}}\xi)\big] \ge 1 - \epsilon, \tag{11b}$$
$$\min_{F_n^{\mathrm{Ed}} \in \mathcal{D}_n^{\mathrm{Ed}}} \ \mathbb{P}_{F_n^{\mathrm{Ed}}} \big[(d_n - \alpha_n^{\mathrm{Ed}}\xi) \le \overline{D}_n\big] \ge 1 - \epsilon \ \Bigg\}, \ \forall n. \tag{11c}$$

Similarly, we rewrite problem (4) of the spatial arbitrageur using (9a), (9b), and (10d) as

$$\underset{p, \alpha^{\mathrm{Ar}}}{\mathrm{Min}} \ \max_{F^{\mathrm{Ar}} \in \mathcal{D}^{\mathrm{Ar}}} \ \mathbb{E}_{F^{\mathrm{Ar}}} \big[\big(C - \tilde{\lambda}(\xi)\big)\big(p + \alpha^{\mathrm{Ar}}\xi\big)\big] \tag{12a}$$
$$\text{s.t.} \ \min_{F^{\mathrm{Ar}} \in \mathcal{D}^{\mathrm{Ar}}} \ \mathbb{P}_{F^{\mathrm{Ar}}} \big[-\overline{P} \le p + \alpha^{\mathrm{Ar}}\xi \big] \ge 1 - \epsilon, \tag{12b}$$
$$\min_{F^{\mathrm{Ar}} \in \mathcal{D}^{\mathrm{Ar}}} \ \mathbb{P}_{F^{\mathrm{Ar}}} \big[p + \alpha^{\mathrm{Ar}}\xi \le \overline{P} \big] \ge 1 - \epsilon. \tag{12c}$$

Lastly, substituting (10a), (10c), and (10d) in (5) yields

$$\underset{\tilde{\lambda}(\xi)}{\mathrm{Max}} \ \tilde{\lambda}(\xi)\Big(p + \alpha^{\mathrm{Ar}}\xi - \sum_{n \in \mathcal{N}} (d_n - \alpha_n^{\mathrm{Ed}}\xi) - L - \xi\Big). \tag{13}$$

3.3 Analytical derivation of market-clearing prices

This section focuses on the unconstrained problem (13), whose optimality condition imposes

$$\frac{\partial \mathcal{L}^{(13)}}{\partial \tilde{\lambda}(\xi)} = (p + \alpha^{\mathrm{Ar}}\xi) - \sum_{n\in\mathcal{N}} (d_n - \alpha_n^{\mathrm{Ed}}\xi) - (L + \xi) = 0, \tag{14a}$$

where $\mathcal{L}^{(13)}$ denotes the Lagrangian function of (13). Given the response of the spatial arbitrageur α^{Ar} as well as the response of elastic demands $\alpha_n^{\mathrm{Ed}}, \forall n$, the equality constraint (14a) holds true for any realization of ξ if

$$\alpha^{\mathrm{Ar}}\xi + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}}\xi = \xi \Leftrightarrow \alpha^{\mathrm{Ar}} + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}} = 1, \tag{14b}$$

$$p - \sum_{n\in\mathcal{N}} d_n - L = 0. \tag{14c}$$

The equality constraints (14b) and (14c) are derived by separating ξ-dependent uncertain and ξ-independent nominal terms in (14a). Thereby, the equality constraints (14b) imposes that the total response of the spatial arbitrageur and the price-elastic demands should be able to fully offset the supply–demand imbalance at the future stage.[10] In addition, the equality constraint (14c) imposes that all nominal demands should be fully supplied.

The analytical procedure from (13) to (14b)-(14c) suggests that one could also decompose the probabilistic market-clearing price $\tilde{\lambda}(\xi)$ to two deterministic variables λ^{B} and λ^{E}. Therefore, we rewrite the optimization problem (13) of the price-setter by a collection of two deterministic optimization problems as

$$\underset{\lambda^{\mathrm{B}}}{\mathrm{Max}}\ \lambda^{\mathrm{B}}\Big(\alpha^{\mathrm{Ar}} + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}} - 1\Big), \tag{14d}$$

$$\underset{\lambda^{\mathrm{E}}}{\mathrm{Max}}\ \lambda^{\mathrm{E}}\Big(p - \sum_{n\in\mathcal{N}} d_n - L\Big). \tag{14e}$$

Since the optimality conditions of (14d) and (14e) are identical to the equality constraints (14b) and (14c), any λ^{E} and λ^{B} are optimal solutions of (14d) and (14e) as long as these optimality conditions are fulfilled. The variable λ^{E} provides the deterministic market-clearing price for the underlying commodity. In addition, λ^{B} provides the payment due to *balancing services*, i.e., the payment to remunerate price-elastic demands and the spatial arbitrageur for their response to any supply–demand imbalance.

---

[10] Note that $\alpha_n^{\mathrm{Ed}}, \forall n$ and α^{Ar} are free variables meaning that they can be either positive or negative and even greater than the absolute value of 1 as long as their summation is equal to 1. Thereby, an elastic demand could, for example, increase her consumption, i.e., $\alpha_n^{\mathrm{Ed}} < 0$, although the local market faces a deficit in supply given by a deviation $\xi > 0$ as long as any other player, e.g., the spatial arbitrageur by $\alpha^{\mathrm{Ar}} > 1$ or another elastic demand, offsets the demand increase.

Eventually, given that (14b) and (14c) hold, we can replace the terms including the price $\tilde{\lambda}(\xi)$ in (10b), (11a) and (12a) as

$$\tilde{\lambda}(\xi)L = \lambda^{\mathrm{E}}L; \qquad \tilde{\lambda}(\xi)\xi = \lambda^{\mathrm{B}}, \tag{15a}$$

$$\tilde{\lambda}(\xi)d_n = \lambda^{\mathrm{E}}d_n; \qquad \tilde{\lambda}(\xi)\alpha_n^{\mathrm{Ed}}\xi = \lambda^{\mathrm{B}}\alpha_n^{\mathrm{Ed}}, \tag{15b}$$

$$\tilde{\lambda}(\xi)p = \lambda^{\mathrm{E}}p; \qquad \tilde{\lambda}(\xi)\alpha^{\mathrm{Ar}}\xi = \lambda^{\mathrm{B}}\alpha^{\mathrm{Ar}}. \tag{15c}$$

4 A tractable Nash game

We revisit our distributionally robust Nash equilibrium problem given the analytical prices derived in Section 3.3.

Price-inelastic demand

The payment of the inelastic demand (10b) recasts as

$$\lambda^{\mathrm{E}}L + \lambda^{\mathrm{B}}, \tag{16}$$

indicating that the inelastic demand is charged at the price λ^{E} for the nominal consumption L. In addition, she pays λ^{B} for the balancing services, as she deviates ξ from her nominal consumption L.

Price-elastic demand

Next, we revisit the optimization problem (11) of the price-elastic demand n. Pursuing an equilibrium solution existence and uniqueness, we make two slight changes. First, we arbitrarily introduce theoretical lower and upper bound A on the participation factor α_n. The rationale behind these bounds is to achieve a compact and closed strategy set, which is required later for the equilibrium solution existence proof. However, we select sufficiently large values for these bounds, and check *a posteriori* that these constraints are non-binding. Second, we add a quadratic regularizer [28] in the form of $c(z, x) = \frac{1}{2}\beta(z + x)^2$ to the objective function, in which β is a sufficiently small positive constant, e.g., 10^{-3}. A sufficiently small value for β will alter negligibly the social welfare of the market in comparison to $\beta = 0$. However, this quadratic regularizer, which can be institutionally interpreted as a transaction cost arising from trades, ensures an identical payoff for identical players. In addition, this regularizer yields a strongly monotone objective function, which is necessary later to achieve a unique equilibrium solution. The revisited problem (11) writes as

$$\Big\{ \underset{d_n, \alpha_n^{\mathrm{Ed}}}{\mathrm{Min}} \big(\lambda^{\mathrm{E}} - U_n\big)d_n - \lambda^{\mathrm{B}}\alpha_n^{\mathrm{Ed}} + c(d_n, \alpha_n^{\mathrm{Ed}}) + \max_{F_n^{\mathrm{Ed}} \in \mathcal{D}_n^{\mathrm{Ed}}} \mathbb{E}_{F_n^{\mathrm{Ed}}}\big[U_n\alpha_n^{\mathrm{Ed}}\xi\big] \tag{17a}$$

$$\text{s.t. (11b)–(11c)}, \tag{17b}$$

$$-A \leq \alpha_n^{\mathrm{Ed}} \leq A \Big\}, \ \forall n, \tag{17c}$$

in which the last ξ-dependent term in the objective function (17a) as well as the distributionally robust chance constraints (11b) and (11c) make the problem still intractable. We follow the convex reformulation technique proposed in [7] for a distributionally robust objective function. In addition, we use the worst-case CVaR constraints as an approximation of distributionally robust chance constraints [9], [12], and therefore, provide—except of the regularizer $c(d_n, \alpha_n^{\mathrm{Ed}})$ in the objective function—a purely linear approximation for (17).

