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Master formula for ε'/ε beyond the Standard Model

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ABSTRACT

We present for the first time a master formula for ε'/ε, the ratio probing direct CP violation in $K \to \pi\pi$ decays, valid in *any* ultraviolet extension of the Standard Model (BSM). The formula makes use of hadronic matrix elements of BSM operators calculated recently in the Dual QCD approach and the ones of the SM operators from lattice QCD. We emphasize the large impact of several scalar and tensor BSM operators in the context of the emerging ε'/ε anomaly. We have implemented the results in the open source code flavio.

The non-conservation of the product of parity (P) and charge-conjugation (C) symmetries in nature, known under the name of CP violation, was established experimentally for the first time in 1964 via $K \to \pi\pi$ decays [1]. Since then, this fundamental phenomenon has been confirmed also in other processes in the quark sector and is rather consistently described by the so-called Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [2,3] within the Standard Model (SM) of elementary particle physics. Currently there are experimental efforts to establish analogous CP violation in the lepton sector.

CP violation proves to be a prerequisite [4] for our present understanding of matter dominance over anti-matter in the universe. However, the CP-violating contribution from the CKM matrix in the SM fails to account for this observation and it remains to be seen whether the CP-violating contributions in the lepton sector will be able to do so. As direct collider searches have not yet revealed any presence of new physics, rare processes in the quark sector remain a good territory to search for new sources of CP violation. This is especially the case for the kaon physics observables ε and ε', which measure indirect and direct CP violation in K^0-$\bar{K}^0$ mixing and K^0 decay into $\pi\pi$, respectively.

Recently, there has been a renewed interest in the ratio ε'/ε [5–25], due to hints for a significant tension between measurements and the SM prediction from the RBC-UKQCD lattice collaboration [26,27] and the Dual QCD approach (DQCD) [28,29]. While on the experimental side the world average from the NA48 [30] and KTeV [31,32] collaborations reads

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}\,, \tag{1}$$

the lattice collaboration [26,27] and the NLO analyses in [33,34] based on their results find $(\varepsilon'/\varepsilon)_{\text{SM}}$ in the ballpark of $(1-2) \times 10^{-4}$, that is by one order of magnitude below the data, but with an error in the ballpark of 5×10^{-4}. An independent analysis based on hadronic matrix elements from DQCD [28,29] gives a strong support to these values and moreover provides an *upper bound* on $(\varepsilon'/\varepsilon)_{\text{SM}}$ in the ballpark of 6×10^{-4}. A different view has been expressed in [35] where, using ideas from chiral perturbation theory but going beyond it, the authors find $(\varepsilon'/\varepsilon)_{\text{SM}} = (15 \pm 7) \times 10^{-4}$ in agreement with the data, albeit with a large uncertainty.

The results from RBC-UKQCD and DQCD motivated several authors to look for various extensions of the SM which could bring the theory to agree with data. For a recent review see [36]. In all the models studied to date, the rescue comes from the modification of the Wilson coefficient of the dominant electroweak left-right (LR) penguin operator Q_8, but also solutions through a modified contribution of the dominant QCD LR penguin operator Q_6 could be considered [10]. However, in generic BSM scenarios, also operators not present in the SM could play an important role. The very recent calculation of the $K \to \pi\pi$ hadronic matrix elements of all BSM four-quark operators, in particular scalar and tensor operators in DQCD [37] and the one of the chromo-magnetic operator by the ETM lattice collaboration [38] and in DQCD [39],

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allow for the first time the study of ε'/ε in an arbitrary extension of the SM. While the matrix element of the chromo-magnetic operator has been found to be much smaller than previously expected, the values of the BSM matrix elements of scalar and tensor operators are found to be in the ballpark of the ones of Q_8, the dominant electroweak penguin operator in the SM. Consequently, these operators could help in the explanation of the emerging ε'/ε anomaly.

As far as short-distance contributions encoded in the Wilson coefficients are concerned, they have been known for the SM operators already for 25 years at the NLO level [40–45] and for the BSM operators two-loop anomalous dimensions have been known [46,47] for almost two decades. First steps towards the NNLO predictions for ε'/ε have been made in [48–51] and the complete NNLO result should be available soon [52].

