Technische Universität München
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Master’s Thesis

First-Mover and Second-Mover Advantage under Uncertainty

Examinar: Prof. Dr. Robert K. Frhr. von Weizsäcker
Chair of Public Finance and Industrial Organization
Supervisor: Dr. Thomas Daske
Course of Study: Master in Management
Submitted by: Moritz P. Thon
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List of Abbreviations

FOC	First order condition
GA	General assumption
NSS	Natural *Stackelberg* Situation
ODE	Ordinary differential equation
PBE	Perfect *Bayes*ian Equilibrium
SPE	Subgame Perfect Equilibrium

1 Introduction

"Is it better to move first, or second – to innovate, or to imitate?" The answer to this question asked by *Rasmusen* and *Yoon* [38] is key to managerial decision-making when firms face the opportunity to expand into a new, unexplored economic market which is accessible also to other competitors. Being first in a new market can yield important *strategic advantages* [24], such as having the possibility to create exclusive contracts or to establish a monopoly. At the same time, however, the first-moving firm has to bare the costs and risks associated with exploring and shaping the market. Contrary to the (potential) strategic advantage of the first-mover, a firm can gain a significant *informational advantage* by waiting until sufficient information about the new market becomes observable due to the first-mover's initial investments and market exploration. In the end, the informational advantage can result in higher profits for the second-mover. Generally, it is difficult to balance advantages and disadvantages associated with the timing of market entry and to rate whether moving first is an economically advisable strategy.

In the context of industrial organization, first- and second-mover scenarios that require an (almost) *irreversible action* or *irreversible commitment* are commonly studied by modelling them as sequential-move duopoly competition [40]. Prominent examples are the former decision of the Austrian Airline Group to serve the Eastern European and Iraqi markets [29] or the international market of DRAM [40]. Both endeavor require huge initial investments and long-term contracts and are therefore irreversible in the short run once the decision to expand has been made. To analyse such non-cooperative sequential-move duopoly competition, where two identical and rivaling firms successively implement their optimal strategic decisions (usually output quantity or price), it is commonly assumed that both firms aim to individually maximize their profits. The resulting equilibrium payoffs of the firms in sequential-move duopoly competition and their rating, however, depend on the specific form of competition. In this thesis, focus thus lies on the analysis and discussion of various forms of sequential-move duopoly market competition from industrial organization and to investigate whether they result in payoff advantages for the first- or second-mover.

As a starting point, the analysis of the classical *Stackelberg* competition [43] – a sequential-move quantity duopoly competition – yields a general first-mover advantage. This first-mover advantage even holds for a wide and economically reasonable class of nonlinear market demand and production cost functions. In contrast the analysis of a sequentialized

Hotelling competition [27] – a sequential-move price duopoly competition with differentiated supply – exhibits a general second-mover advantage. Thus, naturally the question on the fundamental difference between *Stackelberg*, sequentialized *Hotelling* and other forms of sequential-move market competition in the context of first- and second-mover advantages arises.

In 1985 *Gal-Or* gave a characterization of first- and second-mover advantages for a broad range of deterministic payoff functions [17]. Among other things her characterization includes *Stackelberg* and sequentialized *Hotelling* competition as special cases. The Theorem of *Gal-Or* proves that first- and second-mover advantages in environments of complete information only arise due to the specific form of the firms' payoff functions, more precisely due to the sign of their cross derivatives. A bottleneck of *Gal-Or*'s Theorem is that it requires both firms to have access to complete information yielding their deterministic payoff functions. In the context of industrial organization, however, model parameters can often be determined only vaguely or underlie market uncertainties. Thus, it is much more reasonable to assume that some model parameters are stochastically distributed rather than being fixed-valued.

Two years after characterizing first- and second-mover advantages in environments of complete information, *Gal-Or* showed that the assumption of complete information is indeed crucial and cannot be dropped from the preconditions of her theorem [20]. *Gal-Or* extended *Stackelberg* competition to include uncertainties in the marked demand in an economically reasonable and stochastically general manner. Therein, both firms have access to individual private information about the actual market demand, i.e., they individually draw samples from the stochastic distribution of the market demand. The distribution of the market demand itself, however, is not known to the firms. By definition, the first-mover decides on its (optimal) output quantity first. The second-mover can subsequently infer the private information of the first-mover from its optimal output quantity. Consequently, the second-mover has access to both private observations and thus has a significant information advantage. Using this information advantage, the second-mover can increase his payoff which enables (under some conditions on the distribution of the market demand) an expected second-mover advantage. This observation shows that the information asymmetry in the case of uncertainties can generally advantage the second-mover compared to the case of complete information. Yet, only few generalizations and extensions to more complex forms of competition or other model parameters were achieved [11, 35, 40].

Due to their great importance to the field, the first aim of this thesis is to review two key publications by *Gal-Or* [17, 20] in detail. Subsequently, the stochastic framework introduced by *Gal-Or* is used to investigate the influence of uncertainties in further model parameters of *Stackelberg* and sequentialized *Hotelling* competition. As second goal, important further literature [1–3, 5, 6, 9–11, 13, 16, 23, 25, 31, 35, 40] is reviewed with a special focus on their connection to the publications by *Gal-Or*. Throughout this thesis, implications of the presented theoretical results to problems from industrial organization are discussed.

Accordingly, the remainder of this thesis is structured as follows: In Chapter 2 the Theorem of *Gal-Or* [17] characterizing first- and second-mover advantages under complete information is reviewed. At first, required definitions and assumptions, the necessity of continuous strategy spaces as well as the proof of the Theorem are presented in Sections 2.1, 2.2 and 2.3, respectively. Subsequently, the Theorem's implications to *Stackelberg* and sequentialized *Hotelling* competition are studied in Section 2.4.

Chapter 3 deals with the analysis of uncertainties in various model parameters analog to the publication by *Gal-Or* [20]. In Section 3.1, *Stackelberg* competition is generalized to the case of an uncertain market demand as done by *Gal-Or*. Furthermore, the introduced stochastic framework is employed to also investigate uncertainties in the marginal costs of *Stackelberg* and sequentialized *Hotelling* competition in Sections 3.2 and 3.3, respectively.

In Chapter 4, important further literature and their connections to the publications by *Gal-Or* are reviewed. To this end, direct extensions of the work by *Gal-Or* to tripoly and oligopoly competition are discussed in Section 4.1. Subsequently, in Sections 4.2 and 4.3, related forms of competition in which the first-mover completely reveals uncertainties and the occurrence of Natural *Stackelberg* Situations are reviewed.

Finally, an overall summary and a comprehensive discussion of the initial question is given in Chapter 5.

2 First- and Second-Mover Advantage under Complete Information

Game theory is an important tool to analyze strategic interactions between non-cooperating economics agents (players). In this chapter, important game theoretical concepts are reviewed with a special focus on symmetric extensive two-player games. The game theoretical results are used to address the question of first- and second-mover advantages in industrial organization under the premise of complete information. The assumption of complete information is dropped in the subsequent Chapter 3.

2.1 Definitions and Assumptions

In this section, key definitions and assumptions concerning symmetric extensive two-player games are reviewed. Further details on game theory and its applications to industrial organization can be found in the textbooks by *Belleflamme* and *Peitz* [8], *Osborne* and *Rubinstein* [36] or *Shy* [41].

Analyzing first- and second-mover scenarios using sequential-move (extensive) games, it is usually assumed that the sequential movement and the order of the players is exogenously determined (cf. Section 4.3 for a discussion of these assumptions). Denoting the two non-cooperative players by player I and player II, the following general assumption (GA) on their order and behaviour is made:

General assumption 1 (Assumtions on players)
In extensive two-player games, without loss of generality, player I *moves first and player* II *moves second. Furthermore, both players are non-cooperative, act perfectly rational and all information (if not stated otherwise) is common knowledge.*

The profits of player I and player II let be defined by the deterministic *payoff functions* π_1 and π_2, respectively. In this chapter, the players' payoffs only depend on their *strategies* $s_1 \in \mathcal{S}$ and $s_2 \in \mathcal{S}$ (usually output quantity or price) chosen from the total *strategy* *space* $\mathcal{S}$. Both players aim at maximizing their individual payoffs $\pi_1(s_1, s_2)$ and $\pi_2(s_1, s_2)$ by implementing their optimal strategic decisions, where both player assume that the other player mutually does the same. Thus, the common equilibrium concept of non-cooperative extensive games is applied:

Definition 2.1 (Subgame Perfect Equilibrium)
Under GA 1, a Subgame Perfect Equilibrium *(SPE)* $(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \in \mathcal{S}^2$ *of an extensive two-player game with mutual strategy space* $\mathcal{S}$ *and payoff functions* $\pi_1, \pi_2 : \mathcal{S}^2 \to \mathbb{R}$ *is defined by:*

$$s_2^{\mathrm{F}} = g(s_1^{\mathrm{L}}) := \underset{s_2 \in \mathcal{S}}{\operatorname{argmax}}\, \pi_2(s_1^{\mathrm{L}}, s_2) \tag{2.1}$$

and

$$s_1^{\mathrm{L}} := \underset{s_1 \in \mathcal{S}}{\operatorname{argmax}}\, \pi_1(s_1, g(s_1)). \tag{2.2}$$

The function $g : \mathcal{S} \to \mathcal{S}$ *is called the* reaction function *of the second-mover, i.e., of player* II.

With an appropriate equilibrium concept at hand, first- and second-mover advantages of *symmetric* extensive two-player games can be defined in a straight-forward manner:

Definition 2.2 (First- and second-mover advantage)
Under GA 1, the SPE payoffs $\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$ *and* $\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$ *of a symmetric extensive two-player game yield*

- *a* first-mover advantage *iff*[1] $\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$.
- *a* second-mover advantage *iff* $\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) < \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$.

First- and second-mover advantage are called weak *if equal SPE payoffs are possible too.*

2.2 Strategy Spaces

The strategy space $\mathcal{S}$ is either discrete or continuous. Both possibilities are considered and discussed in the following.

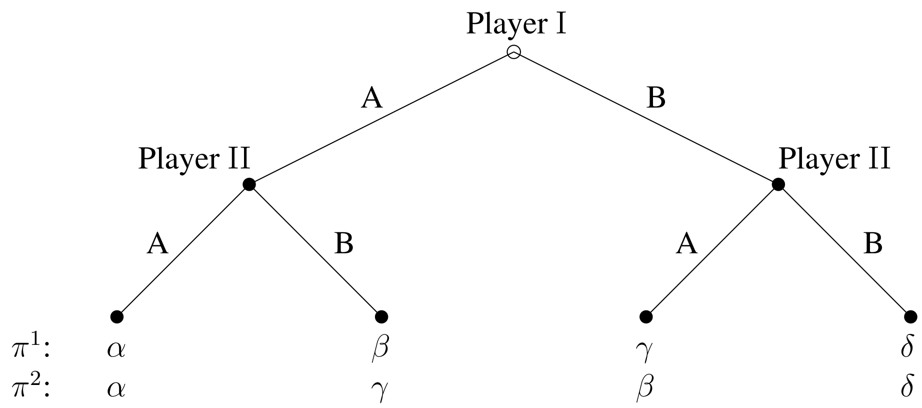

Figure 2.1 Symmetric extensive two-player game with two strategies. At first, player I chooses between strategy A or B. Subsequently, player II chooses strategy A or B as reaction to the strategic descision of player I. The realized payoffs π_1 of player I and π_2 of player II are given by $\alpha, \beta, \gamma, \delta \in \mathbb{R}$.

2.2.1 Discrete Strategy Spaces

As a starting point, symmetric extensive two-player games with a two-element strategy space $\mathcal{S} = \{A, B\}$ are considered, see Figure 2.1. It holds the following Proposition:

Proposition 2.3 (Weak first mover-advantage)
Symmetric extensive two-player games with two strategies as given in Figure 2.1 yield a weak first mover-advantage.

Proof. The reaction function $g : \{A, B\} \to \{A, B\}$ of player II depends on $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ and is given by[2]

$$g(A) = \begin{cases} A & \text{if } \alpha > \gamma \\ B & \text{if } \alpha < \gamma \end{cases} \quad \text{and} \quad g(B) = \begin{cases} A & \text{if } \beta > \delta \\ B & \text{if } \beta < \delta \end{cases} . \tag{2.3}$$

Distinguishing the cases $\alpha \lesseqgtr \gamma$ and $\beta \lesseqgtr \delta$ the SPEs $(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$ are estimated by simple algebraic comparisons:

Suppose $\alpha > \gamma$ and $\beta > \delta$. Then

$$(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = (A, A) \text{ and hence } \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \alpha = \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}), \tag{2.4}$$

1 Abbreviation for “if and only if”.

2 In the reaction function g of player II, the cases $\alpha = \gamma$ and $\beta = \delta$ can be arbitrarily assigned to the strategies A or B. Here and in the follwing, theses special cases are omitted to simplify the exposition.

such that both players I and II have equal SPE payoffs.

Suppose $\alpha > \gamma$ and $\beta < \delta$. Then

$$(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \begin{cases} (A, A) & \text{if } \alpha > \delta \text{ and hence } \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \alpha = \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}), \\ (B, B) & \text{if } \alpha < \delta \text{ and hence } \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \delta = \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}), \end{cases} \tag{2.5}$$

such that both players I and II have equal SPE payoffs.

Suppose $\alpha < \gamma$ and $\beta < \delta$. Then

$$(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = (B, B) \text{ and hence } \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \delta = \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}), \tag{2.6}$$

such that both players I and II have equal SPE payoffs.

Suppose $\alpha < \gamma$ and $\beta > \delta$. Then

$$(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \begin{cases} (A, B) & \text{if } \beta > \gamma \text{ and hence } \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \beta > \gamma = \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}), \\ (B, A) & \text{if } \beta < \gamma \text{ and hence } \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \gamma > \beta = \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}), \end{cases} \tag{2.7}$$

such that player I has a higher SPE payoffs than player II.

Thus, independent of $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ the first-mover always earns at least as much as the second-mover, i.e., $\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \geq \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$.

□

To prove Proposition 2.3, besides the alikeness of the players (and thus the symmetry of their payoffs), no assumptions on the payoffs $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ are required. Despite the simplicity of symmetric extensive two-player games with two strategies, Proposition 2.3 shows that the weak advantage of the first-mover is induced by the asymmetry due to the players' sequential movement.

This weak first-mover advantage, however, can already vanish when the strategy space is enlarged to a three-element strategy space $\mathcal{S} = \{A, B, C\}$. Using simple algebraic comparisons it is straight-forward to show that the strategy pair (C,B) is the unique SPE of the example given in Figure 2.2 together with the restriction $3 < \mu < 10$. The resulting SPE payoffs of player I and player II are $\pi_1(C, B) = \mu$ and $\pi_2(C, B) = 7$, respectively. Thus, depending on the specific value of μ, there is a first- *or* second-mover advantage.