Based on (22) and (23), we write the decision-making problem of the elastic demand n as

$$\Big\{\underset{\Xi_n^{\mathrm{Ed}}}{\mathrm{Min}}\ J_n^{\mathrm{Ed}} = \big(\lambda^{\mathrm{E}} - U_n\big) d_n - \lambda^{\mathrm{B}} \alpha_n^{\mathrm{Ed}} + c(d_n, \alpha_n^{\mathrm{Ed}}) + \phi_n^{\mathrm{Ed}} \rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \sigma_{ni}^{\mathrm{Ed}} \quad (18a)$$

Reformulation of (17a):

$$\text{s.t. } U_n \alpha_n^{\mathrm{Ed}} \widehat{\xi}_{ni}^{\mathrm{Ed}} + \sum_{b \in \mathcal{B}} \gamma_{nbi}^{\mathrm{Ed}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) \leq \sigma_{ni}^{\mathrm{Ed}} \ :\ \zeta_{ni}^{\mathrm{Ed.1a}}, \qquad \forall i, \quad (18b)$$

$$-\phi_n^{\mathrm{Ed}} \leq \sum_{b \in \mathcal{B}} Q_b \gamma_{nbi}^{\mathrm{Ed}} - U_n \alpha_n^{\mathrm{Ed}} \leq \phi_n^{\mathrm{Ed}} \ :\ \underline{\zeta}_{ni}^{\mathrm{Ed.1b}}, \overline{\zeta}_{ni}^{\mathrm{Ed.1b}}, \qquad \forall i, \quad (18c)$$

$$0 \leq \gamma_{nbi}^{\mathrm{Ed}} \ :\ \zeta_{nbi}^{\mathrm{Ed.1c}}, \qquad \forall b, i, (18d)$$

CVaR approximation of (11b):

$$\underline{\tau}_n^{\mathrm{Ed}} + \frac{1}{\epsilon} \Big(\underline{\phi}_n^{\mathrm{Ed}} \rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \underline{\sigma}_{ni}^{\mathrm{Ed}}\Big) \leq 0 \ :\ \zeta_n^{\mathrm{Ed.2a}}, \qquad (18e)$$

$$-d_n + \alpha_n^{\mathrm{Ed}} \widehat{\xi}_{ni}^{\mathrm{Ed}} - \underline{\tau}_n^{\mathrm{Ed}} + \sum_{b \in \mathcal{B}} \underline{\gamma}_{nbi}^{\mathrm{Ed1}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) \leq \underline{\sigma}_{ni}^{\mathrm{Ed}} : \zeta_{ni}^{\mathrm{Ed.2b}}, \quad \forall i, \quad (18f)$$

$$\sum_{b \in \mathcal{B}} \underline{\gamma}_{nbi}^{\mathrm{Ed2}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) \leq \underline{\sigma}_{ni}^{\mathrm{Ed}} \ :\ \zeta_{ni}^{\mathrm{Ed.2c}}, \qquad \forall i, \quad (18g)$$

$$-\underline{\phi}_n^{\mathrm{Ed}} \leq \sum_{b \in \mathcal{B}} Q_b \underline{\gamma}_{nbi}^{\mathrm{Ed1}} - \alpha_n^{\mathrm{Ed}} \leq \underline{\phi}_n^{\mathrm{Ed}} \ :\ \underline{\zeta}_{ni}^{\mathrm{Ed.2d}}, \overline{\zeta}_{ni}^{\mathrm{Ed.2d}}, \qquad \forall i, \quad (18h)$$

$$-\underline{\phi}_n^{\mathrm{Ed}} \leq \sum_{b \in \mathcal{B}} Q_b \underline{\gamma}_{nbi}^{\mathrm{Ed2}} \leq \underline{\phi}_n^{\mathrm{Ed}} \ :\ \underline{\zeta}_{ni}^{\mathrm{Ed.2e}}, \overline{\zeta}_{ni}^{\mathrm{Ed.2e}}, \qquad \forall i, \quad (18i)$$

$$0 \leq \underline{\gamma}_{nbi}^{\mathrm{Ed1}} \ :\ \zeta_{nbi}^{\mathrm{Ed.2f}}, \qquad \forall b, i, (18j)$$

$$0 \leq \underline{\gamma}_{nbi}^{\mathrm{Ed2}} \ :\ \zeta_{nbi}^{\mathrm{Ed.2g}}, \qquad \forall b, i, (18k)$$

CVaR approximation of (11c):

$$\overline{\tau}_n^{\mathrm{Ed}} + \frac{1}{\epsilon} \Big(\overline{\phi}_n^{\mathrm{Ed}} \rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \overline{\sigma}_{ni}^{\mathrm{Ed}}\Big) \leq 0 \ :\ \zeta_n^{\mathrm{Ed.3a}}, \qquad (18l)$$

$$d_n - \alpha_n^{\mathrm{Ed}} \widehat{\xi}_{ni}^{\mathrm{Ed}} - \overline{D}_n - \overline{\tau}_n^{\mathrm{Ed}} + \sum_{b \in \mathcal{B}} \overline{\gamma}_{nbi}^{\mathrm{Ed1}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) \leq \overline{\sigma}_{ni}^{\mathrm{Ed}} : \zeta_{ni}^{\mathrm{Ed.3b}}, \quad \forall i, \quad (18m)$$

$$\sum_{b \in \mathcal{B}} \overline{\gamma}_{nbi}^{\mathrm{Ed2}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) \leq \overline{\sigma}_{ni}^{\mathrm{Ed}} \ :\ \zeta_{ni}^{\mathrm{Ed.3c}}, \qquad \forall i, \quad (18n)$$

$$-\overline{\phi}_n^{\mathrm{Ed}} \leq \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{nbi}^{\mathrm{Ed1}} + \alpha_n^{\mathrm{Ed}} \leq \overline{\phi}_n^{\mathrm{Ed}} \;:\; \underline{\zeta}_{ni}^{\mathrm{Ed.3d}}, \overline{\zeta}_{ni}^{\mathrm{Ed.3d}}, \qquad \forall i, \quad (18o)$$

$$-\overline{\phi}_n^{\mathrm{Ed}} \leq \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{nbi}^{\mathrm{Ed2}} \leq \overline{\phi}_n^{\mathrm{Ed}} \;:\; \underline{\zeta}_{ni}^{\mathrm{Ed.3e}}, \overline{\zeta}_{ni}^{\mathrm{Ed.3e}}, \qquad \forall i, \quad (18p)$$

$$0 \leq \overline{\gamma}_{nbi}^{\mathrm{Ed1}} \;:\; \zeta_{nbi}^{\mathrm{Ed.3f}}, \qquad \forall b, i, (18q)$$

$$0 \leq \overline{\gamma}_{nbi}^{\mathrm{Ed2}} \;:\; \zeta_{nbi}^{\mathrm{Ed.3g}}, \qquad \forall b, i, (18r)$$

Constraint (17c):

$$\left.-A \leq \alpha_n^{\mathrm{Ed}} \leq A \;:\; \underline{\zeta}_n^{\mathrm{Ed.4a}}, \overline{\zeta}_n^{\mathrm{Ed.4a}} \right\}, \; \forall n, \; (18s)$$

where $\Xi_n^{\mathrm{Ed}} = \{d_n, \alpha_n^{\mathrm{Ed}}, \phi_n^{\mathrm{Ed}}, \sigma_{ni}^{\mathrm{Ed}}, \gamma_{nbi}^{\mathrm{Ed}}, \underline{\tau}_n^{\mathrm{Ed}}, \underline{\phi}_n^{\mathrm{Ed}}, \underline{\sigma}_{ni}^{\mathrm{Ed}}, \underline{\gamma}_{nbi}^{\mathrm{Ed1}}, \underline{\gamma}_{nbi}^{\mathrm{Ed2}}, \overline{\tau}_n^{\mathrm{Ed}}, \overline{\phi}_n^{\mathrm{Ed}}, \overline{\sigma}_{ni}^{\mathrm{Ed}}, \overline{\gamma}_{nbi}^{\mathrm{Ed1}}, \overline{\gamma}_{nbi}^{\mathrm{Ed2}}\}$, $\forall n \in \mathcal{N}$, and $|\mathcal{I}_n^{\mathrm{Ed}}|$ returns the cardinality of set $\mathcal{I}_n^{\mathrm{Ed}}$. Symbols followed a colon denote the dual variable of the respective constraint. We will need those dual variables later when we derive the Karush-Kuhn-Tucker conditions in Appendix C.2.

Spatial arbitrageur

Similarly, we revisit the optimization problem (12) of the spatial arbitrageur, yielding

$$\underset{p,\alpha^{\mathrm{Ar}}}{\mathrm{Min}} \; \big(C - \lambda^{\mathrm{E}}\big)p - \lambda^{\mathrm{B}}\alpha^{\mathrm{Ar}} + c(p,\alpha^{\mathrm{Ar}}) + \max_{F^{\mathrm{Ar}}\in\mathcal{D}^{\mathrm{Ar}}} \mathbb{E}_{F^{\mathrm{Ar}}}\big[C\alpha^{\mathrm{Ar}}\xi\big] \qquad (19a)$$

$$\text{s.t. } (12b)\text{–}(12c), \qquad (19b)$$

$$-A \leq \alpha^{\mathrm{Ar}} \leq A, \qquad (19c)$$

whose linear approximation writes as

$$\underset{\Xi^{\mathrm{Ar}}}{\mathrm{Min}} \; J^{\mathrm{Ar}} = \big(C - \lambda^{\mathrm{E}}\big)p - \lambda^{\mathrm{B}}\alpha^{\mathrm{Ar}} + c(p,\alpha^{\mathrm{Ar}}) + \phi^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \sigma_i^{\mathrm{Ar}} \qquad (20a)$$

Reformulation of (19a):

$$\text{s.t. } C\alpha^{\mathrm{Ar}}\widehat{\xi}_i^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} \gamma_{bi}^{\mathrm{Ar}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) \leq \sigma_i^{\mathrm{Ar}} \;:\; \zeta_i^{\mathrm{Ar.1a}}, \qquad \forall i, \quad (20b)$$

$$-\phi^{\mathrm{Ar}} \leq \sum_{b\in\mathcal{B}} Q_b\gamma_{bi}^{\mathrm{Ar}} - C\alpha^{\mathrm{Ar}} \leq \phi^{\mathrm{Ar}} \;:\; \underline{\zeta}_i^{\mathrm{Ar.1b}}, \overline{\zeta}_i^{\mathrm{Ar.1b}}, \qquad \forall i, \quad (20c)$$

$$0 \leq \gamma_{bi}^{\mathrm{Ar}} \;:\; \zeta_{bi}^{\mathrm{Ar.1c}}, \qquad \forall b, i \; (20d)$$

