When Reading Meets Mathematics

Using Eye Movements to Analyze Complex Word Problem Solving

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Abstract

Word problems are a vital part of mathematics education. In the last decades, the goals of mathematics education have shifted towards real-world applications, modelling, and a functional use of mathematics. Complex word problems are a feasible tool for the learning and assessment of these mathematical competencies. They typically include both functional and redundant contextual information, substantial amounts of text, a complex syntax, and multiple representations. Thus, complex word problems provide specific challenges to both students and researchers. In particular, linguistic factors play a pivotal role in complex word problem solving, and mathematical thinking and reading are tightly interwoven in the solution process.

In the research project presented here, the method of eye tracking was used to address a number of these challenges. Knowledge about the process of word problem solving was integrated with previous research on eye movements, taking into account both mathematical thinking and processes of reading as a foundation for a meaningful interpretation of global measures of eye movements during complex word problem solving. This integration had not been utilized in this context before. Based on these considerations, it was investigated how eye movements were associated with solution strategies and how they were related to performance as well as motivational-affective variables. Moreover, the approach provided the possibility to directly compare complex word problem solving in two fundamentally different languages, namely Chinese and German.

This dissertation summarizes two publications that report a total of three studies. The studies all analyzed eye movements during complex word problem solving. The first (N = 17) and second study (N = 42) are reported in Paper A and included the development and evaluation of appropriate global measures of eye movements. Further, they investigated their association with task difficulty and students' performance. The third study (N = 139) reported in Paper B compared word problem solving in Chinese and German and associated emerging reading patterns with achievement, mathematical self-concept, mathematics anxiety and flow experience. Furthermore, studies that were associated with the research project but were not part of this dissertation are briefly discussed.

The results indicated that patterns of eye movements were associated with the solution process of complex word problems and with cognitive and motivational-affective outcomes. Moreover, this association is very similar throughout Chinese and German. Therefore, the approach presented here provides specific possibilities to analyze and interpret complex word problems and offers unique insights into the close interplay between reading and mathematics.

Zusammenfassung

Textaufgaben sind ein wesentlicher Bestandteil von Mathematikunterricht. In den letzten Jahrzehnten haben sich die Ziele des Mathematikunterrichts hin zu realen Anwendungen, Modellieren und einer funktionsbezogenen Nutzung von Mathematik entwickelt. Komplexe Textaufgaben sind ein praktikables Werkzeug zum Erlernen und zur Erhebung dieser mathematischen Kompetenzen. Sie beinhalten typischerweise sowohl funktionale als auch redundante Kontextinformationen, substanzielle Textmengen, eine anspruchsvolle Syntax und verschiedene Repräsentationen. Komplexe Textaufgaben stellen daher sowohl Lernende als auch Forscherinnen und Forscher vor besondere Herausforderungen. Insbesondere sprachliche Faktoren spielen eine zentrale Rolle bei der Lösung komplexer Textaufgaben, und Lesen und mathematisches Denken sind im Lösungsprozess eng miteinander verwoben.

In dem hier vorgestellten Forschungsprojekt wurde die Methode des Eyetracking eingesetzt, um einige dieser Herausforderungen anzugehen. Das Wissen über den Prozess der Lösung von Textaufgaben wurde in die bisherige Forschung zu Blickbewegungen integriert, wobei sowohl mathematisches Denken als auch Leseprozesse als Grundlage für eine sinnvolle Interpretation von globalen Maßen von Blickbewegungen bei der Lösung komplexer Textaufgaben berücksichtigt wurden. Diese Integration war in diesem Zusammenhang bisher noch nicht umgesetzt worden. Auf dieser Grundlage wurde untersucht, wie Blickbewegungen mit Lösungsstrategien in Verbindung gebracht werden können und wie sie mit Performanz sowie motivational-affektiven Variablen zusammenhängen. Darüber hinaus war es dadurch möglich, die Lösung von komplexen Textaufgaben in zwei grundsätzlich unterschiedlichen Sprachen, nämlich Chinesisch und Deutsch, direkt zu vergleichen.

Diese Dissertation fasst zwei Publikationen zusammen, in denen insgesamt drei Studien berichtet werden. Alle Studien untersuchten dabei Blickbewegungen bei der Lösung komplexer Textaufgaben. Die erste (N = 17) und zweite Studie (N = 42) sind in Paper A enthalten und beinhalteten die Entwicklung und Bewertung geeigneter globaler Maße von Blickbewegungen. Außerdem untersuchten sie ihren Zusammenhang mit Aufgabenschwierigkeit und der Leistung der Studierenden. Die dritte Studie (N = 139), die in Paper B berichtet wird, verglich den Lösungsprozess von Textaufgaben auf Chinesisch und Deutsch und die damit verbundenen Lesemuster mit Leistung, mathematischem Selbstkonzept, mathematikbezogener Ängstlichkeit und Flow-Erleben. Darüber hinaus werden Studien kurz diskutiert, die mit dem Forschungsprojekt verbunden, aber nicht Teil dieser Dissertation waren.

Die Ergebnisse zeigten, dass Muster von Blickbewegungen mit dem Lösungsprozess komplexer Textaufgaben und mit motivational-affektiven Personenmerkmalen verbunden waren.

Darüber hinaus ist dieser Zusammenhang auf Chinesisch und Deutsch sehr ähnlich. Der hier vorgestellte Ansatz bietet daher spezifische Möglichkeiten zur Analyse und Interpretation komplexer Textaufgaben und einzigartige Einblicke in das enge Zusammenspiel von Lesen und Mathematik.

Included and Associated Publications

Included publications

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Associated publications

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Associated posters and presentations

Beitlich, J. T., Lehner, M. C., Strohmaier, A. R., Reiss, K. M. (2016). *The Relation of Eye Movements on Mathematical Tasks and Task Difficulty.* Presentation at the International Congress on Mathematical Education. Hamburg.

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Schmitz, A., Karstens, F., Jost, J., Strohmaier, A., Lehner, M. C., Reiss, K. (2018). *Strategien zum Verstehen von Fachtexten.* Symposium at the 6. Tagung der Gesellschaft für Empirische Bildungsforschung (GEBF). Basel, Switzerland.

Strohmaier, A. (2018). *Research in mathematics education with innovative technology - the example of eye tracking and electrodermal activity.* Presentation at the Katholieke Universiteit Leuven. Kortrijk, Belgium.

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Contents

1 Introduction

Word problems are characteristic tasks in school mathematics that can take various shapes and forms. Their occurrence ranges from simple verbal descriptions of basic arithmetic operations to extensive mathematical texts. Thus, they are often much more than just mathematics translated into everyday language. By integrating mathematics, reading, real-world knowledge, personal relevance, and authenticity, solving word problems can include a variety of cognitive processes that go way beyond calculation (Daroczy et al., 2015). This makes it a precious facet of today's mathematics education.

Word problem solving is a key building block to ultimately fulfill the promise of a functional, contemporary and relevant mathematics education (Niss, 2016). It is assumed that it can serve as a predictor of the success with which students will employ mathematics in their life and as an indicator for the quality of mathematics education and educational systems in general. This view on mathematics has gained popularity across the world during the last two decades, accelerated by large-scale studies like the *Programme for International Student Assessment* (PISA; OECD, 2013a). Accordingly, recent research often uses word problems to assess mathematical competence. Today, word problem solving is more important than ever.

Word problems have been a vivid part of research in mathematics education for decades. Nevertheless, recent developments have raised new questions. International comparative studies use longer, more complex word problems than previous research has focused on. Similarly, mathematical modelling that has emerged as a central aspect of mathematics education requires longer word problems, providing a substantial amount of contextual information (Vorhölter et al., 2019). With these developments, reading has become a more and more relevant aspect of word problem solving in particular, and of mathematics education in general (Morgan et al., 2014). Moreover, solution processes have gained importance over outcomes. Evaluating the final result of word problem solving is often merely the tip of the iceberg of the solution process, with large parts of the cognitive activities remaining unseen under the surface.

In the research project presented here, emerging challenges in research on word problem solving were addressed by using the method of eye tracking to analyze complex word problem solving. Eye tracking offers unique possibilities to observe cognitive processes, and elegantly addresses both reading and mathematical thinking. The following introduction serves as a frame for two papers published in the course of this project and puts it into a broader context, also addressing aspects that were investigated in associated publications.

2 Word Problems in Mathematics Education

2.1 Mathematical word problems

Word problems have been a vital part of mathematics education across cultures and centuries (Swetz, 2009). They are considered mathematical tasks in which relevant information is presented as text rather than in mathematical notion (Boonen et al., 2016; Daroczy et al., 2015; Verschaffel et al., 2000). An example for an ancient word problem is found in the Egyptian *Papyrus Rhind* which dates around 1550 BC (Robins & Shute, 1987).