The previous examples with discrete strategy space $\mathcal{S}$ show that symmetric extensive two-player games can but do *not* have to yield an advantage for one of the players. If a first- or second-mover advantage exists, however, it is determined by the specific structure of the payoff functions π_1 and π_2. Hence, no general game theoretical results can be used to investigate the question of first- and second-mover advantages even under the

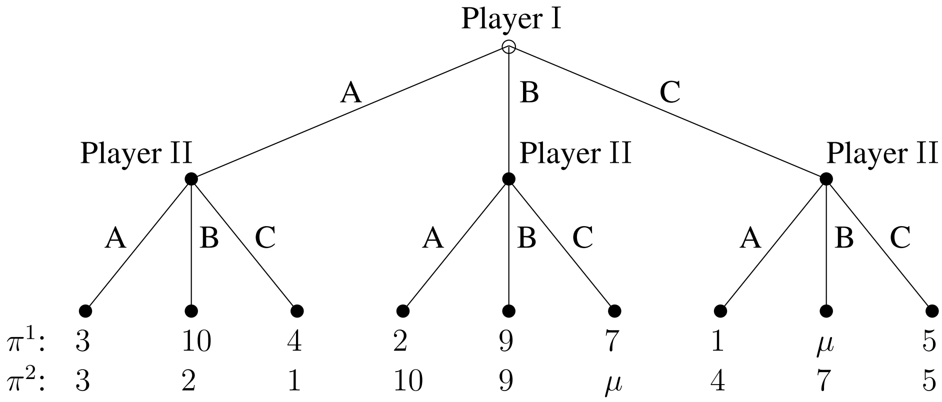

Figure 2.2 Example of a symmetric extensive two-player game with three strategies which can yield a first- or second-mover advantage. At first, player I chooses between strategy A, B and C. Subsequently, player II chooses strategy A, B and C as reaction to the strategic descision of player I. The realized payoffs π_1 of player I and π_2 of player II are specified, whereby $\mu \in \mathbb{R}$. If $3 < \mu < 10$, the unique SPE is given by the strategy pair (C,B).

assumption of complete information. In the remainder of this thesis, thus, only specific payoff functions and continuous strategy spaces are considered that are reasonable in the context of industrial organization.

2.2.2 Continuous Strategy Spaces

Following *Gal-Or* [17], continuous strategies spaces and payoff functions in this thesis are restricted by the following assumptions:

General assumption 2 (Properties of strategies and payoff functions)

(i) The strategies s_i ($i = 1, 2$) are continuous *and within a* non-empty interval*, i.e.,*

$$s_1, s_2 \in \mathcal{S} = [\underline{s}, \overline{s}], \text{ where } \underline{s}, \overline{s} \in \mathbb{R}, \underline{s} < \overline{s}. \tag{2.8}$$

(ii) The payoff functions π_1 and π_2 are two times continuously differentiable *with respect to both strategies, i.e.,*

$$\pi_1, \pi_2 \in \mathrm{C}^2([\underline{s}, \overline{s}]^2; \mathbb{R}). \tag{2.9}$$

(iii) The players I *and* II *are* identical*, i.e.,*

$$\pi_1(u, v) = \pi_2(v, u), \quad \forall u, v \in [\underline{s}, \overline{s}]. \tag{2.10}$$

(iv) The payoff functions π_i *(*$i = 1,2$*) are* strictly concave *in their* own *strategy* s_i*, i.e.*[3]

$$\partial_{s_1 s_1}\pi_1(s_1,s_2), \partial_{s_2 s_2}\pi_2(s_1,s_2) < 0 \quad \forall s_1, s_2 \in]\underline{s}, \overline{s}[. \tag{2.11}$$

(v) π_i *and* $\partial_{s_i}\pi_i$ *(*$i = 1,2$*) are both* strictly monotone *in their* rival's *strategy* s_{-i}[4]*, i.e.,*

$$\begin{aligned}&\partial_{s_2}\pi_1(s_1,s_2), \partial_{s_1 s_2}\pi_1(s_1,s_2) \neq 0, \quad \forall s_1, s_2 \in]\underline{s}, \overline{s}[,\\ &\partial_{s_1}\pi_2(s_1,s_2), \partial_{s_1 s_2}\pi_2(s_1,s_2) \neq 0, \quad \forall s_1, s_2 \in]\underline{s}, \overline{s}[.\end{aligned} \tag{2.12}$$

The assumptions made on the strategies s_1, s_2 and payoff functions π_1, π_2 in GA 2 allow to give a characterization of first- and second-mover advantages. GA 2 (i) – (iii) are required to proof significant results and to allow a meaningful comparison of the players' payoffs. The validity of 2 (iv) – (v) is addressed in the context of industrial organization in Section 2.4.

The assumed smoothness of the payoff functions π_1 and π_2 in GA 2 (ii) allows to derive necessary conditions for *interior* SPEs, i.e., $(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \in]\underline{s}, \overline{s}[^2$, from Definition 2.1:

Proposition 2.4 (FOC and characterization of reaction function)
Under GA 1 – 2, an interior SPE $(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$ *satisfies the following* first order condition *(FOC)*[5]*:*

$$\begin{aligned}0 &= \partial_{s_1}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) - \frac{\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})}{\partial_{s_2 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})},\\ 0 &= \partial_{s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}).\end{aligned} \tag{2.13}$$

Furthermore, the reaction function $g :]\underline{s}, \overline{s}[\to]\underline{s}, \overline{s}[, s_1 \mapsto g(s_1)$ *of player* II *is a well-defined and continuously differentiable mapping satisfying:*

$$\partial_{s_1} g(s_1) = -\frac{\partial_{s_1 s_2}\pi_2(s_1, g(s_1))}{\partial_{s_2 s_2}\pi_2(s_1, g(s_1))}, \quad \forall s_1 \in]\underline{s}, \overline{s}[. \tag{2.14}$$

Proof. Cf. [17]. Let $s_1 \in]\underline{s}, \overline{s}[$ be arbitrary. For $\pi_2(s_1, s_2)$, the standard FOC of an interior extremum [39] s_2^* with respect to maximizing s_2 and fixed s_1 yields $\partial_{s_2}\pi_2(s_1, s_2^*) = 0$. In addition, from Equation (2.11) it follows $\partial_{s_2}(\partial_{s_2}\pi_2(s_1, s_2^*)) \neq 0 \ \forall s_1, s_2^* \in [\underline{s}, \overline{s}]$. Together,

3 $\partial_{s_j}\pi_i$ $(i, j = 1,2)$ denotes the partial derivative of π_i with respect to s_j. $\partial_{s_j s_k}\pi_i$ $(i, j, k = 1,2)$ denotes the second partial derivative of π_i with respect to s_j and s_k.

4 s_{-i} $(i = 1,2)$ denotes the rival's strategy, i.e., in a two-player game $s_{-1} := s_2, s_{-2} := s_1$.

5 The respective second order conditions of an interior SPE can be found in the publication by *Gal-Or* [17].

the arbitrariness of s_1 and the smoothness of π_2 allow to apply the *Implicit Function Theorem* [39] which yields that $\exists_1\ g \in \mathrm{C}^1(]\underline{s}, \overline{s}[;]\underline{s}, \overline{s}[), s_1 \mapsto g(s_1) = s_2^*$ such that

$$\partial_{s_2}\pi_2(s_1, g(s_1)) = 0, \quad \forall s_1 \in]\underline{s}, \overline{s}[. \tag{2.15}$$

Setting $s_1 = s_1^{\mathrm{L}}$ and using $s_2^{\mathrm{F}} = g(s_1^{\mathrm{L}})$ results in the stated FOC for π_2 in Equation (2.13). Furthermore, by taking the total derivative with respect to s_1 it follows

$$0 = \frac{\mathrm{d}}{\mathrm{d}s_1}(\partial_{s_2}\pi_2(s_1, g(s_1))) = \partial_{s_1 s_2}\pi_2(s_1, g(s_1)) + \partial_{s_2 s_2}\pi_2(s_1, g(s_1))\partial_{s_1} g(s_1). \tag{2.16}$$

Thus, by simple rearrangement $(\partial_{s_2 s_2}\pi_2(s_1, s_2)) \neq 0\ \forall s_1, s_2 \in]\underline{s}, \overline{s}[)$ Equation (2.14) is shown. Due to

$$\partial_{s_1} g(s_1^{\mathrm{L}}) \overset{(2.16)}{=} -\frac{\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})}{\partial_{s_2 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})}, \tag{2.17}$$

the FOC for π_1 in Equation (2.13) follows by the standard FOC of an interior extremum [39]

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}s_1}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \frac{\mathrm{d}}{\mathrm{d}s_1}\pi_1(s_1^{\mathrm{L}}, g(s_1^{\mathrm{L}})) \\ &= \partial_{s_1}\pi_1(s_1^{\mathrm{L}}, g(s_1^{\mathrm{L}})) + \partial_{s_2}\pi_1(s_1^{\mathrm{L}}, g(s_1^{\mathrm{L}}))\partial_{s_1} g(s_1^{\mathrm{L}}) \\ &\overset{(2.17)}{=} \partial_{s_1}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) - \partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})\frac{\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})}{\partial_{s_2 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})}. \end{aligned} \tag{2.18}$$

□

The Theorem of *Gal-Or* characterizes payoff advantages of first- and second-mover based on strategic substitutes and complements defined as follows:

Definition 2.5 (Strategic substitutes and complements)
Strategy s_2 *of player* II

(i) is a strategic substitute *to strategy* s_1 *of player* I*, iff they mutually offset each other, i.e.,*

$$\partial_{s_1} g(s_1) < 0, \quad \forall s_1 \in]\underline{s}, \overline{s}[. \tag{2.19}$$

(ii) is a strategic complement *to strategy* s_1 *of player* I*, iff they mutually reinforce each other, i.e.,*

$$\partial_{s_1} g(s_1) > 0, \quad \forall s_1 \in]\underline{s}, \overline{s}[. \tag{2.20}$$

Under the considered general assumptions, strategic substitutes and complements are characterized by the players' payoff functions π_1 and π_2:

Proposition 2.6 (Characterization of strategic substitutes and complements)
Under GA 1 – 2, strategy s_2 is

(i) a strategic substitute to strategy s_1 iff $\exists u, v \in]\underline{s}, \overline{s}[\colon \partial_{s_1 s_2}\pi_2(u, v) < 0$.

(ii) a strategic complement to strategy s_1 iff $\exists u, v \in]\underline{s}, \overline{s}[\colon \partial_{s_1 s_2}\pi_2(u, v) > 0$.

Proof. Cf. [17]. Since $\partial_{s_2 s_2}\pi_2(s_1, s_2) < 0$ and $\partial_{s_1 s_2}\pi_2(s_1, s_2) \neq 0 \; \forall s_1, s_2 \in [\underline{s}, \overline{s}]$, both equivalences are a direct consequence of Proposition 2.4, Equation (2.14).

□

2.3 Theorem of *Gal-Or*

The characterization of first- and second-mover advantages under complete information by *Gal-Or* reads as follows:

Theorem 2.7 (*Gal-Or*, 1985)
Under GA 1 – 2, an interior SPE $(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$ yields:

(i) If s_2 is a strategic substitute to strategy s_1 (iff $\partial_{s_1 s_2}\pi_2 < 0$), then there is a first-mover advantage, i.e.,

$$\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}). \tag{2.21}$$

(ii) If s_2 is a strategic complement to strategy s_1 (iff $\partial_{s_1 s_2}\pi_2 > 0$), then there is a second-mover advantage, i.e.,

$$\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}). \tag{2.22}$$

The following lemma ensures the exposition of the proof.

Lemma 2.8
Under GA 1 – 2, an interior SPE $(s_1^{\mathrm{L}}, s_2^{\mathrm{F}})$ satisfies:

(i) If $\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) < 0$ and $\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) < 0$, then $s_1^{\mathrm{L}} > g(s_2^{\mathrm{F}})$.

(ii) If $\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) < 0$ and $\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > 0$, then $s_1^{\mathrm{L}} < g(s_2^{\mathrm{F}})$.

(iii) If $\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > 0$ and $\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > 0$, then $s_1^{\mathrm{L}} > s_2^{\mathrm{F}}$.

(iv) If $\partial_{s_1 s_2}\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) > 0$ *and* $\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) < 0$*, then* $s_1^{\mathrm{L}} < s_2^{\mathrm{F}}$.

Proof. Cf. [17]. (i) is shown by a proof by contradiction. Thus, suppose $g(s_2^{\mathrm{F}}) \geq s_1^{\mathrm{L}}$. It follows

$$\partial_{s_1}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \overset{(2.11)}{\geq} \partial_{s_1}\pi_1(g(s_2^{\mathrm{F}}), s_2^{\mathrm{F}}) \overset{(2.10)}{=} \partial_{s_2}\pi_2(s_2^{\mathrm{F}}, g(s_2^{\mathrm{F}})) \overset{(2.13)}{=} 0. \tag{2.23}$$

Together with $\partial_{s_2 s_2}\pi_2(s_1, s_2) < 0$ and $\partial_{s_1 s_2}\pi_2(s_1, s_2) < 0 \; \forall s_1, s_2 \in [\underline{s}, \overline{s}]$ it follows from the FOC of the SPE in Equation (2.13) that $\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \geq 0$. This lies in contradiction to the presumptions and thus $s_1^{\mathrm{L}} > g(s_2^{\mathrm{F}})$, finishing the proof of (i). The proof of (ii) results by a replacement of the previous greater-equal signs by smaller-equal signs.

The proof of (iii) is performed also by a proof by contradiction. Thus, suppose $s_2^{\mathrm{F}} \geq s_1^{\mathrm{L}}$. It follows

$$\begin{aligned}\partial_{s_1}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) &\overset{(2.11)}{\geq} \partial_{s_1}\pi_1(s_2^{\mathrm{F}}, s_2^{\mathrm{F}}) \\ &\overset{(2.10)}{=} \partial_{s_2}\pi_2(s_2^{\mathrm{F}}, s_2^{\mathrm{F}}) \overset{\partial_{s_1 s_2}\pi_2 > 0}{\geq} \partial_{s_2}\pi_2(s_1^{\mathrm{F}}, s_2^{\mathrm{F}}) \overset{(2.13)}{=} 0.\end{aligned} \tag{2.24}$$

Together with $\partial_{s_2 s_2}\pi_2(s_1, s_2) < 0$ and $\partial_{s_1 s_2}\pi_2(s_1, s_2) > 0 \; \forall s_1, s_2 \in [\underline{s}, \overline{s}]$ it follows from Equation (2.13) that $\partial_{s_2}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \leq 0$. This lies in contradiction to the presumptions and thus $s_1^{\mathrm{L}} > s_2^{\mathrm{F}}$, finishing the proof of (iii). The proof of (iv) results by an appropriate replacement of greater-equal and smaller-equal signs in the proof of (iii).

□

As a consequence of Lemma 2.8, the proof of the Theorem of *Gal-Or* follows:

Proof of theorem 2.7. Cf. [17]. (i) is shown by

$$\begin{aligned}\pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) = \pi_1(s_1^{\mathrm{L}}, g(s_1^{\mathrm{L}})) &\overset{(2.2)}{\geq} \pi_1(s_2^{\mathrm{F}}, g(s_2^{\mathrm{F}})) \\ &\underset{(2.12)}{\overset{L.\ 2.8\ (i),(ii)}{>}} \pi_1(s_2^{\mathrm{F}}, s_1^{\mathrm{L}}) \overset{(2.10)}{=} \pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}),\end{aligned} \tag{2.25}$$

whereas (ii) follows by

$$\pi_2(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}) \overset{(2.1)}{\geq} \pi_2(s_1^{\mathrm{L}}, s_1^{\mathrm{L}}) \overset{(2.10)}{=} \pi_1(s_1^{\mathrm{L}}, s_1^{\mathrm{L}}) \underset{(2.12)}{\overset{L.\ 2.8\ (iii),(iv)}{>}} \pi_1(s_1^{\mathrm{L}}, s_2^{\mathrm{F}}). \tag{2.26}$$

□

The Theorem of *Gal-Or* shows that there is – depending on the sign of the cross derivative $\partial_{s_1 s_2}\pi_2$ – a first- or second-mover advantage in symmetric extensive two-player games under the premise of deterministic payoff functions and continuous strategy spaces. The validity of the indispensable presumptions given in GA 2 (iv) and (v), however, are yet not assessed. In the following, two fundamental examples from industrial organization are presented, analyzed and reflected with a special focus towards the respective applicability of the Theorem of *Gal-Or*.

