Managing uncertainty in design flood magnitude: Flexible protection strategies vs. safety factors

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Abstract

When planning flood protection, agencies are confronted with uncertainty in the design flood magnitude. In particular, the required capacity may increase in the future and render the protection insufficient. This problem can be adressed by applying a safety factor to the design capacity. We propose a Bayesian quantitative sequential decision model that identifies a cost-optimal safety factor in the face of uncertainty. It takes into account the flexibility of the protection system, i.e. how costly it is to adjust. We focus on the description of the decision model and on the concept of flexibility, investigating only the effect of uncertainty from the historic flood record. Extension to other types of uncertainty is possible. The model is implemented for a catchment in Germany. Various degrees of uncertainty are investigated by using different lengths of historic records. The optimal safety factor decreases with decreasing uncertainty and with increasing system flexibility.

Key words

Bayesian decision analysis, decision support under uncertainty, flexible protection strategies, flood protection, safety factor, natural hazards, uncertainty quantification

1 Introduction

Building flood protection systems against design flood events with a prescribed return period is a common practice in many countries. For example, protection against a 100-year flood is required for most built areas in Germany and several other European countries (Deutsche Vereinigung für Wasserwirtschaft Abwasser und Abfall e.V., 2011; Central European Flood Risk Assessment and Management, 2013); delta regions in the Netherlands have to be protected against up to a 10.000-year flood (Kabat *et al.*, 2005; Bischiniotis *et al.*, 2016). Estimates of the magnitude of a design flood are, however, subject to uncertainty due to limited historic data, uncertainty in projections of climate change impacts or uncertainty in socioeconomic development (Apel *et al.*, 2004; Hall and Solomatine, 2008; Schanze, 2012), as well as complexities, biases and unknowns (Merz *et al.*, 2015). Multiple approaches exist to deal with uncertainties in flood risk estimates (Hallegatte, 2009; Kwakkel *et al.*, 2010; Samuels *et al.*, 2010). These include the application of safety factors to the initial design or the construction of flexible flood protection systems. The safety factor is the ratio between the applied capacity of the flood protection system and the capacity that would be selected without taking into account the uncertainties. Such safety factors are used in practice; for example, in Bavaria, a safety factor of 1.15 – i.e. 15% spare capacity – is applied to the 100-year design flood discharge when planning novel mitigation measures to account for climate change uncertainty (Pohl, 2013; Wiedemann and Slowacek, 2013). Such factors are typically implemented based on rule-of-thumb estimates rather than a rigorous quantitative analysis (KLIWA, 2005, 2006). In particular, they do not consider the flexibility of the system.

Flexible flood protection systems are systems that can be modified without excessive cost in the future, when more information is available or when the demands on the system are altered because of anthropogenic or climate change (Vrijling *et al.*, 2007). Flexible protection systems and systems can be a good solution when uncertainty is high since they allow for shorter planning horizons through sequential planning (U.S. Climate Change Science Program, 2009; Löwe *et al.*, 2017). Figure 1 shows an example of a flood wall that is flexible: a larger foundation than necessary is built initially, at extra cost, to allow for a future extension if needed. An alternative example is the reservation of land for future construction of a retention basin that would further increase the protection of the settlement.

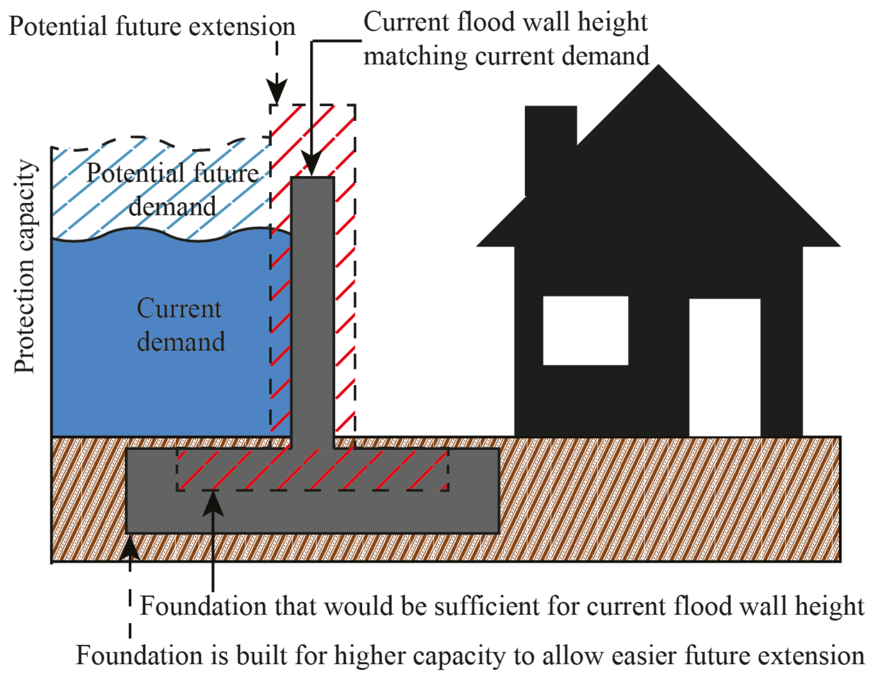

Figure 1. An example of a flexible flood protection measure - a flood wall. Adapted from (Vrijling *et al.*, 2007).

The challenge is in determining the optimal design, i.e. the safety factor, taking into account the flexibility of the flood protection system. It is not economical to apply the same safety factor to flexible systems as to inflexible ones since the former can be adapted in the future without excessive cost. Hence it is necessary to find a relationship between the optimal safety factor and system flexibility. Additionally, one may wish to quantify the value of flexibility, since it typically comes at an additional cost. When the choice is among systems with varying flexibility, questions arise such as: Is it worth investing in a flexible system, which is potentially more expensive initially? Or is it more cost-effective to select an inflexible system and apply a high safety factor (a conservative design) that would be satisfactory under multiple future scenarios?

Classical cost-benefit analysis (Griffin, 1998; Hine and Hall, 2010; Špačková and Straub, 2015) does not take into account the flexibility of the system. It does not address the fact that additional information will be gathered in the future, which may motivate system adjustments. Optimization of flexible flood management under uncertainty in a sequential manner has e.g. been considered in (Woodward *et al.*, 2011; Harvey, Hall and Peppé, 2012; Hino and Hall, 2017). In these studies, uncertainty on the future is expressed by means of selected discrete scenarios with associated probabilities; the optimization is performed among a discrete set of alternative decisions. In contrast, in our approach we represent the uncertainty by continuous random variables, explicilty include the future learning process by means of Bayesian analysis, and consider a continuous space of decision alternatives.

We propose a fully quantitative sequential probabilistic decision model for optimizing flood protection capacity, which considers the flexibility of the flood protection systems. It takes basis in Bayesian decision theory (Raiffa and Schlaifer, 1961; Benjamin and Cornell, 1970; Davis *et al.*, 1972). The theory allows modelling decisions sequentially, thereby accounting for future information (e.g. discharge measurements). The initial capacity is optimized considering possible future adjustments of the capacity at regular intervals. The theory provides a natural way to 1) model decisions using the full spectrum of uncertainty rather than individual scenarios and 2) model a sequential process in which new information that becomes available only in the future can be accounted for. The theory has been used to optimize adaptation to climate change (Hobbs, 1997; Garrè and Friis-Hansen, 2013; Simpson *et al.*, 2016), yet never in the form of a comprehensive study on flood management. Our framework allows one to both quantify the value of flexibility (Špačková and Straub, 2017) – the savings that can be incurred by a more flexible measure – as well as the value of information (Winkler *et al.*, 1983) – i.e. the savings that can be incurred by reducing parameter uncertainty.

For demonstration purposes, only the uncertainty in flood predictions due to limited discharge records is considered in this paper. Such uncertainty is known as statistical uncertainty (Wood and Rodriguez-Iturbe, 1975), but in the following we use the more general term *parameter uncertainty* instead (Kennedy and O'Hagan, 2001; Der Kiureghian and Ditlevsen, 2009). This should emphasize that the framework is applicable more generally to uncertainty in parameters of flood models, for example the parameter space could be extended to include a trend parameter stemming from climate change uncertainty. The Bayesian approach makes it possible to easily include other sources of information, such as climate projections and hydrological conditions (Viglione *et al.*, 2013).

We propose a methodology in Section 2 for identifying the optimal flood protection strategy as a function of the parameter uncertainty, the flexibility of a flood protection system, its cost and discounting functions. The case study in Section 3 allows one to numerically investigate the effect of these factors on the optimal flood protection. Finally, we discuss to what extent these results can be generalized to make overall recommendations on flood protection under parameter uncertainty in Section .

2 Methodology

We begin the methodology by providing background information on uncertainty in design flood estimation and the resulting setup of the proposed framework in Section 2.1. In Section 2.2, models for flexibility and cost of protection systems are presented. Learning and updating of the annual maximum discharge model are described in Section 2.3. Finally, optimization to find the cost-optimal decision is outlined in Section 2.4.

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aiı se ntial decisi ı king in flood ection ning is to determine the ıtimal p cti apacity to ı lemented at p nt, con ring potential future adjus nents ht ev servations c a ges of the syst Decisic n flood protection are con nonly l c T- design floc i rion. This crit ı corres ds to requiring protection gainst od nt ı annual ex ı nce probabilit ual to $1/T$. The flood magnitude is n this c ct d by the di r e, whose annu axima follow a probability distributi ı with ne $\mathbf{\Theta}$ corresspon T-year event is $q^{(T)}$, and it holds

$$1 - F_{Q|\mathbf{\Theta}}(\quad |\boldsymbol{\theta}) = \frac{}{T} \quad \leftrightarrow \quad q^{(T)} = F^{-1}_{Q|\mathbf{\Theta}}\left(1 - \frac{}{T}\middle|\boldsymbol{\theta}\right), \tag{1}$$

e $F_{Q|\mathbf{\Theta}}$ is CDF and $F^{-1}_{Q|\mathbf{\Theta}}$ is the inverse CDF of the annual maximum discharge Q. The n flood t is therefore a function of the discharge distribution parameters, it is written as $(\boldsymbol{\theta})$.

distributi arameters $\mathbf{\Theta}$ are subject to parameter uncertainty, which in a Bayesian analysis ework is ribed by a probability distribution over $\mathbf{\Theta}$. The effect of this uncertainty on the ibution c is illustrated in Figure 2. In most practical applications, this uncertainty is ected, and oint estimate $\hat{\boldsymbol{\theta}}$ is applied, such as the Maximum Likelihood Estimator (MLE) e Maxim A-Posteriori (MAP), which is identical to the MLE in the case of a uniform prior $\mathbf{\Theta}$ (Papp rger and Beven, 2006). To include uncertainty, instead the predictive distribution ıe annual imum discharge Q – the distribution of future Q based on past observations – ld be eva ed (Hall *et al.*, 2011).

re 3 high ts the relation between the inherent variability of the extreme discharges, as ribed by $F_{Q|\mathbf{\Theta}}$, and the uncertainty in the parameters $\mathbf{\Theta}$ reflected by the credible intervals on $F_{Q|\mathbf{\Theta}}$. The latter can be reduced by collecting additional data or improving models, but the former will persist.