CVaR approximation of (12b):

$$\underline{\tau}^{\mathrm{Ar}} + \frac{1}{\epsilon}\Big(\underline{\phi}^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \underline{\sigma}_i^{\mathrm{Ar}}\Big) \leq 0 \;:\; \zeta^{\mathrm{Ar.2a}}, \qquad (20e)$$

$$-\overline{P} - p - \alpha^{\mathrm{Ar}}\widehat{\xi}_i^{\mathrm{Ar}} - \underline{\tau}^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} \underline{\gamma}_{bi}^{\mathrm{Ar1}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) \leq \underline{\sigma}_i^{\mathrm{Ar}} : \zeta_i^{\mathrm{Ar.2b}}, \; \forall i, \quad (20f)$$

$$\sum_{b\in\mathcal{B}} \underline{\gamma}_{bi}^{\mathrm{Ar2}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) \leq \underline{\sigma}_i^{\mathrm{Ar}} \;:\; \zeta_i^{\mathrm{Ar.2c}}, \qquad \forall i, \quad (20g)$$

$$-\underline{\phi}^{\mathrm{Ar}} \le \sum_{b\in\mathcal{B}} Q_b \underline{\gamma}_{bi}^{\mathrm{Ar1}} + \alpha^{\mathrm{Ar}} \le \underline{\phi}^{\mathrm{Ar}} \;:\; \underline{\zeta}_i^{\mathrm{Ar.2d}}, \overline{\zeta}_i^{\mathrm{Ar.2d}}, \qquad \forall i, \quad (20h)$$

$$-\underline{\phi}^{\mathrm{Ar}} \le \sum_{b\in\mathcal{B}} Q_b \underline{\gamma}_{bi}^{\mathrm{Ar2}} \le \underline{\phi}^{\mathrm{Ar}} \;:\; \underline{\zeta}_i^{\mathrm{Ar.2e}}, \overline{\zeta}_i^{\mathrm{Ar.2e}}, \qquad \forall i, \quad (20i)$$

$$0 \le \underline{\gamma}_{bi}^{\mathrm{Ar1}} \;:\; \zeta_{bi}^{\mathrm{Ar.2f}}, \qquad \forall b, i, \quad (20j)$$

$$0 \le \underline{\gamma}_{bi}^{\mathrm{Ar2}} \;:\; \zeta_{bi}^{\mathrm{Ar.2g}}, \qquad \forall b, i, \quad (20k)$$

CVaR approximation of (12c):

$$\overline{\tau}^{\mathrm{Ar}} + \frac{1}{\epsilon}\Big(\overline{\phi}^{\mathrm{Ar}} \rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|} \sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \overline{\sigma}_i^{\mathrm{Ar}}\Big) \le 0 \;:\; \zeta^{\mathrm{Ar.3a}}, \quad (20l)$$

$$p + \alpha^{\mathrm{Ar}} \widehat{\xi}_i^{\mathrm{Ar}} - \overline{P} - \overline{\tau}^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} \overline{\gamma}_{bi}^{\mathrm{Ar1}} \big(H_b - Q_b \widehat{\xi}_i^{\mathrm{Ar}}\big) \le \overline{\sigma}_i^{\mathrm{Ar}} : \zeta_i^{\mathrm{Ar.3b}}, \quad \forall i, \quad (20m)$$

$$\sum_{b\in\mathcal{B}} \overline{\gamma}_{bi}^{\mathrm{Ar2}} \big(H_b - Q_b \widehat{\xi}_i^{\mathrm{Ar}}\big) \le \overline{\sigma}_i^{\mathrm{Ar}} \;:\; \zeta_i^{\mathrm{Ar.3c}}, \qquad \forall i, \quad (20n)$$

$$-\overline{\phi}^{\mathrm{Ar}} \le \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{bi}^{\mathrm{Ar1}} - \alpha^{\mathrm{Ar}} \le \overline{\phi}^{\mathrm{Ar}} \;:\; \underline{\zeta}_i^{\mathrm{Ar.3d}}, \overline{\zeta}_i^{\mathrm{Ar.3d}}, \qquad \forall i, \quad (20o)$$

$$-\overline{\phi}^{\mathrm{Ar}} \le \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{bi}^{\mathrm{Ar2}} \le \overline{\phi}^{\mathrm{Ar}} \;:\; \underline{\zeta}_i^{\mathrm{Ar.3e}}, \overline{\zeta}_i^{\mathrm{Ar.3e}}, \qquad \forall i, \quad (20p)$$

$$0 \le \overline{\gamma}_{bi}^{\mathrm{Ar1}} \;:\; \zeta_{bi}^{\mathrm{Ar.3f}}, \qquad \forall b, i, \quad (20q)$$

$$0 \le \overline{\gamma}_{bi}^{\mathrm{Ar2}} \;:\; \zeta_{bi}^{\mathrm{Ar.3g}}, \qquad \forall b, i, \quad (20r)$$

Constraint (19c):

$$-A \le \alpha^{\mathrm{Ar}} \le A \;:\; \underline{\zeta}^{\mathrm{Ar.4a}}, \overline{\zeta}^{\mathrm{Ar.4a}}, \quad (20s)$$

where $\Xi^{\mathrm{Ar}} = \{p, \alpha^{\mathrm{Ar}}, \phi^{\mathrm{Ar}}, \sigma_i^{\mathrm{Ar}}, \gamma_{bi}^{\mathrm{Ar}}, \underline{\tau}^{\mathrm{Ar}}, \underline{\phi}^{\mathrm{Ar}}, \underline{\sigma}_i^{\mathrm{Ar}}, \underline{\gamma}_{bi}^{\mathrm{Ar1}}, \underline{\gamma}_{bi}^{\mathrm{Ar2}}, \overline{\tau}^{\mathrm{Ar}}, \overline{\phi}^{\mathrm{Ar}}, \overline{\sigma}_i^{\mathrm{Ar}}, \overline{\gamma}_{bi}^{\mathrm{Ar1}}, \overline{\gamma}_{bi}^{\mathrm{Ar2}}\}$.

4.1 Price-setter

Lastly, the optimization problem (13) of the price-setter is revisited by a deterministic problem comprising (14d) and (14e), but with theoretical constraints. This optimization problem writes as

$$\underset{\lambda^{\mathrm{E}},\lambda^{\mathrm{B}}}{\mathrm{Max}} \quad J^{\mathrm{Ps}} = \lambda^{\mathrm{E}}\Big(p - \sum_{n\in\mathcal{N}} d_n - L\Big) + \lambda^{\mathrm{B}}\Big(\alpha^{\mathrm{Ar}} + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}} - 1\Big) \quad (21a)$$

$$\text{s.t.} \quad -\Lambda \le \lambda^{\mathrm{E}} \le \Lambda \;:\; \underline{\zeta}^{\mathrm{Ps.E}}, \overline{\zeta}^{\mathrm{Ps.E}}, \quad (21b)$$

$$-\Lambda \le \lambda^{\mathrm{B}} \le \Lambda \;:\; \underline{\zeta}^{\mathrm{Ps.B}}, \overline{\zeta}^{\mathrm{Ps.B}}. \quad (21c)$$

Recall that the sufficiently large parameter Λ constitutes theoretical bounds, such that the feasible set is closed and compact, which is required later for the proof of the equilibrium solution existence. In our numerical study, we check *a posteriori* that these bounds are inactive.

Definition 1 Based on the decision-making problems (18), (20), and (21), we define the tractable Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$ corresponding to the distributionally robust Nash equilibrium problem (3), (4), and (5). The symbol $\mathcal{Z}$ is the set of all players, and J_i their respective payoff function, i.e., $\{\{J_n^{\text{Ed}}\}_{\forall n \in \mathcal{N}}, J^{\text{Ar}}, J^{\text{Ps}}\}$. The symbol $K = (K_{n_1}^{\text{Ed}} \times \cdots \times K_N^{\text{Ed}} \times K^{\text{Ar}} \times K^{\text{Ps}})$ denotes the strategy set of the game, where K_n^{Ed} is the strategy set of the price-elastic demand $n \in \mathcal{N}$, K^{Ar} is the strategy set of the spatial arbitrageur, and lastly K^{Ps} is the strategy set of the price-setter.

Proposition 1 *For the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$ a Nash equilibrium point exists.*

Proof 1. *We provide the proof in Appendix B.* ■

Proposition 2 *For the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$ an equivalent convex optimization problem exists, whose global solution is unique and, thereby, gives a unique Nash equilibrium point.*

Proof 2. *We provide the proof in Appendix C.* ■

Remark 1 Note that our proofs rely on the affine policy, the worst-case CVaR approximation of distributionally robust chance constraints[11], and the quadratic regularizer. In detail, the affine policy allows for a linear reformulation of distributionally robust objective functions, and—along with the worst-case CVaR approximation—the definition of a tractable and convex Nash game. The quadratic regularizer is needed to obtain strict monotonicity of players' preferences, which we take advantage of to prove uniqueness of the Nash equilibrium point.

Remark 2 This article generalizes the findings in [3] by showing that although different players may have access to different empirical data and are heterogeneously ambiguity-averse, an equivalent optimization to the competitive market equilibrium problem still exists.

5 Numerical results and discussion

This section numerically analyzes the implications of heterogeneous ambiguity aversion on local market-clearing outcomes. To identify the Nash equilibrium point, i.e., market-clearing outcomes, we solve the single optimization

[11] Given that $\epsilon \leq N^{-1}$ in (11b), (11c) as well as in (12b), (12c)—with N noting the number of samples applied—the CVaR representation of distributionally robust chance constraints is in this case according to [12] an exact representation of distributionally robust chance constraints.

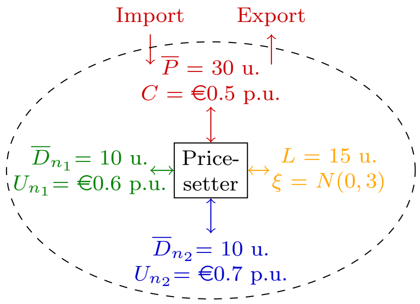


Fig. 5: Case study: A local market with two elastic demands, namely n_1 (green) and n_2 (blue), an aggregated inelastic demand (yellow), and the spatial arbitrageur (red). The maximum consumption as well as the import/export capacity are given in units (u.). The import cost and export revenue as well as the consumption utility are expressed in € per unit (€ p.u.).

problem (26), which is—according to Proposition 2—equivalent to the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$. This optimization problem is a convex quadratic program that can be solved by available commercial solvers such as the Gurobi Optimizer or the IBM CPLEX Optimizer. Without the quadratic regularizer in the objective function, this single optimization problem becomes a linear program. All source codes are available in our online companion [21].