> If a pyramid is 250 cubits high and the side of its base 360 cubits long, what is its seked[1]? (Chace, 1927; p. 96)

One of the major purposes of word problems is to connect mathematics and real-life. It is reasonable to assume that initially, word problems have been used primarily for targeted vocational training, to provide students with problems they will likely face during their typical work life (Swetz, 2009). Due to this inherent association with real-life applications of mathematics, word problem solving is closely related to the activity of *mathematical modelling*. According to Blum and Niss (1991), mathematical modelling refers to the entire process leading from a real-word problem situation to a (mental) mathematical model. It is assumed that this linking of mathematics and the real world does not only provide students with applications for their mathematical knowledge, but that it is a valuable mathematical activity in itself (e.g., Niss, 2016; Schoenfeld, 1991; Verschaffel et al., 2000; Vorhölter et al., 2019).

The definition of mathematical word problems is very broad and covers a variety of different mathematical tasks that differ with regard to their difficulty, content, or their appearance. For the purpose at hand, it is feasible to further distinguish between *prototype* word problems and *complex* word problems. Even though this distinction cannot be utilized rigorously (as will be seen below), it is feasible for a broad categorization.

2.1.1 Prototype word problems

Prototype word problems are problems that follow a simple, linear syntax and are relatively short, typically consisting of about three main clauses. They often refer to one of the four basic arithmetic operations. They might be contextualized, but the context is not of functional importance for the solution of the problem. Prototype word problems might include aspects of

[1] The *seked* was the Egyptian unit of slope. It is inverse to the modern notion of an incline.

modelling, but the focus of the problem is clearly on the mathematical operation. An example can be found in Verschaffel et al. (1992):

> Brian has 32 books. Ralph has 13 books more than Brian has. How many books does Ralph have? (p. 90)

This example very strictly fulfills the definition of a word problem, as it presents the same information as a basic arithmetic task, but in textual rather than in mathematical notion. It resembles the structure of the mathematical notation and contains exactly the information necessary to solve the problem. The context is authentic, but arguably the aspect of mathematical modelling is only marginally relevant for the solution. The function of the contextualization is limited to an illustration, but hardly offers specific challenges or benefits.

Prototype word problems closely resemble what is often described as *dressed up* problems in research on modelling (Leiss et al., 2019). However, dressed up problems are usually characterized solely with regard to the extent to which processes of modelling are required to solve the problem. In contrast, the categorization of prototype and complex word problems given here revolves not primarily around the aspect of mathematical modelling, but also around linguistic features such as length and syntax, and the amount of contextual information given.

2.1.2 Complex word problems

Complex word problems typically include a notable amount of text and show variations in the syntax. They provide superficial or redundant contextual information and do not immediately indicate the required mathematical operation to solve them. Often, multiple representations are used to display contextual information. The contextualization serves a functional purpose, which means that the task requires aspects of modelling and the integration of textual information. As an example serves the following problem from PISA (OECD, 2013b):

> Mount Fuji is a famous dormant volcano in Japan. Toshi wore a pedometer to count his steps on his walk along the Gotemba trail. His pedometer showed that he walked 22500 steps on the way up. Estimate Toshi's average step length for his walk up the 9 km Gotemba trail. Give your answer in centimetres (cm). (p. 20)

With the problem, a sketch of the Mount Fuji is given. In this problem, it is clear that a more complex syntax and context are used. The contextual information exceeds what is necessary to solve the problem and provides linguistic challenges. Moreover, the context and the textual information have a function in this word problem. They offer students the possibility to build a mental model of the situation and to include their knowledge and intuition on length. It also enables them to verify the result, since they might have a broad concept of how long a step might be. Clearly, the problem is very distinct from the equivalent problem in mathematical notion and provides specific challenges, both linguistic and with regard to modelling.

It should be noted that the distinction between prototype and complex word problems is a continuum rather than dichotomous. For example, consider the following example reported by Boonen et al. (2013):

> The area of a rectangular field is 60 square meters. If its length is 10 m, how far would you have traveled if you walked the whole way around the field? (p. 278)

This problem clearly differs from the example by Verschaffel et al. (1992) in that the syntax is more complex, the structure is not as linear, and the question does not immediately indicate the mathematical operation necessary to solve the problem. However, compared to the example from PISA, the context arguably does not serve a particularly functional purpose and seems therefore not essential to the solution process. Further, it does not include superficial information and is still relatively short.

2.2 Mathematical literacy and word problems

In a summary of research on word problem solving, Verschaffel et al. (2000) discussed in detail the role of word problems for mathematics education, particularly in providing real-life applications. The authors argued that the practice of using oversimplified and unrealistic word problems for practicing routine mathematical operations is somewhat problematic, as it neglects a meaningful and authentic relation between mathematics and the real world. An excessive use of these prototype problems will inherently contribute to what Schoenfeld (1991) referred to as "suspension of sense-making" (p. 316). This includes that students acquire a mindset whereby all word problems are solvable, the information included in the problem is sufficient, necessary and helpful for the problem solution, and there is a somewhat *nice* solution, for example whole numbers or insightful statements. This mindset contradicts many features of mathematics in the real world. In contrast, Verschaffel et al. (2000) called for a *modelling perspective* on word problem solving, focusing not solely on the mathematical operation behind the facade of the words, but on all aspects of mathematical modelling. Since then, mathematics education has fundamentally evolved, addressing the challenge to increase the relevance of mathematics education for the real world. This also had great effects on the use of word problems, as will be elaborated in the following.

2.2.1 From skills and routines to competencies

The idea that the goal of school mathematics is not merely the accumulation of content knowledge and the ability to perform mathematical operations has been discussed intensively in the last three decades in politics (e.g., NCTM, 1989, 2000; CCSSI, 2017; Klieme et al., 2003; KMK, 2012; Prenzel, et al., 2016) and research (e.g., DeSeCo, 2002; Greer et al., 2009; Neubrand et al., 2001; Niss, 2016). This development was further promoted by international large-scale assessments like PISA (e.g., OECD, 2016) and the *Trends in International Mathematics and Science Study* (TIMSS; e.g., IEA, 2015). Following this discussion, new ideas about the goals of school mathematics emerged. This included a reconceptualization of the term *mathematical competencies* that took into account a broader and more functional understanding of mathematical abilities (Weinert, 2001a, 2001b). Accordingly, competencies refer to mathematics in a wide range of contexts, in particular the real world. Being competent does not end with cognitive abilities, but students must be motivated and willing to use these abilities. As a consequence, competencies do not exist independent of action but are only observable in contexts (DeSeCo, 2002).

At the same time and inspired by the aforementioned theoretical considerations, a framework that operationalized mathematical competencies was developed in the context of PISA (Niss, 2016). By 2003, when mathematics was the major assessed domain in PISA for the first time, the concept of *mathematical literacy* was elaborated in detail in the study's framework (OECD, 2004). After a revision in 2012, when mathematics was the major domain again, the framework now arguably provides the internationally most established consensus on the goals of mathematics education (OECD, 2013a; Stacey & Turner, 2016). During this development, the framework took into account a variety of international concepts of functional goals of mathematics education (Neubrand et al., 2001; Niss, 2016).

With the recent definition of mathematical literacy, the shift from skills and routines to competencies as goals of mathematics education has reached a peak. With 78 countries and economies participating in PISA 2018, it has a pivotal influence on educational systems and on the understanding of the role of mathematics in society. But with the paradigm of mathematical literacy came new challenges, particularly regarding the question how to assess the achievement of these newly formulated goals of mathematics education. The central assumption that will be discussed here is that complex word problems are a suitable mathematical task to address these goals and that therefore, mathematical literacy is inherently linked to complex word problem solving.

2.2.2 Defining mathematical literacy

In PISA 2012, mathematical literacy is defined as

> […] an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013a; p. 25)

This definition includes a variety of aspects of a recent understanding of the goals of mathematics education. For example, it should be noted that mathematical literacy as defined in the PISA framework is closely related to the concept of mathematical modelling (Blum & Niss, 1991; Stacey, 2016; Vorhölter et al., 2019). Based on this definition, it is argued in the following that complex word problems are the favorable instrument for assessing mathematical literacy for two main reasons: (a) because they enable contextualization and (b) because they allow to focus on solution processes, including mathematical modelling.

2.2.3 Mathematical literacy and contextualization

In line with other preceding frameworks (NCTM, 1989; DeSeCo, 2002), mathematical literacy in PISA focuses on mathematics *in a variety of contexts* and *in the world*. This means that mathematics needs to be situated in the natural, social, and cultural setting in which students live (OECD, 2004; Stacey & Turner, 2016). The idea to put mathematics into context is in the core of mathematical literacy (OECD, 2013a; Stacey, 2016). It refers back to the works of Freudenthal, who stated that "mathematics should be taught within contexts, and I would like the most abstract mathematics taught within the most concrete context." (1981, p. 144). The development of the PISA framework followed this tradition (Neubrand et al., 2001). Thus, mathematical literacy is rooted in the functionality of school mathematics outside of school (De Lange, 2006). Consequently, all items used in PISA include a real world framing to assess not only the skills and routines, but also how successful they can be used in the world (OECD, 2013a). This can only be achieved through additional information that is given in illustrations, through data, or in text. Even though word problems are not the only way to achieve contextualization, written text is by far the most prominent solution in PISA (OECD, 2013b). When multiple representations are used to contextualize the mathematical problem, written information is usually chosen as the reference representation of information by students (Ott et al., 2018). Thus, complex word problems that include notable amounts of text are well-suited to provide contextual information.