Having all these ingredients from long-distance and short-distance contributions at hand, we are in the position to present for the first time a master formula for ε'/ε that can be applied to any ultraviolet extension of the SM. Neglecting isospin breaking corrections, ε'/ε can be written as

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm th} = -\frac{\omega}{\sqrt{2}|\varepsilon_K|}\left[\frac{{\rm Im}A_0}{{\rm Re}A_0} - \frac{{\rm Im}A_2}{{\rm Re}A_2}\right], \tag{2}$$

where $\omega = {\rm Re}A_2/{\rm Re}A_0$ and $A_{0,2}$ are the $K \to \pi\pi$ isospin amplitudes,

$$A_{0,2} = \left\langle (\pi\pi)_{I=0,2} \,\middle|\, \mathcal{H}^{(3)}_{\Delta S=1} \,\middle|\, K^0 \right\rangle. \tag{3}$$

Isospin breaking corrections have been considered in [53,54]. These corrections will affect only the A_0 contributions that are suppressed by $\omega \sim 1/22$. They can only be relevant in NP scenarios in which, similar to the case of the SM, the Wilson coefficients of the operators contributing to A_0 are by more than one order of magnitude larger than those relevant for the A_2 amplitude. Here $\mathcal{H}^{(3)}_{\Delta S=1}$ denotes the $\Delta S = 1$ effective Hamiltonian with only the three lightest quarks ($q = u, d, s$) being dynamical, obtained by decoupling the heavy $W^\pm$, Z^0, and h^0 bosons and the top quark at the electroweak scale $\mu_{\rm ew} \sim m_W$ and the bottom and charm quarks at their respective mass thresholds [55].

Assuming that no particles beyond the SM ones with mass below the electroweak scale exist, any BSM effect is encoded in the Wilson coefficients of the most general $\Delta S = 1$ dimension-six effective Hamiltonian. The values of the Wilson coefficients $C_i(\mu_{\rm ew})$ in this effective Hamiltonian at the electroweak scale with $N_f = 5$ active quark flavours,

$$\mathcal{H}^{(5)}_{\Delta S=1} = -\mathcal{N}_{\Delta S=1} \sum_i C_i\, O_i\,, \tag{4}$$

are connected to those of $\mathcal{H}^{(3)}_{\Delta S=1}$, entering ε'/ε, by the usual QCD and QED renormalization group (RG) evolution. In full generality, three classes of operators can contribute, directly or via RG mixing, to $K \to \pi\pi$ decays:

a. *four-quark operators:*

$$O^q_{XAB} = (\bar{s}^i \Gamma_X P_A d^i)(\bar{q}^j \Gamma_X P_B q^j)\,, \tag{5}$$
$$\widetilde{O}^q_{XAB} = (\bar{s}^i \Gamma_X P_A d^j)(\bar{q}^j \Gamma_X P_B q^i)\,, \tag{6}$$

b. *electro- and chromo-magnetic dipole operators:*

$$O^{(\prime)}_{7\gamma} = m_s (\bar{s}\sigma^{\mu\nu} P_{L(R)} d) F_{\mu\nu}\,, \tag{7}$$
$$O^{(\prime)}_{8g} = m_s (\bar{s}\sigma^{\mu\nu} T^a P_{L(R)} d) G^a_{\mu\nu}\,, \tag{8}$$

c. *semi-leptonic operators:*

$$O^{\ell}_{XAB} = (\bar{s}\,\Gamma_X P_A d)(\bar{\ell}\,\Gamma_X P_B \ell)\,. \tag{9}$$

Here i, j are colour indices, $A, B = L, R$, and $X = S, V, T$ with $\Gamma_S = 1$, $\Gamma_V = \gamma^\mu$, $\Gamma_T = \sigma^{\mu\nu}$.[1] Throughout it is sufficient to consider the case $A = L$, whereas results for the case $A = R$ follow analogously due to parity conservation of QCD and QED. We will choose the overall normalization factor $\mathcal{N}_{\Delta S=1}$ below such that the coefficients C_i are dimensionless.

In the following, we will neglect the electro-magnetic dipole and semi-leptonic operators, which only enter through small QED effects. This leaves 40 four-quark operators for $N_f = 5$ and one chromo-magnetic dipole operator of a given chirality which have to be considered at the electroweak scale. A detailed renormalization group analysis of these operators, model independently and in the context of the Standard Model effective field theory (SMEFT), is performed in [56]. The goal of the present letter is to provide the central result of [56] and [37], the master formula for ε'/ε, in a form that could be used by any model builder or phenomenologist right away without getting involved with the technical intricacies of these analyses.