Let us consider a local market for a general commodity. Figure 5 illustrates the players in the game as well the arbitrarily selected input data. In detail, a spatial arbitrageur is restricted to import and export a given commodity up to a maximum quantity $\overline{P}$ of 30 units at a fixed cost C of €0.5 per unit. This restriction is imposed by the physical network constraints. Two elastic demands, namely n_1 and n_2, may consume a maximum quantity $\overline{D}_n$ of 10 units each. The elastic demand n_1 gains a utility U_{n_1} of €0.6 per unit, while n_2 earns a slightly higher utility U_{n_2} of €0.7 per unit. The aggregated inelastic demand expects to consume $L = 15$ units, while her uncertain deviation ξ follows a multivariate Gaussian distribution $N(\mu, \sigma)$, with a mean of $\mu = 0$ and a standard deviation of $\sigma = 3$.

From $N(\mu, \sigma)$ we draw 10^5 random samples, and provide the spatial arbitrageur as well as the elastic demands n_1 and n_2 with 500 randomly selected samples, the so-called training data. These training samples for different players are not necessarily identical. We will use 10^4 number of the remaining samples later as test data. Given the training data, we solve the Nash equilibrium problem and determine the optimal values for quantities p, d_{n_1}, d_{n_2}, participation factors α^{Ar}, $\alpha^{\text{Ed}}_{n_1}$, $\alpha^{\text{Ed}}_{n_2}$, price λ^{E} and balancing service payment λ^{B}. Given the test data, we compute *a posteriori* the expected out-of-sample disutility of the players. Note that we do not solve another optimization problem for the out-of-sample computations, since the optimal values of the participation

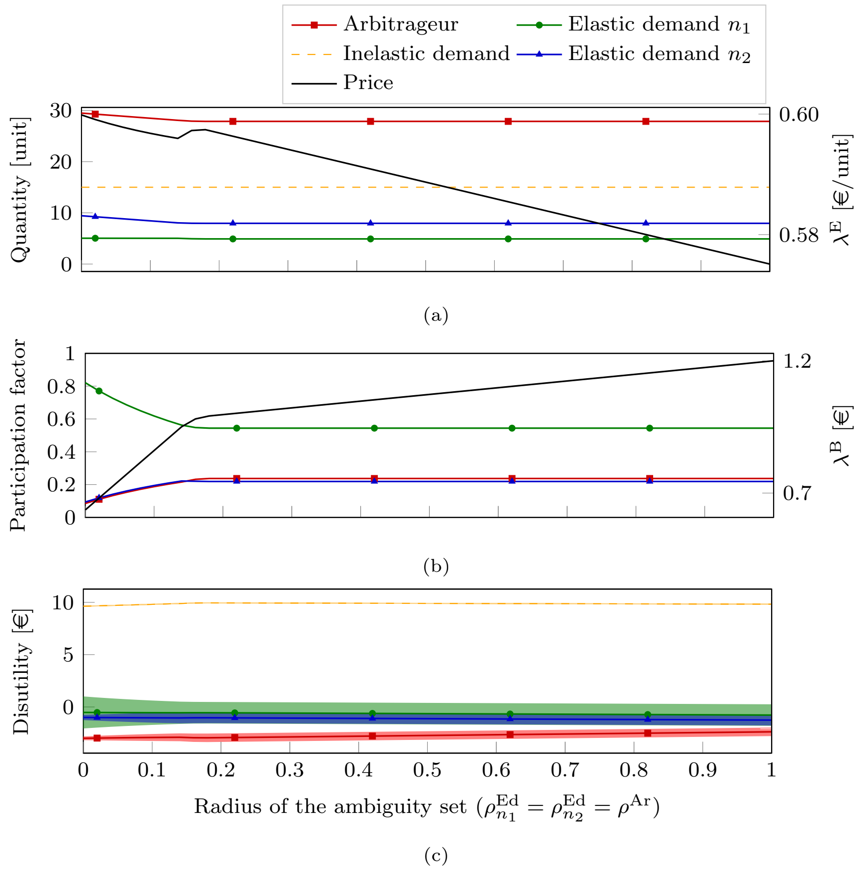


Fig. 6: Evolution of quantities to be traded (plot 6a), participation factors (plot 6b), and expected out-of-sample disutility as well as its standard deviation highlighted by the shaded area (plot 6c) as a function of the radius.

factors have been already determined.[12] We set the regularizer to $\beta = 10^{-6}$, and the violation probability of the chance constraints to $\epsilon = 0.05$.

5.1 The impact of ambiguity aversion

Two elastic demands n_1 and n_2 and the spatial arbitrageur contribute to offsetting any consumption deviation ξ of the inelastic demand from her nominal consumption L. Based on their expectation on ξ the spatial arbitrageur and two elastic demands make an individual trade-off between the quantity of the commodity to be bought and their participation factor. This trade-off highly depends on their individual belief on the deviation ξ. As the ambiguity set for a specific player enlarges, she contributes more actively to balancing services.

[12] We assume that the market applies a real-time schedule determined in the forward stage.

This effect is illustrated in Figure 6, where the radius of all players is assumed to be identical, i.e., $\rho^{\mathrm{Ed}}_{n_1} = \rho^{\mathrm{Ed}}_{n_2} = \rho^{\mathrm{Ar}}$. This assumption will be relaxed later. By increasing the radius, players become more ambiguity-averse. Meanwhile, all players possess the same empirical data, i.e., $\widehat{F}^{\mathrm{Ed}}_{n_1} = \widehat{F}^{\mathrm{Ed}}_{n_2} = \widehat{F}^{\mathrm{Ar}}$, yielding homogeneous ambiguity sets. As the ambiguity aversion of all players increases, all players reduce their quantity of the commodity to be traded as shown in Figure 6a. However, we observe that this decrease is less steep for the elastic demand n_1, since her utility from consumption is slightly lower than that of n_2. The commodity price λ^{E} falls as the ambiguity aversion increases. Figure 6b shows the evolution of the participation factors α^{Ar}, $\alpha^{\mathrm{Ed}}_{n_1}$, and $\alpha^{\mathrm{Ed}}_{n_2}$. As the ambiguity aversion increases the elastic demand n_2 as well as the spatial arbitrageur provide a greater contribution to balancing services. Meanwhile, the elastic demand n_1 proportionally reduces her participation, although starting from a significantly higher value. The balancing price λ^{B} rises as the ambiguity aversion increases. Lastly, we observe in Figure 6c that the expected disutility only slightly changes, whereas its standard deviation, indicated by the shaded area around the expected disutility, is positively correlated to the participation factor.

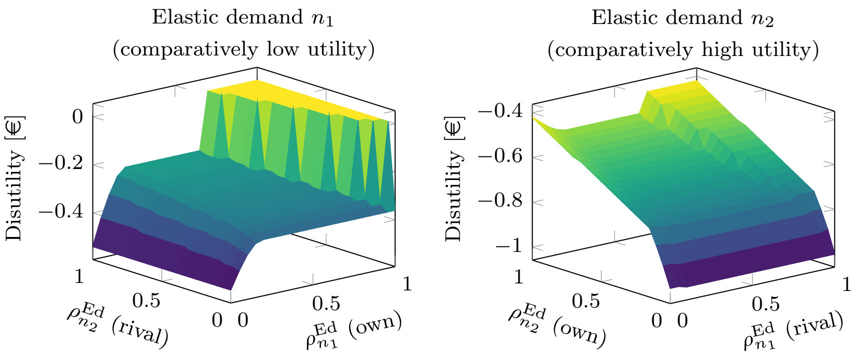
5.2 On heterogeneous ambiguity aversion

In the following we are interested in exploring the impact of heterogeneous ambiguity aversion. For this purpose, we assume the radius of the spatial arbitrageur to be $\rho^{\mathrm{Ar}} = 0.1$. At the same time, we gradually increase the radius of both elastic demands.

Figure 7 illustrates the expected disutility, i.e., the negative utility, of elastic demands n_1 and n_2, respectively, as a function of own as well as rival ambiguity aversion. As own ambiguity aversion of a demand increases her expected disutility increases as well. However, it also depends on the ambiguity aversion of the rival. According to Figure 7a, corresponding to demand n_1, her expected disutility significantly depends on the ambiguity aversion of the elastic demand n_2. Given a high ambiguity aversion of both elastic demands, player n_1 does not earn any utility. In contrast, as shown in Figure 7b, the disutility of the elastic demand n_2 hardly depends on the rival ambiguity aversion. Given a high ambiguity aversion of both elastic demands, she still earns a utility from consumption. These observations highlight that a player with a comparatively low consumption utility is highly exposed to the rival ambiguity aversion.

6 Conclusion

We studied a perfectly competitive local market, in which players trade a single commodity while being subject to the same source of uncertainty. These players could be heterogeneously ambiguity-averse by having individual knowledge about and confidence in empirical data describing the uncertain event. We proposed a generalized formulation of a distributionally robust Nash equilibrium



(a) (b)

Fig. 7: Expected disutility of the elastic demand n_1 and n_2 as a function of the radius of the own and the rival's ambiguity sets.

problem and applied a Wasserstein distance metric to model the ambiguity set of each player. Through the application of distributionally robust chance constraints, an affine policy and a quadratic regularizer, we defined a tractable Nash game. We mathematically proved that for this game an equivalent single optimization problem exists, whose solution is unique. This implies the existence of a unique Nash equilibrium point. Numerical results indicated that a player with a comparatively low consumption utility is highly subject to rival ambiguity aversion.

A Linear approximation

For the linear reformulation, we follow [7] and reformulate a distributionally robust objective function of the form $\max_{F\in\mathcal{D}} \mathbb{E}_F\big[C\alpha\xi\big]$ as

$$\min_{\phi,\sigma_i,\gamma_{bi}} \ \phi\rho + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\sigma_i \tag{22a}$$

$$\text{s.t. } C\alpha\widehat{\xi}_i + \sum_{b\in\mathcal{B}}\gamma_{bi}\big(H_b - Q_b\widehat{\xi}_i\big) \le \sigma_i, \qquad \forall i, \tag{22b}$$

$$\Big|\sum_{b\in\mathcal{B}} Q_b\gamma_{bi} - C\alpha\Big| \le \phi, \qquad \forall i, \tag{22c}$$

$$\gamma_{bi} \ge 0, \qquad \forall b, i, \tag{22d}$$

where ϕ, σ_i and γ_{bi} are auxiliary variables, and $|\mathcal{I}|$ returns the cardinality of set $\mathcal{I}$. Set $\mathcal{B}$ contains the bounds on ξ, i.e., $\max(\widehat{\xi}_i, \forall i)$ and $\min(\widehat{\xi}_i, \forall i)$. Parameter Q_b is a vector of $[1, -1]$. The absolute value $|\cdot|$ in (22c) results from the dual of the infinity-norm applied in (6a).