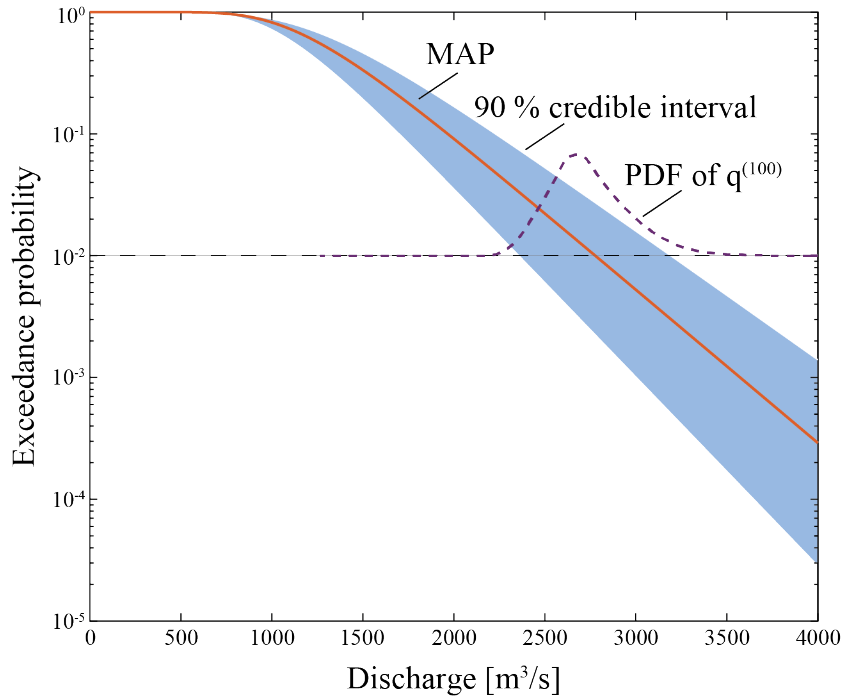

Figure 2. Exceedance probability and associated 90% credible interval (shaded) as a function of discharge. The dashed PDF represents the uncertainty on the 100-year discharge prediction. The figure is based on a 31-year data record from the case study (viz. Section 3) under the assumption of a Gumbel distribution.

In the case where the flood model is learned purely based on past discharge records, the parameter uncertainty is associated with the length of the records, as apparent from Figure 3. The credible interval width decreases as the available data series becomes longer.

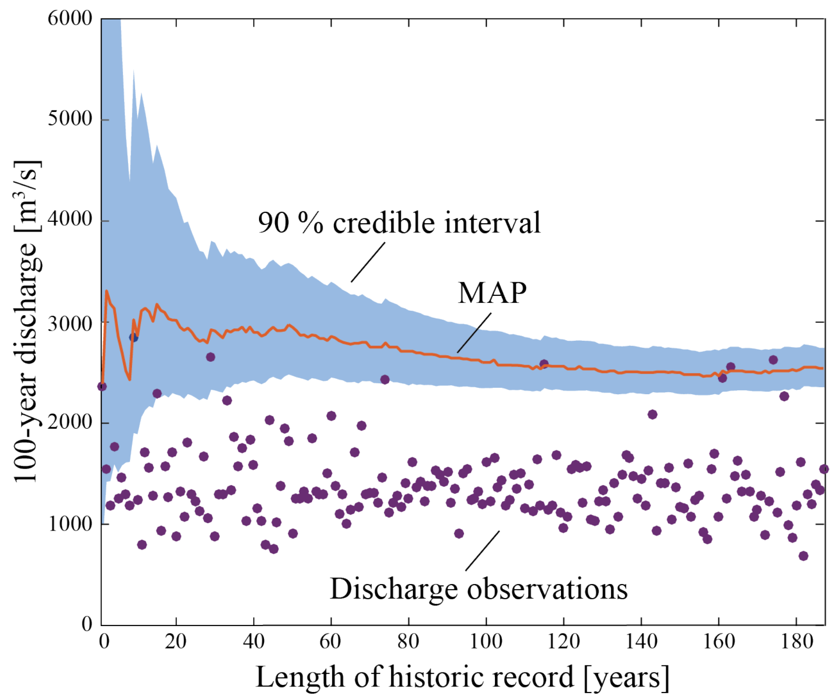

Figure 3. 100-year design discharge estimate (MAP) and associated 90% credible interval as a function of the historic data record length. Annual maximum discharge data (shown as dots) are for a gauge in Wasserburg am Inn, Germany, for the period from 1828 to 2013. The 100-year discharge is estimated under the assumption of a stationary Gumbel distribution, neglecting measurement uncertainty (viz. Section 3).

Before we proceed to quantitatively outline the methodology, we summarize four cornerstones of the proposed framework:

(1) Multiple future decision steps are considered, at which the protection capacity can be revised. Each future decision includes an optimization of the protection capacity.

(2) The flexibility of the protection systems is included as an explicit parameter of the decision problem.

(3) The potential future observations are modelled stochastically. They consist of records of annual maximum discharges.

(4) The annual maximum discharge is described by a probability distribution with parameters $\boldsymbol{\Theta}$.

The timeli ing p n the ca protection $v_1, \ldots, v_{N-1}$ are c Δt. Each s $\ldots, N$ consists of Δt years, during wh Δt discharge nade. These form the observation v or $\mathbf{q}_i$ $q_{(i-1)\times\Delta t+2}, \cdots, q_{i\times\Delta t}]$. At time $t = 0$ rs, a decision on the initial capacity v_0 is ma based on historic discharge records $\mathbf{q}_H$. The observations $\mathbf{q}_N$ in last step of the pla n N have no influence on planning, as the lifetime of the protection ends at time $t = N \times \Delta t$ and no further decision is considered. This seems like a simplification of the problem, as infrastructure is rarely discarded at the end of the planned service life. One may think of this lifetime as the timespan of analysis. It can be more precise to model the flood protection with an infinite lifetime, considering partial or full replacements at future time steps. However, in practice there is little difference among the results of these two model approaches, because costs that are incurred far ahead in the future have only a small impact on the total present value lifetime cost and risk, due to discounting (Rackwitz, 2004). They therefore do not affect the initial decisions, which are the focus of this analysis.

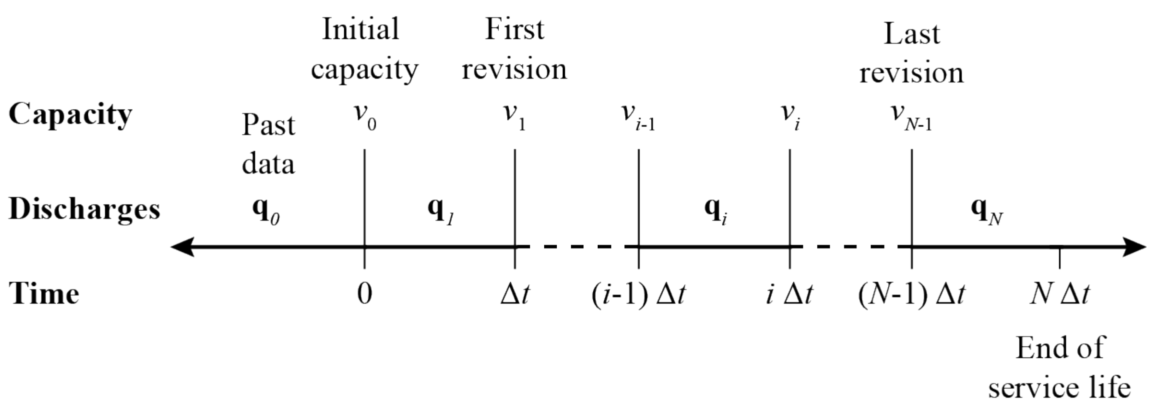

Figure 4. Timeline of decisions on capacity v and observations of discharges $\mathbf{q}$ in an optimization process of N steps.

The key parameters of the proposed framework are summarized in Table 1.

Table 1. Overview of parameters of the decision model

Id.	Parameter/variable	Type	Value domain	Units
Q	Annual max. discharge	Random	$[0, \infty)$	m^3/s
$\mathbf{\Theta}$	Parameters of prob. model of Q	Random	$[0, \infty)$	-
$\mathbf{q}_i$	Future observations of Q	Random	$[0, \infty)$	m^3/s
v_i	Capacity of flood protection system	Decision	$[1, \infty)$	m^3/s
v_{req}	Required protection capacity	Random	$[1, \infty)$	m^3/s
γ_i	Safety factor	Decision	$[1, \infty)$	-
c_i	Protection cost	Deterministic	$[0, \infty)$	€
φ	Flexibility of flood protection system	Decision	$(-\infty, 1]$	-

2.2 Cost and flexibility of flood protection systems

The flexibility φ of flood protection systems is defined in terms of their cost. Following (Špačková and Straub, 2017), it is:

$$\varphi = \frac{c_0(v_i) - c_i(v_{i-1}, v_i)}{c_0(v_{i-1})}, \tag{2}$$

where $c_0(v)$ is the cost of establishing the system to capacity v initially, i.e. when the system is first implemented, and $c_i(v_{i-1}, v_i)$ is the undiscounted cost of adjusting the protection capacity at a future time from v_{i-1} to v_i. $\varphi = 0$ corresponds to an inflexible system and $\varphi = 1$ to a fully flexible system. This definition of flexibility is illustrated in Figure 5.