2.4 Examples from Industrial Organization

In this section, two fundamental examples from industrial organization are reviewed. In an industrial organization context, the beforehand introduced abstract symmetric extensive games of player I and player II correspond to a sequential-move (market) competition of identical firm I and firm II. Further, the strategies s_1 and s_2 represent either the firms' individual output quantities or their respective price decisions.

2.4.1 *Stackelberg* Competition

In this section, the sequential-move *Stackelberg* competition under complete information is considered [43]. Therein, both firms produce non-differentiable products under identical premises. As strategic variables firms I and II individually determine their output quantity, denoted by q_1 and q_2, respectively. Denoting the inverse market demand and production cost functions by $P(q)$ and $C(q)$, respectively, the payoff functions of firms I and II read:

$$\begin{aligned} \pi_1(q_1,q_2) &= P(q_1+q_2)q_1 - C(q_1), \\ \pi_2(q_1,q_2) &= P(q_1+q_2)q_2 - C(q_2). \end{aligned} \tag{2.27}$$

To show the validity of GA 2 (i) – (v) in the context of *Stackelberg* competition, it is assumed that the output quantities q_1 and q_2 can be varied continuously in between $[0,\overline{q}]$, where $\overline{q} > 0$ is a sufficiently large real number[6]. Moreover, it is assumed that the inverse marked demand function $P(q)$ and production cost function $C(q)$ are sufficiently smooth, i.e., at least two times continuously differentiable. Due to the saturation of markets with respect to products it can naturally be inferred that the inverse marked demand $P(q)$ is *strictly decreasing* and *concave*. In addition, *convexity* of the production cost $C(q)$ can be assumed due to limited production capabilities. Together, these assumption yield

$$\partial_q P(q) < 0,\ \partial_{qq}P(q) \le 0,\ \forall q \in]0,2\overline{q}[\quad \text{and} \quad \partial_{qq}C(q) \ge 0,\ \forall q \in]0,\overline{q}[, \tag{2.28}$$

[6] In the case of large output quantities, e.g. in mass production, this assumption is not a restriction. It rather corresponds to a relaxation of the outputs to continuous output quantities.

leading to the following conclusion:

Corollary 2.9 (First-mover advantage in *Stackelberg* competition)
Let the inverse market demand $P(q)$ and production cost $C(q)$ satisfy the conditions given by Equation (2.28). *Then any interior SPE*[7] *of* Stackelberg *competition with payoff functions defined in Equations* (2.27) *yields a first-mover advantage.*

Proof. Using Equation (2.28) it directly follows ($i = 1, 2$):

$$\begin{aligned} \pi_1(q_1, q_2) &\overset{(2.27)}{=} P(q_1 + q_2)q_1 - C(q_2) \overset{(2.27)}{=} \pi_2(q_2, q_1), \\ \partial_{q_i q_i}\pi_i(q_1, q_2) &\overset{(2.27)}{=} \partial_{qq}P(q_1 + q_2)q_i + 2\partial_q P(q_1 + q_2) - \partial_{qq}C(q_i) \underset{q_i>0}{\overset{(2.28)}{<}} 0, \\ \partial_{q_{-i}}\pi_i(q_1, q_2) &\overset{(2.27)}{=} \partial_q P(q_1 + q_2)q_i \underset{q_i>0}{\overset{(2.28)}{<}} 0, \\ \partial_{q_i q_{-i}}\pi_i(q_1, q_2) &\overset{(2.27)}{=} \partial_{qq}P(q_1 + q_2)q_i + \partial_q P(q_1 + q_2) \underset{q_i>0}{\overset{(2.28)}{<}} 0, \quad \forall q_1, q_2 \in]0, \overline{q}[. \end{aligned} \tag{2.29}$$

In particular, all requirements from GA 2 are satisfied and $\partial_{q_1 q_2}\pi_2(q_1, q_2) < 0$. Thus, output quantity q_2 of firm II is a strategic substitute to output quantity q_1 of firm I, cf. Proposition 2.6. Applying Theorem 2.7, it further follows that any interior SPE yields a first-mover advantage.

□

Corollary 2.9 shows, that *Stackelberg* competition yield a first-mover advantage under quite general and natural assumptions on the market, products and payoffs. This finding is further assessed by the following example.

Example 1 (*Stackelberg* competition under complete information)
A frequently found special case of Equation (2.28) fulfilling all requirements of GA 2, is given by a *linear* inverse marked demand function $P(q) = a - bq$ and a *linear* production cost function $C(q) = cq$ with $a, b, c > 0, a > c$. The resulting payoff functions of *Stackelberg* competition (with linear inverse market demand and production cost functions) are

$$\begin{aligned} \pi_1(q_1, q_2) &= (a - b(q_1 + q_2))q_1 - cq_1 = ((a - c) - b(q_1 + q_2))q_1, \\ \pi_2(q_1, q_2) &= (a - b(q_1 + q_2))q_2 - cq_2 = ((a - c) - b(q_1 + q_2))q_2. \end{aligned} \tag{2.30}$$

[7] Taking the plausible assumptions that the production cost function is upward sloping ($\partial_q C(q) > 0$) and that the inverse marked demand tends to zero for a large production surplus ($P(q) \overset{p\to\infty}{\to} 0$), it follows that the payoff functions of both firms have to decrease for large production quantities ($\partial_j \pi_i(q_1, q_2) < 0,\ i, j = 1, 2$ for $q_1, q_2 \to \infty$). Since no restrictions on $\overline{p}$ are required, thus, without loss of generality, each profit-maximizing SPE can in fact be assumed to be an *interior* SPE, i.e., $(p_1^{\mathrm{L}}, p_2^{\mathrm{F}}) \in]0, \overline{p}[^2$.

Therein, the parameters a and c can be interpreted as *market demand* and *marginal cost*, respectively. Using the FOC given by Equation (2.13), the unique interior SPE output quantities $(q_1^{\mathrm{L}}, q_2^{\mathrm{F}})$ are given by

$$\begin{aligned} q_1^{\mathrm{L}} &= \frac{a-c}{2b}, \\ q_2^{\mathrm{F}} &= g(q_1^{\mathrm{L}}) = \frac{a-c}{2b} - \frac{q_1^{\mathrm{L}}}{2} = \frac{a-c}{4b}, \end{aligned} \tag{2.31}$$

which results in SPE payoffs

$$\begin{aligned} \pi_1(q_1^{\mathrm{L}}, q_2^{\mathrm{F}}) &= \frac{(a-c)^2}{8b}, \\ \pi_2(q_1^{\mathrm{L}}, q_2^{\mathrm{F}}) &= \frac{(a-c)^2}{16b}. \end{aligned} \tag{2.32}$$

It is important to note that the SPE output quantities $q_1^{\mathrm{L}}, q_2^{\mathrm{F}}$ are realized iff they and the SPE payoffs $\pi_1(q_1^{\mathrm{L}}, q_2^{\mathrm{F}}), \pi_2(q_1^{\mathrm{L}}, q_2^{\mathrm{F}})$ are positive. Thus, they are realized iff $a > c$ which had been assumed *a posteriori*. This condition corresponds to the indispensable condition that the market allows to at least sell a single quantity with a price above the marginal cost and thus with profit.

Moreover, it holds $\partial_{q_1} g(q_1) = -\frac{1}{2} < 0$ and thus output quantity q_2 of firm II is a strategic substitute to output quantity q_1 of firm I. There hence is a first-mover advantage in accordance with the Theorem of *Gal-Or* (Theorem 2.7): $\pi_1(q_1^{\mathrm{L}}, q_2^{\mathrm{F}}) = 2\ \pi_2(q_1^{\mathrm{L}}, q_2^{\mathrm{F}})$.

2.4.2 Sequentialized *Hotelling* Competition

In this section, a sequentialization of the classical *Hotelling* competition [27] is introduced and analyzed with respect to first- and second-mover advantages.

Example 2 (Sequentialized *Hotelling* competition under complete information)
Two identical firms I and II compete at selling their maximally differentiated products along a "street" with equally distributed customers by sequentially deciding on the prices of their products, denoted by p_1 and p_2, respectively. Without loss of generality, the street is represented by the collection of all positions x in the interval $[0, 1]$, the total amount of customers is 1, and firms I and II are located at the edge positions $x_1 = 0$ and $x_2 = 1$, respectively. Furthermore, it is assumed that the *net utility* u of the products for a customer at position $0 \leq x \leq 1$ is given by

$$u(x) = \begin{cases} \overline{u} - t(x-0)^2 - p_1 & \text{if product of firm I is consumed} \\ \overline{u} - t(1-x)^2 - p_2 & \text{if product of firm II is consumed} \end{cases}. \tag{2.33}$$

Therein, $\overline{u} > 0$ corresponds to the *gross utility* of the products and the influence of the quadratic transport costs is represented by the *transport cost* $t > 0$. Customers naturally prefer products with highest net utility and thus the utility-indifferent *marginal customer* $\hat{x}$ is determined by

$$\overline{u} - t(\hat{x} - 0)^2 - p_1 \overset{(2.33)}{=} \overline{u} - t(1 - \hat{x})^2 - p_2, \tag{2.34}$$

which is equivalent to

$$\hat{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}. \tag{2.35}$$

Denoting the *marginal cost* by c, the payoff functions of the sequentialized *Hotelling* competition (with quadratic transport cost and linear production cost functions) are[8]

$$\begin{aligned} \pi_1(p_1, p_2) &= (\hat{x}(p_1, p_2) - 0)(p_1 - c) \overset{(2.35)}{=} \left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)(p_1 - c), \\ \pi_2(p_1, p_2) &= (1 - \hat{x}(p_1, p_2))(p_2 - c) \overset{(2.35)}{=} \left(\frac{1}{2} - \frac{p_2 - p_1}{2t}\right)(p_2 - c). \end{aligned} \tag{2.36}$$

Using the FOC given by Equation (2.13), the unique interior SPE prices $(p_1^{\mathrm{L}}, p_2^{\mathrm{F}})$ are given by

$$\begin{aligned} p_1^{\mathrm{L}} &= \frac{3}{2}t + c, \\ p_2^{\mathrm{F}} &= g(p_1^{\mathrm{L}}) = \frac{t + c}{2} + \frac{p_1^{\mathrm{L}}}{2} = \frac{5}{4}t + c, \end{aligned} \tag{2.37}$$

which results in SPE payoffs

$$\begin{aligned} \pi_1(p_1^{\mathrm{L}}, p_2^{\mathrm{F}}) &= \frac{9}{16}t, \\ \pi_2(p_1^{\mathrm{L}}, p_2^{\mathrm{F}}) &= \frac{25}{32}t. \end{aligned} \tag{2.38}$$

The SPE prices $p_1^{\mathrm{L}}, p_2^{\mathrm{F}}$ are realized since they and the SPE payoffs $\pi_1(p_1^{\mathrm{L}}, p_2^{\mathrm{F}}), \pi_2(p_1^{\mathrm{L}}, p_2^{\mathrm{F}})$ are strictly positive given that the transport costs are non-vanishing, i.e., if $t > 0$.

Concerning the Theorem of *Gal-Or* (Theorem 2.7), it is straight-forward to verify that the payoff functions given by Equation (2.36) satisfy all conditions stated in GA 2. In particular, it holds $\partial_{p_1 p_2}\pi_2(p_1, p_2) = \frac{1}{2t} > 0$. Thus, price p_2 of firm II is a strategic complement to price p_1 of firm I in accordance with Proposition 2.6: $\partial_{p_1} g(p_1) = \frac{1}{2} > 0$. As a consequence of the Theorem of *Gal-Or*, there has to be a second-mover advantage: $\pi_1(p_1^{\mathrm{L}}, p_2^{\mathrm{F}}) = 0.72\ \pi_2(p_1^{\mathrm{L}}, p_2^{\mathrm{F}})$.

[8] A more general model assuming a linear production cost structure and a general market demand (being equivalent to a general utility function) is developed and analyzed by *Amir* and *Stepanova* [4].

3 First- and Second-Mover Advantage under Uncertainty

In the previous Chapter 2, first- and second-mover advantages were characterized under the assumption of deterministic payoff functions. The aim of this section is to drop this prerequisite by assuming (market) competition environments of *in*complete information. More precisely, the influence of uncertainties to first- and second-mover advantages in sequential-move duopoly competition is addressed. Here, a special focus lies on uncertainties in model parameters which are assumed to be (stochastically) *distributed*. It is assumed that both firms I and II act under equal premises and initially have access to equivalent information. To account for uncertainties in model parameters, the following (abstract) stochastic framework introduced by *Gal-Or* [20] is used:

The stochastic variation of a model parameter is represented by a *random variable*, denoted by u. The random variable u is a superposition of the random variables u_1 and u_2 which correspond to the distribution of the uncertain parameter in the respective market segments of the players I and II, respectively. Both players observe private signals as estimates for the uncertain parameter in their market segments, denoted by x_1 and x_2. Further, the random variables and private signals fulfill:

General assumption 3 (Properties of random variables and private signals)

(i) The random variable u of the uncertain parameter is the mean of the random variables u_1 and u_2 of the market segments, i.e.,

$$u = \frac{u_1 + u_2}{2}. \tag{3.1}$$

(ii) The random variables u_1 and u_2 of the market segments are distributed according to identical prior probability distributions*, such that*

$$\mathrm{E}[u_1] = \mathrm{E}[u_2] = \theta, \quad \mathrm{Var}[u_1] = \mathrm{Var}[u_2] = \sigma \quad \textit{and} \quad \mathrm{Cov}[u_1, u_2] = h, \tag{3.2}$$

where $0 \leq \theta$ and $0 < h \leq \sigma$[9].

[9] h cannot exceed σ, in order to not violate the positive definiteness of the variance-covariance matrix. When $h = \sigma$, both private signals are the sum of the market demand θ and additional "white noise". This case can be interpreted as both firms having access to the same market segment. When $0 < h < \sigma$, firms have access to possibly different market segments that, however, are at least partially correlated [20].

(iii) The private signals x_i ($i = 1, 2$) are unbiased estimators *of the random variables u_i of the market segments with variance $0 < m_i < \infty$*[10]*, i.e.,*

$$\mathrm{E}\left[x_i|u_i\right] = u_i \quad \textit{and} \quad \mathrm{Var}\left[x_i|u_i\right] = m_i, \quad i = 1, 2. \tag{3.3}$$

Furthermore, conditional on the random variables u_i, the private signals x_i are distributed according to identical posterior probability distributions.

(iv) The prior and posterior probability distributions yield linear posterior expected values *with*

$$\begin{aligned} \mathrm{E}\left[u|x_1, x_2\right] &= a_0 + a_1 x_1 + a_2 x_2, \\ \mathrm{E}\left[u|x_1\right] &= b_0 + b_1 x_1, \\ \mathrm{E}\left[u|x_2\right] &= c_0 + c_1 x_2, \\ \mathrm{E}\left[x_2|x_1\right] &= d_0 + d_1 x_1, \\ \mathrm{E}\left[x_1|x_2\right] &= f_0 + f_1 x_2, \end{aligned} \tag{3.4}$$

where $a_0, \ldots, f_1 \in \mathbb{R}$.