As discussed in [9] a distributionally robust chance constraint can be conservatively approximated by a constraint including the CVaR at level ϵ. According to [12, Proposition 1] and [27], a distributionally robust chance constraint of the form $\min_{F\in\mathcal{D}} \mathbb{P}_F\big[B-\alpha\xi \le 0\big] \ge 1-\epsilon$ can be approximated as a CVaR constraint $\max_{F\in\mathcal{D}} \mathbb{P}_F - \mathrm{CVaR}_\epsilon\big[B-\alpha\xi\big] \le 0$. Applying the dual of an infinity norm as our arbitrary choice, such a CVaR constraint reduces to the following set of linear equations:

$$\tau + \frac{1}{\epsilon}\big(\phi^{\mathrm{CVaR}}\rho + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}}\sigma_i^{\mathrm{CVaR}}\big) \le 0, \tag{23a}$$

$$B - \alpha\widehat{\xi}_i - \tau + \sum_{b\in\mathcal{B}}\gamma_{bi}^{\mathrm{b1}}\big(H_b - Q_b\widehat{\xi}_i\big) \le \sigma_i^{\mathrm{CVaR}}, \qquad \forall i, \tag{23b}$$

$$\sum_{b\in\mathcal{B}}\gamma_{bi}^{\mathrm{b2}}\big(H_b - Q_b\widehat{\xi}_i\big) \le \sigma_i^{\mathrm{CVaR}}, \qquad \forall i, \tag{23c}$$

$$\big|\sum_{b\in\mathcal{B}}Q_b\gamma_{bi}^{\mathrm{b1}} + \alpha\big| \le \phi^{\mathrm{CVaR}}, \qquad \forall i, \tag{23d}$$

$$\big|\sum_{b\in\mathcal{B}}Q_b\gamma_{bi}^{\mathrm{b2}}\big| \le \phi^{\mathrm{CVaR}}, \qquad \forall i, \tag{23e}$$

$$\gamma_{bi}^{\mathrm{b1}}, \gamma_{bi}^{\mathrm{b2}} \ge 0, \qquad \forall b, i, \tag{23f}$$

where τ, ϕ^{CVaR}, σ_i^{CVaR}, $\gamma_{bi}^{\mathrm{b1}}$ and $\gamma_{bi}^{\mathrm{b2}}$ are auxiliary variables.

B Proof of proposition 1

This proof is based on [29, Theorem 1], which states that a solution set to the competitive equilibrium problem exists given that the strategy set of each player is convex and compact. In addition, the objective function of each player needs to be continuous. For the game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$ the strategy set K comprising the stratgey set of each player is closed, compact, convex, and non-empty. Moreover, all objective functions $J_{i\in\mathcal{Z}}$ are continuously differentiable. Consequently, a solution to the competitive Nash equilibrium problem exists. ■

C Proof of proposition 2

In the following, we show the existence of an equivalent single optimization problem to the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$, whose optimal solution coincides with the Nash equilibrium point. The rationale behind the proof of this equivalence is that the Karush-Kuhn-Tucker (KKT) conditions of the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$ and of the single optimization problem are identical. In addition, we show that the global solution to the single optimization problem is unique, which implies the existence of a unique Nash equilibrium point.

C.1 Towards a single optimization problem

We first derive the objective function of the single optimization problem based on individual cost functions (16), (18a) and (20a) as

$$\underbrace{\lambda^{\mathrm{E}}L + \lambda^{\mathrm{B}}}_{(16)} + \sum_{n\in\mathcal{N}}\underbrace{\Big(\,(\lambda^{\mathrm{E}} - U_n)d_n - \lambda^{\mathrm{B}}\alpha_n^{\mathrm{Ed}} + c(d_n, \alpha_n^{\mathrm{Ed}}) + \phi_n^{\mathrm{Ed}}\rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|}\sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\sigma_{ni}^{\mathrm{Ed}}\Big)}_{(18a)}$$
$$+ \underbrace{\big(C - \lambda^{\mathrm{E}}\big)p - \lambda^{\mathrm{B}}\alpha^{\mathrm{Ar}} + c(p, \alpha^{\mathrm{Ar}}) + \phi^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}}\sigma_i^{\mathrm{Ar}}}_{(20a)}\,. \tag{24}$$

With the first-order coefficient for λ^{E} and λ^{B} of the price-setter's problem (21) equal to zero, the function (24) reduces to

$$\sum_{n\in\mathcal{N}} \underbrace{\Big(-U_n d_n + c(d_n,\alpha_n^{\mathrm{Ed}}) + \phi_n^{\mathrm{Ed}}\rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|}\sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\sigma_{ni}^{\mathrm{Ed}}\Big)}_{\text{Elastic demand } n}$$
$$+ \underbrace{Cp + c(p,\alpha^{\mathrm{Ar}}) + \phi^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}}\sigma_i^{\mathrm{Ar}}}_{\text{Spatial arbitrageur}}. \tag{25}$$

Based on the function (25) we propose the single optimization problem

$$\underset{\Xi^{\mathrm{Cp}}}{\mathrm{Min}} \sum_{n\in\mathcal{N}} \Big(-U_n d_n + c(d_n,\alpha_n^{\mathrm{Ed}}) + \phi_n^{\mathrm{Ed}}\rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|}\sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\sigma_{ni}^{\mathrm{Ed}}\Big)$$
$$+ Cp + c(p,\alpha^{\mathrm{Ar}}) + \phi^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}}\sigma_i^{\mathrm{Ar}} \tag{26a}$$
$$\text{s.t. } p - \sum_{n\in\mathcal{N}} d_n - L = 0 \;:\; \lambda^{\mathrm{E}}, \tag{26b}$$
$$\alpha^{\mathrm{Ar}} + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}} - 1 = 0 \;:\; \lambda^{\mathrm{B}}, \tag{26c}$$
$$\text{(18b)–(18s)}, \qquad \forall n, \tag{26d}$$
$$\text{(20b)–(20s)}, \tag{26e}$$

where $\Xi^{\mathrm{Cp}} = \{d_n, p, \alpha_n^{\mathrm{Ed}}, \alpha^{\mathrm{Ar}}, \phi_n^{\mathrm{Ed}}, \sigma_{ni}^{\mathrm{Ed}}, \gamma_{nbi}^{\mathrm{Ed}}, \phi^{\mathrm{Ar}}, \sigma_i^{\mathrm{Ar}}, \gamma_{bi}^{\mathrm{Ar}}, \underline{\tau}_n^{\mathrm{Ed}}, \underline{\phi}_n^{\mathrm{Ed}}, \underline{\sigma}_{ni}^{\mathrm{Ed}}, \underline{\gamma}_{nbi}^{\mathrm{Ed1}}, \underline{\gamma}_{nbi}^{\mathrm{Ed2}}, \overline{\tau}_n^{\mathrm{Ed}}, \overline{\phi}_n^{\mathrm{Ed}}, \overline{\sigma}_{ni}^{\mathrm{Ed}}, \overline{\gamma}_{nbi}^{\mathrm{Ed1}}, \overline{\gamma}_{nbi}^{\mathrm{Ed2}}, \underline{\tau}^{\mathrm{Ar}}, \underline{\phi}^{\mathrm{Ar}}, \underline{\sigma}_i^{\mathrm{Ar}}, \underline{\gamma}_{bi}^{\mathrm{Ar1}}, \underline{\gamma}_{bi}^{\mathrm{Ar2}}, \overline{\tau}^{\mathrm{Ar}}, \overline{\phi}^{\mathrm{Ar}}, \overline{\sigma}_i^{\mathrm{Ar}}, \overline{\gamma}_{bi}^{\mathrm{Ar1}}, \overline{\gamma}_{bi}^{\mathrm{Ar2}}\}$. Note that λ^{E} states the dual variable of the equality constraint (26b), while λ^{B} presents the dual variable of the equality constraint (26c).

C.2 Karush-Kuhn-Tucker conditions

We continue by comparing the KKT conditions of the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i\in\mathcal{Z}})$ with those of the single optimization problem (26). The KKT conditions associated with (18) are as follows.