2.2.4 Mathematical literacy and the role of solution processes

The mathematical activities mentioned in the definition of mathematical literacy mostly reflect mathematical processes rather than outcomes, e.g., *employing, interpreting,* or *reasoning*. In PISA 2012, this is further specified. Following the definition, mathematical literacy is organized in three interrelated aspects: mathematical processes, mathematical content, and context. The processes are *formulating situations mathematically*; *employing mathematical concepts, facts, procedures, and reasoning*; and *interpreting, applying and evaluating mathematical outcomes* (OECD, 2013a). In PISA, mathematical content is never assessed in an isolated manner, but always related to one of these processes.

Accordingly, when assessing mathematical literacy, problems must be designed in a way that they specifically address these mathematical processes. For example, to give students the opportunity to formulate a situation mathematically again requires a contextualization. Moreover, when students need to interpret and evaluate mathematical outcomes or reason mathematically, some kind of ambiguity must be present in the problem, which can also be achieved by embedding the problem in a context. In many ways, complex word problems offer the possibility to realize and specifically address the processes covered by the concept of mathematical literacy.

In sum, the goals of mathematical literacy imply that complex word problems are a feasible tool to assess the achievement of a recent understanding of the goals of mathematics education. However, complex word problem solving includes a number of cognitive activities that go beyond modelling. Particularly, the larger amount of text means that reading gains importance in word problem solving. This will be discussed in the following.

2.3 Reading and mathematical word problem solving

In a literature review, Daroczy et al. (2015) analyzed how linguistic factors contribute to word problem difficulty. They conclude that language is of pivotal importance for word problem solving in general, and that therefore, aspects of language need to be taken into account in the analysis of their solution process. With the increase in text length and difficulty in complex word problems, the influence of linguistic factors possibly gains even more importance. This close relation between reading and mathematical thinking in complex word problem solving is illustrated by empirical results. For example, PISA reported a latent correlation between reading and mathematics of r = .86 (OECD, 2014). This means that on average, students who performed well in mathematics also performed substantially better in reading, and vice versa.

Kintsch and Greeno (1985) proposed a process model that can be used to describe how reading and mathematical thinking interact during word problem solving. The model distinguishes between two subprocesses. First, the verbal input is transformed into a conceptual representation of its meaning and then organized to form the *text base*. This step closely resembles decoding processes during reading (Strohmaier, Lehner, et al., 2019). From the text base, the problem model (or *situation model* in other research) is constructed by excluding irrelevant information from the text base and by inferring additional information. This process is very similar to processes of comprehending regular text (Strohmaier, Lehner, et al., 2019). The building of a problem model has particularly shown to be a key predictor of complex word problem solving and modelling (Leiss et al., 2010). In a study by Leiss et al. (2019), the successful building of a suitable situation model depended not only on mathematics performance, but also on reading comprehension. The problem model contains the problem-relevant information in a form suitable for solution. Kintsch and Greeno's (1985) model has since been refined and modified by several researchers, but the basic processes remained fairly similar in most models of word problem solving (e.g., Cummins et al., 1988; Nathan et al., 1992; Nesher & Teubal, 1975; Reusser, 1990), always emphasizing the close relation of reading and mathematical processes.

Regarding the relation between processes of reading and mathematical thinking, two general assumptions can be distinguished. The first assumption is that processes of reading and mathematical thinking are separated and occur sequentially. In this case, students first read the word problem, translate it into a mathematical representation, and finally solve the resulting, purely mathematical problem. This approach is referred to as a *direct translation strategy* (Hegarty et al., 1995). A direct translation strategy might be efficient for some prototype word problems (Pape, 2004), but is prone to inconsistency and linguistic complexity (de Koning et al., 2017; Hegarty et al., 1992; Hegarty et al., 1995; Lewis & Mayer, 1987). The second assumption is that mathematical and linguistic processes constantly interact and occur in parallel. This is referred to as a *problem model strategy* (Hegarty et al., 1992). Here, decoding information and relating it to mathematical concepts happens simultaneously. Accordingly, reading and mathematical thinking is integrated, repeated and combined to form a holistic problem model rather than a mathematical translation of the word problem. This problem model strategy is very similar to the process of building a *situation model* in research on mathematical modelling (Leiss et al., 2010).

Although both strategies have been observed in word problem solving, a direct translation strategy gets more and more ineffective with increasing word problem complexity (de Koning et al., 2017). Prototype word problems are often solved successfully with keyword strategies, but complex word problems usually require a constant integration of text comprehension and mathematical processes (Daroczy et al., 2015). Furthermore, students need to integrate prior knowledge about the context of the item and the mathematical content to successfully form a problem model (Kintsch & Greeno, 1985). For example, the last sentence of an item from PISA reads (see also Figure 2):

> Using Toshi's estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm? (OECD, 2013b; p. 20)

When reading this sentence and building a problem model, it seems reasonable to assume that mathematical, scientific, and linguistic knowledge about the concept of the "estimated speed", the "latest time", or "by 8 pm" are not clearly distinguishable. Therefore, students cannot simply translate the problem and then solve it but need to make use of integrated knowledge.

Since text comprehension and mathematical abilities interact in proficient word problem solving, it is impossible to differentiate from the solution what processes and what individual prerequisites led to the result. Accordingly, mathematical literacy is not merely the sum of mathematical reading comprehension for decoding and arithmetic skills for computation (Daroczy et al., 2015). Their interaction forms a distinct factor that influences the solution process of word problems (Abedi, 2006; Boonen et al., 2016; Boonen et al., 2013). Thus, many students can successfully solve arithmetic tasks and have good text comprehension skills but might struggle when solving word problems and items assessing mathematical literacy (Daroczy et al., 2015).

This has a number of important implications for mathematics education (Daroczy et al., 2015). For example, reading comprehension and language skills should be specifically addressed in mathematics classrooms (Boonen et al., 2016). Moreover, the language in which a mathematical word problem is presented could have an effect on solution processes. For example, it has been assumed that Chinese offers specific benefits for representing mathematics (Galligan, 2001). International large-scale studies use translated versions of complex word problems in order to compare mathematical achievement between countries. However, the question how the translation affects the solution process is not yet fully resolved (Grisay et al., 2007). All these aspects underpin the notion that language plays a key role in complex word problem solving.

2.4 Assessing word problem solving

In mathematics in general and in complex word problem solving in particular, the final solution only reflects a small part of the information that is relevant to understand how students solved the task. A correct or incorrect answer can have a myriad of reasons, ranging from factual knowledge over competencies and strategies to luck, motivation, or anxiety. Accordingly, mathematical literacy specifically emphasizes mastering mathematical processes and affective-motivational aspects of mathematical competencies, extending goals of successful mathematics education beyond factual knowledge (OECD, 2013a; Schiepe-Tiska et al., 2016). If such aspects are not taken into account, it is impossible to fully distinguish the reasons for students' success or failure in solving mathematical tasks. Discriminating the process through which students came to a solution and including motivational-affective outcomes are key prerequisites to make well-founded judgements about comprehensive competencies of the student, ways to foster it, and the validity of the instrument (De Lange, 2007; Jarodzka et al., 2019). Thus, with the shift towards mathematical literacy and complex word problem solving, the need to assess processes rather than outcomes and to include motivational-affective variables was renewed.

A multitude of methods have been explored to analyze processes of learning, with a *Special Interest Group* of the *European Association for Research on Learning and Instruction* (EARLI) focusing solely on their assessment (Jarodzka et al., 2019). For complex word problem solving, methods like think aloud protocols, retrospective interviews or error analyses have been previously used for in-depth analysis of the validity of items from large-scale assessments (De Lange, 2007). However, these methods might be limited in their potential to observe learning processes comprehensively (Jarodzka et al., 2019; Lai et al., 2013). Think-aloud protocols have the inherent deficit that they might obstruct the solution process as they require cognitive resources and might interfere with the solution itself. Both interviews and think-aloud protocols share the limitation that they merely document processes that are consciously reported. Therefore, self-reports might be flawed and selective. Lastly, retrospective reports of all kinds might always be subject to selection and fading. For assessing motivational-affective aspects of word problem solving, physiological measures like *Electrodermal Activity* have been applied, also in research associated with the current project (Strohmaier, Schiepe-Tiska, & Reiss, 2020). Apart from the challenge of assessing learning processes, the close interaction between reading and mathematical thinking in complex word problem solving provides another challenge for assessment. Since reading plays such an important role, it seems desirable to have an assessment method that displays this interaction. The method of eye tracking meets many of these requirements, as will be laid out in chapter 3.

