

Simulation of Tsunamis with the exascale hyperbolic PDE engine ExaHyPE

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1. Introduction

1.1 The ExaHyPE Engine

ExaHyPE is a Horizon 2020 EU project to develop a high-performance engine to solve hyperbolic systems of PDEs using the high-order discontinuous Galerkin finite element method [1]. The projects goals are to develop an engine with flexible support for various applications which shall be tailored towards expected exascale architectures. The end-user is provided with an abstraction of the complicated algorithms to implement the ADER-DG numerical scheme and of the issues related to scalability and parallel adaptive mesh refinement (AMR), which are handled internally by the Peano framework [2]. The engine offers a newly developed a posteriori finite volume (FV) limiter to resolve shock waves and non-physical states [3].

Our research focuses on the simulation of large hazardous tsunami events. State of the art tsunami codes tend to use low order approximate schemes like FV schemes, like the GeoClaw software [4] and low-order Discontinuous Galerkin (DG) methods, like [5] by Stefan Vater et al. . Main reason is the issue of inundation which can be resolved only for low-order representations by the use of elaborate Riemann solvers and limiters. On the other hand, as shown in [6], high-order ADER-DG methods minimize time-and energy-to-solution. The scheme we present in this contribution is a radical simple approach: We utilize the a posterior FV limiter of the ExaHyPE-engine to model deep oceanic areas by a high-order method and keeping a common Riemann solver, while wetting and drying areas by are resolved by a FV method with a suitable but computationally more expensive Riemann solver.

1.2 The Shallow Water Equations and the New Efficient HLLEM Riemann Solver

Tsunamis are modeled by the non-linear hyperbolic Shallow Water Equations. As vertical effects are neglected, they are a sufficient approximation of water flows as long as the wave amplitude is significantly lower than the size of the simulated domain. The equations take conservation of mass and momentum into account, bathymetry is considered by a well-balanced source term. Here h defines the water height u and v the velocities in x or y direction, b the bathymetry and g the gravitational constant.

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$$\begin{pmatrix} h \\ hu \\ hv \end{pmatrix}_t = \begin{pmatrix} hu \\ \frac{1}{2}gh^2 + \frac{hu^2}{h} \\ \frac{hvhv}{h} \end{pmatrix}_x + \begin{pmatrix} hv \\ \frac{hvhv}{h} \\ \frac{1}{2}gh^2 + \frac{hv^2}{h} \end{pmatrix}_y + \begin{pmatrix} 0 \\ b_x gh \\ b_y gh \end{pmatrix}$$

Our FV scheme relies on the new efficient HLLEM Riemann solver formulation for conservative hyperbolic PDEs [7] but is in general extendable to any well-balanced Riemann solver: On an interface between two cells it can be expressed in the following form:

$$D(Q_L, Q_R) = 0.5 \cdot \left(F(Q_L) + F(Q_R) - \begin{pmatrix} 0 \\ 0.5 \cdot g \cdot (h_L + h_R) \cdot D\eta \\ 0.5 \cdot g \cdot (h_L + h_R) \cdot D\eta \end{pmatrix} \right) - 0.5 \cdot s_{max} \begin{pmatrix} D\eta \\ hu_R - hu_L \\ hv_R - hv_L \end{pmatrix}$$

Where F is the flux neglecting hydrostatic pressure and . In the case of two adjacent wet cells the solver reduces to the well known Rusanov flux. If one of the cells is flooded the jump in bathymetry is taken into account. The velocity in the flux is limited to not run into negative water

1.3 The ADER-DG Method and *a posteriori* Finite Volume Limiter

The ADER-DG method fundamentally differs in integration in time from known high-order Runge-Kutta DG methods. Instead of solving the ODE for intermediate steps in time it uses a element local Picard iteration, discretized in time and space [1]. We obtain a high-order well-balanced scheme which only requires one traversal of the whole domain per computed time-step. The scheme uses the Rusanov Flux to solve Riemann problems in space in time. For this paper we adjusted the solver to consider the bathymetry in the diffusive penalty term.