$$\frac{\partial\mathcal{L}_n^{\mathrm{Ed}}}{\partial d_n} = \lambda^{\mathrm{E}} - U_n + \beta d_n + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|}\sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\Big(-\zeta_{ni}^{\mathrm{Ed.2b}} + \zeta_{ni}^{\mathrm{Ed.3b}}\Big) = 0, \qquad \forall n, \tag{27a}$$
$$\frac{\partial\mathcal{L}_n^{\mathrm{Ed}}}{\partial \alpha_n^{\mathrm{Ed}}} = -\lambda^{\mathrm{B}} + \beta\alpha_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|}\sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\Big(\zeta_{ni}^{\mathrm{Ed.1a}}U_n\widehat{\xi}_{ni}^{\mathrm{Ed}} + \underline{\zeta}_{ni}^{\mathrm{Ed.1b}}U_n - \overline{\zeta}_{ni}^{\mathrm{Ed.1b}}U_n$$
$$+ \zeta_{ni}^{\mathrm{Ed.2b}}\widehat{\xi}_{ni}^{\mathrm{Ed}} + \underline{\zeta}_{ni}^{\mathrm{Ed.2d}} - \overline{\zeta}_{ni}^{\mathrm{Ed.2d}} - \zeta_{ni}^{\mathrm{Ed.3b}}\widehat{\xi}_{ni}^{\mathrm{Ed}} - \underline{\zeta}_{ni}^{\mathrm{Ed.3d}} + \overline{\zeta}_{ni}^{\mathrm{Ed.3d}}\Big)$$
$$- \underline{\zeta}_n^{\mathrm{Ed.4a}} + \overline{\zeta}_n^{\mathrm{Ed.4a}} = 0, \qquad \forall n, \tag{27b}$$
$$\frac{\partial\mathcal{L}_n^{\mathrm{Ed}}}{\partial \phi_n^{\mathrm{Ed}}} = \rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|}\sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\Big(-\underline{\zeta}_{ni}^{\mathrm{Ed.1b}} - \overline{\zeta}_{ni}^{\mathrm{Ed.1b}}\Big) = 0, \qquad \forall n, \tag{27c}$$
$$\frac{\partial\mathcal{L}_n^{\mathrm{Ed}}}{\partial \sigma_{ni}^{\mathrm{Ed}}} = \sum_{i\in\mathcal{I}_n^{\mathrm{Ed}}}\frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} - \zeta_{ni}^{\mathrm{Ed.1a}} = 0, \qquad \forall n, i, \tag{27d}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \gamma_{nbi}^{\mathrm{Ed}}} = \zeta_{ni}^{\mathrm{Ed.1a}}\big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) - \underline{\zeta}_{ni}^{\mathrm{Ed.1b}} Q_b + \overline{\zeta}_{ni}^{\mathrm{Ed.1b}} Q_b - \zeta_{nbi}^{\mathrm{Ed.1c}} = 0, \quad \forall n, b, i, \tag{27e}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \underline{\tau}_n^{\mathrm{Ed}}} = \zeta_n^{\mathrm{Ed.2a}} - \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \zeta_{ni}^{\mathrm{Ed.2b}} = 0, \quad \forall n, \tag{27f}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \underline{\phi}_n^{\mathrm{Ed}}} &= \zeta_n^{\mathrm{Ed.2a}} \frac{1}{\epsilon} \rho_n^{\mathrm{Ed}} \\ &+ \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \Big(- \underline{\zeta}_{ni}^{\mathrm{Ed.2d}} - \overline{\zeta}_{ni}^{\mathrm{Ed.2d}} - \underline{\zeta}_{ni}^{\mathrm{Ed.2e}} - \overline{\zeta}_{ni}^{\mathrm{Ed.2e}} \Big) = 0, \quad \forall n,\end{aligned} \tag{27g}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \underline{\sigma}_{ni}^{\mathrm{Ed}}} = \frac{1}{\epsilon} \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \zeta_n^{\mathrm{Ed.2a}} - \zeta_{ni}^{\mathrm{Ed.2b}} - \zeta_{ni}^{\mathrm{Ed.2c}} = 0, \quad \forall n, i, \tag{27h}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \underline{\gamma}_{nbi}^{\mathrm{Ed1}}} = \zeta_{ni}^{\mathrm{Ed.2b}}\big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) - \underline{\zeta}_{ni}^{\mathrm{Ed.2d}} Q_b + \overline{\zeta}_{ni}^{\mathrm{Ed.2d}} Q_b - \zeta_{nbi}^{\mathrm{Ed.2f}} = 0, \quad \forall n, b, i, \tag{27i}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \underline{\gamma}_{nbi}^{\mathrm{Ed2}}} = \zeta_{ni}^{\mathrm{Ed.2c}}\big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) - \underline{\zeta}_{ni}^{\mathrm{Ed.2e}} Q_b + \overline{\zeta}_{ni}^{\mathrm{Ed.2e}} Q_b - \zeta_{nbi}^{\mathrm{Ed.2g}} = 0, \quad \forall n, b, i, \tag{27j}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \overline{\tau}_n^{\mathrm{Ed}}} = \zeta_n^{\mathrm{Ed.3a}} - \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \zeta_{ni}^{\mathrm{Ed.3b}} = 0, \quad \forall n, \tag{27k}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \overline{\phi}_n^{\mathrm{Ed}}} &= \zeta_n^{\mathrm{Ed.3a}} \frac{1}{\epsilon} \rho_n^{\mathrm{Ed}} \\ &+ \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \Big(- \underline{\zeta}_{ni}^{\mathrm{Ed.3d}} - \overline{\zeta}_{ni}^{\mathrm{Ed.3d}} - \underline{\zeta}_{ni}^{\mathrm{Ed.3e}} - \overline{\zeta}_{ni}^{\mathrm{Ed.3e}} \Big) = 0, \quad \forall n,\end{aligned} \tag{27l}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \overline{\sigma}_{ni}^{\mathrm{Ed}}} = \frac{1}{\epsilon} \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \zeta_n^{\mathrm{Ed.3a}} - \zeta_{ni}^{\mathrm{Ed.3b}} - \zeta_{ni}^{\mathrm{Ed.3c}} = 0, \quad \forall n, i, \tag{27m}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \overline{\gamma}_{nbi}^{\mathrm{Ed1}}} = \zeta_{ni}^{\mathrm{Ed.3b}}\big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) - \underline{\zeta}_{ni}^{\mathrm{Ed.3d}} Q_b + \overline{\zeta}_{ni}^{\mathrm{Ed.3d}} Q_b - \zeta_{nbi}^{\mathrm{Ed.3f}} = 0, \quad \forall n, b, i, \tag{27n}$$

$$\frac{\partial \mathcal{L}_n^{\mathrm{Ed}}}{\partial \overline{\gamma}_{nbi}^{\mathrm{Ed2}}} = \zeta_{ni}^{\mathrm{Ed.3c}}\big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) - \underline{\zeta}_{ni}^{\mathrm{Ed.3e}} Q_b + \overline{\zeta}_{ni}^{\mathrm{Ed.3e}} Q_b - \zeta_{nbi}^{\mathrm{Ed.3g}} = 0, \quad \forall n, b, i, \tag{27o}$$

$$0 \le -U_n \alpha_n^{\mathrm{Ed}} \widehat{\xi}_{ni}^{\mathrm{Ed}} - \sum_{b \in \mathcal{B}} \gamma_{nbi}^{\mathrm{Ed}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) + \sigma_{ni}^{\mathrm{Ed}} \perp \zeta_{ni}^{\mathrm{Ed.1a}} \ge 0, \quad \forall n, i, \tag{27p}$$

$$0 \le \phi_n^{\mathrm{Ed}} + \sum_{b \in \mathcal{B}} Q_b \gamma_{nbi}^{\mathrm{Ed}} - U_n \alpha_n^{\mathrm{Ed}} \perp \underline{\zeta}_{ni}^{\mathrm{Ed.1b}} \ge 0, \quad \forall n, i, \tag{27q}$$

$$0 \le -\sum_{b \in \mathcal{B}} Q_b \gamma_{nbi}^{\mathrm{Ed}} + U_n \alpha_n^{\mathrm{Ed}} + \phi_n^{\mathrm{Ed}} \perp \overline{\zeta}_{ni}^{\mathrm{Ed.1b}} \ge 0, \quad \forall n, i, \tag{27r}$$

$$0 \le \gamma_{nbi}^{\mathrm{Ed}} \perp \zeta_{nbi}^{\mathrm{Ed.1c}} \ge 0, \quad \forall n, b, i, \tag{27s}$$

$$0 \le -\underline{\tau}_n^{\mathrm{Ed}} - \frac{1}{\epsilon}\Big(\underline{\phi}_n^{\mathrm{Ed}} \rho_n^{\mathrm{Ed}} + \frac{1}{|\mathcal{I}_n^{\mathrm{Ed}}|} \sum_{i \in \mathcal{I}_n^{\mathrm{Ed}}} \underline{\sigma}_{ni}^{\mathrm{Ed}}\Big) \perp \zeta_n^{\mathrm{Ed.2a}} \ge 0, \quad \forall n, \tag{27t}$$

$$0 \le d_n - \alpha_n^{\mathrm{Ed}} \widehat{\xi}_{ni}^{\mathrm{Ed}} + \underline{\tau}_n^{\mathrm{Ed}} - \sum_{b \in \mathcal{B}} \underline{\gamma}_{nbi}^{\mathrm{Ed1}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) + \underline{\sigma}_{ni}^{\mathrm{Ed}} \perp \zeta_{ni}^{\mathrm{Ed.2b}} \ge 0, \quad \forall n, i, \tag{27u}$$

$$0 \le -\sum_{b \in \mathcal{B}} \underline{\gamma}_{nbi}^{\mathrm{Ed2}} \big(H_b - Q_b \widehat{\xi}_{ni}^{\mathrm{Ed}}\big) + \underline{\sigma}_{ni}^{\mathrm{Ed}} \perp \zeta_{ni}^{\mathrm{Ed.2c}} \ge 0, \quad \forall n, i, \tag{27v}$$

$$0 \le \underline{\phi}_n^{\mathrm{Ed}} + \sum_{b \in \mathcal{B}} Q_b \underline{\gamma}_{nbi}^{\mathrm{Ed1}} - \alpha_n^{\mathrm{Ed}} \perp \underline{\zeta}_{ni}^{\mathrm{Ed.2d}} \ge 0, \quad \forall n, i, \tag{27w}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \underline{\gamma}^{\mathrm{Ed1}}_{nbi} + \alpha^{\mathrm{Ed}}_n + \underline{\phi}^{\mathrm{Ed}}_n \perp \overline{\zeta}^{\mathrm{Ed.2d}}_{ni} \ge 0, \qquad \forall n, i, \tag{27x}$$

$$0 \le \underline{\phi}^{\mathrm{Ed}}_n + \sum_{b\in\mathcal{B}} Q_b \underline{\gamma}^{\mathrm{Ed2}}_{nbi} \perp \underline{\zeta}^{\mathrm{Ed.2e}}_{ni} \ge 0, \qquad \forall n, i, \tag{27y}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \underline{\gamma}^{\mathrm{Ed2}}_{nbi} + \underline{\phi}^{\mathrm{Ed}}_n \perp \overline{\zeta}^{\mathrm{Ed.2e}}_{ni} \ge 0, \qquad \forall n, i, \tag{27z}$$

$$0 \le \underline{\gamma}^{\mathrm{Ed1}}_{nbi} \perp \zeta^{\mathrm{Ed.2f}}_{nbi} \ge 0, \qquad \forall n, b, i, \tag{27aa}$$

$$0 \le \underline{\gamma}^{\mathrm{Ed2}}_{nbi} \perp \zeta^{\mathrm{Ed.2g}}_{nbi} \ge 0, \qquad \forall n, b, i, \tag{27ab}$$