For the inflexible system ($\varphi = 0$), the cost of increasing the capacity to v_i is identical to an entirely new system with this capacity. For fully flexible systems ($\varphi = 1$), the cost of increasing the protection to v_i equals the difference in cost between building to v_{i-1} initially and building to v_i initially, i.e. no additional costs are incurred by building in two (or more) steps.

For optimization purposes, it is useful to express the adjustment cost as a function of flexibility. Based on Eq. (2), the costs of adjusting the capacity from v_{i-1} to v_i equals

$$c_i(\varphi, v_{i-1}, v_i) = c_0(v_i) - \varphi \times c_0(v_{i-1}). \tag{3}$$

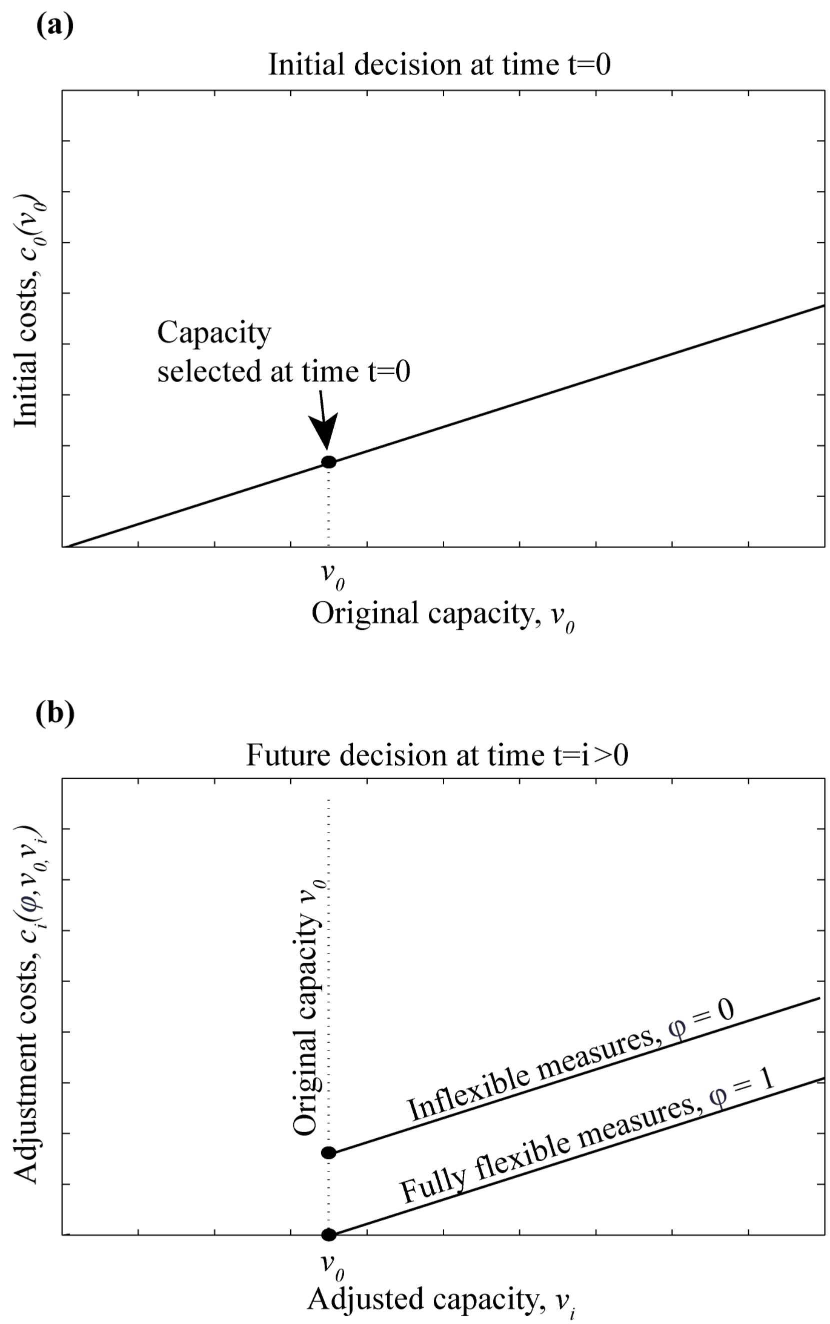

Figure 5. Illustration of the costs for fully flexible vs. inflexible systems (Špačková and Straub, 2017): **(a)** Initial cost $c_0$when the system is first implemented. **(b)** Cost c_i for adjustment of the system to a higher capacity v_i. For the inflexible systems, the adjustment costs are equal to the costs c_0 incurred when implementing directly capacity v_i. For the fully flexible system ($\varphi = 1$), the adjustment costs are equal to the difference between the initial costs of the unadjusted and the adjusted system.

2.3 yesian analysis o extreme disc arge and determi ation of required protection capa

This on summarizes e Bayesian a alysis of the ann al maximum discharge Q. The parar $\boldsymbol{\Theta}$ of the probabi ty distributio of Q are learned fr n the data.

Initia nnual maximum lischarge dat $\mathbf{q}_H = [q_1, \dots, q_M]$ f om M years are available. The poste oint PDF of the p ameters $\boldsymbol{\theta}$ is etermined from thes data as:

$$f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\mathbf{q}_H) \;\; L(\boldsymbol{\theta}|\mathbf{q}_H) \times f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}), \tag{4}$$

wher $\boldsymbol{\theta}$) is the prior stribution of e parameters and $L(\boldsymbol{\theta}|\mathbf{q}_H)$ is the likelihood of the parar given the obse ations.The di charge maxima can be assumed to be independent betw dividual years (C les, 2004). N glecting measurement error, the likelihood function in Eq. (ormulated as

$$L(\boldsymbol{\theta}|\mathbf{q}_H) \;\; \prod_{j=1}^{M} f_{Q|\boldsymbol{\Theta}}(q_j|\boldsymbol{\theta}), \tag{5}$$

with f_{Q} eing the PDF of Q for given par meters $\boldsymbol{\theta}$.

After t itial design and implementation f flood protection, one continues to observe discharge maxim s sketched in Figure 4, each pla ning step i corresponds to a time interval with length Δt wit w observations $\mathbf{q}_i = [q_{(i-1)\times\Delta t+}, \dots, q_{i\times\Delta t}]$. These data are included in the Bayesian analys sequentially updating the esti ate of $\boldsymbol{\Theta}$. The posterior joint PDF of $\boldsymbol{\Theta}$ at time $t = i \times \Delta t$

$$f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i) \propto L \;\; \boldsymbol{\theta}|\mathbf{q}_i) \times f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\boldsymbol{q}_H, \boldsymbol{q}_1, \dots, \boldsymbol{q}_{i-1}), \tag{6}$$

where $f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\boldsymbol{q}_H, \boldsymbol{q}_1, \dots, \boldsymbol{q}_{i-1})$ is the joint F DF of $\boldsymbol{\Theta}$ obtained in the previous step of the sequential updating at time $t = (i-1) \times \Delta t$ and $L(\boldsymbol{\theta}|\mathbf{q}_i)$ is the likelihood function describing the observations $\mathbf{q}_i$, which is defined analogously to Eq. (5).

The conditional probability of observing discharges $\mathbf{q}_{i+1}$ in time step $i+1$ for given history of previous discharges $\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i$ is the predictive distribution

$$f_Q(\mathbf{q}_{i+1}|\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i) = \int_{\boldsymbol{\Theta}} f_{Q|\boldsymbol{\Theta}}(\mathbf{q}_{i+1}|\boldsymbol{\theta}) \times f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i)\, \mathrm{d}\boldsymbol{\theta}, \tag{7}$$

where we use the convention $\int_{\boldsymbol{\Theta}} \mathrm{d}\boldsymbol{\theta} = \int_{\Theta_1} \dots \int_{\Theta_n} \mathrm{d}\theta_1 \dots \mathrm{d}\theta_n$.

The required p otection acity at time $t = i \times \Delta t$ is found MAP estimate $\widehat{\boldsymbol{\theta}}_i$ defined as

$$\widehat{\boldsymbol{\theta}}_i = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\mathbf{q}_H, \mathbf{q}_1, .. \tag{8}$$

The minimal r quired p ction capacity at time step i is v_{req} esponding to the T-year flood event det rmined parameters $\widehat{\boldsymbol{\theta}}_i$. Following Eq. (1), i

$$\left(\widehat{\boldsymbol{\theta}}_i\right) = q^{(T)}\left(\widehat{\boldsymbol{\theta}}_i\right) = F_{Q|\boldsymbol{\theta}}^{-1}\left(1 - \frac{}{T}\middle|\widehat{\boldsymbol{\theta}}_i\right) \tag{9}$$

where the inve se CDF $F_{Q|\boldsymbol{\theta}}^{-1}$ is evaluated with parameters $\widehat{\boldsymbol{\theta}}_i$.

It follows fr m Eqs. (8)-(9) that the required capacity $v_{req}\left(\widehat{\boldsymbol{\theta}}_i\right)$ is a function of the data $\mathbf{q}_H, \mathbf{q}_1, \dots,$ $_i$:

$$v_{re}\ (\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i) = F_{Q|\boldsymbol{\theta}}^{-1}\left(1 - \frac{}{T}\middle|\underset{\boldsymbol{\theta}}{\operatorname{argmax}}(f_{\boldsymbol{\Theta}|Q}(\boldsymbol{\theta}|\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i))\right), \tag{10}$$

The safety fact r is defined in this context as the ratio between the implemented capacity and the required protec ion capacity:

$$\gamma = \frac{}{v_{req}}. \tag{11}$$

For example, $\gamma = 1.1$ corresponds to including a 10% reserve above the minimum required protection capacity.