2.5 Summary

In this chapter, mathematical literacy was introduced as one of the most established examples of a recent conceptualization of the goals of mathematics education. It was argued that this conceptualization promotes the use of complex word problems within classrooms as well as for assessing the success of mathematics education. This is because they allow to embed mathematics in the real world and address various mathematical processes that go beyond calculation, particularly mathematical modelling. However, the close association with processes of reading and the need to take into account the solution process rather than just the outcome provides a number of open questions. For example, identifying under what circumstances reading and mathematical thinking occur sequentially or in parallel and how this affects the solution process and the outcome is an important question for understanding the way complex word problems are tackled. When educational systems from countries with different languages are compared, it is of profound importance to understand the nature and effects of these differences. Moreover, motivational-affective variables should be taken into account for a comprehensive understanding of complex word problem solving. However, methods to meet these open questions need to fulfill some prerequisites. In particular, process data should be gathered, while at the same time providing an assessment environment that is as unobtrusive as possible. Based on these considerations, it will be argued in the following that eye tracking provides a promising method to tackle some uniquely informative and relevant questions regarding complex word problem solving.

3 Observing Eye Movements

The eyes play a pivotal role in decoding visual information. Therefore, they provide a link between the real world and the brain, and the observation of eye movements offers unique insights into a broad range of cognitive processes (Holmqvist et al., 2011; Just & Carpenter, 1980; Rayner, 1998). Accordingly, the method of eye tracking that records these movements has been used in numerous studies from various domains (see Duchowski, 2007; for an overview). In the following, the method's potential for complex word problem solving will be elaborated. First, some general theoretical and practical considerations will be introduced that highlight the unique potential of eye tracking. Second, the association between eye movements and cognitive processes will be discussed, transferring key findings from research on eye movements during reading to complex word problem solving. Third, a brief overview of previous related research is given, narrowing down towards the specific application presented here. In conclusion, it will be argued that eye tracking offers a way to analyze complex word problem in a new and uniquely informative way.

3.1 General benefits of eye tracking

Eye tracking provides a number of benefits that distinguish its potential for research from other methods (Duchowski, 2007). Referring to the research presented here, three specific benefits will be briefly summarized.

a) Assessing solution processes rather than outcomes

As was laid out earlier, mathematical literacy and complex word problem solving emphasize the importance of the solution process compared to the solution itself. Eye tracking is a process measure, as data is collected continuously throughout the recording. Process measures enable researchers to distinguish events that occur during the solution process and are not visible in the result, like the order of information processing, solution strategies, or occurring problems (Jarodzka et al., 2019). Compared to other possibilities to assess processes of learning like interviews or think-aloud protocols, eye tracking provides a relatively unobtrusive method, because it does not require participants to interrupt their work or to spend cognitive resources on self-observation and communication. Moreover, data is collected live and not retrospectively.

b) Observing the interaction between reading and mathematical thinking

Reading and mathematical thinking are important aspects of complex word problem solving. Within the respective domains, it has been shown that eye movements are associated with both processes (Rayner et al., 1998; Strohmaier et al., in press). Thus, reading and mathematical thinking potentially influence eye movements jointly. If this is the case, eye tracking should offer the possibility to capture this interaction and thereby observe complex word problem solving holistically. In comparison, other methods will often have to focus on one of the two aspects. For example, analyzing students' solutions could indicate mathematical errors and misconceptions, but is possibly less informative regarding aspects of reading. On the other hand, think-aloud protocols could reveal issues during the reading process, but might disrupt mathematical thinking.

c) Making specific cognitive processes visible

Eye tracking provides researchers with the possibility to directly observe cognitive processes that would otherwise be challenging to visualize (Strohmaier et al., in press). For example, research on the subconscious mental associations of space and numbers (*spatial-numerical associations;* SNA), like the *mental number line* have benefited substantially from the observation of eye movements (e.g., Sullivan et al., 2011). Similarly, eye tracking could offer insights into the process of complex word problem solving that are unique to the method, particularly if they are related to reading, which is a highly automated and subconscious process (Rayner et al., 2012).

3.2 Local and global measures of eye movements

During reading, the human eye moves continually. Two fundamental kinds of eye movements can be distinguished. During *fixations*, the eyes remain relatively still on one point. During a fixation, visual acuity is very good in the central region of vision, which is about 2° (Rayner, 1998)[2]. Outside this area, acuity gets poorer. Therefore, in order to process larger amounts of visual information, the eyes fixate only for about 100 ms to 500 ms before quickly moving on to the next fixation location at a speed of up to 500°/s (Holmqvist et al., 2011). These extremely quick movements between fixations are called *saccades*. Vision is almost completely suppressed during saccades (Matin, 1974). Saccades are ballistic movements, which means that they cannot be controlled after they are initiated. Fixations and saccades are the building

[2] At a typical reading distance of 50 cm, 2° of vision correspond to the area of a circle with a diameter of 1.75 cm.

blocks for characterizing eye movements (Holmqvist et al., 2011; Rayner et al., 2012). From these fundamental events, various measures of eye movements can be derived that are assumed to be associated with different cognitive processes (see Holmqvist et al., 2011, for an overview). For the present research, it was distinguished between *local* and *global* measures of eye movements (Strohmaier, Lehner, et al., 2019).

Local measures refer to specific elements of the stimulus like words, illustrations, or numbers. These measures typically relate to the *eye-mind assumption* formulated by Just and Carpenter (1980). It states that "the eye remains fixated on a word as long as the word is being processed" (p. 330) and has since been interpreted as a more general rule of *visual focus equals cognitive focus.* According to the eye-mind assumption, local measures like the number of fixations on specific areas are assumed to provide researchers with a distribution of attention on the stimulus. In contrast, global measures of eye movements relate to the stimulus as a whole and are typically standardized by the text length. Global measures of eye movements include, but are not limited to, the *mean fixation duration*, the *number of saccades per word*, the *reading speed*, the *frequency of blinks*, and the *proportion of regressions*[3]. While local measures typically illustrate the distribution of attention, global measures refer to more holistic cognitive processes (Strohmaier, Lehner, et al., 2019). They are based on the complex cognitive processes that occur during information processing, which have been investigated in detail by research on eye movements during reading (Rayner, 1998). In the following, it will be argued that global measures of eye movements are a powerful tool to analyze complex word problem solving and that they offer key advantages over local measures. To that end, it will first be described how they are related to cognitive processes in general, and in complex word problem solving in particular.

3.3 Global measures of eye movements and cognitive processes

Research on reading provides elaborated models that explain the association between eye movements and cognitive processes (e.g., Reichle et al., 1998). According to these models and supported by a large number of research, it is assumed that the two key factors controlling eye movements during reading are *word recognition* and *text comprehension* (Clifton et al., 2016).

The process of word recognition involves several simultaneous cognitive activities (for a detailed summary, see Rayner & Liversedge, 2011). When a word is somewhat near to the center

[3] Regressions are saccades that are directed backwards in relation to the regular reading direction.

of the current fixation, it is previewed peripherally and decisions about whether and where to fixate next are made. The time it takes the brain to initiate a saccade (the *saccade latency*) is around 200 ms. Waiting for the eyes to finish decoding on each fixation before initiating the next saccade would lead to a substantial waiting time of 200 ms per fixation, which would almost double the total reading time. To counteract this, the brain needs to proactively initiate the eyes to move on before the current fixation is finished and all information is fully decoded. Thus, it has to count on the eyes to finish word recognition before the saccade is carried out 200 ms later. This means that the brain needs to initiate the saccade, its direction and its length all based on provisional information and that it cannot stop or alter the saccadic movement once it is initiated. During the actual fixation, the brain immediately starts to analyze the word. To optimize this process, the reader's brain heavily relies on preliminary information about the currently fixated word, gathered from prior knowledge, contextual information, the surrounding text, and prevision (Rayner et al., 2012). Accordingly, simply decoding the written information in a *bottom-up* way (from the text to the brain) is not sufficient for efficient word recognition, but *top-down* processes (from the brain to the text) need to be integrated. The most important variables that influence word recognition speed and success are word frequency, word predictability, and word length: Words that are more frequent in the language of the text (e.g., "the", "and", or "have") are fixated shorter and skipped more often than less common words. The same is true for words that can be predicted by the context (e.g., „Wonderland" in "Alice's Adventures in Wonderland") and for shorter words (Clifton et al., 2016; Rayner & Liversedge, 2011). Words that are harder to recognize get reread more often and sometimes require more than one fixation. Primarily, challenges in word recognition therefore lead to an increase of the mean fixation duration and a decrease in reading speed. Further, it increases the number of saccades and regressions.