Writing

$$\left(\frac{\varepsilon'}{\varepsilon}\right) = \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm BSM}, \tag{10}$$

our formula allows to calculate automatically $(\varepsilon'/\varepsilon)_{\rm BSM}$ once the Wilson coefficients of all contributing operators are known at the electroweak scale $\mu_{\rm ew}$. It reads as follows:

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm BSM} = \sum_i P_i(\mu_{\rm ew})\, {\rm Im}\left[C_i(\mu_{\rm ew}) - C'_i(\mu_{\rm ew})\right], \tag{11}$$

where

$$P_i(\mu_{\rm ew}) = \sum_j \sum_{I=0,2} p^{(I)}_{ij}(\mu_{\rm ew}, \mu) \left[\frac{\langle Q_j(\mu)\rangle_I}{\text{GeV}^3}\right]. \tag{12}$$

The sum in (11) extends over the Wilson coefficients C_i of the linearly independent four-quark and chromo-magnetic dipole operators listed in Table 1. The C'_i are the Wilson coefficients of the corresponding chirality-flipped operators obtained by replacing $P_L \leftrightarrow P_R$. The relative minus sign accounts for the fact that their $K \to \pi\pi$ matrix elements differ by a sign. Among the contributing operators are also operators present already in the SM but their Wilson coefficients in (11) include only BSM contributions.

The dimensionless coefficients $p^{(I)}_{ij}(\mu_{\rm ew}, \mu)$ include the QCD and QED RG evolution from $\mu_{\rm ew}$ to $\mu \sim \mathcal{O}(1\,\text{GeV})$ for each Wilson coefficient as well as the relative suppression of the contributions to the $I = 0$ amplitude due to ${\rm Re}A_2/{\rm Re}A_0 \ll 1$ for the matrix elements $\langle Q_j(\mu)\rangle_I$ of all the operators Q_j present at the low-energy scale. The index j includes also i so that the effect of self-mixing is included. We refer the reader to [56] for the numerical values of the $p^{(I)}_{ij}(\mu_{\rm ew}, \mu)$ and $\langle Q_j(\mu)\rangle_I$ for our choice of the set of Q_j. The details given there allow to easily account for future updates of the matrix elements. The $P_i(\mu_{\rm ew})$ do not depend on μ to the considered order, because the μ-dependence cancels between matrix elements and the RG evolution operator. Moreover, it should be emphasized that their values are *model-independent* and depend only on the SM dynamics below the electroweak scale, which includes short distance contributions down to μ and the

[1] For Γ_T there is only $P_A = P_B$ in four dimensions but not $P_A \neq P_B$.

Table 1
Table of $\Delta S=1$ operators contributing to $(\varepsilon'/\varepsilon)_{\rm BSM}$ with coefficients $P_i(\mu_{\rm ew})$ for $\mu_{\rm ew}=160\,{\rm GeV}$, and corresponding suppression scales. The Hamiltonian is normalized as $\mathcal{H}^{(5)}_{\Delta S=1}=-\sum_i \frac{C_i(\mu_{\rm ew})}{(1\,{\rm TeV})^2}\,O_i$.