2.4 Optimization of the protection capacity

The goal is to find a cost-optimal strategy ensuring adequate protection capacity over the lifetime of the system, taking into account the flexibility of the system. Protection must be increased whenever the actual capacity is smaller than the required protection capacity (T-year discharge). The minimum capacity is v_{req}, but it can be increased beyond the required protection capacity if this is expected to be cost effective over the remaining lifetime of the protection. The total lifetime cost of the flood protection system, in function of the N decisions on protection capacity $v_0, \dots, v_{N-1}$, is:

$$c_{tot}(\varphi, v_0, \dots, v_{N-1}) = c_0(\varphi, v_0) + \sum_{i=1}^{N-1} d_i \times c_i(\varphi, v_{i-1}, v_i), \tag{12}$$

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$_0$ and $c_i(\varphi, v_{i-1}, v_i\ )$ are the
cting the societal preference for

$$(13)$$

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, $v_{req}(\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i)$, exceeds the existing protection
increased to

$$_q(\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i), \quad (15)$$

t every modification. The protection for the final step $v_{req}(\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_{N-1}) > v_{N-2}$, then protection is increased to $v_{N-1} = v_{req}(\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_{N-1})$ only, since there is no benefit in incorporating a reserve at the end of the lifetime of the protection.

With this heuristic, the decisions at all time steps are fully determined for given safety factor γ and discharge data $\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i$. To summarize,

$$v_i(\gamma, \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i) = \begin{cases} \quad > v_{i-1} \text{and } i \in [1, \dots, N-2] \\ v_{req}(\mathbf{q}_H, \mathbf{q}_1, \dots \quad \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i) > v_{i-1} \text{and } i = N-1 \\ v_{i-1}, \quad \text{wise} \end{cases} \tag{16}$$

The only decision to be made is the ce the capacities to be built are determined according to Eq. (16). The randomne the discharges $\mathbf{q}_1, \dots, \mathbf{q}_{N-1}$. Once the safety factor γ is fixed, the decision tree th an event tree. The corresponding expected total lifetime cost is determined as

$$\begin{aligned} \mathrm{E}[c_{tot} \mid \varphi, \gamma] = \; & \quad f_Q(\mathbf{q}_1, \dots, \mathbf{q}_i \mid \mathbf{q}_H) \times \\ & c_i\big(\varphi, v_{i-1}(\gamma, \mathbf{q}_H, \mathbf{q}_1, \quad , \mathbf{q}_1, \dots, \mathbf{q}_i)\big)\big] \mathrm{d}\mathbf{q}_1 \dots \mathrm{d}\mathbf{q}_i, \end{aligned} \tag{17}$$

Where $c_0(.)$ and $c_i(.)$ are initial and (see Section 2.2), d_i is the discounting factor of Eq. (13), $f_Q(\mathbf{q}_1, \dots, \mathbf{q}_i \mid \mathbf{q}_H)$ is the observations $\mathbf{q}_1, \dots, \mathbf{q}_i$ given the past annual maximum discharge data $\mathbf{q}_H$ and $v_i(\gamma, \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i)$ is the selected capacity following Eq. (16). Evaluation of Eq. (17) by Monte Carlo sampling is straightforward.

The optimal safety factor γ^{opt} for a given flexibility φ is the one minimizing the expected lifetime cost:

$$\gamma^{opt}(\varphi) = \underset{\gamma}{\operatorname{argmin}} \, \mathrm{E}[c_{tot} \mid \varphi, \gamma]. \tag{18}$$

2.4.2 Backwards induction optimization

Backwards induction optimizes safety factors individually for each decision step, finding the optimal solution recursively from the last decision to the initial one. It explores the full solution space and leads to more cost-effective solutions than the heuristic approach. It does, however, come at a significantly higher computational cost. The decision process is vizualised in Figure 6, with three exemplary discharges / capacities highlighted amongst a continuous spectrum.

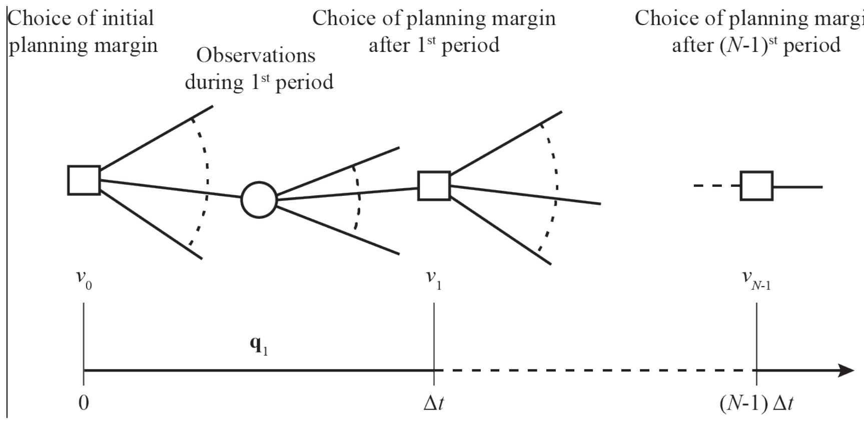


Figure 6. Decision tre ds induction optimization. Squares represent protection decisions an observations of annual maximum discharge.

The optimal safety fact or capacity v_i after the i^{th} step is the one minimizing the expected lifetime cost, previous decisions and data:

$$\gamma_i^{opt}(\varphi, v_0, .., v_{i-1} \quad \underset{\gamma_i}{\operatorname{argmin}} \mathrm{E}[c_{tot}\,|\varphi, \gamma_i, v_0, .., v_{i-1}, \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i]. \quad (19)$$

When optimizing the the one on capacity v_{N-1}, the $v_0, .., v_{N-2}$ and $\mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_{N-1}$ are kno e cost is known deterministically:

$$\mathrm{E}[c_{tot}\,|\varphi, v_0 \quad {}_H, \mathbf{q}_1, \dots, \mathbf{q}_{N-1}] = c_{tot}(\varphi, v_0\,, \dots, v_{N-1}), \quad (20)$$

with $c_{tot}(\varphi, v_0\,, \dots, v_{N-}$ q. (12).

The expected lifetime c on capacity v_i at time $i\Delta t$ is calculated recursively from the expected lifetime c Δt:

$$\begin{aligned} &\mathrm{E}[c_{tot}\,|\varphi, \gamma_i, v_0, .., v_{i-} \\ &\quad = \int f_Q(\quad {}_i) \\ &\quad \times \mathrm{E}\big[c_{to} \quad {}_H), .., v_i(\gamma_i, \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i), \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_{i+1}\big]\,\mathrm{d}\mathbf{q}_{i+1}, \end{aligned} \quad (21)$$

where $f_Q(\mathbf{q}_{i+1}|\mathbf{q}_H, \mathbf{q}_1,$ litional probability of making observations $\mathbf{q}_{i+1}$ given observations $\mathbf{q}_H, \mathbf{q}_1, \dots,$ e previously (viz. Eq. (7)) and $\mathrm{E}\big[\mathrm{c_{tot}}\,|\varphi, \gamma_{i+1}^{opt}, v_0, .., v_i(\gamma_i, \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_i), \mathbf{q}_H, \mathbf{q}_1, \dots, \mathbf{q}_{i+1}\big]$ is the expected lifetime cost known from the previous step of the recursive relation evaluated at γ_{i+1}^{opt}, i.e. assuming that the optimal safet elected future. In analogy to Eq. (15), $v_i(\gamma_i, \mathbf{q}_H, \mathbf{q}_1, \ldots, \mathbf{q}_i) = \gamma_i \times v_{req}(\mathbf{q}_H, \mathbf{q}_1, \ldots, \mathbf{q}_i)$.

Finally, the optim l initial safe s

$$\gamma_0^{opt}(\quad \underset{\gamma_0}{\operatorname{argmin}} \mathrm{E}[\mathrm{c_{tot}} \,|\varphi, \gamma_0, \mathbf{q}_H]. \tag{22}$$

The expected tota lifetime cos ated as

$$\mathrm{E}[\mathrm{c_{tot}} \,|\varphi, \gamma_0, \mathbf{q}_H] = \int \quad) \times \mathrm{E}\left[\mathrm{c_{tot}} \,|\varphi, \gamma_1^{opt}, v_0(\gamma_0, \mathbf{q}_H), \mathbf{q}_H, \mathbf{q}_1\right] \mathrm{d}\, \mathbf{q}_1, \tag{23}$$

where $f_Q(\mathbf{q}_1|\mathbf{q}_H)$ is the cond obability of making observation $\mathbf{q}_1$ given the historic data $\mathbf{q}_H$ and $\mathrm{E}\left[\mathrm{c_{tot}} \,|\varphi, \gamma_1^{opt}, v_0(\quad _H, \mathbf{q}_1\right]$ is the expected lifetime cost known from the previous step of tl e recursive r aluated at γ_1^{opt} using Monte Carlo sampling. In analogy to Eq. (14), $v_0(\gamma_0, \mathbf{q}_H) = \gamma_0 \times v_{req}(\mathbf{q}_H)$.

Because of the Markovian nature of the uncertainty, the decision problem corresponds to the class of partially observable Markovian decision processes (POMDP). The full solution could therefore alternatively be found by application of POMDP algorithms for discrete decision problems, as suggested in (Špačková and Straub, 2017). However, the continuous domains of random variables and optimization parameters would need to be discretized, leading to a computationally costly POMDP solution. The proposed tailored solution strategy is computationally efficient, because it exploits the specifics of the investigated problem. The computational implementation is given in the appendix.

3 Case study

Application of the proposed model is demonstrated by a case study based on gauge data obtained at the river Inn, in the town of Wasserburg am Inn, Germany. This case study is purposley kept simple to better investigate and demonstrate the workings of the proposed methodoloy and to enable the reader to reproduce the analysis. Extension to more complex applications is discussed in Section 4.

3.1 Catchment characteristics and assumptions

At gauge Wasserburg am Inn, discharge data have been recorded for 186 years; these are reported in the Annex. To highlight the effect of parameter uncertainty, we compare results obtained using different lengths of the discharge records: 31, 62, 93, 124, 155 and the complete 186 years. In the first case, only the most recent 31 years of the annual maxima are used to mimic an application to a ca hment where few data are available an paramet ncertainty is large. I the l case, th full ata set is used to epresent a catchment here lo ecords are available nd th paramet unc tainty is thus sig ificantly lower. The mple m and standard devia on of he annu max na for different l ngths of the discharge ecords summarized in Table 2. Fo implicit we r glect the uncertai ty arising from meas ement e s, which are likely t be lar for old reco ds. Furthermore, t e small observed line trend o 58 $\frac{\mathrm{m}^3}{\mathrm{s}}$/year is disreg rded, e. the da are ssumed to be stat nary. These choices ere ma o facilitate interpret ion o he resul and enhance the repr ducibility of the stud includi e measurement unce tainty nd a tren does not fundamentally change the implemen tion no results, as discussed later.