Further, eye movements during reading are influenced by text comprehension. Higher order characteristics of a text such as its semantic or grammatical structure influence the reading process. For example, in the moment an element of the text is not immediately understood by the reader, the regular reading process might get interrupted (Clifton et al., 2016; Rayner et al., 2004). The eyes might further investigate the critical word (or critical region) until the problem has been resolved. This would lead to an increase in the number of saccades and regressions, longer fixation duration and a decrease in reading speed. Alternatively, the reader might execute a regression to reread previous segments of text – this leads to an increase in the number of saccades and proportion of regressions and again, a decrease in reading speed. The more of these unexpected words, structures, or references occur, the more the reading is disrupted. The plausibility, predictability, and complexity of the content of a sentence are factors that influence the possibility and amount of disruption (Clifton et al., 2016). Text comprehension also depends on individual factors such as reading abilities, working memory span, motivation and individual reading intention (Radach & Kennedy, 2004; Rayner et al., 2004). Compared to word recognition that mainly affects mean fixation duration, text comprehension should be reflected mainly through the number of saccades and proportion of regressions.

3.4 Global measures of eye movements in complex word problem solving

For the current research project, the assumptions regarding the relation between eye movements and cognitive processes from research on eye movements during reading were transferred to complex word problem solving (Strohmaier, Lehner, et al., 2019). This builds on the assumption that word recognition and text comprehension are closely related to aspects of word problem solving. According to the process model by Kintsch and Greeno (1985; see section 2.3), a word problem first needs to be transformed to the text base. This requires the transformation of the visual input into a conceptual representation. This appears very similar to the process of word recognition that also features the transformation of a visual stimulus into a mental representation (Clifton et al., 2016). This observation is in line with findings that linguistic factors on the word-level influence word problem difficulty, for example if the word problem consists of longer, unfamiliar, or ambiguous words (Daroczy et al., 2015; Hegarty et al., 1995). Because of these similarities, it is assumed that the process of building a text base is reflected in eye movements similarly to the process of word recognition. Accordingly, it is assumed that the process of building a text base predominantly affects the mean fixation duration and reading speed (Strohmaier, Lehner et al., 2019).

Further, it is assumed that the process of text comprehension during reading is associated with the process of building a problem model in word problem solving (Leiss et al., 2019; Strohmaier, Lehner et al., 2019). In mathematical word problems, typical aspects of text comprehension, for example the semantic structure, the order of information, the placement of the question, or the presence of distractors influences the difficulty of a word problem (Boonen et al., 2013; Daroczy et al., 2015). According to Kintsch and Greeno's (1985) model, top-down comprehension processes are responsible for organizing the text base, exclude irrelevant information, and infer missing information required to build a problem model, which is very similar to top-down text comprehension processes (Goldmann & Rakestraw, 2000). Similar to reading, word problem solving therefore reflects an epistemic interaction between text and reader. Thus, the building of a problem model is assumed to affect parameters of eye movements similarly as text comprehension processes during reading. Accordingly, the number of saccades and proportion of regressions should be related to processes of problem model building (Strohmaier, Lehner, et al., 2019).

3.5 Eye tracking in mathematics education research

Eye tracking has been increasingly implemented in research on learning in general (see Lai et al., 2013; for a review) and in mathematics education in particular (Barmby et al., 2014; Hartmann, 2015; Lilienthal & Schindler, 2019; Mock et al., 2016; Perttula, 2017; Schindler et al., 2018). In a contemporary systematic literature review, we recently reviewed the use of the method in mathematics education (Strohmaier et al., in press).

In the review, specific benefits that eye tracking holds for research in mathematics education were identified. Based on a total of 160 reviewed studies, we concluded that eye tracking provides unique possibilities to investigate learning processes in mathematics. In particular, observing eye movements offered ways to reveal mental representations and solution strategies without interfering with the process of learning or solving problems. Moreover, with increasing technological and methodological developments and a growing interest in the method, the amount of studies in mathematics education that use eye tracking has greatly increased during the last decade (Strohmaier et al., in press). According to the review, mathematics education researchers interpreted eye movements in various ways. However, a majority of studies build their interpretation of eye movements on the eye-mind assumption and implemented local measures of eye movements. Other studies mapped eye movements directly onto mental representations. The review also showed that relatively few studies used global measures of eye movements (Strohmaier et al., in press).

Studies on word problem solving reflect some of the earliest occasions where eye tracking was used in mathematics education research. The first study was reported by Terry (1921), who analyzed fundamental characteristics of eye movements during word problem solving and related them to early models of eye movements during reading. However, no further research building on his findings and using eye tracking was conducted during the following decades. The method received new attention during the 1980s, when De Corte and Verschaffel (1986a, 1986b) used eye tracking to analyze addition and subtraction word problems. In following studies, they addressed both linguistic and mathematical characteristics of word problems to investigate their interrelation, for example to illustrate Lewis & Mayer's (1987) *consistency hypothesis* (De Corte et al., 1990; Verschaffel et al., 1992). Hegarty, Mayer, and Green (1992) as well as Hegarty, Mayer, and Monk (1995) continued this line of research, relating word problem solving to students' mathematical skills. Van der Schoot et al. (2009) further analyzed the consistency hypothesis and distinguished between fine-grained linguistic aspects of word problems. In a different approach, Fry (1988) used eye tracking to analyze the relation between eye movements during word problem solving and spatial visualization abilities.

All of these aforementioned studies investigated prototype word problems and utilized local measures of eye movements. They strongly implied that eye tracking can provide insights into the interplay between processes of reading and mathematical thinking during word problem solving but are not directly transferable to complex word problem solving. For example, analyzing the relative amount of attention on the relational term in addition or subtraction problems can reveal keyword strategies in prototype word problems (e.g., Hegarty et al., 1992). Critically, in complex word problems the operation is often not indicated by a single relational term and the relative amount of attention is not comparable between problems of differing length and appearance. For example, one problem might contain more irrelevant contextual information. In this case, the number of fixations on relevant keywords or numbers gets obscured. Moreover, the process of modelling is typically less pronounced in prototype word problem solving (Verschaffel et al., 2000) and arguably less reflected in local measures of eye movements (Strohmaier, Lehner, et al., 2019). In order to analyze and compare complex word problems, global measures of eye movements thus seem to hold a unique potential, as they are standardized and indicate cognitive processes beyond the distribution of attention.

Recent studies considered more complex word problems, yet they did not focus on reading and solution processes but on the integration of illustrations (Dewolf et al., 2015; Sajka & Rosiek, 2015), formulae (Kohlhase et al., 2018), or figures (Lee & Wu, 2018). The study that is most related to the current research was reported by Stolinska et al. (2014), who used eye tracking to investigate differences in the reading behavior of experts and novices in word problems. They found that experts tended to show longer saccades, which was in line with previous research on saccadic eye movements during reading. However, their findings were not directly linked to cognitive processes of word problem solving.

3.6 Summary

In the current chapter, eye tracking was introduced as a method to analyze complex word problem solving in a novel way. It was argued that in general, it provides the possibility to analyze solution processes rather than outcomes, that it is a promising approach to investigate the interplay of reading and mathematical thinking, and that it offers unique insights into cognitive processes (Strohmaier et al., in press). For the purpose of this research project, it was distinguished between local and global measures of eye movements. Local measures are a feasible tool to analyze prototype word problems, where problems follow a simple structure and eye movements can be compared based on this structure. For complex word problems that differ with regard to their length and appearance and emphasize processes of modeling, standardized global measures seem to hold an additional potential. Global measures of eye movements have been of particular interest in research on eye movements during reading and are associated with underlying cognitive processes. By juxtaposing a model of eye movements during reading and a process model of word problem solving, it was argued that a similar relation can be assumed between global measures of eye movements and complex word problem solving (Clifton et al., 2016; Kintsch & Greeno, 1985; Rayner & Liversedge, 2011).

Existing studies in mathematics education provide evidence that observing eye movements can be a meaningful method for analyzing mathematical thinking. Moreover, our systematic literature review illustrated that in previous research, eye tracking provided more than just a visualization of the distribution of attention, and that more elaborate processes like mental effort or strategy use could be analyzed if appropriate measures were used (Strohmaier et al., in press). However, none of the studies reported by the review explicitly integrated models of eye movements during reading and models of complex word problem solving and used global measures of eye movements. Accordingly, the specific interplay between these aspects has not been holistically addressed. In sum, the observation of global measures of eye movements offers a promising approach to analyze complex word solving.

4 The Present Research

Summing up, the starting point for the research project reported here was the development of innovative goals of mathematics education during the last decades, which was exemplified by the concept of mathematical literacy. This shift initiated an increasing importance of complex word problems, particularly as assessment items in large scale studies like PISA. Based on the mathematics framework from PISA, it was argued that complex word problems are the most feasible tool to assess mathematical literacy, particularly because they allow for a real-world contextualization, a focus on solution processes, and a modelling perspective. However, it is an immanent feature of complex word problems that processes of reading and mathematical thinking interact during their solution. For all these reasons, the assessment of process measures during complex word problem solving can possibly offer valuable insights and foster our understanding of the prerequisites and characteristics of mathematical competencies and mathematical literacy. Few research has systematically made use of the potential to use eye tracking to analyze the process of complex word problem solving so far, and to our knowledge, none of the studies specifically addressed the interplay of reading and mathematical thinking. Hence, the following overarching research question was posed:

RQ 0: How can the observation of eye movements be used to meaningfully analyze the solution process of complex word problems, taking into account the interaction between reading and mathematical thinking?