Class	O_i	P_i	$\frac{\Lambda}{\rm TeV}$	SMEFT
A)	$O^u_{VLL}=(\bar s^i\gamma_\mu P_L d^i)(\bar u^j\gamma^\mu P_L u^j)$	-4.3 ± 1.0	65	✓
	$O^u_{VLR}=(\bar s^i\gamma_\mu P_L d^i)(\bar u^j\gamma^\mu P_R u^j)$	-126 ± 9	354	✓
	$\widetilde O^u_{VLL}=(\bar s^i\gamma_\mu P_L d^j)(\bar u^j\gamma^\mu P_L u^i)$	1.5 ± 1.7	38	✓
	$\widetilde O^u_{VLR}=(\bar s^i\gamma_\mu P_L d^j)(\bar u^j\gamma^\mu P_R u^i)$	-436 ± 34	659	✓
	$O^d_{VLL}=(\bar s^i\gamma_\mu P_L d^i)(\bar d^j\gamma^\mu P_L d^j)$	2.3 ± 0.6	48	✓
	$O^d_{VLR}=(\bar s^i\gamma_\mu P_L d^i)(\bar d^j\gamma^\mu P_R d^j)$	123 ± 10	350	✓
	$O^d_{SLR}=(\bar s^i P_L d^i)(\bar d^j P_R d^j)$	-214 ± 19	462	✓
	$O^s_{VLL}=(\bar s^i\gamma_\mu P_L d^i)(\bar s^j\gamma^\mu P_L s^j)$	-0.4 ± 0.1	18	✓
	$O^s_{VLR}=(\bar s^i\gamma_\mu P_L d^i)(\bar s^j\gamma^\mu P_R s^j)$	-0.32 ± 0.05	17	✓
	$O^s_{SLR}=(\bar s^i P_L d^i)(\bar s^j P_R s^j)$	-0.0 ± 0.1	6	✓
	$O^c_{VLL}=(\bar s^i\gamma_\mu P_L d^i)(\bar c^j\gamma^\mu P_L c^j)$	0.7 ± 0.1	25	✓
	$O^c_{VLR}=(\bar s^i\gamma_\mu P_L d^i)(\bar c^j\gamma^\mu P_R c^j)$	0.7 ± 0.1	26	✓
	$\widetilde O^c_{VLL}=(\bar s^i\gamma_\mu P_L d^j)(\bar c^j\gamma^\mu P_L c^i)$	0.2 ± 0.1	13	✓
	$\widetilde O^c_{VLR}=(\bar s^i\gamma_\mu P_L d^j)(\bar c^j\gamma^\mu P_R c^i)$	0.4 ± 0.2	20	✓
	$O^b_{VLL}=(\bar s^i\gamma_\mu P_L d^i)(\bar b^j\gamma^\mu P_L b^j)$	-0.30 ± 0.03	17	✓
	$O^b_{VLR}=(\bar s^i\gamma_\mu P_L d^i)(\bar b^j\gamma^\mu P_R b^j)$	-0.28 ± 0.03	16	✓
	$\widetilde O^b_{VLL}=(\bar s^i\gamma_\mu P_L d^j)(\bar b^j\gamma^\mu P_L b^i)$	-0.0 ± 0.1	4	✓
	$\widetilde O^b_{VLR}=(\bar s^i\gamma_\mu P_L d^j)(\bar b^j\gamma^\mu P_R b^i)$	-0.1 ± 0.1	8	✓
B)	$O_{8g}\;=m_s(\bar s\sigma^{\mu\nu}T^a P_L d)\,G^a_{\mu\nu}$	-0.3 ± 0.1	18	✓
	$O^s_{SLL}=(\bar s^i P_L d^i)(\bar s^j P_L s^j)$	-0.05 ± 0.02	7	
	$O^s_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^i)(\bar s^j\sigma^{\mu\nu} P_L s^j)$	0.15 ± 0.05	12	
	$O^c_{SLL}=(\bar s^i P_L d^i)(\bar c^j P_L c^j)$	0.2 ± 0.1	15	✓
	$O^c_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^i)(\bar c^j\sigma^{\mu\nu} P_L c^j)$	0.14 ± 0.05	11	✓
	$\widetilde O^c_{SLL}=(\bar s^i P_L d^j)(\bar c^j P_L c^i)$	0.2 ± 0.1	14	✓
	$\widetilde O^c_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^j)(\bar c^j\sigma^{\mu\nu} P_L c^i)$	5.4 ± 1.8	73	✓
	$O^b_{SLL}=(\bar s^i P_L d^i)(\bar b^j P_L b^j)$	0.4 ± 0.1	18	
	$O^b_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^i)(\bar b^j\sigma^{\mu\nu} P_L b^j)$	0.11 ± 0.03	10	
	$\widetilde O^b_{SLL}=(\bar s^i P_L d^j)(\bar b^j P_L b^i)$	0.3 ± 0.1	18	
	$\widetilde O^b_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^j)(\bar b^j\sigma^{\mu\nu} P_L b^i)$	13.5 ± 4.3	116	
C)	$O^u_{SLL}=(\bar s^i P_L d^i)(\bar u^j P_L u^j)$	-74 ± 16	272	✓
	$O^u_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^i)(\bar u^j\sigma^{\mu\nu} P_L u^j)$	162 ± 36	402	✓
	$\widetilde O^u_{SLL}=(\bar s^i P_L d^j)(\bar u^j P_L u^i)$	15.6 ± 3.3	124	✓
	$\widetilde O^u_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^j)(\bar u^j\sigma^{\mu\nu} P_L u^i)$	509 ± 108	713	✓
D)	$O^d_{SLL}=(\bar s^i P_L d^i)(\bar d^j P_L d^j)$	87 ± 16	295	
	$O^d_{TLL}=(\bar s^i\sigma_{\mu\nu} P_L d^i)(\bar d^j\sigma^{\mu\nu} P_L d^j)$	-191 ± 35	436	
E)	$O^u_{SLR}=(\bar s^i P_L d^i)(\bar u^j P_R u^j)$	266 ± 21	515	
	$\widetilde O^u_{SLR}=(\bar s^i P_L d^j)(\bar u^j P_R u^i)$	60 ± 5	244	✓

long distance contributions represented by the hadronic matrix elements. The BSM dependence enters our master formula in (11) *only* through the Wilson coefficients $C_i(\mu_{\rm ew})$ and $C_i'(\mu_{\rm ew})$. That is, even if a given P_i is non-zero, the fate of its contribution depends on the difference of these two coefficients. In particular, in models with exact left-right symmetry this contribution vanishes as first pointed out in [57].