As high order methods run into shocks even for smooth initial conditions and unphysical states can occur the engine's offers a new *a posteriori* FV limiter [3]. The solution on cells that are detected as non-physical, in our case dry, get rolled back to the previous time-step and projected onto a FV patch. These patches conserving the CFL condition so the current time-step size does not need to be changed. The solution is recomputed in FV representation, the polynomial solution reconstructed from the new FV result.

2. Numerical results

All presented result were generated on the SuperMUC supercomputer using a single Haswell node and TBB parallelization on 28 cores provided by the ExaHyPE Engine.

2.1 Solitary Wave on a Simple Beach

The one dimensional single wave on a simple beach [8] benchmark is based on laboratory data. The experiment resembles a coast line by using a linearly sloping bottom topography towards the beach. A tsunami like wave traveling onto the coast is initiated. A graphic representation of the experiment can be seen in Fig..

We use this benchmark to see how exact our FV solver is able to reproduce real inundation events.

A comparison between our simulated results and the reference data can be seen in Fig. 1.

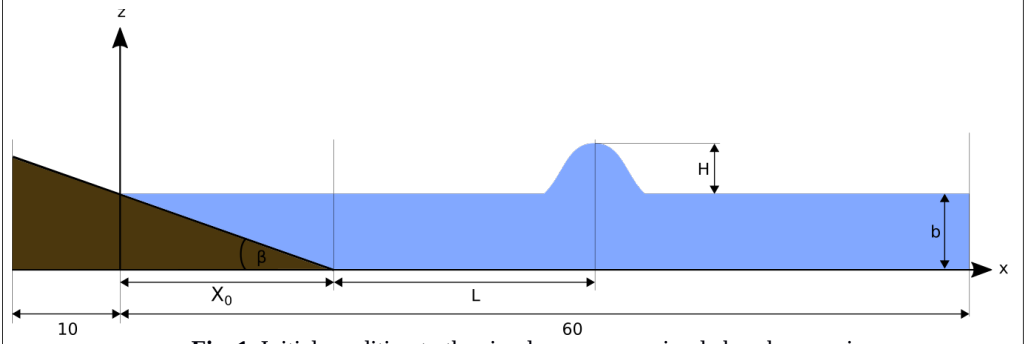


Fig. 1. Initial condition to the simple wave on a simple beach scenario.