Paper A served the purpose to develop, implement and evaluate the methodology to tackle this overarching question and to investigate fundamental characteristics of eye movements during complex word problem solving. Specifically, one of the goals of this paper was to analyze how global measures of eye movements are meaningful with regard to the solution process of the mathematical word problem. These measures were further used to investigate how reading and mathematical thinking interact during word problem solving by analyzing how they relate to performance and perceived task difficulty. This way, a tool to analyze and compare solution processes of complex word problems was developed that went beyond identifying a distribution of attention and referred to cognitive processes involved in Kintsch and Greeno's (1985) model of word problem solving. Hence, the research questions addressed in Paper A were as follows:

RQ 1.1: Are global measures of eye movements related to the perceived and empirical difficulty of mathematical word problems and therefore associated with the solution process?

RQ 1.2: Does the relation between these measures and the perceived difficulty of a word problem differ between students of varying performance?

Based on the results from Paper A (see section 6.1, for a summary), Paper B included a study that further developed the method and tackled subsequent research questions. Since global measures of eye movements are independent of the appearance of the word problem, they allowed for conducting a cross-linguistic comparative study to investigate differences in word problem solving in Chinese and German. This was done to address the question if and how the language in which a complex word problem is presented affects the solution process in a novel and uniquely informative way. To this end, global measures of eye movements were used to identify patterns of eye movements and were associated with solution strategies. These patterns were then compared between Chinese and German and validated by analyzing relations to cognitive and affective-motivational variables. This way we took into account that recent operationalizations of the goals of mathematics education expanded their scope beyond cognitive abilities towards an inclusion of motivational-affective outcomes (Schiepe-Tiska et al., 2016). Accordingly, the research questions formulated in Paper B were as follows:

RQ 2.1: Do eye movements associated with cognitive processes during word problem solving differ between students solving word problems in German and in Chinese?

RQ 2.2: What reading patterns can be distinguished by clustering eye movement patterns in the two groups? How similar are these reading patterns and how are they distributed?

RQ 2.3: How are reading patterns associated with mathematical performance, flow experience, mathematical self-concept, and mathematical anxiety?

Apart from the two main publications included here, a number of related studies were conducted, in part making use of the same methodology. In addition from the research questions formulated above, these studies revolved about more specific aspects of the research project and included preliminary analysis. These results are briefly summarized in section 6.3.

5 Methodology

5.1 Participants

In all three studies, adults were recruited as participants. This allowed us to conduct all studies in a controlled environment at universities and without additional permissions from parents. It was assumed that fundamental observations regarding eye movements are fairly similar between younger and older participants (Rayner et al., 2012; for a critical discussion of this limitation, see Paper A and B). Participants gave informed consent and were paid about 15€ for their participation.

In the study reported in Paper A, participants in the first study were 17 members of the academic staff from the Technical University of Munich. Participants in the second study were 42 undergraduate students. The study reported in Paper B was conducted with two groups of 72 and 67 undergraduate students. For all studies, participants were recruited that were not involved in mathematics in their program or profession to avoid a bias in the sample with regard to mathematical abilities and motivational-affective variables.

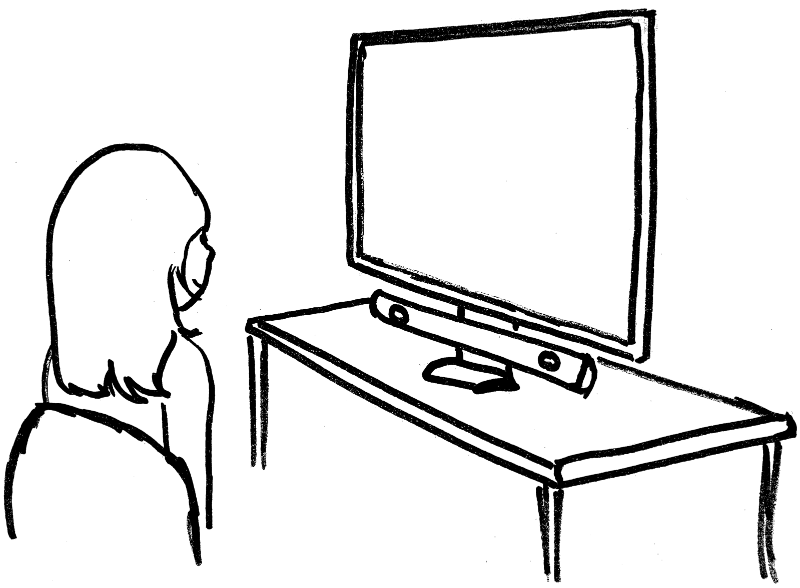

Figure 1. Illustration of the setup of the eye tracker.

5.2 Apparatus

All studies were conducted with the same SMI RED 500 eye-tracker. The device provided a sampling rate of 500 Hz. This eye tracker was a remote device. Thus, no head rest or other movement restriction was needed during the experiments. This provided a more comfortable testing environment but came at the price of slightly decreased accuracy and precision. However, this decreased precision is less problematic when global measures of eye movements are assessed. Participants were seated in front of a regular 22-inch monitor with a resolution of 1680 x 1056 pixel in a distance of about 70 centimeters (see Figure 1). The word problems were presented on one page at the computer screen, and students could solve them without time limits. For data collection of the group of Taiwanese students in Paper B, the apparatus was brought to a research facility in Taipei to ensure data comparability.

5.3 Stimuli and instruments

In all studies, published word problems form PISA were used (OECD, 2013b). An example item is displayed in Figure 2. All items reflected complex word problems according to the definition applied here. For all studies, items were selected that reflected all four content areas addressed in PISA (*change and relationships*, *space and shape*, *quantity*, and *uncertainty and data*; OECD, 2016). To assure the best comparability possible, most of the studies reported here used the same items. Items from PISA are developed to assess mathematical literacy from 15-year-olds. However, the studies reported here as well as other research clearly show that adults are still remarkably challenged by the more difficult items (Strohmaier, Lehner, et al., 2019).

Using items from PISA had three major advantages. First, it raised the possibility to directly relate the interpretation of the results to PISA. Second, all published items from PISA have undergone a careful process of item development that assured us that they were relating to the concept of mathematical literacy and further provided information about their expected difficulty. Third, it was possible to access all official translations of the items, which allowed for a cross-linguistic comparison based on high-quality translations. Items from PISA always include an introduction text that is not related to the mathematical problem itself. Usually, the topic of the item and the introduction text remains the same for several successive items, which are called a unit. The content of the items refers to mathematical concepts that students of all participating school systems should have acquired at the age of 15. The development of the contents is closely coordinated with the curricula of the participating countries and reflects a broad consensus (Niss, 2016). Most items are not published, as they are successively used to assure comparability between the assessments. However, a broad selection of example items from previous studies is available online and free to use.

Apart from the mathematical word problems, various additional variables were assessed throughout the studies reported here. If available, we used scales from PISA and their official translations (e.g., for mathematical self-concept and mathematics anxiety). If the scale had never been used in PISA, instruments were used that had been previously reported in renowned publications (e.g., for flow experience). Thus, no items or scales had to be developed.

5.4 Eye tracking measures and data analysis

The manufacturer’s software SMI iView X and SMI Experiment Center were used for stimuli presentation and data recording. SMI BeGaze was used for event detection (to cluster fixations and detect blinks) and supplemented by own analytical algorithms implemented in *SPSS* (to

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.

Question 2: CLIMBING MOUNT FUJI

The Gotemba walking trail up Mount Fuji is about 9 kilometres (km) long.

Walkers need to return from the 18 km walk by 8 pm.

Toshi estimates that he can walk up the mountain at 1.5 kilometres per hour on average, and down at twice that speed. These speeds take into account meal breaks and rest times.

Using Toshi’s estimated speeds, what is the latest time he can begin his walk so that he can return by 8 pm?

Figure 2. Example published item that was used in the studies. (Adapted from PISA 2012 Released Mathematics Items (p. 20), by OECD (2013). Copyright (2013b) by the OECD. Used under CC BY-NC-SA 3.0 IGO)

calculate saccades and regressions). For technical details about the parameters of event detection and how the global measures were derived, see Paper B. In Paper A, statistical analyses included correlational analyses and multilevel regression models to account for the nested structure of the data. In Paper B, we conducted independent cluster analyses in both language groups to identify reading patterns, and analyses of variance (ANOVAs) to analyze the relation to both cognitive and affective-motivational variables. In both papers, analyses were controlled for gender to account for possible systematic differences, particularly with regard to motivational-affective variables.