In the context of industrial organization, GA 3 (i) and (ii) correspond to the assumption of equal premises for both players which includes the quantitative equality of the market segments u_1 and u_2 in size and structure. GA 3 (iii) implies that the private signals are observed using unbiased methods, such as carefully performed and analyzed market surveys. The latter assumption GA 3 (iv) possesses a restriction to the applicability of the stochastic framework in industrial organization. The validity of GA 3 (iv) requires strong assumptions on the prior and posterior probability distributions that generally do not have to be satisfied. However, several pairs of prior-posterior probability distributions satisfy the linearity assumption, such as the Gamma-Poisson, Beta-Binomial and Normal-Normal distributions [12, 20, 40].

The coefficients $a_0, \ldots, f_1$ of the posterior expected values in Equation (3.4) are uniquely determined by the assumptions in GA 3.

Proposition 3.1 (Properties of private signals and parameters of posterior expected values) *Under GA 3, it holds:*

[10] In this thesis the variance of the market segments is restricted to finite values, i.e., $m_i < \infty$ ($i = 1, 2$). This corresponds to the reasonable assumption that the private observations of both players are *not* indefinitely noisy and possess at least some connection to the actual market segments. Some aspects of the cases $m_1 = \infty$ or $m_2 = \infty$ are discussed in [20].

(i) The private signals x_1 *and* x_2 *of both players* I *and* II *have the following expected values (*$i = 1, 2$*):*

$$\begin{aligned} \mathrm{E}\left[x_i\right] &= \theta, \\ \mathrm{E}\left[x_i^2\right] &= \theta^2 + \sigma + m_i, \\ \mathrm{E}\left[x_1 x_2\right] &= \theta^2 + h. \end{aligned} \tag{3.5}$$

(ii) The coefficients $a_0, \dots, f_1 \in \mathbb{R}$ *in GA 3 (iv) are uniquely determined by*

$$\begin{aligned} \mathrm{E}\left[u|x_1, x_2\right] &= \underbrace{\frac{\theta}{2}\frac{(\sigma-h)(m_1+m_2)+2m_1m_2}{(\sigma+m_1)(\sigma+m_2)-h^2}}_{=a_0} + \underbrace{\frac{1}{2}\frac{(\sigma+h)(\sigma+m_2-h)}{(\sigma+m_1)(\sigma+m_2)-h^2}}_{=a_1} x_1 \\ &\quad + \underbrace{\frac{1}{2}\frac{(\sigma+h)(\sigma+m_1-h)}{(\sigma+m_1)(\sigma+m_2)-h^2}}_{=a_2} x_2, \\ \mathrm{E}\left[u|x_1\right] &= \underbrace{\frac{\theta}{2}\frac{\sigma+2m_1-h}{\sigma+m_1}}_{=b_0} + \underbrace{\frac{1}{2}\frac{\sigma+h}{\sigma+m_1}}_{=b_1} x_1, \\ \mathrm{E}\left[u|x_2\right] &= \underbrace{\frac{\theta}{2}\frac{\sigma+2m_2-h}{\sigma+m_2}}_{=c_0} + \underbrace{\frac{1}{2}\frac{\sigma+h}{\sigma+m_2}}_{=c_1} x_2, \end{aligned} \tag{3.6}$$

$$\begin{aligned} \mathrm{E}\left[x_2|x_1\right] &= \underbrace{\theta\frac{\sigma+m_1-h}{\sigma+m_1}}_{=d_0} + \underbrace{\frac{h}{\sigma+m_1}}_{=d_1} x_1, \\ \mathrm{E}\left[x_1|x_2\right] &= \underbrace{\theta\frac{\sigma+m_2-h}{\sigma+m_2}}_{=f_0} + \underbrace{\frac{h}{\sigma+m_2}}_{=f_1} x_2. \end{aligned} \tag{3.7}$$

Proof. Using the *Law of Total Expectation* [44], the expected private signals are

$$\mathrm{E}\left[x_i\right] = \mathrm{E}\left[\mathrm{E}\left[x_i|u_i\right]\right] \overset{(3.3)}{=} \mathrm{E}\left[u_i\right] \overset{(3.2)}{=} \theta, \quad i = 1, 2. \tag{3.8}$$

Applying the *Law of Total Variance* [44] results in

$$\mathrm{Var}\left[x_i\right] = \mathrm{E}\left[\mathrm{Var}\left[x_i|u_i\right]\right] + \mathrm{Var}\left[\mathrm{E}\left[u_i|x_i\right]\right] \underset{(3.3)}{\overset{(3.2)}{=}} \sigma + m_i, \quad i = 1, 2, \tag{3.9}$$

and thus by the definition of the variance it holds:

$$\mathrm{E}\left[x_i^2\right] = \mathrm{E}\left[x_i\right]^2 + \mathrm{Var}\left[x_i\right] \underset{(3.9)}{\overset{(3.8)}{=}} \theta^2 + \sigma + m_i, \quad i = 1, 2. \tag{3.10}$$

By the definition of the covariance it follows

$$\mathrm{E}[x_1 x_2] = \mathrm{E}[x_1]\,\mathrm{E}[x_2] + \mathrm{Cov}[x_1, x_2] \overset{(3.2)}{\underset{(3.8)}{=}} \theta^2 + h, \tag{3.11}$$

which finishes the proof of (i). The proof of (ii) is based on equations (3.8)–(3.11) and results in [14]. Since it is only of technical nature it is omitted in this thesis. A detailed proof can be found in the appendix of [20].

□

In the case of complete information, the payoff functions π_1 and π_2 of the player I and II only depend on the players' strategies s_1 and s_2. Introducing uncertainties, however, the payoffs additionally depend on the random variable u, i.e., $\pi_1 = \pi_1(s_1, s_2, u)$ and $\pi_2 = \pi_2(s_1, s_2, u)$.

Both players have access to their private signals x_1 and x_2 as estimates for the uncertain parameter in their market segment u_1 and u_2. In a non-cooperative *Bayes*ian extensive two-player game, both players aim to maximize their *expected payoffs* under the knowledge of their private signals. Hence, profit-maximizing optimal strategies under uncertainty $(s_1^{\mathrm{L,un}}, s_2^{\mathrm{F,un}})$ of player I and player II are in fact functions of the observed private signals x_1 and x_2, respectively. Thus, the SPE introduced in Definition 2.1 for environments of complete information must be generalized in the case of uncertainties:

Definition 3.2 (Perfect *Bayes*ian Equilibrium)
Under GA 1, a Perfect Bayesian Equilibrium *(PBE)* $(s_1^{\mathrm{L,un}}, s_2^{\mathrm{F,un}})$ *of a* Bayes*ian extensive two-player game with random variable* u *and private signals* x_1 *and* x_2 *is defined by:*

$$s_2^{\mathrm{F,un}} = G(x_2, s_1^{\mathrm{L,un}}) := \underset{s_2 \in \mathcal{S}}{\mathrm{argmax}}\ \mathrm{E}\left[\pi_2(s_1^{\mathrm{L,un}}, s_2, u) | s_1^{\mathrm{L,un}}, x_2\right] \tag{3.12}$$

and

$$s_1^{\mathrm{L,un}} = H(x_1) := \underset{s_1 \in \mathcal{S}}{\mathrm{argmax}}\ \mathrm{E}\left[\pi_1(s_1, G(x_2, s_1), u) | x_1\right], \tag{3.13}$$

where G *and* H *denote the* reaction functions *of player* I *and player* II*, respectively.*

The definition of PBEs is not based on restricting requirements on the strategy space $\mathcal{S}$. In particular, the strategy space can be discrete or continuous. Discrete strategy spaces, however, do not to yield further insight if uncertainties in the payoffs are introduced. More precisely, in the previous examples with discrete strategy spaces (Figures 2.1 and 2.2) all results were gained by simple algebraic comparisons of the specific discrete payoff values. Introducing uncertainties in the players' payoffs would hence only shift the payoffs under complete information by the expected values of the random variables. Thus, the consideration of potential uncertainties in the payoffs does not lead to any alteration of first- and second-mover advantages compared to the case of complete information. Summarizing, discrete strategy spaces remain inappropriate to address the question of first- and second-mover advantage also in the case of uncertain model parameters.

An analog to the Theorem of *Gal-Or* (Theorem 2.7) characterizing *expected* first- and second-mover advantages under the premise of uncertainties in model parameters and continuous strategy spaces does not exist and is beyond the scope of this thesis. Thus, in this chapter, the beforehand introduced *Stackelberg* and sequentialized *Hotelling* competition (Examples 1 and 2) are generalized by introducing uncertainties in various model parameters. The resulting competitions under uncertainty are analyzed with respect to first- and second-mover advantages.

3.1 *Stackelberg* Competition under Uncertain Market Demand

As a first step, *Stackelberg* competition under complete information (Example 1) is generalized to include uncertainties in the market demand as done by *Gal-Or* [20]. In particular, the following example is concerned with the question of whether the general first-mover advantage of *Stackelberg* competition under complete information (cf. Corollary 2.9) remains valid also in the case of incomplete information. In the context of industrial organization, the symmetric extensive *Bayes*ian games of player I and player II correspond to the sequential-move competition of identical firm I and firm II under uncertainty.

Example 3 (*Stackelberg* competition under uncertain market demand)
Stackelberg competition under uncertain market demand are frequently considered [6, 11, 15, 16, 18, 20, 22, 24–26, 31, 33, 34, 37, 40, 45]. As proposed by *Gal-Or* [20], the market demand is decomposed into a static and a stochastic contribution. Denoting the static contribution by a and the stochastic contribution by u, the payoff functions of *Stackelberg* competition (with linear inverse market demand and production cost functions) under uncertain demand are given by

$$\begin{aligned}\pi_1(q_1,q_2,u) &= ((a+u)-b(q_1+q_2))q_1 - cq_1 = ((a-c+u)-b(q_1+q_2))q_1,\\ \pi_2(q_1,q_2,u) &= ((a+u)-b(q_1+q_2))q_2 - cq_2 = ((a-c+u)-b(q_1+q_2))q_2,\end{aligned} \tag{3.14}$$

cf. Equation (2.30). Therein, $a, b, c > 0$ are fixed-valued model parameters. Analog to the case of complete information (cf. Example 1), it is assumed that the static market demand exceeds the marginal cost, i.e., $a > c$. The expected payoff $\mathrm{E}\left[\pi_2(q_1,q_2,u)|q_1,x_2\right]$

of firm II under the knowledge of the private signal x_2 and the output quantity $q_1 = H(x_1)$ of firm I is required to compute the FOCs of the PBE. To this end, it is assumed that the reaction function H of firm I is *continuously differentiable* and *invertible*[11]. Together with the linearity of the present model the FOC of Equation (3.12) for firm II is:

$$\begin{aligned} 0 =& \frac{\mathrm{d}}{\mathrm{d}q_2} \mathrm{E}\left[\pi_2(q_1, q_2, u) | q_1, x_2\right] \\ \overset{(3.14)}{=}& (a-c) - b(q_1 + 2q_2) + \mathrm{E}\left[u | q_1, x_2\right] \\ =& (a-c) - b(q_1 + 2q_2) + \mathrm{E}\left[u | x_1 = H^{-1}(q_1), x_2\right] \\ \overset{(3.6)}{=}& (a-c) - b(q_1 + 2q_2) + a_0 + a_1 x_1 + a_2 x_2 \\ =& (a-c) - b(q_1 + 2q_2) + a_0 + a_1 H^{-1}(q_1) + a_2 x_2. \end{aligned} \tag{3.15}$$

By simple rearrangement, the reaction function $G(x_2, q_1)$ of firm II follows

$$q_2 = G(x_2, q_1) = \frac{a-c}{2b} - \frac{q_1}{2} + \frac{a_0 + a_1 H^{-1}(q_1) + a_2 x_2}{2b}. \tag{3.16}$$

and its partial derivative with respect to q_1 follows:

$$\begin{aligned} \partial_{q_1} G(x_2, q_1) =& -\frac{1}{2} + \frac{a_1}{2b} \partial_{q_1} H^{-1}(q_1) = -\frac{1}{2} + \frac{a_1}{2b} \frac{1}{\partial_{x_1} H(H^{-1}(q_1))} \\ =& -\frac{1}{2} + \frac{a_1}{2b} \frac{1}{\partial_{x_1} H(x_1)}. \end{aligned} \tag{3.17}$$

Using Equation (3.16), the FOC of Equation (3.13) for firm I is given by:

$$\begin{aligned} 0 =& \frac{\mathrm{d}}{\mathrm{d}q_1} \mathrm{E}\left[\pi_1(q_1, G(x_2, q_1), u) | x_1\right] \\ \overset{(3.14)}{=}& (a-c) - b(2q_1 + \mathrm{E}\left[G(x_2, q_1) | x_1\right] - q_1 \mathrm{E}\left[\partial_{q_1} G(x_2, q_1) | x_1\right]) + \mathrm{E}\left[u | x_1\right] \\ \underset{(3.17)}{\overset{(3.16)}{=}}& \frac{a-c}{2} - b q_1 - \frac{a_0 + a_1 x_1 + a_2 \mathrm{E}\left[x_2 | x_1\right]}{2} - q_1 \frac{a_1}{2} \frac{1}{\partial_{x_1} H(x_1)} + \mathrm{E}\left[u | x_1\right] \\ \overset{(3.6)}{=}& \frac{a-c}{2} - b q_1 - \frac{a_0 + a_1 x_1 + a_2 (d_0 + d_1 x_1)}{2} - q_1 \frac{a_1}{2} \frac{1}{\partial_{x_1} H(x_1)} + (b_0 + b_1 x_1). \end{aligned} \tag{3.18}$$

Finally, the substitution of q_1 by $H(x_1)$ and the multiplication with $\partial_{x_1} H(x_1)$ results in an ordinary differential equation (ODE) determining the reaction function $H(x_1)$ of firm I:

$$\begin{aligned} 0 = \left(\frac{a - c - a_0 - a_2 d_0 + 2b_0}{2} \right) \partial_{x_1} H(x_1) + (-b) H(x_1) \partial_{x_1} H(x_1) \\ + \left(\frac{-a_1 - a_2 d_1 + 2b_1}{2} \right) x_1 \partial_{x_1} H(x_1) + (-\frac{a_1}{2}) H(x_1). \end{aligned} \tag{3.19}$$

11 In fact, it is possible to prove that H under no circumstances is a decreasing function, see [20]. In the context of industrial organization, however, it reasonable to assume smooth one-to-one relationships between the PBE output quantities and the firms' private signals.

The solution of the nonlinear ODE is a special case of the following proposition:

Proposition 3.3 (Solution of nonlinear ODE)
The nonlinear ODE

$$0 = \mu_1 \partial_{x_1} H(x_1) + \mu_2 H(x_1) \partial_{x_1} H(x_1) + \mu_3 x_1 \partial_{x_1} H(x_1) + \mu_4 H(x_1) + \mu_5 x_1 + \mu_6, \tag{3.20}$$

with coefficients $\mu_1, \ldots, \mu_6 \in \mathbb{R}$ *has two linear solutions*

$$H(x_1) = A_0 + A_1 x_1, \tag{3.21}$$

Therein, the coefficients A_0 *and* A_1 *are:*

$$\begin{aligned} A_0 &= -\frac{\mu_1}{\mu_2} + \frac{2}{\mu_2} \frac{\mu_1\mu_4 - \mu_2\mu_6}{\mu_4 - \mu_3 \pm \sqrt{(\mu_4+\mu_3)^2 - 4\mu_2\mu_5}}, \\ A_1 &= -\frac{\mu_3}{\mu_2} + \frac{2}{\mu_2} \frac{\mu_3\mu_4 - \mu_2\mu_5}{\mu_4 - \mu_3 \pm \sqrt{(\mu_4+\mu_3)^2 - 4\mu_2\mu_5}}, \end{aligned} \tag{3.22}$$

where both discriminants have to be introduced with equal signs.