The numerical values of the $P_i(\mu_{\rm ew})$ are collected in Table 1 for

$$\mu_{\rm ew}=160\,{\rm GeV}\,,\qquad \mathcal{N}_{\Delta S=1}=(1\,{\rm TeV})^{-2}\,. \tag{13}$$

They have been calculated with the flavio package [58], where we have implemented general BSM contributions to ε'/ε. As seen in (12), the P_i depend on the hadronic matrix elements $\langle Q_j(\mu)\rangle_I$ and the RG evolution factors $p^{(I)}_{ij}(\mu_{\rm ew},\mu)$. The numerical values of the hadronic matrix elements rely on lattice QCD in the case of SM operators [26,27] and on results for scalar and tensor operators obtained in DQCD [37]. The matrix element of the chromo-magnetic dipole operator is from [38] and [39] which agree with each other.

The operators in Table 1 have been grouped into five distinct classes.

Class A: All hadronic matrix elements can be expressed in terms of the ones of SM operators calculated by lattice QCD [26,27].

Class B: All operators contribute only through RG mixing into the chromo-magnetic operator O_{8g} so that only one hadronic matrix element is involved and taken from [38,39].

Class C: RLRL type operators with flavour $(\bar sd)(\bar uu)$ that contribute via BSM matrix elements [37] or by generating the chromo-magnetic dipole matrix element [38,39] through mixing.

Class D: RLRL type operators with flavour $(\bar sd)(\bar dd)$ that contribute via BSM matrix elements [37] or the chromo-magnetic dipole matrix element [38,39].

Class E: RLLR type operators with flavour $(\bar sd)(\bar uu)$ that contribute exclusively via BSM matrix elements [37].

Besides the P_i, we provide in the next-to-last column of Table 1 the suppression scale Λ that would generate $(\varepsilon'/\varepsilon)_{\rm BSM}=10^{-3}$ for $C_i=(1\,{\rm TeV})^2/\Lambda^2$. It gives an indication of the maximal scale probed by ε'/ε for any given operator.

Among the 40 four-quark operators present in $\mathcal{H}^{(5)}_{\Delta S=1}$, four have been omitted in Table 1, namely $O^{b,c}_{SLR}$ and $\widetilde O^{b,c}_{SLR}$, since they neither contribute directly nor via RG mixing at the level considered, i.e. they have $P_i=0$.

In models with a mass gap above the electroweak scale, $v\ll\Lambda$, where v is the Higgs vacuum expectation value and Λ the BSM scale, some of the operators in Table 1 are not generated at leading order in an expansion in v/Λ. As discussed in more detail in [56], these operators violate hypercharge, that is conserved in the SMEFT above the electroweak scale.[2] In the rightmost column of Table 1, we have indicated whether the operator can arise from a tree-level matching of SMEFT at dimension six onto the $\Delta S=1$ effective Hamiltonian (cf. [59,60]).

Inspecting the results in Table 1, the following comments are in order.

- The largest P_i values in Class A can be traced back to the large values of the matrix elements $\langle Q_{7,8}\rangle_2$, the dominant electroweak penguin operators in the SM, and the enhancement by $1/\omega\approx22$ of the $I=2$ contributions.
- The small P_i values in Class B are the consequence of the fact that each one is proportional to $\langle O_{8g}\rangle_0$, which has recently been found to be much smaller than previously expected [38,39]. Moreover, as $\langle O_{8g}\rangle_2=0$, all contributions in this class are suppressed by the factor $1/\omega$ relative to contributions from other classes.
- The large P_i values in Classes C and D can be traced back to the large hadronic matrix elements of scalar and tensor operators calculated recently in [37]. Due to the smallness of $\langle O_{8g}\rangle_0$, the contribution of the chromo-magnetic dipole operator is negligible.
- While the operators in Classes D have sizable P_i, they violate hypercharge as discussed above, so they do not arise in a tree-level matching from SMEFT at dimension six.
- While the $I=0$ matrix elements of the operators in Class E cannot be expressed in terms of SM ones, the $I=2$ matrix elements can, and the large P_i values can be traced back to the large SM matrix elements $\langle Q_{7,8}\rangle_2$.