We st the model pla ibility (MacKay, 199 when g the annual discha e ma ma with Gun el, Fréchet, Wei ull and a GEV distri tion. T Gumbel distribution fou to be th mos plausible one ove he entire historic rec d and is applied in the follow ng. It describe by l cation parameter μ and scale parameter β: $\mathbf{\theta} = (\mu, \beta)$. The joint PDF of μ and β are foun foll ving Eq. (4). A diffuse uniform distribution is used as the prior. The magnitude of the 10 year discharge is computed using Eq. (9). Table 2 summarizes Maximum A-posteriori Estimate (MA) and standard deviations for the parameters of the Gumbel distribution as well as th esti ate of the 100-year discharge. The sample standard deviation of the annual maximu disc arge represents the inherent randomness of discharge maxima; the standard deviations of μ and σ reflect the parameter uncertainty caused by the limited length of available discharge records and is therefore reduced with increasing data size. The corresponding 90% credible interval for the 100-year-discharge associated with different record length are shown in Figure 3.

Table 2. Case study data propert

Data			Fitted probabilistic model					
Discharge series length [years]	Annual maximum discharge [m³/s]		ɔarameter β [m³/s] Gumbel scale		Gumbel locɛ ɔn parameter μ [m³/s]		100-year design discharge [m³/s]	
	Mean	*Std.-dev.*	*MAP*	*Std.-dev.*	*MAP*	*Std.-dev.*	*From Eq.* (9)	*Std.-dev.*
31	1471	502	352	147	1255	138	2876	808
62	1449	445	343	96	1250	114	2820	555
93	1424	399	302	70	1248	96	2641	419
124	1416	375	282	58	1252	79	2550	344
155	1398	355	268	50	1244	68	2477	296
186	1398	379	284	49	1233	66	2539	291

[a]Annual maximum discharge at tl Vasserburg am Inn gauge, Bavaria, Germany: sample statistics (heading: Data) and parameters (itted Gumbel model (heading: Fitted probabilistic model) for different lengths of the historic d ıarge records.

We consider the designed flood ıtection systems to have a lifetime of 80 years, which is the average lifetime for technical flc protection in Germany (Bund / Länder-Arbeitsgemeinschaft Wasser, 2005). An initial capacit f the protection system, which is expressed in the form of the design discharge, must be selecte The decision on the protection capacity will be revised every 20 years, taking into account the scharge records that will be available at these points in time, which reduce the uncertainty of t annual maximum discharge parameters.

As dyke increase costs rise line / with height (Kok *et al.*, 2008; Jonkman *et al.*, 2013) and capacity has a quadratic depender on height, we chose a square root cost function for increasing flood protection capacity at time $i\Delta t$:

$$c_i(\varphi, v_{i-1}, v_i) = \begin{cases} \sqrt{v_i} - \varphi \times \sqrt{v_{i-1}}, & v_{i-1} < v_i \\ 0. & v_{i-1} = v_i \end{cases} \tag{24}$$

φ is the flexibility, v_{i-1} is the original capacity and v_i is the adjusted capacity. The costs of the initial design v_1 are also calculated through Eq. (24), with $v_0 = 0$.

Figure 7 shows the cost function $c_i(\varphi, v_{i-1}, v_i)$ for multiple values of flexibilities when increasing protection from $v_{i-1} = 1000\ \mathrm{m}^3/\mathrm{s}$ to a higher capacity.

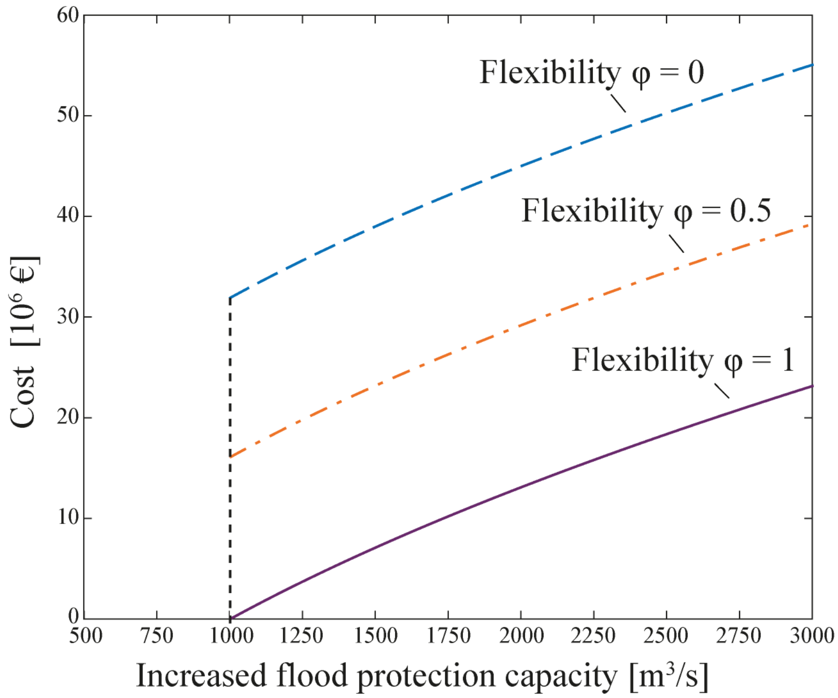


Figure 7. Adjustment cost functions for increasing from $1000\ m^3/s$ to some higher capacity for three different values of flexibility.

A discounting factor (viz. Eq. (13)) of 2% is used, corresponding to the lower bound for technical flood protection proposed in the literature (Bund / Länder-Arbeitsgemeinschaft Wasser, 2005). Costs are discounted to time $\mathrm{t} = 0$.

3.2 Comparison of results from heuristic optimization and backward induction

Results of the safety factor optimization for the flood protection system in Wasserburg am Inn are presented in this section for the case of 31 years of historic records, i.e. assuming a high parameter uncertainty. Results are computed with the two approaches: the (sub-optimal) heuristic optimization that provides a strategy with constant safety factor applied throughout the whole lifetime of the protection system (Section 2.4.1), and the backward induction approach, in which the safety factors are optimized individually for each decision time and flood records (Section 2.4.2).

Figure 8 shows optimal initial safety factor and associated expected lifetime cost obtained with backwards induction. For systems with low flexibility, it is recommendable to adopt a conservative approach; the recommended safety factor takes values up to 1.22, corresponding to a reserve of

22% over the current estimate of the 100-year discharge. For systems with increased flexibility, the optimal safety factor is lower, because the systems can more easily be adjusted later when new data indicates that the 100-year discharge is higher than originally estimated. For $\varphi = 1$, the recommended safety factor is 1 because adjustment, if necessary, can be done without overhead cost. There is, however, the caveat that this adjustment would only be done at the next revision and thus, some decision makers may prefer to add a reserve even in the case of full flexibility. In addition, a freeboard is always included to mitigate small fluctuations, e.g. in wave height.

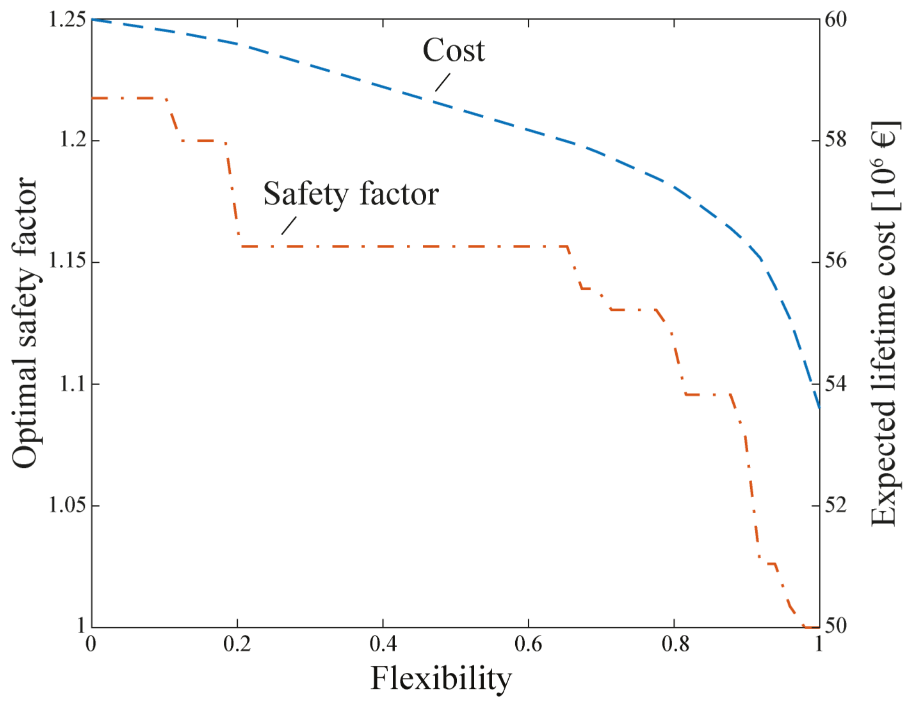


Figure 8. Results of backward induction for the flood protection, for an initial flood estimate based on 31 years of historic discharge records: Optimal initial safety factor and expected discounted lifetime cost depending on flexibility.

The expected discounted lifetime cost decreases with flexibility, from 60.0×10^6 € for no flexibility to 53.6×10^6 € for full flexibility. The difference in the lifetime costs for systems with different flexibility is the value of flexibility. If a fully flexible system with $\varphi = 1$ is implemented, the expected net present value of lifetime savings would be 6.4×10^6 € over the non-flexible system with $\varphi = 0$.