Proof. By introducing Equations (3.21) and (3.22) into the ODE given by Equation (3.20), the validity of the proposition is verified in a straight-forward manner.

□

The solution of the ODE determining the reaction function $H(x_1)$ of firm I in Equation (3.19) follows from Proposition 3.3 by substituting $\mu_1 = \frac{a-c-a_0-a_2d_0+2b_0}{2}$, $\mu_2 = -b$, $\mu_3 = \frac{-a_1-a_2d_1+2b_1}{2}$, $\mu_4 = -\frac{a_1}{2}$ and $\mu_5 = \mu_6 = 0$. Since the linear solution with positive discriminants results in the trivial solution $H(x_1) = 0$, the sought PBE output quantity $q_1^{\mathrm{L,un}}$ of firm I is[12]

$$q_1^{\mathrm{L,un}} = H(x_1) = A_0 + A_1 x_1, \tag{3.23}$$

where the coefficients A_0 and A_1 possess negative discriminants and thus after some manipulations:

$$\begin{aligned} A_0 &= -\frac{\mu_1}{\mu_2} - \frac{\mu_1\mu_4}{\mu_2\mu_3} \overset{(3.6)}{=} \frac{h}{2b} \frac{\sigma + m_1 - h}{(\sigma+m_1)(\sigma+m_2) - h^2} \left(a - c + \frac{\theta}{2} \frac{\sigma + 2m_1 - h}{\sigma + m_1} \right), \\ A_1 &= -\frac{\mu_3}{\mu_2} - \frac{\mu_4}{\mu_2} \overset{(3.6)}{=} \frac{h}{4b} \frac{\sigma + m_1 - h}{(\sigma+m_1)(\sigma+m_2) - h^2} \frac{\sigma + h}{\sigma + m_1}. \end{aligned} \tag{3.24}$$

[12] The linearity of the reaction function H of firm I *a posteriori* reinforces the presumed smoothness and invertibility of H.

Introducing the inverse reaction function $H^{-1}(q_1) = \frac{q_1 - A_0}{A_1}$ of firm I into the definition of the reaction function $G(x_2, q_1^{\mathrm{L,un}})$ yields the PBE output quantity $q_2^{\mathrm{F,un}}$ of firm II

$$\begin{aligned} q_2^{\mathrm{F,un}} = G(x_2, q_1^{\mathrm{L,un}}) &\overset{(3.16)}{=} \frac{a-c}{2b} - \frac{q_1^{\mathrm{L,un}}}{2} + \frac{a_0 + a_1 \frac{q_1^{\mathrm{L,un}} - A_0}{A_1} + a_2 x_2}{2b} \\ &= \underbrace{\frac{1}{2b}\left(a - c + a_0 - a_1 \frac{A_0}{A_1}\right)}_{=:B_0} + \underbrace{\frac{a_2}{2b}}_{=:B_1} x_2 + \underbrace{\left(\frac{1}{2b}\frac{a_1}{A_1} - \frac{1}{2}\right)}_{=:B_2} q_1^{\mathrm{L,un}}, \end{aligned} \tag{3.25}$$

where after some manipulations:

$$\begin{aligned} B_0 &\underset{(3.24)}{\overset{(3.6)}{=}} \frac{h}{2b}\,\frac{\sigma + m_1 - h}{(\sigma + m_1)(\sigma + m_2) - h^2}\left(a - c - \frac{\theta}{2}\,\frac{\sigma - h}{h}\right), \\ B_1 &\underset{(3.24)}{\overset{(3.6)}{=}} \frac{\sigma + h}{4b}\,\frac{\sigma + m_1 - h}{(\sigma + m_1)(\sigma + m_2) - h^2}, \\ B_2 &\underset{(3.24)}{\overset{(3.6)}{=}} -\frac{1}{2} + \frac{\sigma + m_2 - h}{\sigma + m_1 - h}\,\frac{\sigma + m_1}{h}. \end{aligned} \tag{3.26}$$

Using Proposition 3.1 (i), the *expected* PBE output quantity of firm I is[13]

$$\begin{aligned} \mathrm{E}\left[q_1^{\mathrm{L,un}}\right] &\overset{(3.23)}{=} A_0 + A_1\,\mathrm{E}\left[x_1\right] \overset{(3.5)}{=} A_0 + A_1\theta \\ &\overset{(3.24)}{=} \frac{h}{2b}\,\frac{\sigma + m_1 - h}{(\sigma + m_1)(\sigma + m_2) - h^2}\left(a - c + \theta\right). \end{aligned} \tag{3.27}$$

Using $B_0 + B_1\theta \underset{(3.26)}{\overset{(3.24)}{=}} A_0 + A_1\theta \overset{(3.27)}{=} \mathrm{E}\left[q_1^{\mathrm{L,un}}\right]$ it follows the expected PBE output quantity of firm II:

$$\begin{aligned} \mathrm{E}\left[q_2^{\mathrm{F,un}}\right] &\overset{(3.25)}{=} B_0 + B_1\,\mathrm{E}\left[x_2\right] + B_2\,\mathrm{E}\left[q_1^{\mathrm{L,un}}\right] \\ &\overset{(3.5)}{=} B_0 + B_1\theta + B_2\,\mathrm{E}\left[q_1^{\mathrm{L,un}}\right] = \mathrm{E}\left[q_1^{\mathrm{L,un}}\right](1 + B_2) \\ &\underset{(3.26)}{\overset{(3.27)}{=}} \frac{h}{2b}\,\frac{\sigma + m_1 - h}{(\sigma + m_1)(\sigma + m_2) - h^2}\left(a - c + \theta\right) \\ &\qquad\qquad \left(\frac{1}{2} + \frac{\sigma + m_2 - h}{\sigma + m_1 - h}\,\frac{\sigma + m_1}{h}\right). \end{aligned} \tag{3.28}$$

Substituting $q_1^{\mathrm{L,un}} = H(x_1)$ and $q_2^{\mathrm{L,un}} = G(x_2, H(x_1))$ into the payoff functions in Equation (3.14), taking their expected value (with respect to x_1 and x_2) and using the

[13] It is important to note that the following expected values are taken with respect to x_1 and x_2. The previous expected values were taken with respect to u.

properties of the expected private signals in Proposition 3.1 (i) yields the expected payoff of firms I

$$\begin{aligned} \mathrm{E}\left[\pi_1(H(x_1), G(x_2, H(x_1)), u)|x_1\right] = \left(\frac{1}{2} + \frac{\sigma+m_2-h}{\sigma+m_1-h}\,\frac{\sigma+m_1}{h}\right) \\ \frac{h^2}{4b}\,\frac{(\sigma+m_1-h)^2}{((\sigma+m_1)(\sigma+m_2)-h^2)^2}\left((a-c+\theta)^2+\frac{1}{4}\frac{(\sigma+h)^2}{\sigma+m_1}\right), \end{aligned} \tag{3.29}$$

and the expected payoff of firm II

$$\begin{aligned} &\mathrm{E}\left[\pi_2(H(x_1), G(x_2, H(x_1)), u)|H(x_1), x_2\right] = \\ &\qquad \mathrm{E}\left[\pi_1(H(x_1), G(x_2, H(x_1)), u)|x_1\right]\left(\frac{1}{2} + \frac{\sigma+m_2-h}{\sigma+m_1-h}\,\frac{\sigma+m_1}{h}\right) \\ &\qquad + \frac{1}{16b}\,\frac{(\sigma+h)^2}{\sigma+m_1}\,\frac{(\sigma+m1-h)^2}{(\sigma+m_1)(\sigma+m_2)-h^2}. \end{aligned} \tag{3.30}$$

Comparing the PBE output quantities and payoffs of both firms leads to the following observations:

Proposition 3.4 (First- and second-mover advantage in *Stackelberg* competition under uncertain market demand)
Under GA 1 and 3, the unique interior PBE output quantities $(q_1^{\mathrm{L,un}}, q_2^{\mathrm{F,un}})$ *of* Stackelberg *competition under uncertain market demand (Example 3) satisfying* $a > c$ *yields:*

(i) If $\theta < 4(a-c)$ *and* $\frac{\theta}{2(a-c)+\theta}\sigma < h < \frac{2}{3}\sigma$*, then the PBE output quantities* $q_1^{\mathrm{L,un}}$ *and* $q_2^{\mathrm{F,un}}$ *are realized if the observed private signals* x_1 *and* x_2 *are sufficiently large, in particular if they are positive, i.e.,*

$$\begin{aligned} q_1^{\mathrm{L,un}} = H(x_1) > 0 \quad \text{and} \quad q_2^{\mathrm{L,un}} = G(x_2, q_1^{\mathrm{L,un}}) > 0, \\ \forall x_1 > \underbrace{-\frac{A_0}{A_1}}_{<0},\ x_2 > \underbrace{-\frac{B_0}{B_1}}_{<0}. \end{aligned} \tag{3.31}$$

(ii) If $0 < h < \frac{2}{3}\sigma$*, then the reaction function* $G(x_2, q_1^{\mathrm{L,un}})$ *of the second-mover is upward sloping with respect to the output quantity of the first-mover, i.e.,*

$$\partial_{q_1} G(x_2, q_1^{\mathrm{L,un}}) = B_2 > 0. \tag{3.32}$$

(iii) If $0 < h < \frac{2}{3}\sigma$*, then there is an expected second-mover advantage induced by a higher expected PBE output quantity of the second-mover, i.e.,*

$$\begin{aligned} \mathrm{E}\left[q_2^{\mathrm{F,un}}\right] > \mathrm{E}\left[q_1^{\mathrm{L,un}}\right] > 0, \\ \mathrm{E}\left[\pi_2(q_1^{\mathrm{L,un}}, q_2^{\mathrm{F,un}}, u)|q_1^{\mathrm{L,un}}, x_2\right] > \mathrm{E}\left[\pi_1(q_1^{\mathrm{L,un}}, q_2^{\mathrm{F,un}}, u)|x_1\right] > 0. \end{aligned} \tag{3.33}$$

Proof. (i) Let be $x_1 > -\frac{A_0}{A_1},\ x_2 > -\frac{B_0}{B_1}$. Since $a > c$ and $0 < h \leq \sigma$ it follows that A_0 and A_1 are positive. Thus,

$$q_1^{\mathrm{L,un}} = H(x_1) \overset{(3.23)}{=} A_0 + A_1 x_1 > 0, \tag{3.34}$$

due to $x_1 > -\frac{A_0}{A_1}$. Since $B_1 > 0$ and $q_1^{\mathrm{L,un}} > 0$, the positivity of

$$q_2^{\mathrm{F,un}} = G(x_2, q_1^{\mathrm{L,un}}) \overset{(3.23)}{=} B_0 + B_1 x_2 + B_2 q_1^{\mathrm{L,un}} \tag{3.35}$$

follows for all $x_2 > -\frac{B_0}{B_1}$ if B_0 and B_2 are positive too. The positivity of B_0 and B_2 are equivalent to

$$\frac{\theta}{2(a-c)+\theta}\sigma < h \quad \text{and} \quad \frac{h}{2}\frac{\sigma + m_1 - h}{\sigma + m_1} < \sigma + m_2 - h. \tag{3.36}$$

Since $\frac{h}{2}\frac{\sigma+m_1-h}{\sigma+m_1}$ is an increasing function with respect to m_1 and $\sigma + m_2 - h$ is an increasing function with respect to m_2, the latter inequality is satisfied for all values of $m_1 > 0$ and $m_2 > 0$ if

$$\frac{h}{2} < \sigma - h \quad \Leftrightarrow \quad h < \frac{2}{3}\sigma. \tag{3.37}$$

Thus, it is required that $\frac{\theta}{2(a-c)+\theta}\sigma < h < \frac{2}{3}\sigma$, which only is satisfiable iff $\theta < 4(a-c)$. With the positivity of $q_1^{\mathrm{L,un}}$ and $q_2^{\mathrm{F,un}}$ the positivities of the payoffs follow immediately from Equation (3.14). Thus, both firms seek to realize their PBE output quantities to expect (positive) profits.

(ii) The slope of the reaction function of firm II is given by

$$\partial_{q_1} G(x_2, q_1) = B_2, \tag{3.38}$$

and its positivity follows iff $B_2 > 0$. As shown in the proof of (i), B_2 is positive for all values of $m_1 > 0$ and $m_2 > 0$ given that $0 < h < \frac{2}{3}\sigma$.

(iii) Since $\frac{1}{16b}\frac{(\sigma+h)^2}{\sigma+m_1}\frac{(\sigma+m1-h)^2}{(\sigma+m_1)(\sigma+m_2)-h^2} > 0$, Equations (3.28) and (3.30) directly reveal the expected second-mover advantage given that

$$\left(\frac{1}{2} + \frac{\sigma + m_2 - h}{\sigma + m_1 - h}\frac{\sigma + m_1}{h}\right) = 1 + B_2 > 1. \tag{3.39}$$

Again, the positivity of B_2 is guaranteed for all $m_1 > 0$ and $m_2 > 0$ if $0 < h < \frac{2}{3}\sigma$.

□

If the firms' private signals x_1 and x_2 (that estimate the stochastic contributions of the market demand) are not restricted to positive values, e.g., if the prior and posterior probability distributions of the random variables are normal distributed (and thus $x_1, x_2 \in \mathbb{R}$), the estimated PBE output quantities $q_1^{\mathrm{L,un}}$ and $q_2^{\mathrm{F,un}}$ can be negative preventing their realization by the firms[14]. In addition, the output quantity $q_2^{\mathrm{F,un}}$ of the second-mover can be negative if a precondition of Proposition 3.4 (i) is violated. In particular, the scenario of a large expected market demand u compared to its static fraction a, more precisely $\mathrm{E}\left[u\right] = \theta \gg 4(a-c)$, can be of great practical relevance. It is important to note, however, that the PBE output quantity $q_1^{\mathrm{L,un}}$ of firm I is already realizable if $a + \theta > c$ and $x_1 > -\frac{A_0}{A_1}$. Hence, forms of competition under such premises violating a precondition of Proposition 3.4 (i) can lead to a non-participation of the second-mover and a monopoly situation for the first-mover.

According to Proposition 3.4 (iii), however, if the second-mover participates in *Stackelberg* competition under uncertain market demand, he can expect a higher profit (independent of the variances m_1 and m_2 of the observed private signals x_1 and x_2) if the observed market segments u_1 and u_2 are sufficiently uncorrelated, i.e., if $\mathrm{Cov}\left[u_1, u_2\right] = h < \frac{2}{3}\sigma$. In such cases, the informative value for the second-mover gained by the inference of the first-mover's private signal x_1 from the first-mover's PBE output quantity $q_1^{\mathrm{L,un}}$ by $x_1 = H^{-1}(q_1^{\mathrm{L,un}})$ is significant. This "*better information effect*" [20] enables the second-mover to overcome the first-mover's advantage shown for environments of complete information (cf. Example 1). Moreover, there exists an additional "*conjectural variation effect*" [20] that is beneficial to the second-mover. This effect is captured by $B_2 + \frac{1}{2} = \frac{\sigma+m_2-h}{\sigma+m_1-h}\frac{\sigma+m_1}{h}$, which measures the amount by which the slope of the reaction function $\partial_{q_1} G(x_2, q_1^{\mathrm{L,un}})$ of the second-mover in *Stackelberg* competition under uncertain marked demand exceeds the slope $\partial_{q_1} g(q_1^{\mathrm{L}})$ in the respective competition under complete information. This conjectural effect alters the sign of the slope of the reaction function of firm II in the case of $0 < h < \frac{2}{3}\sigma$ to positive. Thus, the statement of the Theorem of *Gal-Or* (Theorem 2.7) remains valid in the present example under uncertainty, i.e., the output quantities of both firms are strategic complements and there is an expected second-mover advantage.