$$0 \le -\overline{\tau}^{\mathrm{Ed}}_n - \frac{1}{\epsilon}\big(\overline{\phi}^{\mathrm{Ed}}_n \rho^{\mathrm{Ed}}_n + \frac{1}{|\mathcal{I}^{\mathrm{Ed}}_n|}\sum_{i\in\mathcal{I}^{\mathrm{Ed}}_n} \overline{\sigma}^{\mathrm{Ed}}_{ni}\big) \perp \zeta^{\mathrm{Ed.3a}}_n \ge 0, \qquad \forall n, \tag{27ac}$$

$$\begin{aligned} 0 \le & -d_n + \alpha^{\mathrm{Ed}}_n \widehat{\xi}^{\mathrm{Ed}}_{ni} + \overline{D}_n + \overline{\tau}^{\mathrm{Ed}}_n \\ & - \sum_{b\in\mathcal{B}} \overline{\gamma}^{\mathrm{Ed1}}_{nbi}\big(H_b - Q_b \widehat{\xi}^{\mathrm{Ed}}_{ni}\big) + \overline{\sigma}^{\mathrm{Ed}}_{ni} \perp \zeta^{\mathrm{Ed.3b}}_{ni} \ge 0, \qquad \forall n, i, \end{aligned} \tag{27ad}$$

$$0 \le -\sum_{b\in\mathcal{B}} \overline{\gamma}^{\mathrm{Ed2}}_{nbi}\big(H_b - Q_b \widehat{\xi}^{\mathrm{Ed}}_{ni}\big) + \overline{\sigma}^{\mathrm{Ed}}_{ni} \perp \zeta^{\mathrm{Ed.3c}}_{ni} \ge 0, \qquad \forall n, i, \tag{27ae}$$

$$0 \le \overline{\phi}^{\mathrm{Ed}}_n + \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}^{\mathrm{Ed1}}_{nbi} + \alpha^{\mathrm{Ed}}_n \perp \underline{\zeta}^{\mathrm{Ed.3d}}_{ni} \ge 0, \qquad \forall n, i, \tag{27af}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \overline{\gamma}^{\mathrm{Ed1}}_{nbi} - \alpha^{\mathrm{Ed}}_n + \overline{\phi}^{\mathrm{Ed}}_n \perp \overline{\zeta}^{\mathrm{Ed.3d}}_{ni} \ge 0, \qquad \forall n, i, \tag{27ag}$$

$$0 \le \overline{\phi}^{\mathrm{Ed}}_n + \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}^{\mathrm{Ed2}}_{nbi} \perp \underline{\zeta}^{\mathrm{Ed.3e}}_{ni} \ge 0, \qquad \forall n, i, \tag{27ah}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \overline{\gamma}^{\mathrm{Ed2}}_{nbi} + \overline{\phi}^{\mathrm{Ed}}_n \perp \overline{\zeta}^{\mathrm{Ed.3e}}_{ni} \ge 0, \qquad \forall n, i, \tag{27ai}$$

$$0 \le \overline{\gamma}^{\mathrm{Ed1}}_{nbi} \perp \zeta^{\mathrm{Ed.3f}}_{nbi} \ge 0, \qquad \forall n, b, i, \tag{27aj}$$

$$0 \le \overline{\gamma}^{\mathrm{Ed2}}_{nbi} \perp \zeta^{\mathrm{Ed.3g}}_{nbi} \ge 0, \qquad \forall n, b, i, \tag{27ak}$$

$$0 \le A + \alpha^{\mathrm{Ed}}_n \perp \underline{\zeta}^{\mathrm{Ed.4a}}_n \ge 0, \qquad \forall n \tag{27al}$$

$$0 \le -\alpha^{\mathrm{Ed}}_n + A \perp \overline{\zeta}^{\mathrm{Ed.4a}}_n \ge 0, \qquad \forall n \tag{27am}$$

where $\mathcal{L}^{\mathrm{Ed}}_n$ is the Lagrangian function of (18).

The KKT conditions corresponding to (20) write

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial p} = C - \lambda^{\mathrm{E}} + \beta p + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \Big(-\zeta^{\mathrm{Ar.2b}}_i + \zeta^{\mathrm{Ar.3b}}_i \Big) = 0, \tag{28a}$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \alpha^{\mathrm{Ar}}} = & -\lambda^{\mathrm{B}} + \beta \alpha^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \Big(\zeta^{\mathrm{Ar.1a}}_i C \widehat{\xi}^{\mathrm{Ar}}_i + \underline{\zeta}^{\mathrm{Ar.1b}}_i C - \overline{\zeta}^{\mathrm{Ar.1b}}_i C \\ & - \zeta^{\mathrm{Ar.2b}}_i \widehat{\xi}^{\mathrm{Ar}}_i - \underline{\zeta}^{\mathrm{Ar.2d}}_i + \overline{\zeta}^{\mathrm{Ar.2d}}_i + \zeta^{\mathrm{Ar.3b}}_i \widehat{\xi}^{\mathrm{Ar}}_i + \underline{\zeta}^{\mathrm{Ar.3d}}_i - \overline{\zeta}^{\mathrm{Ar.3d}}_i \Big) \\ & - \underline{\zeta}^{\mathrm{Ar.4a}} + \overline{\zeta}^{\mathrm{Ar.4a}} = 0, \end{aligned} \tag{28b}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \phi^{\mathrm{Ar}}} = \rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \Big(-\underline{\zeta}^{\mathrm{Ar.1b}}_i - \overline{\zeta}^{\mathrm{Ar.1b}}_i \Big) = 0, \tag{28c}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \sigma^{\mathrm{Ar}}_i} = \sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|} - \zeta^{\mathrm{Ar.1a}}_i = 0, \qquad \forall i \tag{28d}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \gamma^{\mathrm{Ar}}_{bi}} = \zeta^{\mathrm{Ar.1a}}_i \big(H_b - Q_b \widehat{\xi}^{\mathrm{Ar}}_i\big) - \underline{\zeta}^{\mathrm{Ar.1b}}_i Q_b + \overline{\zeta}^{\mathrm{Ar.1b}}_i Q_b - \zeta^{\mathrm{Ar.1c}}_{bi} = 0, \qquad \forall b, i \tag{28e}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \underline{\tau}^{\mathrm{Ar}}} = \zeta^{\mathrm{Ar.2a}} - \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \zeta_i^{\mathrm{Ar.2b}} = 0, \tag{28f}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \underline{\phi}^{\mathrm{Ar}}} = \zeta^{\mathrm{Ar.2a}}\frac{1}{\epsilon}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \Big(-\underline{\zeta}_i^{\mathrm{Ar.2d}} - \overline{\zeta}_i^{\mathrm{Ar.2d}} - \underline{\zeta}_i^{\mathrm{Ar.2e}} - \overline{\zeta}_i^{\mathrm{Ar.2e}}\Big) = 0, \tag{28g}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \underline{\sigma}_i^{\mathrm{Ar}}} = \frac{1}{\epsilon}\frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\zeta^{\mathrm{Ar.2a}} - \zeta_i^{\mathrm{Ar.2b}} - \zeta_i^{\mathrm{Ar.2c}} = 0, \qquad \forall i, \tag{28h}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \underline{\gamma}_{bi}^{\mathrm{Ar1}}} = \zeta_i^{\mathrm{Ar.2b}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) - \underline{\zeta}_i^{\mathrm{Ar.2d}}Q_b + \overline{\zeta}_i^{\mathrm{Ar.2d}}Q_b - \zeta_{bi}^{\mathrm{AR.2f}} = 0, \qquad \forall b,i, \tag{28i}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \underline{\gamma}_{bi}^{\mathrm{Ar2}}} = \zeta_i^{\mathrm{Ar.2c}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) - \underline{\zeta}_i^{\mathrm{Ar.2e}}Q_b + \overline{\zeta}_i^{\mathrm{Ar.2e}}Q_b - \zeta_{bi}^{\mathrm{Ar.2g}} = 0, \qquad \forall b,i, \tag{28j}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \overline{\tau}^{\mathrm{Ar}}} = \zeta^{\mathrm{Ar.3a}} - \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \zeta_i^{\mathrm{Ar.3b}} = 0, \tag{28k}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \overline{\phi}^{\mathrm{Ar}}} = \zeta^{\mathrm{Ar.3a}}\frac{1}{\epsilon}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \Big(-\underline{\zeta}_i^{\mathrm{Ar.3d}} - \overline{\zeta}_i^{\mathrm{Ar.3d}} - \underline{\zeta}_i^{\mathrm{Ar.3e}} - \overline{\zeta}_i^{\mathrm{Ar.3e}}\Big) = 0, \tag{28l}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \overline{\sigma}_i^{\mathrm{Ar}}} = \frac{1}{\epsilon}\frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\zeta^{\mathrm{Ar.3a}} - \zeta_i^{\mathrm{Ar.3b}} - \zeta_i^{\mathrm{Ar.3c}} = 0, \qquad \forall i, \tag{28m}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \overline{\gamma}_{bi}^{\mathrm{Ar1}}} = \zeta_i^{\mathrm{Ar.3b}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) - \underline{\zeta}_i^{\mathrm{Ar.3d}}Q_b + \overline{\zeta}_i^{\mathrm{Ar.3d}}Q_b - \zeta_{bi}^{\mathrm{Ar.3f}} = 0, \qquad \forall b,i, \tag{28n}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ar}}}{\partial \overline{\gamma}_{bi}^{\mathrm{Ar2}}} = \zeta_i^{\mathrm{Ar.3c}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) - \underline{\zeta}_i^{\mathrm{Ar.3e}}Q_b + \overline{\zeta}_i^{\mathrm{Ar.3e}}Q_b - \zeta_{bi}^{\mathrm{Ar.3g}} = 0, \qquad \forall b,i, \tag{28o}$$

$$0 \le -C\alpha^{\mathrm{Ar}}\widehat{\xi}_i^{\mathrm{Ar}} - \sum_{b\in\mathcal{B}}\gamma_{bi}^{\mathrm{Ar}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) + \sigma_i^{\mathrm{Ar}} \perp \zeta_i^{\mathrm{Ar.1a}} \ge 0, \qquad \forall i, \tag{28p}$$