Note that the expected lifetime cost includes both initial building costs and anticipated retrofitting costs. For each value of flexibility, the minimal lifetime cost based on a balance of these two components is given and this minimal cost decreases smoothly with increasing flexibility. However, the optimal safety factor that leads to minimal lifetime cost is not continously decreasing with flexibility because only a discrete set of possible protection levels is considered. Figure 9 compares results of the backwards induction with those from the heuristic optimization. Figure 9 (a) shows the recommended initial safety factor. The backwards induction optimization recommends a higher initial safety factor than the heuristic one. This is sensible because the safety factor for the backwards induction optimization is allowed to vary over time. The optimal safety factor is likely to decrease at later time steps when the uncertainty in the estimation of the 100-year discharge is smaller and the remaining lifetime is shorter. In the case of the heuristic optimization, a constant safety factor is applied throughout the lifetime and it is thus lower than the initial safety factor found with the backwards induction. For these reasons, the solution of the heuristic optimization is suboptimal. The discrepancy in the safety factor recommendation between the two approaches reduces as the flexibility of the protection system increases. For full flexibility ($\varphi = 1$), both approaches correctly recommend a safety factor of 1.

Figure 9 (b) shows a comparison of the expected lifetime costs incurred when following the recommendation of the two approaches. As expected, the costs of the optimal solution found with the heuristic approach exceed those found with backward induction, which confirms that the heuristic optimization leads to a suboptimal solution. The discrepancy reduces with increased flexibility of the protection system and for full flexibility, the two approaches make the same recommendation and the associated costs are equal.

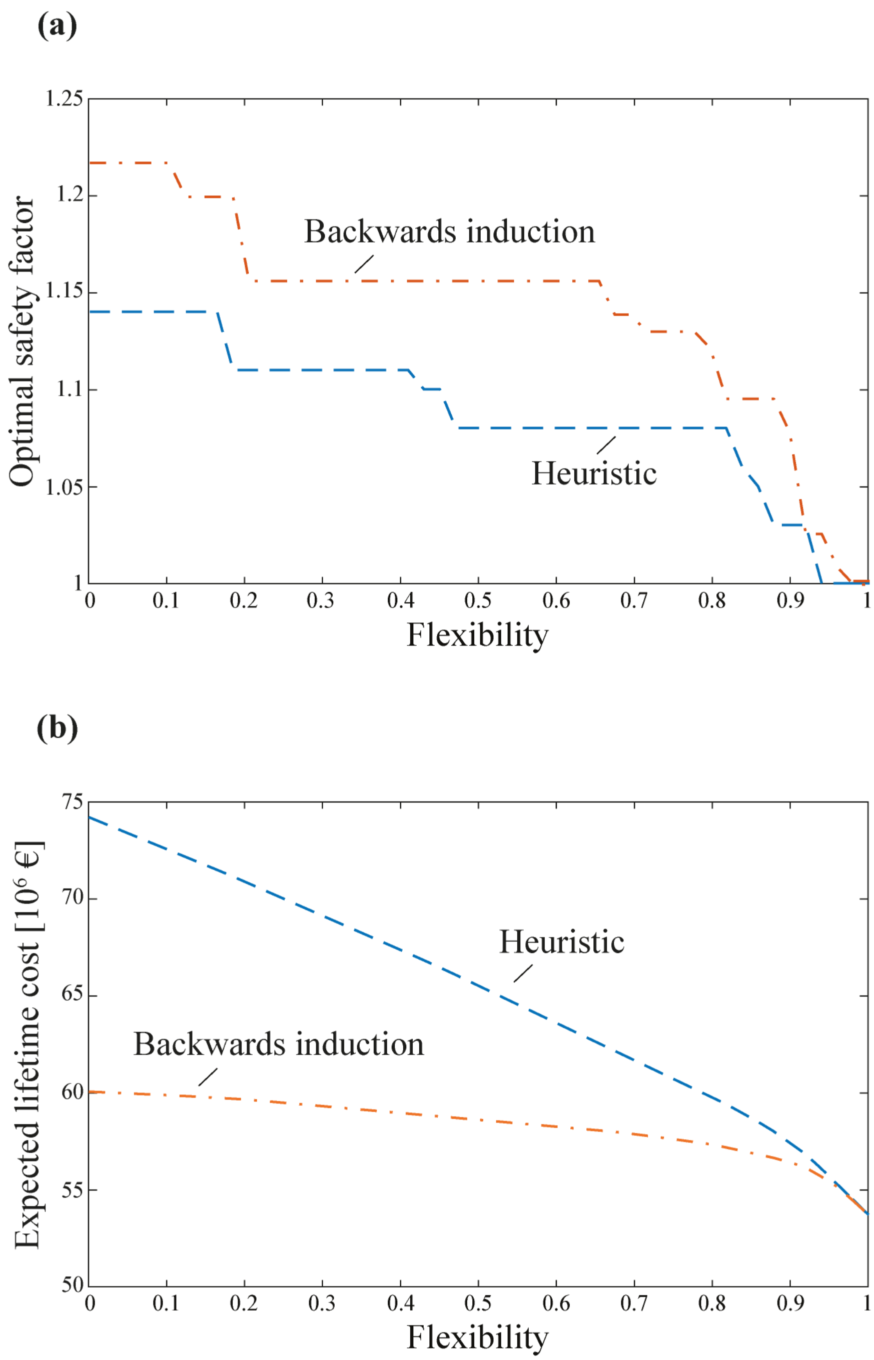


Figure 9. Comparison of heuristic and backwards-induction optimization approach for the case of a 31 years flood record. **(a)** Optimal initial safety factor and **(b)** expected lifetime cost.

3.3 Dependence of optimal initial protection capacity on parameter uncertainty

We investigate the influence of parameter uncertainty – here caused by finite data record length – on the optimal safety factor and protection capacity. The following results were obtained by backwards induction.

Figure 10 shows the optimal safety factor recommendation depending on flexibility for varying lengths of flood records. As expected, the optimal initial protection increases with increasing parameter uncertainty (i.e. with decreasing flood records).

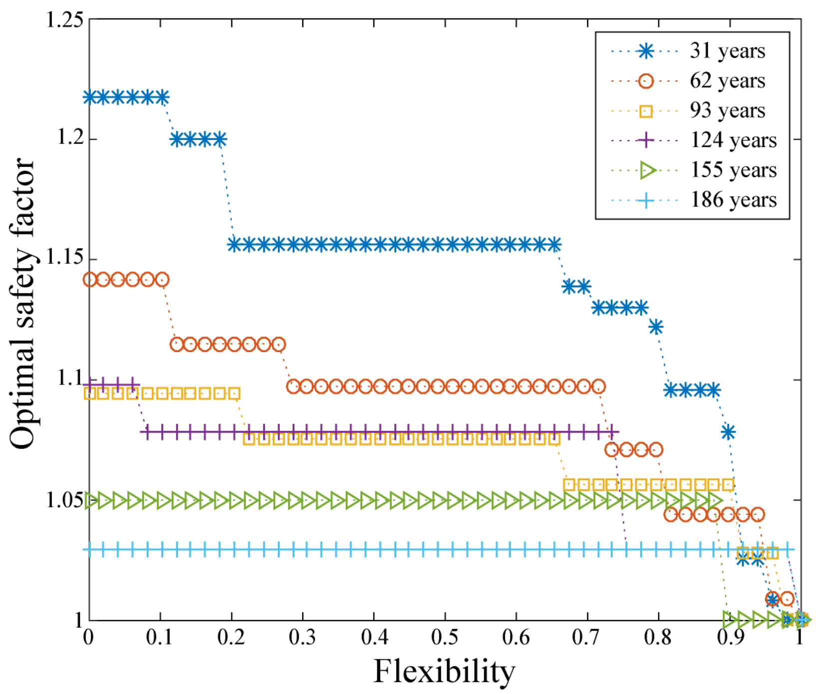


Figure 10. Optimal initial safety factor as a function of the flexibility of the protection system, for varying length of the initial discharge record (associated with varying parameter uncertainty).

Figure 11 shows the expected discounted lifetime cost associated with the safety factor recommendations. Cost consistently decreases as the parameter uncertainty decreases. This is consistent with the results of Figure 10. Lower uncertainty leads to lower safety factors, and hence to lower design costs.

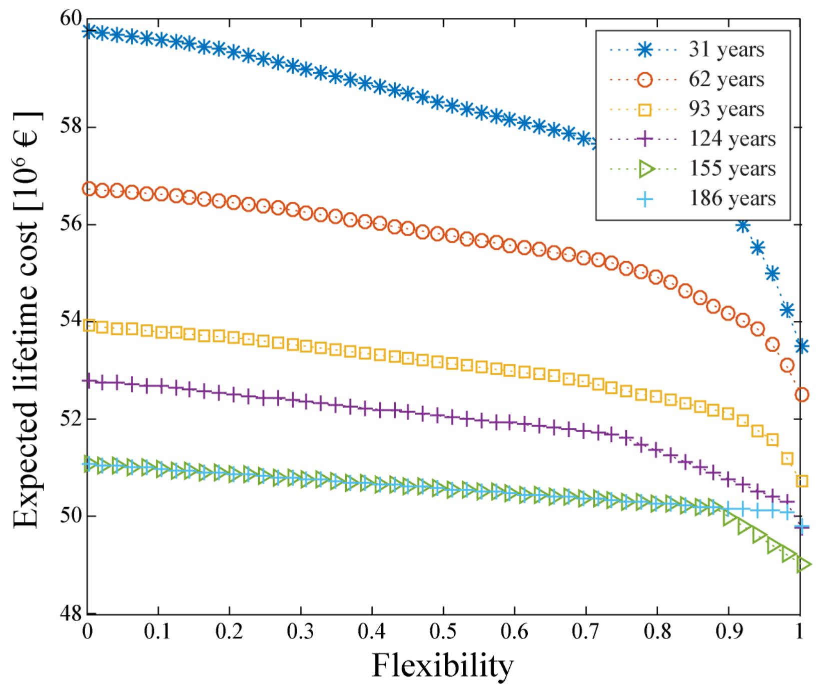


Figure 11. Expected lifetime cost for varying lengths of the historic data record.

For highly flexible measures, the dependence of the expected lifetime cost on the parameter uncertainty is lower, because flexible measures require low or no reserve even under high uncertainty. Flexibility is thus especially beneficial under high uncertainty: the value of flexibility calculated using an initial estimate based on 31 years of data is 6.4×10^6 € (as already discussed in the previous section), as opposed to the value of flexibility associated with the case of 186 years of data, which is 1.3×10^6 €.