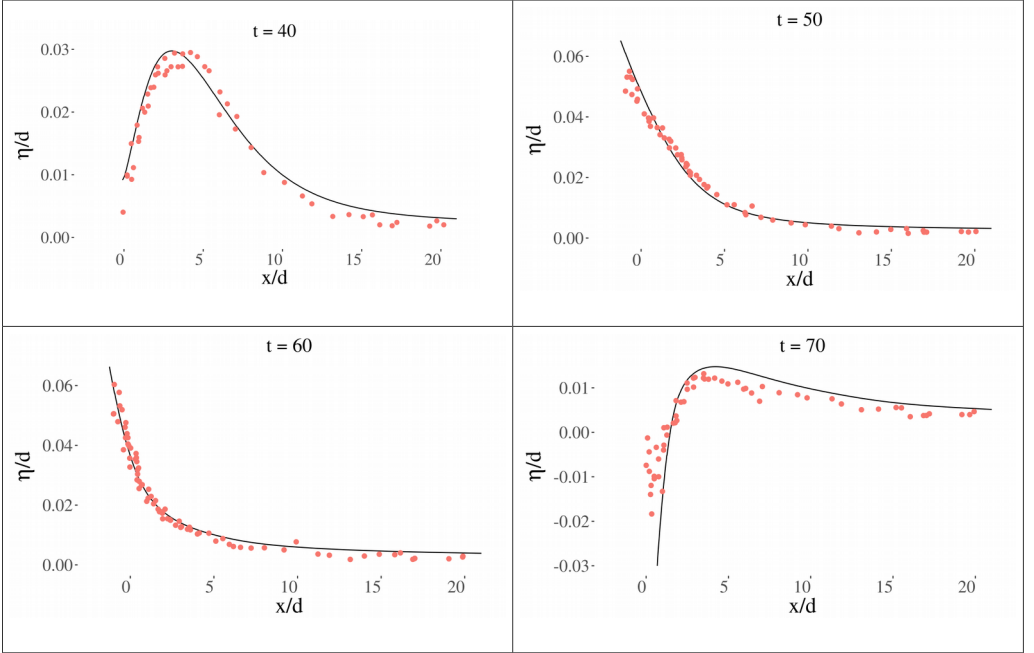


Fig. 2. Comparison of the evolution of the simple wave of a simple beach benchmark.

As measurements are provided dimensionless the result only depends on the ratio H/d , which in our case is taken as 0.0185 and the angle $\cot \beta = 19.85$. The simulation was performed on a two dimensional grid of 616734 cells, in y direction the solution was taken constant. Our FV scheme captures the run up. Amplitude and temporal scale also fit sufficiently, we conclude that our FV solver approximates inundated areas accurately.

2.2 Oscillating lake

The oscillating lake scenario is a analytically defined periodic solution. It consists of a water droplet which traverses periodically circular through a dry basin [9]. This scenario leads to a continuously ongoing wet dry front, which states an error-prone problem for our scheme. It is used to verify two main

characteristics: That the induced bathymetry is applied correct to the numerical scheme and that the resolution of wetting and drying cells is sufficiently accurate.

In Fig. 3 the water height of the numerical result of our FV solver simulated on a uniform grid with $2e+6$ cells showed. The error comparison of the water height in Tbl. 1 shows numerical convergence of our scheme towards the analytic solution.

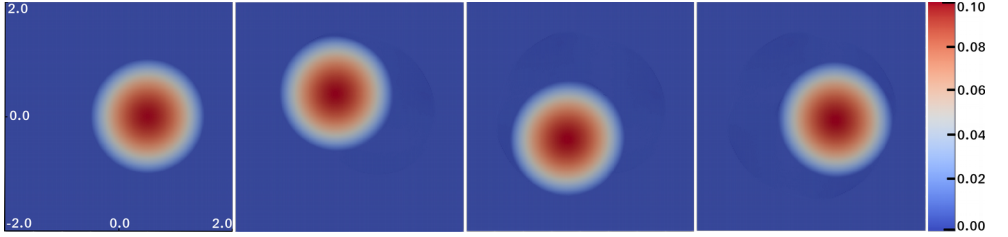


Fig. 3. Water height of the numerical simulation of one periodic phase of the oscillating lake scenario. Results are showed after 0 , 0.33, 0.66 and a whole phase.

Rel. Error	0,049382716	0,0082304527	0,0027434842
dx	3,460631E-06	3,608596E-08	3,882806E-09

Tbl. 1: Numerical error for various mesh sizes

2.3. Reproduction of Tsunami Events

To see how well our numeric scheme performs at the simulation of real tsunami events we performed a reproduction of actual buoy measurements. The observed tsunami is the Tohoku event of 2011. The displacement caused by the earthquake is taken from [10] and applied static as initial condition using the ASAGI framework [11]. We compared an ADER-DG method of Order 3 including the FV limiter against a pure FV method. Areas that have to be limited are indicated by a bathymetry threshold, as soon as it is higher than -100m the FV solver is used.