Almost all existing BSM analyses of ε'/ε in the literature are based on the contributions of operators from Class A or the chromo-magnetic dipole operator. Table 1 shows that also other

[2] As an exception, the hypercharge constraint can be avoided for the operator $\widetilde O^u_{SLR}$, if in the intermediate SMEFT the dimension-six operator with right-handed modified $W^\pm$ couplings ($\mathcal{O}_{Hud}$ in the basis of [70]) is generated, as for example in a left-right symmetric UV completion of the SM due to tree-level W_L–W_R mixing. The $\widetilde O^u_{SLR}$ is then generated at the electroweak scale by the tree-level $W^\pm$ exchange of a single insertion of $\mathcal{O}_{Hud}$ with a dimension-four SM coupling of $W^\pm$ and quarks.

operators, in particular the ones in Class C, could be promising to explain the emerging ε'/ε anomaly and can play an important role in constraining BSM scenarios.

However, in a concrete BSM scenario, the Wilson coefficients with the highest values of P_i could vanish or be suppressed by small couplings. Moreover, additional constraints on Wilson coefficients can come in SMEFT and from correlations with other observables as discussed in more detail in [56].

Next, we would like to comment on the accuracy of the values of the P_i listed in Table 1. As far as short distance contributions to the P_i are concerned, they have been calculated in the leading logarithmic approximation to RG improved perturbation theory using the results of [61–65]. Although the inclusion of next-to-leading corrections is possible already now, such contributions are renormalization scheme dependent and can only be cancelled by the one of the hadronic matrix elements. While in the case of SM operators this dependence has been included in the DQCD calculations in [66], much more work still has to be done in the case of BSM operators.

The uncertainties from the matrix elements depend on the operator classes in Table 1. In the P_i column, we have given the uncertainties obtained from varying the individual matrix elements, assuming them to be uncorrelated. In Class A, they stem from the lattice matrix elements. Here we point out that due to the enhancement of the $I = 2$ contributions by the factor $1/\omega \approx 22$, the largest P_i are dominated by the $I = 2$ matrix elements, which are known to 5–7% accuracy from lattice QCD [27]. For the small P_i in Class A, in some cases there are cancellations between contributions from different matrix elements, leading to larger relative uncertainties. The matrix elements of four-quark BSM operators entering Classes C–E have only been calculated recently in DQCD approach [37] and it will still take some time before lattice QCD will be able to provide results for them. Previous results of DQCD imply that it is a successful approximation of low-energy QCD and that the uncertainties in the largest P_i are at most at the level of 20%. While not as precise as ultimate lattice QCD calculations, DQCD offered over many years an insight in the lattice results and often, like was the case of the $\Delta I = 1/2$ rule [67] and the parameter $\hat{B}_K$ [68], provided results almost three decades before this was possible with lattice QCD. The agreement between results from DQCD and lattice QCD is remarkable, in particular considering the simplicity of the former approach compared to the sophisticated and computationally demanding numerical lattice QCD one. The most recent example of this agreement was an explanation by DQCD of the pattern of values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ entering ε'/ε obtained by lattice QCD [28,29] and of the pattern of lattice values for BSM parameters B_i in K^0-$\bar{K}^0$ mixing [69]. This should be sufficient for the exploration of new phenomena responsible for the hinted ε'/ε anomaly. Similar comments apply to the hadronic matrix element of the chromo-magnetic dipole operator, entering mainly the P_i in Class B, that was recently calculated in DQCD in [39] and found to be in agreement with the lattice QCD result from [38]. Since the P_i in Class B only receive a single contribution, their relative uncertainties mirror the relative uncertainty of the chromo-magnetic matrix element, that was estimated at 30% in [39].

The usefulness of our master formula is twofold. First, it opens the road to an efficient search for BSM scenarios behind the ε'/ε anomaly and through the values of P_i in Table 1 indicates which routes could be more successful than others. This will play an important role if the ε'/ε anomaly will be confirmed by more precise lattice QCD calculations. Second, it allows to put strong constraints on models with new sources of CP violation, in many cases probing scales up to hundreds of TeV, as shown in Table 1. Thus, even if future lattice QCD calculations within the SM will confirm the data on ε'/ε, our master formula will be instrumental in putting strong constraints on the parameters of a multitude of BSM scenarios.

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