14 To (almost always) enforce the positivity of normal distributed private singals x_1 and x_2 (with a probability of $99.7\,\%$) one can restrict their standard deviation by assuming $\sqrt{\sigma} < \frac{a-c+\theta}{3}$, cf. [11, 35, 40].

3.2 *Stackelberg* Competition under Uncertain Marginal Cost

As another example under incomplete information, *Stackelberg* competition under complete information (Example 1) is generalized to include uncertainties in the marginal cost. To this end, the stochastic framework as introduced by *Gal-Or* [20] is employed.

Example 4 (*Stackelberg* competition under uncertain marginal cost)
Stackelberg competition under uncertain marginal cost are studied in few publications [1, 2, 7, 19, 21, 32]. As in the previous Example 3, the marginal cost is decomposed into a static and a stochastic contribution which are denoted by c and u, respectively. The payoff functions of *Stackelberg* competition (with linear inverse market demand and production cost functions) under uncertain marginal cost are thus given by

$$\begin{aligned}\pi_1(q_1,q_2,u) &= (a-b(q_1+q_2))q_1-(c+u)q_1 = ((a-c-u)-b(q_1+q_2))q_1,\\ \pi_2(q_1,q_2,u) &= (a-b(q_1+q_2))q_2-(c+u)q_2 = ((a-c-u)-b(q_1+q_2))q_2,\end{aligned} \tag{3.40}$$

cf. Equations (2.30). Therein, $a,b,c > 0, a > c$ are fixed-valued model parameters. Comparing the payoff functions under uncertain marginal cost with the beforehand payoff function under uncertain marked demand in Equation (3.14), the only difference lies in the reversal of the sign of the random variable u. Thus, in an analog manner as in Example 3, it follows that the PBE output quantities $(\tilde{q}_1^{\mathrm{L,un}}, \tilde{q}_2^{\mathrm{F,un}})$ of *Stackelberg* competition under uncertain marginal cost are given by

$$\begin{aligned}\tilde{q}_1^{\mathrm{L,un}} &= H(x_1) = \tilde{A}_0+\tilde{A}_1x_1,\\ \tilde{q}_2^{\mathrm{F,un}} &= G(x_2,\tilde{q}_1^{\mathrm{L,un}}) = \tilde{B}_0+\tilde{B}_1x_2+\tilde{B}_2\tilde{q}_1^{\mathrm{L,un}}.\end{aligned} \tag{3.41}$$

Therein, the coefficients $\tilde{A}_0$, $\tilde{A}_1$, $\tilde{B}_0$, $\tilde{B}_1$, $\tilde{B}_2$ are estimated by reversing the signs of a_0, a_1, a_2, b_0 and b_1 compared to the calculation of the coefficients A_0, A_1, B_0, B_1, B_2 in Equations (3.24) and (3.26). Thus, they are given by:

$$\begin{aligned}\tilde{A}_0 &= \frac{h}{2b}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2}\left(a-c-\frac{\theta}{2}\frac{\sigma+2m_1-h}{\sigma+m_1}\right),\\ \tilde{A}_1 &= -\frac{h}{4b}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2}\,\frac{\sigma+h}{\sigma+m_1},\\ \tilde{B}_0 &= \frac{h}{2b}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2}\left(a-c+\frac{\theta}{2}\,\frac{\sigma-h}{h}\right),\\ \tilde{B}_1 &= -\frac{\sigma+h}{4b}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2},\\ \tilde{B}_2 &= -\frac{1}{2}+\frac{\sigma+m_2-h}{\sigma+m_1-h}\,\frac{\sigma+m_1}{h}.\end{aligned} \tag{3.42}$$

Besides in the term $a-c$, the first four coefficients $\tilde{A}_0, \tilde{A}_1, \tilde{B}_0$ and $\tilde{B}_1$ have reversed signs compared to A_0, A_1, B_0 and B_1, respectively. It is important to note, however, that the last coefficient $\tilde{B}_2$ is identical to the coefficient B_2.

Analog to the estimations in Example 3, Equations (3.41) and (3.42) yield the expected PBE output quantities of both firms

$$\begin{aligned}
\mathrm{E}\left[\tilde{q}_1^{\mathrm{L,un}}\right] &\underset{(3.42)}{\overset{(3.41)}{=}} \frac{h}{2b}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2}\,(a-c-\theta)\,,\\
\mathrm{E}\left[\tilde{q}_2^{\mathrm{F,un}}\right] &\underset{(3.42)}{\overset{(3.41)}{=}} \frac{h}{2b}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2}\,(a-c-\theta)\\
&\qquad\left(\frac{1}{2}+\frac{\sigma+m_2-h}{\sigma+m_1-h}\,\frac{\sigma+m_1}{h}\right),
\end{aligned} \tag{3.43}$$

and their expected PBE payoffs:

$$\begin{aligned}
\mathrm{E}\left[\pi_1(H(x_1),G(x_2,H(x_1)),u)|x_1\right] &\underset{(3.42)}{\overset{(3.41)}{=}} \left(\frac{1}{2}+\frac{\sigma+m_2-h}{\sigma+m_1-h}\,\frac{\sigma+m_1}{h}\right)\\
&\frac{h^2}{4b}\,\frac{(\sigma+m_1-h)^2}{((\sigma+m_1)(\sigma+m_2)-h^2)^2}\left((a-c-\theta)^2+\frac{1}{4}\frac{(\sigma+h)^2}{\sigma+m_1}\right),\\
\mathrm{E}\left[\pi_2(H(x_1),G(x_2,H(x_1)),u)|H(x_1),x_2\right] &\underset{(3.42)}{\overset{(3.41)}{=}}\\
&\mathrm{E}\left[\pi_1(H(x_1),G(x_2,H(x_1)),u)|x_1\right]\left(\frac{1}{2}+\frac{\sigma+m_2-h}{\sigma+m_1-h}\,\frac{\sigma+m_1}{h}\right)\\
&+\frac{1}{16b}\,\frac{(\sigma+h)^2}{\sigma+m_1}\,\frac{(\sigma+m1-h)^2}{(\sigma+m_1)(\sigma+m_2)-h^2}.
\end{aligned} \tag{3.44}$$

Comparing the PBE output quantities and payoffs of both firms leads to observations similar to Proposition 3.4:

Proposition 3.5 (First- and second-mover advantage in *Stackelberg* competition under uncertain marginal cost)
Under GA 1 and 3, the unique interior PBE output quantities $(\tilde{q}_1^{\mathrm{L,un}}, \tilde{q}_2^{\mathrm{F,un}})$ *of* Stackelberg *competition under uncertain marginal cost (Example 4) satisfying* $0<\theta<a-c$ *yields:*

(i) If $0<h<\frac{2}{3}\sigma$*, then the PBE output quantities* $\tilde{q}_1^{\mathrm{L,un}}$ *and* $\tilde{q}_2^{\mathrm{F,un}}$ *are realized if the observed private signals* x_1 *and* x_2 *are sufficiently small, i.e.,*

$$\tilde{q}_1^{\mathrm{L,un}} = H(x_1) > 0 \quad \text{and} \quad \tilde{q}_2^{\mathrm{L,un}} = G(x_2, \tilde{q}_1^{\mathrm{L,un}}) > 0,$$
$$\forall x_1 < \underbrace{-\frac{\tilde{A}_0}{\tilde{A}_1}}_{>0},\; x_2 < \underbrace{-\frac{\tilde{B}_0}{\tilde{B}_1}}_{>0}. \tag{3.45}$$

(ii) If $0 < h < \frac{2}{3}\sigma$*, then the reaction function* $G(x_2, \tilde{q}_1^{\mathrm{L,un}})$ *of the second-mover is upward sloping with respect to the output quantity of the first-mover, i.e.,*

$$\partial_{q_1} G(x_2, \tilde{q}_1^{\mathrm{L,un}}) = \tilde{B}_2 > 0. \tag{3.46}$$

(iii) If $0 < h < \frac{2}{3}\sigma$*, then there is an expected second-mover advantage induced by a higher expected PBE output quantity of the second-mover, i.e.,*

$$\begin{aligned} \mathrm{E}\left[\tilde{q}_2^{\mathrm{F,un}}\right] > \mathrm{E}\left[\tilde{q}_1^{\mathrm{L,un}}\right] > 0, \\ \mathrm{E}\left[\pi_2(\tilde{q}_1^{\mathrm{L,un}}, \tilde{q}_2^{\mathrm{F,un}}, u) | \tilde{q}_1^{\mathrm{L,un}}, x_2\right] > \mathrm{E}\left[\pi_1(\tilde{q}_1^{\mathrm{L,un}}, \tilde{q}_2^{\mathrm{F,un}}, u) | x_1\right] > 0. \end{aligned} \tag{3.47}$$

Proof. (i) Let be $x_1 < -\frac{\tilde{A}_0}{\tilde{A}_1}$ and $x_2 < -\frac{\tilde{B}_0}{\tilde{B}_1}$. Since $a > c + \theta$ and $0 < h \leq \sigma$ it follows that $\tilde{A}_0 > 0$ and $\tilde{A}_1 < 0$. Hence, it follows

$$\tilde{q}_1^{\mathrm{L,un}} = H(x_1) \overset{(3.41)}{=} \tilde{A}_0 + \tilde{A}_1 x_1 > 0, \tag{3.48}$$

due to $x_1 < -\frac{\tilde{A}_0}{\tilde{A}_1}$. Since $\tilde{B}_0 > 0$, $\tilde{q}_1^{\mathrm{L,un}} > 0$ and $\tilde{B}_1 < 0$, the positivity of

$$\tilde{q}_2^{\mathrm{F,un}} \overset{(3.41)}{=} G(x_2, \tilde{q}_1^{\mathrm{L,un}}) = \tilde{B}_0 + \tilde{B}_1 x_2 + \tilde{B}_2 \tilde{q}_1^{\mathrm{L,un}} \tag{3.49}$$

follows for all $x_2 < -\frac{\tilde{B}_0}{\tilde{B}_1}$ if $\tilde{B}_2$ is positive. Following the proof for the positivity of $B_2 = \tilde{B}_2$ in Proposition 3.4 (i), the positivity of $\tilde{B}_2$ is ensured if $0 < h < \frac{2}{3}\sigma$. With the positivity of $\tilde{q}_1^{\mathrm{L,un}}$ and $\tilde{q}_2^{\mathrm{F,un}}$ the positivities of the payoffs follow immediately from Equation (3.40). Thus, both firms seek to realize their PBE output quantities to expect (positive) profits.

Due to $\tilde{B}_2 = B_2$, the proofs of (ii) and (iii) are matching the proofs of Proposition 3.4 (ii) and (iii), respectively.

□

Note that $-\frac{\tilde{A}_0}{\tilde{A}_1} > 0$ and $-\frac{\tilde{B}_0}{\tilde{B}_1} > 0$. Thus, both firms realize their PBE output quantities if their private signals x_1 and x_2 (that estimate the stochastic contributions of their marginal costs) have sufficiently small *positive* values. The reaction functions $H(x_1)$ and $G(x_2, \tilde{q}_1^{\mathrm{L,un}})$ of both firms are downward sloping with respect to the private signals. Thus, a restriction of the private signals to positive values, e.g., if their prior-posterior probability distributions are Gamma-Poisson or Beta-Binomial distributed, easily results in negative PBE output quantities $\tilde{q}_1^{\mathrm{L,un}}$ and $\tilde{q}_2^{\mathrm{F,un}}$ which prevents their realization by the firms. Furthermore, it is important to note that – in contrast to environments of uncertain market demand (cf. Example 3) – the feasibility of the firms' PBE output quantities does not favor the first-mover. *Stackelberg* competition under uncertain marginal cost does not allow the first-mover to establish a monopoly situation by a non-participation of the second-mover.

According to Proposition 3.5 (iii), if both firms participate in *Stackelberg* competition under uncertain marginal cost, a second-mover advantage is expected if the observed market segments u_1 and u_2 are sufficiently uncorrelated, i.e., if $\mathrm{Cov}\,[u_1, u_2] = h < \frac{2}{3}\sigma$. The coefficient $\tilde{B}_2$ determining the slope $\partial_{q_1} G(x_2, \tilde{q}_1^{\mathrm{L,un}})$ of the reaction function of firm II is identical to the coefficient B_2 of the competition under an uncertain market demand (cf. Example 3). Thus, the discussion of the latter in terms of the *better information effect* and the *conjectural variation effect* remains valid without restrictions and is not repeated here.

3.3 Sequentialized *Hotelling* Competition under Uncertain Marginal Cost

The analyses of Examples 1, 3 and 4 have shown that incomplete information in *Stackelberg* quantity competition give an information advantage to the second-mover. Independent of the parameter in which uncertainty is assumed, the second-mover can be enabled to earn higher expected profits compared to the first-mover as opposed to the case of complete information.

This section is concerned with the question of whether this observation remains valid also in the case of price competition in which a general second-mover advantage already exists under the premise of complete information. To this end, the stochastic framework developed by *Gal-Or* [20] is employed to investigate the effects of uncertainty in the marginal cost of the sequentialized *Hotelling* competition introduced in Examples 2.

Example 5 (Sequentialized *Hotelling* competition under uncertain marginal cost)
As in the previous Examples 3 and 4, the uncertain marginal cost is decomposed into a static and a stochastic contribution, denoted by c and u, respectively. Thus, the payoff functions of the sequentialized *Hotelling* competition (with quadratic transport cost and linear production cost functions) under uncertain marginal cost are given by

$$\begin{aligned} \pi_1(p_1, p_2, u) &= \left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)(p_1 - (c + u)), \\ \pi_2(p_1, p_2, u) &= \left(\frac{1}{2} - \frac{p_2 - p_1}{2t}\right)(p_2 - (c + u)), \end{aligned} \tag{3.50}$$

cf. Equation (2.36). Therein, $t > 0$ and $c > 0$ are fixed-valued model parameters. The reaction functions, expected PBE prices and expected PBE payoffs of both firms are derived analog to the procedure in Example 3. Here, only key steps and the main results are reviewed.