6 Summary of Publications

6.1 Paper A: *Eye movements during mathematical word problem solving - Global measures and individual differences*.

In Paper A, the assumption was investigated that global measures of eye movements are associated with mathematical word problem solving in a similar way that they are associated with cognitive processes during reading. In the introduction we presented, compared, and discussed a model of word problem solving (Kintsch & Greeno, 1985) as well as models of reading (Rayner et al., 2012) to build a strong theoretical foundation for our claims. Ultimately, the goal of the paper was to establish global measures of eye movements as a tool to analyze mathematical word problem solving. To test if the global measures proposed by us were related to cognitive processes during word problem solving, we analyzed the relation between eye movements, the perceived difficulty of a task and its solution rate. We conducted two experiments with adults and undergraduate students (N = 17 and N = 42), solving complex mathematical word problems form a pool of published items from PISA. Experiment 1 showed that more difficult items were read with longer fixations, more saccades, more regressions, and slower, with correlations ranging from r = 0.70 to r = 0.86. Multilevel modelling in experiment 2 revealed that for the number of saccades and the proportion of regressions, the relationship was stronger for low-performing students, with performance explaining up to 37% of the variance between students. These two measures are primarily associated with building a problem model. We discussed how this approach enables the use of eye tracking in complex mathematical word problem solving and contributes to our understanding of the role of reading in mathematics.

In line with the goal of the paper, we found evidence that global measures of eye movements are associated with cognitive processes during word problem solving. Therefore, using these measures seems a particularly promising approach for analyzing complex word problem solving, where processes of reading and mathematical thinking constantly interact.

The study design was developed by all authors. Data collection was supervised by all authors. I handled and analyzed the data, wrote the manuscript and implemented the suggestions and remarks by co-authors and reviewers. I supervised the submission process and serve as a corresponding author for this publication.

6.2 Paper B: *Comparing eye movements during mathematical word problem solving in Chinese and German*

Paper B reflects an application of the methodology and findings from Paper A. With the possibility to use global measures of eye movements, we addressed one key gap in research on word problem solving, namely a cross-linguistic comparison of complex word problem solving (Daroczy et al., 2015; Grisay et al., 2007). Based on previous research, it is assumed that the language in which a word problem is presented could affect its solution process (Leung, 2017). In particular, East-Asian, non-alphabetic languages are assumed to provide specific benefits for mathematics compared to Indo-European, alphabetic languages (Galligan, 2001). By analyzing students' eye movements we analyzed word problem solving processes in Chinese and German. 72 German and 67 Taiwanese undergraduate students solved PISA word problems in their own language. Results showed differences in eye movements of students, between the two languages. Moreover, independent cluster analyses revealed three clusters of reading patterns based on eye movements in both languages. Corresponding reading patterns emerged in both languages that were similarly and significantly associated with performance and motivational-affective variables. They explained more variance among students in these variables than the languages alone.

Our analyses show that eye movements of students during reading differ between the two languages, but very similar reading patterns exist in both languages. This result supports the assumption that the language alone is not a sufficient explanation for differences in students' mathematical achievement, but that reading patterns are more strongly related to performance. Moreover, the association with motivational-affective variables clearly supports the efforts to stronger include these variables when addressing complex mathematical tasks like word problem solving.

With regard to the overarching research questions of this project, the results supported the assumption that eye tracking can be a feasible tool for analyzing complex word problem solving which provides specific benefits. In this case, they allowed the comparison of word problems that were identical with regard to their content, but fundamentally differed in their appearance.

The study design was developed by all authors. Data collection was supervised by all authors. I handled and analyzed the data, wrote the manuscript and implemented the suggestions and remarks by co-authors and reviewers. I supervised the submission process and serve as a corresponding author for this publication.

6.3 Associated publications

Apart from the two main papers reported here, a brief overview of further publications that were associated with the research project will be given in the following.

As a third full paper in an international journal, we submitted a systematic literature review on the use of eye tracking in mathematics education. During the work on the present project, it became obvious that such a review is not yet available, but that the increasing number of eye tracking studies in the field of mathematics education became more and more of a challenge to overlook. In an international cooperation, we addressed this issue by providing a review of 160 studies published between 1921 and 2018 (Strohmaier et al., in press).

The research project included three publications in proceedings of the *PME* and the *AERA*. In 2016, we first presented fundamental observations on eye movements on complex word problems and correlations with several outcome variables based on pilot data (Strohmaier et al., 2016b). In 2017, we analyzed the relation between global measures of eye movements and the mathematical self-concept and mathematics anxiety (Strohmaier et al., 2017). In this study as well as in 2018, we used global measures of eye movements to investigate how the contextual framing of the testing situation affects word problem solving (Strohmaier & Reiss, 2018).

Further, five papers were published in the proceedings of the annual conference of the *GDM*. This conference was the first occasion to present preliminary results from a pilot study on the use of global measures of eye movements. For this presentation, we were awarded the price for the best poster (Strohmaier et al., 2016a). In the following years, two symposia on cognitive processes during the reading of mathematical texts and new advances in the use of eye tracking in mathematics education were initiated (Strohmaier, Lehner, et al., 2018; Strohmaier, Obersteiner, & Schindler, 2019). In these symposia, overviews of the method of eye tracking and preliminary results from our studies were presented (Strohmaier, MacKay, et al., 2019; Strohmaier, Tatsidou, & Reiss, 2018).

In these related publications, different approaches and applications of our framework were implemented and evaluated. Overall, eye tracking provided a substantial and unique benefit in most cases, opening new ways to analyze complex word problem solving. Thereby, these associated studies tested our approach beyond the core implications reported in the two main studies. In particular, we addressed motivational-affective variables and manipulations in the situational context and put our methods in a broader perspective by systematically situating the research within the international community. This goal was further supported by presentations and discussions during international conferences and research visits.

7 Discussion

In the studies summarized here, complex word problem solving was analyzed in a unique and novel way through the observation of eye movements. In general, the use of eye movements reflects a well-established method in mathematics education (Strohmaier et al., in press). Yet, in the present studies, the use of the method went beyond previous research and offered the possibility to tackle research questions that have previously not been addressed in this context. This way, the research presented here provided new insights regarding the process of complex word problem solving. In the following, the implementation of eye tracking for this purpose as well as implications from the resulting findings will be discussed.

7.1 Global measures of eye movements

Our theoretical considerations implied that investigating global measures of eye movements reflected a valuable tool for addressing some of the remaining gaps in research on word problem solving, specifically to take into account processes of reading. At the same time, it allowed to utilize the methodological benefit of eye tracking as a method to gather meaningful data on solution processes rather than the mere outcomes. No studies had previously adopted global measures of eye movements from research on reading to analyze mathematical word problem solving. Before the studies presented here were conducted, it was unclear if and how these measures of eye movements used here could be meaningfully associated with mathematical thinking during word problem solving.

To build a theoretical framework for the use of global measures of eye movements, they were carefully linked to previous research. Since there is a large body of research on the interpretation of eye movements during reading (e.g., Rayner, 1998; Rayner et al., 2012), a promising approach was to identify similarities in the processes of reading and mathematical thinking. Based on the process model of word problem solving by Kintsch and Greeno (1985), it seemed plausible to argue that reading and word problem solving are indeed closely related cognitive activities. Therefore, the same eye movement measures should be meaningful indicators of cognitive processes both during reading and during word problem solving.

Our empirical findings supported this assumption. They showed that global measures of eye movements were related to task difficulty and students' performance. In other words, the way that the eyes worked to extract information from the word problem differed between easier and harder problems. Specifically, participants showed longer mean fixation durations, more saccades, more regressions and a slower reading speed in more difficult word problems (Strohmaier, Lehner, et al., 2019). This was in line with what had been observed for non-mathematical texts of varying difficulty. Models of reading associate these measures with higher effort in word recognition and text comprehension (Rayner & Liversedge, 2011). Based on our considerations, we argued that in complex word problem solving, these measures are influenced by students' increased effort in building a text base and a problem model (Kintsch & Greeno, 1985). Thus, our findings indicated that global measures of eye movements were indeed associated with processes of word problem solving. At the same time, they provided the possibility to compare word problems of different length and appearance because they did not refer to local elements of the problem but to the text as a whole. This reflects a valuable advantage of the use of global measures of eye movements over the use of local measures in previous research.

7.2 Complex word problem solving

The development and evaluation of global measures of eye movements was a major goal in the research project presented here. Yet, it ultimately served the purpose to observe and analyze the solution process of complex word problems in a novel way and to investigate research questions that had not been previously addressed. Since the first results indicated that the measures used in our studies were meaningfully associated with mathematical thinking, they provided the possibility to analyze complex word problem solving from a new perspective.