3.4 Sensitivity to cost function and discounting

We investigate the sensitivity of the results to the cost function and the discounting rate. Results presented above are for the square root cost function of Eq. (24) and a discounting rate of 2%. In Figure 12, the effect of the cost function on the optimal safety factor is shown. In addition to the previously used square root function $\sqrt{v_i} - \varphi \times \sqrt{v_{i-1}}$, a linear and a logarithmic cost function are applied. For the considered domain $v_{i-1} < v_i$, the linear function is

$$c_i(\varphi, v_{i-1}, v_i) = v_i - \varphi \times v_{i-1}, \tag{25}$$

and the logarithmic function is

$$c_i(\varphi, v_{i-1}, v_i) = \log(v_i) - \varphi \times \log(v_{i-1}). \tag{26}$$

The different cost functions result in the same qualitative dependence on flexibility, but the quantitative recommendation is influenced by the choice of the cost function. The differences are here due to the smaller marginal costs of the logarithmic cost function, which favor larger capacities. The linear cost function has higher marginal costs than the square root function, hence it leads to a slightly lower safety factor.

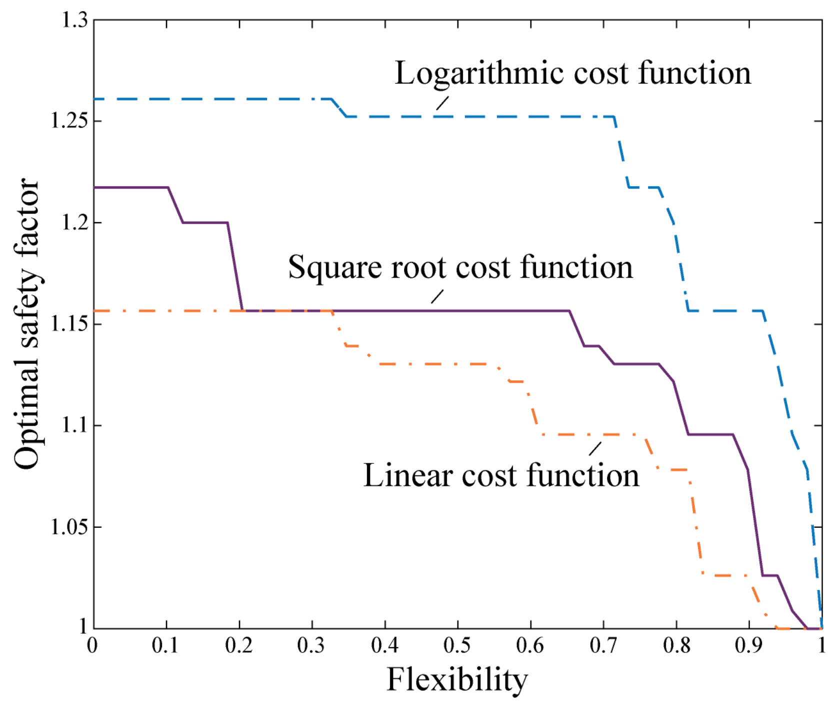
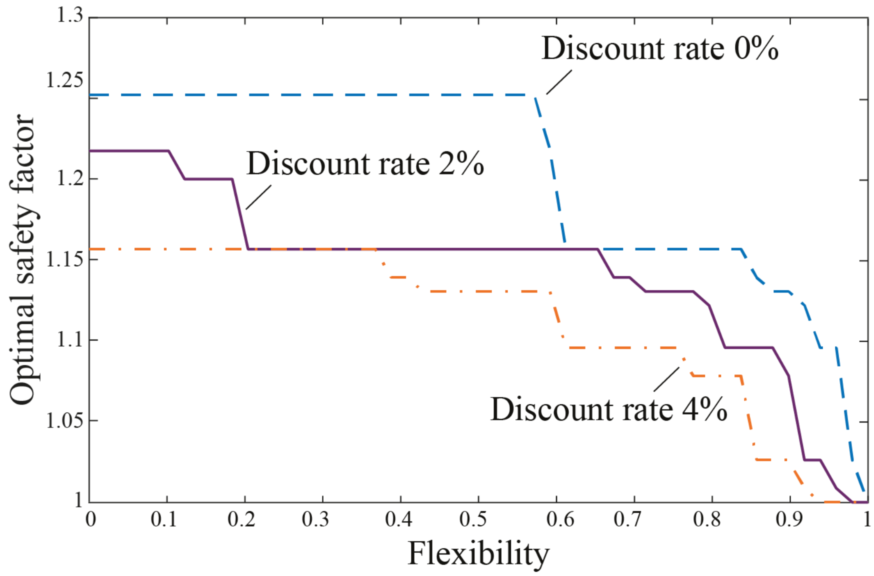


Figure 12. Dependence of optimal initial safety factor on the cost function for the case of 31 years of discharge records. The choice of the cost function influences the magnitude of the safety factor but not the qualitative dependence on flexibility.

Lastly, we investigate the effect of varying the discount rate. Results are shown in Figure 13 for the case of 31 years of discharge records and the square root cost function of Eq. (24). Both the optimal safety factor (a) and the expected lifetime cost (b) exhibit a dependence on the discount rate, as expected, but qualitatively results remain unchanged. A higher discounting rate leads to reduced safety factor recommendation and smaller total expected costs (net present values). Delaying decisions is more attractive financially when the discounting rate is higher.

(a)



(b)

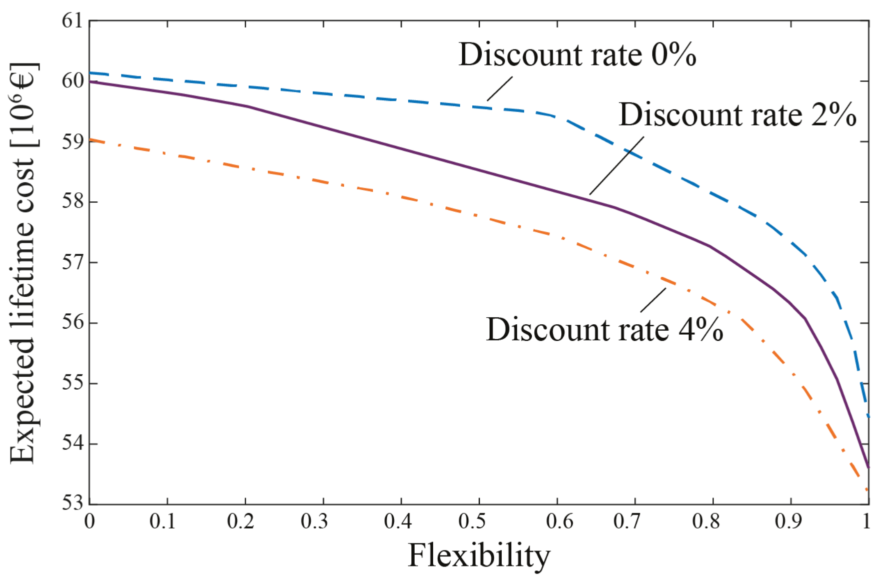


Figure 13. Optimal safety factor **(a)** and expected lifetime cost **(b)** for the 31-year historic data record with varying rate of discounting.

4 Discussion

We implemented the proposed decision framework for a flood model base discharge records; parameter uncertainty is thereby associated with the limite records. This choice was made to focus on the presentation of the methodolog facilitate the interpretation of results and to enable a clearer presentati implementations of the methodology, one might often need to extend the prob two aspects: Firstly, to include non-stationarity and associated uncertainty, aris and anthropogenic changes; secondly, to consider additional informatio hydrological models in combination with regional climate data (McMillan *et al.*,

Including additional sources of uncertainty in the model is straightforward, if thi be quantified in parametric form. The size of the stochastic parameter vector $\boldsymbol{\Theta}$ when including uncertain trend parameters in the case of climate change, yet thi fundamental difficulty. $\boldsymbol{\Theta}$ can simply be defined to contain the parameters extreme value distribution, e.g. a Gumbel-distribution with a location parameter $\mu = \mu_0 + t \times \mu_1$. Furthermore, the conclusions that can be drawn from the numerical results presented in this paper are not expected to change fundamentally. The outcomes of the analysis are dependend on the magnitude of the parameter uncertainty, but not on their origin. A case study using the presented methodology under climate change conditions forms part of (Dittes *et al.*, 2017).

The optimization is based on a backwards induction, which explicitly accounts for all possible development paths over the lifetime of the system. The decision on the optimal capacity is thereby freely revised at regular intervals. In addition, we present a heuristic approach to the optimization, in which a constant safety factor is applied over the lifetime of the protection. Since it reduces the solution space, it can only lead to a sub-optimal solution, but it is significantly easier to understand and implement, and can, therefore, be applied to verify results and communicate the approach. One has the possibility to further improve the heuristic optimization, by choosing alternative, possibly closer to optimal, heuristics. For example, the results obtained with the complete optimization indicate that the safety factor should be reduced as the end of service life approaches, which is to be expected, as fewer adjustments will be necessary when the time span is shorter. This could be included in the heuristics by introducing a 'reduction factor' that lowers safety factor with time. One variation of this that could contribute significantly to more optimal outcomes would be to allow the planning margin at initial construction to be larger than the latter ones (Straub and Špačková, 2016).

As a side-effect, the proposed framework also provides an estimate of the value of reducing parameter uncertainty in the flood models. This is observed in Figure 10: For an inflexible system ($\varphi = 0$) that is to be planned based on 33 years of discharge data, having an additional 122 years of data would reduce the total expected life-cycle cost by 15% (from 60×10^6 € to 51.2×10^6 €). These findings underline the great benefits of longer records (Rogger *et al.*, 2011; Kjeldsen *et al.*, 2014). Our numbers are indicative of the so-called *value of information*, which can be found by comparing the total expected life-cycle costs computed for different degrees of parameter uncertainty. Overall, the value of information is higher for non-flexible systems, for which parameter uncertainty is more critical, and reduces to zero for fully flexible systems. Similarly, one can determine the *value of flexibility*, by calculating the difference in total expected life-cycle cost between a flexible and an inflexible system. The value of flexibility is higher when parameter uncertainty is high.

The definition of flexibility used throughout the paper is a simplification: Flexibility is not a fixed value independent of the capacity, as implied by Eq. (2). Consider an initially low sheet pile wall: extending it slightly is straightforward (with flexibility close to 1) but if it is extended beyond a certain height, the base may need to be fortified, leading to large costs and low flexibility. In fact, for an actual project, one would not determine flexibility and then determine the optimal design based on this flexibility value, rather one would work directly with the cost functions for new systems c_0 and upgrades c_i. The motivation for working with the concept of flexibility here is that it allows to make general conclusions, which can inform decision making without a detailed analysis.