The FOCs of the PBE defined by

$$\begin{aligned} 0 =& \frac{\mathrm{d}}{\mathrm{d}p_2} \mathrm{E}\left[\pi_2(p_1, p_2, u) | p_1, x_2\right], \\ 0 =& \frac{\mathrm{d}}{\mathrm{d}p_1} \mathrm{E}\left[\pi_1(p_1, G(x_2, p_1), u) | x_1\right], \end{aligned} \tag{3.51}$$

yield the governing equations for the reaction function $G(x_2, p_1)$ of firm II and its derivative with respect to p_1

$$\begin{aligned} G(x_2, p_1) &= \frac{t+c}{2} + \frac{p_1}{2} + \frac{a_0 + a_1 H^{-1}(p_1) + a_2 x_2}{2}, \\ \partial_{p_1} G(x_2, p_1) &= \frac{1}{2} + \frac{a_1}{2} \frac{1}{\partial_{x_1} H(x_1)}, \end{aligned} \tag{3.52}$$

as well as a nonlinear ODE determining the reaction function $H(x_1)$ of firm I:

$$\begin{aligned} 0 =& (3t + 2c + a_0 + a_2 d_0 + b_0) \partial_{x_1} H(x_1) - 2H(x_1) \partial_{x_1} H(x_1) \\ &+ (a_1 + a_2 d_1 + b_1)\, x_1 \partial_{x_1} H(x_1) + a_1 H(x_1) - a_1 b_1 x_1 - a_1 c - a_1 b_0. \end{aligned} \tag{3.53}$$

Two linear solutions of the ODE are follow from Proposition 3.3 by substituting $\mu_1 = 3t + 2c + a_0 + a_2 d_0 + b_0$, $\mu_2 = -2$, $\mu_3 = a_1 + a_2 d_1 + b_1$, $\mu_4 = a_1$, $\mu_5 = -a_1 b_1$, $\mu_6 = -a_1 c - a_1 b_0$. Noting that $\mu_3 \mu_4 = \mu_2 \mu_5$, however, only the linear solution

$$p_1^{\mathrm{L,un}} = H(x_1) = \bar{A}_0 + \bar{A}_1 x_1 \tag{3.54}$$

with positive discriminants signs in the coefficients $\bar{A}_0$ and $\bar{A}_1$ remains as well-defined solution. Hence, the two coefficients are computed to:

$$\begin{aligned} \bar{A}_0 =& -\frac{\mu_1}{\mu_2} + \frac{1}{\mu_2} \frac{\mu_1 \mu_4 - \mu_2 \mu_6}{\mu_4 - \mu_3} \overset{(3.6)}{=} \frac{1}{2} \left(3t + 2c + \theta \frac{\sigma + 2m_1 - h}{\sigma + m_1} \right) \\ &+ \frac{3t}{2} \frac{(\sigma + m_1)(\sigma + m_2 - h)}{h(\sigma + m_1 - h) + (\sigma + m_1)(\sigma + m_2) - h^2}, \\ \bar{A}_1 =& -\frac{\mu_3}{\mu_2} \overset{(3.6)}{=} \frac{1}{2} \frac{\sigma + h}{\sigma + m_1}. \end{aligned} \tag{3.55}$$

By introducing the PBE price $p_1^{\mathrm{L,un}}$ of firm I into Equation (3.52), the PBE price $p_2^{\mathrm{F,un}}$ of firm II follows

$$p_2^{\mathrm{F,un}} = G(x_2, p_1^{\mathrm{L,un}}) = \bar{B}_0 + \bar{B}_1 x_2 + \bar{B}_2 p_1^{\mathrm{L,un}}, \tag{3.56}$$

where the coefficients $\bar{B}_0$, $\bar{B}_1$ and $\bar{B}_2$ are given by:

$$\begin{aligned}\bar{B}_0 =& \frac{1}{2}\left(t + c + a_0 - a_1\frac{\bar{A}_0}{\bar{A}_1}\right), \overset{(3.6)}{=} \frac{t+c}{2} + \frac{\theta}{4}\,\frac{(\sigma-h)(m_1+m_2)-2m_1m_2}{(\sigma+m_1)(\sigma+m2)-h^2}\\ &+ \frac{1}{4}\,\frac{(\sigma+m_1)(\sigma+m_2-h)}{(\sigma+m_1)(\sigma+m_2)-h^2}\left(3t+2c+\theta\frac{\sigma+2m_1-h}{\sigma+m_1}\right.\\ &\left. + 3t\frac{(\sigma+m_1)(\sigma+m_2-h)}{h(\sigma+m_1)+(\sigma+m_1)(\sigma+m_2)-h^2}\right), \qquad (3.57)\\ \bar{B}_1 =& \frac{a_2}{2} \overset{(3.6)}{=} \frac{\sigma+h}{4}\,\frac{\sigma+m_1-h}{(\sigma+m_1)(\sigma+m_2)-h^2},\\ \bar{B}_2 =& \frac{1}{2} + \frac{a_1}{2\bar{A}_1} \underset{(3.55)}{\overset{(3.6)}{=}} \frac{1}{2} + \frac{\sigma+m_1}{2}\,\frac{\sigma+m_2-h}{(\sigma+m_1)(\sigma+m_2)-h^2}.\end{aligned}$$

As a consequence of the PBE prices $p_1^{\mathrm{L,un}} = H(x_1)$ and $p_2^{\mathrm{L,un}} = G(x_2, H(x_1))$, which depend on the observed private signals x_1 and x_2, the *expected* PBE prices are estimated to:

$$\begin{aligned}\mathrm{E}\left[p_1^{\mathrm{L,un}}\right] &\overset{(3.54)}{=} \bar{A}_0 + \bar{A}_1\,\mathrm{E}[x_1] \overset{(3.5)}{=} \bar{A}_0 + \bar{A}_1\theta\\ &\overset{(3.55)}{=} \frac{3}{2}t + c + \theta + \frac{3t}{2}\,\frac{(\sigma+m_1)(\sigma+m_2-h)}{h(\sigma+m_1-h)+(\sigma+m_1)(\sigma+m_2)-h^2},\\ \mathrm{E}\left[p_2^{\mathrm{F,un}}\right] &\overset{(3.56)}{=} \bar{B}_0 + \bar{B}_1\,\mathrm{E}[x_2] + \bar{B}_2\,\mathrm{E}\left[p_1^{\mathrm{L,un}}\right] \overset{(3.5)}{=} \bar{B}_0 + \bar{B}_1\theta + \bar{B}_2\,\mathrm{E}\left[p_1^{\mathrm{L,un}}\right] \qquad (3.58)\\ &\overset{(3.57)}{=} \frac{5}{4}t + c + \theta + \frac{3t}{4}\underbrace{\frac{(\sigma+m_1)(\sigma+m_2-h)}{h(\sigma+m_1-h)+(\sigma+m_1)(\sigma+m_2)-h^2}}_{=:\chi}.\end{aligned}$$

Substituting $p_1^{\mathrm{L,un}} = H(x_1)$ and $p_2^{\mathrm{L,un}} = G(x_2, H(x_1))$ into the payoff functions in Equation (3.50), taking their expected value (with respect to x_1 and x_2) and using the properties of the expected private signals in Proposition 3.1 (i), the expected payoffs of both firms follow. Using the above introduced abbreviation χ, they are:

$$\begin{aligned}\mathrm{E}\left[\pi_1(H(x_1), G(x_2, H(x_1)), u)|x_1\right] &= \frac{9}{16}t\left(1-\chi^2\right),\\ \mathrm{E}\left[\pi_2(H(x_1), G(x_2, H(x_1)), u)|H(x_1), x_2\right] &= \frac{25}{32}t\left(1+\frac{6}{5}\chi+\frac{9}{25}\chi^2\right) \qquad (3.59)\\ &+ \frac{1}{32t}\,\frac{(\sigma+h)^2(\sigma+m_1-h)^2}{(\sigma+m_1)((\sigma+m_1)(\sigma+m_2)-h^2)}.\end{aligned}$$

The analysis of the PBE prices and payoffs of both firms yields the following observations:

Proposition 3.6 (Second-mover advantage in sequentialized *Hotelling* competition under uncertain marginal cost)
Under GA 1 and 3, the unique interior PBE prices $(p_1^{\mathrm{L,un}}, p_2^{\mathrm{F,un}})$ *of the sequentialized* Hotelling *competition under uncertain marginal cost (Example 5) yields:*

(i) The PBE prices $p_1^{\mathrm{L,un}}$ *and* $p_2^{\mathrm{F,un}}$ *are realized if the observed private signals* x_1 *and* x_2 *are sufficiently large, in particular if they are positive, i.e.,*

$$\begin{aligned} p_1^{\mathrm{L,un}} = H(x_1) > 0 \quad \text{and} \quad p_2^{\mathrm{L,un}} = G(x_2, p_1^{\mathrm{L,un}}) > 0, \\ \forall x_1 > \underbrace{-\frac{\bar{A}_0}{\bar{A}_1}}_{<0},\ x_2 > \underbrace{-\frac{\bar{B}_0}{\bar{B}_1}}_{<0}. \end{aligned} \tag{3.60}$$

(ii) The reaction function $G(x_2, p_1^{\mathrm{L,un}})$ *of the second-mover is upward sloping with respect to the price of the first-mover, i.e.,*

$$\partial_{p_1} G(x_2, p_1^{\mathrm{L,un}}) = \bar{B}_2 > 0. \tag{3.61}$$

(iii) There is an expected second-mover advantage induced by a lower expected PBE price of the second-mover, i.e.,

$$\begin{aligned} 0 < \mathrm{E}\left[p_2^{\mathrm{F,un}}\right] < \mathrm{E}\left[p_1^{\mathrm{L,un}}\right], \\ \mathrm{E}\left[\pi_2(p_1^{\mathrm{L,un}}, p_2^{\mathrm{F,un}}, u) | p_1^{\mathrm{L,un}}, x_2\right] > \mathrm{E}\left[\pi_1(p_1^{\mathrm{L,un}}, p_2^{\mathrm{F,un}}, u) | x_1\right] > 0. \end{aligned} \tag{3.62}$$

Proof. Since $0 < h \leq \sigma$, the coefficients $\bar{A}_0, \bar{A}_1, \bar{B}_0, \bar{B}_1$ and $\bar{B}_2$ are positive. Hence, (i) and (ii) follow from Equations (3.54) and (3.56) similar to the proof of Proposition 3.4 (i) and (ii), respectively.

The prove of (iii) results from Equations (3.58) and (3.59) by noting that $\frac{1}{32t} \frac{(\sigma+h)^2(\sigma+m_1-h)^2}{(\sigma+m_1)((\sigma+m_1)(\sigma+m_2)-h^2)} > 0$ and given that $0 < \chi = \frac{(\sigma+m_1)(\sigma+m_2-h)}{h(\sigma+m_1-h)+(\sigma+m_1)(\sigma+m_2)-h^2} < 1$. The positivity of χ follows from $0 < h \leq \sigma$ and also does its boundedness by 1 which is equivalent to:

$$\begin{aligned} (\sigma + m_1)(\sigma + m_2 - h) < h(\sigma + m_1 - h) + (\sigma + m_1)(\sigma + m_2) - h^2 \\ \Leftrightarrow \quad -h\sigma - hm_1 < h\sigma + hm_1 - 2h^2 \\ \Leftrightarrow \quad 0 < 2h\left(\sigma + m_1 - h\right). \end{aligned} \tag{3.63}$$

□

Proposition 3.6 shows that the second-mover retains his second-mover advantage independent of the properties of the random variable u. Given that the observed private signals x_1 and x_2 (that estimate the stochastic contributions of the firms' marginal costs) are positive, e.g., if their prior-posterior probability distributions are Gamma-Poisson or Beta-Binomial distributed, both firms realize their PBE prices. As in Examples 3 and 4, the informative value for the second-mover gained by the inference of the first-mover's private signal x_1 from the first-mover's PBE price $p_1^{\mathrm{L,un}}$ by $x_1 = H^{-1}(p_1^{\mathrm{L,un}})$ is significant. This information advantage allows the second-mover to further increase his expects profit (by a factor of $\frac{6}{5}\chi + \frac{9}{25}\chi^2$) and decrease the first-mover's expected profits (by a factor of $-\chi^2$) compared to environments of complete information, cf. Equations (2.38) and (3.59).

The enlarged second-mover advantage by the better information effect is again supported by the conjectural variation effect captured by $\bar{B}_2 - \frac{1}{2} = \frac{\sigma+m_1}{2} \frac{\sigma+m_2-h}{(\sigma+m_1)(\sigma+m_2)-h^2}$. The conjectural variation effect measures the amount by which the slope of the reaction function $\partial_{p_1} G(x_2, p_1^{\mathrm{L,un}})$ of the second-mover in the sequentialized *Hotelling* competition under uncertain marginal cost exceeds the slope $\partial_{p_1} g(p_1^{\mathrm{L}})$ of the respective competition under complete information. In the previous Examples 3 and 4 the conjectural effect could alter the sign of the slope of the reaction function of firm II from negative to positive. In the present Example 5, it further increases the slope of the reaction function in which explains the enlarged payoff advantage of the second-mover.

It is important to note that the statement of the Theorem of *Gal-Or* (Theorem 2.7) remains valid also in the present example under uncertainty, i.e., the prices of both firms are strategic complements and there is an expected second-mover advantage.

4 Review of Further Literature

The previous Chapters 2 and 3 reviewed two major contributions by *Gal-Or* addressing the question of first- and second-mover advantages. The developed theoretical concepts and results were applied to various forms of *Stackelberg* and sequentialized *Hotelling* competition. Since the later publication by *Gal-Or* in 1987 [20] further work related to first- and second-mover advantages has been published which is briefly summarized and discussed in this chapter.

4.1 Tripoly and Oligopoly Competition

A conceptual simple extension of work by *Gal-Or* presented in this thesis [20] (cf. Example 3) was performed by *Shinkai* [40] who considered a *three*-firm *Stackelberg* competition under uncertainty. *Shinkai* used a very similar stochastic framework as introduced by *Gal-Or* (but without market segmentation) to investigate a sequential-move tripoly quantity competition under uncertain market demand. Nevertheless, his results are surprising. The results show that the conjecture that the second-mover can generally expect higher profits compared to the first-mover due to his information advantage is no longer valid a the three-firm scenario. In contrast, the second-mover earns the lowest expected profits while the third-mover earns the highest. *Shinkai* explains this surprising non-monotone expected payoff pattern "by the fact that the first-mover and the third-mover are strategic substitutes at the equilibrium in our three player *Stackelberg* game under incomplete information, although the first- and the second-movers or the second- and the third-movers are strategic complements, respectively" [40][15].

The study of sequential-move n-player *Stackelberg* oligopoly competition under complete information (generalization of Example 1) by *Boyer* and *Moreaux* [9], and *Anderson* and *Engers* [5] yield that the strategies of *all* n firms are substitutes with respect to the prior

[15] Following the working paper by *Cumbul* [11], the latter observation by *Shinkai* remains valid also in the sequential-move n-firm generalization. While the strategies of succeeding firms are complements, the strategies with respect to the remaining prior firms are substitutes. *Cumbul*, however, claims that the expected last-mover advantage found by *Gal-Or* and *Shinkai* breaks down and alters into an expected first-mover advantage if the number of sequential-move firms n is bigger or equal to five.

firms. In analogy to the Theorem of *Gal-Or* (cf. Theorem 2.7) it is thus not surprising (but is yet not proven under general preconditions on the payoff functions of the n-players) that there is a strictly monotonically decreasing payoff pattern with respect to the ranking of the firms, which leaves the first-mover with the highest and the *last*-mover the least profits. In comparison with the three-firm competition under uncertain market demand by *Shinkai* [40] it is evident that the information advantage of the non-first-movers remains even if more than two firms participate in *Stackelberg* competition under uncertainty. This advantage, however, is only predominant for the last-mover and the intermediate firms suffer from the strategic dominance of prior firms[16].

Another important extension of work by *Gal-Or* was performed by *Nakamura* [35], where a sequential-move quantity competition of a single first-mover and n simultaneous-moving second-movers was studies. All firms act under an uncertain market demand which is represented by a very similar stochastic framework as developed by *Gal-Or* (but without market segmentation). In contrast to the results by *Gal-Or*, the first-mover's and second-movers' estimates for the spread of the market demand from their private signals (in this thesis denoted by m_1 and m_2) can yield a non-positive payoff for the first-mover. Thus, the single first-mover may not participate in the sequential-move competition if the number of second-movers is too large. Moreover, the spread of the private signals determines if the strategies of the first-mover and the second-movers are strategic substitutes or complements, and if there is a resulting first- or second-mover advantage, respectively. The findings by *Nakamura* agree with conclusions from *Gal-Or* [20] that a second-mover benefits in a sequential-move competition from the better information effect and the conjectural variation effect (cf. Examples 3 and 4). Thus, the accumulated information advantage of the second-movers is strong and enables them to enforce a non-participation of the first-mover in the sequential-move competition.