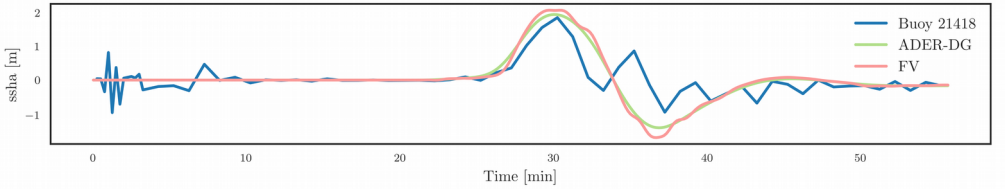


Fig. 4. Measured buoy data compared with numerical solutions of the Limiter ADER-DG method and a Finite Volume method as comparison.

The pure FV simulation was run on $4.5e+6$ elements, which is equivalent to a resolution of 700m. The limited ADER-DG method on $5e+5$ elements which, because of the high order, results in a resolution of approximately 525m. A comparison of measurements and numerical results for a buoy placed about 550km away from the center of the Tsunami can be seen in Fig. 4. We see that both methods reconstruct

the wave sufficiently enough. The high order property of the ADER-DG method results in a smoother result, while the FV solver shows various oscillations. Fig. 5 compares the actual tsunami wave including bathymetry against the the used method 5 minutes after the initial earthquake event. A large fraction of the domain is simulated with the high order method while problematic dry areas are treated by the FV limiter.

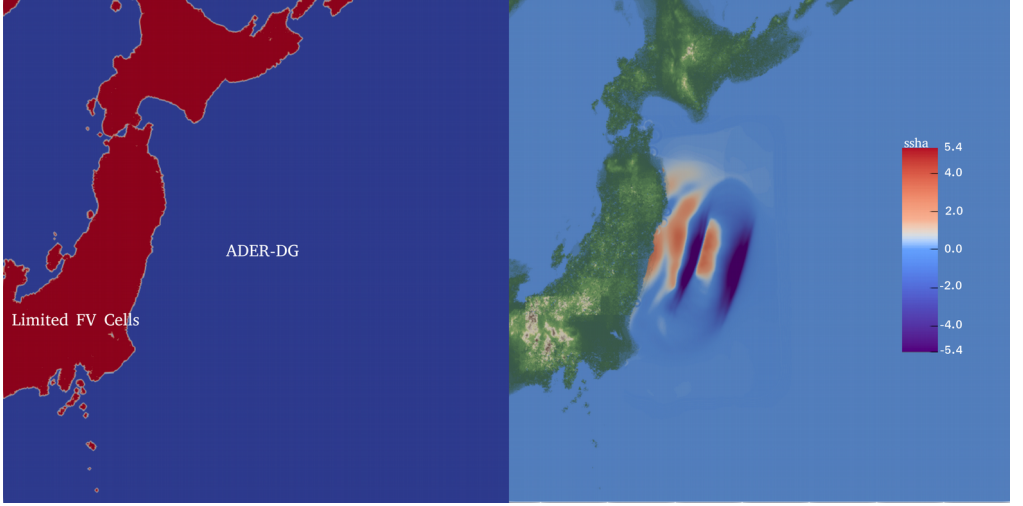


Fig. 5. The Tohoku tsunami. On the left are the domains showed which are solved as FV or ADER-DG cells. On the right is the tsunami 5 min after the initial event.

3. Conclusion and further work

Main characteristic of the here presented ADER-DG method with FV limiter is the fact that we are able to simulate tsunami waves with a high-order scheme, while still being able to include coasts and inundated land. The quality of the simulated tsunami shows that our method can compete with low order methods, while we can expect a high advantage in terms of time and energy to solution.

In the future we will try to confirm those expectations in a larger comparison with other methods.

The ExaHyPE-engine allows to extend simulations by a MPI parallelization easily, using this feature we plan to perform simulations of higher accuracy in the near future. The engine also provides a simple interface for the implementation of dynamic adaptive mesh refinement (AMR) using this we can resolve areas of high interest, like coasts, buoys or the initial displaced domain with higher accuracy while keeping unaffected areas in a low resolution.

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