The methodology and results in this paper are valid for a rule-based approach to flood protection planning, which requires compliance with pre-defined protection levels (here the 100-year event). The cost of exceedance or failure of the flood protection system is not considered as part of the optimization. While this corresponds to current practice in many countries, its application leads to suboptimal flood protection (Hutter and Schanze, 2010; Merz *et al.*, 2010; Vrijling, Schweckendiek and Kanning, 2011; Nillesen and Kok, 2015; Tsimopoulou, Kok and Vrijling, 2015). Ideally, decisions should be risk-based, i.e. found by an optimization of costs vs. risks. This is in line with the European flood risk directive on the assessment and management of flood risks 2007/60 (European Parliament and European Council, 2007). Some authorities complement the simpler rule-based planning with the more optimal risk-based planning by using a ‘zoning’ approach, with different regions being assigned a different design flood (Kind, 2014). An initial investigation towards a risk-based assessment is reported in (Špačková and Straub, 2017), taking basis in partially observable Markovian decision processes.

Conclusions

We present a Bayesian decision framework for optimizing flood protection systems in the face of parameter uncertainty. It accounts for the flexibility of protection systems, i.e. the ability to adjust the flood protection capacity in the future to ensure that the system complies with a fixed protection criterion (commonly the 100-year event) at all times. Based on the initial uncertainty, the framework returns a quantitative recommendation for the initial capacity as well as associated costs. It thereby anticipates discharge data and other information that will become available in the future. We apply the framework to a case study with varying length of discharge records to investigate the effect of parameter uncertainty on protection recommendations.

The following conclusions on the planning of flood protection systems can be drawn from the numerical investigations: (1) The higher the uncertainty, the higher the recommended safety factor (reserve) and expected lifetime cost. (2) The more flexible the system, the lower the recommended safety factor and the lifetime cost. (3) The economic value of discharge data is considerable since it reduces uncertainty and thus leads to lower requirements on safety factors. (4) Using flexible systems can be financially beneficial, in particular for planning in a rule-based regulatory framework under high uncertainty.

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Appendix – Computational implementation

We outline the computational implementation of the presented methodology here. Steps 1-5 serve for generating $S_1 \times S_2 \ldots \times S_{N-1}$ random samples of future discharge data and the corresponding required protec ... pacities. For co ... ational ... elect $S_1 \geq$ ... $\geq \cdots \geq S_{N-1}$. The ... 6 and 7 optimiz ... protect ... duction.

For given disch ... ecords $\mathbf{q}_H$ and ... g identi ... tribution $f_{Q|\boldsymbol{\theta}}$:

1. Compu ... required protect ... apacity ... ng Eq. (10).
2. Genera ... 1 … S_1 samples ... e param ... $\boldsymbol{\theta}|\mathbf{q}_H)$ (viz. Eq. (4)).
3. For eac ... ple $\boldsymbol{\theta}_s$, randoml ... erate an ... $\mathbf{q}_{1,s}$ fro ... $_{|\boldsymbol{\theta}}(\mathbf{q}|\boldsymbol{\theta}_s)$.
4. **Loop** f ... me steps $i = 2, \ldots,$ ... 1:
    a. ... npute the required ... ection c ... $\mathbf{q}_{i-1,s})$ for step i ... owing Eq. (10).
    b. ... erate $s = 1 \ldots S_i$ s ... es of the ... $(\boldsymbol{\theta}|\mathbf{q}_H, \mathbf{q}_{1,s}, \ldots, \mathbf{q}_{i-1,s})$ ... . Eq. (6)).
    c. ... each sample $\boldsymbol{\theta}_s$, ra ... nly gen ... ischarges $\mathbf{q}_{i,s}$from $f_{Q|\boldsymbol{\theta}}(\mathbf{q}|\boldsymbol{\theta}_s)$.

    Endloop

5. Compute the required protect ... apacity $v_{req}(\mathbf{q}_H, \mathbf{q}_{1,s}, \ldots, \mathbf{q}_{N-1,s})$ for step N following Eq. (10).

One can now find the optimal saf ... actor and the associated expected total life-cycle cost through backwards induction:

6. Discretize capacity v into K v ... s. Consider all K^N possible combinations (scenarios) of required protection capacities v_{req} at the N planning times. Determine the Probability Mass Function (PMF) of these scenarios using the samples generated in steps 1-5, considering that the capacity is not reduced.

7. **Loop** for each value of flexibility φ
    a. For each of the K^Nscenarios of required protection capacity, compute costs and discount them to t_0 following Eqs. (12) and (13).
    b. Optimize the final safety factor $\gamma_{N-1}^{opt}(a, v_0, \ldots, v_{N-2}, \mathbf{q}_H, \mathbf{q}_1, \ldots, \mathbf{q}_{N-1})$ at the last decision time t_{N-1} conditionally on previous capacities $v_0, \ldots, v_{N-2}$ and observations $\mathbf{q}_H, \mathbf{q}_1, \ldots, \mathbf{q}_{N-1}$ following Eqs. (19) and (20).
    c. **Loop** starting with $i = N - 2$ and continuing backwards to $i = 1$:
        i. Optimize safety factor $\gamma_{N-1}^{opt}(a, v_0, \ldots, v_{i-1}, \mathbf{q}_H, \mathbf{q}_1, \ldots, \mathbf{q}_i)$ at decision time t_i conditional on previous capacities$v_0, \ldots, v_{i-1}$ and observations $\mathbf{q}_H, \mathbf{q}_1, \ldots, \mathbf{q}_i$ following Eqs. (19) and (21).

Endloop

d. Find the optimal initial safety factor $\gamma_0^{opt}(a, \mathbf{q}_H)$ following Eqs. (22) and (23).

Endloop

For the case study shown in Section 3, we used $S_1 = 700$, $S_2 = 100$ and $S_3 = 40$. From repeatedly performing the analysis, we find a percentage error in the order of 1.5% in both the optimal initial safety factor and the expected lifetime cost.

Figure captions

Figure 1. An example of a flexible flood protection measure - a flood wal (Vrijling *et al.*, 2007).

Figure 2. Exceedance probability and associated 90% credible interval (sl of discharge. The dashed PDF represents the uncertainty on the 100-year dis he figure is based on a 31-year data record from the case study (viz. Section assumption of a Gumbel distribution.

Figure 3. 100-year design discharge estimate (MAP) and associated 90% a function of the historic data record length. Annual maximum discharge da are for a gauge in Wasserburg am Inn, Germany, for the period from 1828 to discharge is estimated under the assumption of a stationary Gumbel distri measurement uncertainty (viz. Section 3).

Figure 4. Timeline of decisions on capacity v and observations of dischar optimization process of N steps.

Figure 5. Illustration of the costs for fully flexible vs. inflexible systems (b, 2017): **(a)** Initial cost $c_0$when the system is first implemented. **(b)** Cost c_i e system to a higher capacity v_i. For the inflexible systems, the adjustment e costs c_0 incurred when implementing directly capacity v_i. For the fully fle 1), the adjustment costs are equal to the difference between the initial costs o l the adjusted system.

Figure 6. Decision tree for the full optimization with three exemplary cap amongst a continuous spectrum. Branches that violate the constraint $v_i \geq v_{req}(\boldsymbol{q}_H, \boldsymbol{q}_1, \dots, \boldsymbol{q}_i)$ are not shown.

Figure 7. Adjustment cost functions for increasing from $1000\ m^3/s$ to some higher capacity for three different values of flexibility.

Figure 8. Results of backward induction for the flood protection, for an initial flood estimate based on 31 years of historic discharge records: Optimal initial safety factor and expected discounted lifetime cost depending on flexibility.

Figure 9. Comparison of heuristic and backwards-induction optimization approach for the case of a 31 years flood record. **(a)** Optimal initial safety factor and **(b)** expected lifetime cost.

Figure 10. Optimal initial safety factor as a function of the flexibility of the protection system, for varying length of the initial discharge record (associated with varying parameter uncertainty).

Figure 11. Expected lifetime cost for varying lengths of the historic data record.

Figure 12. Dependence of optimal initial safety factor on the cost function for the case of 31 years of discharge records. The choice of the cost function influences the magnitude of the safety factor but not the qualitative dependence on flexibility.

Figure 13. Optimal safety factor **(a)** and expected lifetime cost **(b)** for the 31-year historic data record with varying rate of discounting.

Tables

Table 1. Overview of parameters of the decision model

Id.	Parameter/variable	Type	Value domain	Units
Q	Annual max. discharge	Random	$[0, \infty)$	m³/s
$\mathbf{\Theta}$	Parameters of prob. model of Q	Random	$[0, \infty)$	-
$\mathbf{q}_i$	Future observations of Q	Random	$[0, \infty)$	m³/s
v_i	Capacity of flood protection system	Decision	$[1, \infty)$	m³/s
v_{req}	Required protection capacity	Random	$[1, \infty)$	m³/s
γ_i	Safety factor	Decision	$[1, \infty)$	-
c_i	Protection cost	Deterministic	$[0, \infty)$	€
φ	Flexibility of flood protection system	Decision	$(-\infty, 1]$	-

Table 2. Case study data properties[a]

Data			*Fitted probabilistic model*					
Discharge series length [years]	Annual maximum discharge [m³/s]		Gumbel scale parameter β [m³/s]		Gumbel location parameter μ [m³/s]		100-year design discharge [m³/s]	
	Mean	*Std.-dev.*	*MAP*	*Std.-dev.*	*MAP*	*Std.-dev.*	*From Eq.* (9)	*Std.-dev.*
31	1471	502	352	147	1255	138	2876	808
62	1449	445	343	96	1250	114	2820	555
93	1424	399	302	70	1248	96	2641	419
124	1416	375	282	58	1252	79	2550	344
155	1398	355	268	50	1244	68	2477	296
186	1398	379	284	49	1233	66	2539	291

[a]Annual maximum discharge at the Wasserburg am Inn gauge, Bavaria, Germany: sample statistics (heading: Data) and parameters of fitted Gumbel model (heading: Fitted probabilistic model) for different lengths of the historic discharge records.