$$0 \le \phi^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} Q_b\gamma_{bi}^{\mathrm{Ar}} - C\alpha^{\mathrm{Ar}} \perp \underline{\zeta}_i^{\mathrm{Ar.1b}} \ge 0, \qquad \forall i, \tag{28q}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b\gamma_{bi}^{\mathrm{Ar}} + C\alpha^{\mathrm{Ar}} + \phi^{\mathrm{Ar}} \perp \overline{\zeta}_i^{\mathrm{Ar.1b}} \ge 0, \qquad \forall i, \tag{28r}$$

$$0 \le \gamma_{bi}^{\mathrm{Ar}} \perp \zeta_{bi}^{\mathrm{Ar.1c}} \ge 0, \qquad \forall b,i \tag{28s}$$

$$0 \le -\underline{\tau}^{\mathrm{Ar}} - \frac{1}{\epsilon}\Big(\underline{\phi}^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}}\underline{\sigma}_i^{\mathrm{Ar}}\Big) \perp \zeta^{\mathrm{Ar.2a}} \ge 0, \tag{28t}$$

$$0 \le \overline{P} + p + \alpha^{\mathrm{Ar}}\widehat{\xi}_i^{\mathrm{Ar}} + \underline{\tau}^{\mathrm{Ar}} - \sum_{b\in\mathcal{B}}\underline{\gamma}_{bi}^{\mathrm{Ar1}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) + \underline{\sigma}_i^{\mathrm{Ar}} \perp \zeta_i^{\mathrm{Ar.2b}} \ge 0, \qquad \forall i, \tag{28u}$$

$$0 \le -\sum_{b\in\mathcal{B}}\underline{\gamma}_{bi}^{\mathrm{Ar2}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) + \underline{\sigma}_i^{\mathrm{Ar}} \perp \zeta_i^{\mathrm{Ar.2c}} \ge 0, \qquad \forall i, \tag{28v}$$

$$0 \le \underline{\phi}^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} Q_b\underline{\gamma}_{bi}^{\mathrm{Ar1}} + \alpha^{\mathrm{Ar}} \perp \underline{\zeta}_i^{\mathrm{Ar.2d}} \ge 0, \qquad \forall i, \tag{28w}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b\underline{\gamma}_{bi}^{\mathrm{Ar1}} - \alpha^{\mathrm{Ar}} + \underline{\phi}^{\mathrm{Ar}} \perp \overline{\zeta}_i^{\mathrm{Ar.2d}} \ge 0, \qquad \forall i, \tag{28x}$$

$$0 \le \underline{\phi}^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} Q_b\underline{\gamma}_{bi}^{\mathrm{Ar2}} \perp \underline{\zeta}_i^{\mathrm{Ar.2e}} \ge 0, \qquad \forall i, \tag{28y}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \underline{\gamma}_{bi}^{\mathrm{Ar2}} + \underline{\phi}^{\mathrm{Ar}} \perp \overline{\zeta}_i^{\mathrm{Ar.2e}} \ge 0, \qquad \forall i, \tag{28z}$$

$$0 \le \underline{\gamma}_{bi}^{\mathrm{Ar1}} \perp \zeta_{bi}^{\mathrm{Ar.2f}} \ge 0, \qquad \forall b, i, \tag{28aa}$$

$$0 \le \underline{\gamma}_{bi}^{\mathrm{Ar2}} \perp \zeta_{bi}^{\mathrm{Ar.2g}} \ge 0, \qquad \forall b, i, \tag{28ab}$$

$$0 \le -\overline{\tau}^{\mathrm{Ar}} - \frac{1}{\epsilon}\big(\overline{\phi}^{\mathrm{Ar}}\rho^{\mathrm{Ar}} + \frac{1}{|\mathcal{I}^{\mathrm{Ar}}|}\sum_{i\in\mathcal{I}^{\mathrm{Ar}}} \overline{\sigma}_i^{\mathrm{Ar}}\big) \perp \zeta^{\mathrm{Ar.3a}} \ge 0, \tag{28ac}$$

$$0 \le -p - \alpha^{\mathrm{Ar}}\widehat{\xi}_i^{\mathrm{Ar}} + \overline{P} + \overline{\tau}^{\mathrm{Ar}} - \sum_{b\in\mathcal{B}} \overline{\gamma}_{bi}^{\mathrm{Ar1}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) + \overline{\sigma}_i^{\mathrm{Ar}} \perp \zeta_i^{\mathrm{Ar.3b}} \ge 0, \qquad \forall i, \tag{28ad}$$

$$0 \le -\sum_{b\in\mathcal{B}} \overline{\gamma}_{bi}^{\mathrm{Ar2}}\big(H_b - Q_b\widehat{\xi}_i^{\mathrm{Ar}}\big) + \overline{\sigma}_i^{\mathrm{Ar}} \perp \zeta_i^{\mathrm{Ar.3c}} \ge 0, \qquad \forall i, \tag{28ae}$$

$$0 \le \overline{\phi}^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{bi}^{\mathrm{Ar1}} - \alpha^{\mathrm{Ar}} \perp \underline{\zeta}_i^{\mathrm{Ar.3d}} \ge 0, \qquad \forall i, \tag{28af}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{bi}^{\mathrm{Ar1}} + \alpha^{\mathrm{Ar}} + \overline{\phi}^{\mathrm{Ar}} \perp \overline{\zeta}_i^{\mathrm{Ar.3d}} \ge 0, \qquad \forall i, \tag{28ag}$$

$$0 \le \overline{\phi}^{\mathrm{Ar}} + \sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{bi}^{\mathrm{Ar2}} \perp \underline{\zeta}_i^{\mathrm{Ar.3e}} \ge 0, \qquad \forall i, \tag{28ah}$$

$$0 \le -\sum_{b\in\mathcal{B}} Q_b \overline{\gamma}_{bi}^{\mathrm{Ar2}} + \overline{\phi}^{\mathrm{Ar}} \perp \overline{\zeta}_i^{\mathrm{Ar.3e}} \ge 0, \qquad \forall i, \tag{28ai}$$

$$0 \le \overline{\gamma}_{bi}^{\mathrm{Ar1}} \perp \zeta_{bi}^{\mathrm{Ar.3f}} \ge 0, \qquad \forall b, i, \tag{28aj}$$

$$0 \le \overline{\gamma}_{bi}^{\mathrm{Ar2}} \perp \zeta_{bi}^{\mathrm{Ar.3g}} \ge 0, \qquad \forall b, i, \tag{28ak}$$

$$0 \le A + \alpha^{\mathrm{Ar}} \perp \underline{\zeta}^{\mathrm{Ar.4a}} \ge 0, \tag{28al}$$

$$0 \le -\alpha^{\mathrm{Ar}} + A \perp \overline{\zeta}^{\mathrm{Ar.4a}} \ge 0, \tag{28am}$$

where $\mathcal{L}^{\mathrm{Ar}}$ is the Lagrangian function of (20).

Lastly, the KKT conditions of (21) write

$$\frac{\partial \mathcal{L}^{\mathrm{Ps}}}{\partial \lambda^{\mathrm{E}}} = p - \sum_{n\in\mathcal{N}} d_n - L - \underline{\zeta}^{\mathrm{Ps.E}} + \overline{\zeta}^{\mathrm{Ps.E}} = 0, \tag{29a}$$

$$\frac{\partial \mathcal{L}^{\mathrm{Ps}}}{\partial \lambda^{\mathrm{B}}} = \alpha^{\mathrm{Ar}} + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}} - 1 - \underline{\zeta}^{\mathrm{Ps.B}} + \overline{\zeta}^{\mathrm{Ps.B}} = 0, \tag{29b}$$

$$0 \le \Lambda + \lambda^{\mathrm{E}} \perp \underline{\zeta}^{\mathrm{Ps.E}} \ge 0, \tag{29c}$$

$$0 \le -\lambda^{\mathrm{E}} + \Lambda \perp \overline{\zeta}^{\mathrm{Ps.E}} \ge 0, \tag{29d}$$

$$0 \le \Lambda + \lambda^{\mathrm{B}} \perp \underline{\zeta}^{\mathrm{Ps.B}} \ge 0, \tag{29e}$$

$$0 \le -\lambda^{\mathrm{B}} + \Lambda \perp \overline{\zeta}^{\mathrm{Ps.B}} \ge 0, \tag{29f}$$

where $\mathcal{L}^{\mathrm{Ps}}$ is the Lagrangian function of (21).

Similarly, the KKT conditions of the single optimization problem (26) are given by

$$\text{(27a)–(27am)}, \tag{30a}$$

$$\text{(28a)–(28am)}, \tag{30b}$$

$$p - \sum_{n\in\mathcal{N}} d_n - L = 0, \tag{30c}$$

$$\alpha^{\mathrm{Ar}} + \sum_{n\in\mathcal{N}} \alpha_n^{\mathrm{Ed}} - 1 = 0, \tag{30d}$$

Note that the KKT conditions of the single optimization problem (26) is a collection of the KKT conditions corresponding to optimization problems (18) and (20) with two additional equality constraints, namely (30c) and (30d). However, given that the constraints on λ^{E} and λ^{B} in the price-setters' optimization problem (21) are non-binding, the equality constraints (30c) and (30d) are equivalent to the derivatives with respect to λ^{E} (29a) and λ^{B} (29b) of the price-setter's problem.

Consequently, for non-binding constraints on λ^{E} and λ^{B} the solution of the single optimization problem (26) is equivalent to the solution of the Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$, and vice versa.

C.3 Uniqueness of the Nash equilibrium point

We note that the objective function (26a) of the single optimization problem is strictly convex given by the quadratic term $c(d_n, \alpha_n^{\mathrm{Ed}})$ and $c(p, \alpha^{\mathrm{Ar}})$ indicating strict monotonicity of players' preferences [30]. Owing to strict convexity of the objective function (26a) and the convex and compact strategy set (26b)–(26e), the single optimization problem (26) yields a unique solution. Since (26) is equivalent to the original Nash game $\Gamma(\mathcal{Z}, K, \{J_i\}_{\forall i \in \mathcal{Z}})$, the Nash equilibrium point is also unique. ■

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