The discussed *Stackelberg*-type forms of competition with an uncertain market demand by *Cumbul* [11], *Gal-Or* [20], *Nakamura* [35] and *Shinkai* [40] have in common that the concept of strategic substitutes and complements remains key to rate the expected payoffs of the firms and thus to address the question of first-, second- and last-mover advantages. As shown by *Shinkai* [40], however, uncertainties can lead to complex reaction functions where the strategy of a firm can be a strategic substitute with respect to the strategies of some firms *and* a strategic complement with respect to others. Yet, a characterization of strategic substitutes and complements for general non-deterministic payoff functions (comparable to Proposition 2.6 for deterministic payoff functions) is not achieved. Furthermore, a rating of first-, second- and last-mover advantages under the premise of uncertainties comparable to the Theorem of *Gal-Or* (cf. Theorem 2.7) remains to be shown.

16 According to the working paper by *Cumbul* [11], the expected profits of firms are strictly decreasing with respect to their ranking until the second last-mover which thus always earns the least. Depending on the number of participating firms n, the expected profit of the last-mover can can range from the most (if $n \leq 4$) down to the second least (if n is large).

4.2 Revelation of Uncertainty by First-Mover's Action

The stochastic framework developed by *Gal-Or* assumes that the two firms individually observe private signals as estimates for the uncertain parameter. The realized value of the distributed parameter, however, is unknown to both firms. It is drawn from the probability distribution *a posteriori* to the players' strategic decisions. In the context of industrial organization, this assumption is reasonable for the first-mover which has to explore the new market. The entry and exploration of the first-mover, however, might lead to a revelation of the market's uncertainties such that the second-mover might act under the premise of complete information.

Liu [31] and *Ferreira et al.* [16] investigated such a *revealing Stackelberg* competition under the assumption of an uniformly distributed uncertain market demand. Their works show that the complete revelation of the uncertainty by the first-mover's action generally still favors the first-mover (as does the case of complete information). There only is an expected second-mover advantage iff the market demand is sufficiently spread *and* the revealed demand (after the first-mover's action) is far from its expected value. On the first glance, this results seems surprising since having complete information represents an even stronger information advantage for the second-mover compared to "just" being able to infer the private signal of the first-mover from its output quantity decision. Still, this lies in line with the observation by *Gal-Or* [20] that the first-mover can higher his and reduce the second-mover's expected payoff by directly revealing his private instead of letting it be inferred by the second-mover. Following *Gal-Or*, the incentive of the first-mover to directly reveal his private information to the follower is explained by the fact that "the follower is as well informed as he would have been with direct revelation, but the leader loses his preemptive capability when indirect inferences arise" [20]. Analog, the preemptive capability of the first-mover is preserved when the second-mover is provided directly with complete information about the market demand.

4.3 Natural *Stackelberg* Situation

Following *Shinkai* [40], *Stackelberg* competition is appropriate to describe situations with *irreversible action* or *irreversible commitment*. They can, however, also occur when they are beneficial for both firms and if they can agree on a mutually advantageous distribution of roles, called a *Natural Stackelberg Situation* (NSS) [1]. In the literature, the existence of NSSs are commonly studied by following the approaches of *Dowrick* [13] or *Hamilton* and *Slutsky* [23], where games with a structure as shown in Figure 4.1 were considered. At first,

Player I \ Player II	Move first (F)	Move second(S)
Move first (F)	*Cournot* competition or *Stackelberg* warfare	*Stackelberg* competition
Move second (S)	*Stackelberg* competition	Delayed competition or *Cournot* competition

Figure 4.1 Structures of games of leadership commonly found in the literature: At first, both firms simultaneously choose between moving first (F) or moving second (S). Subsequently, the strategy pairs (F,S) or (S,F) result in sequential-move *Stackelberg* competition subgames. The choices (F,F) or (S,S) either lead to a simultaneous-move *Cournot* competition, *Stackelberg* warfare or delayed competition as subgames, respectively. If (F,S) or (S,F) is the only Subgame Perfect (*Bayes*ian) Equilibrium of the game, it is called a Natural *Stackelberg* Situation.

both firms simultaneously choose between the strategies of moving first (F) or moving second (S). If the firms mutually agree on a distribution of their roles, i.e., if they either choose the strategy pair (F,S) or (S,F), the firms subsequently participate in a sequential-move *Stackelberg* competition subgame where the strategies are actually implemented. If the firms do not mutually agree on their roles and if there is an *observable delay* [23] between the choosing of roles and the subsequent subgame, both firms either enter a simultaneous-move *Cournot* competition (strategy pair (F,F)) or delay their competition (strategy pair (S,S)). If the choosing of roles already induces an *action commitment*, i.e., if the firms' strategy implementations are done simultaneous to their choosing of roles, both firms either enter a *Stackelberg warfare* [13] (strategy pair (F,F)) or a *Cournot* competition (strategy pair (S,S)). Modelling the entry timing of two firms by such *games of leadership*, a mutually beneficial NSS corresponds to a situation where the strategy pair (F,S) or (S,F) is the only Subgame Perfect (*Bayes*ian) Equilibrium.

The existence of a NSSs strongly depends on the specific expected payoffs of the respective subgames. If the two firms I and II are identical the payoff structure of the game of leadership is symmetric, i.e., $\pi_1(u, v) = \pi_2(v, u)\ \forall u, v = \mathrm{F}, \mathrm{S}$. Thus, both firms will come to the same conclusion that being the first- or second-mover is beneficial. Hence, absolutely equal firms will naturally not agree on a sequential-move competition unless they are exogenously forced into it [13]. A minimum differentiation between the two firms must thus exist to yield a NSS.

As one of the first analyzing NSSs, *Boyer* and *Moreaux* [10] considered a game of leadership where the subgames were price competition under complete information with varying marginal costs (in this thesis denoted by c) of the two firms. In their work they “show that if costs are identical or similar, then both firms will prefer the role of follower; if there is a significant cost differential between the firms, then the non-cooperative equilibrium can only be of two types: either the less efficient firm will act as the leader, selling a limited quantity at a low price, and the more efficient firm as the follower, selling to the residual demand at a higher price, or the more efficient firm acting as leader will drive the less efficient firm out of the market by adopting a limit pricing strategy, but in so doing that firm makes less profits than if it acts as follower” [10]. An extension of the work by *Boyer* and *Moreaux* to more general inverse market demand and production cost functions was done by *Amir* and *Grillo* [3] showing that the beforehand conclusion can remain valid also under the presumption of more complex market models fulfilling *log-concavity* and *log-convexity* conditions.

Similar results were found by *Albæk* [1, 2] when he investigated quantity and price competition subgames with differentiable goods produced under uncertainty in their marginal costs. The assumed difference in the costs, however, is not on its expected value but only on the spread of its uncertainty, i.e., the variance of the random variable. The firms are thus identical when removing the assumption of incomplete information and replacing it by their completely informed antagonists. Still, this small distinction of the two firms can be sufficient to yield a NSS in quantity competition (but not price), where the firm with the greater cost variance is the first-mover [1]. It is important to note that the expected profits of *both* firms resulting from sequential-move *Stackelberg* competition under uncertainty are greater than the expected profits in the simultaneous-move *Cournot* competition under uncertainty.

Appelbaum and *Weber* studied the existence of NSSs in quantity competition in which there is an uncertainty in the inverse market demand function [6]. The employed market model is a generalization of Example 3. They assumed a linear cost structure and an abstract inverse market demand with a positive slope with respect to the random variable. In contrast to the previous works under uncertainties in the marginal cost, the analysis of their game of leadership shows that uncertainties in the market demand generally do *not* result in NSSs but only the simultaneous-move subgames are subgame PBEs. More precisely, *Appelbaum* and *Weber* showed that the strategy-pair (L,L) always is a subgame PBE, whereas (E,E) requires a small marginal costs to be another equilibrium strategy-pair. This result is in line with the finding by *Gal-Or* [20] (cf. Example 3) that the expected second-mover advantage in *Stackelberg* competition under uncertain demand is induced by the information advantage of the second-mover. Thus, this naturally leads to the waiting strategy (L) for both firms to avoid a sequential-move competition in which he expects to earns less than the second-mover.

NSSs arising from an uncertain market demand were also investigated by *Hirokawa* and *Sasaki* [25] under the assumption that the market demand is completely revealed after the first-mover (cf. Section 4.2). In addition, the subgame determined by the choices of both firms is a *multiperiod* competition which is repeated infinitely (with a discount on future payoffs). Under these assumptions *Hirokawa* and *Sasaki* showed that – in contrast to the results by *Appelbaum* and *Weber* [6] – NSSs can also arise in the case of an uncertain market demand.

The results concerned the existence of NSSs rely on the specific choices of the market model and the stochastic framework in a complex manner. Nevertheless, the works by the various authors show that "it is not unreasonable to expect that firms in a duopoly framework will tend to coordinate on their mutually advantageous role distribution even if they compete in a non-cooperative strategic way in terms of both prices and quantities" [10].

5 Summary

In this thesis, various forms of sequential-move competition were analyzed with respect to first- and second-mover advantages in the context of industrial organization.

As a first step, symmetric extensive two-player games with only two strategic options were investigated showing that they generally favor the first-mover independent of the players' payoff functions. This observation, however, is no longer valid in the case of larger discrete or continuous strategic spaces. Subsequently, the Theorem of *Gal-Or* [17] (cf. Theorem 2.7) characterizing first- and second-mover advantages under the premise of complete information by the slope of the reaction function of the second-mover was reviewed in detail. As a consequence of the theorem it is evident that under the assumption of complete information only the specific form of the firms' payoff functions determines the favorable market entry timing. Hence, in environments of complete information no payoff asymmetry due to firms' sequential movement is introduced. The analysis of the classic *Stackelberg* competition (cf. Example 1) and a sequentialized *Hotelling* competition (cf. Example 2) indicates that duopoly quantity competition generally favor the first-mover, while duopoly price competition yield a benefit for the second-mover.

In industrial organizations, however, complete information is barely available and managers usually have to act in environments of uncertainty. To respect this lack of complete information, it is much more reasonable to assume that model parameters are stochastically distributed, rather than being fixed-valued. Thus, as a second step, the stochastic framework developed by *Gal-Or* [20] was introduced and applied to *Stackelberg* competition under uncertain marked demand (cf. Example 3), *Stackelberg* competition uncertain marginal cost (cf. Example 4) and sequentialized *Hotelling* competition under uncertain marginal cost (cf. Example 5). In contrast to complete information, the study of different forms of competition under uncertainty indicates that the asymmetry due to the firms' sequential movement generally favors the second-mover independent the specific market structure (*Stackelberg* or *Hotelling*), the strategic variable (quantity or price) or the parameter which is uncertain (market demand or marginal cost).

The comparison of various forms of *Stackelberg* competition (Examples 1, 3 and 4) shows that incomplete information may enable the second-mover to earn higher expected profits compared to the first-mover (if the market segments are sufficiently uncorrelated) as opposed to the case of complete information. Similar, the investigation of sequentialized

Hotelling competition under complete information and uncertainty (Examples 2 and 5) shows that the second-mover can further increase his payoff advantage under incomplete information. In all shown examples under uncertainty, the second-mover benefits from the firms' asymmetric movement by the better information effect [20] which he gains from the inference of the first-mover's private signal from the first-mover's profit-maximizing strategic decision.

Accompanied by the information advantage of the second-mover comes the conjectural variation effect [20], which measures the increase of the slope of the reaction function of the second-mover under uncertainty compared to the competition under complete information. More precisely, the study of Examples 3 – 5 shows that the slope of the reaction functions of the second-mover under uncertainty $\partial_{s_1} G(x_2, s_1^{\mathrm{L,un}})$ differs to the slope of the respective reaction functions under complete information $\partial_{s_1} g(s_1^{\mathrm{L}})$ only by some positive, fixed-valued constant $K(u)$ which only depends on properties of the random variable u, i.e.,

$$\partial_{s_1} G(x_2, s_1^{\mathrm{L,un}}) = \partial_{s_1} g(s_1^{\mathrm{L}}) + K(u). \tag{5.1}$$

Together, the better information effect and the conjectural variation effect lead to the conclusion that the statement of the Theorem of *Gal-Or* remains valid in all of the presented examples under uncertainty. Thus, in all examples, strategic substitutes imply a first-mover advantage, whereas strategic complements induce a second-mover advantage. This is not straight-forward to see from the Theorem of *Gal-Or* since one of its major requirements is violated by the non-parity of the *de facto* payoff functions $\mathrm{E}\left[\pi_1(s_1, s_2, u)|x_1\right]$ and $\mathrm{E}\left[\pi_2(s_1, s_2, u)|s_1, x_2\right]$ of the firms. If the Theorem of *Gal-Or* remains valid also for more general non-deterministic payoff functions or if the observation only is a consequence of the simple payoff functions that were considered in the examples cannot be answered within the scope of this thesis. It seems reasonable, however, that the concept of strategic substitutes and complements remains key to address the question of first- and second-mover advantages also under the precondition of incomplete information.

As final step, important further literature addressing the question of first- and second-mover advantages was reviewed. It is remarkable that in the literature many results on first- and second-mover advantages under uncertainty are based on the study of *Stackelberg* competition under uncertain market demand [6, 11, 15, 16, 18, 20, 22, 24–26, 31, 33, 34, 37, 40, 45]. Only few publications consider an uncertain marginal cost [1, 2, 7, 19, 21, 32]. Even less studied are sequential-move forms of price competition under uncertainty [1, 2], such as the developed sequentialized *Hotelling* competition (cf. Example 5). Moreover, no appropriate and applicable theoretical foundation yet exists for competitions under incomplete information, which is even close to the relevance of the Theorem of *Gal-Or* for environments of complete information.

Important extensions of work by *Gal-Or* are by *Shinkai* [40] and *Cumbul* [11] who studied tripoly and oligopoly *Stackelberg* competition under uncertain market demand. Their work shows that if the information advantage exceeds the strategic advantage of the first-mover (such as in *Stackelberg* competition with sufficiently uncorrelated market segments or sequentialized *Hotelling* competition), then it is beneficial to a firm that *all* information is revealed to it. This observation thus generally results in a last-mover advantage. Furthermore it follows that if a firm is not able to be the very last-mover, e.g. if new firms always enter the market, then it is economically favorable for the firm to be the first- or early-mover instead of being an intermediate- or late-mover. Similar, the works by *Liu* [31] and *Ferreira et al.* [16] show that it is generally more beneficial for a firm to be the first-mover if the uncertainty is completely revealed by its action. As in the case of complete information, the first-mover's preemptive capability is preserved in a revealing competiton such that his strategic advantage usually prevails the information advantage of the second-mover.

Much of the related literature and the examples that were studied in this thesis assumed a sequential-move structure of the competition that was exogenously given. As investigated by many authors [1–3, 6, 10, 13, 23, 25, 40], however, this exogenously given structure does not have to represent a restriction to the applicability of the results. Sequential-move forms of competition can occur not only due to differences in the implementation speeds of firms' market entry, but they can also be a natural consequence of the (not perfectly identical) firms' strive for maximizing their individual profits.

The initial question of whether it is economically advisable to be the first- or second-moving firm in a new market is often intuitively answered by the "long-standing hypothesis that firms that enter a market early tend to have higher performance than their followers" [42]. Many findings of this thesis, however, support the view by *Kerin et al.* that "the belief that entry order automatically endows first movers with immutable competitive advantages and later entrants with overwhelming disadvantages is naîve in light of conceptual and empirical evidence" [28]. "Thus, pioneering may prove advantageous to some firms in some circumstances, but it is not necessarily a superior strategy for all entrants" [30]. Summarized, the answer to this question strongly depends on the specific market conditions.

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Moritz Thon