7.2.1 Eye movements and performance

One key question was how eye movements and the associated cognitive processes differ between students of varying performance. In Paper A, we found that lower performing students showed a higher relation between task difficulty and global measures of eye movements. We interpreted this as an indicator that with increasing task difficulty, weaker students struggled more and more with the extraction of information from the word problem and with successfully integrating it into a problem model (Strohmaier, Lehner, et al., 2019). Similarly, Paper B showed that students with a pattern of eye movements that is typical for struggling reading also performed worse in the word problems (Strohmaier, Schiepe-Tiska, Chang, et al., 2020). In line with previous findings on prototype word problems (Hegarty et al., 1995; van der Schoot et al., 2009) this further implied that the association between reading and mathematical thinking is of particular importance for lower achieving students. This supports efforts to integrate language training in mathematics classrooms from early on (Boonen et al., 2016), particularly regarding word problem solving. It is assumed that by providing students with the tools to decode written information efficiently, they will also be supported during complex word problem solving.

7.2.2 Affective-motivational outcomes

Apart from addressing mathematical achievement, we further followed a more recent understanding of the goals of mathematics education by including affective-motivational variables in many analyses during this research project (Schiepe-Tiska et al., 2016). The concept of competencies and mathematical literacy specifically includes this perspective (OECD, 2013a; Weinert, 2001a, 2001b). By accounting for self-concept, mathematical anxiety, and flow experience, we took a comprehensive perspective on the goals of mathematics education and word problem solving in particular. Our results support the assumption that these affective-motivational outcomes are relevant when addressing complex word problem solving, as they were uniquely connected to eye movements and thus, their solution process. This indicates that for example, students with higher mathematical anxiety approach complex word problem differently, irrespective of their mathematical abilities (Strohmaier, Schiepe-Tiska, Chang, et al., 2020; Strohmaier et al., 2017). Again, this calls for a comprehensive view on word problem solving that focuses not solely on the cognitive mathematical aspects of word problem solving but takes into account how affective and motivational student characteristics influence the way that complex word problems are approached and solved. For example, students with negative affective-motivational characteristics might tend to apply a direct translation strategy in order not to miss any crucial information, which might in turn hinder them in building a comprehensive problem model (Strohmaier, Schiepe-Tiska, Chang, et al., 2020).

7.2.3 Strategies in solving complex word problems

Our results further indicated that eye movements are not only directly associated with cognitive effort, but that they also reflect the strategy that students employ during complex word problem solving. In Paper B, we combined six different measures of eye movements through cluster analysis in order to identify patterns of eye movements that we named *reading patterns*. Although these patterns were solely defined through eye movements, they were meaningfully and significantly related to both cognitive and motivational-affective outcomes. Based on our findings, we proposed that these reading patterns reflect typical strategies in tackling complex word problems. Specifically, we argued that the reading patterns identified a group of struggling readers and group of students that used a problem model strategy and a direct translation strategy, respectively (Strohmaier, Schiepe-Tiska, Chang, et al., 2020). Our findings further extend previous findings on strategy use in prototype word problems (de Konig et al., 2017; Hegarty et al., 1992; Hegarty et al., 1995). They imply that a modelling perspective is the favorable approach for tackling complex word problems not only because it supports sense-making, but also because it is associated with better performance and more positive affective-motivational student characteristics (Strohmaier, Schiepe-Tiska, Chang et al., 2020). Assessing these reading patterns seems to hold a valuable potential for future research. For example, comparing reading patterns in different age groups and with regard to long-term effects could be very informative to understand in detail the relation between language and mathematics in word problem solving.

7.2.4 Cross-linguistic similarities

In Paper B, the question if the solution of complex word problems differs between Chinese and German was investigated (Galligan, 2001; Grisay et al., 2007), building on findings from Paper A. After identifying reading patterns, we compared their distribution and relation to performance and affective-motivational variables between the languages. The patterns seemed to be rather universal, as we found similar reading patterns in our independent samples of Taiwanese and German students. Thus, basic processes and strategies during complex word problem solving seemed to exist irrespective of the language of the problem. Moreover, reading patterns were related to cognitive and motivational-affective variables similarly in both groups. Regarding these variables, including reading patterns explained more variance among the group of students than the language, which further supports the notion that accounting for eye movements can provide unique and valuable insights into the solution process of complex word problems. The direct comparison of solution processes between two fundamentally different languages is a key benefit of the implementation of global measures of eye movements. This cross-linguistic comparison provided valuable insights regarding the comparability of translated versions of word problems and could be further employed to other languages and populations.

7.2.5 The fundamental association between reading and mathematical thinking

Apart from the findings regarding complex word problems, our results contribute to the research on the role of language in mathematics in general. In the studies reported here, eye movements that were known to reflect cognitive processes during reading were also associated with mathematical thinking. This supports the notion that reading and mathematical thinking go hand in hand during word problem solving. This had previously been reported by a large body of research, including studies on mathematical modelling (Boonen et al., 2016; Daroczy et al., 2015; Leiss et al., 2019). However, the results reported here extend these findings, illustrating that even the process of visual decoding is already integrating reading and mathematical thinking. For example, the mean fixation duration during reading is known to be influenced by word length, word predictability, and word frequency (Rayner et al., 2012). However, it also emerged as an indicator for perceived word problem difficulty in our studies and was presumably associated with the underlying mathematical solution strategy. This means that in the process of decoding a word, aspects of mathematical thinking already occur. Though this is fully in line with previous research, it has not been directly observed in that way. Overall, these observations substantiate the notion that reading and mathematical thinking are inseparable in word problem solving.

7.3 Methodological considerations

Independent of findings regarding the association between reading and mathematical thinking, the research presented here might be considered valuable from a mere methodological perspective. It was guided by the spirit that technological and methodological innovations should not be a reason to conduct research in itself, but that their key lies in developing well-founded and creative theoretical links between methods and research questions to utilize their potential in the best way possible. Eye tracking is used in a variety of domains and offers many different approaches to tackle various research questions (Duchowski, 2007). This makes it necessary and promising to integrate methodological advances from a variety of domains. In our studies, we largely build on research on eye movements during reading, which arguably reflects the most elaborated field of eye tracking research (Rayner, 1998). This way, we were able to integrate theoretical and technical advances that had not been used to this end before in mathematics education research but were at the same time verified and established in a different domain. This practical approach opens new paths in a very efficient way. We believe that the exchange and integration of shared methods is a key benefit of interdisciplinary research that provides new possibilities and perspectives that are at the same time reasonable and sound.

7.4 Implications

The research presented here was designed to specifically address aspects of complex word problem solving in mathematics, but it might provide various other implications and possibilities for future research. In general, the approach offers the possibility to compare eye movements on mathematical texts even when they differ with regard to their appearance. Comparing word problems in different languages was one such example, but other manipulations like the syntactical structure of the word problem or the amount of information given to solve a problem could be analyzed from a new perspective. Many of these questions have been previously addressed with other methods and for prototype word problems (Daroczy et al., 2015), but the integration of global measures of eye movements could provide unique insight into the effects on solution processes rather than outcomes.

Future research could extend the findings reported here by including participants from a broader range of age and from more levels of mathematical achievement. The recent studies focused on adults, but at the same time indicated that individual differences and differences between specific groups of participants exist. Thus, it seems necessary to conduct additional research with younger students to replicate and generalize the findings. The approach presented here might also prove valuable for specific groups of students with regard to the interaction of reading and mathematical thinking, for example to further investigate relations between dyscalculia and dyslexia or to address possible influences of bilingualism on word problem solving. Regarding these approaches, it could be argued that any cognitive process that affects language processing should, to some extent, also be relevant for solving mathematical word problems.

The papers reported here focused on the method of eye tracking. Arguably, it provides specific benefits and is superior to many methods regarding the compromise between offering a direct link to cognitive processes, while at the time being relatively unobtrusive with regard to the cognitive processes during the task (Strohmaier et al., in press). However, the triangulation with other methods, for example think-aloud protocols, interviews, or videography might provide additional clues on the interpretation of eye movements and on underlying cognitive processes. For example, students could be interviewed after the experiment regarding their strategy choice or might explain their eye movements in retrospect. This way, the assumptions about the interpretation of eye movements could be further verified.

Lastly, the use of global measures of eye movements is not limited to complex word problems but could be used to investigate various forms of mathematical texts, for example in schoolbooks. Moreover, the approach might be transferred to other domains, since the integration of reading and domain-specific cognitive processes should be a universal challenge for students in other areas as well.

8 Conclusion

Word problem solving has been one of the most typical elements of mathematics for millennia and arguably has gained new importance during the last decades. The research project presented here sought to address this renewed attention and proposed ways to tackle the challenges and possibilities that accompany it.

Summing up the research that was presented here, two main perspectives on the findings emerge. On the one hand, the methodological approach to use global measures of eye movements proved inspiring and fruitful. Their joint influence on eye movements indicates that in many ways, reading and mathematical thinking share common cognitive processes and patterns of information processing. On the other hand, the findings provided insights into the process of solving complex word problems that had not been observed in that way before, including strategy use and a cross-linguistic comparison. In sum, the implications from this research substantiate on a fundamental level what research on the role of language in mathematics education has long proclaimed: In word problem solving, mathematics and language are inseparable.

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A Appendix

Note:
For copyright reasons, the appendices are not included in this publication of the dissertation.